

# Amplitude Modulation

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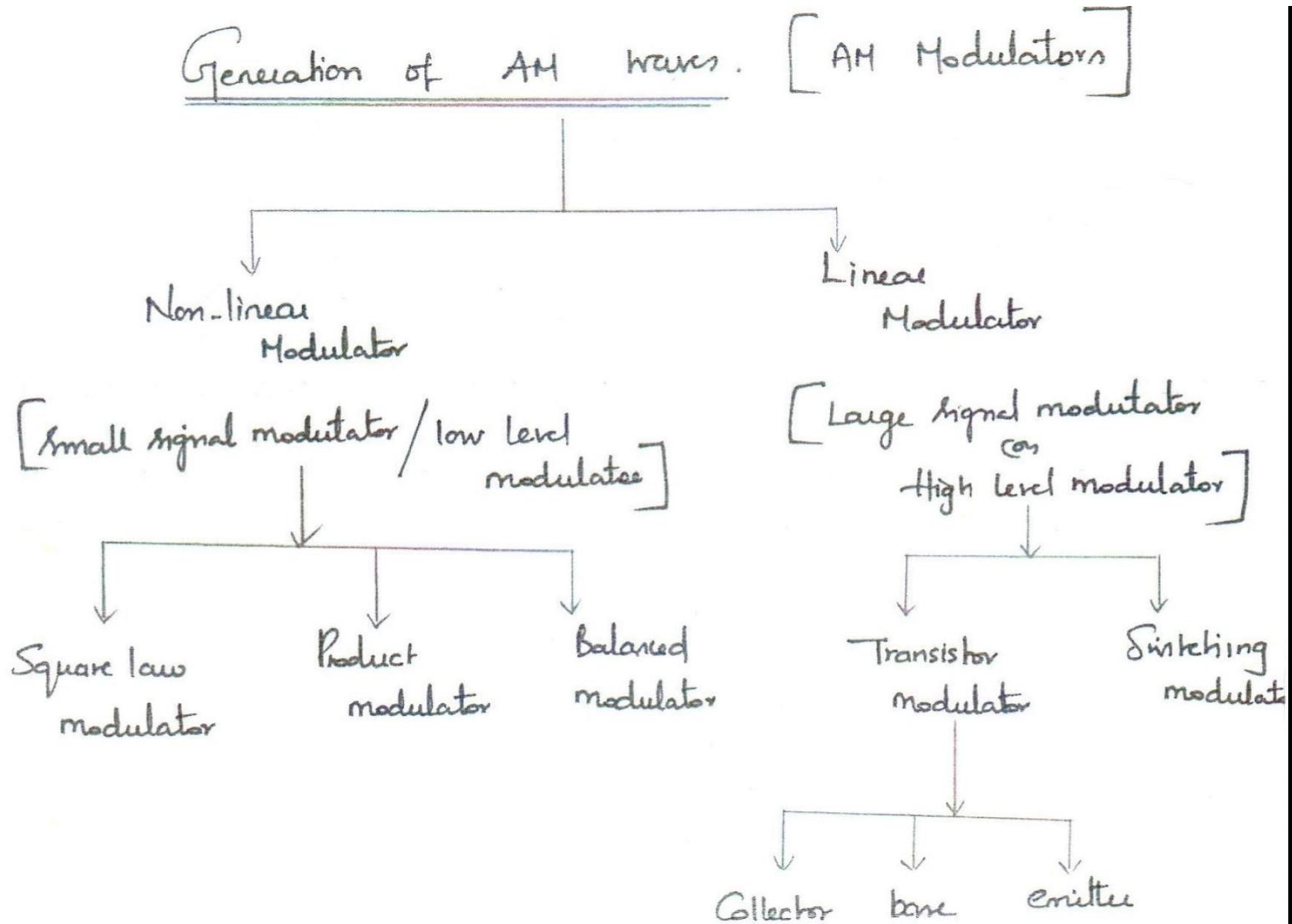
- ❖ Filter Method

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# Classification of Generation of DSB-FC



