ECE202 Analog Communication

Unit – 2
Angle Modulation

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Angle Modulation

The angle (frequency (or) phase) of the Carrier signal is Varied according to the message signal, then it is called angle modulation here the amplitude of the Carrier signal is Constant.

Types of angle modulation
- Frequency modulation (FM)
- phase modulation (PM)

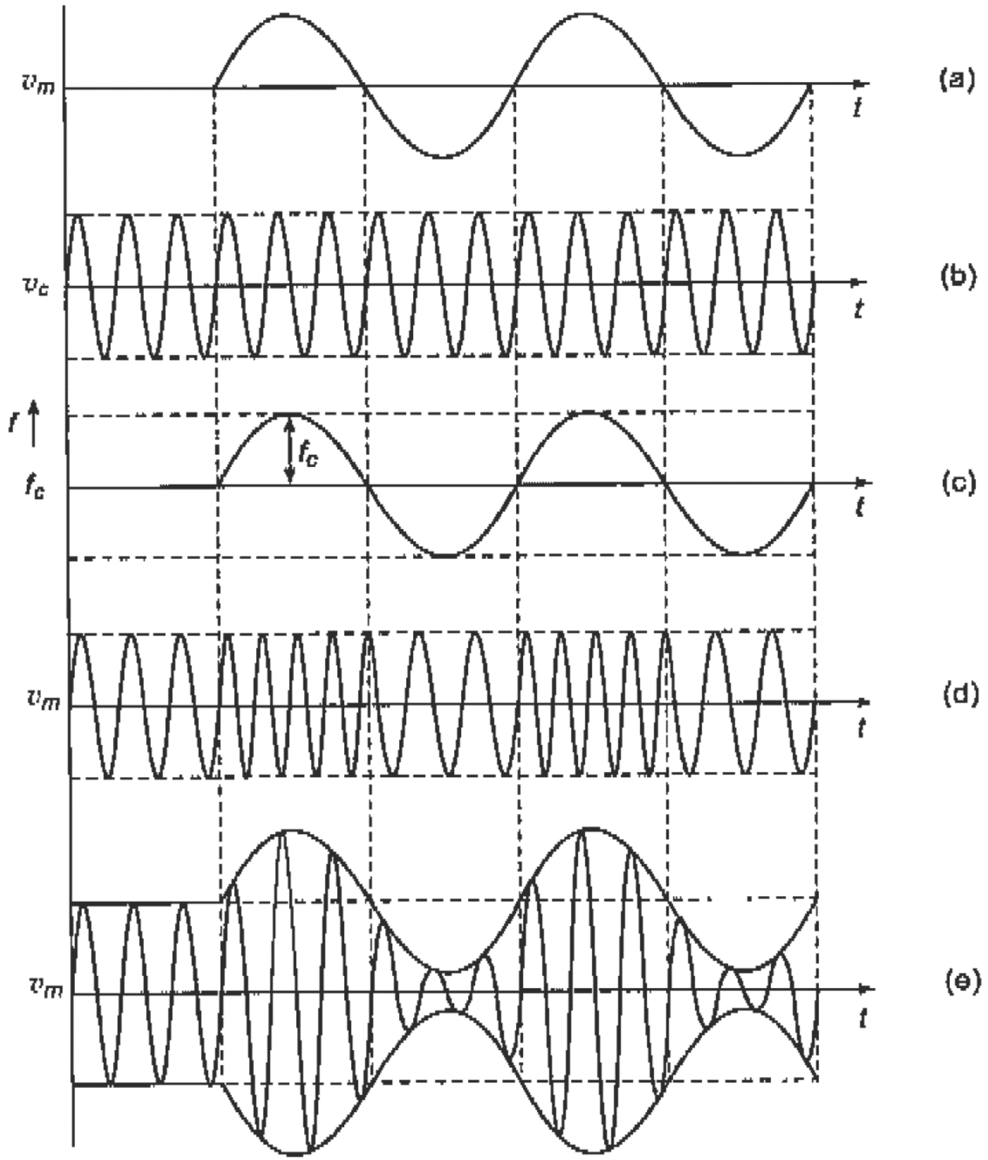
Frequency modulation [FM] :

When frequency of the Carrier signal Varies as per the amplitude Variations of modulating signal, then it is Called FM.
In FM, amplitude of the modulated Carrier remains Constant.

phase modulation [PM]

When the phase of the Carrier Varies as per amplitude Variations of modulating signal, then it is Called phase modulation (PM).
In PM, amplitude of the modulated Carrier remains Constant.

Representation of FM signal



AM and FM Signals. (a) Message, (b) Carrier, (c) Frequency deviation, (d) FM and (e) AM.

Frequency Modulation – Time domain representation

$$E_{FM}(t) = E_c \cos f_i(t) \quad \text{—} \quad (1)$$

$f_i(t)$ – Instantaneous frequency.

$$f_i(t) = \omega_c t + 2\pi \int_0^t k_f \, E_m(t) . dt$$

$$= \omega_c t + 2\pi \int_0^t k_f \, E_m \cos \omega_m t . dt$$

$$= \omega_c t + 2\pi k_f E_m \int_0^t \cos \omega_m t . dt$$

$$= \omega_c t + 2\pi k_f E_m . \left[\frac{\sin \omega_m t}{\omega_m} \right]_0^t$$

Frequency Modulation – Time domain representation

$$= \omega_c t + 2\pi k_f \frac{E_m}{\omega_m} \cdot \sin \omega_m t$$

$$= \omega_c t + \frac{2\pi k_f E_m}{2\pi f_m} \sin \omega_m t$$

$$\therefore f_i(t) = \left[\omega_c t + \frac{k_f E_m}{f_m} \sin \omega_m t \right] \quad \text{—— (2)}$$

Sub equ (2) in (1)

$$E_{FM}(t) = E_c \cos \left[\omega_c t + \frac{k_f E_m}{f_m} \sin \omega_m t \right]$$

$$\boxed{E_{FM}(t) = E_c \cos \left[\omega_c t + m_f \sin \omega_m t \right]} \quad \text{—— (3)}$$

Frequency Modulation – Modulation Index

- The modulation index of FM, m_f is given by

$$m_f \ (or)\ \beta = \frac{(\text{maximum})\ \text{frequency deviation}}{\text{modulating frequency}} = \frac{\Delta_f}{f_m}$$

$k_f \rightarrow$ Frequency deviation sensitivity (or) frequency deviation

(or) deviation sensitivity (a constant)

$$\boxed{\Delta f = k_f \cdot E_m} \Rightarrow \text{Frequency deviation}$$

$$\boxed{m_f = \frac{k_f E_m}{f_m} = \frac{\Delta f}{f_m}} \Rightarrow \text{Modulation index of Frequency modulation}$$

Frequency Modulation – Time domain representation

Deviation Ratio : [DR]

$$DR = \frac{\text{Maximum frequency deviation}}{\text{Maximum modulating index}} = \frac{\Delta f\,(max)}{f_m\,(max)}$$

$\Delta f\,(max)$ is Constant Value = 75kHz

if $f_{m(max)}$ = 15kHz

$$DR = \frac{75kHz}{15kHz} = 5.$$

Frequency Modulation – Time domain representation

Percentage of Modulation of FM:

$$\% \text{ of modulation} = \frac{\Delta f \text{ (actual)}}{\Delta f \text{ (max.)}} \times 100$$

Eg. if Δf (actual) = 20 kHz. & Δf (max) = 75 kHz (Constant

$$\left.\begin{array}{l}\text{Percentage} \\ \text{of modulation}\end{array}\right\} = \frac{20\text{ kHz}}{75\text{ kHz}} \times 100 = 26.67\%$$

Numerical

In a FM system, when the audio frequency (AF) is 500 Hz, and the AF voltage is 2.4 V, the deviation is 4.8 kHz. If the AF voltage is now increased to 7.2 V, what is the new deviation? If the AF voltage is further raised to 10 V while the AF is dropped to 200 Hz, what is the deviation? Find the modulation index in each case.

Solution

Case 1 $f_{m1} = 500$ Hz, $V_{m1} = 2.4$ V and $\delta_{f1} = 4.8$ kHz.

Using this we can compute the proportionality constant k_f given by $k_f = \dfrac{\delta_{f1}}{V_{m1}} = \dfrac{4.8}{2.4} = 2$ kHz/V

The modulation index $m_{f1} = \dfrac{\delta_{f1}}{f_{m1}} = \dfrac{4.8}{0.5} = 9.6$

Case 2 $f_{m2} = 500$ Hz, $V_{m2} = 7.2$ V

$\delta_{f2} = k_f \times V_{m2} = 2 \times 7.2 = 14.4$ kHz.

The modulation index $m_{f2} = \dfrac{\delta_{f2}}{f_{m2}} = \dfrac{14.4}{0.5} = 28.8$

Case 3 $f_{m3} = 200$ Hz, $V_{m2} = 10$ V

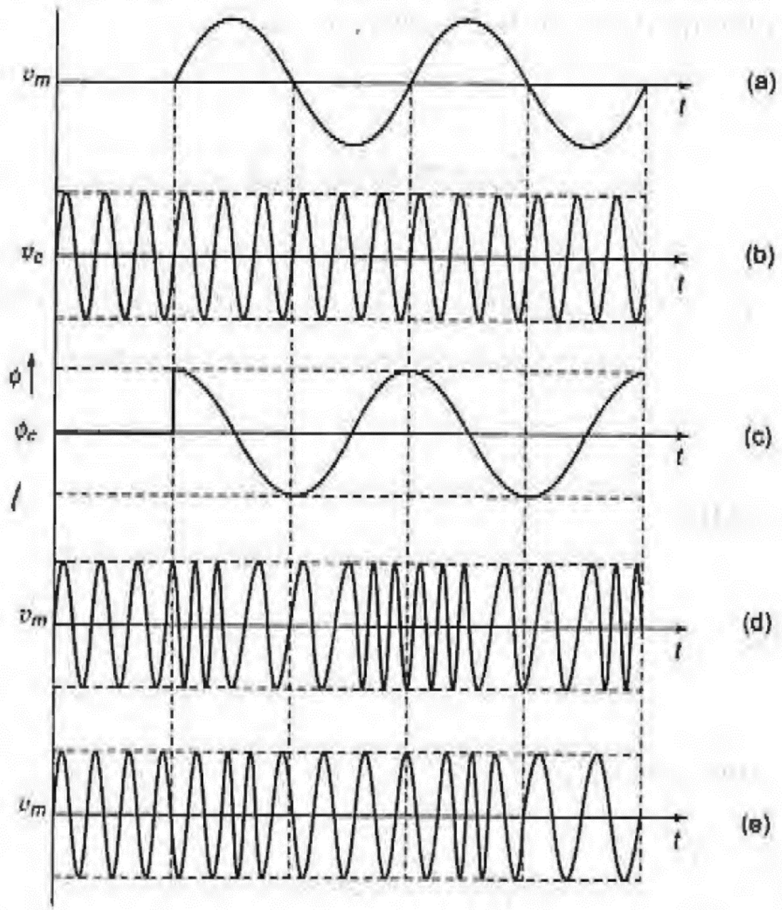
$\delta_{f3} = k_f \times V_{m3} = 2 \times 10 = 20$ kHz.

The modulation index $m_{f3} = \dfrac{\delta_{f3}}{f_{m3}} = \dfrac{20}{0.2} = 100$

Note that the change in modulating frequency made no difference to the deviation since it is independent of the modulating frequency. Alternatively, the modulating frequency change did have to be taken into account in the modulation index calculation.

Phase Modulation (PM)

- *Phase Modulation* is a system in which the amplitude of the modulated carrier is kept constant, while its phase and rate of phase change are varied by the modulating signal.



PM and FM Signals. (a) Message, (b) Carrier, (c) Phase deviation, (d) PM and (e) FM.

Phase Modulation – Time domain representation

$$E_{PM}(t) = E_c \cos \theta_i(t) \quad \text{--- (1)}$$

$\theta_i(t) \rightarrow$ Instantaneous phase

$$\theta_i(t) = (\omega_c t + k_p E_m(t))$$

$k_p \rightarrow$ phase sensitivity / phase deviation.
deviation

$$\theta_i(t) = [\omega_c t + k_p E_m \cos \omega_m t]$$

$$\theta_i(t) = [\omega_c t + m_p \cos \omega_m t] \quad \text{--- (2)}$$

Phase Modulation – Time domain representation

Sub equ (2) in equ (1)

$$E_{PM}(t) = E_c \cos [w_c t + m_p \cos w_m t] \quad \text{—— (3)}$$

$m_p = k_p E_m$

$m_p \rightarrow$ modulation index of phase modulation.

$\therefore m_p \propto E_m$.

Equ (3) $\rightarrow$ repr time domain repr of phase modulated signal.

(or)

$$\therefore E_{PM}(t) = E_c \cos [w_c t + m_p \sin w_m t]$$

phase deviation:

The phase angle of the carrier signal varies from its unmodulated signal during modulation process is known as phase deviation.

$$\boxed{k_p = \frac{m_p}{E_m}}$$

k_p (or) $\Delta\theta$.

FM and PM Relations

Phase Modulation(PM)

It is a form of angle modulation in which the angle $\theta_i(t)$ is varied **linearly** with the message signal m(t),

$$\theta_i(t) = 2\pi f_C\, t + \phi(t)$$

carrier

not a constant

Where $\phi(t) = k_P m(t)$

$$\boxed{\therefore \theta_i(t) = 2\pi f_C t + k_P m(t)} \quad (2.3)$$

The term $2\pi f_C t$ represents the angle of the carrier ; the constant k_P **represents the phase sensitivity** of the modulator , expressed in radians per volt on the assumption that m(t) is a voltage waveform.

The phase modulated signal s(t) is thus described in the time domain by

$$\boxed{S(t) = A_C \cos[2\pi f_C t + k_P m(t)]} \quad (2.4)$$

FM and PM Relations

Frequency modulation(FM)

Frequency modulation is that from of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the message signal m(t):

$$\boxed{f_i(t) = f_c + k_f m(t)} \quad (2.5)$$

The term f_c represents the frequency of the unmodulated carrier and the constant k_f represents the **frequency sensitivity** of the modulator, expressed in hertz per volt on assumption that m(t) is a voltage waveform .

Integration equation (2.5)with respect to time we get,(after multiplying by2 π).

$$\boxed{\theta_i(t) = 2\pi f_c + 2\pi k_f \int_0^t m(\tau)d\tau} \quad (2.6)$$

FM and PM Relations

W.K.T,

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = 2\pi f_i(t)$$

Integrating on both Sides w.r.t. to 't'

$$\theta(t) = \int_0^t 2\pi f_i(t) \cdot dt$$

$$\theta(t) = \int_0^t 2\pi \left[f_c + K_f m(t) \right] dt$$

$$= \int_0^t 2\pi f_c \, dt + \int_0^t 2\pi K_f \, m(t) \cdot dt$$

$$= 2\pi f_c \int_0^t (1) \, dt + 2\pi K_f \int_0^t m(t) \cdot dt$$

$$\boxed{\theta(t) = 2\pi f_c t + 2\pi K_f \int_0^t m(t) \cdot dt}$$

FM and PM Relations

$$s(t) = A_C \cos(\theta_i(t))$$

$$\boxed{s(t) = A_C \cos\left[2\pi f_C t + 2\pi k_f \int_0^t m(c)dc\right]} \qquad (2.7)$$

*In summary (see equation 2.4 & 2.7)

Angle modulated wave

$$s(t) = A_C \cos(2\pi f_c t + \phi(t)) \qquad (2.8)$$

Where

$$\phi(t) = \begin{cases} k_p m(t) & \leftarrow \text{PM} \\ 2\pi k_f \int_0^t m(c)dc & \leftarrow \text{FM} \end{cases} \qquad (2.9)$$

$$\downarrow$$

$$\frac{d\phi(t)}{dt} = \begin{cases} k_P \dfrac{d}{dt} m(t) & \leftarrow \text{PM} \\ 2\pi k_f m(t) & \leftarrow \text{FM} \end{cases} \qquad (2.10)$$

❖ **An FM wave is defined by**

$$S(t) = A_c\cos[10\pi t + \sin(4\pi t)]$$

find the instantaneous frequency of S(t).

