Numerical

An audio frequency signal $5\sin 2\pi(1000)t$ is used to amplitude modulate a carrier of $100\sin 2\pi(10^6)t$. Assume modulation index of 0.4. Find

i. Sideband frequencies

ii. Bandwidth required

iii. Amplitude of each sideband

iv. Total power delivered to a load of 100Ω

Sol:- Given:

$$A_m = 5, \quad A_c = 100, \quad \mu = 0.4, \quad f_m = 1000 Hz, \quad f_c = 1 \times 10^6 Hz.$$

i) Sideband frequencies:

$$f_{USB} = f_c + f_m = 1MHz + 1000Hz = 1.001 MHz$$

$$f_{LSB} = f_c - f_m = 1MHz - 1000Hz = 999\,000 Hz = 0.999 MHz$$

ii) Amplitude of each Sideband frequencies:

$$\frac{\mu A_c}{2} = \frac{0.4 \times 100}{2} = 20V.$$

∴ Amplitude of upper & lower Sideband is 20V.

iii) Bandwidth required:

$$BW = 2f_m = 2 \times 1KHz = 2KHz$$

or

$$BW = f_{USB} - f_{LSB} = 1.001MHz - 999000\,Hz = 2KHz$$

iv) Total power delivered to a load of 100 Ω:

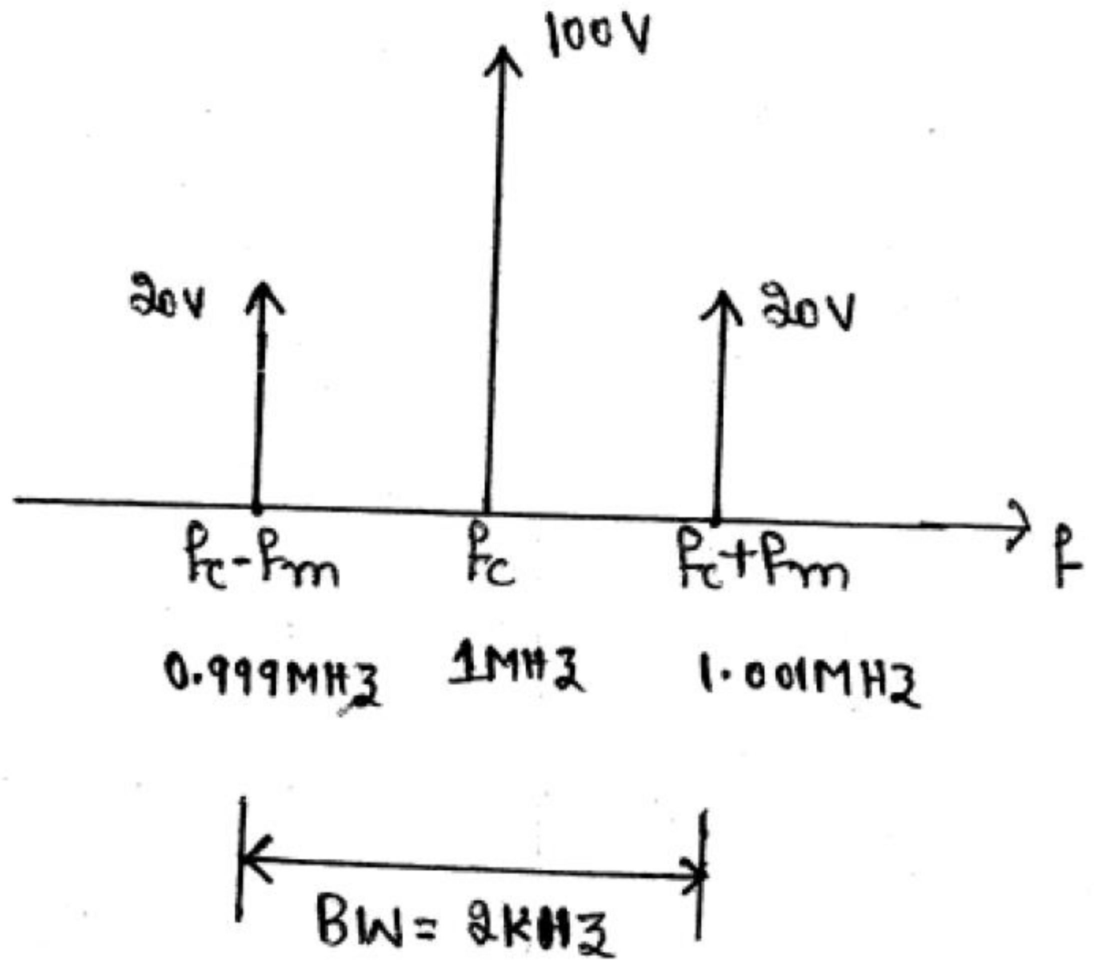
W.K.T

$$P_T = P_c\left[1+\frac{\mu^2}{2}\right] = \frac{A_c^2}{2R}\left[1+\frac{\mu^2}{2}\right]$$

$$= \frac{(100)^2}{2\times100}\left[1+\frac{(0.4)^2}{2}\right]$$

$$\boxed{P_T = 54W}$$

v) Spectrum of AM wave:



Problem

A carrier wave $4\sin(2\pi \times 500 \times 10^3 t)$ volts is amplitude modulated by an audio wave $[0.2 \sin 3(2\pi \times 500t) + 0.1 \sin 5(2\pi \times 500t)]$ volts. Determine the upper and lower sideband and sketch the complete spectrum of the modulated wave. Estimate the total power in the sideband.

