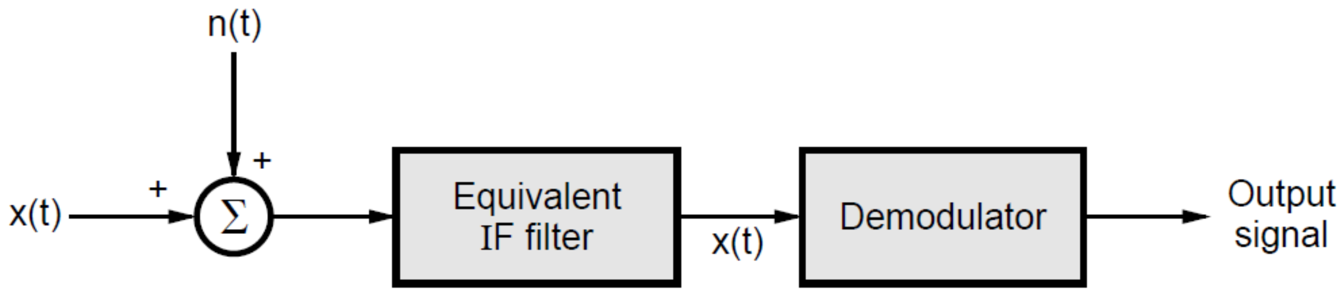
ECE202 Analog Communication

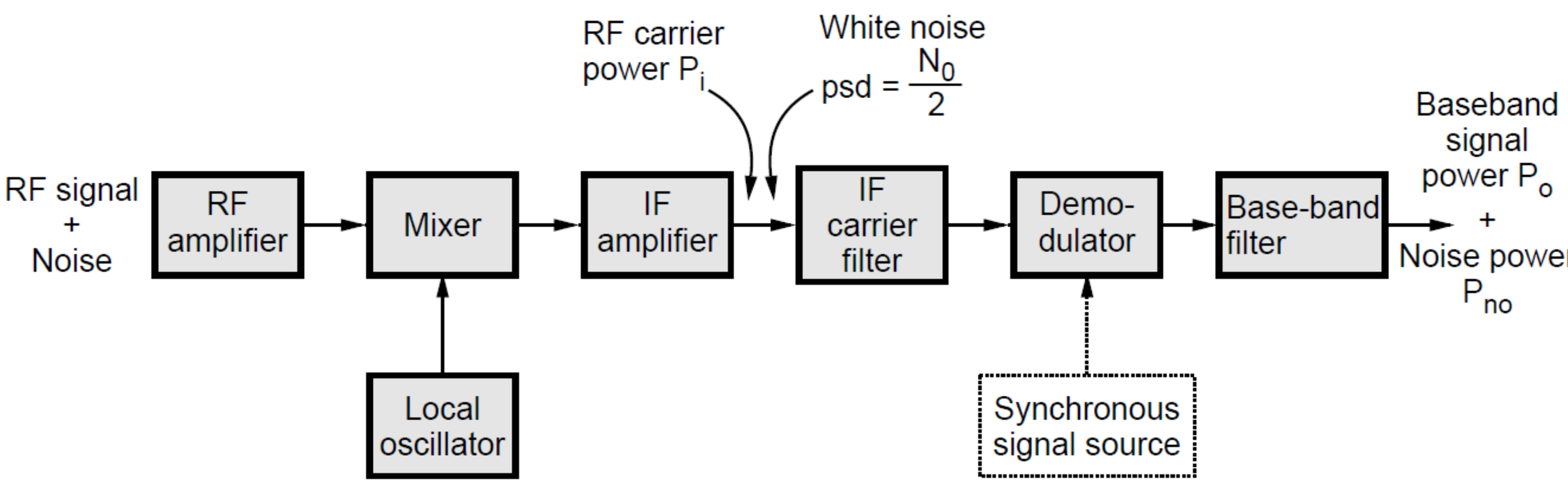
Unit – 4
Noise Analysis in AM and FM

Dr. A. Rajesh

AM/FM Receiver with Noise



receiver model



superheterodyne principle

AM Receiver with Noise

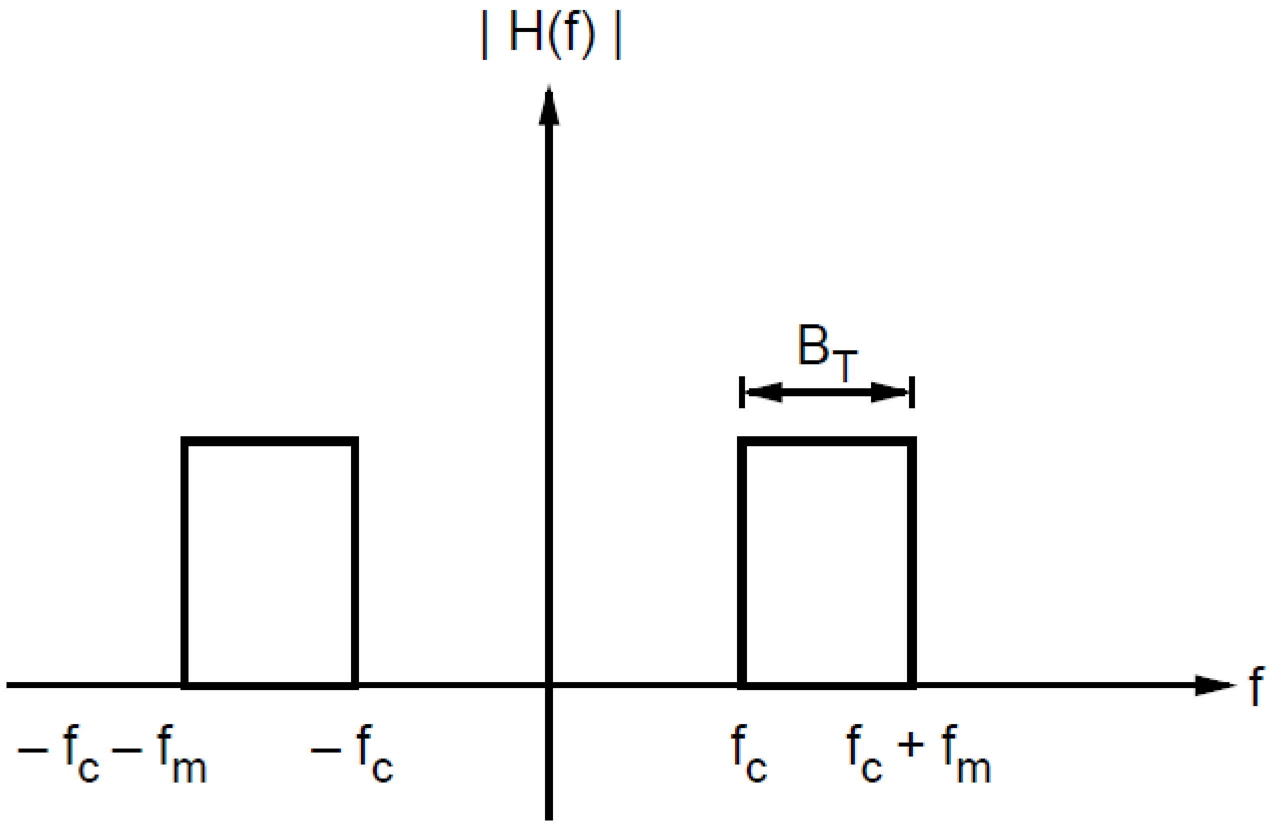


Fig. Ideal characteristic of IF filter

Noise in AM and FM

Performance Measures for Noise in AM and FM

Output Signal to Noise Ratio $(SNR)_o$

It is defined as,

$$(SNR)_o = \frac{\text{Average power of message signal at the receiver output}}{\text{Average power of noise at the receiver output}}$$

- Output signal to noise ratio is used to measure fidelity of the received message.
- As long as the noise and demodulator output are additive, output signal to noise ratio is good measure.
- Output signal at noise ratio depends upon types of modulation and demodulation.