Sol :- W.K.T $f_i(t) = \frac{1}{2\pi} \frac{d}{dt}\theta(t)$ Hz &

$$S(t) = A_c \cos[\theta(t)]$$

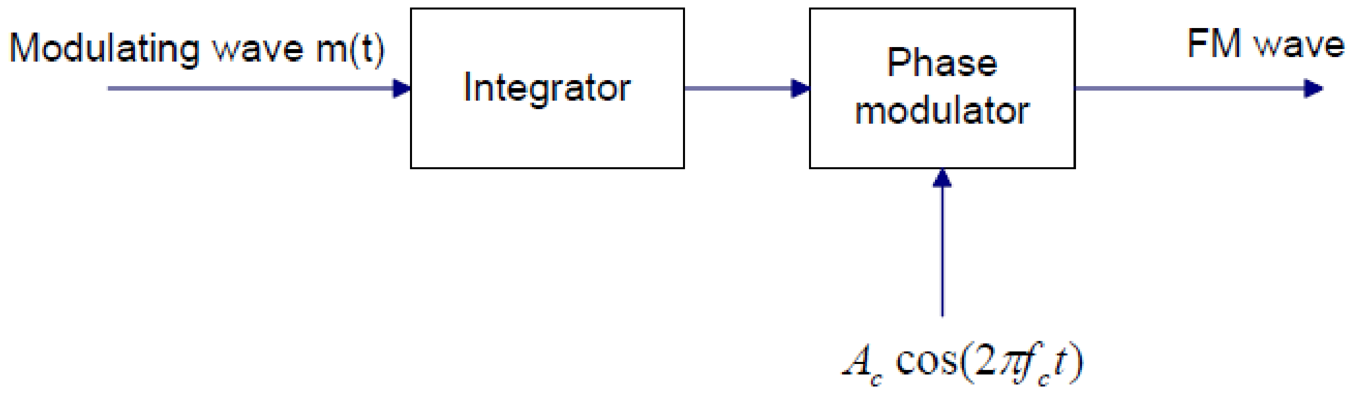
$$\therefore \theta(t) = 10\pi t + \sin 4\pi t$$

$$f_i(t) = \frac{1}{2\pi} \frac{d}{dt}[10\pi t + \sin 4\pi t] \text{ Hz}$$

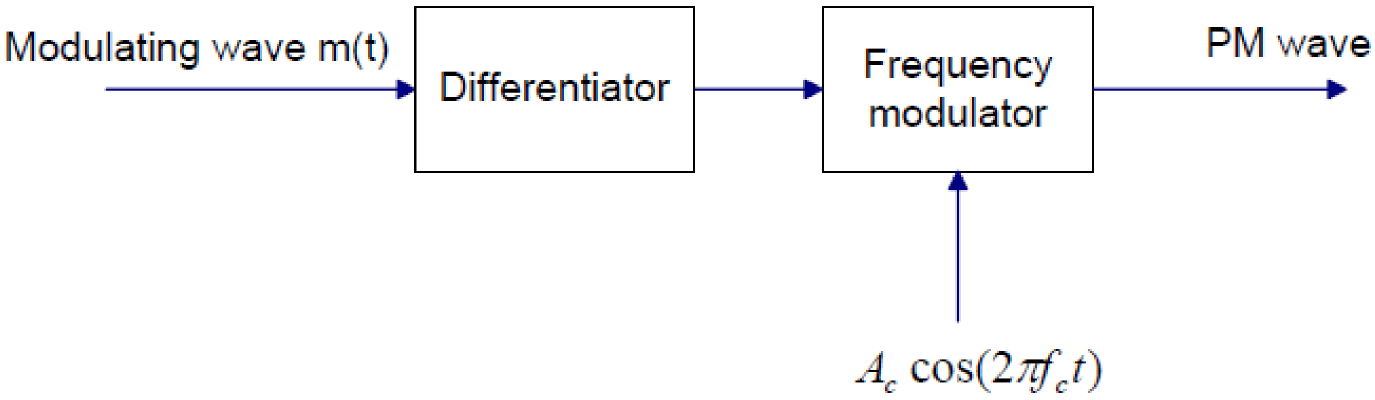
$$= \frac{1}{\cancel{2\pi}}[\overset{5}{\cancel{10\pi}} + \cos(4\pi t) \cdot \overset{2}{\cancel{4\pi}}] \text{ Hz}$$

$$\boxed{f_i(t) = 5 + 2\cos(4\pi t) \text{ Hz}}$$

FM and PM Generation

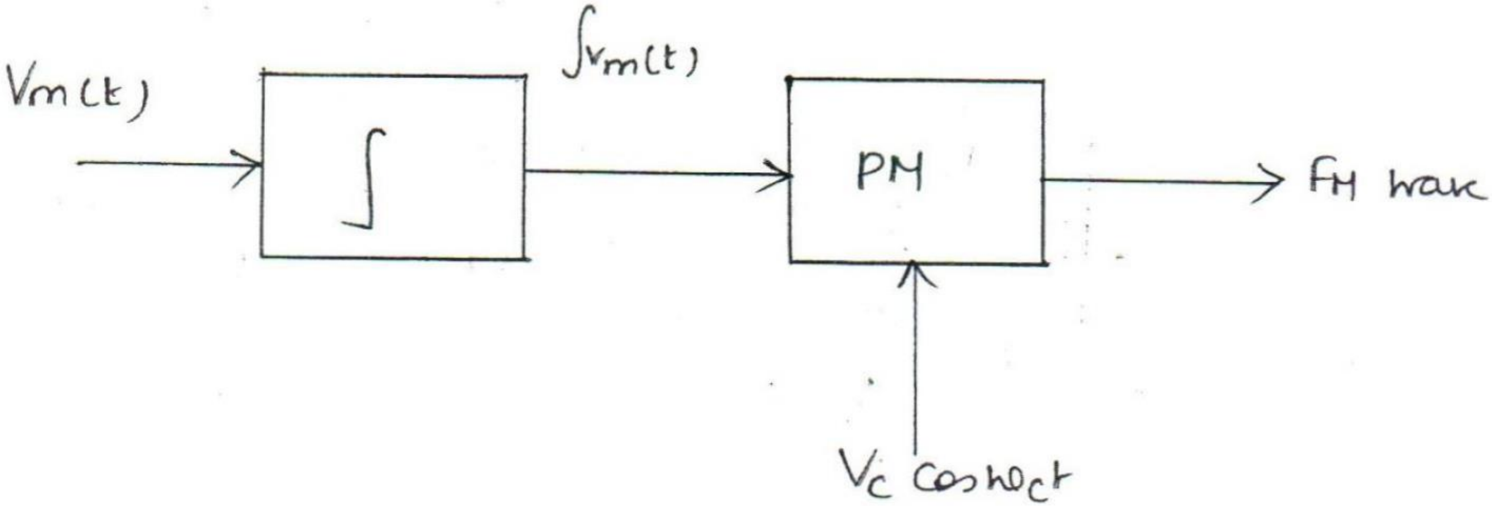


(Scheme for generating an FM wave)



(Scheme for generating a PM wave)

Generation of FM from PM

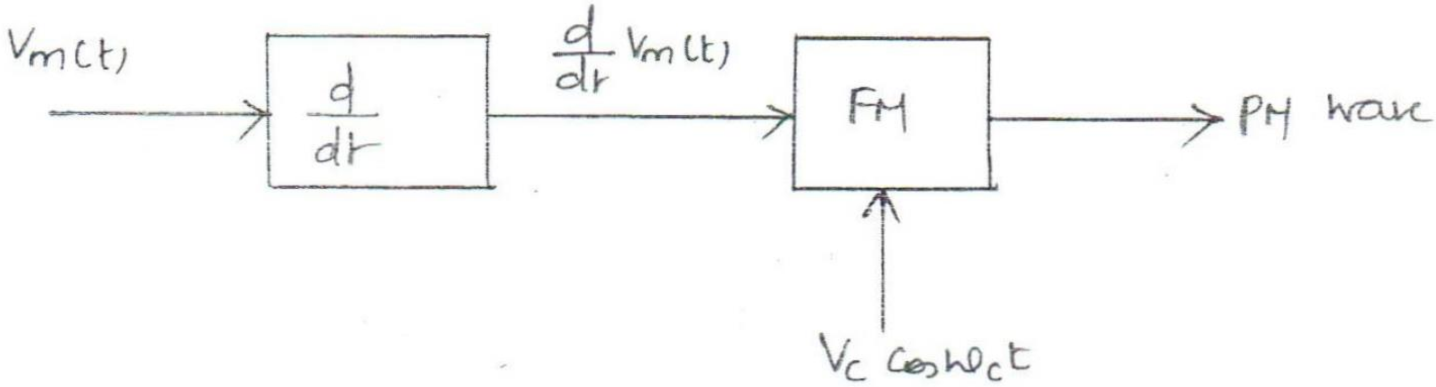


$$V_{PM}(t) = \overset{E_c}{V_c} \cos\left[\omega_c t + k_p \overset{E_m}{V_m} \cos\omega_m t\right]$$

$$V_{FM}(t) = \overset{E_c}{V_c} \cos\left[\omega_c t + k_p \int_0^t \overset{E_m}{V_m} \cos\omega_m t\right]$$

$$\boxed{V_{FM}(t) = \overset{E_c}{V_c} \cos\left[\omega_c t + \frac{k_p \overset{E_m}{V_m}}{\omega_m} \sin\omega_m t\right]}$$

Generation of PM from FM



$$V_{FM}(t) = \overset{E_c}{V_c} \cos\left[\omega_c t + \frac{k_f \overset{E_m}{V_m}}{\omega_m} \sin \omega_m t\right]$$

$$V_{PM}(t) = \overset{E_c}{V_c} \cos\left[\omega_c t + \frac{k_f \overset{E_m}{V_m}}{\omega_m} \frac{d}{dt}(\sin \omega_m t)\right]$$

$$= \overset{E_c}{V_c} \cos\left[\omega_c t + \frac{k_f \overset{E_m}{V_m}}{\cancel{\omega_m}} \cdot \cos \omega_m t \cdot \cancel{\omega_m}\right]$$

$$\boxed{V_{PM}(t) = \overset{E_c}{V_c} \cos\left[\omega_c t + k_f \overset{E_m}{V_m} \cos \omega_m t\right]}$$

Spectral Analysis of FM

We propose two simple cases for the spectral analysis of an FM signal:

(1) A single tone modulation that produces a narrow FM signal.
(2) A single tone modulation that produces wideband FM signal.

$$\Rightarrow \theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$$

β is measured in radians.

$$\beta = \frac{\Delta f}{f_m}$$

Note: $\beta = m_f$
$A_c = E_c$

The FM signal is given by: $s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)$ (2.17).

Depending on the value of the modulation index β, we may distinguish two cases of frequency modulation:

- **Narrowband FM,** for which β is small compared to one radian.
- **Wideband FM,** for which β is large compared to one radian.

The quantity $\Delta f = k_f A_m$ is called the '**frequency deviation**', representing the **maximum departure** of the instantaneous frequency of the FM signal from the carrier frequency f_c.

$$\boxed{\Delta f = k_f A_m} \qquad (2.14).$$

Types of FM

Depending on the Value of modulation index, FM is classified into 2 types.

1. Narrowband FM
2. Wideband FM

Narrowband FM:

When the Value of modulation index m_f is Smaller than one radian in a FM, It's Called as narrowband FM, (i.e) $m_f << 1$.

Let the message S/g be rep^r as,

$$e_m(t) = E_m \sin \omega_m t$$

Let the Carrier S/g be rep^r as,

$$e_c(t) = E_c \sin(\omega_c t + \phi)$$

Narrowband FM

$$e_c(t) = E_c \sin\theta \quad , \quad \text{where } \theta = (\omega_c t + \phi)$$

$$\frac{d\theta}{dt} = \omega_c \longrightarrow \text{angular frq. of Carrier s/g}$$

After frequency modulation,

$$\omega_i = \omega_c + k\, e_m(t)$$

$$= \omega_c + k\, E_m \sin\omega_m t$$

The frequency deviation is maximum, when

$$\sin\omega_m t = \pm 1 \quad , \quad \text{hence } \omega_i = \omega_c \pm k E_m .$$

Narrowband FM

The frequency deviation is proportional to the amplitude of modulating voltage, hence it can be written as,

$$2\pi \Delta f = k E_m.$$

$$\therefore \omega_i = \omega_c \pm 2\pi\Delta f \sin \omega_m t$$

$$\phi_i = \int \omega_i \, dt$$

$$= \int (\omega_c \pm 2\pi\Delta f \sin \omega_m t) \, dt$$

$$\phi_i = \omega_c t \pm \frac{\Delta f}{f_m} \sin \omega_m t.$$

Narrowband FM

$$e_{FM}(t) = E_c \sin \phi_i t$$

$$= E_c \sin\left(\omega_c t \pm \frac{\delta f}{f_m} \sin \omega_m t\right)$$

$$= E_c \sin(\omega_c t \pm m_f \sin \omega_m t)$$

$$e_{FM}(t) = E_c \sin \omega_c t \cdot \cos(m_f \sin \omega_m t) + E_c \cos \omega_c t \cdot \sin(m_f \sin \omega_m t)$$

For Narrow band FM assume the modulation index m_f is small compared to one radian, hence we may use the following approximation

$$\cos(m_f \sin \omega_m t) = 1$$

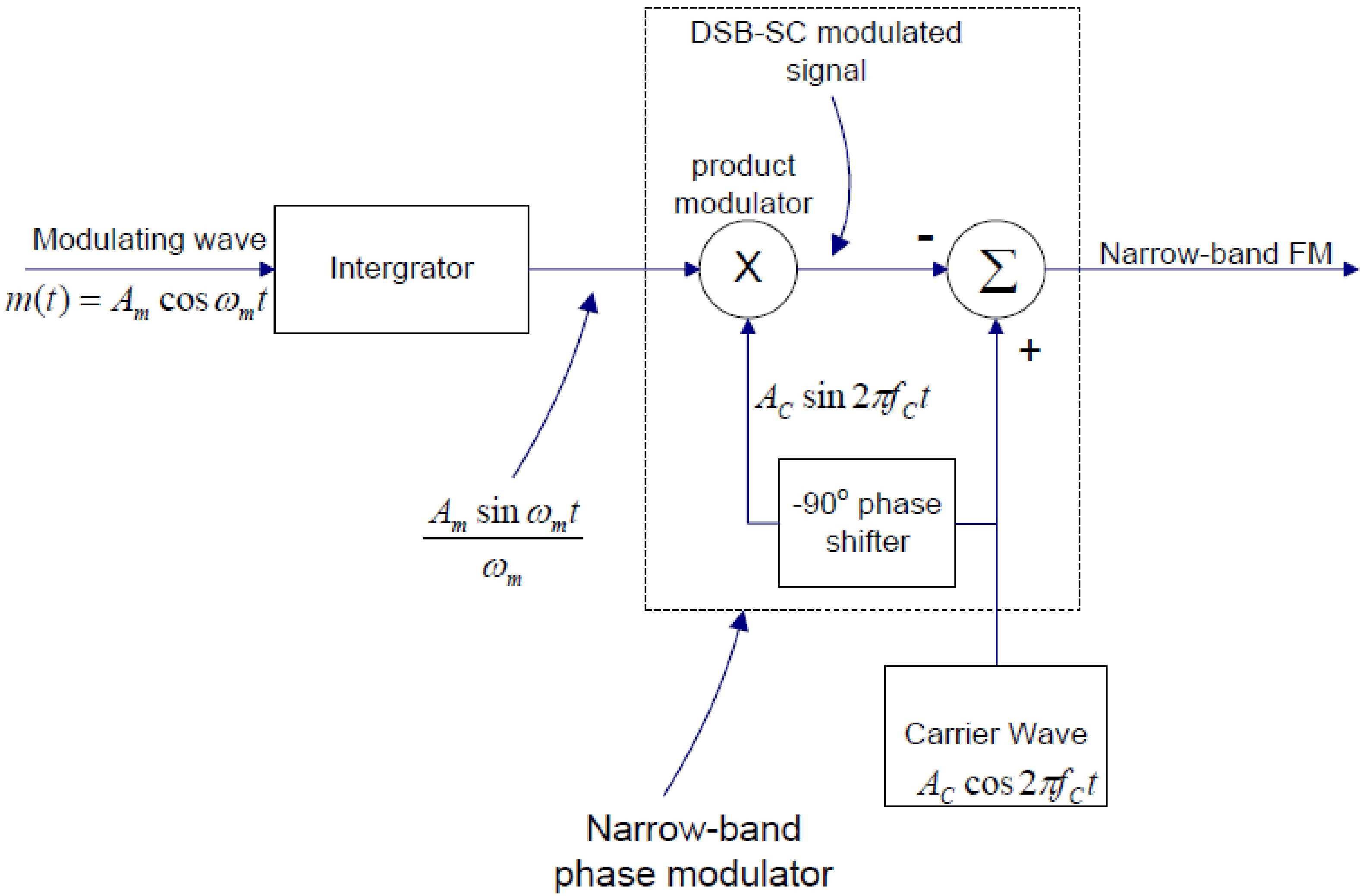
$$\sin(m_f \sin \omega_m t) = m_f \sin \omega_m t$$

[bz $\cos\theta = 1$ & $\sin\theta = \theta$, if 'θ' is small]

∴ Hence we get,

$$\boxed{e_{NBFM}(t) = E_c \sin \omega_c t + E_c \cos \omega_c t \, (m_f \sin \omega_m t)}$$

Narrowband FM – Example with Cosine Input



Method for generating narrowband FM signal.

Fig 2.4.

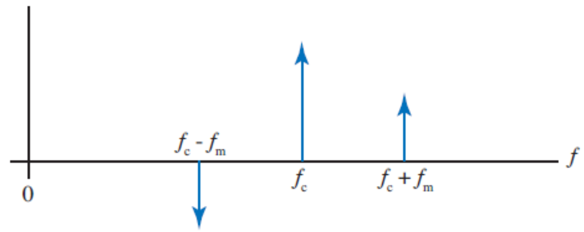
Narrowband FM

- $s(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t).\sin(2\pi f_m)t$

Narrowband FM signal

Note: $\beta = m_f$

$A_c = E_c$



Single tone NBFM spectra

- $$s(t) = A_c \cos 2\pi f_c t + \frac{1}{2}\beta A_c\left[\cos 2\pi(f_c + f_m)t - \cos 2\pi(f_c - f_m)t\right] \qquad (2.10)$$

Narrowband FM signal

- Consider an AM signal equation (page26, equation12)

$$s(t) = A\cos(2\pi f_c t) + \frac{MA_c}{2}\left[\cos 2\pi(f_c + f_m)t + \cos 2\pi(f_c - f_m)t\right] \qquad (2.11)$$

μ – modulation factor of AM

AM signal

Comparing equation 2.10$ 2.11, we see that in the case of sinusoidal modulation, the Basic difference between an AM signal $ a narrowband FM signal is that the algebraic sign of the lower side frequency in the narrowband FM is reversed.
Thus, a narrowband FM signal requires essentially the same transmission band width (i.e.2 f_m) as the AM signal.