Given : $c(t) = 4\sin(2\pi \times 500 \times 10^3 t) \rightarrow A_c = 4V,\ f_c = 500\,KHz$

$m(t) = \underset{\uparrow\, A_{m1}}{0.2} \sin 2\pi(\underset{\uparrow\, f_{m1}}{1500})t + \underset{\uparrow\, A_{m2}}{0.1} \sin 2\pi(\underset{\uparrow\, f_{m2}}{2500})t \rightarrow$

The message signal consists of two sinewaves.

$A_{m1} = 0.2V, \quad f_{m1} = 1500\,Hz$

$A_{m2} = 0.1V, \quad f_{m2} = 2500\,Hz$

* USB & LSB :-

i) $USB_1 = (f_c + f_{m1}) = 500\,KHz + 1.5\,KHz = 501.5\,KHz$

$LSB_1 = (f_c - f_{m1}) = 500\,KHz - 1.5\,KHz = 498.5\,KHz$

ii) $USB_2 = (f_c + f_{m_2}) = 500\,KHz + 2.5\,KHz = 502.5\,KHz$

$LSB_2 = (f_c - f_{m_2}) = 500\,KHz - 2.5\,KHz = 497.5\,KHz.$

* Modulation Index of individual modulating Signals:

i) Modulation Index for 1st Signal $\mu_1 = \frac{A_{m_1}}{A_c} = \frac{0.2}{4} = 0.05$

ii) Modulation Index for 2nd Signal $\mu_2 = \frac{A_{m_2}}{A_c} = \frac{0.1}{4} = 0.025$

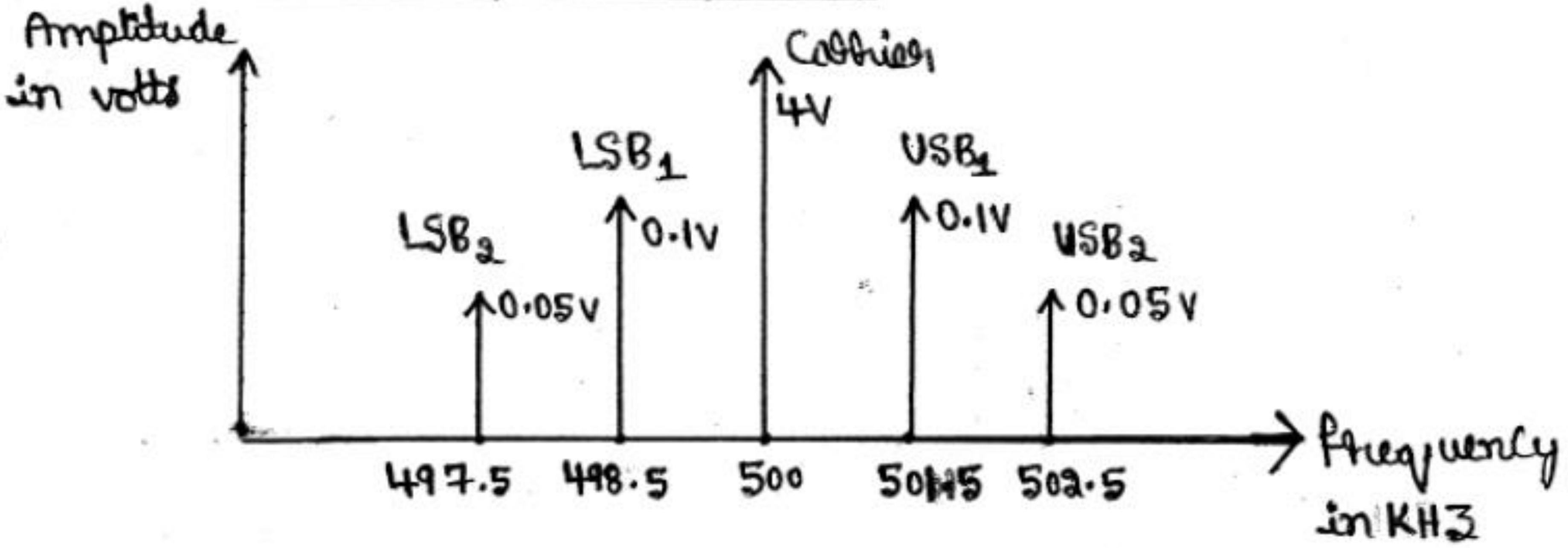
* Sideband amplitudes:

In general, amplitude of each Sideband is given by $\frac{\mu A_c}{2}$

i) Amplitude of USB_1 & LSB_1 will be: $\frac{\mu_1 A_c}{2} = \frac{0.05 \times 4}{2} = 0.1V$

ii) Amplitude of USB_2 & LSB_2 will be: $\frac{\mu_2 A_c}{2} = \frac{0.025 \times 4}{2} = 0.05V$