Noise in AM and FM

Performance Measures for Noise in AM and FM

Channel Signal to Noise Ratio $(SNR)_c$

It is defined as,

$$(SNR)_c = \frac{\text{Average power of message signal at the receiver input}}{\text{Average power of noise in message bandwidth at the receiver input}}$$

- This signal to noise ratio may be viewed as the signal to noise ratio of baseband transmission i.e. without modulation.
- Since both the message signal power as well as noise power measured at the receiver input above ratio can be called as input signal to noise ratio $(SNR)_i$.

Noise in AM and FM

Performance Measures for Noise in AM and FM

Figure of Merit

The ratio of output signal to noise ratio to channel signal to noise ratio is called figure of merit. i.e.,

$$\text{Figure of merit} = \frac{(SNR)_o}{(SNR)_c} = \frac{(SNR)_o}{(SNR)_i}$$

- Here $(SNR)_c$ basically represents input signal to noise ratio $(SNR)_i$.
- It is used to compare the performance of different modulation systems.
- The higher figure of merit indicates better noise performance.

Noise in AM and FM

Noise in CW modulation System:

In any Comm. sly the message signal from the transmitter reaches the receiver through the channel. Noise is introduced in the signal while travelling through the channel.

Some assumptions are made in order to obtain a basic understanding of the way in which noise affects the performance of receiver they are,

1. Channel model
2. Receiver model.

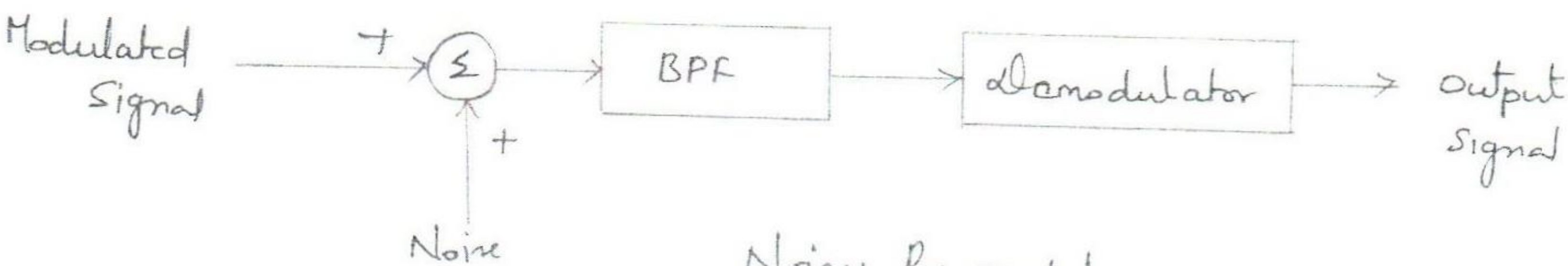
AM Receiver with Noise

① · Channel model :

Assuming a distortionless Comm. channel additive white Gaussian noise (AWGN) is introduced.

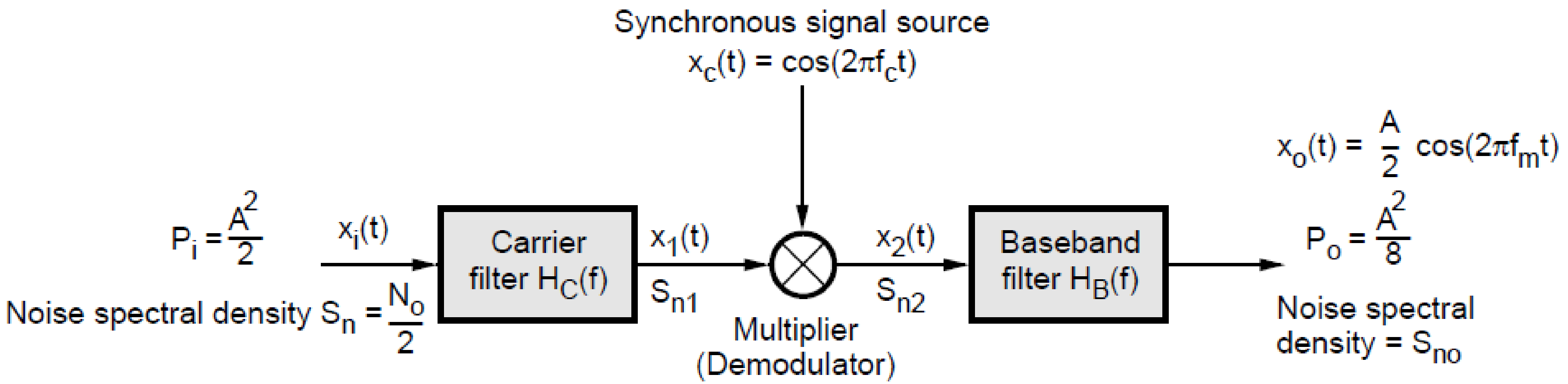
② · Receiver model :

Receiver model is based on the assumption that it Consists of an ideal BPF & an ideal demodulator. The BPF is used to minimise the effect of ch. noise. The receiver model is as shown below.