Wideband FM

$$V_{FM}(t) = E_c \cos(\omega_c t + m_f \sin\omega_m t) \quad \text{— (1)}$$

Expⁿ form,

$$V_{FM}(t) = Re\left[E_c \, e^{j(\omega_c t + m_f \sin\omega_m t)}\right] \quad \text{— (2)}$$

$$x(t) = E_c \, e^{j m_f \sin\omega_m t} \quad \text{— (3)}$$

Sub equ (3) in (2)

$$V_{FM}(t) = Re\left[x(t) \, e^{j\omega_c t}\right] \quad \text{— (4)}$$

$x(t)$ can be expᵈ with the help of Fourier series,

$$x(t) = \sum_{n=-\alpha}^{\alpha} C_n \, e^{j 2\pi f_m n t} \quad \text{— (4)}$$

Where,

$$C_n = f_m \int_{-f_m/2}^{f_m/2} x(t) \, e^{-j 2\pi f_m n t} \cdot dt \quad \text{— (5)}$$

Wideband FM

sub ③ in ⑤

$$C_n = f_m \int_{-f_m/2}^{f_m/2} E_c\, e^{j m_f \sin \omega_m t} \cdot e^{-j 2\pi f_m n t} \cdot dt$$

let, $y = 2\pi f_m t$, hence limits will change from $-\pi$ to π

$$\frac{dy}{dt} = 2\pi f_m \,, \qquad dt = \frac{dy}{2\pi f_m}$$

$$C_n = f_m \int_{-\pi}^{\pi} E_c\, e^{j(m_f \sin y - ny)} \cdot \frac{dy}{2\pi f_m}$$

$$C_n = \frac{\cancel{f_m}\, E_c}{2\pi \cancel{f_m}} \cdot \int_{-\pi}^{\pi} e^{j(m_f \sin y - ny)} \cdot dy$$

Wideband FM

The abov integral is known as the n^{th} order Bessel fn' of the first kind. & its gn. is

$$C_n = E_c J_n(m_f) \quad \text{——— (6)}$$

$$J_n(m_f) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(m_f \sin y - ny)}\,dy$$

Sub (6) in (4)

$$x(t) = \sum_{n=-\alpha}^{\alpha} E_c J_n(m_f)\, e^{j2\pi f_m n t} \quad \text{——— (7)}$$

Sub (7) in (1)

$$V_{FM}(t) = Re\left[\sum_{n=-\alpha}^{\alpha} E_c J_n(m_f)\, e^{j2\pi f_m n t} \cdot e^{j\omega_c t}\right]$$

$$= E_c \sum_{n=-\alpha}^{\alpha} Re\left(J_n(m_f)\, e^{j2\pi(nf_m + f_c)t}\right]$$

Wideband FM

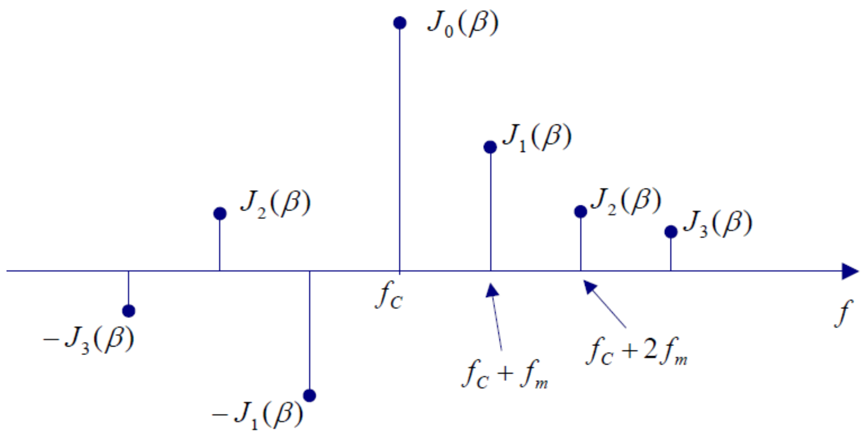
$$= E_c \sum_{n=-\alpha}^{\alpha} J_n(m_f) \cos\left(2\pi(f_c + nf_m)t\right)$$

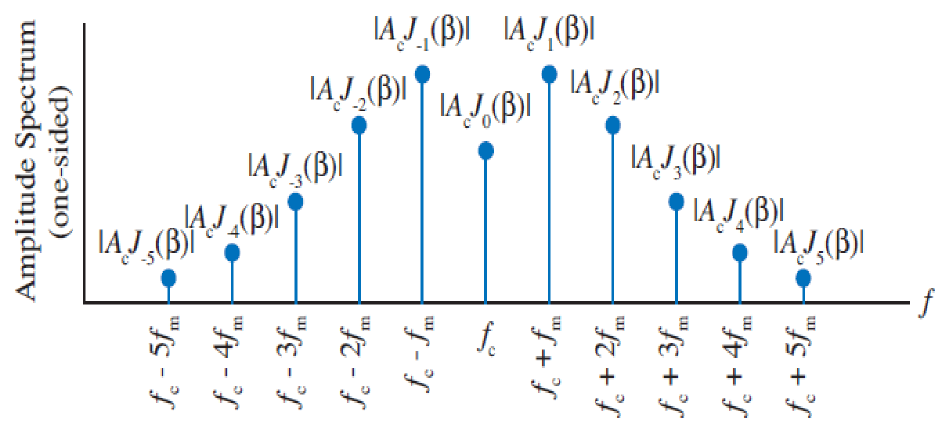
$$V_{WBFM}(t) = E_c \sum_{n=-\alpha}^{\alpha} J_n(m_f) \cos\left[2\pi(f_c + nf_m)t\right] \quad \text{--- } (8)$$

Fourier series.

Taking ~~FT~~ of equ (8)

$$\boxed{V_{WBFM}(t) = \frac{E_c}{2} \sum_{n=-\alpha}^{\alpha} J_n(m_f) \left\{ \delta(f - (f_c - nf_m)) + \delta(f + (f_c + nf_m)) \right.}$$





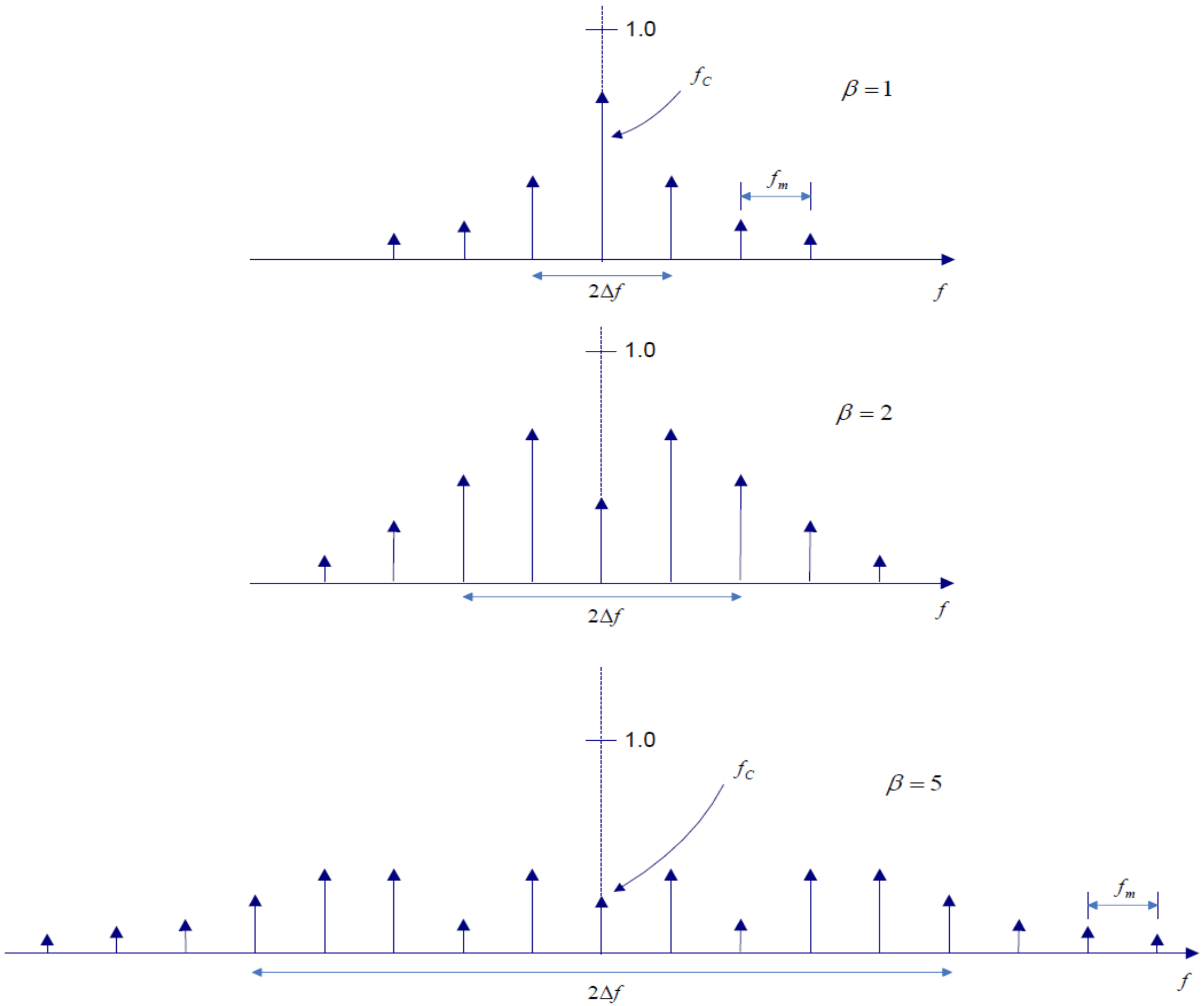
Bessel Coefficients

General form,

$$J_n(m_f) = \left(\frac{m_f}{2}\right)^n \left[\frac{1}{n!} - \frac{(m_f/2)^2}{1!\,(n+1)!} + \frac{(m_f/2)^4}{2!\,(n+2)!} + \cdots\right]$$

m_f	$J_0(m_f)$	$J_1(m_f)$	$J_2(m_f)$	$J_3(m_f)$	$J_4(m_f)$	$J_5(m_f)$	$J_6(m_f)$
0	1	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—
0.5	0.98	0.24	0.03	—	—	—	—
1	0.77	0.44	0.11	0.02	—	—	—
2	0.22	0.58	0.35	0.13	0.03	—	—

Wideband FM – Spectrum of FM signal



Carrier and sideband amplitudes for different modulation indices of FM signals

Modulation index	Sideband amplitude																
	Carrier	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.00	1.00																
0.25	0.98	0.12															
0.5	0.94	0.24	0.03														
1.0	0.77	0.44	0.11	0.02													
1.5	0.51	0.56	0.23	0.06	0.01												
2.0	0.22	0.58	0.35	0.13	0.03												
2.41	0	0.52	0.43	0.20	0.06	0.02											
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01										
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01										
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02									
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02								
5.53	0	−0.34	−0.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01							
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02							
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02						
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03					
8.65	0	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02				
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01			
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01		
12.0	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01

Bessel Coefficients - Properties

i). For small values of m_f $(m_f < 1)$, $J_0(m_f) \approx 1$

ii). For some values of m_f, the carrier component disappear completely, these values are called eign values.

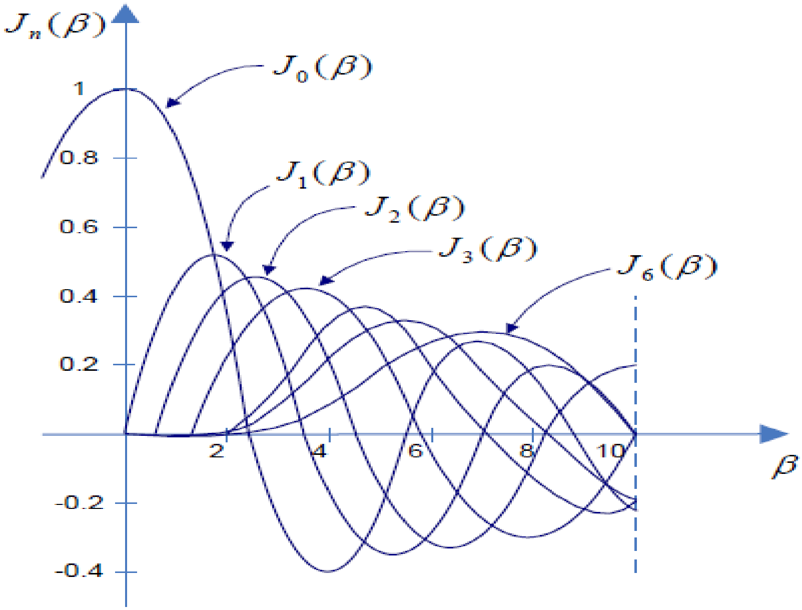
iii). $J_1(m_f) \approx \frac{m_f}{2}$

iv). $J_n(m_f) \approx 0 \quad n > 2$

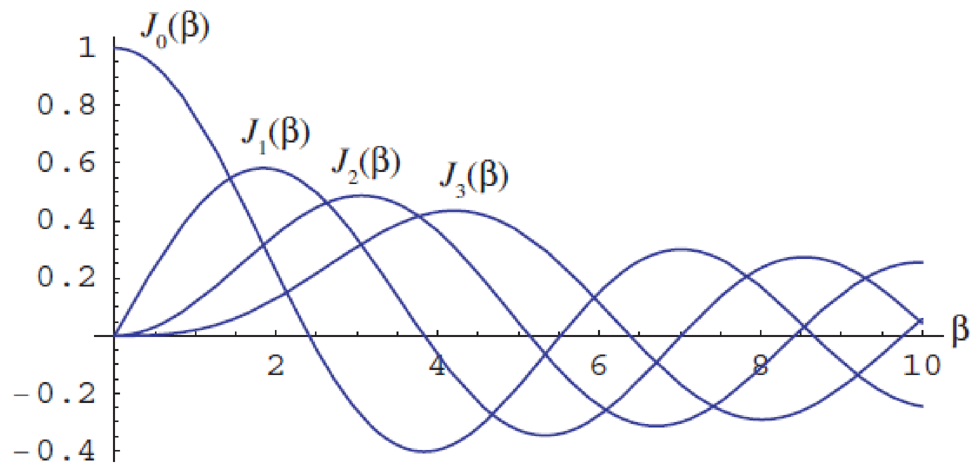
v). $\sum_{n=-\alpha}^{\alpha} J_n^2(m_f) = 1$.

Wideband FM - Spectrum

Figure below shows the Bessel function $J_n(\beta)$ versus modulation index β for different positive integer value of n.



Bessel function for n=0 to n=6



Bessel function of order 0–3 plotted

- The zeros of the Bessel functions are important in spectral analysis

First five Bessel function zeros for order 0 – 5
$J_0(\beta) = 0$
2.40483, 5.52008, 8.65373, 11.7915, 14.9309
$J_1(\beta) = 0$
3.83171, 7.01559, 10.1735, 13.3237, 16.4706
$J_2(\beta) = 0$
5.13562, 8.41724, 11.6198, 14.796, 17.9598

Comparison of NBFM vs WBFM

S.No	Narrow band FM	Wideband FM
1.	Modulation index $m_f < 1$	Modulation index $m_f > 1$
2.	Spectrum Contains 2-Sidebands & Carrier	Spectrum Contains infinite number of sidebands & Carrier.
3.	$BW = 2f_m$	$BW = 2(\delta + f_m)$
4.	Maximum deviation : 5 kHz	Maximum deviation : 75 kHz
5.	Range of modulating frequency 30 Hz to 3 kHz	Range of modulating frequency is 30 kHz to 5 MHz
6.	Noise Suppression is less	Noise Suppression is more
7.	It's used for mobile Communication	It's used for broadcasting & entertainment

Power Distribution of FM

Total power, $P_t = P_c + P_{sidebands}$

$$P_t = P_c + \left[P_1 + P_2 + P_3 + \cdots + P_n\right] \quad \text{——} \quad (1)$$

$$P_c = \frac{V_c^2 J_0^2(m_f)}{2R}$$

$$P_1 = \frac{V_{1\,USB}^2}{2R} + \frac{V_{1\,LSB}^2}{2R}$$

$$P_1 = \frac{V_c^2 J_1^2(m_f)}{2R} + \frac{V_c^2 J_1^2(m_f)}{2R}$$

Power Distribution of FM

$$P_1 = \frac{2V_c^2 J_1^2(m_f)}{2R} = \frac{V_c^2 J_1^2(m_f)}{R}$$

$||^{ly}$, $$P_2 = \frac{V_c^2 J_2^2(m_f)}{R}$$

$\therefore$ equ (1) becomes,

$$P_t = \frac{V_c^2}{2R} J_0^2(m_f) + \frac{V_c^2}{R} J_1^2(m_f) + \dots$$

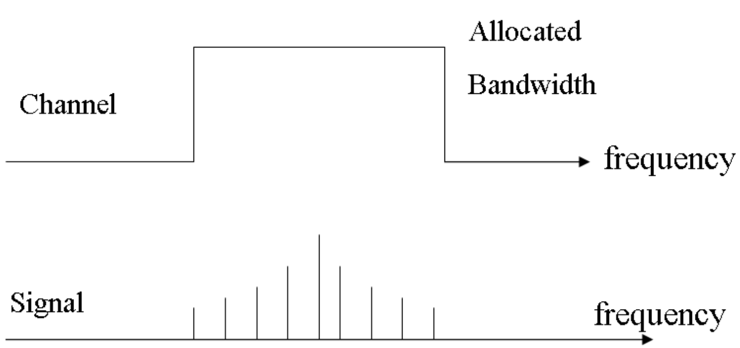
$$= \frac{V_c^2}{2R}\left[J_0^2(m_f) + 2J_1^2(m_f) + 2J_2^2(m_f) + \dots\right]$$

$$P_t = \frac{V_c^2}{2R} \sum_{n=0}^{\alpha} J_n^2(m_f)$$

$\therefore$ $\boxed{P_t = \frac{V_c^2}{2R}}$ watts, $\because \sum_{n=0}^{\alpha} J_n^2(m_f) \approx 1$

Power in FM Signals

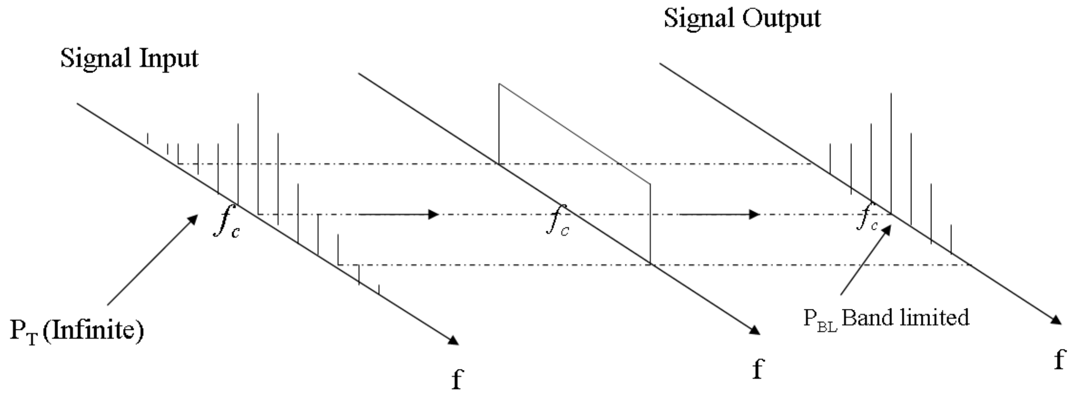
- Now consider – if we generate an FM signal, it will contain an infinite number of sidebands.
- However, if we wish to transfer this signal, *e.g.* over a radio or cable, this implies that we require an infinite bandwidth channel.
- Even if there was an infinite channel bandwidth it would not all be allocated to one user.
- Only a limited bandwidth is available for any particular signal. Thus we have to make the signal spectrum fit into the available channel bandwidth.
- We can think of the signal spectrum as a 'train' and the channel bandwidth as a tunnel – obviously we make the train slightly less wider than the tunnel if we can.