* Complete Spectrum of AM Signal:



* Total power in the Sidebands:-

WKT, the total power in the Sidebands is given by

$$P_{SB} = P_{USB} + P_{LSB} = P_c\left(\frac{\mu^2}{2}\right)$$

for two Signals,

$$P_{SB} = P_c\left(\frac{\mu_t^2}{2}\right)$$

Where,

μ_t = total modulation Index =

i.e. $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.05)^2 + (0.025)^2} = 0.0559$

$$\therefore \quad P_{SB} = P_c\left[\frac{\mu_t^2}{2}\right]$$

$$= \frac{A_c^2}{2R}\left[\frac{\mu_t^2}{2}\right]$$

$$= \frac{(4)^2}{2R}\left[\frac{(0.0559)^2}{2}\right]$$

W.K.T. $P_c = \dfrac{A_c^2}{2R}$

$$= \frac{\cancel{16}\,8}{\cancel{2}R}\left[1.56 \times 10^{-3}\right]$$

$$= \frac{8}{R}\left[1.56 \times 10^{-3}\right]$$

$$\boxed{P_{SB} = \frac{0.0125}{R}}$$

An amplitude modulated signal is given by

$$S(t) = [10\cos(2\pi \times 10^6 t) + 5\cos(2\pi \times 10^6 t)\cos(2\pi \times 10^3 t) + 2\cos(2\pi \times 10^6 t)\cos(4\pi \times 10^3 t)]$$

Find i) total modulated power ii) Sideband power and iii) net modulation index.

Sol :-

WKT

$$\boxed{S(t) = A_c\left[1 + \mu_1 \cos 2\pi f_1 t + \mu_2 \cos 2\pi f_2 t\right] \cos 2\pi f_c t} \longrightarrow (1)$$

Given

$$S(t) = \left[10\cos(2\pi\times10^6 t) + 5\cos(2\pi\times10^6 t)\cos(2\pi\times10^3 t) + 2\cos(2\pi\times10^6 t)\cos(4\pi\times10^3 t)\right]$$

$$S(t) = 10\cos(2\pi\times10^6 t)\left[1 + \frac{5}{10}\cos(2\pi\times10^3 t) + \frac{2}{10}\cos(4\pi\times10^3 t)\right]$$

$$\boxed{S(t) = 10\cos(2\pi\times10^6 t)\left[1 + 0.5\cos(2\pi\times10^3 t) + 0.2\cos(4\pi\times10^3)t\right]} \longrightarrow (2)$$

Comparing eq (1) & (2), we get

$A_c = 10V$, $\mu_1 = 0.5$, $\mu_2 = 0.2$, $f_1 = 1\times10^3 Hz$, $f_2 = 2\times10^3 Hz$, $f_c = 1\times10^6 Hz$

* Net modulation index $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.5)^2 + (0.2)^2}$

$$\boxed{\mu_t = 0.538}$$

* Carrier power $P_c = \frac{A_c^2}{2R} = \frac{(10)^2}{2 \times 1}$ $R = 1\,\Omega$

$$\boxed{P_c = 50\,W}$$

* Sideband power $P_{SB} = P_{USB} + P_{LSB} = \frac{\mu_t^2}{2} P_c = \frac{(0.538)^2}{2} 50$

$$\boxed{P_{SB} = 7.25\,W}$$

* Total modulated power

$$P_T = P_c\left[1 + \frac{\mu_1^2}{2}\right]$$

$$= 50\left[1 + \frac{0.538^2}{2}\right]$$

$$\boxed{P_T = 57.25\,W}$$

OR

$$P_T = P_c + P_{SB}$$

$$= 50W + 7.25W$$

$$\boxed{P_T = 57.25\,W}$$

Consider a message signal $m(t)=20\cos(2\pi t)$volts and a carrier signal $c(t)=50\cos(100\pi t)$volts.

i. Sketch to scale resulting AM wave for 75% modulation.

ii. Find the power delivered across a load of 100Ω due to this AM wave.

Given :- $A_m = 20V$, $f_m = 1Hz$, $A_c = 50V$, $f_c = 50Hz$, $\mu = 0.75$ & $R = 100\Omega$

WKT AM wave is given by

$$S(t) = A_c\left[1 + \mu \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

$$\therefore \boxed{S(t) = 50\left[1 + 0.75 \cos 2\pi(1)t\right] \cos 2\pi(50)t.}$$