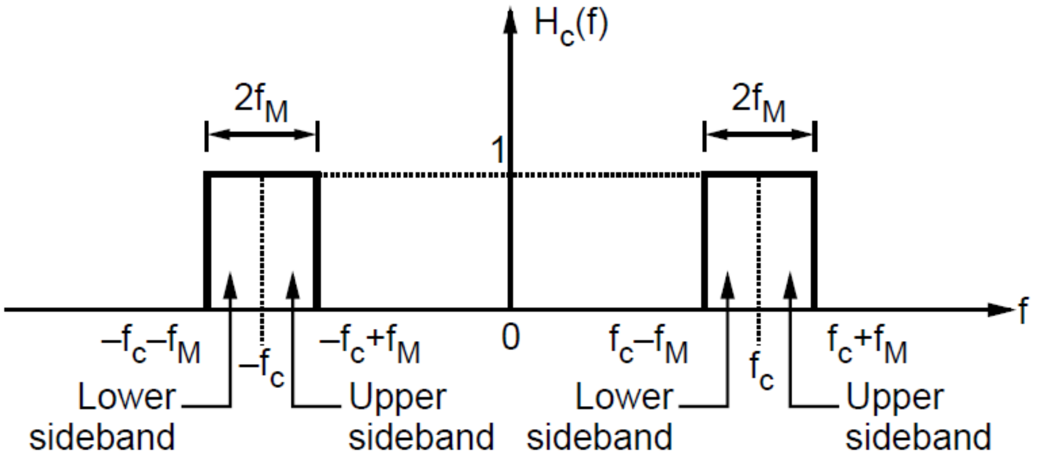
Noisy Rx model

Noise in DSB-SC Receiver

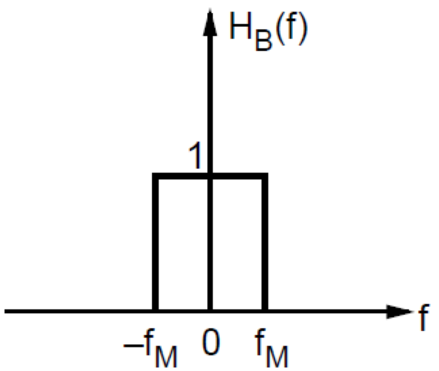


A synchronous demodulator

Noise in DSB-SC Receiver

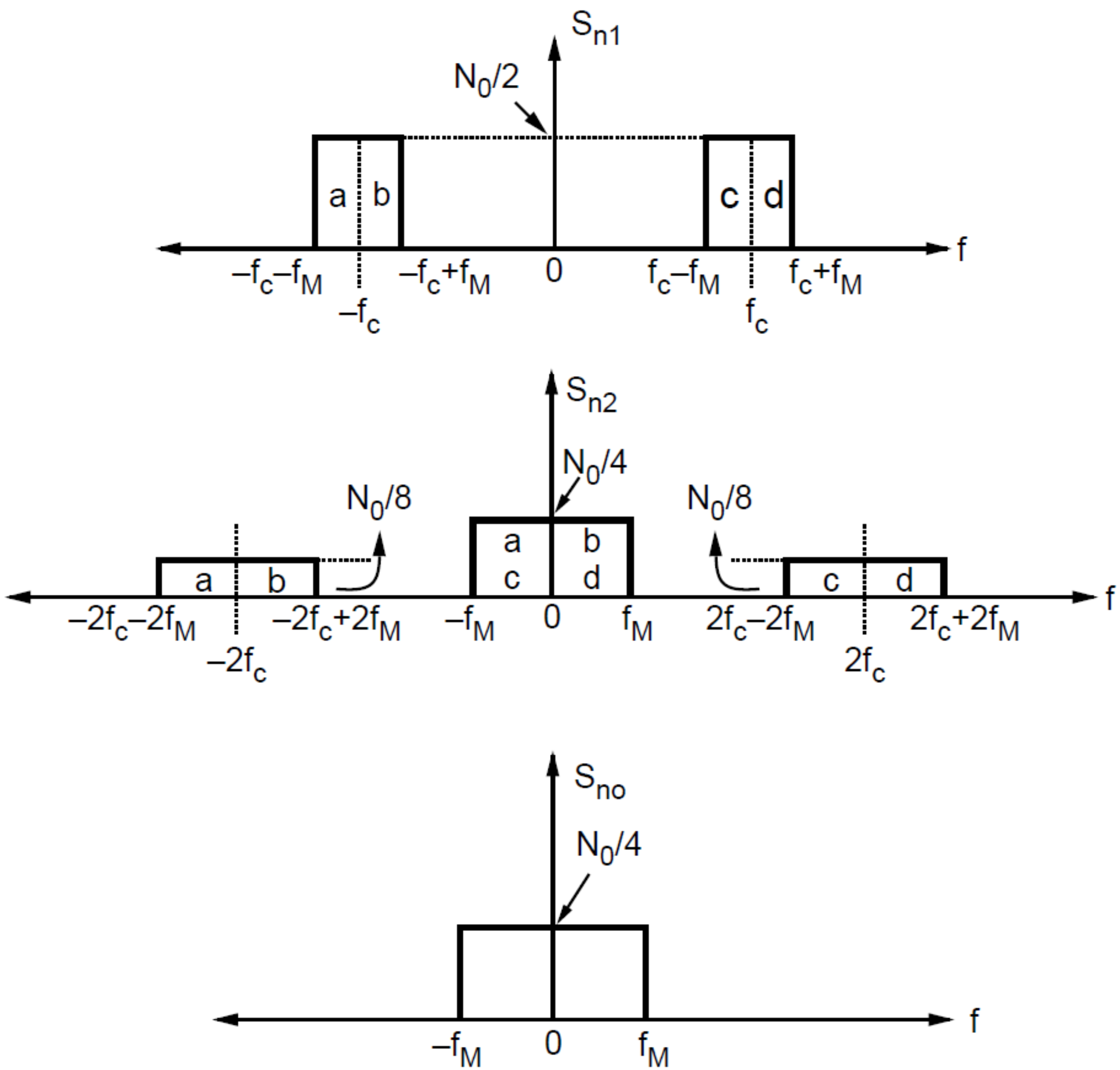


(a) Bandpass range of carrier filter for DSB-SC transmission. The bandwidth of this filter is increased to $2f_M$ to pass both sidebands of x_i (t)



(b) Response of baseband low pass filter. The response of this filter is same as SSB-SC. It passes frequencies less than f_M

Noise in DSB-SC Receiver



(a) Spectral density at the output of carrier filter for DSB-SC transmission.

(b) After multiplication with the carrier, the spectral components of S_{n2} are shifted by $\pm f_c$ and reduced in amplitude by 4

(c) Spectral density of noise at the output of baseband filter. Frequencies less than f_M are only passed

Noise in DSB-SC Receiver

Calculation of Noise Power :

The noise power is given as,

$$P_{no} = \int_{-\infty}^{\infty} S_n(f)\ df$$

white noise of power spectral density $S_n = \dfrac{N_0}{2}$

$$P_{no} = \int_{-f_M}^{f_M} \frac{N_0}{4}\ df$$

$$\therefore \qquad P_{no} = \frac{N_0 f_M}{2}$$

Noise in DSB-SC Receiver

Calculation of Signal Power :

$$x_i(t) = \frac{A}{\sqrt{2}} \cos[2\pi(f_c + f_m)t] + \frac{A}{\sqrt{2}} \cos[2\pi(f_c - f_m)t]$$

The normalized power of the received signal will be,

$$P_i = \left(\frac{A/\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{A/\sqrt{2}}{\sqrt{2}}\right)^2$$

$$P_i = \frac{A^2}{2}$$

Here note that we have used the Parseval's power theorem to obtain the power due to two sidebands. That is the powers due to individual components add to give total power.

