Amplitude Modulation

1. Introduction
   - Time domain description
   - Frequency domain description
   - Power and Current Relations

2. AM (DSB-FC) Generation
   - Square Law Modulator
   - Switching Modulator

3. DSB-SC Generation
   - Balanced Modulator
   - Ring Modulator

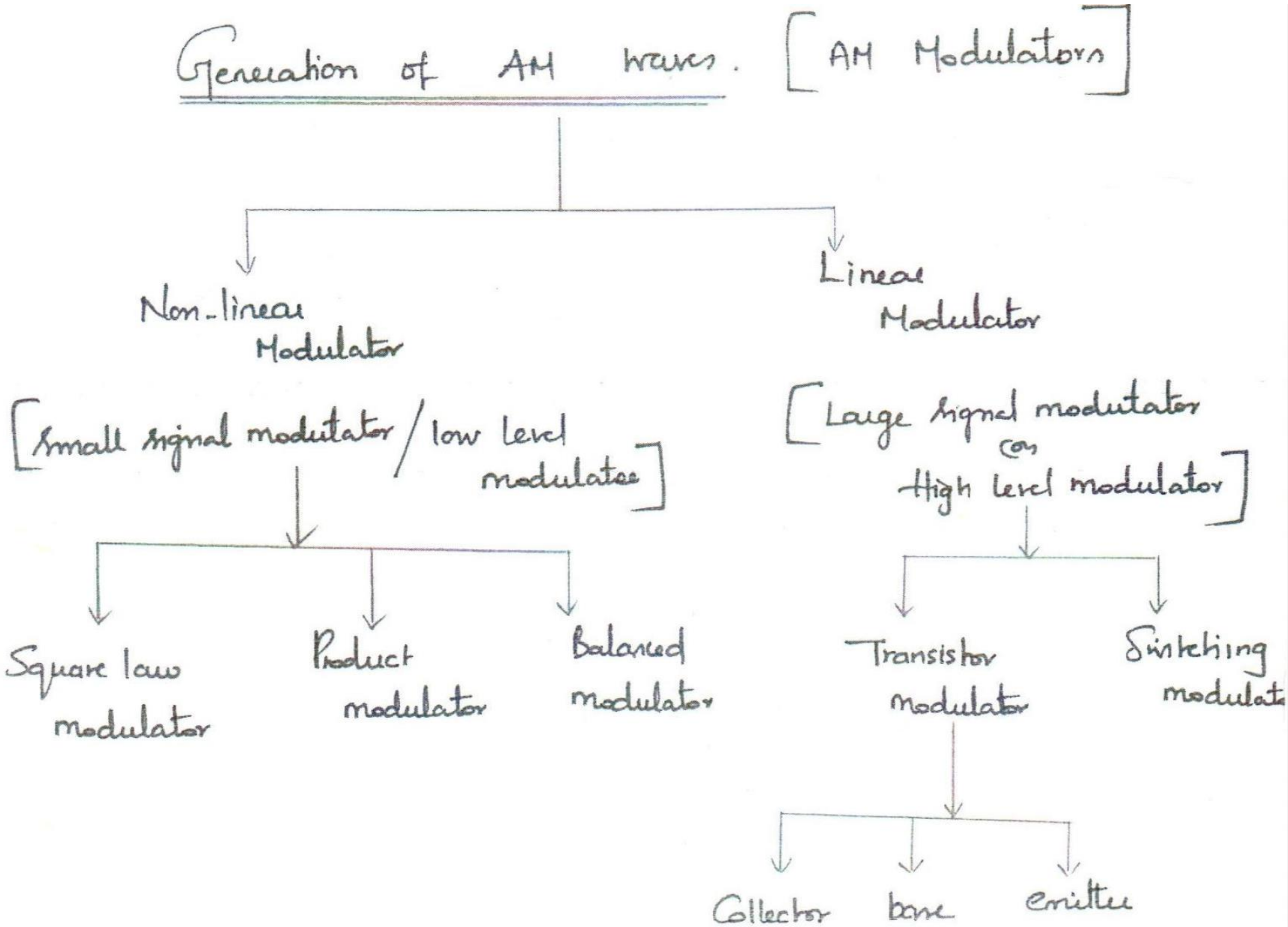
4. SSB Generation
   - Filter Method
   - Phase Shift Method
   - Third Method

5. VSB Generation
   - Filter Method

6. Hilbert Transform
   - Pre-envelope
   - Complex envelope

7. Comparison of AM Techniques

Classification of Generation of DSB-FC



Square-Law Modulator

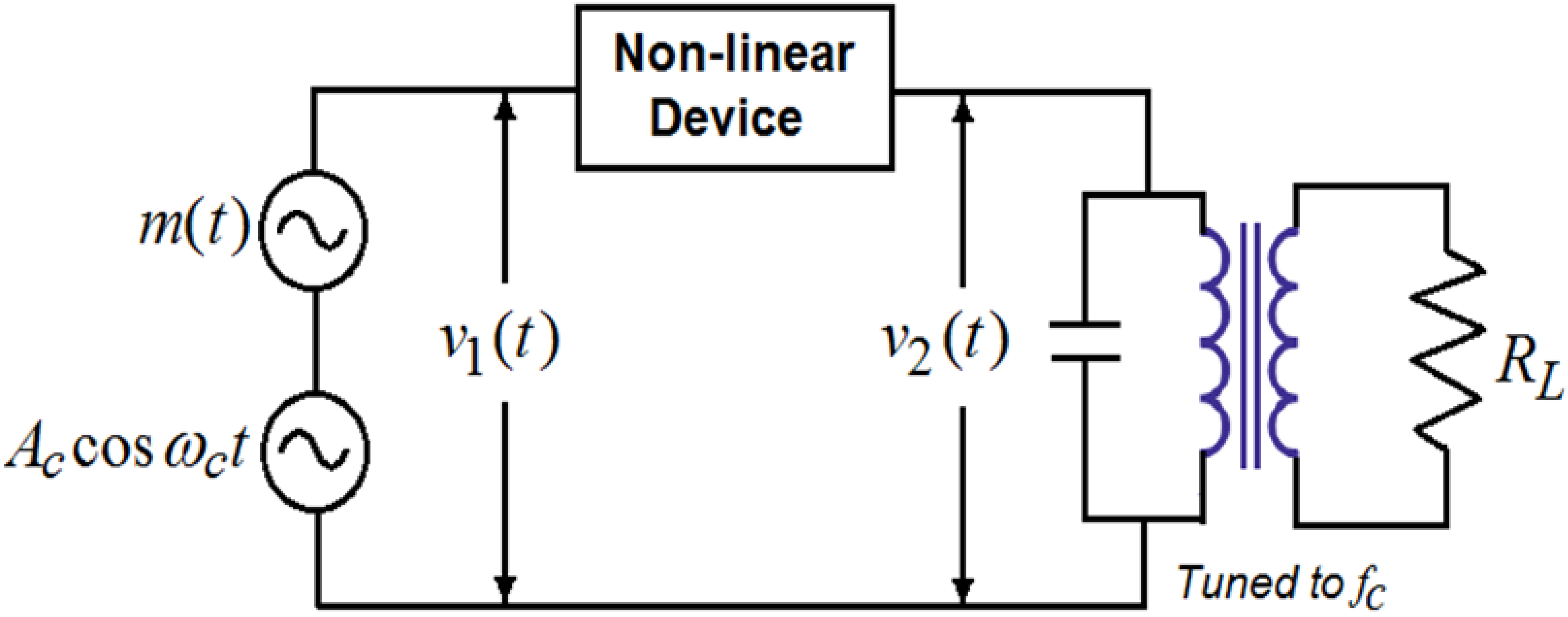
When the output of a device is not directly proportional to input throughout the operation, the device is said to be non-linear. The Input-Output relation of a non-linear device can be expressed as

$$V_0 = a_0 + a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3 + a_4 V_{in}^4 + ...$$

When the in put is very small, the higher power terms can be neglected. Hence the output is approximately given by $V_0 = a_o + a_1 V_{in} + a_2 V_{in}^2$

When the output is considered up to square of the in put, the device is called a square law device and the square law modulator is as shown in the figure

Square-Law Modulator



The diode-resistor equation is given by

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

Where a_1 and a_2 are constants.

Square-Law Modulator …

The output of the non-linear device (diode)

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$ where $$v_1(t) = m(t) + A_c \cos \omega_c t$$

$$v_2(t) = a_1\left[m(t) + A_c \cos \omega_c t\right] + a_2\left[m(t) + A_c \cos \omega_c t\right]^2$$

$$= a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right]\cos \omega_c t + a_1 m(t) + a_2 m^2(t) + a_2 A_c \cos^2 \omega_c t$$

The first term is the desired AM wave with amplitude sensitivity

$$k_a = \frac{2a_2}{a_1}$$

Remaining unwanted three terms are removed by appropriate filtering.

Square-Law Modulator …

If $m(t) = A_m \cos 2\pi f_m t$, we get

$$V_2(t) = s(t) = a_1 A_c \left[1 + 2\frac{a_2}{a_1} A_m \cos 2\pi f_m t \right] \cos 2\pi f_c t$$

Comparing this with the standard representation of AM signal,

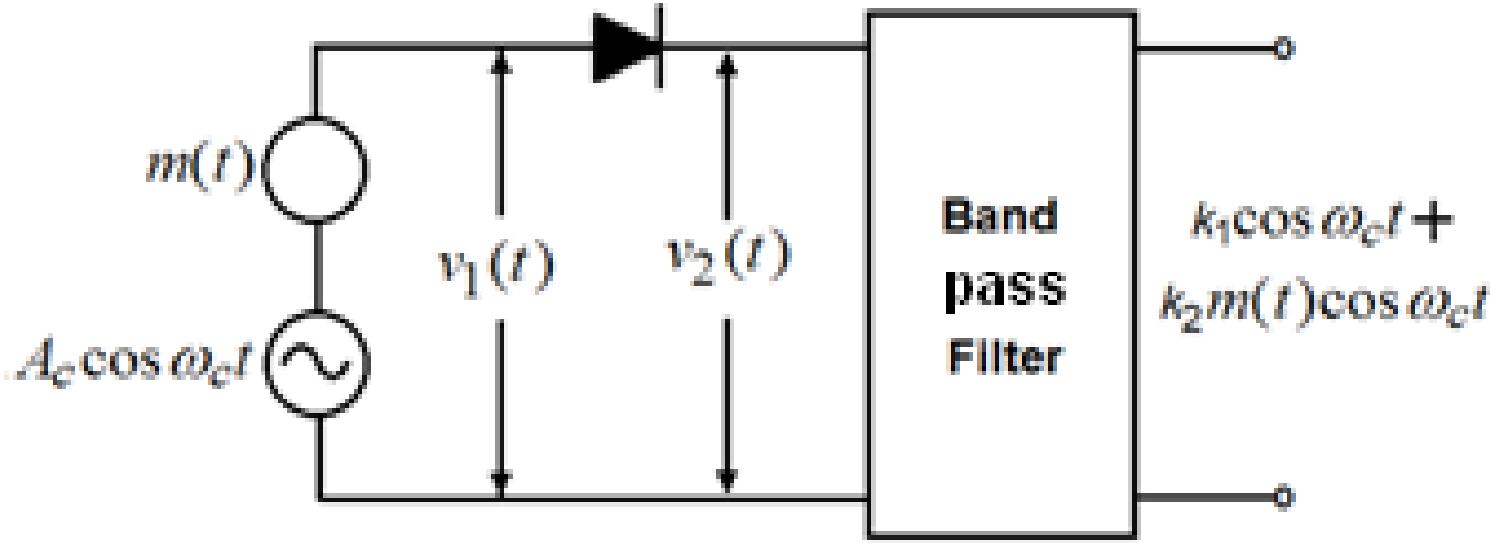
$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

Therefore modulation index of the output signal is given by

$$m = 2\frac{a_2}{a_1} A_m$$

The output AM signal is free from distortion and attenuation only when $(f_c - W) > 2W$ or $f_c > 3W$.

Switching Modulator

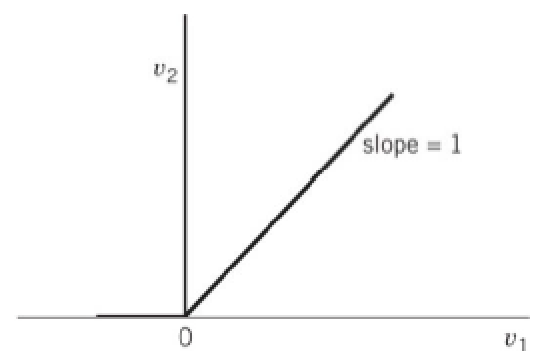


The input voltage to the diode ,

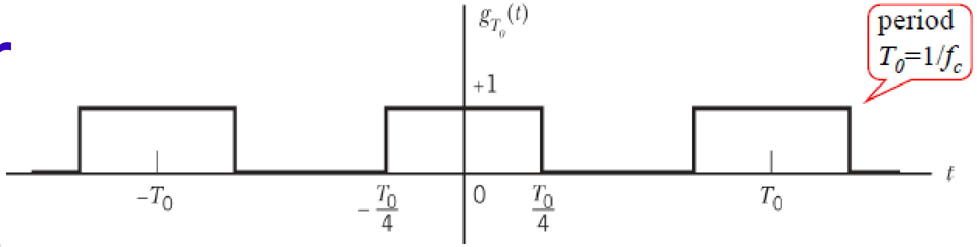
$$v_1(t) = m(t) + A_c \cos \omega_c t$$ where $\| m(t) \| << A_c$.

Then

$$v_2(t) = \begin{cases} v_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$$



Switching Modulator



$$v_2(t) = \left[m(t) + A_c \cos \omega_c t\right] g_{T_0}(t)$$

$$g_{T_0}(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos \omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \dots.\right]$$

$$v_2(t) = \left[m(t) + A_c \cos \omega_c t\right] g_{T_0}(t)$$

$$= \frac{m(t)}{2} + \frac{A_c}{2}\cos \omega_c t + \frac{2}{\pi} m(t) \cos \omega_c t + \dots$$

The desired components $\frac{A_c}{2}\cos \omega_c t + \frac{2}{\pi} m(t) \cos \omega_c t$

Switching Modulator

$$\boxed{g_{T_0}(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\left(2\pi \frac{n}{T_0} t\right) + b_n \sin\left(2\pi \frac{n}{T_0} t\right)\right)}$$

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_{T_0}(t)\,dt$$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} g_{T_0}(t)\cos\left(2\pi \frac{n}{T_0} t\right) dt$$

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} g_{T_0}(t)\sin\left(2\pi \frac{n}{T_0} t\right) dt$$

$$b_n = \frac{2}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \sin\left(2\pi \frac{n}{T_0} t\right) dt = -\frac{2}{T_0}\left.\frac{\cos\left(2\pi \frac{n}{T_0} t\right)}{2\pi \frac{n}{T_0}}\right|_{-\frac{T_0}{4}}^{\frac{T_0}{4}}$$

$$= -\frac{1}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right) - \cos\left(-\frac{n\pi}{2}\right)\right] = 0$$

$$a_0 = \frac{1}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} dt = \frac{1}{2}$$

$$a_n = \frac{2}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \cos 2\pi \frac{n}{T_0} t\,dt = \frac{2}{T_0}\left.\frac{\sin 2\pi \frac{n}{T_0} t}{2\pi \frac{n}{T_0}}\right|_{-\frac{T_0}{4}}^{\frac{T_0}{4}}$$

$$= \frac{1}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) - \sin\left(-\frac{n\pi}{2}\right)\right] = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{2}{(2m-1)\pi}\sin\left(\frac{(2m-1)\pi}{2}\right)$$

$$= \frac{2}{(2m-1)\pi}\left[-\cos(m\pi)\right]$$

$$= \frac{2}{(2m-1)\pi}\left[-(-1)^m\right] = \frac{2}{(2m-1)\pi}(-1)^{m+1}$$

$$= \frac{2}{(2m-1)\pi}(-1)^{m-1}$$

$$g_{T_0}(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\left[2\pi f_c t(2n-1)\right]$$

Switching Modulator

The required AM signal centered at f_c can be separated using band pass filter. The lower cutoff-frequency for the band pass filter should be between w and f_c-w and the upper cut-off frequency between f_c+w and $2f_c$. The filter output is given by the equation

$$S(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi}\frac{m(t)}{A_c}\right]\cos 2\pi f_c t$$

For a single tone information, let $m(t) = A_m \cos(2\pi f_m t)$

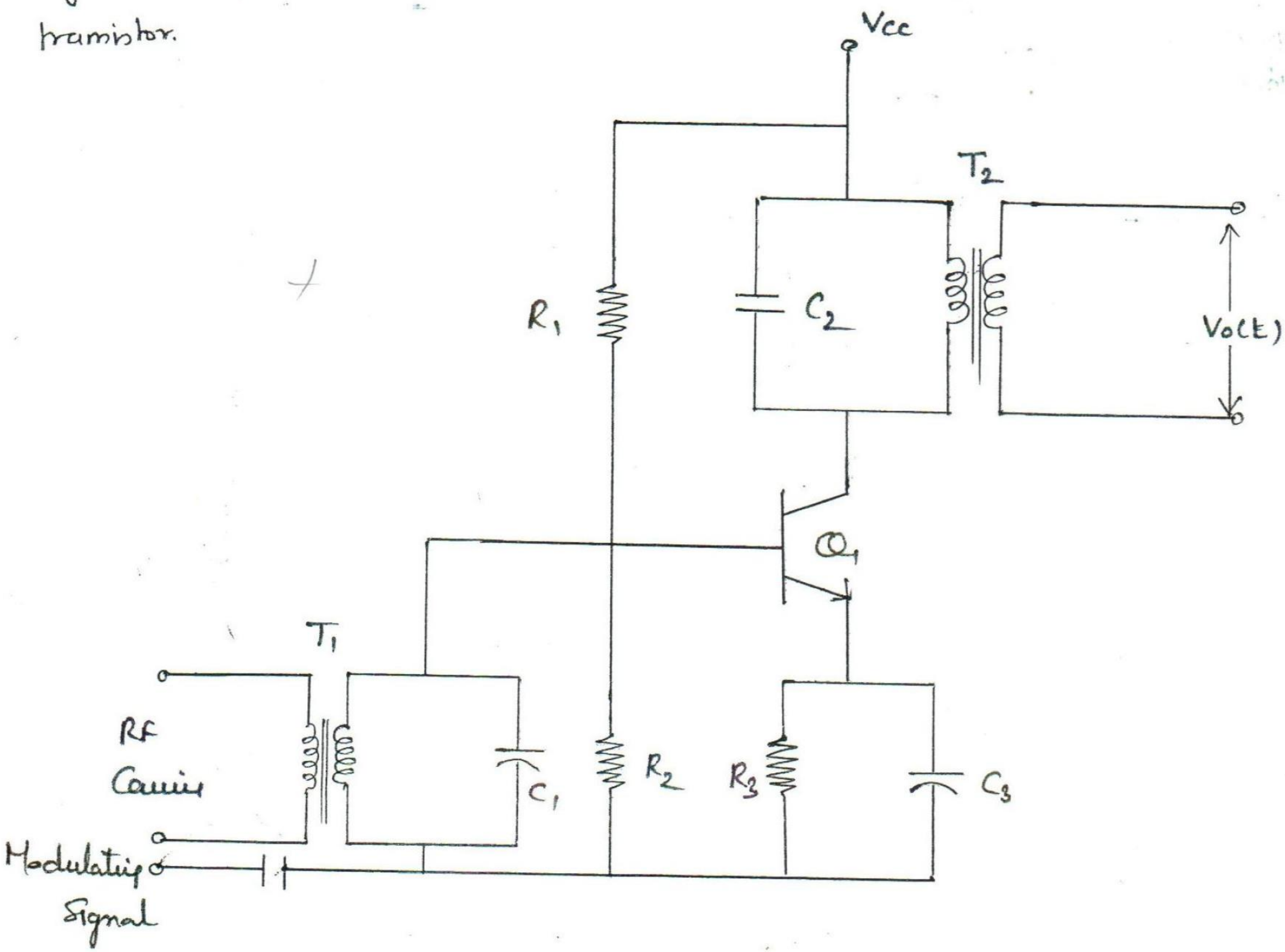
$$S(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi}\frac{A_m}{A_c}\cos 2\pi f_m t\right]\cos 2\pi f_c t$$