# Square-Law Modulator

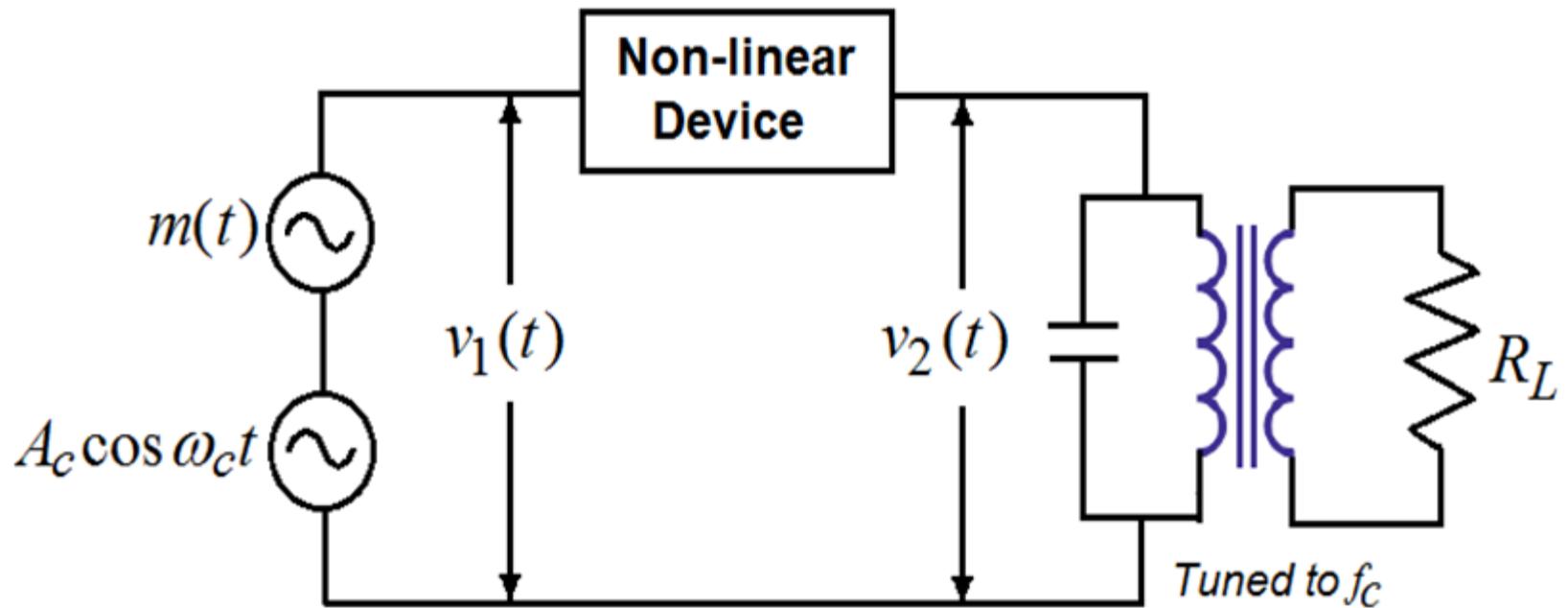
When the output of a device is not directly proportional to input throughout the operation, the device is said to be non-linear. The Input-Output relation of a non-linear device can be expressed as

$$V_0 = a_0 + a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3 + a_4 V_{in}^4 + \dots$$

When the input is very small, the higher power terms can be neglected. Hence the output is approximately given by  $V_0 = a_0 + a_1 V_{in} + a_2 V_{in}^2$

When the output is considered up to square of the input, the device is called a square law device and the square law modulator is as shown in the figure

# Square-Law Modulator



The **diode-resistor** equation is given by

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

Where  $a_1$  and  $a_2$  are constants.

# Square-Law Modulator ...

The output of the non-linear device (diode)

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \text{ where } v_1(t) = m(t) + A_c \cos \omega_c t$$

$$\begin{aligned} v_2(t) &= a_1 [m(t) + A_c \cos \omega_c t] + a_2 [m(t) + A_c \cos \omega_c t]^2 \\ &= a_1 A_c \left[ 1 + \frac{2a_2}{a_1} m(t) \right] \cos \omega_c t + a_1 m(t) + a_2 m^2(t) + a_2 A_c \cos^2 \omega_c t \end{aligned}$$

The **first term** is the desired AM wave with amplitude sensitivity

$$k_a = \frac{2a_2}{a_1}$$

Remaining unwanted three terms are removed by appropriate filtering.