Noise in DSB-SC Receiver

Calculation of Signal Power :

The signal $x_1(t)$ at the output of carrier filter will be same as $x_i(t)$. This $x_1(t)$ is then multiplied with carrier $x_c(t)$ in the demodulator. Hence the output of demodulator $x_2(t)$ will be,

$$x_2(t) = x_1(t) \cdot \cos(2\pi f_c t)$$

$$= \frac{A}{\sqrt{2}} \cos[2\pi(f_c + f_m)t] \cos(2\pi f_c t) + \frac{A}{\sqrt{2}} \cos$$

$$[2\pi(f_c - f_m)t] \cos(2\pi f_c t)$$

$$= \frac{A}{2\sqrt{2}} \cos[2\pi(2f_c + f_m)t] + \frac{A}{2\sqrt{2}} \cos(2\pi f_m t)$$

$$+ \frac{A}{2\sqrt{2}} \cos[2\pi(2f_c - f_m)t] + \frac{A}{2\sqrt{2}} \cos(2\pi f_m t)$$

$$\cos(x)\cos(y) = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$

Noise in DSB-SC Receiver

Calculation of Signal Power : when this signal $x_2(t)$ is passed through the baseband lowpass filter, then the frequencies less than f_M are only passed. That means the frequencies $2f_c + f_m$ and $2f_c - f_m$ are not passed.

$$x_0(t) = \frac{A}{2\sqrt{2}} cos\,(2\,\pi\, f_m\; t) + \frac{A}{2\sqrt{2}} cos\,(2\,\pi\, f_m\; t)$$

$$\therefore \qquad x_0(t) = \frac{A}{\sqrt{2}}\; cos\,(2\,\pi\, f_m\; t)$$

The normalized power of output signal will be,

$$P_o = \left(\frac{A/\sqrt{2}}{\sqrt{2}}\right)^2$$

$$\therefore \qquad P_o = \frac{A^2}{4}$$

Noise in DSB-SC Receiver

Signal to Noise Ratio

Now let us evaluate the signal to noise power ratio of DSB-SC system in presence of white gaussian noise. It is given as,

$$\left(\frac{S}{N}\right)_{output} = \frac{P_o}{P_{no}}$$

$$\left(\frac{S}{N}\right)_{output} = \frac{A^2/4}{N_0 f_M/2}$$

$$= \frac{A^2}{2 N_0 f_M}$$

Since input signal power $P_i = \dfrac{A^2}{2}$

$$\left(\frac{S}{N}\right)_{output} = \frac{P_i}{N_0 f_M}$$

Noise in DSB-SC Receiver

Figure of Merit for DSB-SC receiver (Synchronous detection)

- The input signal power will be, $S_i = \dfrac{A^2}{2}$

- The input noise power over the baseband band width of $2\,f_M$ (including positive and negative frequencies) will be,

$$N_i = N_0 f_M$$

- Hence input signal to noise ratio will be,

$$\frac{S_i}{N_i} = \frac{A^2/2}{N_0 f_M} = \frac{A^2}{2\,N_0 f_M}$$

- Hence figure of merit for DSB-SC will be,

$$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{\dfrac{A^2}{2\,N_0 f_M}}{\dfrac{A^2}{2\,N_0 f_M}}$$

$$\therefore \quad \boxed{\gamma = 1}$$

- Since figure of merit is 1 for DSB-SC reception, there is no improvement in signal to noise ratio.

Noise in DSB-FC

Channel SNR for AM Signal

Consider the AM transmission that has both the sidebands and a carrier. Such modulated signal is mathematically represents as,

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

Here $A_c \cos(2\pi f_c t)$ is the carrier signal.

$m(t)$ is the message signal.

k_a determines percentage modulation (modulation index).

$$P_{total} = P_c\left(1 + \frac{m^2}{2}\right).$$

carrier power $P_c = \dfrac{A_c^2}{2}$

$$\text{Modulated signal power} = \frac{A_c^2}{2}\left(1 + \frac{m^2}{2}\right)$$

$$= \frac{A_c^2}{2}\left(1 + \frac{k_a^2}{2}\right) \qquad \text{Here } k_a = m.$$

Noise in DSB-FC

$\frac{k_a^2}{2}$ is normalized power of message signal

If 'P' is the average power of message signal, then above equation becomes,

$$\text{Modulated signal power} = \frac{A_c^2}{2}\left(1+k_a^2 P\right)$$

Earlier we have proved that if the message bandwidth is 'B', then average noise power in this bandwidth will be,

$$\text{Average noise power} = N_o B$$

Here $\frac{N_o}{2}$ is the power spectral density of white Gaussian noise.

channel signal to noise ratio is obtained as,

$$(SNR)_c = \frac{\text{Modulated signal power}}{\text{Average noise power}}$$

$$= \frac{\frac{A_c^2}{2}\left(1+k_a^2 P\right)}{N_o B}$$

$$= \frac{A_c^2\left(1+k_a^2 P\right)}{2N_o B}$$

Noise in DSB-FC

Output SNR for Envelop Detection

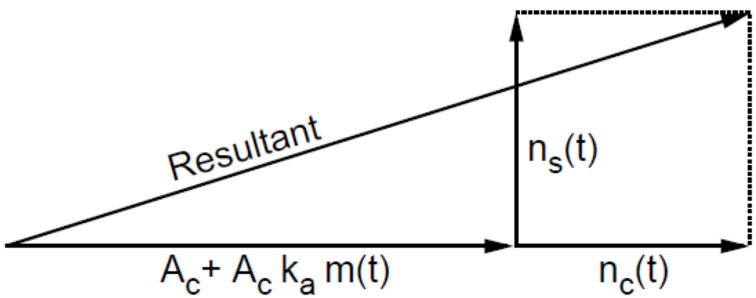
The envelope detector consists of modulated message signal $s(t)$ plus noise $n(t)$. i.e.,

$$x(t) = s(t) + n(t)$$

Representing $n(t)$ in terms of inphase and quadrature components,

$$\begin{aligned} x(t) &= s(t) + n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \\ &= A_c[1 + k_a m(t)]\cos 2\pi f_c t + n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \\ &= [A_c + A_c k_a m(t) + n_c(t)]\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \end{aligned}$$

Fig. shows the phasor representation of $x(t)$.



Phasor diagram of AM plus noise

Noise in DSB-FC

Output SNR for Envelop Detection

The resultant is the envelope of $x(t)$, i.e. output of envelope detector.

$$y(t) = \sqrt{[A_c + A_c k_a m(t) + n_c(t)]^2 + [n_s(t)]^2}$$

When signal power is large compared to noise power, then $n_s(t)$ and $n_c(t)$ will be very small compared to $A_c[1 + k_a m(t)]$.

Therefore above equation can be approximated as,

$$y(t) = A_c + A_c k_a m(t)$$

The first term in above equation is A_c.

It is the carrier amplitude and it is removed with the help of blocking capacitor

$$y(t) = A_c k_a m(t)$$

The power of above signal is the average power at receiver output i.e.,

$$\text{Power at receiver output} = \frac{A_c^2 k_a^2 P}{2}$$

Here P is the average power of message signal m(t).

Noise in DSB-FC

Output SNR for Envelop Detection

And the noise power over the bandwidth 'B' is,

$$\text{Noise power at receiver output} = N_o B$$

Here $\frac{N_o}{2}$ is the power spectral density of white noise.