Therefore modulation index, $m = \frac{4}{\pi}\frac{A_m}{A_c}$

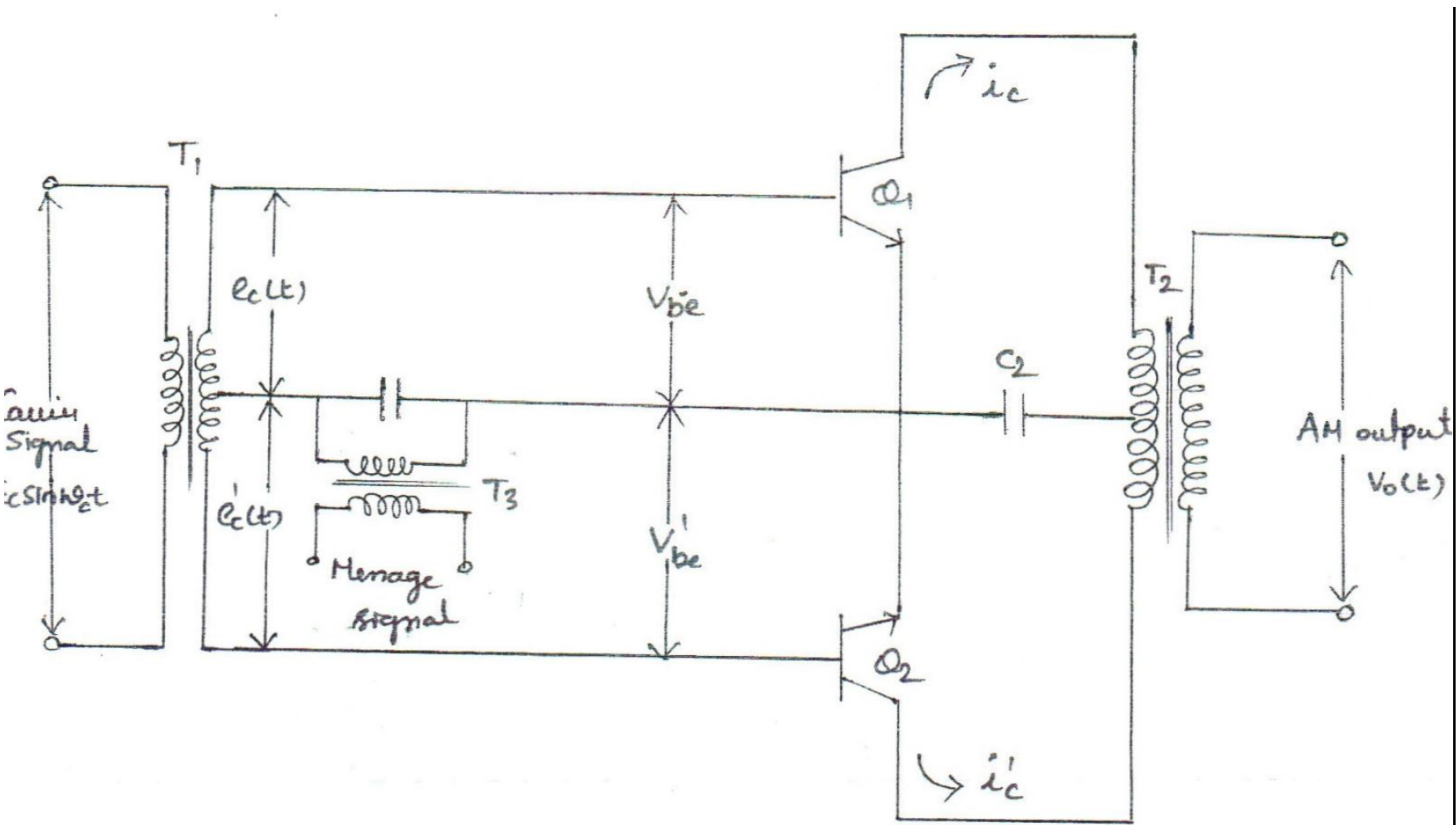
The output AM signal is free from distortions and attenuations only when fc-w>w or fc>$2w$.

Square law modulator using transistor

This circuit is used in the low level modulation. The modulation signal is applied to the emitter and RF Carrier at the base of the transistor.



Balanced modulator



Here two non-linear transistors are connected in the balanced mode. It is assumed that the two transistors are identical & the circuit is symmetrical. The operation of the transistor used are confined to operate in the non-linear region of its transfer characteristics.

Balanced modulator

Where, $e_c(t) = E_c \sin \omega_c t$ —— (1)

let, the message signal be,

$$e_m(t) = E_m \sin \omega_m t \quad \text{——} \quad (2)$$

I/p voltage across the transistor Q_1 is given by,

$$V_{be} = e_c(t) + e_m(t)$$

$$V_{be} = E_c \sin \omega_c t + E_m \sin \omega_m t \quad \text{——} \quad (3)$$

Balanced modulator

The carrier voltage across the base winding of the centre tap transformer as shown in fig. are equal and opposite in phase that is $e_c(t) = e_c'(t)$.

I/p voltage across the transistor Q_2 is given by,

$$V_{be}' = e_c'(t) + e_m(t)$$

$$V_{be}' = -E_c \sin \omega_c t + E_m \sin \omega_m t \quad \text{—— (4)}$$

By the non-linearity relationship of the transistor the collector current can be written as,

$$i_c = a V_{be} + b V_{be}^2 \quad \text{—— (5)}$$

$$i_c' = a V_{be}' + b V_{be}'^2 \quad \text{—— (6)}$$

Balanced modulator

Sub equ (3) in (5), we get

$$i_c = a\left[E_c \sin\omega_c t + E_m \sin\omega_m t\right] + b\left[E_c \sin\omega_c t + E_m \sin\omega_m t\right]^2$$

$$i_c = aE_c \sin\omega_c t + aE_m \sin\omega_m t + bE_c^2 \sin^2\omega_c t + bE_m^2 \sin^2\omega_m t$$

$$+ 2bE_m E_c \sin\omega_c t \sin\omega_m t \quad \text{—— (7)}$$

Sub equ (4) in (6)

$$i_c' = a\left[-E_c \sin\omega_c t + E_m \sin\omega_m t\right] + b\left[E_m \sin\omega_m t - E_c \sin\omega_c t\right]^2$$

$$= -aE_c \sin\omega_c t + aE_m \sin\omega_m t + bE_m^2 \sin^2\omega_m t + bE_c^2 \sin^2\omega_c t$$

$$- 2bE_m E_c \sin\omega_c t \sin\omega_m t \quad \text{—— (8)}$$

Balanced modulator

The o/p AM Voltage (V_o) is given by,

$$V_o = k(i_c - i_c') \quad \text{——} \quad (9)$$

This is because both the Current flows in the Opposite direction k is a Constant depending on impedance & other Circuit parameters.

Sub equ (7) & (8) in equ (9).

$$V_o = k\Big[\big(a E_c \sin \omega_c t + a E_m \sin \omega_m t + b E_c^2 \sin^2 \omega_c t + b E_m^2 \sin^2 \omega_m t$$

$$+ 2b E_m E_c \sin \omega_m t \sin \omega_c t\big) - \big(-a E_c \sin \omega_c t + a E_m \sin \omega_m t$$

$$+ b E_m^2 \sin^2 \omega_m t + b E_c^2 \sin^2 \omega_c t - 2b E_m E_c \sin \omega_c t \sin \omega_m t\big)$$

Balanced modulator

$$V_o = k\Big[aE_c \sin\omega_c t + a\cancel{E_m \sin\omega_m t} + b\cancel{E_c^2 \sin^2\omega_c t} + b\cancel{E_m^2 \sin^2\omega_m t}$$

$$+ 2bE_mE_c \sin\omega_m t \sin\omega_c t + aE_c \sin\omega_c t - a\cancel{E_m \sin\omega_m t}$$

$$- b\cancel{E_m^2 \sin^2\omega_m t} - b\cancel{E_c^2 \sin^2\omega_c t} + 2bE_mE_c \sin\omega_m t \sin\omega_c t\Big]$$

$$V_o = k\left[2aE_c \sin\omega_c t + 4bE_mE_c \sin\omega_m t \sin\omega_c t \right]$$

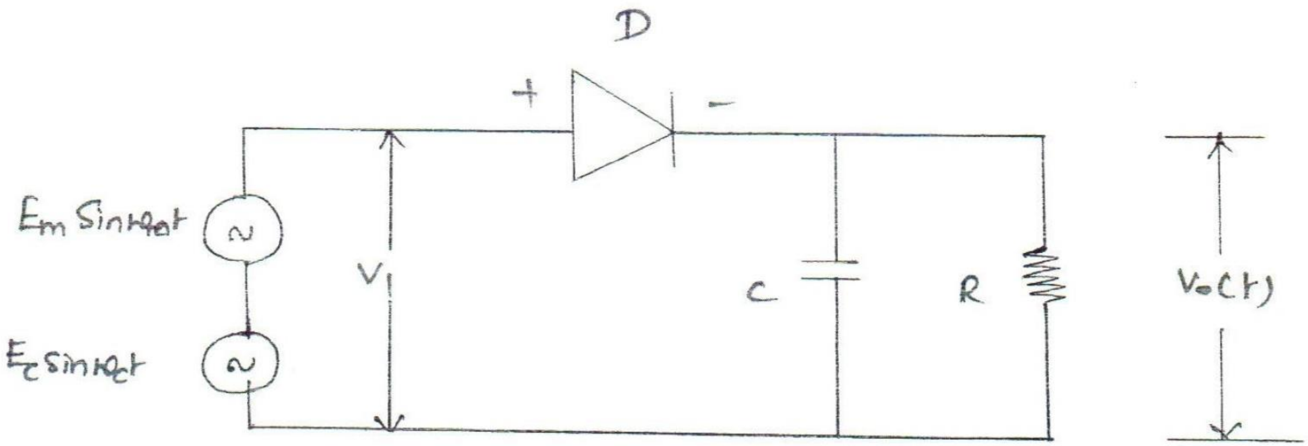
$$V_o = 2kaE_c \sin\omega_c t + 4kbE_mE_c \sin\omega_m t \sin\omega_c t$$

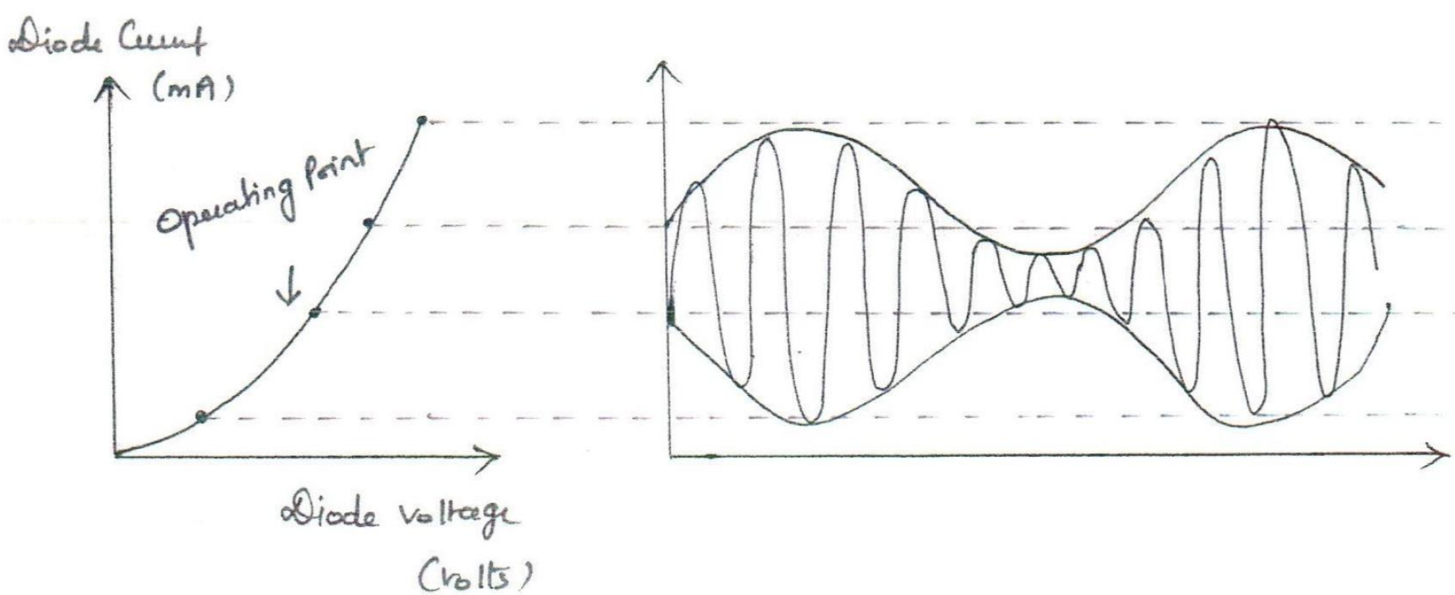
$$V_o = 2kaE_c\left[1 + \frac{2bE_m}{a}\sin\omega_m t\right]\sin\omega_c t.$$

$m_a = \dfrac{2bE_m}{a}$ is the modulation index of AM.

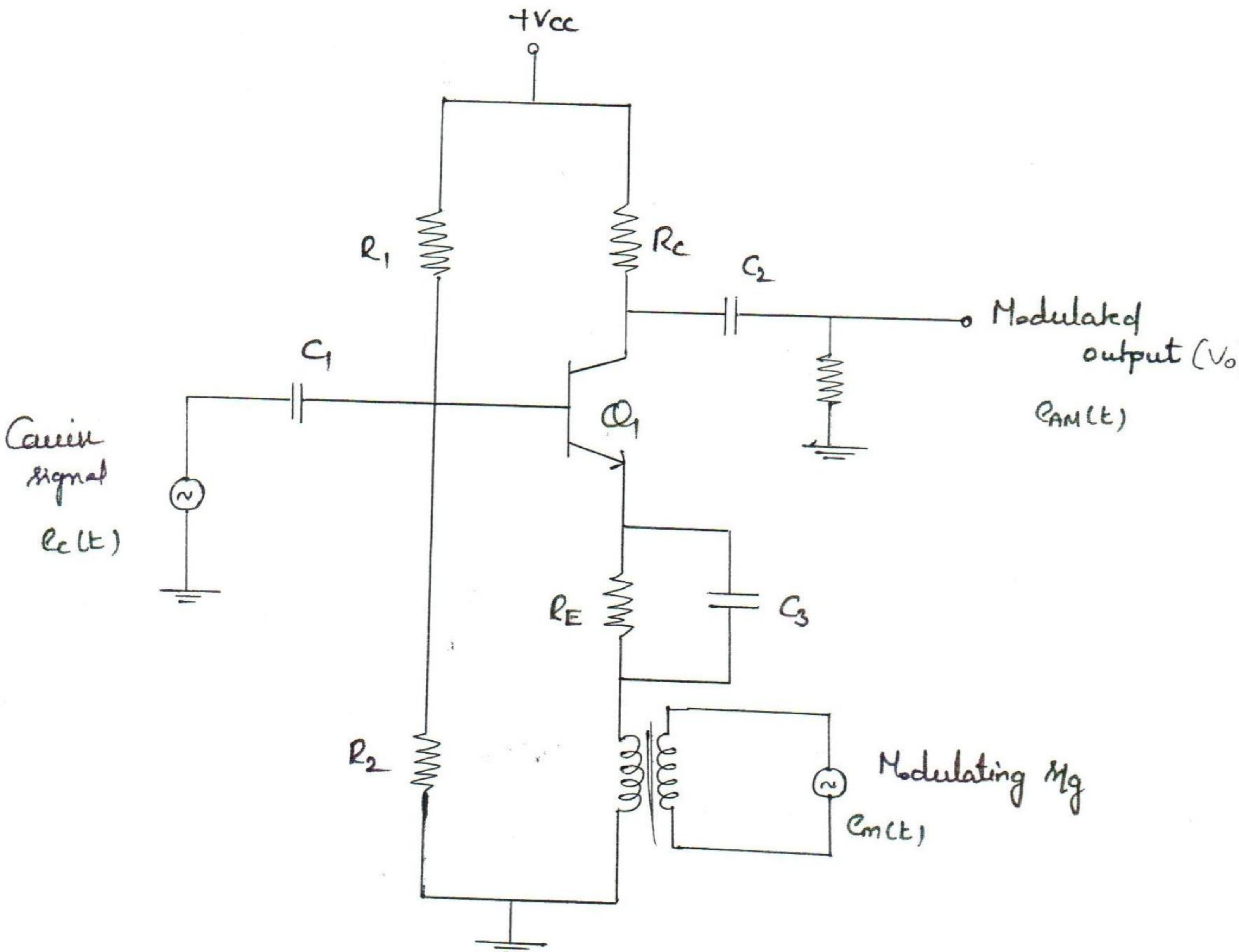
$$\therefore \boxed{V_o = 2kaE_c\left[1 + m_a \sin\omega_m t\right]\sin\omega_c t}$$