i) $A_{max} = A_c(1+\mu) = 50(1+0.75) = \underline{87.5V}$

$A_{min} = A_c(1-\mu) = 50(1-0.75) = \underline{12.5V}$

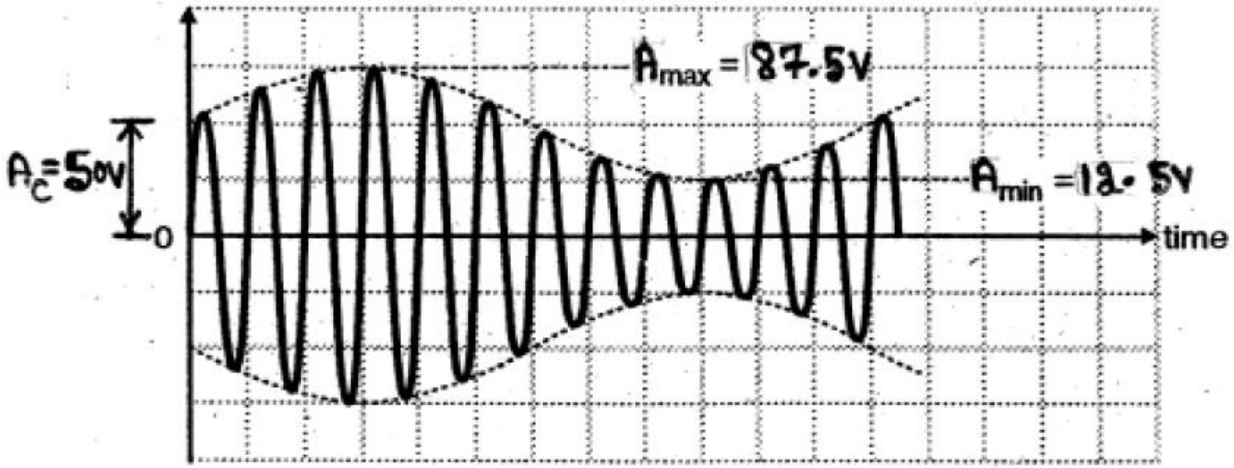


Fig. : AM wave for m = 0.75

ii) $P_T = P_c\left[1 + \frac{\mu^2}{2}\right]$

$* \; P_c = \frac{A_c^2}{2R} = \frac{50^2}{2 \times 100} = \underline{12.5\,W}$

$P_T = 12.5\left[1 + \frac{0.75^2}{2}\right]$

$\boxed{P_T = 16.015\,W}$.

A carrier wave with amplitude 12V and frequency 10 MHz is amplitude modulated to 50% level with a modulated frequency of 1 KHz. Write down the equation for the above wave and sketch the modulated signal in frequency domain.

Given : $A_c = 12V$, $f_c = 10 MHz$, $\mu = 0.5$, $f_m = 1 KHz$

Sol :-

WKT AM wave is given by :

$$S(t) = A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$\boxed{S(t) = 12[1 + 0.5 \cos 2\pi(1\times10^3) t] \cos 2\pi(10\times10^6) t}$$

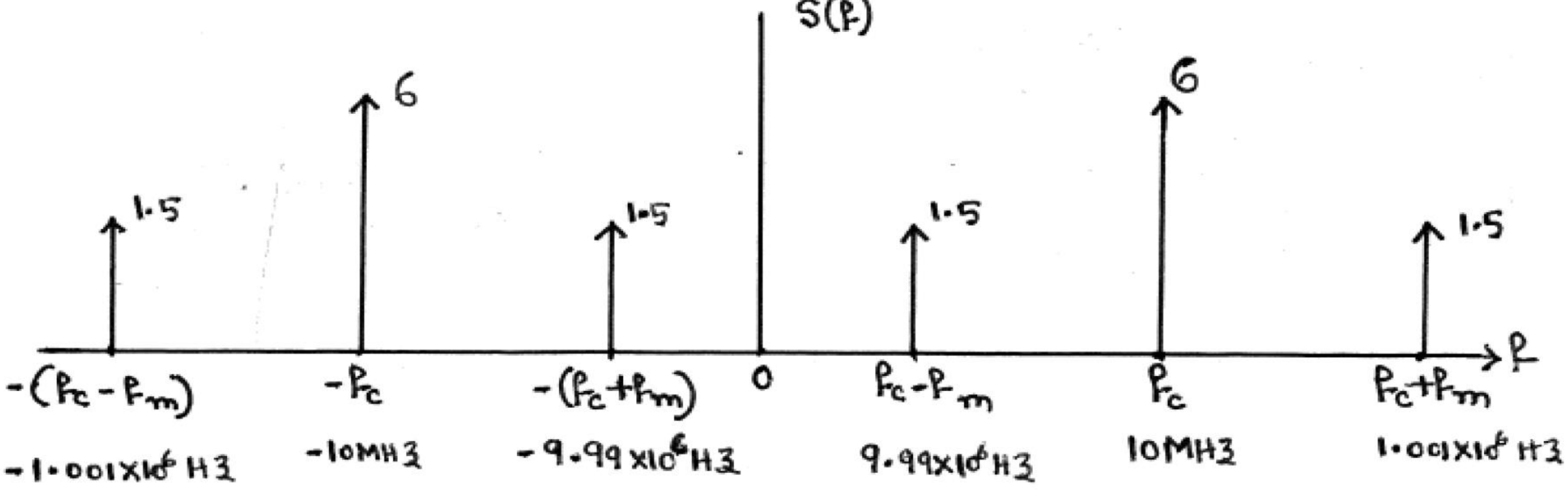
$$S(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi(f_c + f_m)t + \frac{\mu A_c}{2} \cos 2\pi(f_c - f_m)t$$

$$S(t) = 12 \cos 2\pi(10\times10^6 t) + \frac{0.5\times12}{2} \cos 2\pi (10\times10^6 + 1\times10^3) t$$
$$+ \frac{0.5\times12}{2} \cos 2\pi (10\times10^6 - 1\times10^3) t$$

$$S(t) = 12 \cos 2\pi(10\times10^6 t) + 3 \cos 2\pi (1.001\times10^6 t) + 3 \cos 2\pi (9.99\times10^6 t) \longrightarrow (1)$$