Power in FM Signals

- However, many signals (*e.g.* FM, square waves, digital signals) contain an infinite number of components.
- If we transfer such a signal via a limited channel bandwidth, we will lose some of the components and the output signal will be distorted.
- If we put an infinitely wide train through a tunnel, the train would come out distorted, the question is how much distortion can be tolerated?

Generally speaking, spectral components decrease in amplitude as we move away from the spectrum 'centre'.



Carson's Rule

It states that bandwidth of FM signal is equal to twice the sum of frequency deviation and maximum modulating frequency.

Maximum modulating frequency

$$\text{Bandwidth} = 2(\Delta f + f_m)$$

freq deviation

Numerical

Find the carrier and modulating frequencies, the modulation index, and the maximum deviation of the FM represented by the voltage equation $v = 12\sin(6 \times 10^8 t + 5\cos 1250t)$. What power will this FM wave dissipate in a 10 Ω resistor?

Solution

$$f_c = \frac{6 \times 10^8}{2\pi}\ 95.5\ \text{MHz.}$$

$$f_m = \frac{1250}{2\pi}\ 199\ \text{Hz.}$$

$$m_f = 5.$$

$$\delta_f = m_f f_m = 5 \times 199 = 995\ \text{Hz.}$$

$$P = \frac{V_{rms}^2}{R} = \frac{\left(12/\sqrt{2}\right)^2}{10} = \frac{72}{10} = 7.2\ \text{W.}$$

Numerical

(1) The Carrier is frequency modulated with the Sinusoidal signal of 2kHz resulting in a max. freq. deviation of 5kHz. Calculate the modulation index.

Sol.

$f_m = 2\text{kHz}$, $\Delta f = 5\text{kHz}$

$$m_f = \frac{\Delta f}{f_m} = \frac{5}{2} = 2.5$$

Numerical on FM

(2). An FM wave is repr by the equation $V_{FM}(t) = 10 \sin\left[8\times10^6 t + 6 \sin 3\times10^4 t\right]$

Calculate,

i). modulating frequency, (ii). Carrier frequency

iii). Modulating index, (iv). frequency deviation

Sol.

$$e_{FM}(t) = E_c \sin\left[2\pi f_c t + \frac{kE_m}{2\pi f_m} \cdot \sin 2\pi f_m t\right]$$

$$m_f = \frac{kE_m}{2\pi f_m}.$$

$$e_{FM}(t) = E_c \sin\left[2\pi f_c t + m_f \sin 2\pi f_m t\right]$$

$E_c = 10$, $2\pi f_c = 8\times10^6$, $2\pi f_m = 3\times10^4$, $m_f = 6$.

Numerical on FM

i) $2\pi f_m = 3\times10^4$

$$f_m = \frac{3\times10^4}{2\pi} = 4.774 \text{ kHz}$$

ii). $2\pi f_c = 8\times10^6$

$$f_c = \frac{8\times10^6}{2\pi} = 1.273 \text{ MHz}$$

iii). Modulation Index $m_f = 6$

(iv). $m_f = \dfrac{\Delta f}{f_m}$,

$$\Delta f = m_f \cdot f_m = 6\times4.774\times10^3 = 28.644 \text{ kHz}$$

Numerical on FM

(3). The Carrier is frequency modulated with a sinusoidal s/g of 2kHz resulting in a max. deviation of 5kHz. Find the Bandwidth of the modulated s/g

$f_m = 2kHz$

$\Delta f = 5kHz$

$BW = 2[\Delta f + f_m] = 2[5\times10^3 + 2\times10^3]$

$$\boxed{BW = 14kHz}$$

Generation of FM Wave

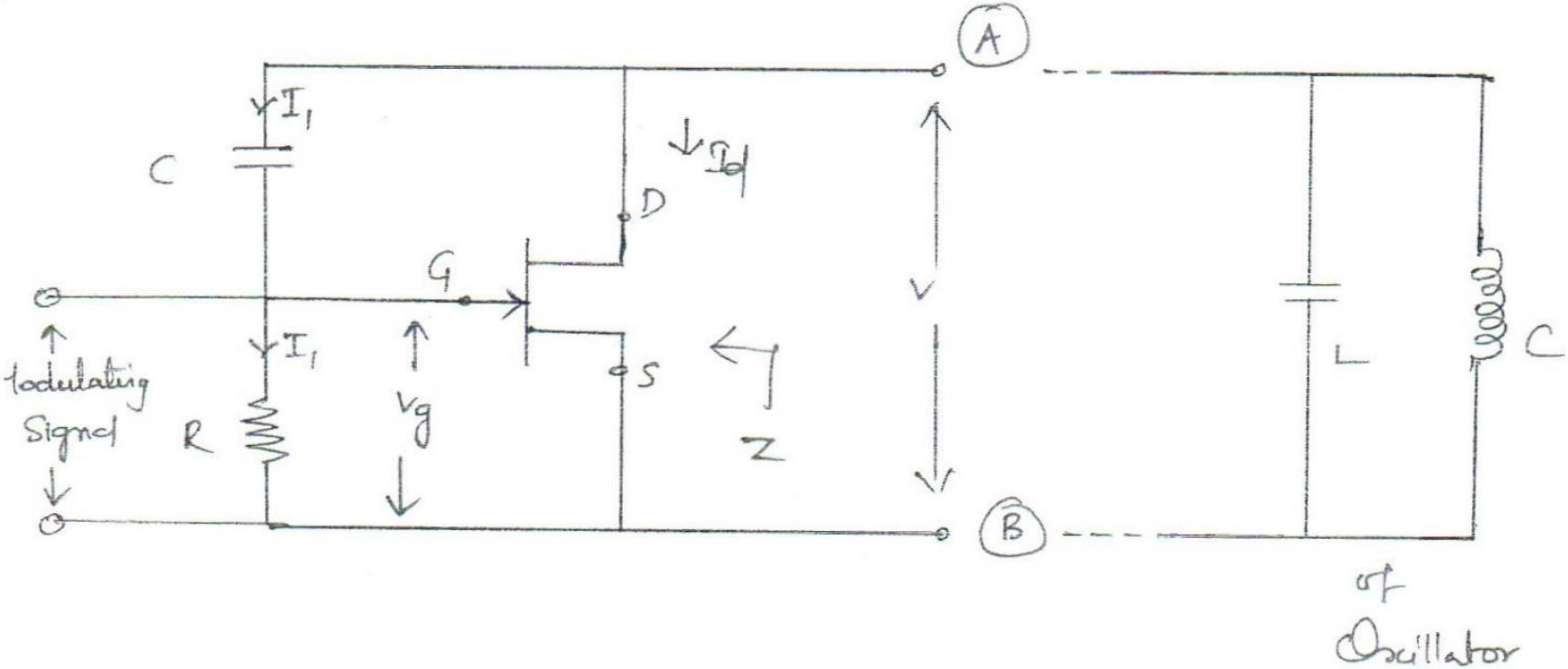
The primary requirement of FM generator is a variable output frequency with variation proportional to the instantaneous amplitude of the modulating voltage.

The other requirement is that the frequency deviation which is independent of modulating frequency.

Methods of FM generation

- Direct method [frequency of the carrier varies directly proportional to modulating voltage]
  - Reactance modulator
  - Varactor diode modulator
- In-direct method [
  - Armstrong method.

FET Reactance Modulator



Reactance of the drain & Source Controlled by gate Voltage (v_g) (or modulating signal voltage.

$$Z = \frac{V}{g_m v_g} \quad \text{--- (1)}$$

$V_g \rightarrow$ gate Voltage

$g_m \rightarrow$ transconductance

FET Reactance Modulator

To find v_g,

$$V_g = I_1 R \quad \text{---} \quad (2)$$

$$I_1 = \frac{V}{R + \frac{1}{j\omega C}}$$

Reactance, $R < C$

$$I_1 = V j\omega C \quad \text{---} \quad (3)$$

Sub equ (3) in (2)

$$v_g = V j\omega C R \quad \text{---} \quad (4)$$

Sub (4) in (1)

$$f_c = \frac{1}{2\pi\sqrt{L C_{eq}}}$$

$$Z = \frac{\cancel{V}}{g_m j\omega \cancel{V} \cdot C R}$$

If voltage of modulating signal varies, (Z) reactance of FET varies (1)

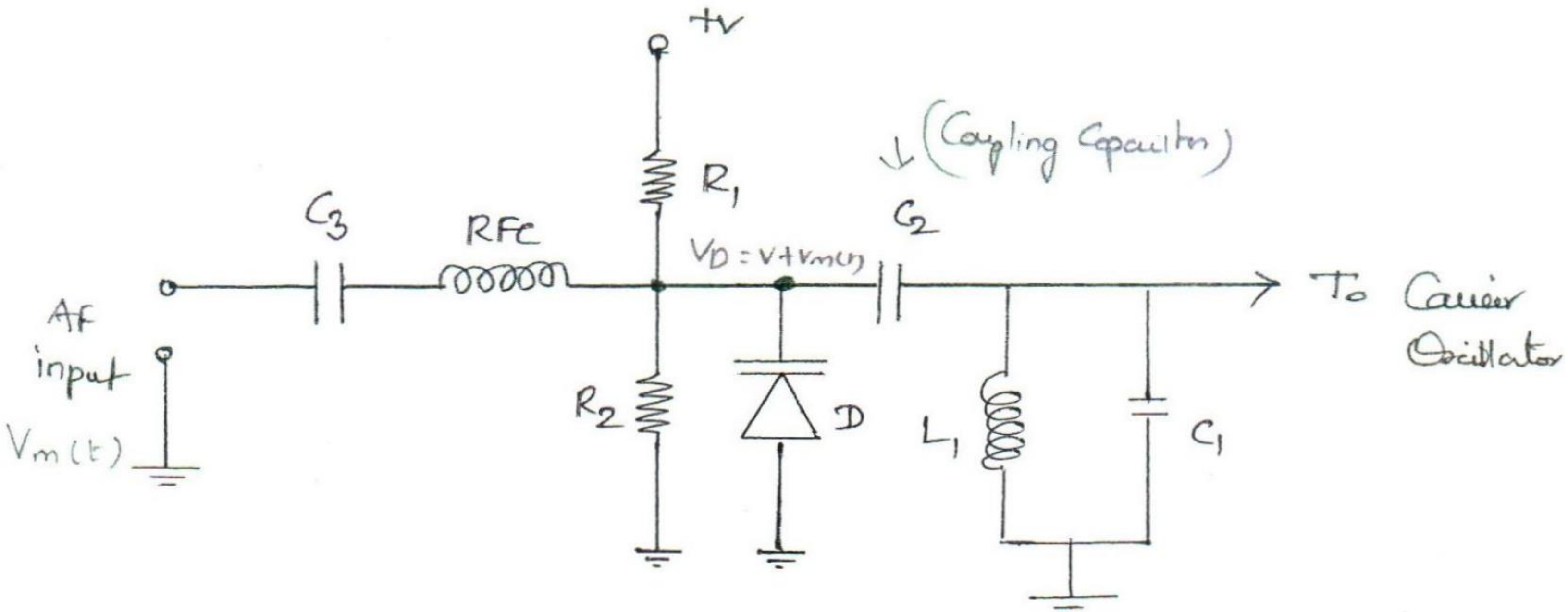
$$\boxed{Z = \frac{1}{j\omega \cdot C_{eq}}}$$

$$\therefore C_{eq} = g_m C R.$$

If Z - varies C_{eq} varies.

If C_{eq} varies then f_c is varies, hence FM wave is generated.

FM using Varactor Diode



Case-(i):

If modulating signal Voltage Increases

Capacitance of Varactor diode decreases

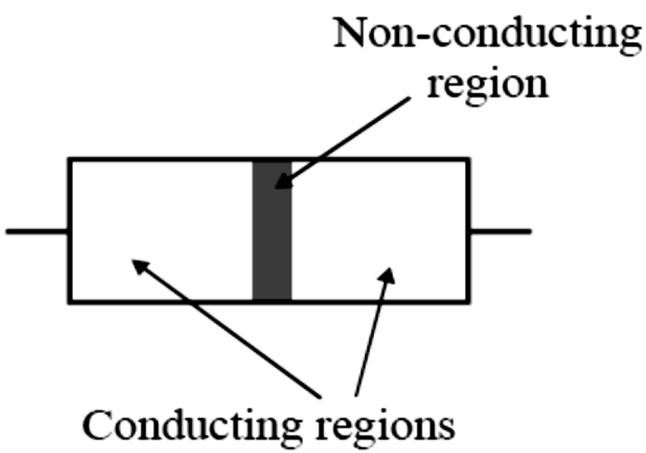
$$C_d = \frac{k}{\sqrt{V_D}}. \qquad V_D = V + V_m(t).$$

frequency of Oscillator Circuit increases.

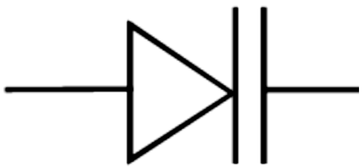
$$f_c = \frac{1}{2\pi\sqrt{L C_{eq}}}, \quad C_{eq} = C_1 + C_d.$$

Varactor Diode Modulator

Varactor diode



$$c = \varepsilon A / d$$



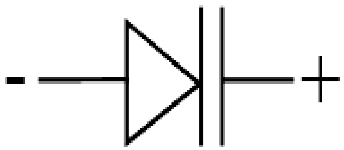
Symbol

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

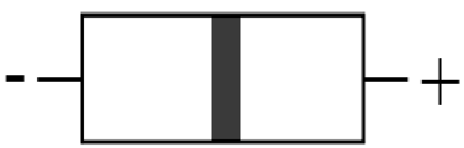
where

- ε is the absolute permittivity
- ε_0 is the vacuum permittivity = 8.854×10^{-12} F.m^{-1}
- ε_r is the relative permittivity
- A is the plate area
- d is the distance between plates

Low voltage applied

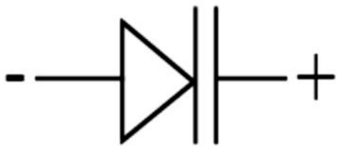


Narrow non-conducting region



- more capacitance.