Square Law modulator using Diode





Low Level AM modulator / Emitter Modulator



Low Level AM modulator / Emitter Modulator

Gain of the transistor stage is depends upon the emitter current at quiescent point.

$$i_e = I_e + E_m \sin \omega_m t \quad \text{—— (1)}$$

Voltage gain of the transistor is proportional to the emitter current.

$$A_v \propto i_e$$

$$A_v = k\, i_e \quad \text{—— (2)}$$

$k \rightarrow$ Constant.

$$A_v = \frac{V_o}{V_i} \quad \text{—— (3)}$$

$$A_v V_i = V_o \quad \text{—— (4)}$$

Low Level AM modulator / Emitter Modulator

$$V_i = E_c \sin \omega_c t \quad \text{——} \quad (5)$$

Sub (1) in (2)

$$A_v = k\,(I_e + E_m \sin \omega_m t) \quad \text{——} \quad (6)$$

Sub (5) & (6) in (4)

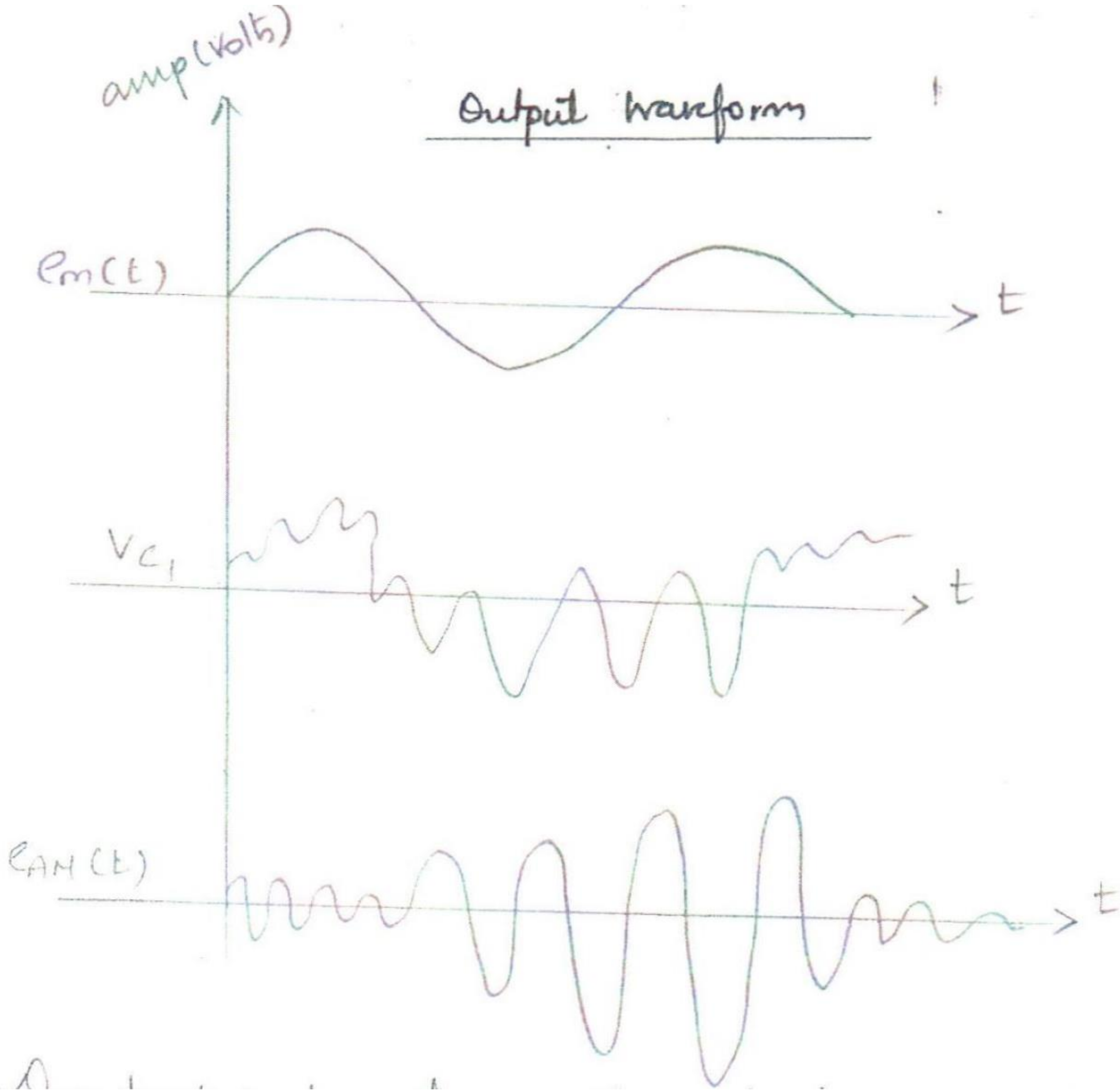
$$V_o = k\,(I_e + E_m \sin \omega_m t)\, E_c \sin \omega_c t$$

$$= k\, I_e E_c \sin \omega_c t + k\, E_m E_c \sin \omega_m t \sin \omega_c t$$

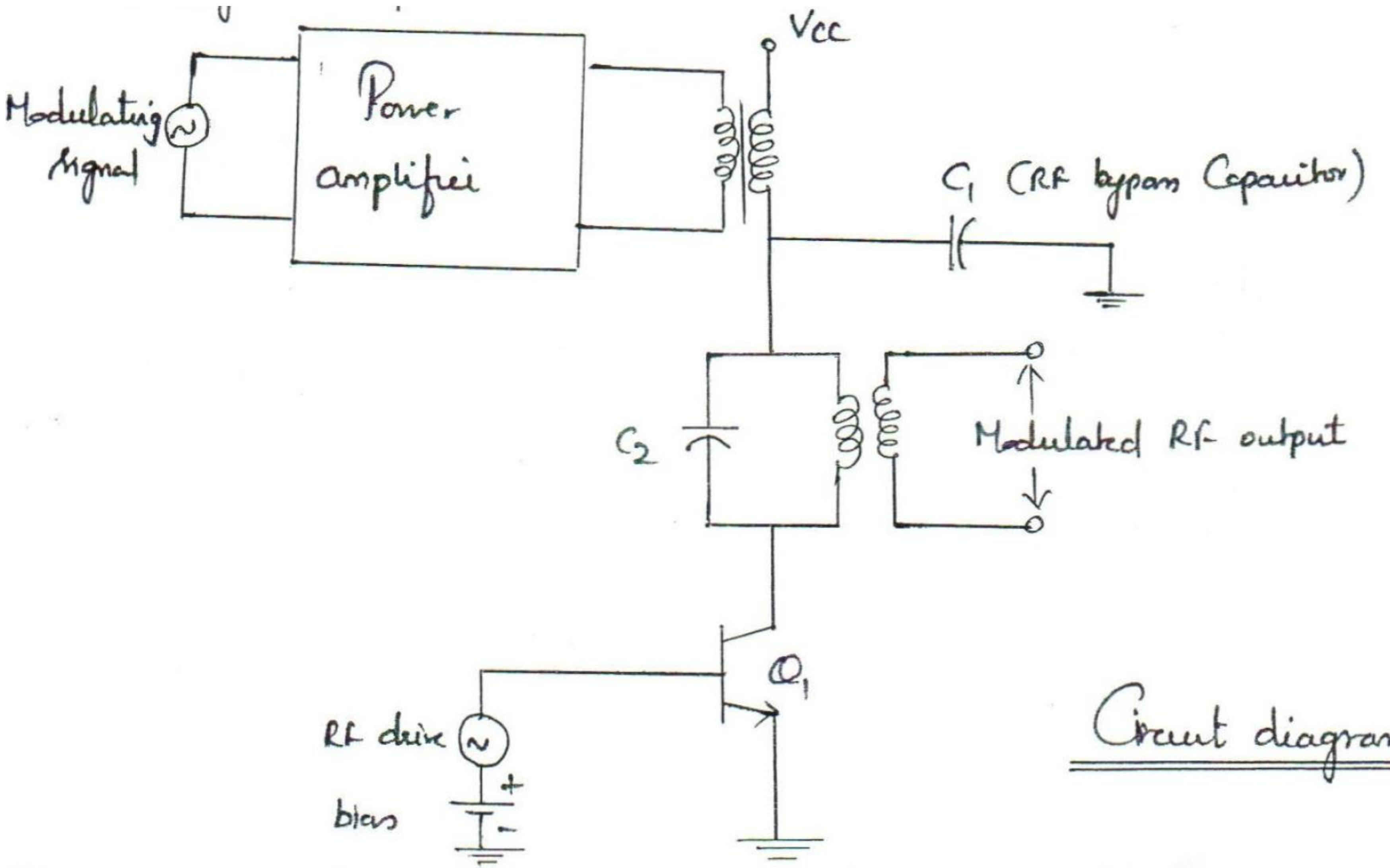
$$V_o = k\, I_e E_c \sin \omega_c t + \frac{k\, E_m E_c}{2} \left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t \right]$$

Thus AM wave is generated.

Low Level AM modulator / Emitter Modulator



High Level AM modulator / Collector Modulator



Circuit diagram

High Level AM modulator / Collector Modulator

Case -(i). without message signal:

→ Circuit will acts as a class-'c'-tuned amplifier

$$V_o(t) = V_{CC}\ E_c(t)$$

$$V_o(t) = V_{CC}\ E_c\ \sin\omega_c t.$$

Case -(ii). with message signal:

→ Supply Voltage will change with respect to message signal

$$V_{CC}' = V_{CC} + V_m \sin\omega_m t$$

$$V_o(t) = V_{CC}'\ V_c \sin\omega_c t$$

High Level AM modulator / Collector Modulator

$$= (V_{cc} + V_m \sin \omega_m t) V_c \sin \omega_c t$$

$$V_o(t) = V_{cc} V_c \sin \omega_c t + V_m V_c \sin \omega_c t \sin \omega_m t$$

$$V_o(t) = V_{cc} V_c \sin \omega_c t + \frac{V_m V_c}{2} \left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t \right]$$

Thus the AM wave is generated.

Adv:- Efficiency is high because class 'c' ampr.

High o/p Power

Detection / Demodulation of AM Signal

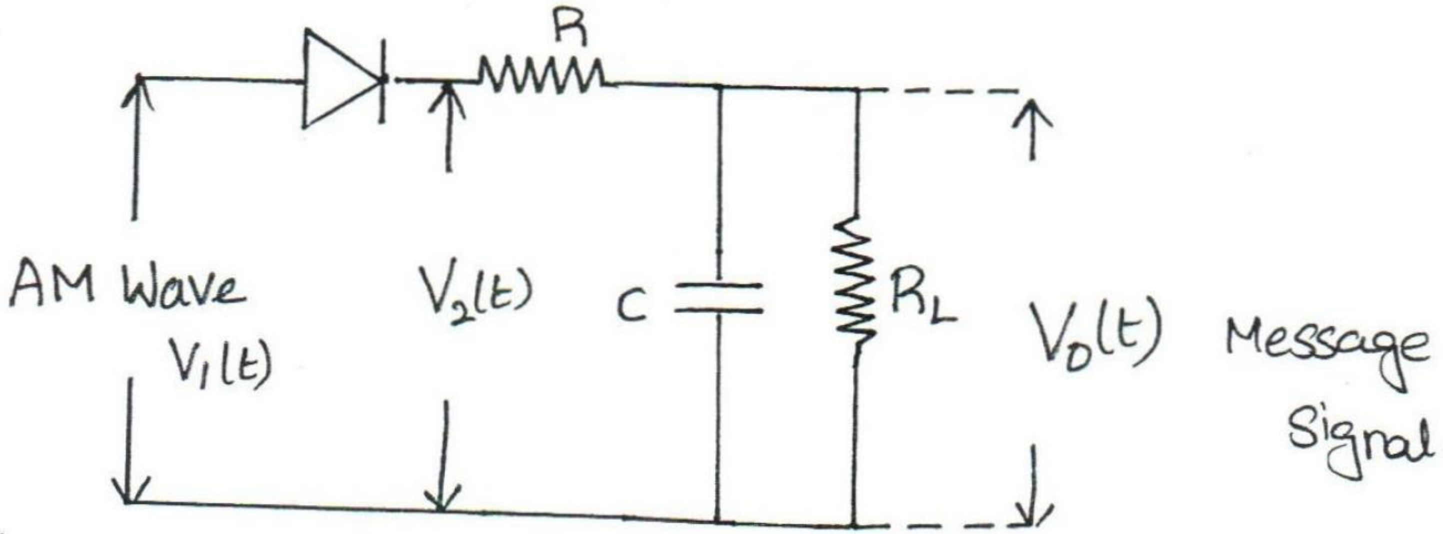
Demodulation / detection :

It is the process in which the modulating voltage is recovered back from the modulated signal. This is the reverse process of modulation taking place in the receiver.

AM detectors

- Linear detector
  - Synchronous (or) Coherent detector
  - Envelope (or) Non-coherent detector
- Non-linear detector
  - Square-law detector
  - Enclave detector

Square Law Detector



Non Linear (V-I (or)) Relationship Between Input and Output,

$$V_2(t) = aV_1(t) + b\,V_1^2(t) \quad \text{—} \; ①$$

$V_1(t) \rightarrow$ I/P of diode (AM Wave)

$V_2(t) \rightarrow$ O/P of diode.

Square Law Detector

$$V_1(t) = E_c\,[1 + m_a \sin \omega_m t]\sin \omega_c t \quad \to (2')$$

Sub (2') in (1)

$$V_2(t) = a\left[E_c(1 + m_a \sin \omega_m t)\right]\sin \omega_c t +$$

$$b\left(E_c[1 + m_a \sin \omega_m t]\sin \omega_c t\right)^2$$

$$= a E_c \sin \omega_c t + a\, m_a E_c \sin \omega_m t \sin \omega_c t$$

$$+ b\left(E_c[1 + m_a \sin \omega_m t]\right)^2 \sin^2 \omega_c t$$

Square Law Detector

$$= aE_c \sin\omega_c t + a m_a E_c \sin\omega_m t$$

$$+ \left[bE_c^2 + E_c^2 b\, 2 m_a \sin\omega_m t \right.$$

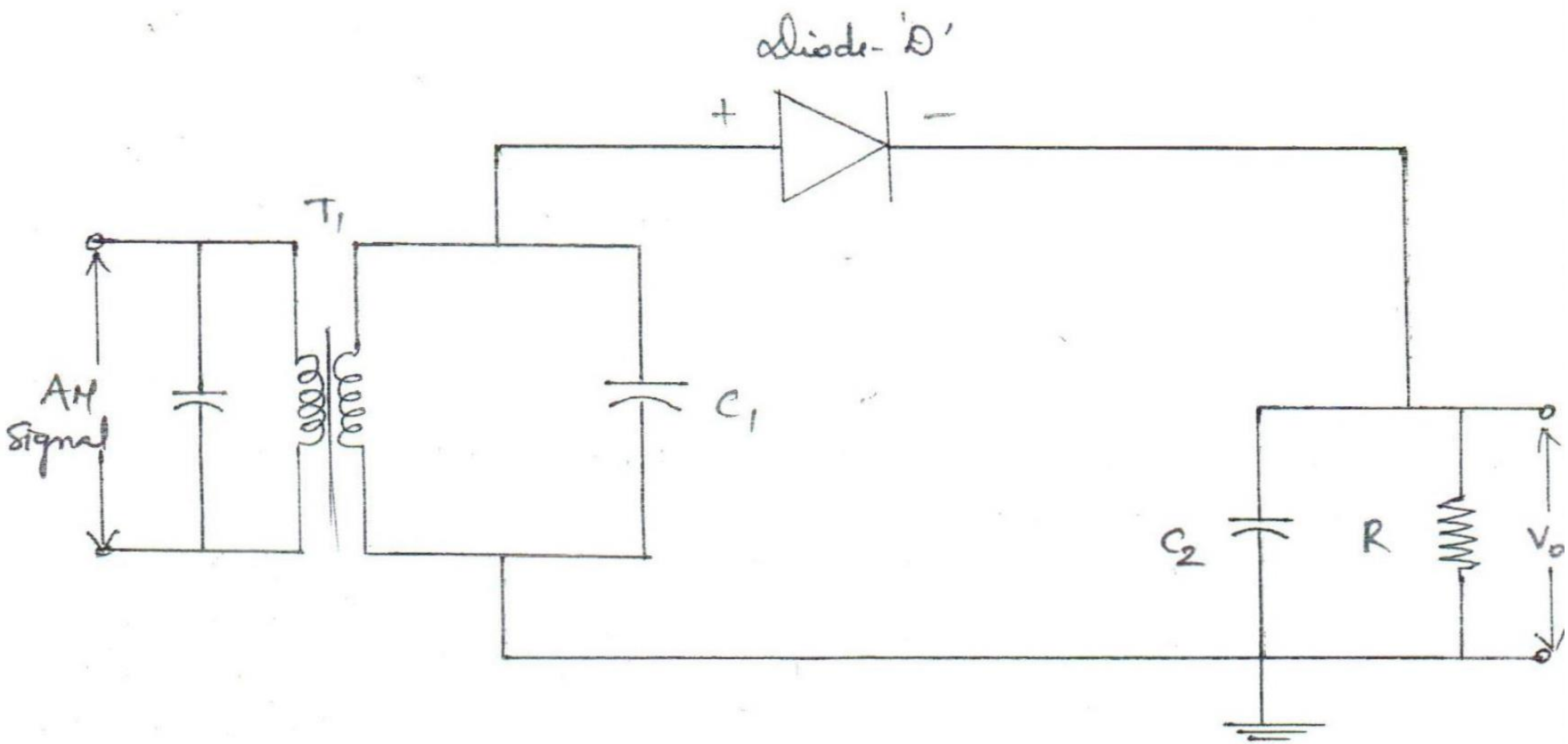
$$\left. + bE_c^2 m_a^2 \sin^2\omega_m t \right] \left(\frac{1 - \cos 4\pi f_c t}{2} \right)$$

out of these terms only desired term is extracted by using Low pass filter.