Taking FT on both Side of eq (1), we get

$$S(f) = \frac{12}{2}[\delta(f - 10\times10^6) + \delta(f + 10\times10^6)] + \frac{3}{2}[\delta(f - 1.001\times10^6) + \delta(f +$$
$$1.001\times10^6) + \frac{3}{2}[\delta(f - 9.99\times10^6) + \delta(f + 9.99\times10^6)]$$



S(f)
6
6
1.5
1.5
1.5
1.5
f
-(fc-fm)
-fc
-(fc+fm)
0
fc-fm
fc
fc+fm
-1.001X10^6 Hz
-10MHz
-9.99X10^6 Hz
9.99X10^6 Hz
10MHz
1.001X10^6 Hz

Consider a message signal $m(t)=20\cos(2\pi t)$volts and a carrier signal $c(t)=50\cos(100\pi t)$volts.

i. The resulting AM wave for 75% modulation.

ii. Sketch the Spectrum of this AM wave

iii. Find the power developed across the load of 100Ω.

Given : $m(t) = 20 \cos 2\pi t \ V$, $c(t) = 50 \cos 100\pi t$ & $\mu = 0.75$

$\omega_m = 2\pi$	$\omega_c = 100\pi$
$\cancel{2\pi} f_m = \cancel{2\pi}$	$\cancel{2\pi} f_c = \cancel{100\pi}\ 50$
$f_m = 1Hz$	$f_c = 50Hz$

i) WKT

$$S(t) = A_c\left[1 + \mu \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

$$S(t) = 50\left[1 + 0.75 \cos 2\pi(1) t\right] \cos 2\pi(50) t$$

$$S(t) = 50 \cos 2\pi(50) t + 37.5 \underline{\cos 2\pi(50) t \cdot \cos 2\pi(1) t}$$

$$\boxed{\cos A \cdot \cos B = \frac{1}{2} \cos(A-B) + \frac{1}{2} \cos(A+B)}$$

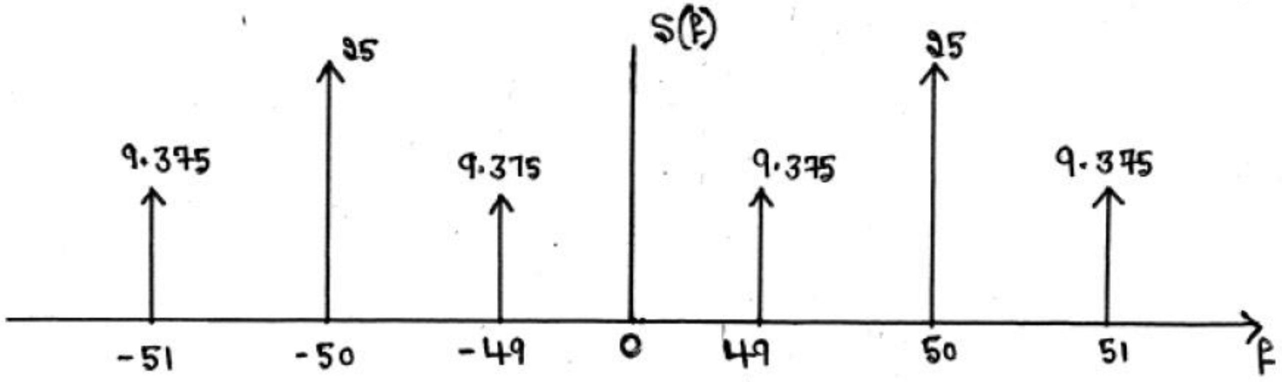
$$S(t) = 50 \cos 2\pi(50) t + \frac{37.5}{2} \cos 2\pi(50-1) t + \frac{37.5}{2} \cos 2\pi(50+1) t$$

$$\boxed{S(t) = 50 \cos 2\pi(50) t + 18.75 \cos 2\pi(49) t + 18.75 \cos 2\pi(51) t} \longrightarrow (1)$$

ii) Taking FT of eq ①, we get

$$S(f) = \frac{50}{2}\left[\delta(f-50)+\delta(f+50)\right] + \frac{18.75}{2}\left[\delta(f-49)+\delta(f+49)\right] + \frac{18.75}{2}\left[\delta(f-51)+\delta(f+51)\right]$$

$$\boxed{S(f) = 25\left[\delta(f-50)+\delta(f+50)\right] + 9.375\left[\delta(f-49)+\delta(f+49)\right] + 9.375\left[\delta(f-51)+\delta(f+51)\right]}$$



iii) $P_T = P_c\left(1+\frac{\mu^2}{2}\right)$

$P_c = \frac{A_c^2}{2R} = \frac{(50)^2}{2\times 100} = \underline{12.5\,W}$

$P_T = 12.5\left(1+\frac{0.75^2}{2}\right) = \underline{16\,W}$

The antenna current of an AM broadcast transmitter modulated to a depth of 40% by an audio sine wave is 11A. It increases to 12A as a result of sinusoidal modulation by another audio sine wave. What is the modulation index due to second wave?