Increased voltage applied

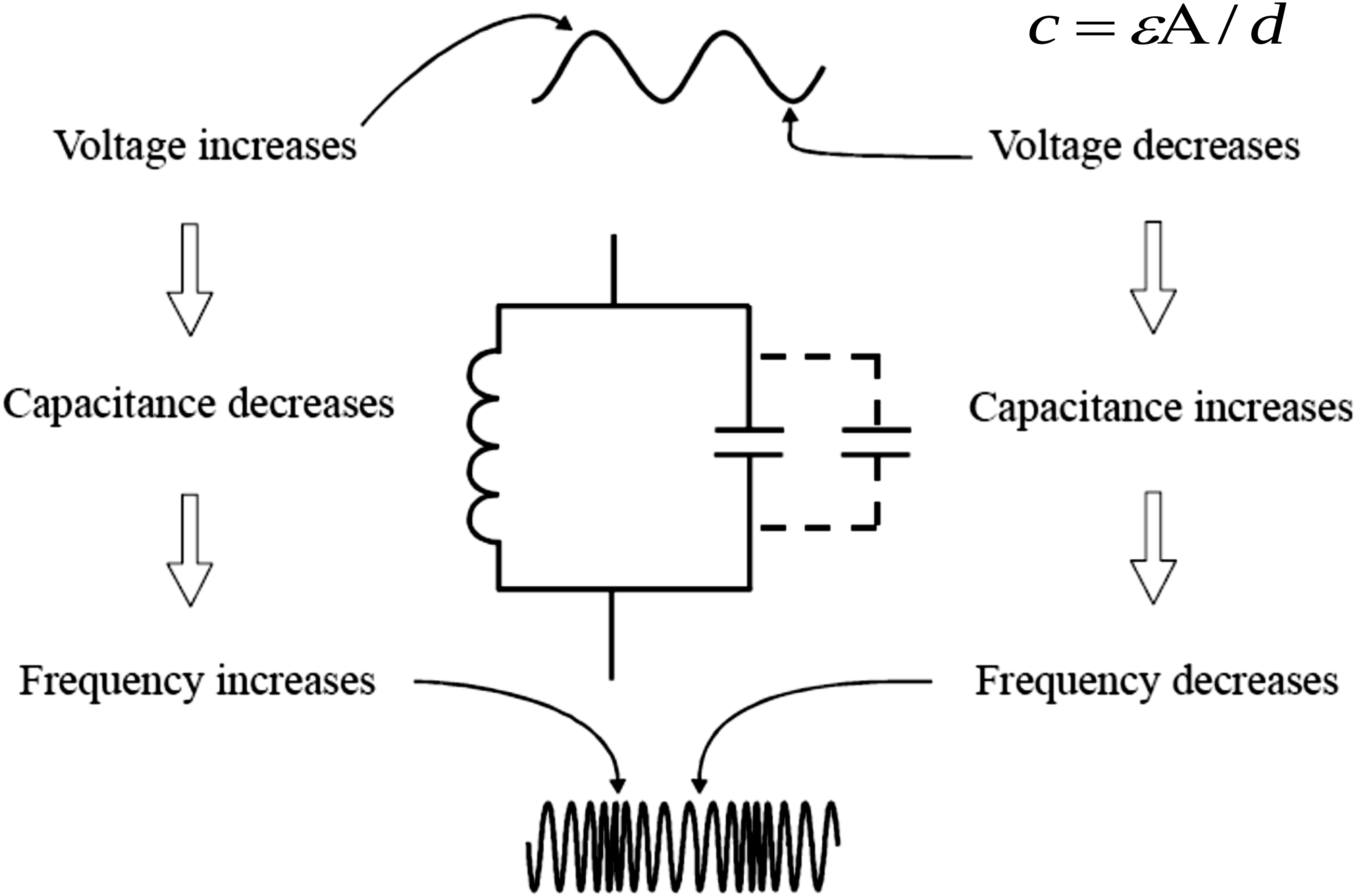


Wider non-conducting region

- less capacitance.

Varactor Diode FM Modulator

$$c = \varepsilon A / d$$



FM using Varactor Diode

Case (iii)

If voltage of modulating signal decreases,

Capacitance of Varactor diode Increases.

Frequency of Oscillator Circuit decreases.

Capacitance of Varactor diode

$$C_d = \frac{k}{\sqrt{V_D}} \quad \text{---} \quad (1)$$

$k \rightarrow$ Constant

$V_D \rightarrow$ Instantaneous Voltage across the diode.

$$V_D = V_0 + V_m(t) \quad \text{---} \quad (2)$$

$$V_D = V_0 + V_m \sin \omega_m t \quad \text{---} \quad (3)$$

$V_0 \rightarrow$ Supply voltage

FM using Varactor Diode

The total Capacitance of the Oscillator tank Circuit is $(C_o + C_d)$ & the instantaneous frequency of Oscillator ω,

$$\omega_i = \frac{1}{\sqrt{L_o(C_o+C_d)}} \quad \text{——} \quad (4)$$

sub (1) in (4)

$$\omega_i = \frac{1}{\sqrt{L_o\left(C_o+\frac{k}{\sqrt{V_D}}\right)}} \quad \text{——} \quad (5)$$

Thus the frequency modulated s/g is generated.

FM using Varactor Diode

The radio frequency choke (RFC) has high reactance at the carrier frequency to prevent carrier signal from getting into the modulating s/g.

Application :

1). Automatic frequency Control (AFC)

2). Remote tuning.

Indirect method of FM Generation – Armstrong Method

Drawbacks in direct method :

i). LC Oscillator Circuits are used to generate Carrier Signal.

ii). The Carrier signal generated from LC Oscillator is not stable for broadcasting & Comm. purpose

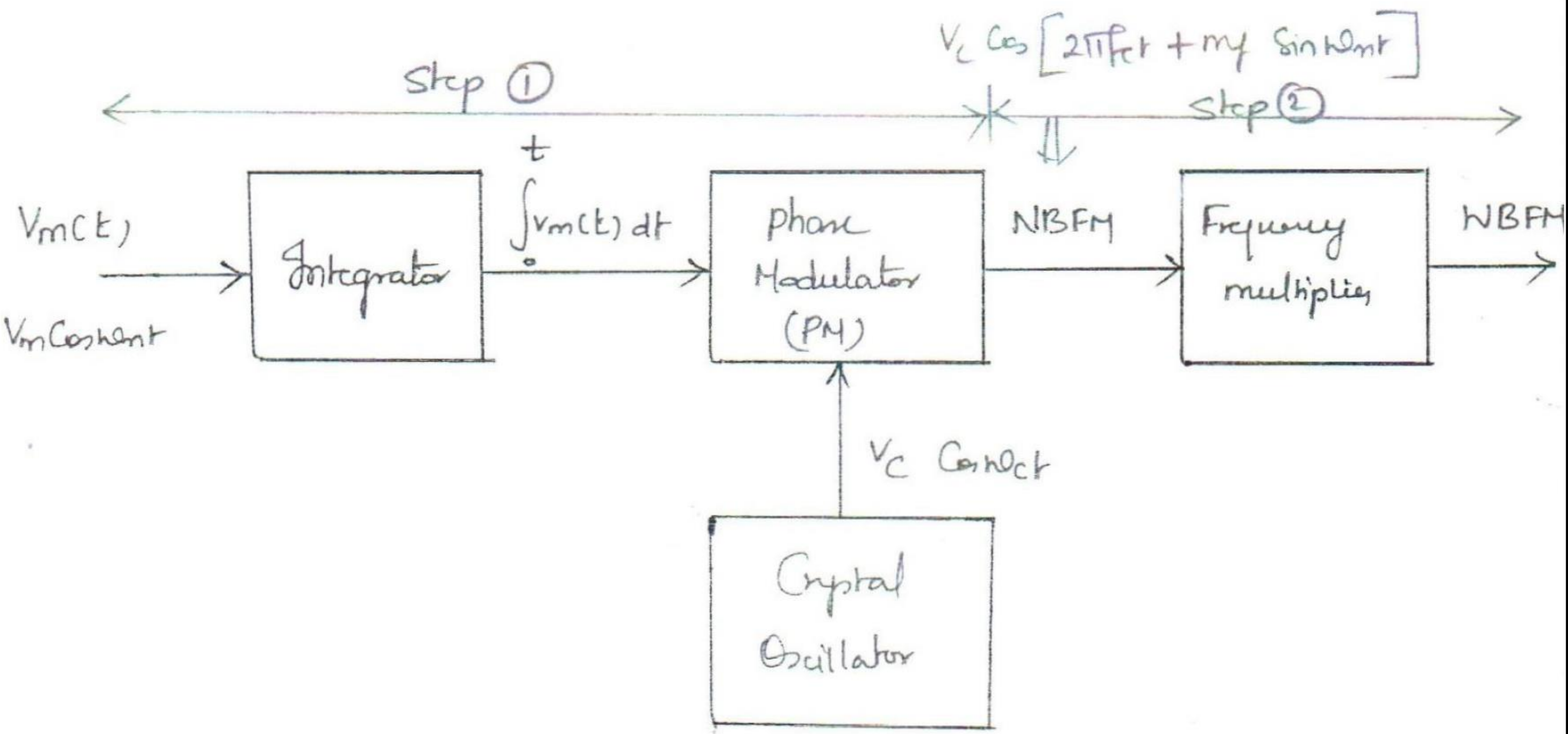
iii). Due to non-linear characteristics of FET & Varactor diode distortion will occur in the output.

Indirect method of FM Generation – Armstrong Method

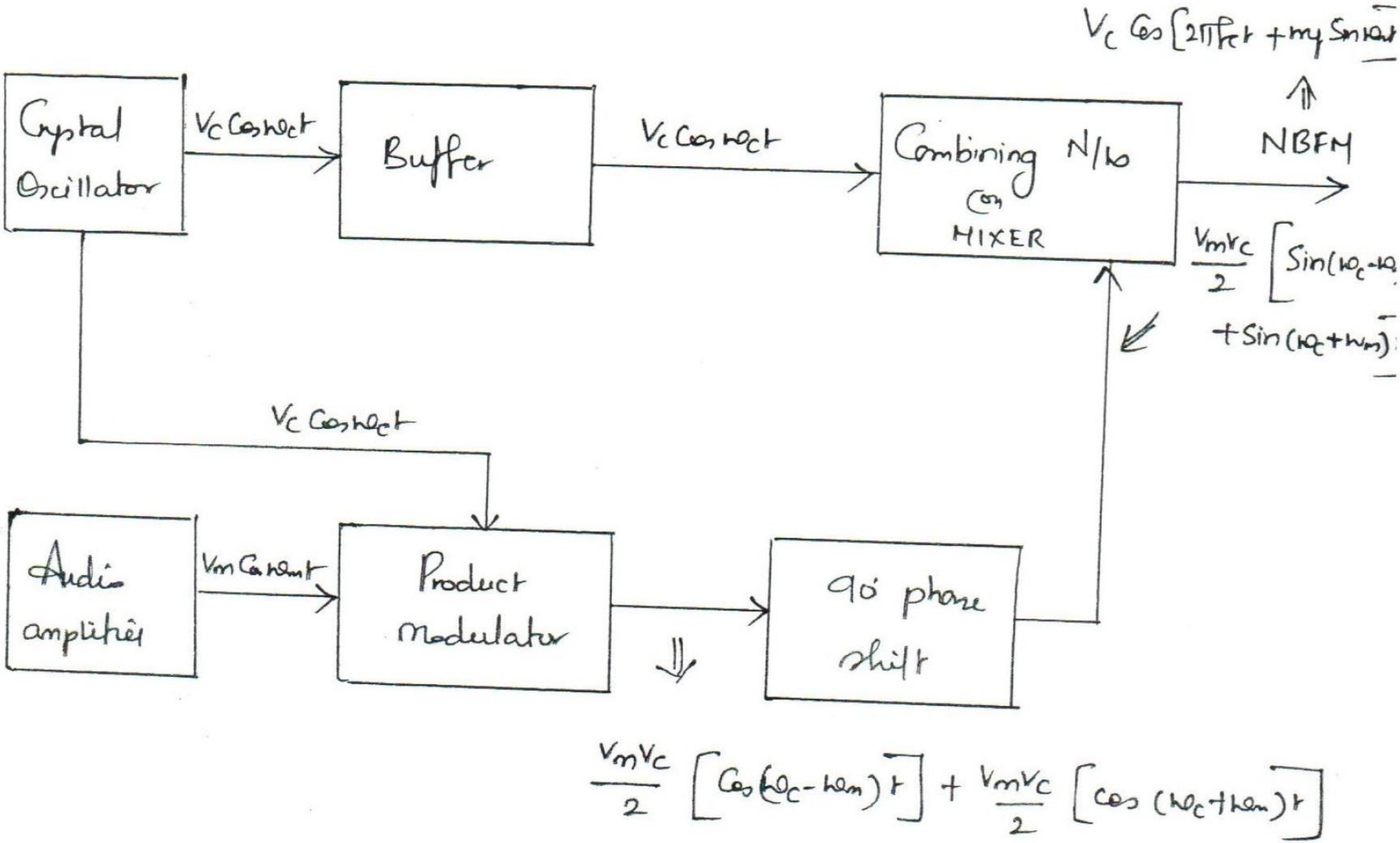
The indirect method is made up of two stages:

i). Generation of NBFM using phase modulation

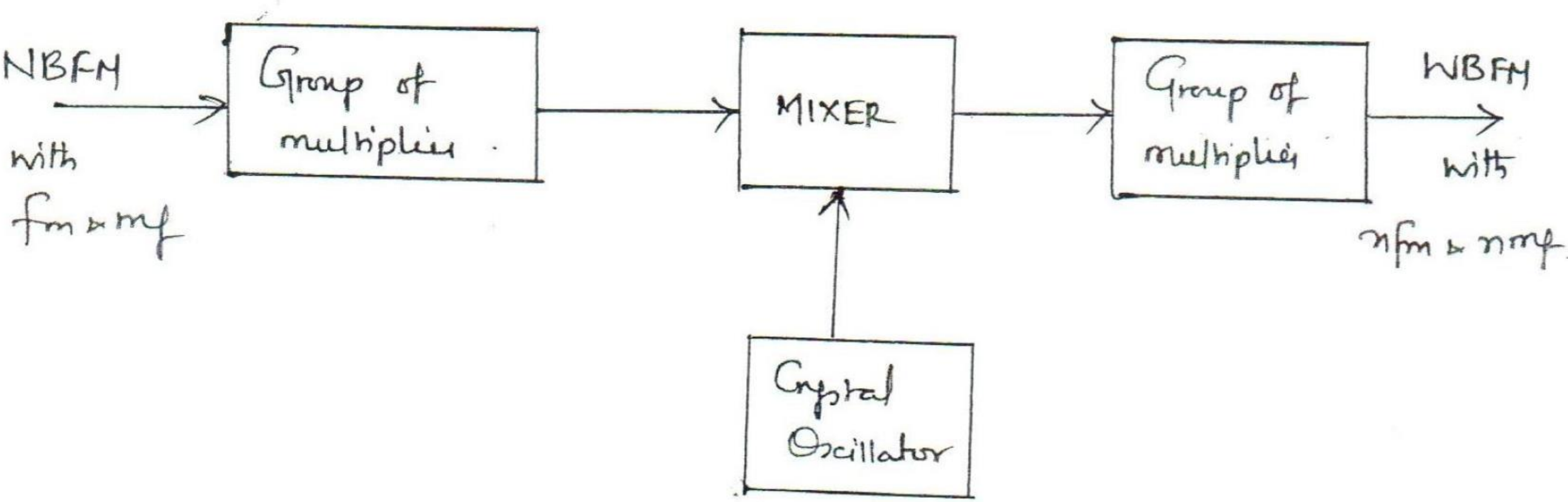
ii). NBFM is converted into WBFM by using Frequency multiplier & mixer.



Generation of NBFM signal from Phase Modulation



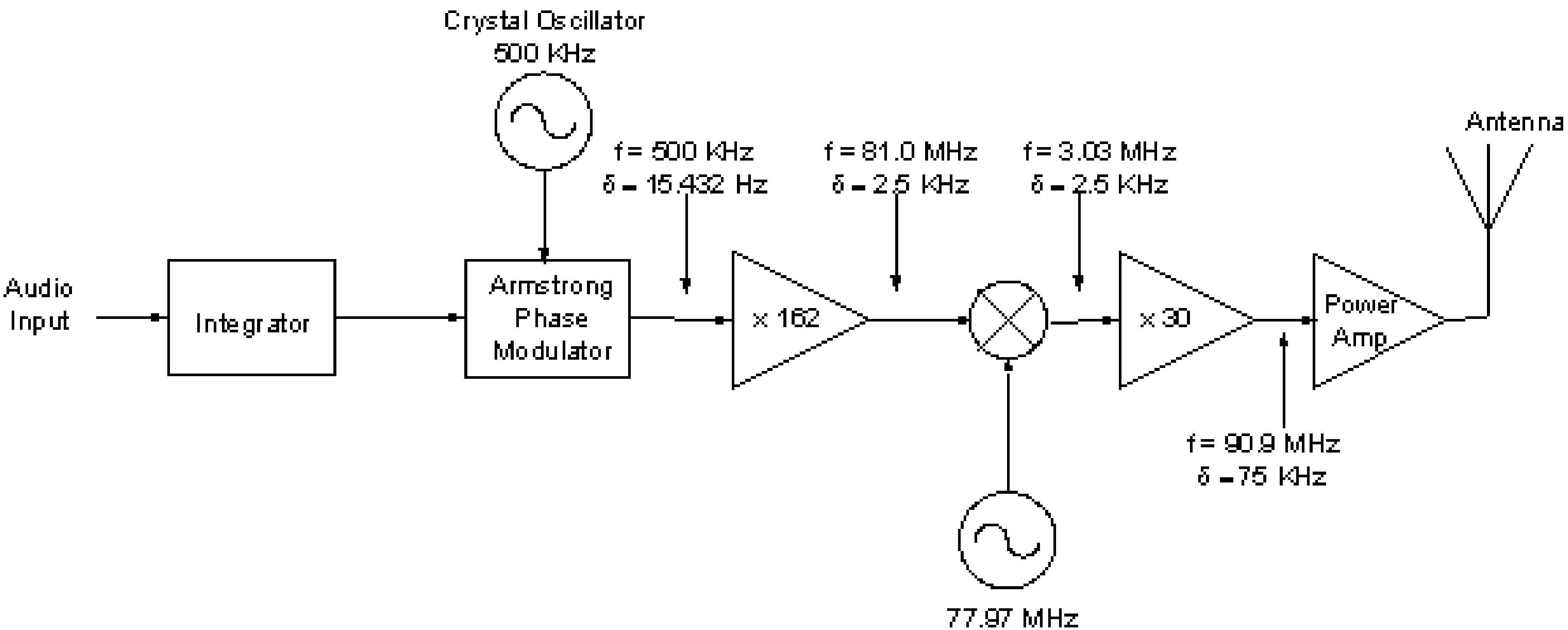
Frequency Multiplier



After Frequency multiplier

$$V_{FM}(t) = V_c \cos\left[2\pi n f_c t + n m_f \sin 2\pi n f_m t\right].$$

Indirect FM Generation – System Design



FM Demodulator / Detector

To detect the FM signal, it is necessary to have a circuit whose output voltage varies linearly with a frequency of input signal. The circuit used is called frequency discriminator which converts FM signal into its corresponding AM signal. Then the modulating signal is obtained from AM signal by envelope detector.