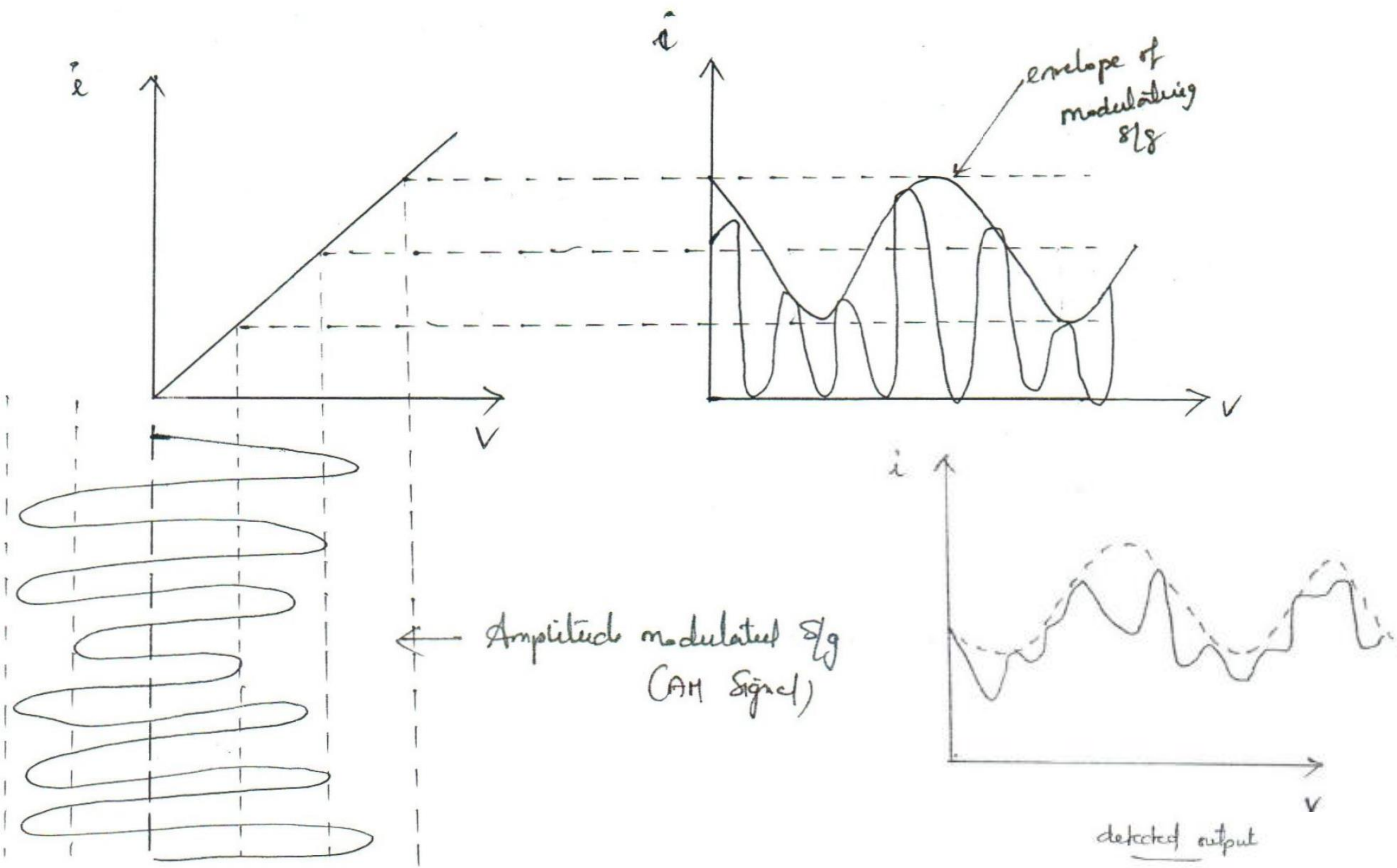
$$\boxed{V_0(t) = bE_c^2 m_a \sin\omega_m t}$$

Thus message signal recovered at the output of Detector.

Envelope / Diode / Linear Detector



Envelope / Diode / Linear Detector



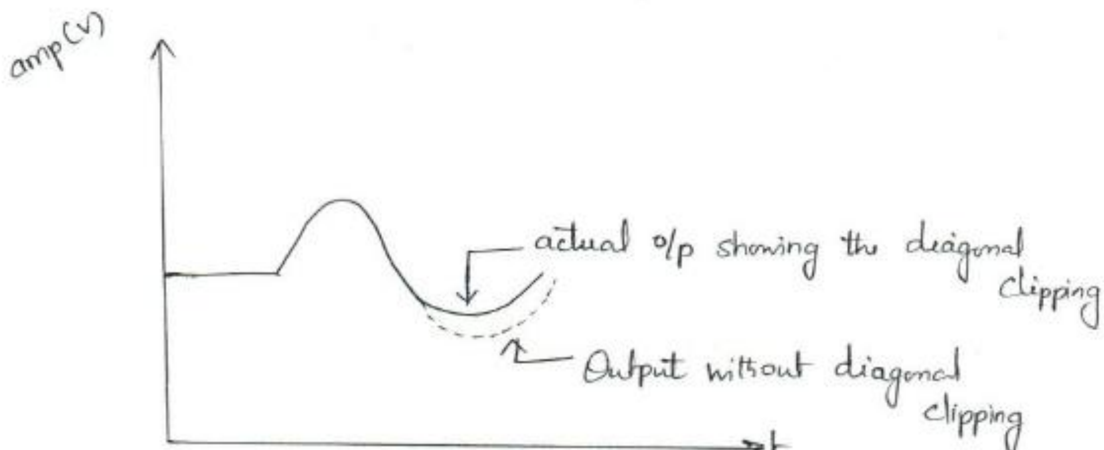
Distortion in Envelope Detector

1). Diagonal clipping:

*. This type of distortion occurs when the RC time Constant of load Circuit is too Long large

* If RC time Constant is too large, it Can't follow the fast changes in the modulating envelope

* As a Result of distortion in diagonal of detected signal is clipped out



Distortion in Envelope Detector

2) Negative - Peak clipping:

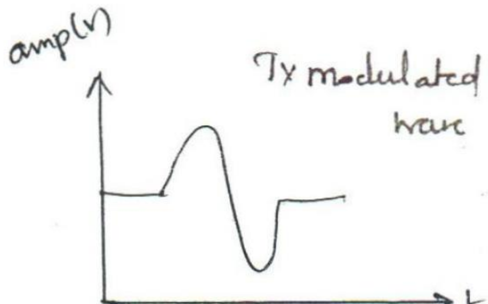
Modulation index of detected message is greater than modulation index of Input AM.

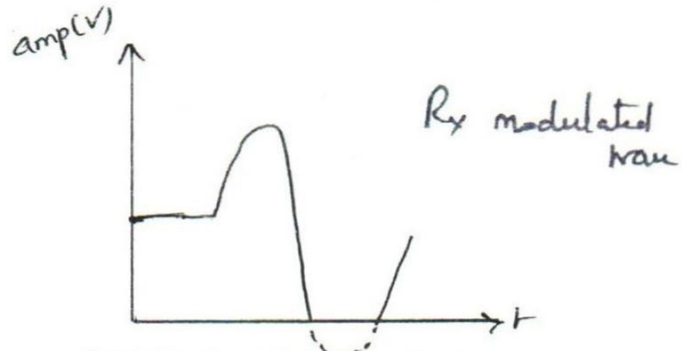
I/p AM (ma) < Detected message (ma)

If modulation index (ma) of message is high

Em > Ec ma > 1, (ie) Over modulation.

Hence negative peak clipping will take place as a result of Over modulation





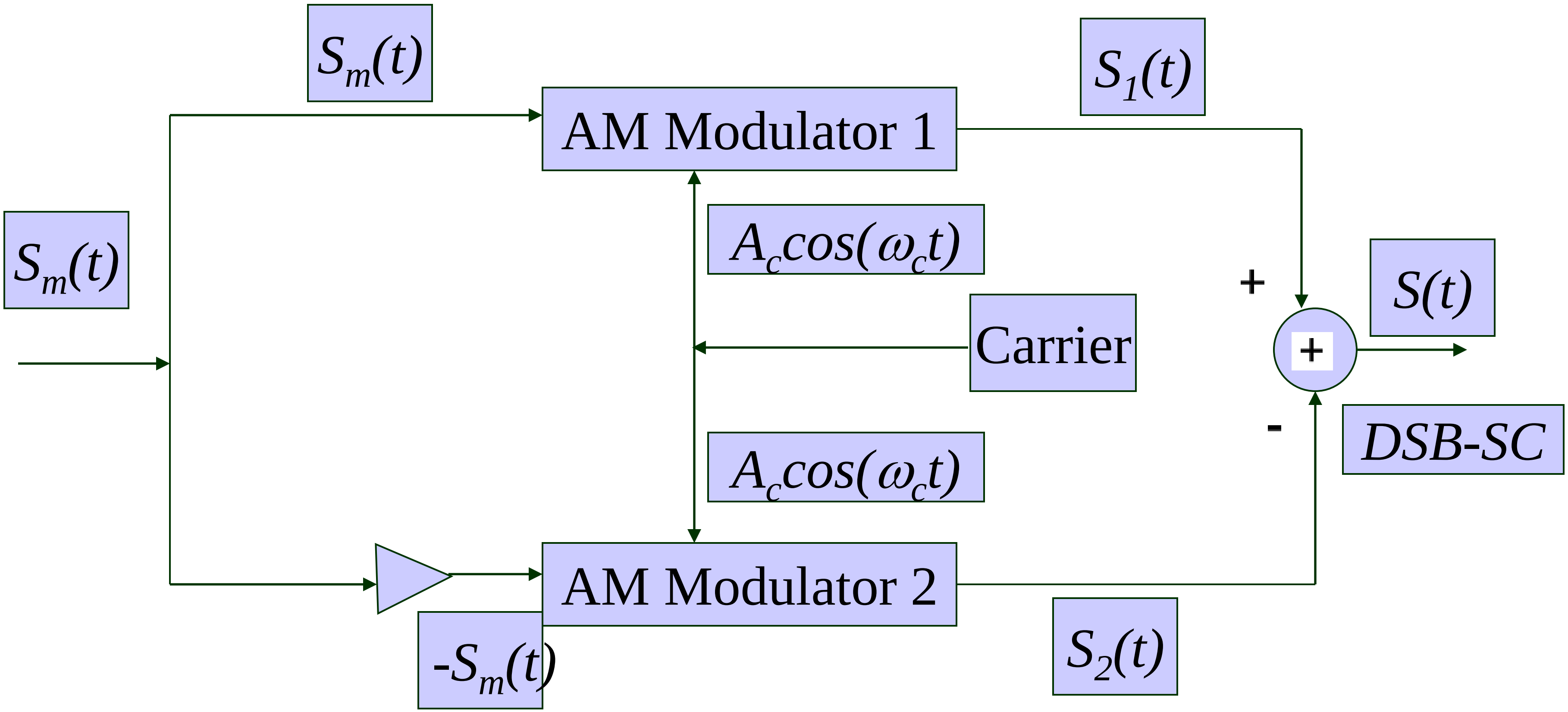
Selection of RC time Constant:

$$\boxed{\frac{1}{f_m} << RC << \frac{1}{f_c}}$$

To avoid distortion present in envelope detector, RC time Constant should be as high as $1/f_c$ & as low as $1/f_m$

DSB-SC Generation

BALANCED MODULATOR



DSB-SC Generation

- The two modulators are identical except for the sign reversal of the input to one of them. Thus,

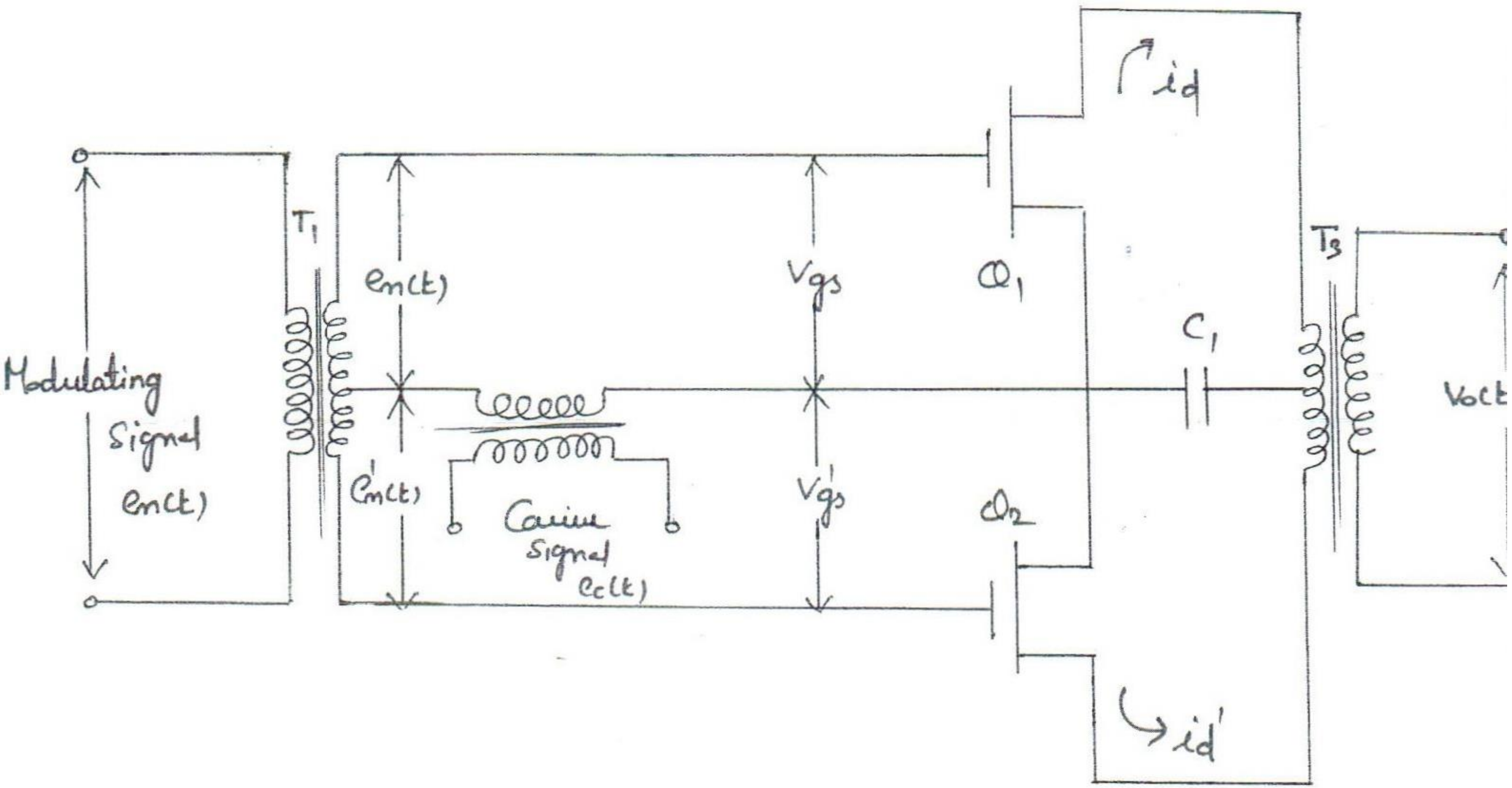
$$s_1(t) = A_c(1 + m\cos(\omega_m t))\cos(\omega_c t)$$

$$s_2(t) = A_c(1 - m\cos(\omega_m t))\cos(\omega_c t)$$

$$\begin{aligned} s(t) &= s_1(t) - s_2(t) \\ &= 2mA_c \cos(\omega_m t)\cos(\omega_c t) \end{aligned}$$

DSB-SC Generation

Balanced Modulator using FET



DSB-SC Generation

Balanced Modulator using FET

The voltage across the FET gates can be written as,

$$V_{gs} = e_c(t) + e_m(t)$$

$$= E_c \sin \omega_c t + E_m \sin \omega_m t \quad \text{—— (1)}$$

$$V_{gs}' = E_c \sin \omega_c t - E_m \sin \omega_m t \quad \text{—— (2)}$$

The drain currents of FET can be written as,

$$i_d = a_1 V_{gs} + a_2 V_{gs}^2 \quad \text{—— (3)}$$

$$i_d' = a_1 V_{gs}' + a_2 V_{gs}'^2 \quad \text{—— (4)}$$

DSB-SC Generation

Balanced Modulator using FET

Now, the o/p of DSBSC Signal $V_o(t)$ is,

$$V_o(t) = k\,(i_d - i_d') \quad \text{——} \quad (5)$$

Sub equ (3) & (4) in equ (5)

we get, $V_o(t) = k\left[(a_1 v_{gs} + a_2 v_{gs}^2) - (a_1 v_{gs}' + a_2 v_{gs}'^2)\right]$

$$V_o(t) = k\left[a_1 (v_{gs} - v_{gs}') + a_2 (v_{gs}^2 - v_{gs}'^2)\right]$$

$$= k\left[a_1 (E_c \sin\omega_c t + E_m \sin\omega_m t - E_c \sin\omega_c t + E_m \sin\omega_m t)\right]$$

$$+ a_2\left[(E_c \sin\omega_c t + E_m \sin\omega_m t)^2 - (E_c \sin\omega_c t - E_m \sin\omega_m t)^2\right]$$

DSB-SC Generation

Balanced Modulator using FET

$$= k\left[2a_1 E_m \sin \omega_m t + a_2 \left(4 E_m E_c \sin \omega_m t \sin \omega_c t\right)\right]$$

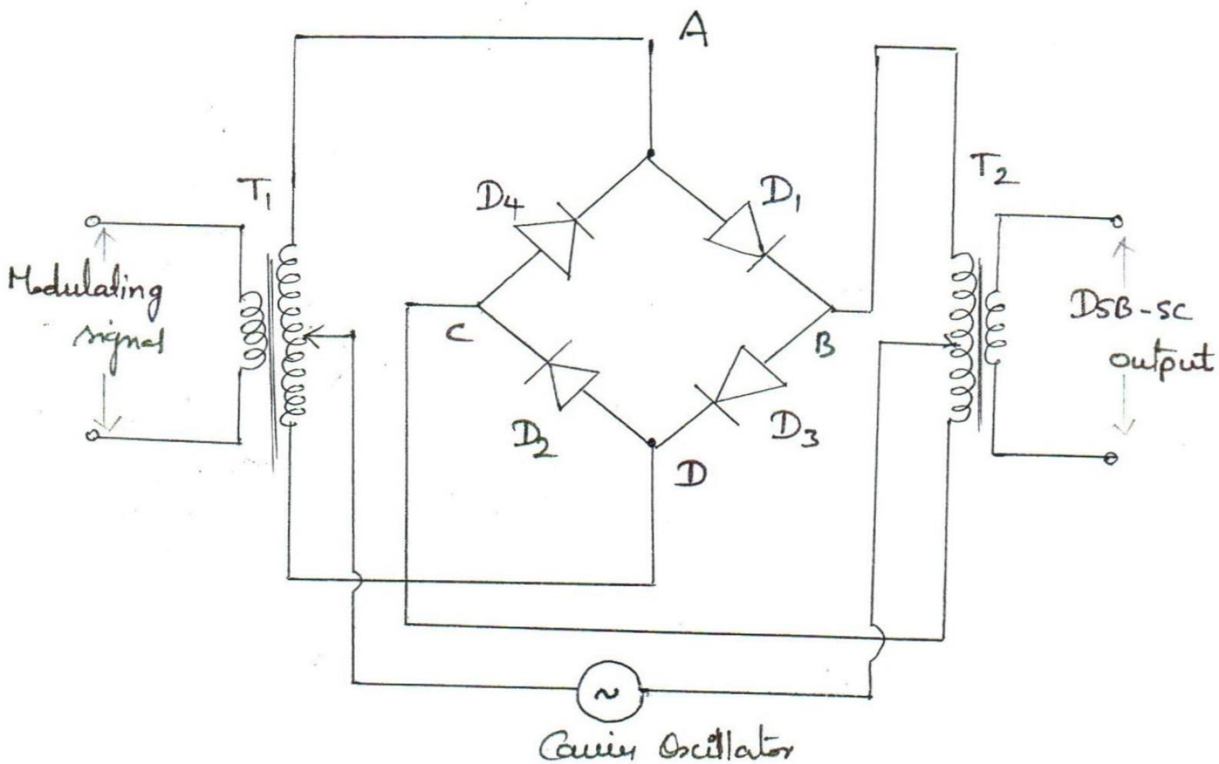
$$= 2ka_1 E_m \sin \omega_m t \left[1 + \frac{2a_2 E_c}{a_1} \cdot \frac{\sin \omega_c t}{\sin \omega_m t} \cdot \sin \omega_m t\right]$$

$$\therefore V_o(t) = 2ka_1 E_m \sin \omega_m t \left[1 + \frac{2a_2 E_c}{a_1} \sin \omega_c t\right] \quad \text{—— (b)}$$

This is output, $V_o(t)$ is applied to the Bandpass filter whose centre frequency is $\pm \omega_c$. Then we will get the output as two side band signals.