# Square-Law Modulator ...

If  $m(t) = A_m \cos 2\pi f_m t$ , we get

$$V_2(t) = s(t) = a_1 A_c \left[ 1 + 2 \frac{a_2}{a_1} A_m \cos 2\pi f_m t \right] \cos 2\pi f_c t$$

Comparing this with the standard representation of AM signal,

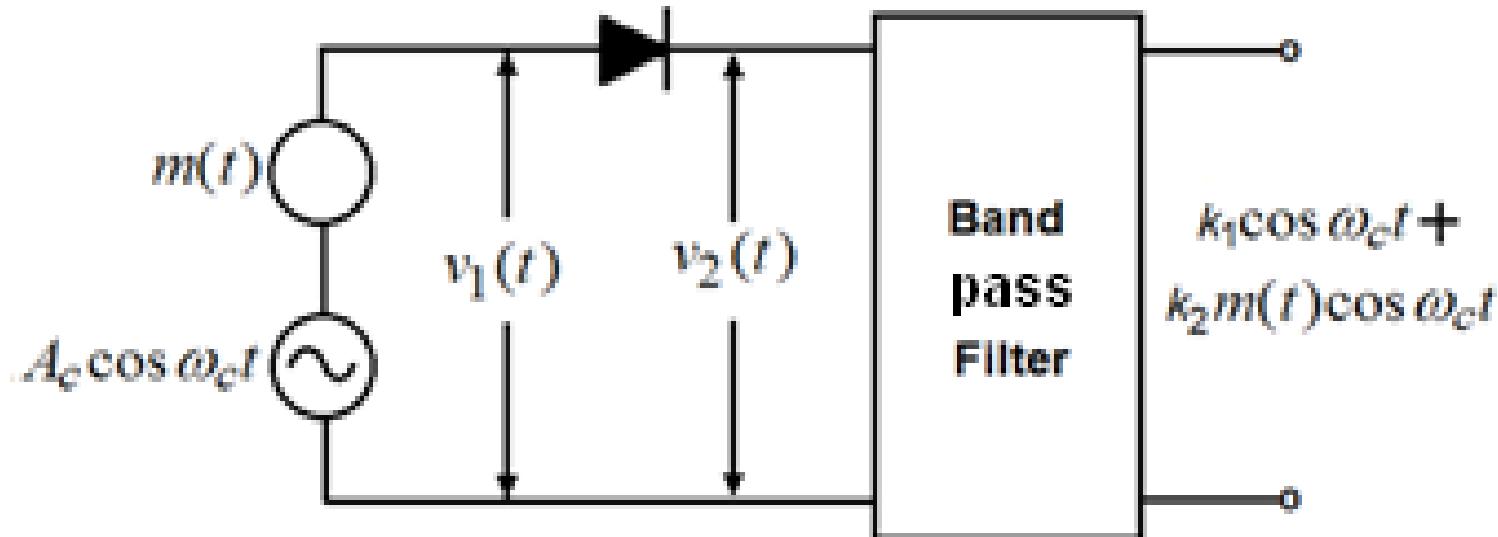
$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

Therefore modulation index of the output signal is given by

$$m = 2 \frac{a_2}{a_1} A_m$$

The output AM signal is free from distortion and attenuation only when  $(f_c - W) > 2W$  or  $f_c > 3W$ .