Therefore output signal to noise ratio will be,

$$(SNR)_o = \frac{\text{Power at receiver output}}{\text{Noise power at receiver output}} = \frac{\frac{A_c^2 k_a^2 P}{2}}{N_o B}$$

$$= \frac{A_c^2 k_a^2 P}{2 N_o B}$$

Noise in DSB-FC

SNR performance of envelop (AM) detection is 3 dB worse than that of DSB system

the input signal power is,

$$P_i = \frac{A_c^2}{2}(1 + k_a^2 P)$$

$$\therefore \quad A_c^2 = \frac{2P_i}{1 + k_a^2 P}$$

Putting for this A_c^2

$$(SNR)_o = \frac{2P_i}{1 + k_a^2 P} \cdot \frac{k_a^2 P}{2N_o B}$$

$$= \frac{k_a^2 P}{1 + k_a^2 P} \cdot \frac{P_i}{N_o f_m}$$ Here $B = f_m$ is bandwidth

$\left(\frac{S}{N}\right)_{output}$ of DSB-SC is $\frac{P_i}{N_o f_m}$. Hence ,

$$\left(\frac{S}{N}\right)_{o,\ envelope(AM)} = \frac{k_a^2 P}{1 + k_a^2 P} \cdot \left(\frac{S}{N}\right)_{o,\ DSB\text{-}SC}$$

Noise in DSB-FC

SNR performance of envelop (AM) detection is 3 dB worse than that of DSB system

Here $k_a^2 P \leq 1$. Assuming $k_a^2 P = 1$ above equation becomes,

$$\left(\frac{S}{N}\right)_{o,\ envelope(AM)} = \frac{1}{2}\left(\frac{S}{N}\right)_{o,\ DSB\text{-}SC}$$

$$10\log_{10}\left\{\left(\frac{S}{N}\right)_{o,\ envelop(AM)}\right\} = \left\{10\log_{10}\left[\left(\frac{1}{2}\right)\cdot\left(\frac{S}{N}\right)_{o,DSB\text{-}SC}\right]\right\}$$

$$\therefore \qquad \left(\frac{S}{N}\right)_{o,envelop(AM)} dB = -3\,dB + \left(\frac{S}{N}\right)_{o,DSB\text{-}SC} dB$$

Thus SNR of envelop or AM detection is 3 dB more than that of DSB-SC.

Noise in DSB-FC

Figure of Merit for Envelope Detection

The figure of merit is given as,

$$\text{Figure of merit} = \frac{(SNR)_o}{(SNR)_c}$$

Putting expressions for $(SNR)_o$ from equation (4.9.26) and $(SNR)_c$ from equation (4.9.23) in a above equation,

$$\text{Figure of merit} = \frac{\dfrac{A_c^2 k_a^2 P}{2N_o B}}{\dfrac{A_c^2\left(1+k_a^2 P\right)}{2N_o B}}$$

$$= \frac{k_a^2 P}{1+k_a^2 P}$$

- The above equation indicates that figure of merit for envelope detection is always less than unity.
- The figure of merit for DSBSC or SSB is equal to unity. This means noise performance of DSBSC and SSB is better than AM receiver with envelope detection.

Noise in DSB-FC

Threshold Effect

Definition :

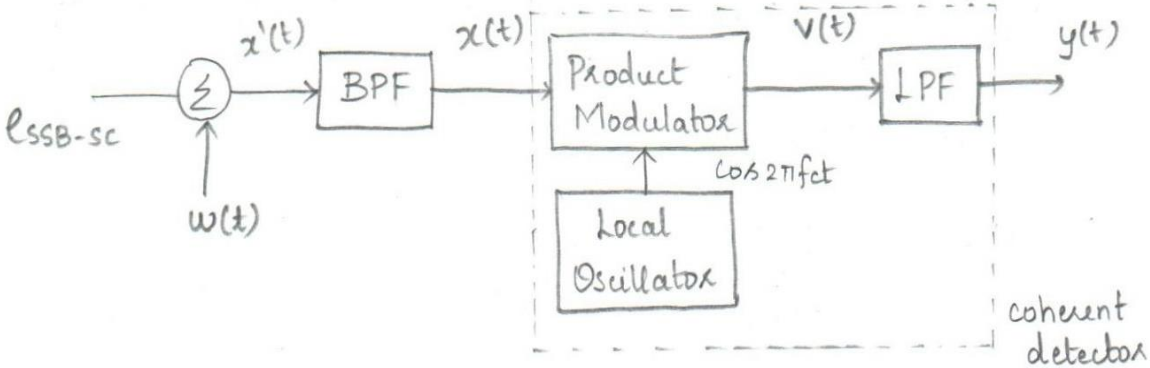
When the carrier to noise ratio reduces below certain value, the message information is lost. The performance of envelope detector deteriorates rapidly and it has no proportion to carrier to noise ratio. This is called threshold effect.

- Every nonlinear receiver exhibits threshold effect. Coherent receivers do not have threshold effect.
- The detector output does not depend only on message signal m(t), rather it is the function of noise also.

When the noise is higher compared to signal, the noise dominates the performance of the receiver.

Noise in SSB-SC

Noise performance in SSB-SC using coherent detector.



$$e_{SSB-SC}(t) = \frac{E_c}{2}\cos 2\pi f_c t\, e_m(t) + \frac{E_c}{2}\sin 2\pi f_c t\, e_{mh}(t)\, x_h(t).$$

Since in $e_{mh}(t)$, small noise is present, it is neglected, then

$$e_{SSB-SC}(t) = \frac{E_c}{2}\cos 2\pi f_c t\, e_m(t)$$

$$x'(t) = e_{SSB-SC}(t) + w(t)$$

$$= \frac{E_c}{2}\cos 2\pi f_c t\, e_m(t) + w(t)$$

$$x'(t) = \underbrace{\frac{E_c e_m(t)}{2}\cos 2\pi f_c t}_{S} + \underbrace{w(t)}_{N}$$

$$(SNR)_c = \frac{P_{si}}{P_{ni}}$$

Noise in SSB-SC

To find P_{si}

$$P_{si} = \frac{E_c\, e_m(t)}{2} \cos 2\pi f_c t$$

$$= \frac{1}{4} E_c^2 P$$

$$P_{si} = \frac{E_c^2 P}{4}$$

To find P_{ni}:

$$P_{ni} = \int_{-\infty}^{\infty} S_N(f)\, df = \int_{0}^{w} \frac{N_0}{2}\, df \qquad [\text{BW of SSB-SC} - f_m]$$

$$P_{ni} = \frac{N_0}{2} w$$

$$(SNR)_c = \frac{\frac{E_c^2 P}{4}}{\frac{N_0 w}{2}} = \frac{E_c^2 P}{2 N_0 w}$$

Noise in SSB-SC

$$v(t) = x(t) \cdot \cos 2\pi f_c t$$

$$x(t) = e_{SSB-SC}(t) + n(t)$$

$$x(t) = \frac{E_c}{2} \cos 2\pi f_c t \, e_m(t) + \left[n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \right]$$

$$v(t) = \left[\frac{E_c}{2} \cos 2\pi f_c t \, e_m(t) + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \right] \times \left[\cos 2\pi f_c t \right]$$

$$v(t) = \frac{E_c}{2} \cos^2 2\pi f_c t \, e_m(t) + n_I(t) \cos^2 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \cos 2\pi f_c t$$

$$= \frac{E_c}{2} \left[\frac{1 + \cos 4\pi f_c t}{2} e_m(t) \right] + \frac{n_I(t)\left(1 + \cos 4\pi f_c t\right)}{2} - \frac{n_Q(t) \sin 4\pi f_c t}{2}$$

high frequency

$$v(t) = \frac{E_c e_m(t)}{4} + \cancel{\frac{E_c \cos 4\pi f_c t \, e_m(t)}{2}} + \frac{n_I(t)\left(1 + \cancel{\cos 4\pi f_c t}\right)}{2} - \cancel{\frac{n_Q(t) \sin 4\pi f_c t}{2}}$$

$$y(t) = \frac{E_c e_m(t)}{4} + \frac{n_I(t)}{2}$$

Noise in SSB-SC

$$P_{so} = \frac{E_c^2 P}{8}$$

$$P_{no} = \int_0^{w} S_N(f)\,df = \int_0^{w} \frac{N_I(t)}{2}\,df$$

$$= \int_0^{w} \frac{N_o}{4}\,df$$

$$= \frac{N_o w}{4}$$

$$(SNR)_o = \frac{P_{so}}{P_{no}} = \frac{E_c^2 P/8}{N_o w/4} = \frac{E_c^2 P}{2 N_o w}$$

$$FOM = \frac{(SNR)_o}{(SNR)_c} = 1$$

Noise Performance Analysis in FM Systems

Capture Effect

The FM system minimizes the effects of noise interference.