DSB-SC Generation

Ring Modulator / Diode bridge modulator



A ring modulator using four diodes are shown in fig.
In a ring modulator circuit four diodes are connected in the form of a ring in which all four diodes point in the same manner. All the four diodes in the ring are controlled by a square wave carrier signal applied through a center-tapped transformer.

DSB-SC Generation

Ring Modulator / Diode bridge modulator

Circuit Operation : The Circuit Operation Can be explained for two different Conditions,

1) With only the Carrier signal applied to the Circuit without the message Signal

2) With a Sinusoidal message signal applied to the Circuit along with the Carrier signal.

Case 1

i) For (+ve) half-cycle of the Carrier signal } Diodes D_1 & D_2 are fwd biased & They Conduct Produces the o/p current.

DSB-SC Generation

Ring Modulator / Diode bridge modulator

(ii) For -ve-half Cycle of the Carrier signal } Diodes D_1 & D_2 are Reverse biased & the diodes D_3 & D_4 are fwd biased.

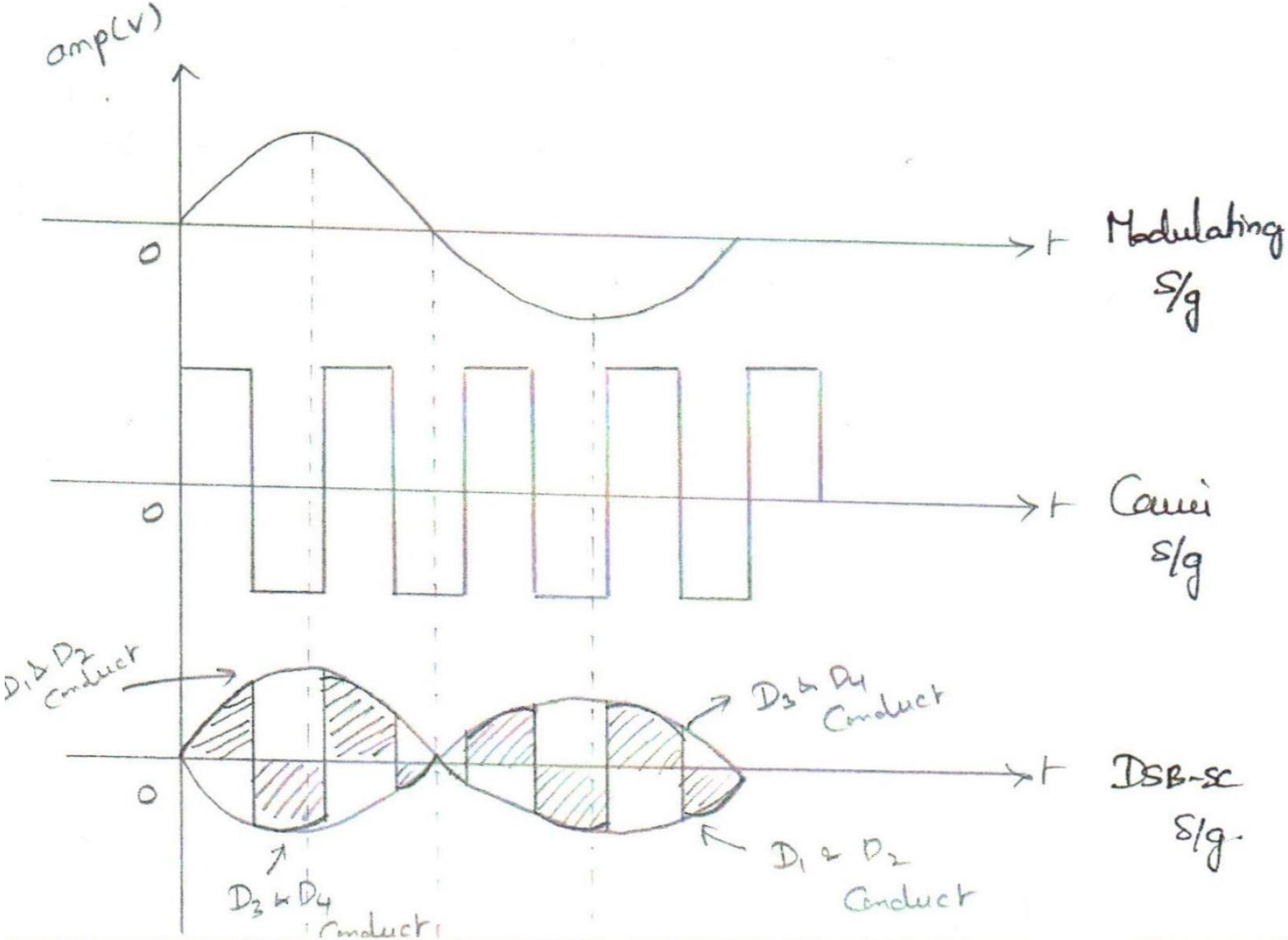
Case 2

i). For (+ve)-half Cycle of the modulating signal } Diodes D_1 & D_2 are fwd biased & they will Connect the secondary of T_1 to the primary of T_2.

(ii). During negative half-cycle ⇒ Diodes D_3 & D_4 are fwd biased and they will Connect the secondary of T_1 to the Primary of T_2 with reverse Connection.

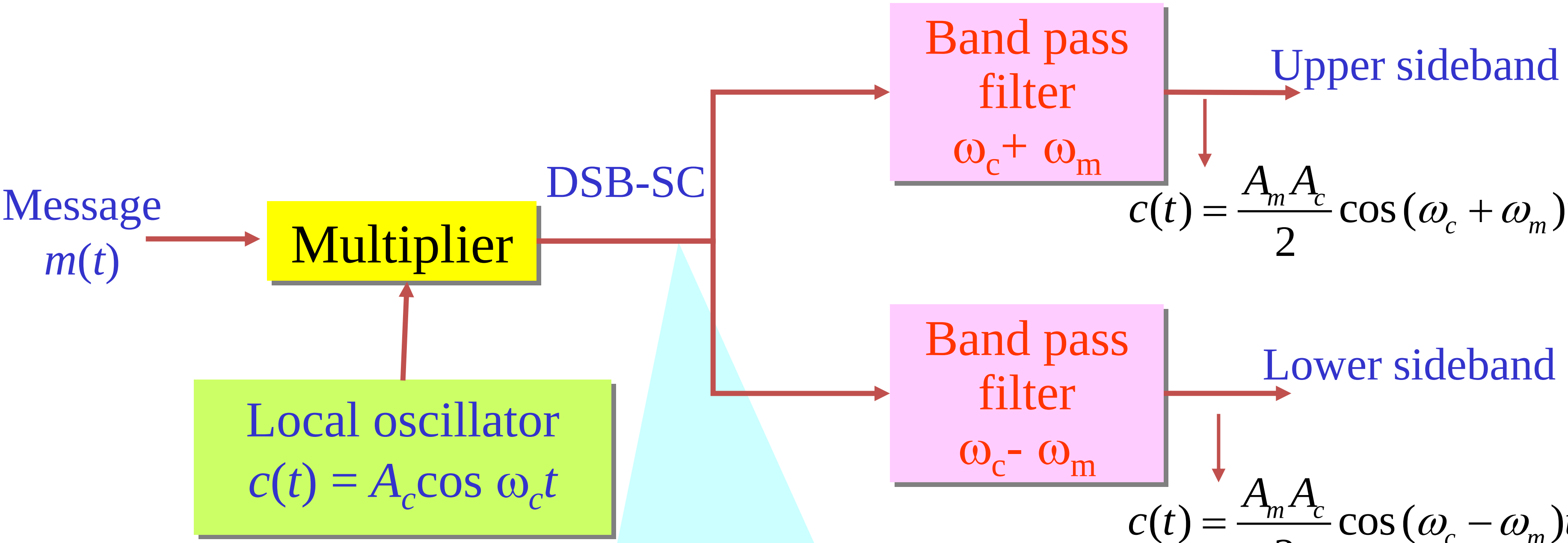
DSB-SC Generation

Ring Modulator / Diode bridge modulator



SSB Generation

Filter Method

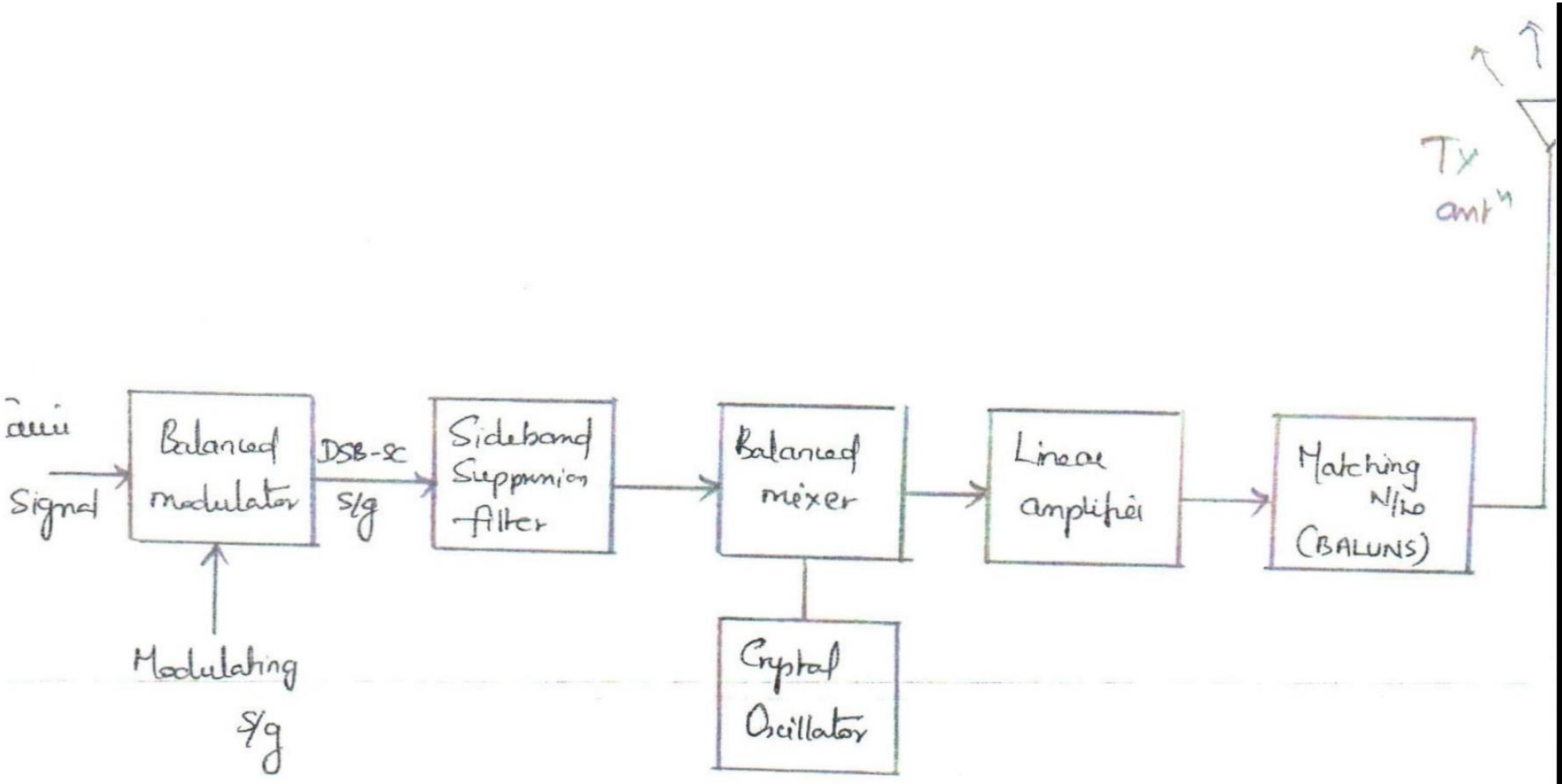


$$c(t) = A_m \cos \omega_m t \;\; A_c \cos \omega_c t$$

$$= \frac{A_m A_c}{2} \cos(\omega_c + \omega_m)t + \frac{A_m A_c}{2} \cos(\omega_c - \omega_m)t$$