# Switching Modulator

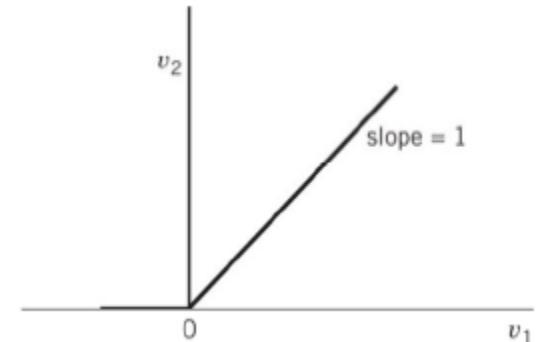


The input voltage to the diode ,

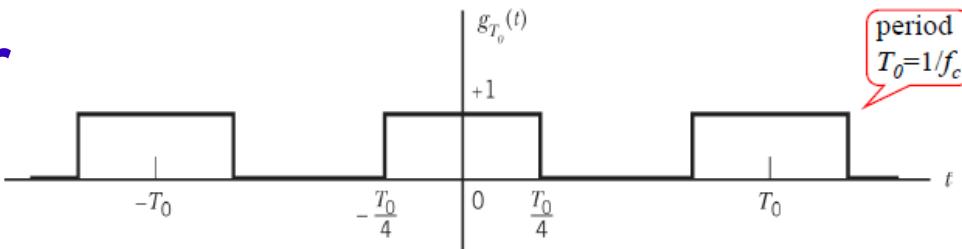
$$v_1(t) = m(t) + A_c \cos \omega_c t \quad \text{where } \|m(t)\| \ll A_c .$$

Then

$$v_2(t) = \begin{cases} v_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$$



# Switching Modulator



$$v_2(t) = [m(t) + A_c \cos \omega_c t] g_{T_0}(t)$$

$$g_{T_0}(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \cos \omega_c t - \frac{1}{3} \cos 3\omega_c t + \frac{1}{5} \cos 5\omega_c t - \dots \right]$$

$$v_2(t) = [m(t) + A_c \cos \omega_c t] g_{T_0}(t)$$

$$= \frac{m(t)}{2} + \frac{A_c}{2} \cos \omega_c t + \frac{2}{\pi} m(t) \cos \omega_c t + \dots$$

The desired components

$$\frac{A_c}{2} \cos \omega_c t + \frac{2}{\pi} m(t) \cos \omega_c t$$

# Switching Modulator

$$g_{T_0}(t) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos\left(2\pi \frac{n}{T_0} t\right) + b_n \sin\left(2\pi \frac{n}{T_0} t\right) \right)$$

$$a_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_{T_0}(t) dt$$

$$a_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} g_{T_0}(t) \cos\left(2\pi \frac{n}{T_0} t\right) dt$$

$$b_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} g_{T_0}(t) \sin\left(2\pi \frac{n}{T_0} t\right) dt$$

$$b_n = \frac{2}{T_0} \int_{-\frac{T_0}{4}}^{-\frac{T_0}{4}} \sin\left(2\pi \frac{n}{T_0} t\right) dt = -\frac{2}{T_0} \left. \frac{\cos\left(2\pi \frac{n}{T_0} t\right)}{2\pi \frac{n}{T_0}} \right|_{-\frac{T_0}{4}}^{\frac{T_0}{4}}$$

$$= -\frac{1}{n\pi} \left[ \cos\left(\frac{n\pi}{2}\right) - \cos\left(-\frac{n\pi}{2}\right) \right] = 0$$

$$g_{T_0}(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t (2n-1)]$$

$$a_0 = \frac{1}{T_0} \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} dt = \frac{1}{2}$$

$$\begin{aligned} a_n &= \frac{2}{T_0} \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \cos 2\pi \frac{n}{T_0} t dt = \frac{2}{T_0} \frac{\sin 2\pi \frac{n}{T_0} t}{2\pi \frac{n}{T_0}} \Big|_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \\ &= \frac{1}{n\pi} \left[ \sin\left(\frac{n\pi}{2}\right) - \sin\left(-\frac{n\pi}{2}\right) \right] = \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \\ &= \frac{2}{(2m-1)\pi} \sin\left(\frac{(2m-1)\pi}{2}\right) \\ &= \frac{2}{(2m-1)\pi} [-\cos(m\pi)] \\ &= \frac{2}{(2m-1)\pi} [-(-1)^m] = \frac{2}{(2m-1)\pi} (-1)^{m+1} \\ &= \frac{2}{(2m-1)\pi} (-1)^{m-1} \end{aligned}$$

# Switching Modulator

The required AM signal centered at  $f_c$  can be separated using band pass filter. The lower cutoff-frequency for the band pass filter should be between  $w$  and  $f_c-w$  and the upper cut-off frequency between  $f_c+w$  and  $2f_c$ . The filter output is given by the equation

$$S(t) = \frac{A_c}{2} \left[ 1 + \frac{4}{\pi} \frac{m(t)}{A_c} \right] \cos 2\pi f_c t$$