Types of FM demodulators

- Frequency discriminator
  - Balanced slope detector
    (or)
    Round Travis detector
- phase discriminator
  - i) Foster-Seeley discriminator
  - ii). Ratio detector.
- PLL FM detector

FM Demodulator / Detector

- Demodulation should provide an output signal whose amplitude is dependent on the instantaneous carrier frequency deviation and whose frequency is dependent on the rate of the carrier frequency change

$$V_m \;\alpha\; \frac{df_c}{dt} \quad \text{and} \quad f_m \;\alpha\; \frac{df_c}{dt}$$

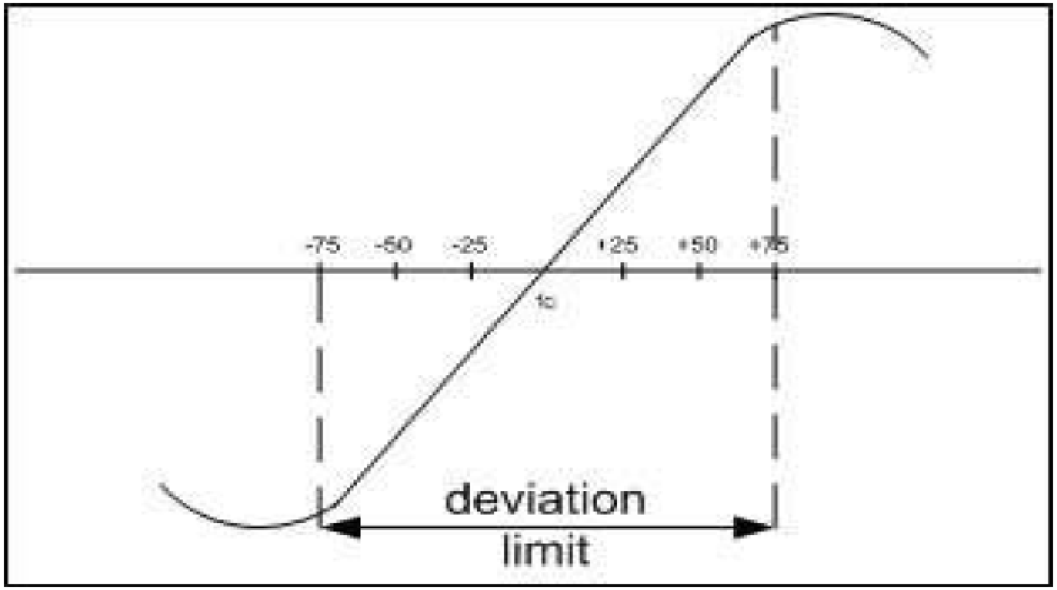


Figure 5.2 : FM Characteristics curve

Frequency Discrimination / (FM-to AM Conversion)

- Components
  - Bandpass Limiter: Consists of **Hard Limiter** & **BP Filter**
  - Discriminator (frequency discriminator gain: K_{FD} V/rad - assume unity)
  - Envelope Detector

$$v_2(t) = -V_L\left[\omega_c + \frac{d\theta(t)}{dt}\right]\sin[\omega_c t + \theta(t)]$$

$$v_{in}(t) = A(t)\cos[\omega_c t + \theta(t)]$$

$$v_1(t) = V_L\cos[\omega_c t + \theta(t)]$$

$$v_{out}(t) = \left|-V_L\left[\omega_c + \frac{d\theta(t)}{dt}\right]\right|$$

$$\theta(t) = K_f\int_{-\infty}^{t} m(t_1)\,dt_1$$

Note: Df=Kf
Freq. deviation sensitivity

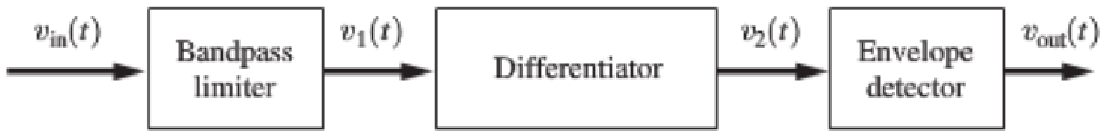
THE OUTPUT WILL BE:

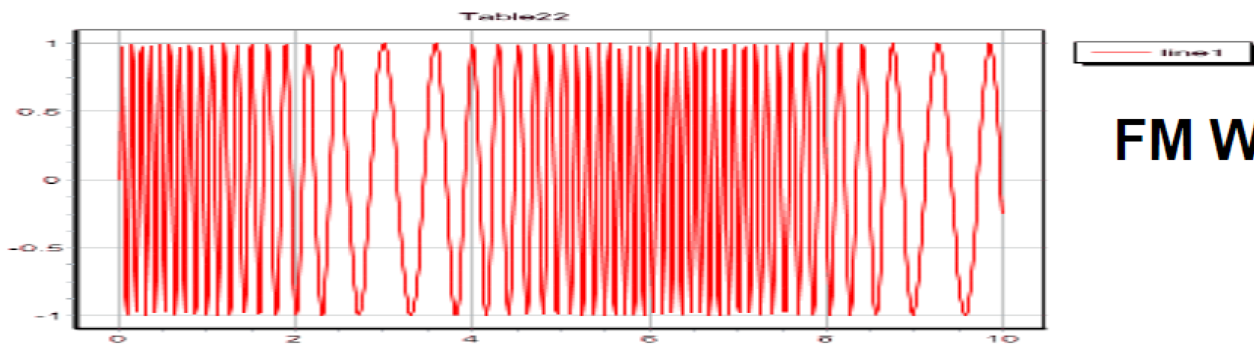
$$v_{out}(t) = V_L\left[\omega_c + \frac{d\theta(t)}{dt}\right]$$

$$v_{out}(t) = V_L\omega_c + V_L K_f m(t)$$

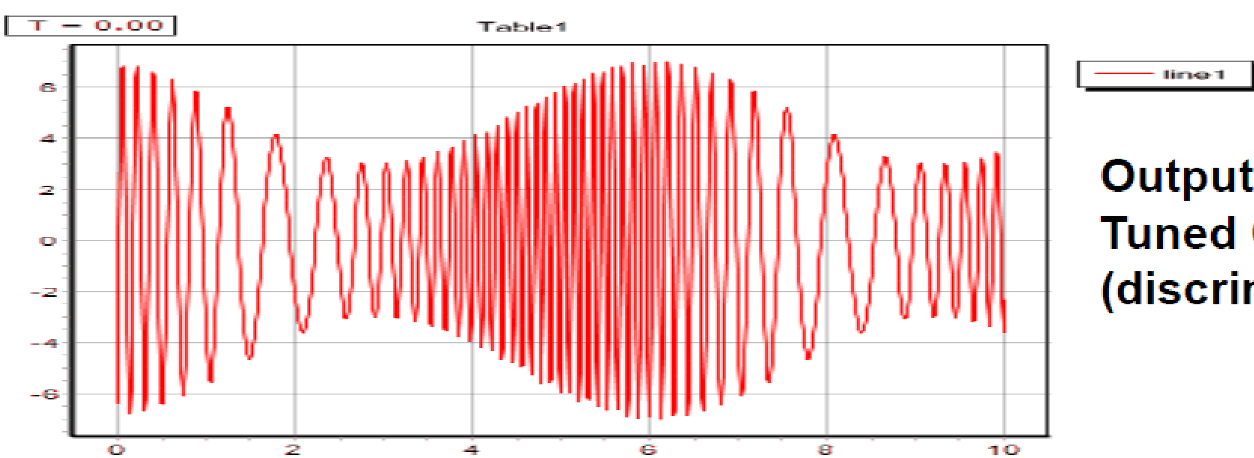
DC Component can be blocked
by an AC coupled circuit

Frequency Discrimination / (FM-to AM Conversion)





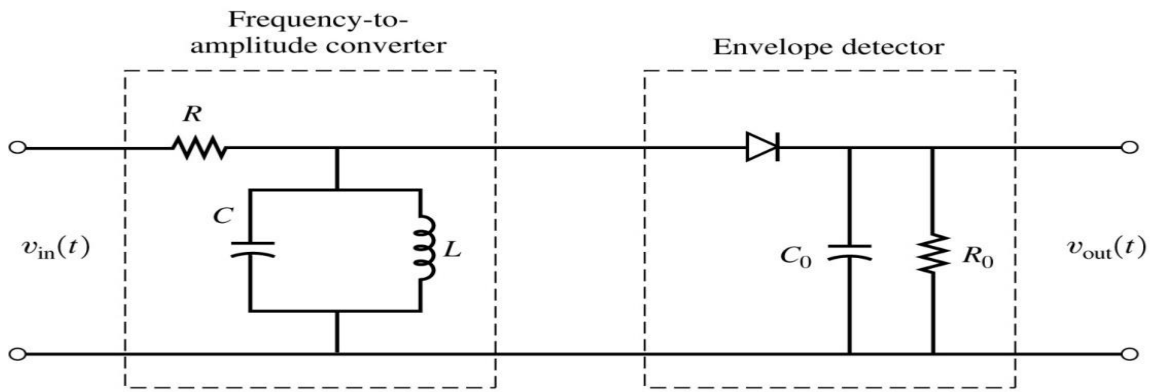
FM Wave



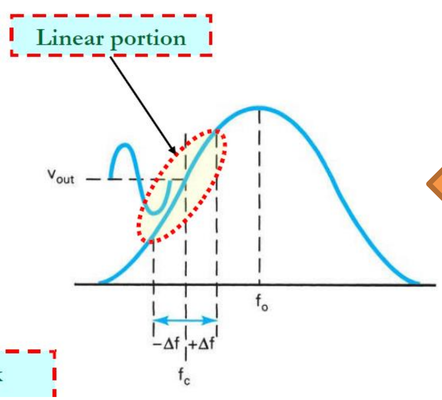
Output of Tuned Circuit (discriminator)

Frequency Discrimination / Slope Detector Circuit

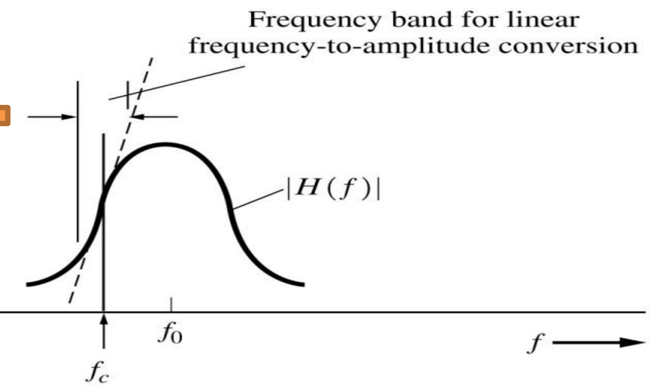
➢ In practice the differentiator can be approximated by a **slope detector** that has a linear frequency-to-amplitude transfer characteristic over the bandwidth BW



(a) Circuit Diagram of a Slope Detector

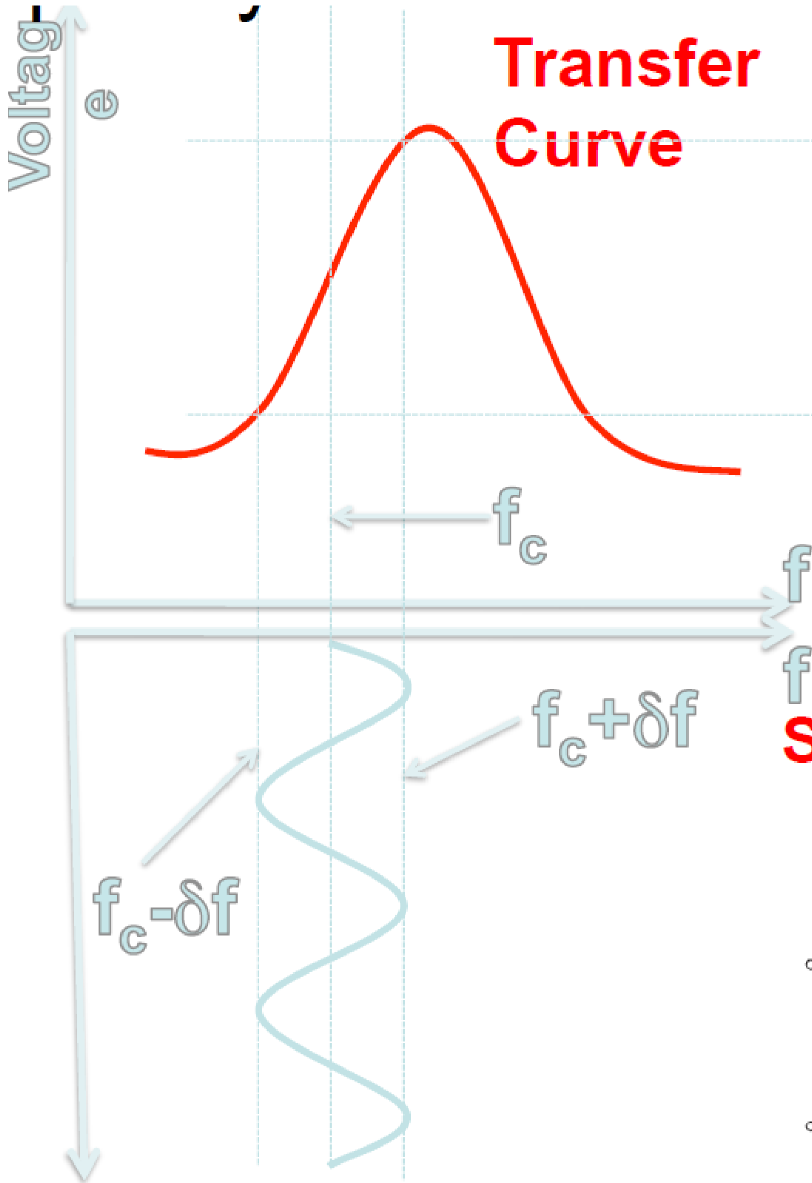


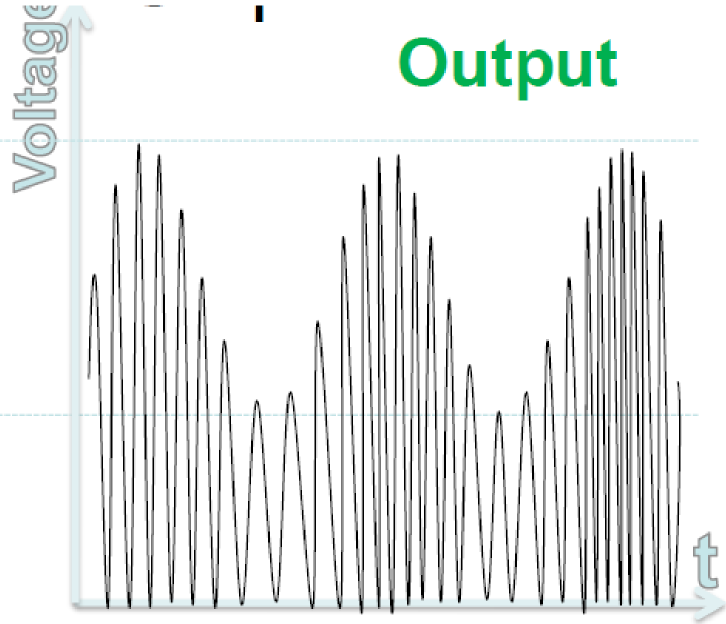
voltage-versus-frequency curve.