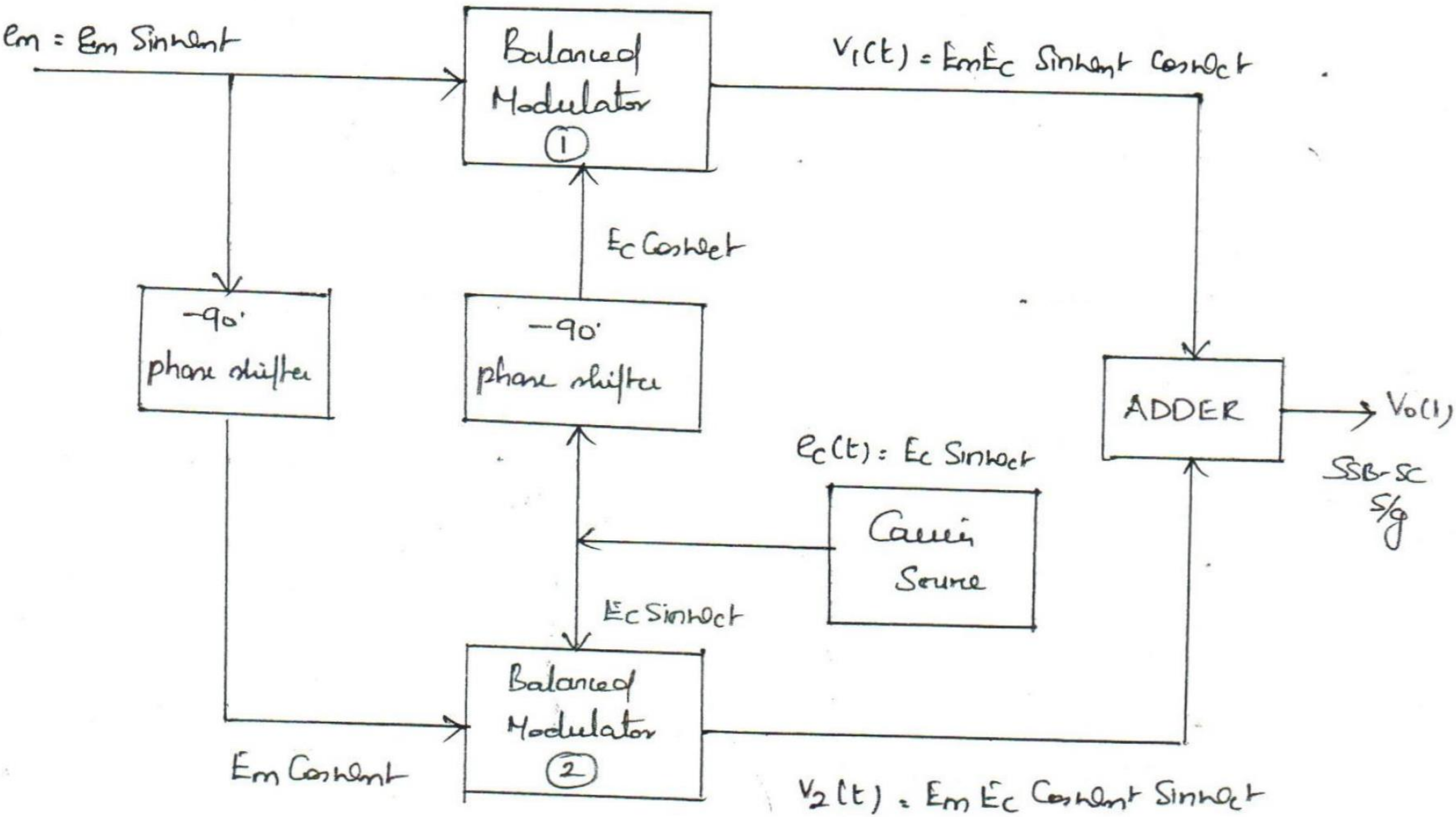
SSB Generation

Filter Method



SSB Generation

Phase Shift Method / Hartley Method



SSB Generation

Phase Shift Method / Hartley Method

Output of adder is,

$$V_o(t) = V_1(t) + V_2(t)$$

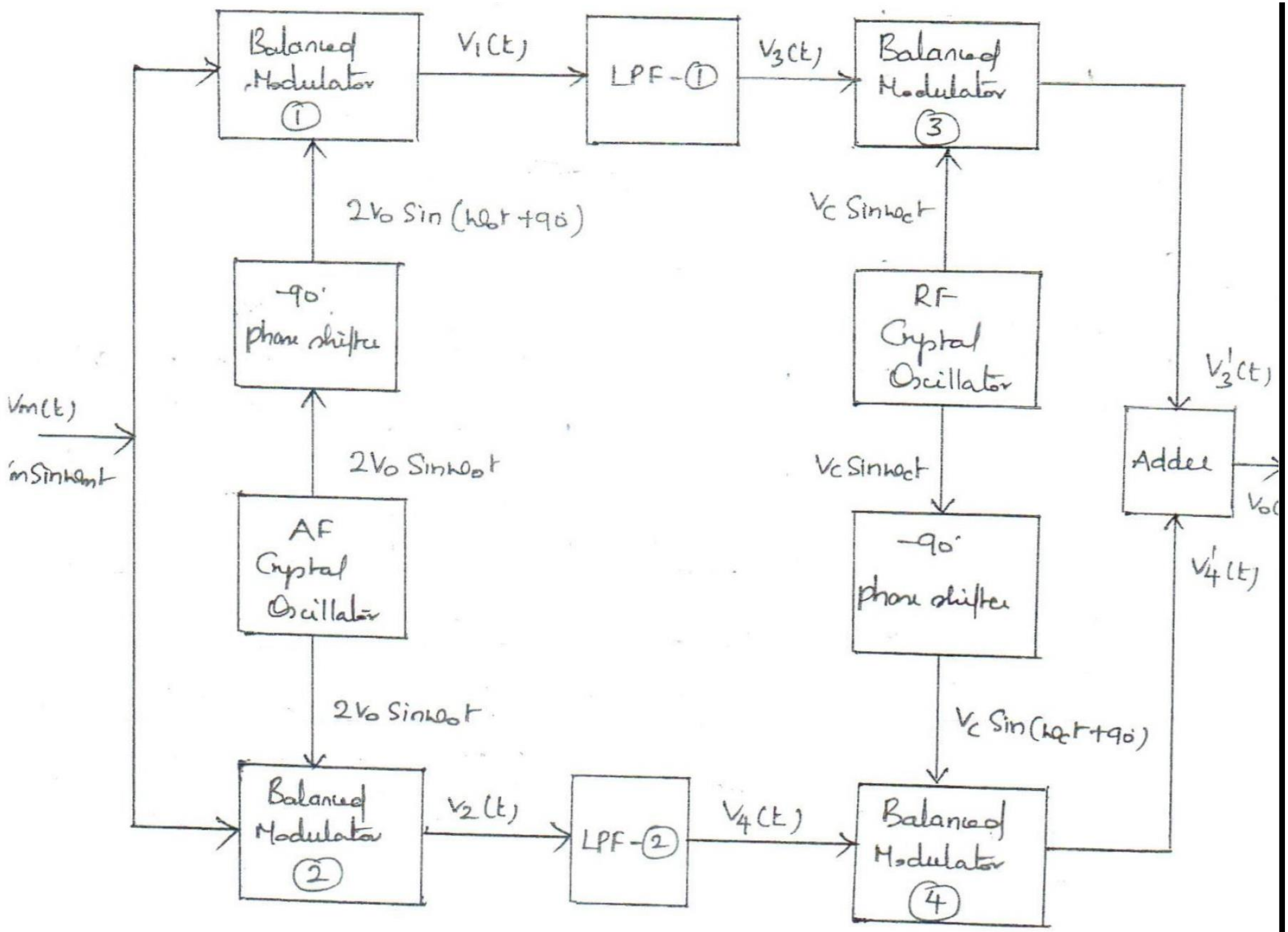
$$V_o(t) = E_m E_c \sin\omega_m t \cos\omega_c t + E_m E_c \cos\omega_m t \sin\omega_c t$$

$$= E_m E_c \left[\sin\omega_m t \cos\omega_c t + \cos\omega_m t \sin\omega_c t \right]$$

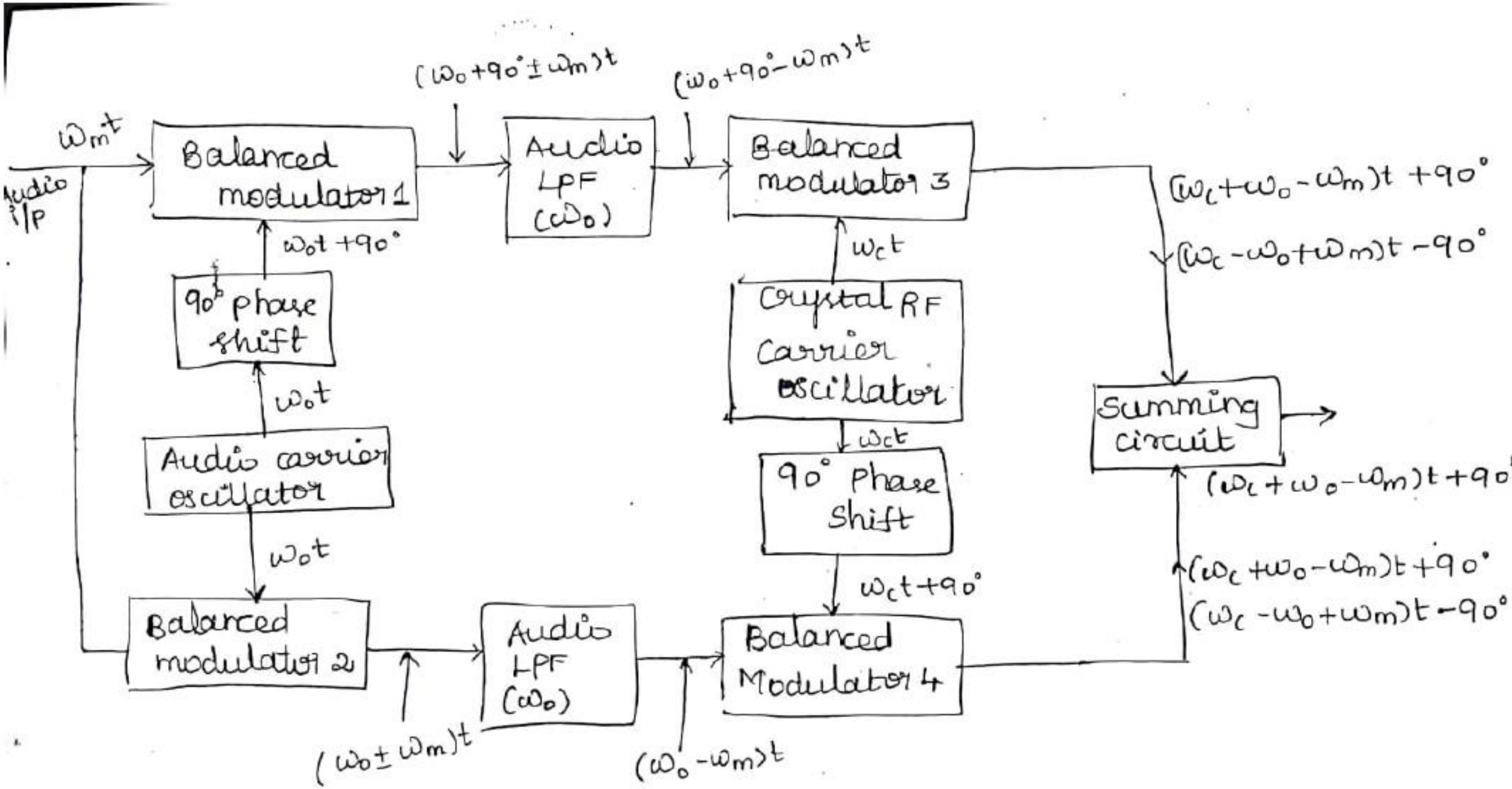
$$V_o(t) = E_m E_c \left[\sin(\omega_c + \omega_m) t \right] //$$

SSB Generation

Weavers Method or Modified Phase Shift Method or Third Method



SSB Generation



Output of SSB signal:

The modulating s/g $\quad V_m(t) = V_m \sin\omega_m t \longrightarrow (1)$

AF carrier s/g $\quad V_o(t) = 2V_o \sin\omega_o t \longrightarrow (2)$

R.F carrier s/g $\quad V_c(t) = 2V_c \sin\omega_c t \longrightarrow (3)$

SSB Generation

∴ Output of Balanced modulator 1 :-

$$= 2 V_0 \sin(\omega_0 t + 90) V_m \sin \omega_m t$$

$$= V_m V_0 [\cos((\omega_0 t - \omega_m t) + 90) - \cos[(\omega_0 t + \omega_m t) + 90)]$$

Output of Balanced modulator 2 :-

$$= 2 V_0 \sin \omega_0 t \, V_m \sin \omega_m t$$

$$= V_m V_0 [\cos(\omega_0 t - \omega_m t) - \cos(\omega_0 t + \omega_m t)] \rightarrow (5)$$

The low pass filters in the BM1 & BM2 eliminates the upper sidebands of modulator.

output of LPF_1 is $V_m V_0 \cos((\omega_0 t - \omega_m t) + 90)$

Output of LPF_2 is $V_m V_0 \cos(\omega_0 - \omega_m)t$.

$\boxed{\text{Assume } V_m = V_0 = V_c = 1}$

Output of Balanced modulator 3:-

$$= 2 \sin \omega_c t \cos(\omega_0 t - \omega_m t + 90) \rightarrow (6)$$

It is in the form of $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$$\therefore \sin[(\omega_c + \omega_0 - \omega_m)t + 90] + \sin[(\omega_c - \omega_0 + \omega_m)t + 90] \rightarrow (7)$$

SSB Generation

Output of Balanced modulator 4:-

$$= 2\sin(\omega_c t + 90)\cos(\omega_0 - \omega_m)t$$

$$= \sin[(\omega_c + \omega_0 - \omega_m)t + 90] + \sin[(\omega_c - \omega_0 + \omega_m)t + 90] \rightarrow 8$$

From eq 7 & 8, the output of summer circuit is,

$$V_0 = \sin[(\omega_c + \omega_0 - \omega_m)t + 90] + \sin[(\omega_c - \omega_0 + \omega_m)t - 90] +$$
$$\sin[(\omega_c + \omega_0 - \omega_m)t + 90] + \sin[(\omega_c - \omega_0 + \omega_m)t + 90$$

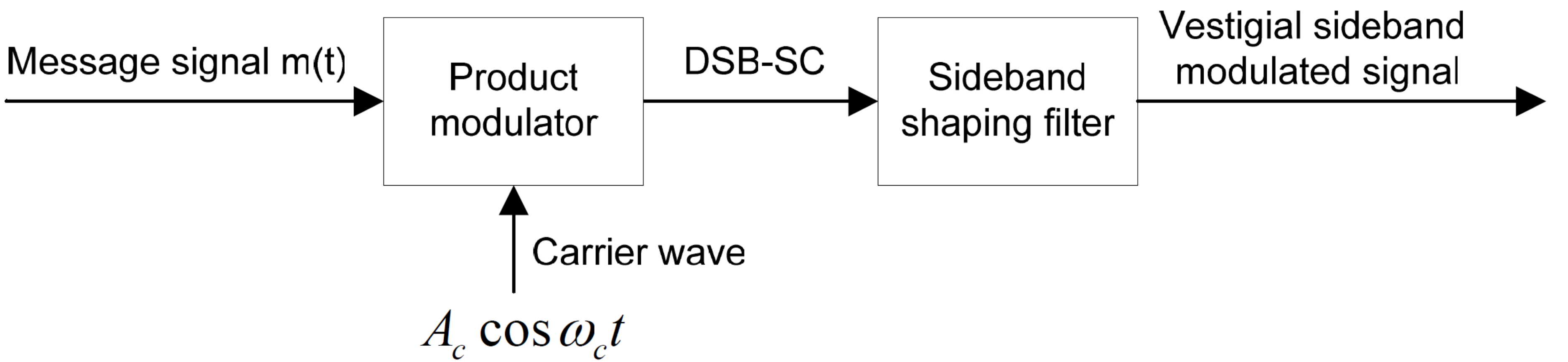
$$= 2\sin[(\omega_c + \omega_0 - \omega_m)t + 90] \longrightarrow 9$$

$$\boxed{V_0 = 2\cos(\omega_c + \omega_0 - \omega_m)t.}$$

The final RF output frequency is $f_c + f_0 - f_m$ which is essentially the lower sideband of RF carrier $f_c + f_0$.

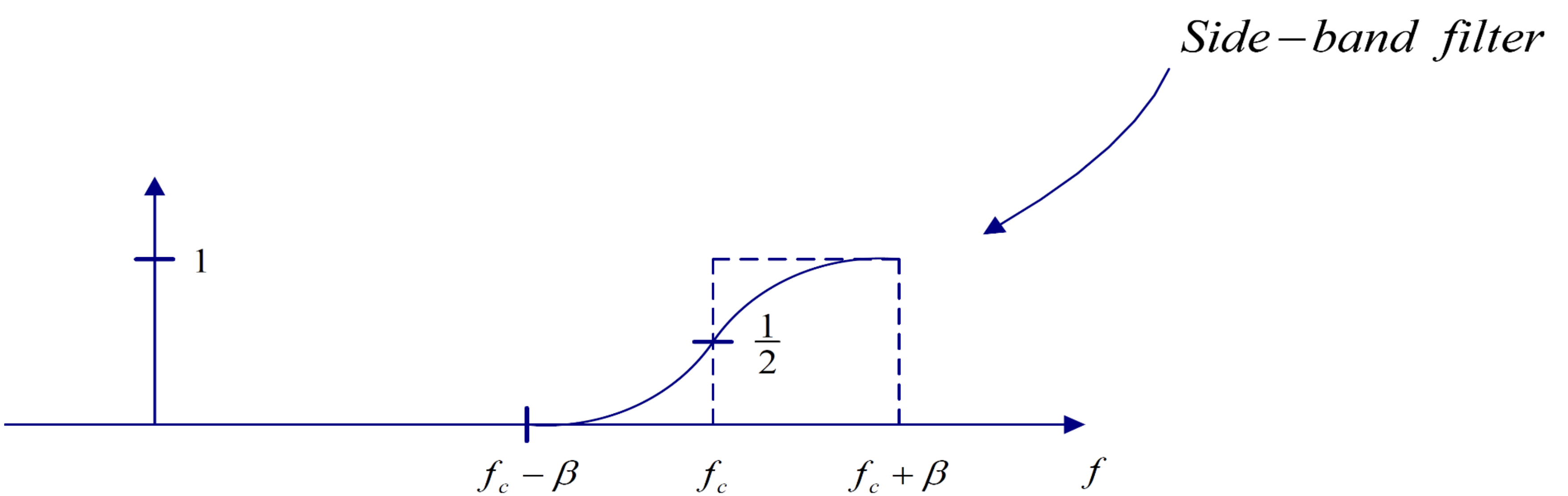
VSB Generation

Filter Method

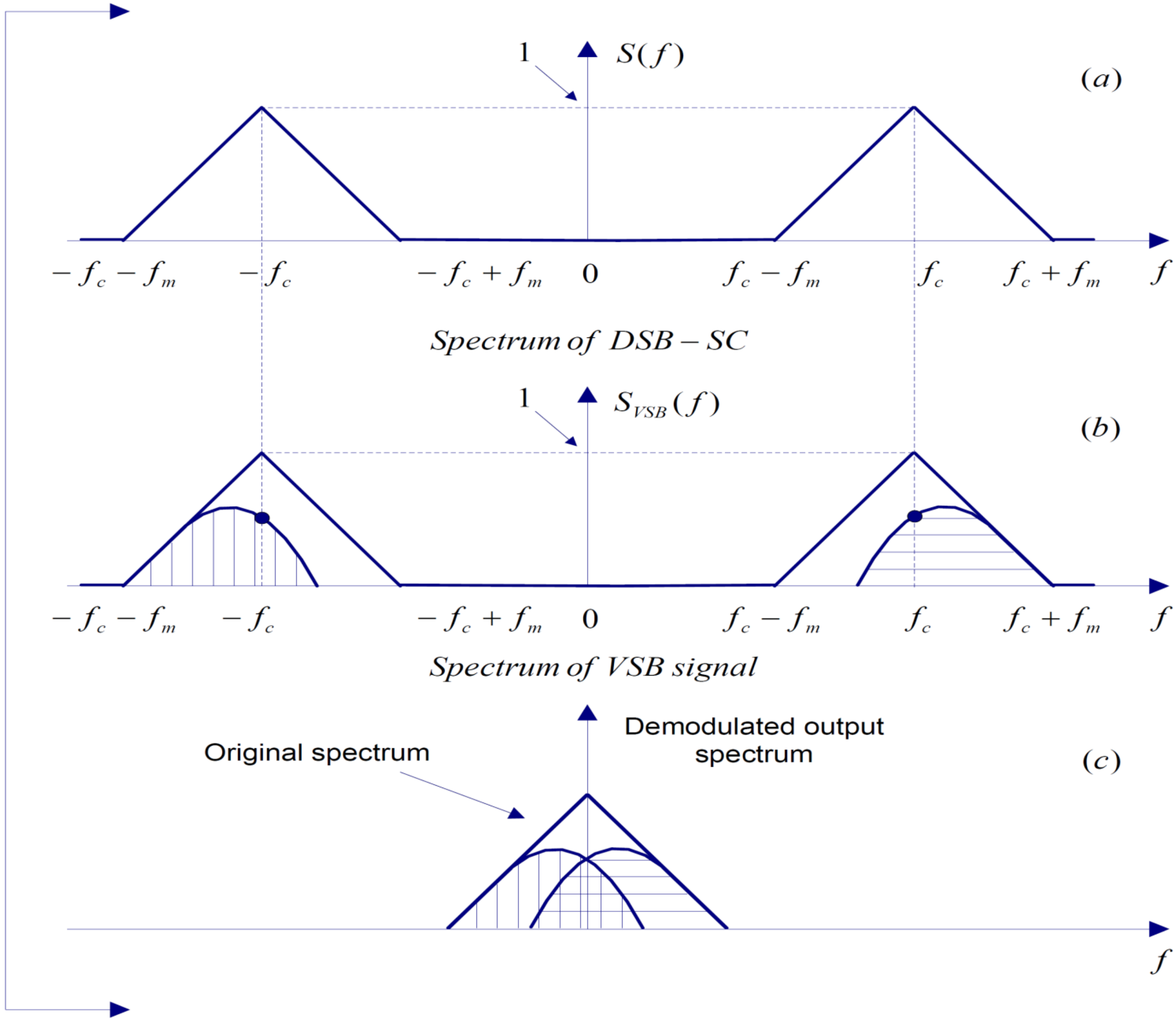


In VSB modulation, one sideband is passed almost completely whereas just a **trace** or vestige, of the other sideband is retained.