For a single tone information, let  $m(t) = A_m \cos(2\pi f_m t)$

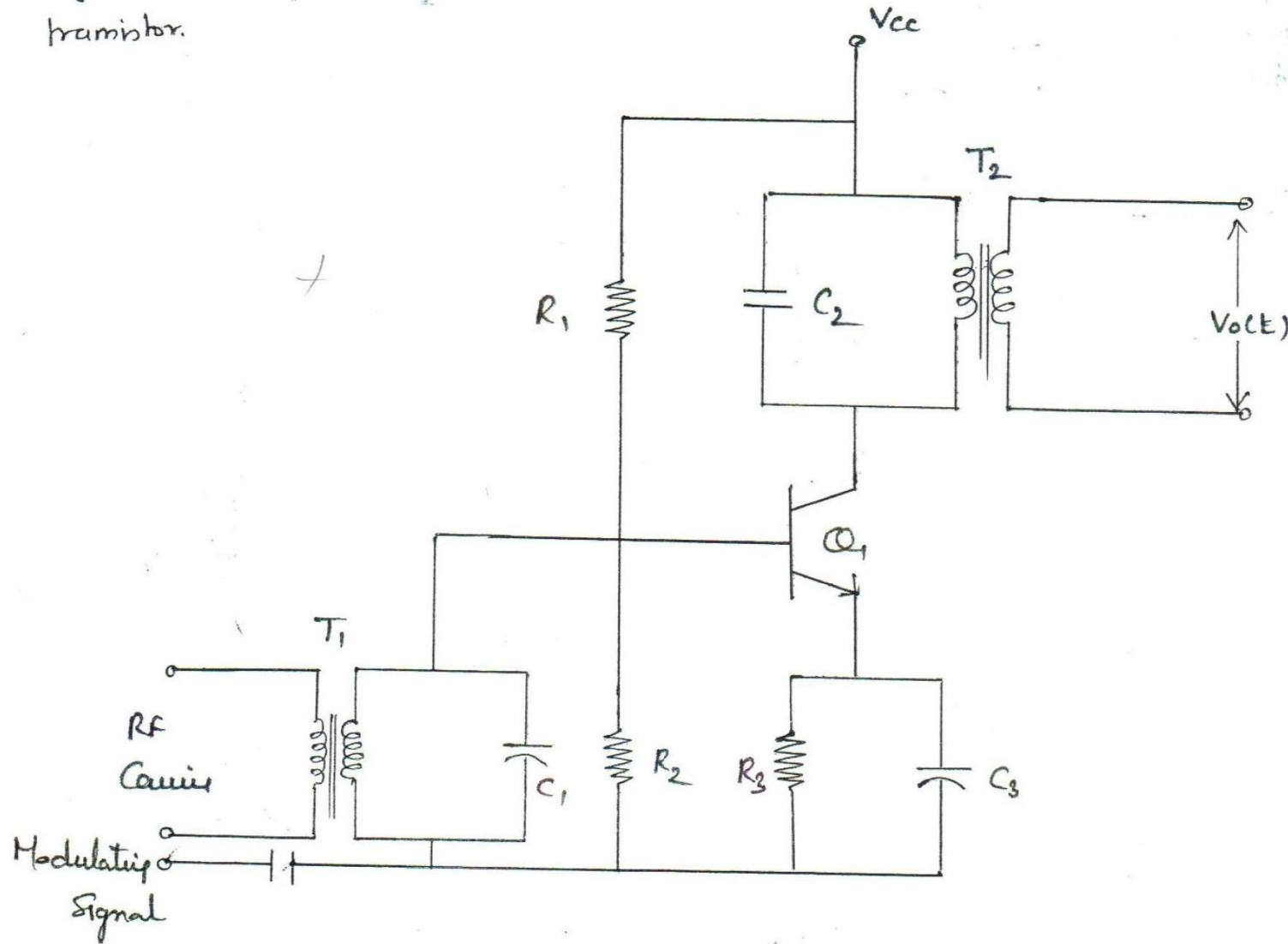
$$S(t) = \frac{A_c}{2} \left[ 1 + \frac{4}{\pi} \frac{A_m}{A_c} \cos 2\pi f_m t \right] \cos 2\pi f_c t$$