Given :

i) $\mu_1 = 0.4$, $I_{t1} = 11A$, $I_c = ?$

ii) $I_{t2} = 12A$, $\mu_2 = ?$

Sol :

i) $I_{t1} = I_c \sqrt{1 + \frac{\mu_1^2}{2}}$

$$I_c = \frac{I_{t1}}{\sqrt{1 + \frac{\mu_1^2}{2}}} = \frac{11}{\sqrt{1 + \frac{0.4^2}{2}}}$$

$$\boxed{I_c = 10.58A}$$

ii) $$I_{\pm 2} = I_c \sqrt{1 + \frac{\mu_t^2}{2}}$$

$$I_{\pm 2}^2 = I_c^2 \left(1 + \frac{\mu_t^2}{2}\right)$$

$$\frac{I_{\pm 2}^2}{I_c^2} = 1 + \frac{\mu_t^2}{2}$$

$$\frac{\mu_t^2}{2} = \left(\frac{I_{\pm 2}^2}{I_c^2}\right) - 1$$

$$\frac{\mu_t^2}{2} = \left(\frac{12^2}{10.58^2}\right) - 1$$

$$\frac{\mu_t^2}{2} = 1.286 - 1$$

$$\frac{\mu_t^2}{2} = 0.286$$

$$\mu_t^2 = 0.572$$

$$\boxed{\mu_t = 0.757}$$

* WKT $$\mu_t = \sqrt{\mu_1^2 + \mu_2^2}$$

$$\mu_t^2 = \mu_1^2 + \mu_2^2$$

$$\mu_2^2 = \mu_t^2 - \mu_1^2$$

$$= 0.757^2 - 0.4^2$$

$$\mu_2^2 = 0.4130$$

$$\boxed{\mu_2 = 0.642}$$

* A multitone modulating signal has the following time-domain form:

$$m(t)=E_1\cos 2\pi f_1 t + E_2\cos 2\pi f_2 t + E_3\cos 2\pi f_3 t \text{ volts} \quad \text{where } E_1 > E_2 > E_3$$

$f_3 > f_2 > f_1$

i. Give the time – domain expression for the conventional AM wave.

ii. Draw the amplitude spectrum for the AM wave obtained in part i. Also find the minimum transmission bandwidth.

Sol:-

(a) The time-domain expression for the conventional AM wave is

$$s(t) = A_c[1 + K_a m(t)] \cos 2\pi f_c t \rightarrow (1)$$

Substituting the value of $m(t)$ in eq (1), we get

$$s(t) = A_c[1 + \underline{K_a E_1} \cos 2\pi f_1 t + \underline{K_a E_2} \cos 2\pi f_2 t + \underline{K_a E_3} \cos 2\pi f_3 t] \times \cos 2\pi f_c t$$

W.K.T, $\mu_1 = K_a E_1$, $\mu_2 = K_a E_2$ & $\mu_3 = K_a E_3$

$$s(t) = A_c[1 + \mu_1 \cos 2\pi f_1 t + \mu_2 \cos 2\pi f_2 t + \mu_3 \cos 2\pi f_3 t] \cos 2\pi f_c t.$$

$$s(t) = A_c \cos 2\pi f_c t + \mu_1 A_c \cos 2\pi f_c t \cdot \cos 2\pi f_1 t + \mu_2 A_c \cos 2\pi f_c t \cdot \cos 2\pi f_2 t + \mu_3 A_c \cos 2\pi f_c t \cdot \cos 2\pi f_3 t$$

W.K.T

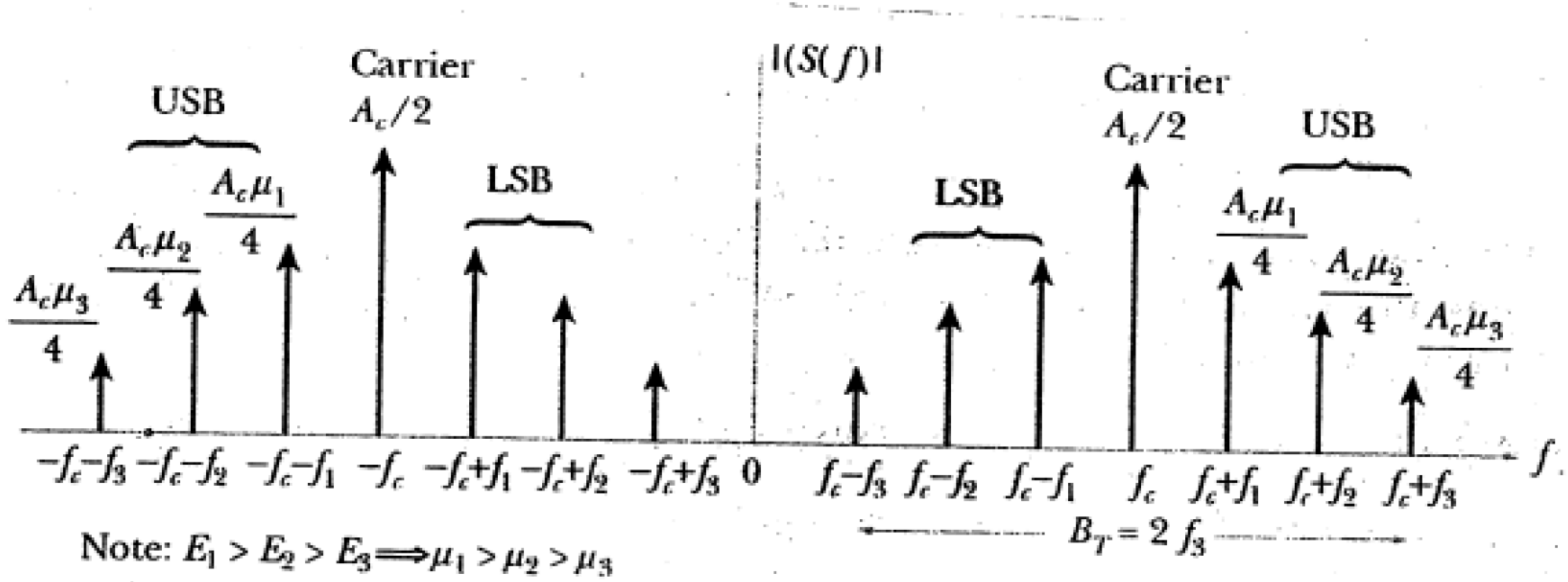
$$\boxed{\cos A \cdot \cos B = \frac{1}{2} \cos(A+B) + \frac{1}{2} \cos(A-B)}$$