This can be effective when interference is weak compared to FM signal.

But if the interference is stronger than FM signal, then FM receiver locks to interference.

This suppresses FM signal.

When noise interference as well as FM signal are of equal strength, then the FM receiver locking fluctuates between them.

This phenomenon is called capture effect.

Noise Performance Analysis in FM Systems

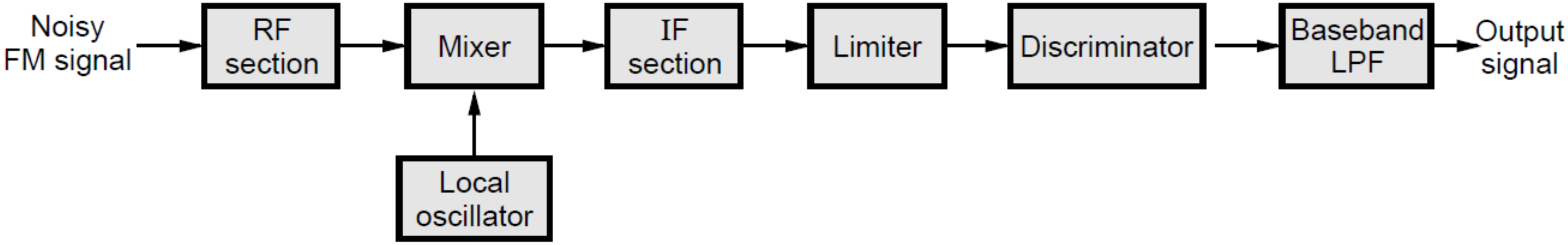


Fig. 4.10.1 Block diagram of FM superheterodyne receiver

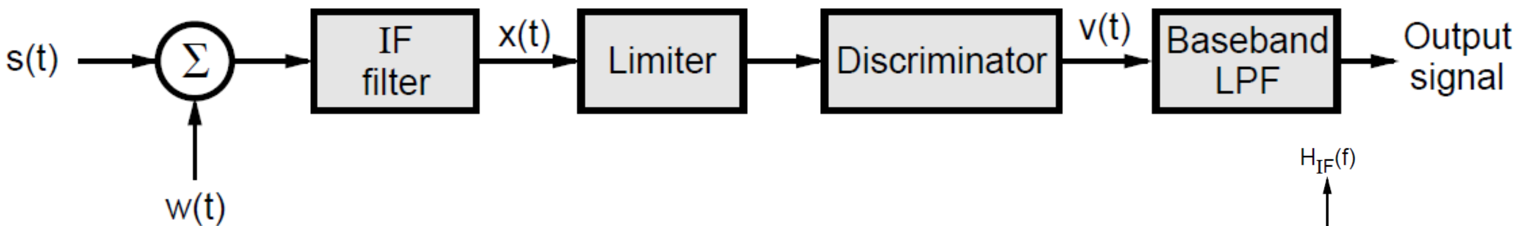


Fig. 4.10.2 FM receiver model

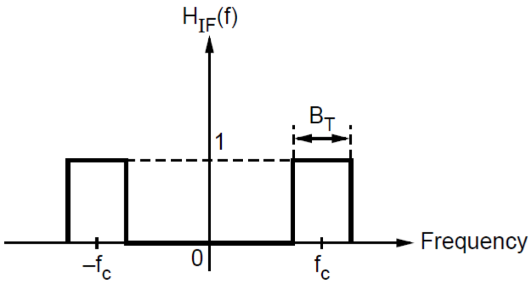


Fig. 4.10.3 IF filter characteristic

The FM signal at the input is $s(t)$.

It has carrier frequency f_c and bandwidth B_T

$w(t)$ is Gaussian noise of zero mean and power spectral density of $\dfrac{N_o}{2}$.

Noise Performance Analysis in FM Systems

The narrowband noise $n(t)$ at the output of IF filter can be represented in terms of inphase and quadrature components as,

$$n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

This noise can also be represented in terms of envelope and phase components as,

$$n(t) = r(t)\cos[2\pi f_c t + \psi(t)]$$

Here, $$r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

$$\psi(t) = \tan^{-1}\left[\frac{n_s(t)}{n_c(t)}\right]$$

The frequency modulated signal at the output of IF filter is represented mathematically as,

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

Here A_c is amplitude of carrier

$\phi(t)$ is instantaneous phase deviation.

The phase deviation and modulating signal $m(t)$ are related as,

$$\phi(t) = 2\pi k_f \int_0^t m(t)\, dt$$

Here k_f is the frequency sensitivity.

Noise Performance Analysis in FM Systems

The signal $x(t)$ at the output of IF filter is given as,

$$x(t) = s(t) + n(t)$$

Putting expressions from equation

$$x(t) = A_c \cos[2\pi f_c t + \phi(t)] + r(t) \cos[2\pi f_c t + \psi(t)]$$

Fig. 4.10.4 shows the phasor diagram of above equation. Note that the phase shift between the vectors A_c and $r(t)$ is $\psi(t) - \phi(t)$.

If $r(t)$ is small compared to A_c, then the relative phase $\theta(t)$ can be approximately given as,

$$\theta(t) \cong \phi(t) + \frac{r(t)}{A_c} \sin[\psi(t) - \phi(t)]$$

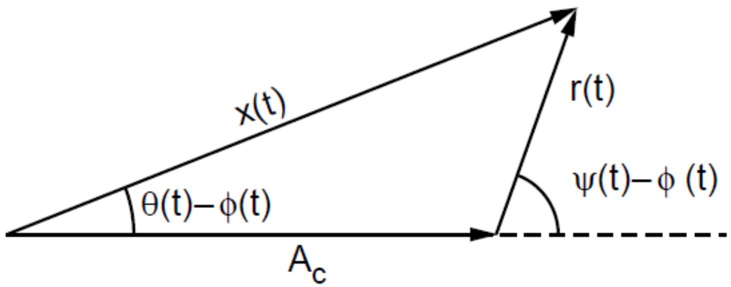


Fig. 4.10.4 Phasor diagram

Noise Performance Analysis in FM Systems

$$\theta(t) = 2\pi k_f \int_0^t m(t)dt + \frac{r(t)}{A_c}\sin[\psi(t)-\phi(t)]$$

The instantaneous output of discriminator is $v(t)$. It is equal to the derivative of relative phase $\theta(t)$ divided by 2π. i.e.,

$$v(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t) = \frac{1}{2\pi}\left\{2\pi k_f m(t) + \frac{d}{dt}\frac{r(t)}{A_c}\sin[\psi(t)-\phi(t)]\right\}$$

$$= k_f m(t) + \frac{1}{2\pi A_c}\frac{d}{dt}\{r(t)\sin[\psi(t)-\phi(t)]\}$$

Here note that the output depends upon the message term $k_f m(t)$ and the noise term $n_d(t)$. i.e.,

$$n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}\{r(t)\sin[\psi(t)-\phi(t)]\}$$