The key to VSB is the sideband filter, a typical transfer function being shown below:



VSB



1
$S(f)$
(a)
$-f_c - f_m$
$-f_c$
$-f_c + f_m$
0
$f_c - f_m$
f_c
$f_c + f_m$
f
Spectrum of DSB – SC
1
$S_{VSB}(f)$
(b)
$-f_c - f_m$
$-f_c$
$-f_c + f_m$
0
$f_c - f_m$
f_c
$f_c + f_m$
f
Spectrum of VSB signal
Original spectrum
Demodulated output spectrum
(c)
f

VSB

In this method, first we will generate DSBSC wave with the help of the product modulator. Then, apply this DSBSC wave as an input of sideband shaping filter. This filter produces an output, which is VSBSC wave.

The modulating signal $m(t)$ and carrier signal $A_c \cos(2\pi f_c t)$ are applied as inputs to the product modulator. Hence, the product modulator produces an output, which is the product of these two inputs.

Therefore, the output of the product modulator is

$$p(t) = A_c \cos(2\pi f_c t) m(t)$$

Apply Fourier transform on both sides

$$P(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

The above equation represents the equation of DSBSC frequency spectrum.

VSB

Let the transfer function of the sideband shaping filter be $H(f)$. This filter has the input $p(t)$ and the output is VSBSC modulated wave $s(t)$. The Fourier transforms of $p(t)$ and $s(t)$ are $P(t)$ and $S(t)$ respectively.

Mathematically, we can write $S(f)$ as

$$S(f) = P(f)\,H(f)$$

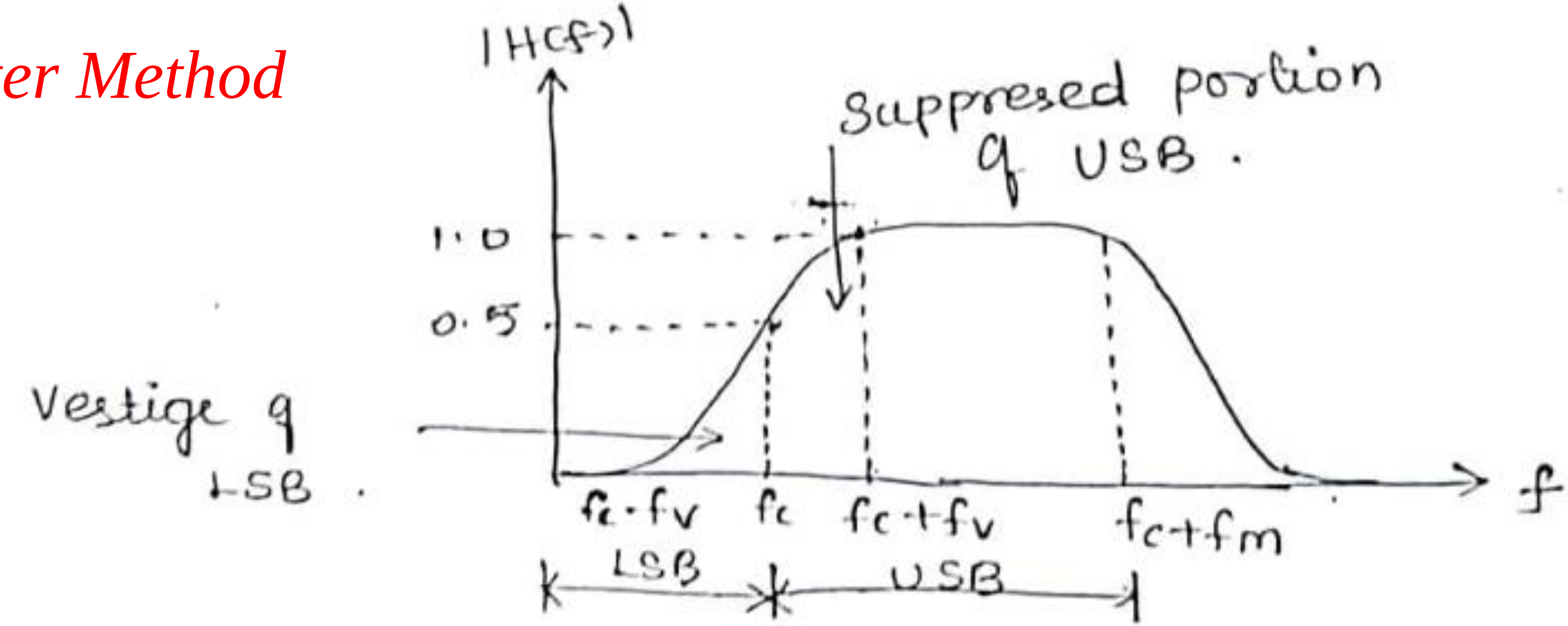
Substitute $P(f)$ value in the above equation.

$$S(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right] H(f)$$

The above equation represents the equation of VSBSC frequency spectrum.

VSB Generation

Filter Method



The portion from fc to fc+fm is USB. The portion from fc to fc+fv is suppressed partially.

The portion from fc to fc−fm is LSB. The portion from fc−fv to fc is to be transmitted as vestige.

<u>Transmission Bandwidth :-</u>

The transmission Bandwidth of VSB modulation is

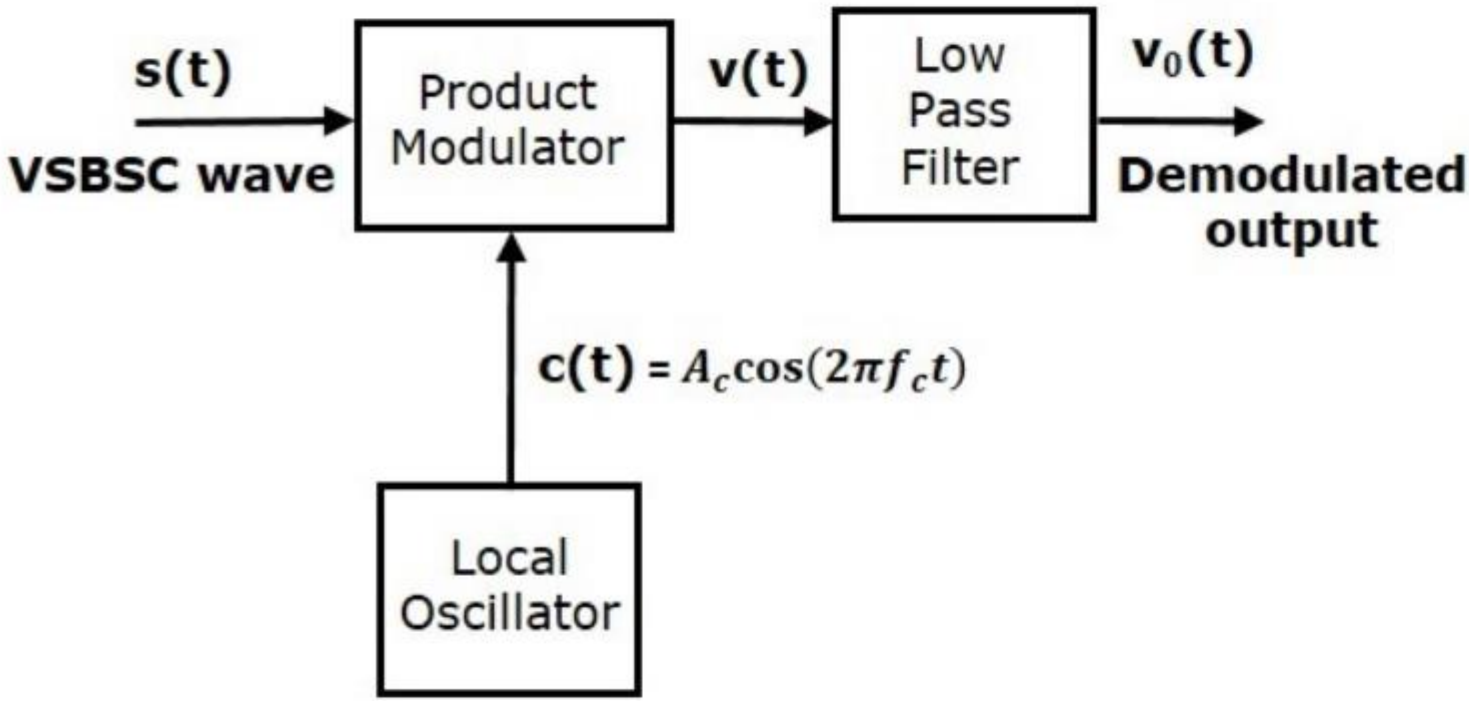
$B.T = f_m + f_v \rightarrow (1)$

f_v → width of vestigial sideband

f_m → message bandwidth.

Demodulation of VSB

Demodulation of VSBSC wave is similar to the demodulation of SSBSC wave. Here, the same carrier signal (which is used for generating VSBSC wave) is used to detect the message signal. Hence, this process of detection is called as **coherent** or **synchronous detection**. The VSBSC demodulator is shown in the following figure.



In this process, the message signal can be extracted from VSBSC wave by multiplying it with a carrier, which is having the same frequency and the phase of the carrier used in VSBSC modulation. The resulting signal is then passed through a Low Pass Filter. The output of this filter is the desired message signal.

Demodulation of VSB

Let the VSBSC wave be $s(t)$ and the carrier signal is $A_c \cos(2\pi f_c t)$.

From the figure, we can write the output of the product modulator as

$$v(t) = A_c \cos(2\pi f_c t) s(t)$$

Apply Fourier transform on both sides

$$V(f) = \frac{A_c}{2}[S(f - f_c) + S(f + f_c)]$$

We know that $S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)] H(f)$

From the above equation, let us find $S(f - f_c)$ and $S(f + f_c)$.

$$S(f - f_c) = \frac{A_c}{2}[M(f - f_c - f_c) + M(f - f_c + f_c)] H(f - f_c)$$

Demodulation of VSB

$$\Rightarrow S(f-f_c) = \frac{A_c}{2}[M(f-2f_c)+M(f)]H(f-f_c)$$

$$S(f+f_c) = \frac{A_c}{2}[M(f+f_c-f_c)+M(f+f_c+f_c)]H(f+f_c)$$

$$\Rightarrow S(f+f_c) = \frac{A_c}{2}[M(f)+M(f+2f_c)]H(f+f_c)$$

Substitute, $S(f-f_c)$ and $S(f+f_c)$ values in $V(f)$.

$$V(f) = \frac{A_c}{2}[\frac{A_c}{2}[M(f-2f_c)+M(f)]H(f-f_c)+$$

$$\frac{A_c}{2}[M(f)+M(f+2f_c)]H(f+f_c)]$$

$$\Rightarrow V(f) = \frac{{A_c}^2}{4}M(f)[H(f-f_c)+H(f+f_c)]$$

$$+\frac{{A_c}^2}{4}[M(f-2f_c)H(f-f_c)+M(f+2f_c)H(f+f_c)]$$

In the above equation, the first term represents the scaled version of the desired message signal frequency spectrum. It can be extracted by passing the above signal through a low pass filter.

$$V_0(f) = \frac{{A_c}^2}{4}M(f)[H(f-f_c)+H(f+f_c)]$$

Example of VSB

Example: Let the message signal be the sum of tow sinusoids.

$$m(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

The message signal is multiplied by a carrier $\cos \omega_c t$ to form the DSB signal.

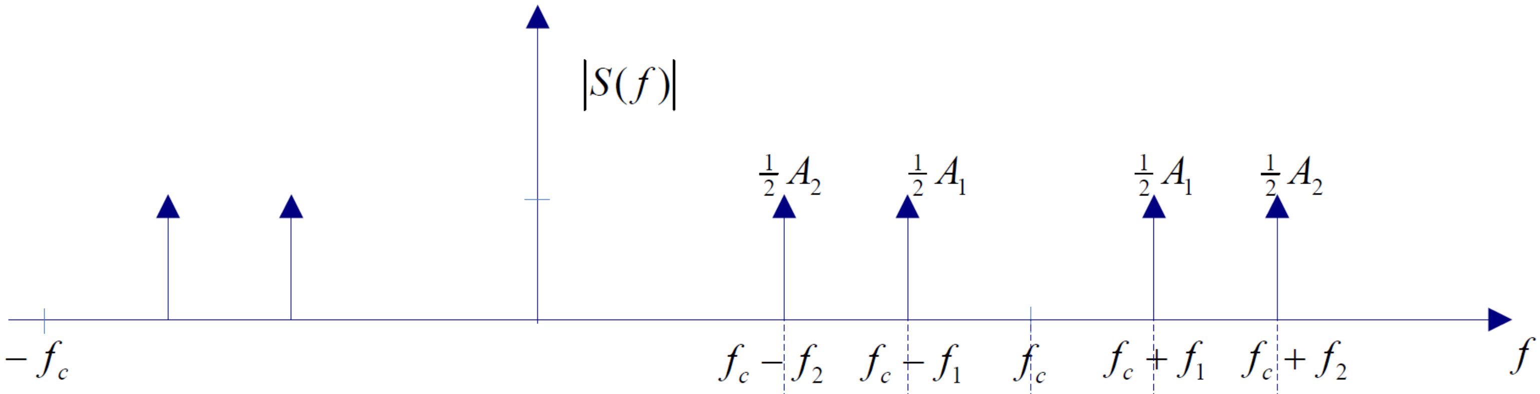
DSB-SC

$$s(t) = (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) \cos \omega_c t$$

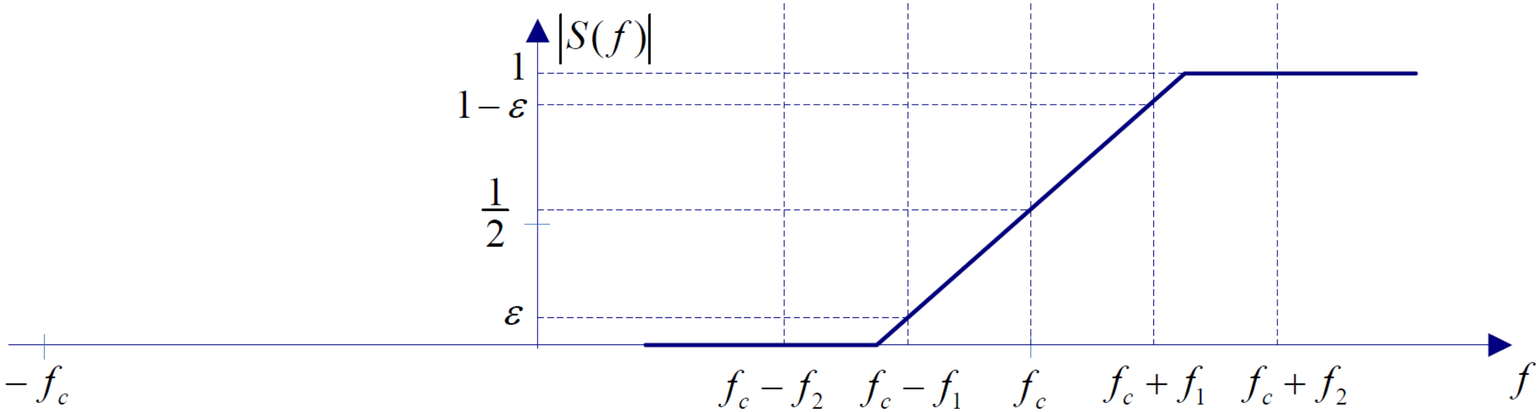
$$= \frac{1}{2} A_1 \cos(\omega_c - \omega_1)t + \frac{1}{2} A_2 \cos(\omega_c - \omega_2)t + \frac{1}{2} A_2 \cos(\omega_c + \omega_2)t + \frac{1}{2} A_1 \cos(\omega_c + \omega_1)t$$

Single sideband spectrum is shown below.

Example of VSB

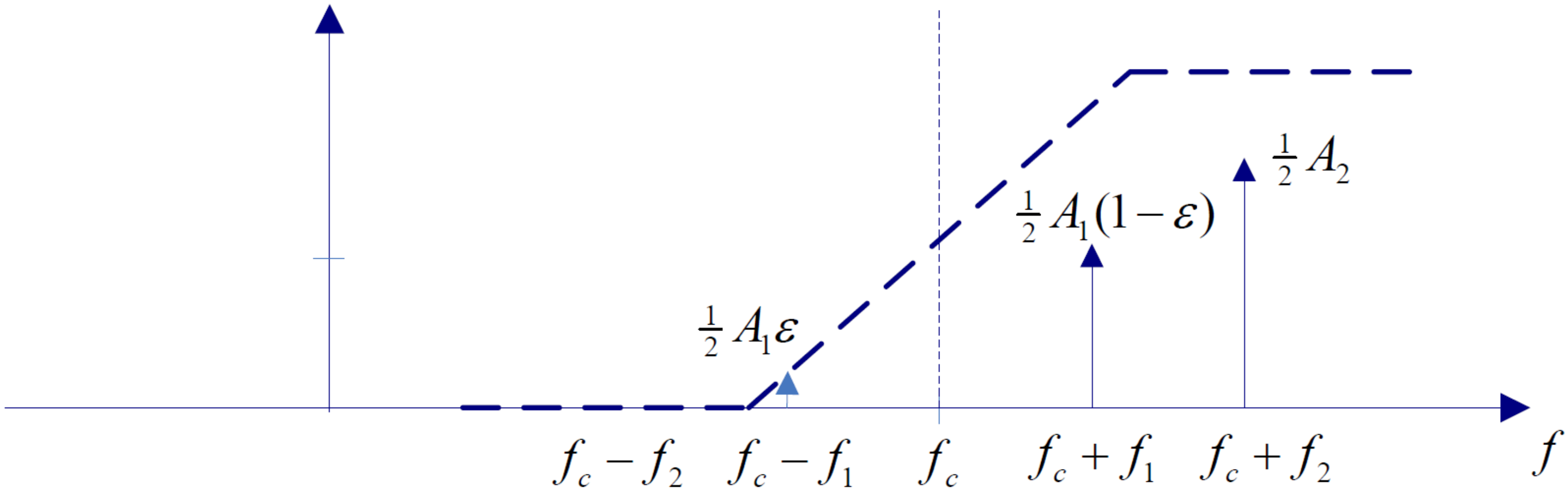


A vestigial sideband filter is then used to generate the VSB signal.