Therefore modulation index,  $m = \frac{4}{\pi} \frac{A_m}{A_c}$

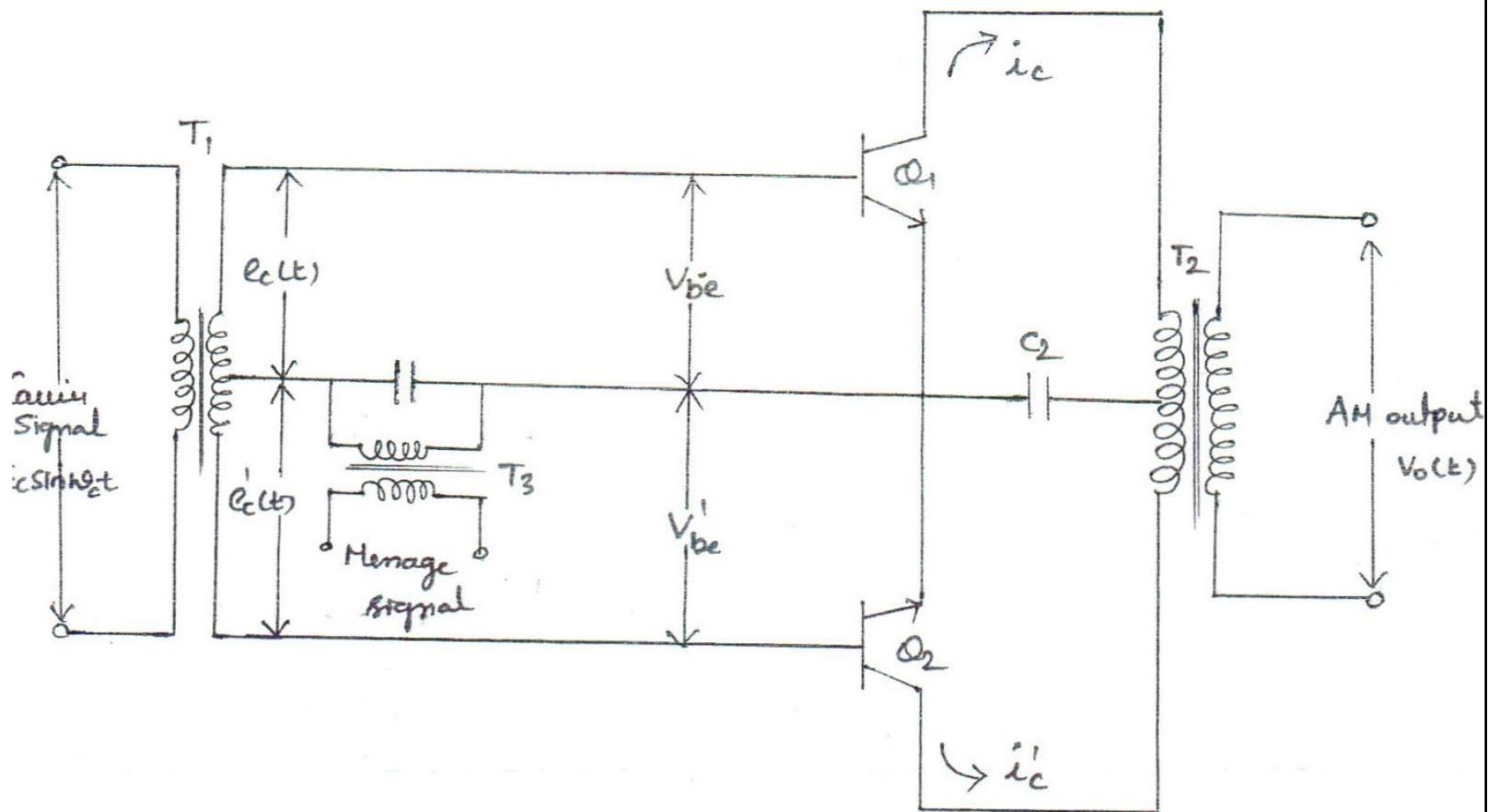
The output AM signal is free from distortions and attenuations only when  $f_c-w > w$  or  $f_c > 2w$ .

# Square law modulator using transistor

This circuit is used in the low level modulation. The modulation signal is applied to the emitter and RF carrier at the base of the transistor.