Noise Performance Analysis in FM Systems

In the above equation, the term $\phi(t)$ is function of modulating signal. But $\psi(t)$ is uniformly distributed over an internal 0 to 2π. Hence $n_d(t)$ will be independent of $\phi(t)$. Therefore above equation can be approximately written as,

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} r(t) \sin\psi(t)$$

From equation we can write,

$$n_s(t) = r(t) \sin\psi(t) \qquad \text{and} \qquad n_c(t) = r(t) \cos\psi(t)$$

Hence equation becomes,

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} n_s(t)$$

Noise Performance Analysis in FM Systems

Output Signal Power

the output of discriminator is,

$$v(t) = k_f m(t) + n_d(t)$$

The output signal component from the filter will be $k_f m(t)$. Hence output signal power will be,

Output signal power $= k_f^2 P$

Here 'P' is average power of massage signal $m(t)$.

Noise Performance Analysis in FM Systems

Output Noise Power

note that noise $n_d(t)$ at the discriminator output is proportional to time derivative of quadrature component of noise $n_s(t)$. i.e.,

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} n_s(t)$$

The differentiation property of Fourier transform states that,

$$\frac{d}{dt} x(t) \leftrightarrow j2\pi f X(f)$$

This means $\frac{d}{dt} x(t)$ can be obtained by passing *x(t)* through a filter with transfer function $j2\pi f$ (i.e. differentiator). Similarly $n_d(t)$ can be obtained by passing $n_d(t)$ through a filter with transfer function,

$$H(f) = \frac{1}{2\pi A_c} \cdot j2\pi f = j\frac{f}{A_c}$$

Noise Performance Analysis in FM Systems

If $S_{N_s}(f)$ is psd of $n_s(t)$ and $S_{N_d}(f)$ is psd of $n_d(t)$, then we can write,

$$S_{N_d}(f) = |H(f)|^2 \, S_{N_s}(f)$$

$$= \frac{f^2}{A_c^2} S_{N_s}(f)$$

The IF filter has ideal bandpass characteristic. Hence the quadrature component of narrowband noise $n_s(t)$ will also have similar characteristic. It is shown in Fig. 4.10.2(a). Then density of $n_d(t)$ will be mathematically expressed as,

$$S_{N_d}(f) = N_o \cdot \frac{f^2}{A_c^2}, |f| \leq \frac{B_T}{2}$$

This noise $n_d(t)$ is passed through the low pass filter of bandwidth equal to message bandwidth W. Hence the output noise $n_o(t)$ will have the power spectral density as

$$S_{N_o}(f) = N_o \frac{f^2}{A_c^2}, \qquad |f| = W$$

Noise Performance Analysis in FM Systems

The average noise power can be obtained by integrating the power spectral density given by above equation. i.e.,

$$\text{Output noise power} = \int_{-W}^{W} S_{N_o}(f)\, df$$

$$= \frac{N_o}{A_c^2} \int_{-W}^{W} f^2\, df$$

$$= \frac{2 N_o W^3}{3 A_c^2}$$

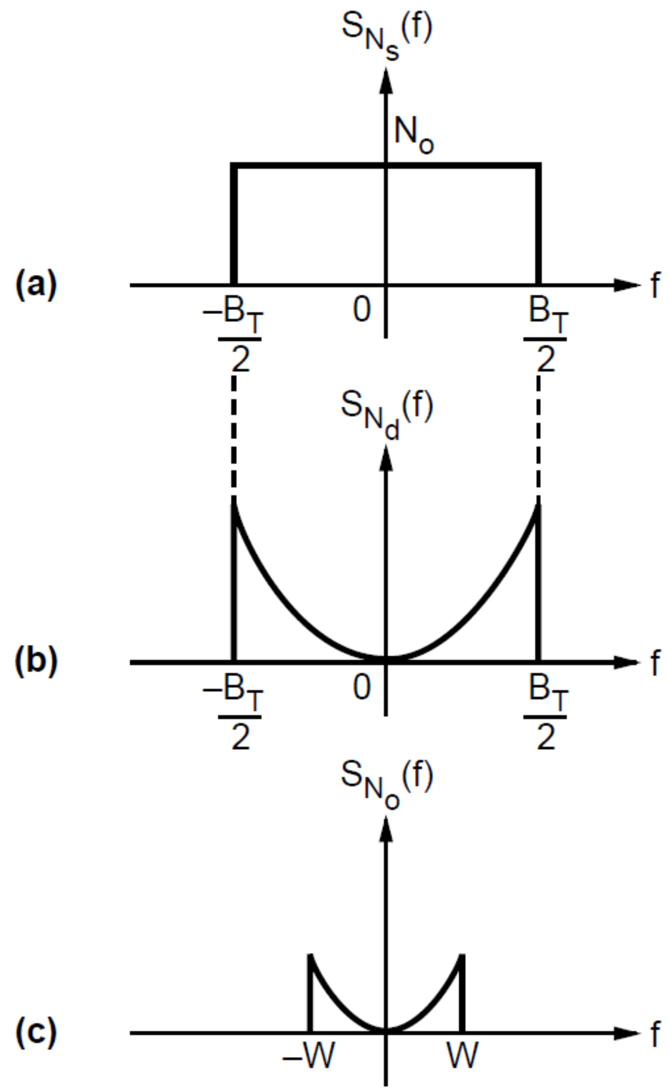


Fig. 4.10.2 psds of noise in FM receiver

Noise Performance Analysis in FM Systems

Output Signal to Noise Ratio

The output signal to noise ratio is given as,

$$(SNR)_o = \frac{\text{Average output signal power}}{\text{Average output noise power}}$$

$$(SNR)_o = \frac{k_f^2 P}{\dfrac{2N_o W^3}{3A_c^2}} = \frac{3A_c^2 k_f^2 P}{2N_o W^3}$$

Channel Signal to Noise Ratio

The average power in the modulated signal given to receiver is $\frac{A_c^2}{2}$. The average noise power in the message bandwidth is $N_o W$. Hence channel signal to noise ratio is given as,

$$(SNR)_c = \frac{\dfrac{A_c^2}{2}}{N_o W} = \frac{A_c^2}{2N_o W}$$

Noise Performance Analysis in FM Systems

Figure of Merit

From channel signal to noise ratio and output signal to noise ratio we can obtain figure of merit for FM receiver. i.e.,

$$\text{Figure of merit} = \frac{(SNR)_o}{(SNR)_c} = \frac{\dfrac{3A_c^2 k_f^2 P}{2N_o W^3}}{\dfrac{A_c^2}{2N_o W}}$$

$$= \frac{3k_f^2 P}{W^2}$$

We know that $P = \dfrac{A_c^2}{2}$

$$\therefore \text{Figure of merit} = \frac{3\,k_f^2 \cdot \dfrac{A_c^2}{2}}{W^2} = \frac{3}{2}\frac{(k_f A_c)^2}{W^2}$$

We know that frequency deviation, $\delta = k_f A_c$. Hence

$$\text{Figure of merit} = \frac{3}{2}\frac{\delta^2}{W^2}$$

$$= \frac{3}{2}\beta^2, \text{ Where } \beta = \frac{\delta}{W} \text{ is the modulation index.}$$

Conclusions

- The deviation ratio is proportional $\dfrac{k_f}{W}$. Hence the figure of merit is the quadratic function of deviation ratio.
- The transmission bandwidth B_T is proportional to deviation ratio. Hence increase in bandwidth B_T, increases the figure of merit FM system.