A vestigial sideband filter is then used to generate the VSB signal.

Example of VSB



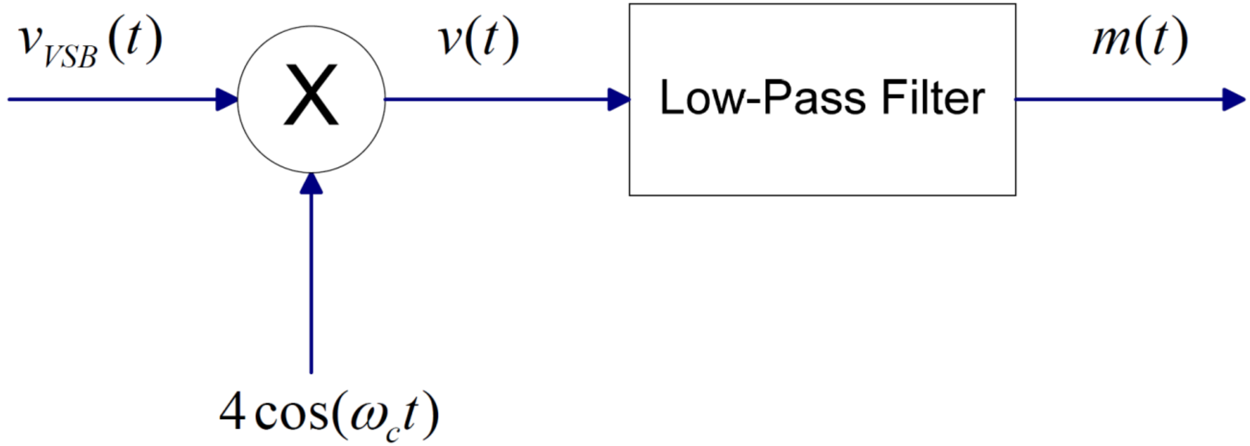
The spectrum of the VSB signal is

$$VSB_{spectrum} = |S(f)|.|H(f)|$$

$$v_{VSB}(t) = \frac{1}{2}A_1\varepsilon\cos(\omega_c - \omega_1)t + \frac{1}{2}A_1(1-\varepsilon)\cos(\omega_c + \omega_1)t + \frac{1}{2}A_2\cos(\omega_c + \omega_2)t$$

Example of VSB

Demodulation:



$$v(t) = v_{VSB}(t).4\cos\omega_c t =$$

$$2A_1\varepsilon\cos(\omega_c - \omega_1)t.\cos\omega_c t + 2A_1(1-\varepsilon)\cos(\omega_c + \omega_1)t.\cos\omega_c t + 2A_2\varepsilon\cos(\omega_c + \omega_2)t.\cos\omega_c t$$

$$= A_1\varepsilon[\cos(2\omega_c - \omega_1)t + \cos\omega_1 t] + A_1(1-\varepsilon)[\cos(2\omega_c + \omega_1)t + \cos\omega_1 t] + A_2[\cos(2\omega_c + \omega_2)t + \cos\omega_2 t]$$

After low-pass filtering,

$$m(t) = A_1\varepsilon\cos\omega_1 t + A_1(1-\varepsilon)\cos\omega_1 t + A_2\cos\omega_2 t$$

$\Rightarrow \boxed{m(t) = A_1\cos\omega_1 t + A_2\cos\omega_2 t}$, which is the assumed message signal.

Hilbert Transform

Hilbert transform of a signal $x(t)$ represented as,

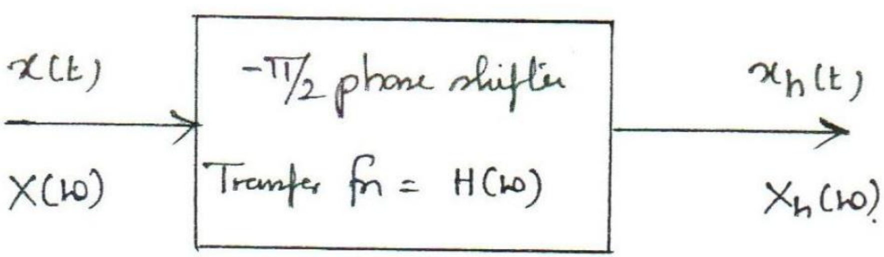
$x_h(t)$ or $H(x(t))$.

It's used to provide $-\pi/2$ or -90° phase shift for every component present in the signal $x(t)$.

Mathematically,

$$H[x(t)] = x_h(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\cdot d\tau$$

$$x_h(t) = \frac{1}{\pi}\cdot x(t)\cdot\frac{1}{t}$$



phase shifting S/y.

Hilbert Transform

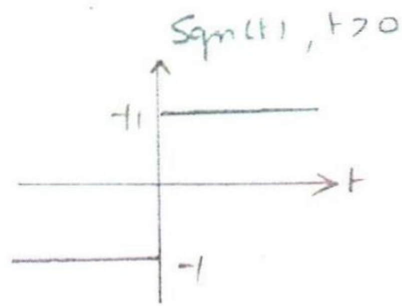
Characteristics of phase shifting S/y:

1. Magnitude of the freq. Components present in $x(t)$ remains unchange, when it is passed through the S/y. (ie) $H(\omega) = 1$

2. The phase of the (+ve) freq. Compⁿ is shifted by $-\pi/2$ (or $-90°$).

Hence, $H(\omega) = -j \cancel{X(\omega)}\, \mathrm{Sgn}(\omega)$.

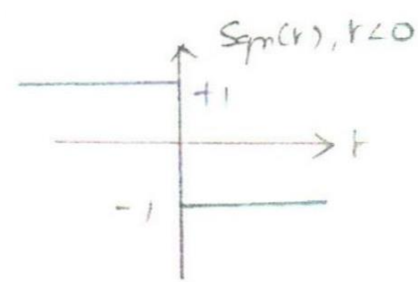
$$X_h(\omega) = X(\omega) \cdot H(\omega) \quad \text{--- (1)}$$



$$H(\omega) = -j\, \mathrm{Sgn}(\omega) \quad \text{--- (2)}$$

Sub equ (2) in (1)

$$X_h(\omega) = -j\, X(\omega) \cdot \mathrm{Sing}(\omega) \quad \text{--- (3)}$$



Hilbert Transform

Derivation of $H(\omega)$:

$$H(\omega) = e^{j\theta(\omega)} \quad \text{--- (4)} \qquad \because H(\omega) = |H(\omega)| \cdot e^{j\theta(\omega)}$$

$$= 1 \cdot e^{j\theta(\omega)}$$

$$\theta(\omega) = \begin{cases} \pi/2 & \text{for } \omega < 0 \\ -\pi/2 & \text{for } \omega > 0 \end{cases}$$

hence, $$H(\omega) = \begin{cases} e^{j\pi/2}, & \omega < 0 \\ e^{-j\pi/2}, & \omega > 0 \end{cases} \quad \text{--- (5)}$$

Sub eqn (5) in (4)

$$e^{j\pi/2} = \cos\pi/2 + j\sin\pi/2 = j$$

$$e^{-j\pi/2} = \cos\pi/2 - j\sin\pi/2 = -j$$

Hilbert Transform

$$\therefore \quad H(\omega) = \begin{cases} j, & \omega < 0 \\ -j, & \omega > 0 \end{cases}$$

$$\frac{H(\omega)}{j} = \begin{cases} 1, & \omega < 0 \\ -1, & \omega > 0 \end{cases}$$

$$\therefore \quad H(\omega) = -j \, \mathrm{Sgn}(\omega)$$

$$\therefore \quad x_h(\omega) = -j \, x(\omega) \, . \, \mathrm{Sgn}(\omega) \, .$$

Taking Inverse Fourier transform

$$F^{-1}\left[x_h(\omega)\right] = x_h(t) = F^{-1}\left[-j x(\omega) \, \mathrm{Sgn}(\omega)\right]$$

$$\boxed{x_h(t) = x(t) \, . \, \frac{1}{\pi t}}$$

Hilbert Transform - Properties

(1). A S/g $x(t)$ & its hilbert transform $x_h(t)$ have the same amplitude spectrum and same auto correlation fn.

(2). $x(t)$ & $x_h(t)$ are orthogonal

(ie) $\int_{-\alpha}^{\alpha} x(t)\, x_h(t) = 0.$ [dot product of i/p × o/p is zero

(3). Hilbert transform of $x_h(t)$ is $-x(t)$

Pre-envelope or Analytic Signal

* Useful in deriving the general exp^r of the SSB-SC S/g.

The preenvelope of real valued s/g $x(t)$ is defined as.

$$x_p(t) = x(t) + j\,x_h(t).$$

Obviously, the pre-envelop $x_p(t)$ is a Complex Valued S/g

The real part of $x_p(t)$ is $x(t)$ and the imaginary part is its hilbert transform $x_h(t)$.

Pre-envelope or Analytic Signal

The use of phasors simplifies manipulation of ac currents and voltages, ||ly pre-envelope simplifies the handling of bandpass signals and system. pre-envelope of the signal g(t) is defined as,

$$\boxed{g_+(t) = g(t) + j\hat{g}(t)}$$

$\hat{g}(t)$ → H.T of g(t)

$g(t)$ → real part of pre-envelope

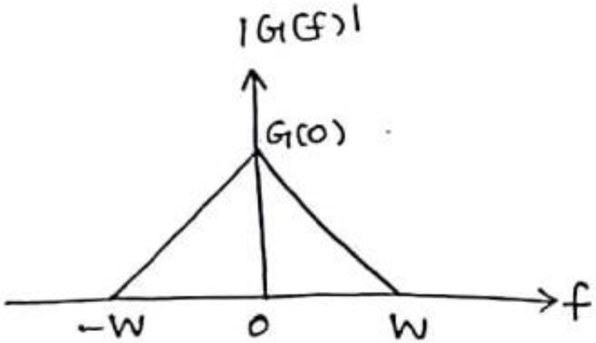
$g_+(t)$ → pre-envelope of s/g g(t).

Pre-envelope or Analytic Signal

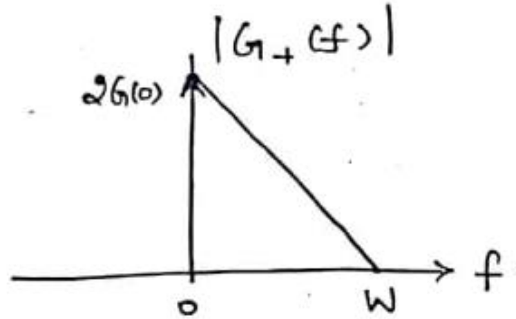
∴ consider the spectrum of pre-envelope,

$$G_+(f) = G(f) + j\,[-j\,\mathrm{sgn}(f)]\,G(f).$$

$$= \begin{cases} 2G(f), & f>0 \\ G(0), & f=0 \\ 0, & f<0. \end{cases}$$



Amplitude spectrum of low pass s/g g(t).



Amplitude spectrum of pre-envelope $g_+(t)$.

The pre-envelope of a low pass signal has no negative frequency component.

Complex Envelope

The new quality based on the analytic s/g, called the Complex envelope is defined as

$$x(t) = x(t)\, e^{j 2\pi f_c t}.$$

Taking Fourier transform,

$$F\left[x(t)\right] = X(\omega)$$

$$X(\omega) = \begin{cases} 2X(f+f_c) & \text{for } f>0 \\ H(0) & \text{for } f=0 \\ 0 & \text{for } f<0 \end{cases}$$

The Complex envelope is just the low pass S/g, Part of the analytic s/g. The analytic low pass S/g has been multiplied by the Complex exponential at the Carrier S/g.

Complex Envelope

consider a bandpass s/g $g(t)$, the spectral components of $g(t)$ are negligible outside the frequency band $f_c - W$ to $f_c + W$. and their signal bandwidth is much less than the carrier frequency $(2W << f_c)$. Bandpass signals satisfying this constraint are called "narrowband bandpass s/g"

The complex envelope of $\tilde{g}(t)$ of such a bandpass s/g $g(t)$ is defined interms of its pre-envelope.

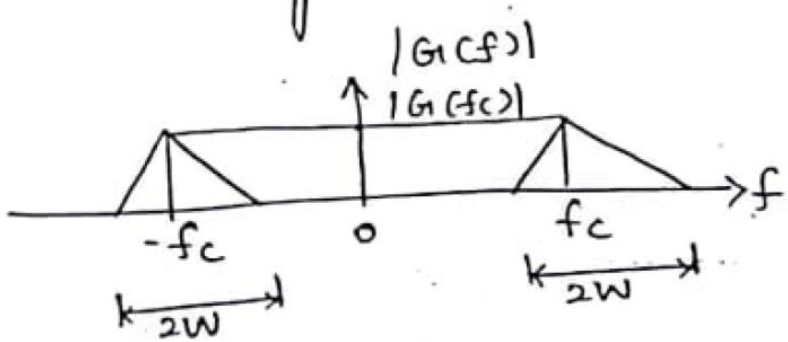
$$g_+(t) = \tilde{g}(t) \exp(j2\pi f_c t)$$

$\tilde{g}(t) \rightarrow$ complex envelope

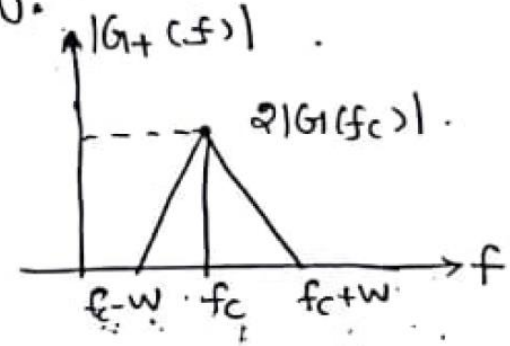
$g_+(t) \rightarrow$ Pre envelope of $g_+(t)$

Complex Envelope

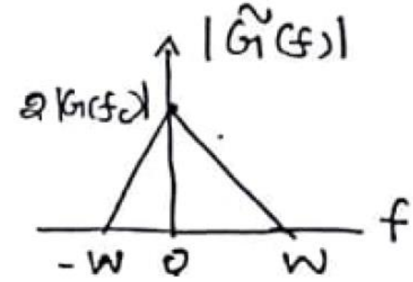
It is obvious that complex envelope spectrum $\tilde{G}(f)$ should be identical to pre envelope spectrum $G_+(f)$ but shifted down the frequency axis by an amount f_c. So it is a spectrum centred around the zero frequency and extending from $-W$ to $+W$.



Amplitude Spectrum of Bandpass s/g $g(t)$



Amplitude spectrum of pre envelope $g_+(t)$



Amplitude spe[ctrum] of complex Envelope $\tilde{g}$(…

Complex Envelope

$$g(t) = \mathrm{Re}\,[g_+(t)].$$

$$= \mathrm{Re}\,[\tilde{g}(t)\exp(j2\pi f_c t)].$$

In general, $\tilde{g}(t) = g_i(t) + j g_q(t)$

$g_q(t)$ → Quadrature component

$g_i(t)$ → In phase component

$$\therefore\ g(t) = \mathrm{Re}\,[\{g_i(t) + j g_q(t)\}\exp(j2\pi f_c t)]$$

$$= \mathrm{Re}\,[\{g_i(t) + j g_q(t)\}[\cos(2\pi f_c t) + j\sin(2\pi f_c t)]]$$

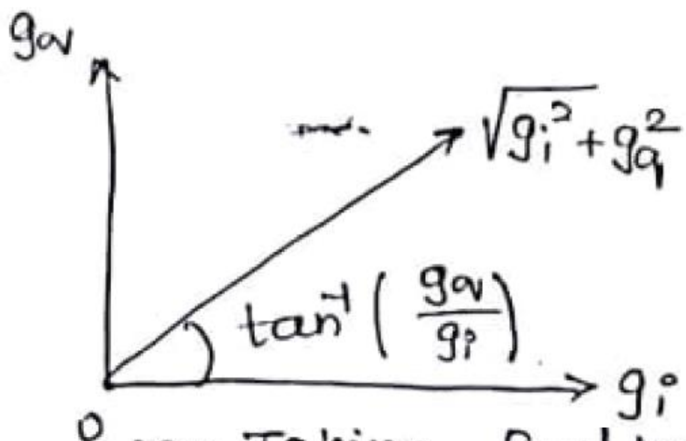
$$\boxed{g(t) = g_i(t)\cos(2\pi f_c t) - g_q(t)\sin(2\pi f_c t).}$$

Complex Envelope

Phasor Representation :-

$$\text{Amplitude} = \sqrt{g_i^2 + g_q^2}$$

$$\text{Phase angle} = \tan^{-1}\left(\frac{g_q}{g_i}\right)$$



Inphase component :-

$$g_i(t) = \text{Re}[\tilde{g}(t)]$$

$$= \text{Re}[g_+(t)\, e^{-j2\pi f_c t}]$$

(By Taking Real part) $= \text{Re}[\{g(t) + j\hat{g}(t)\}\, e^{-j2\pi f_c t}]$

$$\boxed{g_i(t) = g(t)\cos(2\pi f_c t) + \hat{g}(t)\sin(2\pi f_c t)}.$$

Representation of Quadrature component :-

$$g_q(t) = \text{Img}[\tilde{g}(t)]$$

(By Taking Img part)

$$\boxed{g_q(t) = \hat{g}(t)\cos(2\pi f_c t) - g(t)\sin(2\pi f_c t)}.$$

Parameters / Description	AM with Carrier	DSB-SC-AM	SSB-SC-AM	VSB-AM
Method	Carrier and both Sideband	Only Sidebands	Only one sideband	One Sideband and Part of other sideband
Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW < 2f_m$
Power transmitted	$P_t = P_c\left[1+\frac{m_a^2}{2}\right]$	$P_t = P_c\left[\frac{m_a^2}{2}\right]$	$P_t = \frac{P_c m_a^2}{4}$	$P_{t_{SSB}} < P_{t_{VSB}} < P_{t_{DSB}}$
Power Saving (or) Transmission efficiency	33.33%	66.66%	83%	75%
Generation	Not difficult	Not difficult	Complex	Complex
Application	AM broadcasting	Carrier telephony	Police wireless, mobile etc.,	Television and high speed data transmission