$$s(t) = A_c \cos 2\pi f_c t + \frac{\mu_1 A_c}{2} \cos 2\pi (f_c - f_1) t + \frac{\mu_1 A_c}{2} \cos 2\pi (f_c + f_1) t$$
$$+ \frac{\mu_2 A_c}{2} \cos 2\pi (f_c - f_2) t + \frac{\mu_2 A_c}{2} \cos 2\pi (f_c + f_2) t$$
$$+ \frac{\mu_3 A_c}{2} \cos 2\pi (f_c - f_3) t + \frac{\mu_3 A_c}{2} \cos 2\pi (f_c + f_3) t \rightarrow (2)$$

b) Taking Fourier transform on both sides of equation (2), we get

$$S(f) = \frac{A_c}{2} \left[\delta(f - f_c) + \delta(f + f_c) \right] + \frac{\mu_1 A_c}{4} \left\{ \delta[f - (f_c - f_1)] + \delta[f + (f_c - f_1)] \right\}$$
$$+ \frac{\mu_1 A_c}{4} \left\{ \delta[f - (f_c + f_1)] + \delta[f + (f_c + f_1)] \right\} + \frac{\mu_2 A_c}{4} \left\{ \delta[f - (f_c - f_2)] + \delta[f + (f_c - f_2)] \right\}$$
$$+ \frac{\mu_2 A_c}{4} \left\{ \delta[f - (f_c + f_2)] + \delta[f + (f_c + f_2)] \right\} + \frac{\mu_3 A_c}{4} \left\{ \delta[f - (f_c - f_3)] + \delta[f + (f_c - f_3)] \right\}$$
$$+ \frac{\mu_3 A_c}{4} \left\{ \delta[f - (f_c + f_3)] + \delta[f + (f_c + f_3)] \right\}$$

The amplitude Spectrum is shown below



The maximum frequency is f_3.

$\therefore$ The transmission bandwidth $\boxed{B_T = 2f_3}$

Problem

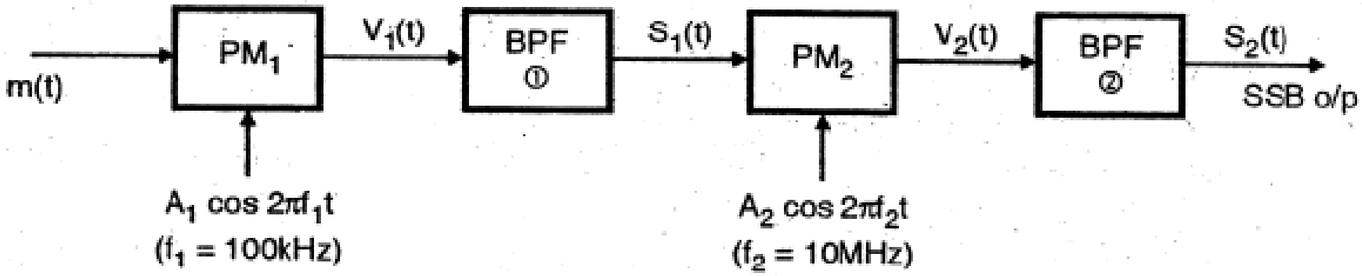
Consider a two-stage product modulator with a BPF after each product modulator, where i/p signal consists of a voice signal occupying the frequency band 0.3 to 3.4 KHz. The two oscillator frequencies have values f1=100 KHz and f2=10 MHz. Specify the following:

i. **Sidebands of DSB-SC modulated waves appearing at the two product modulator output.**
ii. **Sidebands of SSB modulated waves appearing at BPF outputs.**
iii. **The pass bands of the two BPFs.**