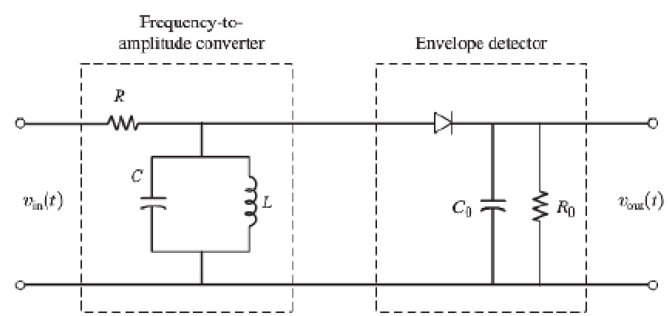
(b) Magnitude of Filter Transfer Function

Frequency Discrimination / Slope Detector Circuit





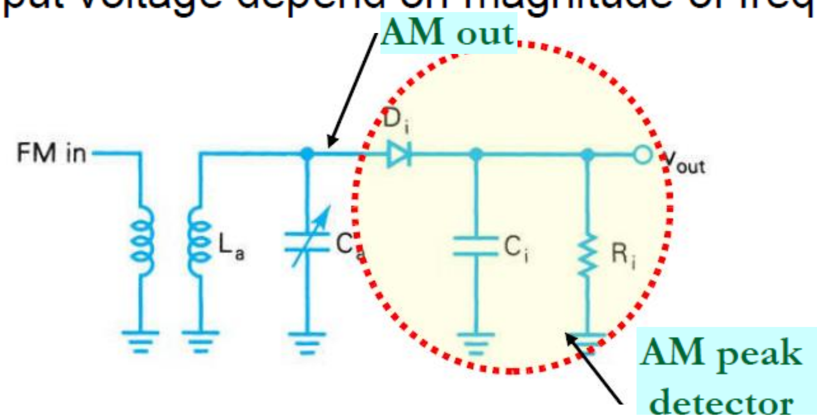
Slope Detector Transfer Characteristics



Frequency Discrimination / Slope Detector Circuit

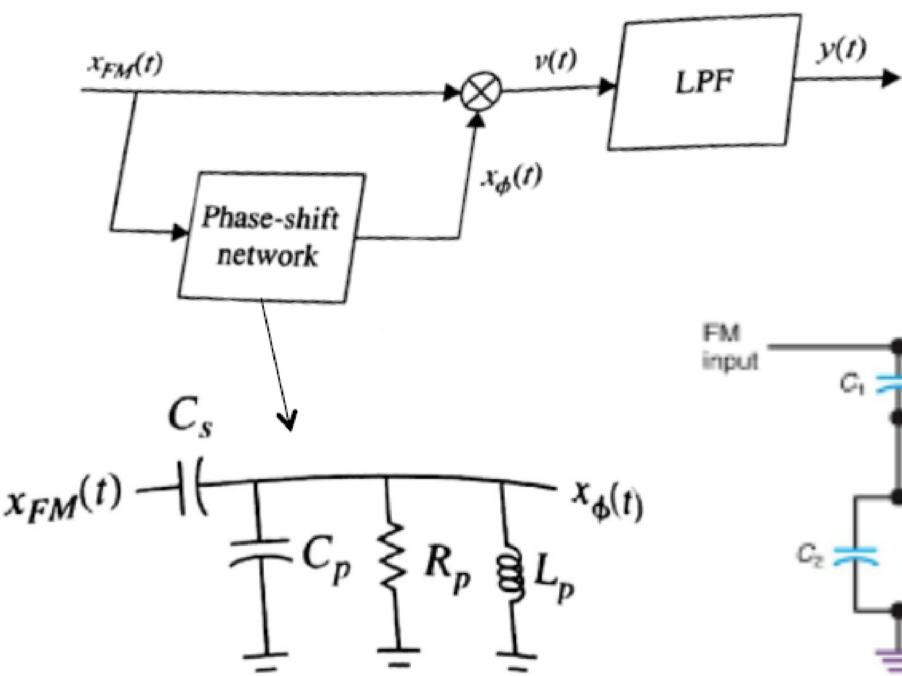
Circuit Operation

- L_a & C_a (tuned circuit) produce o/put voltage (amplitude varies) which is proportional to the i/put freq. (FM in)
  - - -> AM characteristic
- Maximum o/put voltage occurs at resonant freq of tank circuit, f_o and its o/put decrease proportionately as the i/put freq deviates below & above f_o
- IF center frequency (f_c) falls in the center of the ***most linear*** portion of the voltage-versus-frequency curve (Figure 5.3(b))
- When IF deviates above f_c, output voltage increase and when IF deviated below f_c, output voltage decrease.
- The tuned circuit converts frequency variations to amplitude variations (FM-to-AM conversion).
- D_i, C_i and R_i - - -> simple peak detector that converts amplitude variations to o/put voltage (operate as AM Diode Detector)
  - o/put voltage varies at a rate equal to i/put frequency
  - Amplitude of o/put voltage depend on magnitude of freq changes

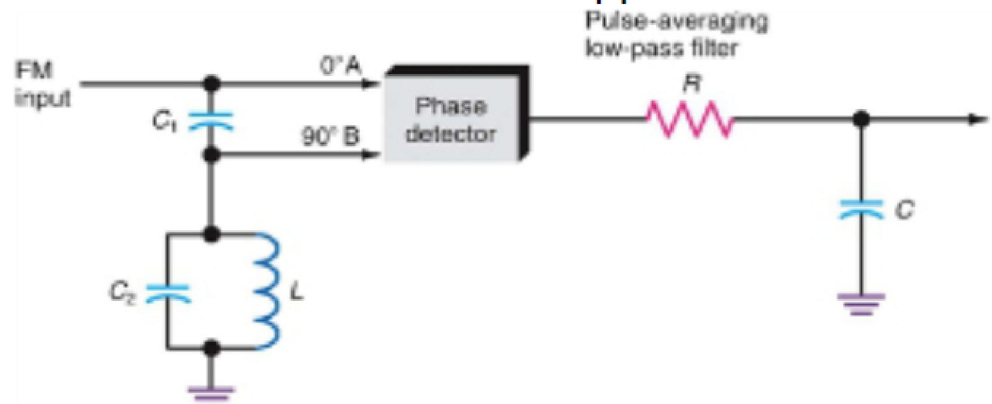


Phase Shift Discriminator / Quadrature Detector

- Very common in TV receivers
- It uses a phase shift circuit
- It converts the instantaneous frequency deviation in an FM signal to phase shift and then detects the changes of phase
  - **Cs** results in -90 deg. Shift
  - The **tuned circuit→** additional phase shift proportional to instantaneous frequency deviation from fc



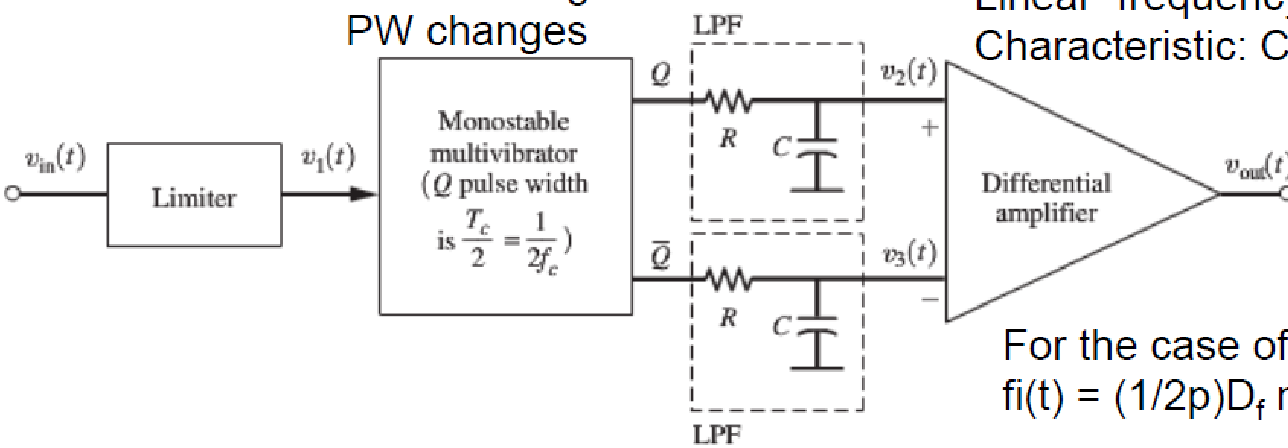
Another approach



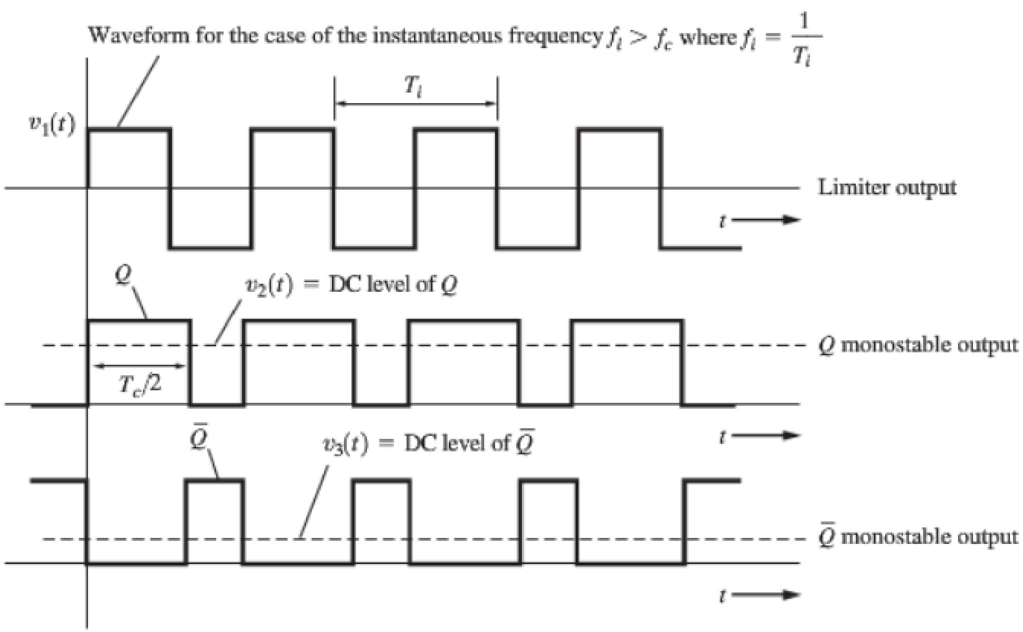
Phase Shift Discriminator / Balanced zero-crossing FM detector

Free-running fc
PW changes

Linear frequency-to-voltage
Characteristic: C[fi(t) – fc]



For the case of FM:
$f_i(t) = (1/2p) D_f\, m(t)$

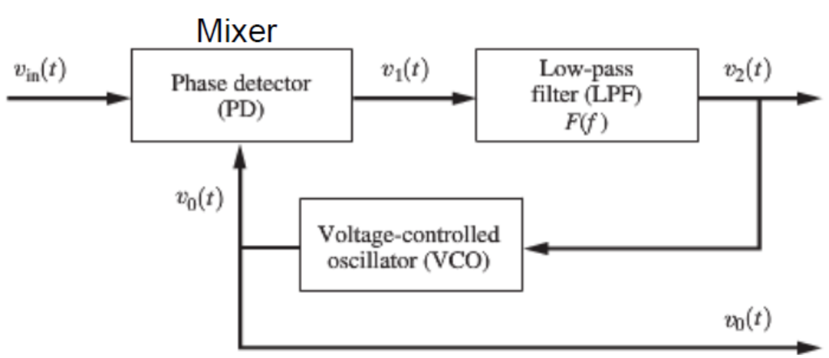


IF fi > fc → Tc>Ti
Qdc > Qdc → Vout > 0

IF fi < fc → Tc<Ti
Qdc < Qdc → Vout < 0

Phase Locked Loops (PLL)

- FM demodulation can be accomplished quite simply with a phase-locked loop (PLL).
- *PLL FM demodulator* is probably the simplest and easiest to understand.
- A PLL frequency demodulator requires no tuned circuits and automatically compensates for changes in the carrier frequency due to instability in the transmit oscillator.
  - Applications: Frequency synthesizer, TV, Demodulators, clock recovery circuits, multipliers, etc.
  - Basic Idea: A negative feedback control system
  - Basic Components: PD, Loop Filter (LPF), VCO
  - Types: Analog / Digital
  - Operation: when it is locked it will track the input frequency: $w_{out}=w_{in}$



Phase Locked Loops (PLL)

VCO: oscillator that produces a period waveform with a frequency that may be varied around free running frequency, f_o.

$\Rightarrow$ VCO output frequency = f_o when $v_2(t) = 0$.

Phase Detector (PD): output is a function of the phase difference between incoming signal $v_o(t)$ and $v_{in}(t)$.

PLL has two modes:

- Narrowband mode: tracks average frequency of $v_{in}(t)$.
- Wideband mode: tracks instantaneous frequency of $v_{in}(t)$.

Lock: When the PLL tracks the (average or instantaneous) frequency of $v_{in}(t)$.

Hold-in range: When the PLL is in lock, the range of frequency of $v_{in}(t)$ to remain in lock. (Also called the lock range.)

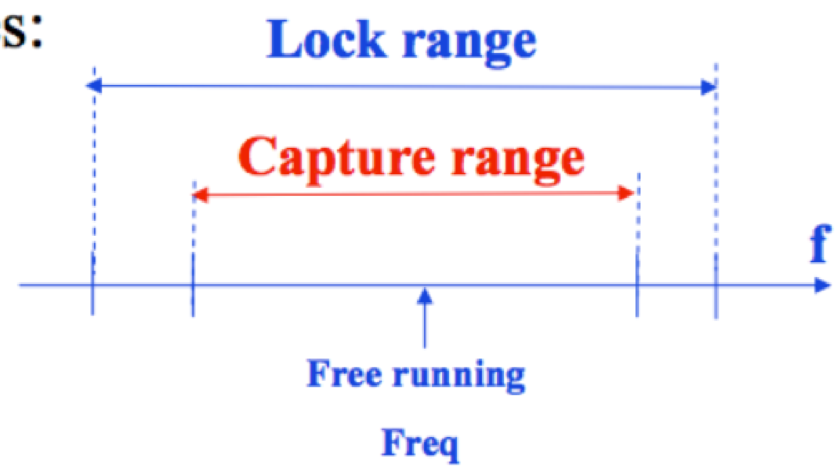
Pull-in range: When the PLL is not in lock, the range of frequency of $v_{in}(t)$ to capture a lock. (Also called the capture range.)

Maximum locked sweep range: When the PLL is in lock, the maximum rate of change of the frequency of $v_{in}(t)$ to remain in lock.

A PLL can be made in analog (APLL) or digital (DPLL) circuits.

PLL Characteristics

- A PLL has basically three states:
  - Free running
  - Capture
  - Lock range (hold-in range)



- Capture Range:
  - The range of input frequencies for which an initially unlocked loop will lock on an input signal.
- Lock Range (or hold-in range):
  - The range of frequencies over which the loop will remain in phase-locked mode with respect to the incoming FM wave.
  - Generally, the lock range is larger than the capture range, because capture is more difficult.

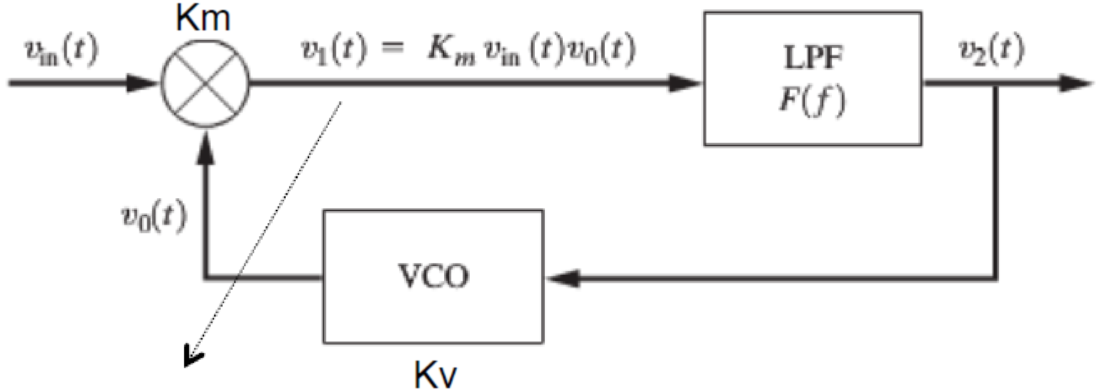
Phase Locked Loops (PLL)

When locked, that is when no phase error → exactly 90 deg. Diff (90 deg. out of phase)

$$v_{\text{in}}(t) = A_i \sin[\omega_0 t + \theta_i(t)]$$

$$v_0(t) = A_0 \cos[\omega_0 t + \theta_0(t)]$$

$$\theta_0(t) = K_v \int_{-\infty}^{t} v_2(\tau)\, d\tau$$



$$v_1(t) = K_m A_i A_0 \sin[\omega_0 t + \theta_i(t)] \cos[\omega_0 t + \theta_0(t)]$$

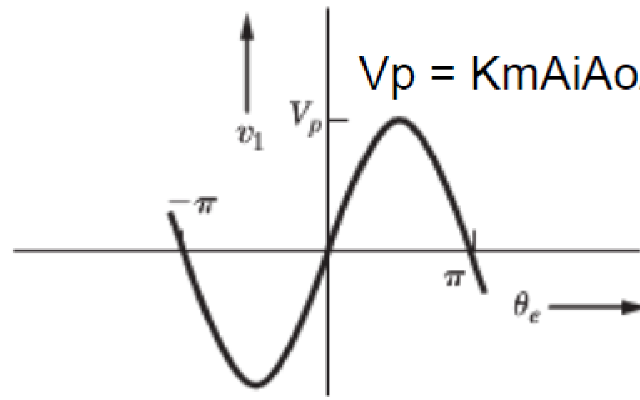
$$= \boxed{\frac{K_m A_i A_0}{2} \sin[\theta_i(t) - \theta_0(t)]} + \frac{K_m A_i A_0}{2} \sin[2\omega_0 t + \theta_i(t) + \theta_0(t)]$$

$$K_d[\sin\ \theta_e(t)]$$

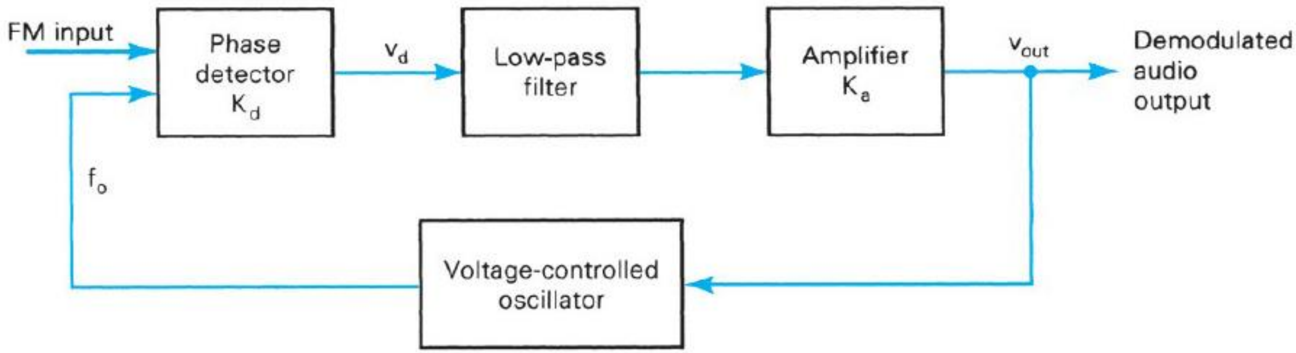
Phase detector constant gain V/rad

Km is the gain of the multiplier

Vp = KmAiAo/2=Kd



FM Demodulator using PLL



(a)

- If the IF amplitude is sufficiently limited prior to reaching the PLL and the loop is properly compensated, the PLL loop gain is constant and equal to K_V.
- after frequency lock had occurred the VCO would track frequency changes in the input signal by maintaining a phase error at the input of the phase comparator.
- Therefore, if the PLL input is a deviated FM signal and the VCO natural frequency is equal to the IF center frequency, the correction voltage produced at the output of the phase comparator and fed back to the input of the VCO is proportional to the frequency deviation and is, thus, the demodulated information signal.
- Therefore, the demodulated signal can be taken directly from the output of the internal buffer and is mathematically given as $V_{out} = \Delta f\, K_d\, K_a$

Pre-emphasis and De-emphasis

- Pre and de-emphasis circuits are used only in frequency modulation.
  - Pre-emphasis is used **at transmitter** and de-emphasis **at receiver**.