Noise Performance Analysis in PM Systems

Noise in Phase Modulation

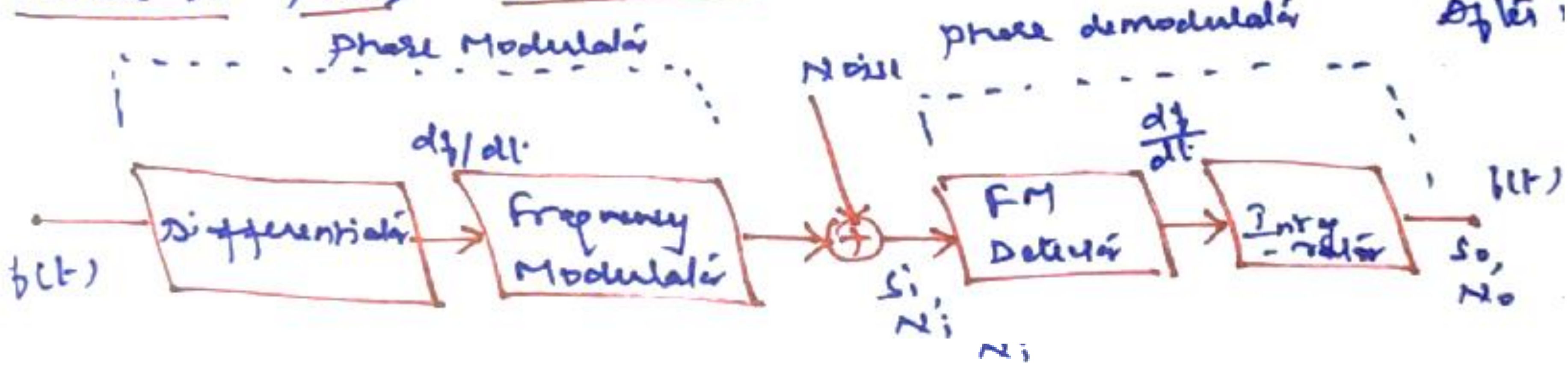


Figure of Merit (8pM)

→ pM can be derived by adopting the procedure used for frequency Modulation.

$$S_i = \frac{A^2}{2} \quad -(1)$$

$$N_i = \eta f_m \quad -(2)$$

Noise Performance Analysis in PM Systems

24

The signal power at the FM detector o/p is proportional to $d\phi/dt$. and, after integrator ckt, the o/p becomes $\phi(t)$.

$$S_o = k^2 k_p^2 \phi^2(t) \quad -(3)$$

Noise signal at the o/p of the detector is similar to FM

$$n_o(t) = \frac{k \dot{n}_s(t)}{A} \quad -(4)$$

After integration

$$n_o(t) = \frac{k n_s(t)}{A} \quad -(5)$$

Noise Performance Analysis in PM Systems

Power density spectrum of $n_o(t)$ is given by

$$S_{n_o}(\omega) = \frac{K^2}{A^2} S_{n_s}(\omega) \quad -(6)$$

$S_{n_s}(\omega)$ equation can be obtained from DSBSC derivation

$$S_{n_s}(\omega) = \eta, \quad |\omega| \leq \omega_m \quad -(7)$$

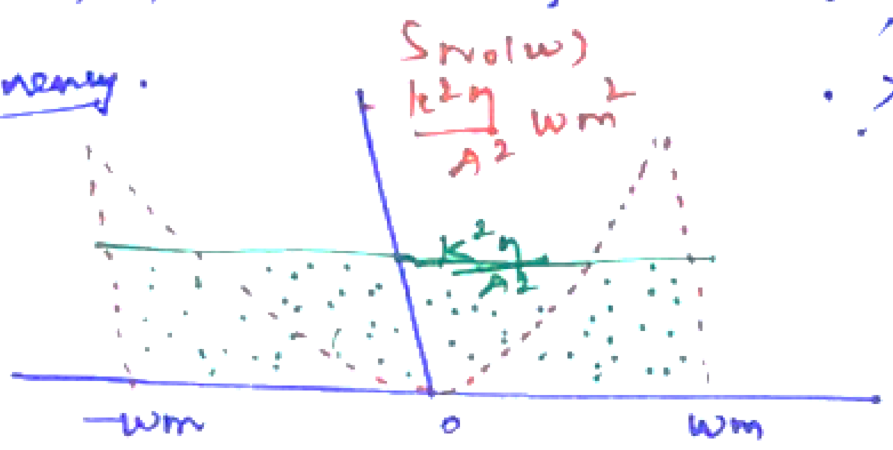
Noise Performance Analysis in PM Systems

Sub (7) in (6)

$$S_{n_o}(\omega) = \frac{k^2}{A^2}\eta \quad -(8)$$

$$S_{n_o}(\omega) = \begin{cases} \frac{k^2}{A^2}\eta, & |\omega| \le \omega_m \\ 0 & \text{elsewhere} \end{cases} \quad -(9)$$

In PM, Power density is independent of frequency.



Noise Performance Analysis in PM Systems

Output Noise Power

$$N_o = \frac{1}{\pi}\int_0^W S_{n_o}(\omega)\,d\omega \quad -(10)$$

$$N_o = \frac{1}{\pi}\int_0^W \frac{k^2}{A^2}\eta\,d\omega \quad -(11)$$

$$N_o = \frac{2k^2\eta f_m}{A^2} \quad -(12)$$

Noise Performance Analysis in PM Systems

Output signal to Noise Ratio can be obtained by (3), (12)

$$\left(\frac{S_o}{N_o}\right) = \frac{k^2 k_p^2 f^2(t)}{\frac{A^2 k_p^2 f^2(t)}{2\eta f_m}}$$

$$\left(\frac{S_o}{N_o}\right) = \frac{A^2 k_p^2 f^2(t)}{2\eta f_m} \quad -(13)$$

figure of merit $\gamma_{PM} = \frac{(S_o/N_o)}{(S_i/N_i)}$

$$\boxed{\gamma_{PM} = k_p^2 f^2(t)} \quad -(14)$$

Noise Performance FM vs PM Systems

$$\gamma_{PM} = k_p^2 \, \overline{m^2(t)}$$

$$\gamma_{PM} = \frac{1}{2} m_f^2 \quad -(1)$$

$$\gamma_{FM} = \frac{3}{2} m_f^2 \quad -(2)$$

$$\frac{\gamma_{FM}}{\gamma_{PM}} = \frac{\frac{3}{2} m_f^2}{\frac{1}{2} m_f^2} = 3.$$

$$\therefore \boxed{\frac{\gamma_{FM}}{\gamma_{PM}} = 3.} \quad -(3)$$