Given :-

$f_1 = 100KHz$, $f_2 = 10MHz$

$m(t) = 0.3KHz$ to $3.4KHz$



o/p of PM1 :

$\boxed{f_c = f_1 = 100KHz, \quad f_m = 0.3KHz \text{ to } 3.4KHz}$

The PM1 o/p consists of two Sidebands as follows

$LSB = f_c - f_m = 100KHz - (0.3KHz \text{ to } 3.4KHz)$

$\boxed{LSB = 99.7KHz \text{ to } 99.6KHz}$

$USB = f_c + f_m = 100KHz + (0.3KHz \text{ to } 3.4KHz)$

$\boxed{USB = 100.3KHz \text{ to } 103.4KHz}$

o/p of BPF1 :- Assume that this BPF1 passes only the USB.

$$\boxed{S_1(t) = 100.3\,KHz \text{ to } 103.4\,KHz}$$

o/p of PM2 :-

$$\boxed{f_2 = f_c = 10MHz \;,\; S_1(t) = f_m = 100.3\,KHz \text{ to } 103.4\,KHz}$$

The PM2 o/p Consists of two Sidebands as follows:

$$LSB = f_c - f_m = 10MHz - (100.3\,KHz \text{ to } 103.4\,KHz)$$

$$\boxed{LSB = 9.899\,MHz \text{ to } 9.8966\,MHz}$$

$$USB = f_c + f_m = 10MHz + (100.3\,KHz \text{ to } 103.4\,KHz)$$

$$\boxed{USB = 10.1003\,MHz \text{ to } 10.1034\,MHz}$$

o/p of BPF2 :- Assume that this BPF2 passes only the USB

$$\boxed{S_2(t) = 10.1003\,\text{MHz} \text{ to } 10.1034\,\text{MHz}}$$

Guard band of BPF :-

Guard band is defined as the <u>highest</u> frequency component of <u>LSB</u> to the <u>lowest</u> frequency component of <u>USB</u>.

$$\boxed{\text{Guard band of BPF1} = 99.7\,\text{KHz to } 100.3\,\text{KHz}}$$

$$\boxed{\text{Guard band of BPF2} = 9.8997\,\text{MHz to } 10.1003\,\text{MHz}}$$

Parameter	LSB	USB
O/P of PM1- V_1 (t)	99.7 KHz to 99.6 KHz	100.3 KHz to103.4 KHz
O/P 0f BPF1 – S_1 (t)	-	100.3 KHz to103.4 KHz
O/P of PM1- V_2 (t)	9.899 MHz to 9.8966 MHz	10.1003 MHz to 10.1034 MHz
O/P 0f BPF1 – S_2 (t)	-	10.1003 MHz to 10.1034 MHz

Guard Band of BPF-1 = 99.7 KHz to 100.3 KHz

Guard Band of BPF-2 = 9.8997 MHz to10.1003 MHz

For the rectangular pulse shown in fig ①, evaluate its hilbert - transform.

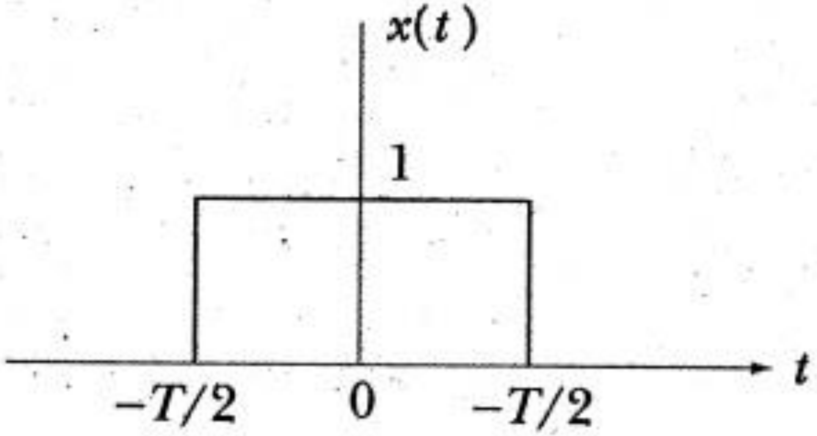


fig ①: Rectangular pulse.

Sol :-

From Fig ①, $x(t) = \begin{cases} 1, & |t| < T/2 \\ 0, & |t| > T/2 \end{cases}$

WKT

$$\begin{aligned}
\hat{x}(t) &= x(t) * \frac{1}{\pi t} \\
&= \int_{-\infty}^{\infty} \frac{x(\tau)}{\pi(t-\tau)} d\tau \\
&= \int_{-T/2}^{T/2} \frac{1}{\pi(t-\tau)} d\tau \\
&= -\frac{1}{\pi} \int_{-T/2}^{T/2} \frac{1}{\tau - t} d\tau \\
&= -\frac{1}{\pi} \left[\ln(\tau - t)\right]\Big|_{\tau=-T/2}^{T/2} \\
&= -\frac{1}{\pi} \left[\ln\left(\frac{T}{2} - t\right) - \ln\left(-\frac{T}{2} - t\right)\right] \\
&= \frac{1}{\pi} \ln\left[\frac{t + \frac{T}{2}}{t - \frac{T}{2}}\right]
\end{aligned}$$