Pre-emphasis

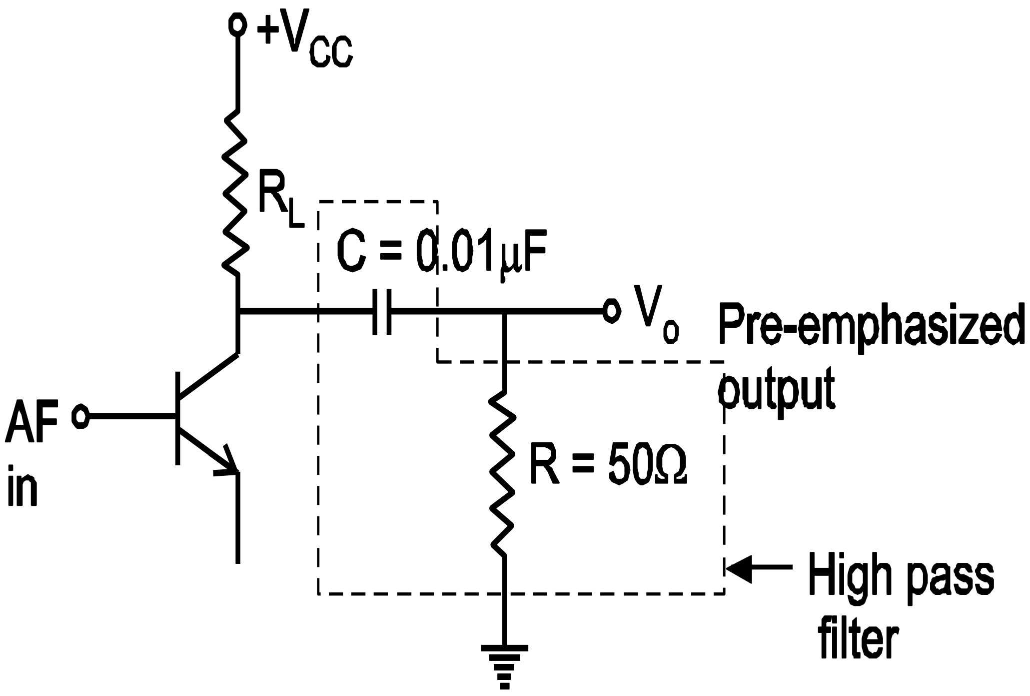
- In FM, the noise has a greater effect on the higher modulating frequencies.
- This effect can be reduced by increasing the value of modulation index (m_f), for higher modulating frequencies.
- This can be done by increasing the deviation 'δ' and 'δ' can be increased by increasing the amplitude of modulating signal at higher frequencies.

Definition:

The artificial boosting of higher audio modulating frequencies in accordance with prearranged response curve is called pre-emphasis.

- Pre-emphasis circuit is a high pass filter as shown in Fig. 1

Pre-emphasis Circuit



- As shown in Fig. , AF is passed through a high-pass filter, before applying to FM modulator.

Pre-emphasis

- As modulating frequency (f_m) increases, capacitive reactance decreases and modulating voltage goes on increasing.
- $f_m \propto$ Voltage of modulating signal applied to FM modulate
- Boosting is done according to pre-arranged curve as shown in Fig. .

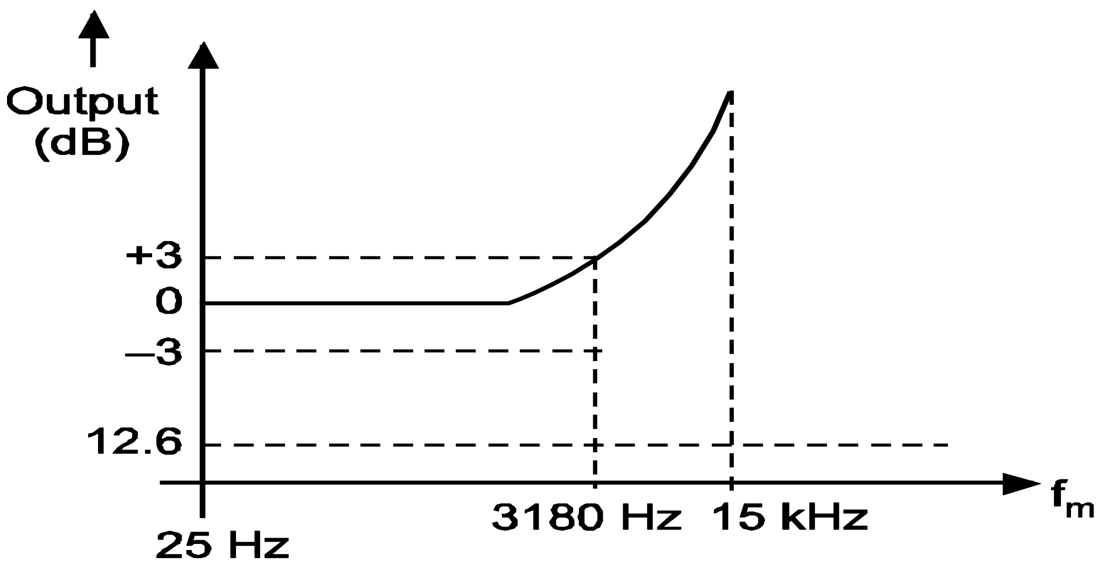


Fig. : Pre-emphasis Curve

Pre-emphasis

- The time constant of pre-emphasis is at 50 μs as per standards.
- In systems employing American FM and TV standards, networks having time constant of 75 μsec are used.
- **The pre-emphasis is used at FM transmitter** as shown in Fig.

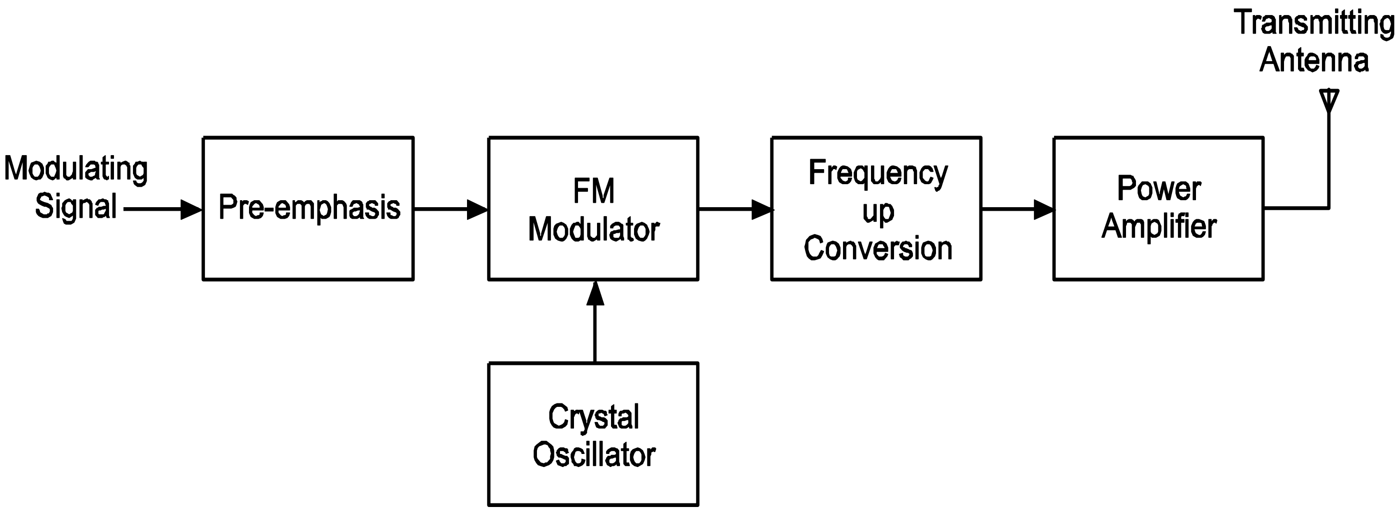


Fig. : FM Transmitter with Pre-emphasis

De-emphasis

- De-emphasis circuit is **used at FM receiver**.

Definition:

The artificial boosting of higher modulating frequencies in the process of pre-emphasis is nullified at receiver by process called de-emphasis.

- De-emphasis circuit is a low pass filter shown in Fig.

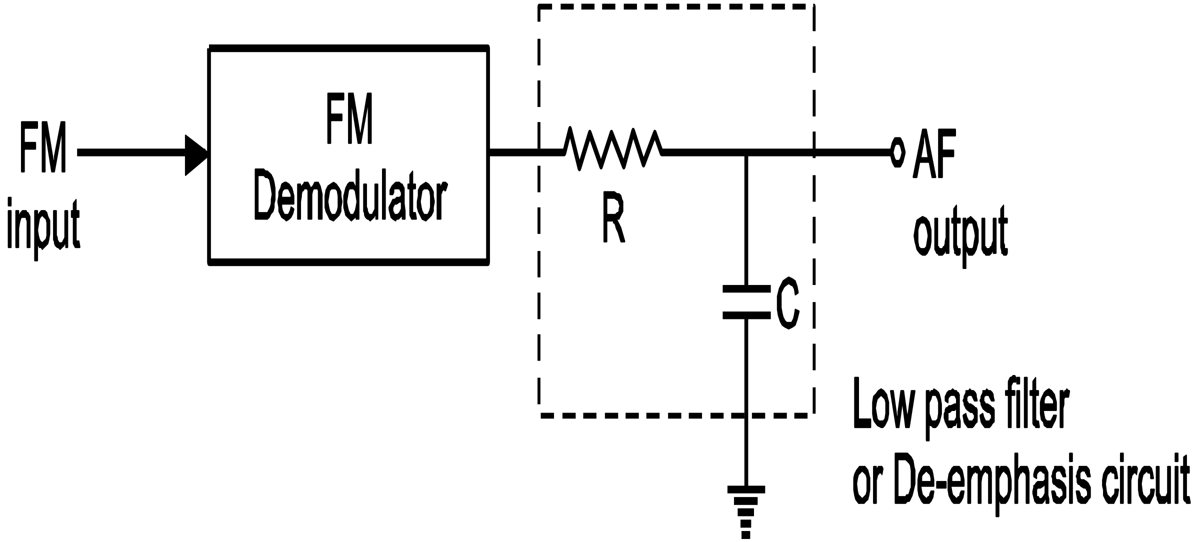


Fig. : De-emphasis Circuit

De-emphasis

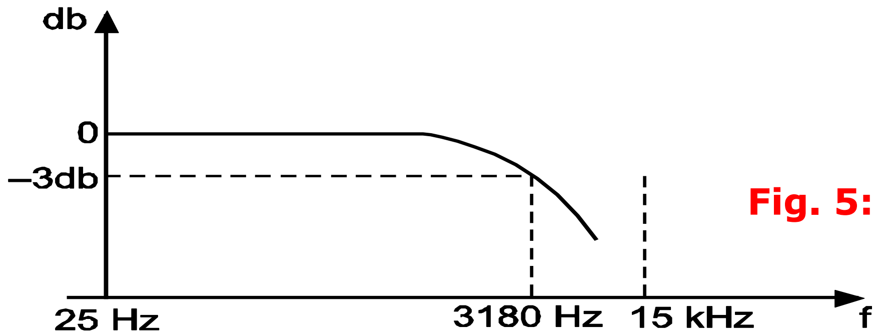


Fig. 5: De-emphasis Curve

- As shown in Fig., de-modulated FM is applied to the de-emphasis circuit (low pass filter) where with increase in f_m, capacitive reactance X_c decreases. So that output of de-emphasis circuit also reduces .
- Fig. shows the de-emphasis curve corresponding to a time constant 50 μs. A 50 μs de-emphasis corresponds to a frequency response curve that is 3 dB down at frequency given by,

$$f = 1/2\pi RC$$
$$= 1/2\pi \times 50 \times 10^{-6}$$
$$= 3180 \text{ Hz}$$

De-emphasis

- The de-emphasis circuit is used after the FM demodulator at the FM receiver shown in Fig.

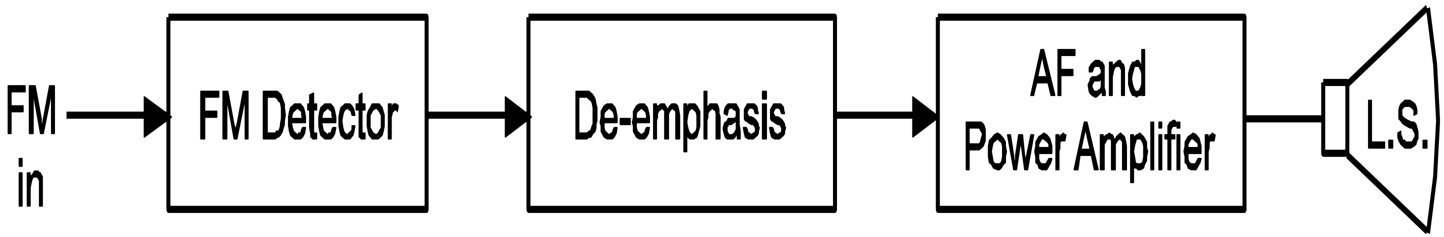
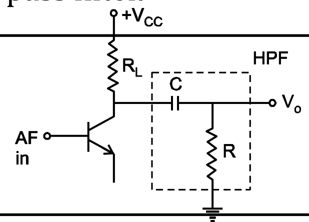
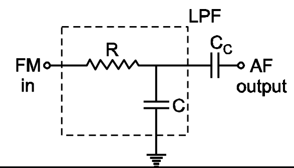
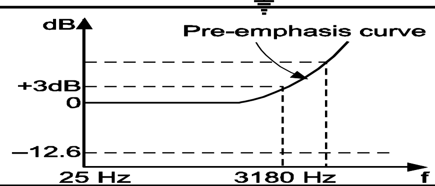
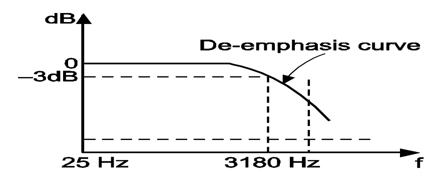
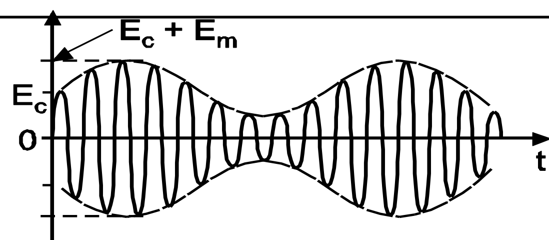
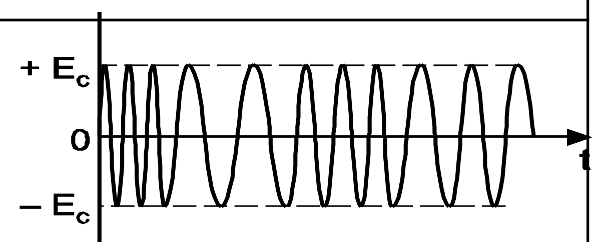


Fig. : De-emphasis Circuit in FM Receiver

Comparison between Pre-emphasis and De-emphasis

Parameter	Pre-emphasis	De-emphasis
1. Circuit used	High pass filter.	Low pass filter.
2. Circuit diagram		
3. Response curve		
4. Time constant	T = RC = 50 μs	T = RC = 50 μs
5. Definition	Boosting of higher frequencies	Removal of higher frequencies
6. Used at	FM transmitter	FM receiver.

Comparison between AM and FM

Parameter	AM	FM
1. Definition	Amplitude of carrier is varied in accordance with amplitude of modulating signal keeping frequency and phase constant.	Frequency of carrier is varied in accordance with the amplitude of modulating signal keeping amplitude and phase constant.
2. Constant parameters	Frequency and phase.	Amplitude and phase.
3. Modulated signal	 AM Wave	 FM Wave
4. Modulation Index	m=Em/Ec	m = δ/ fm
5. Number of sidebands	Only two	Infinite and depends on m_f.
6. Bandwidth	$BW = 2f_m$	$BW = 2\,(\delta + f_{m\,(max)})$
7. Application	MW, SW band broadcasting, video transmission in TV.	Broadcasting FM, audio transmission in TV.