# Balanced modulator



Here two non-linear transistors are connected in the balanced mode. It is assumed that the two transistors are identical & the circuit is symmetrical. The operation of the transistor used are confined to operate in the non-linear region of its transfer characteristics.

# Balanced modulator

Where,  $e_c(t) = E_c \sin \omega_c t$  — ①

let, the message signal be,

$$e_m(t) = E_m \sin \omega_m t \quad \text{— } ②$$

I/p voltage across the transistor Q<sub>1</sub> is given by,

$$V_{be} = e_c(t) + e_m(t)$$

$$V_{be} = E_c \sin \omega_c t + E_m \sin \omega_m t \quad \text{— } ③$$

# Balanced modulator

The Caucie Voltage across the base winding of the Centre tap transformer as shown in fig. are equal and opposite in phase that is  $e_b(t) = -e_c(t)$ .

I/p Voltage across the transistor  $\text{Q}_2$  is given by,

$$V_{be}^1 = e_c^1(t) + e_m(t)$$

$$V_{be}^1 = -E_c \sin \omega_c t + E_m \sin \omega_m t \quad \text{--- (4)}$$

By the non-linearity relationship of the transistor the Collector Current can be written as,

$$\dot{i}_c = a V_{be} + b V_{be}^2 \quad \text{--- (5)}$$

$$\dot{i}_c^1 = a V_{be}^1 + b V_{be}^{1^2} \quad \text{--- (6)}$$

# Balanced modulator

Sub equ ③ in ⑤, we get

$$\dot{i}_c = a \left[ E_c \sin \omega_c t + E_m \sin \omega_m t \right] + b \left[ E_c \sin \omega_c t + E_m \sin \omega_m t \right]^2$$

$$\dot{i}_c = a E_c \sin \omega_c t + a E_m \sin \omega_m t + b E_c^2 \sin^2 \omega_c t + b E_m^2 \sin^2 \omega_m t$$

$$+ 2b E_m E_c \sin \omega_c t \sin \omega_m t \quad — ⑦$$

Sub equ ④ in ⑥

$$\dot{i}_c' = a \left[ -E_c \sin \omega_c t + E_m \sin \omega_m t \right] + b \left[ E_m \sin \omega_m t - E_c \sin \omega_c t \right]^2$$

$$= -a E_c \sin \omega_c t + a E_m \sin \omega_m t + b E_m^2 \sin^2 \omega_m t + b E_c^2 \sin^2 \omega_c t$$

$$- 2b E_m E_c \sin \omega_c t \sin \omega_m t \quad — ⑧$$

# Balanced modulator

The o/p AM Voltage ( $V_o$ ) is given by,

$$V_o = k(i_c - i'_c) \quad \text{--- (9)}$$

This is because both the Current flows in the Opposite direction  
 $k$  is a Constant depending on impedance & other Circuit parameters.

Sub equ (7) & (8) in equ (9).

$$\begin{aligned} V_o = k & \left[ (a E_c \sin \omega_c t + a E_m \sin \omega_m t + b E_c^2 \sin^2 \omega_c t + b E_m^2 \sin^2 \omega_m t \right. \\ & \left. + 2b E_m E_c \sin \omega_m t \sin \omega_c t) - (-a E_c \sin \omega_c t + a E_m \sin \omega_m t \right. \\ & \left. + b E_m^2 \sin^2 \omega_m t + b E_c^2 \sin^2 \omega_c t - 2b E_m E_c \sin \omega_c t \sin \omega_m t) \right] \end{aligned}$$

# Balanced modulator

$$V_o = k \left[ \cancel{a E_c \sin \omega_c t} + \cancel{a E_m \sin \omega_m t} + \cancel{b E_c^2 \sin^2 \omega_c t} + \cancel{b E_m^2 \sin^2 \omega_m t} \right. \\ \left. + 2b E_m E_c \sin \omega_m t \sin \omega_c t + a E_c \sin \omega_c t - a E_m \sin \omega_m t \right. \\ \left. - b E_m^2 \sin^2 \omega_m t - b E_c^2 \sin^2 \omega_c t + 2b E_m E_c \sin \omega_m t \sin \omega_c t \right]$$

$$V_o = k \left[ 2a E_c \sin \omega_c t + 4b E_m E_c \sin \omega_m t \sin \omega_c t \right]$$

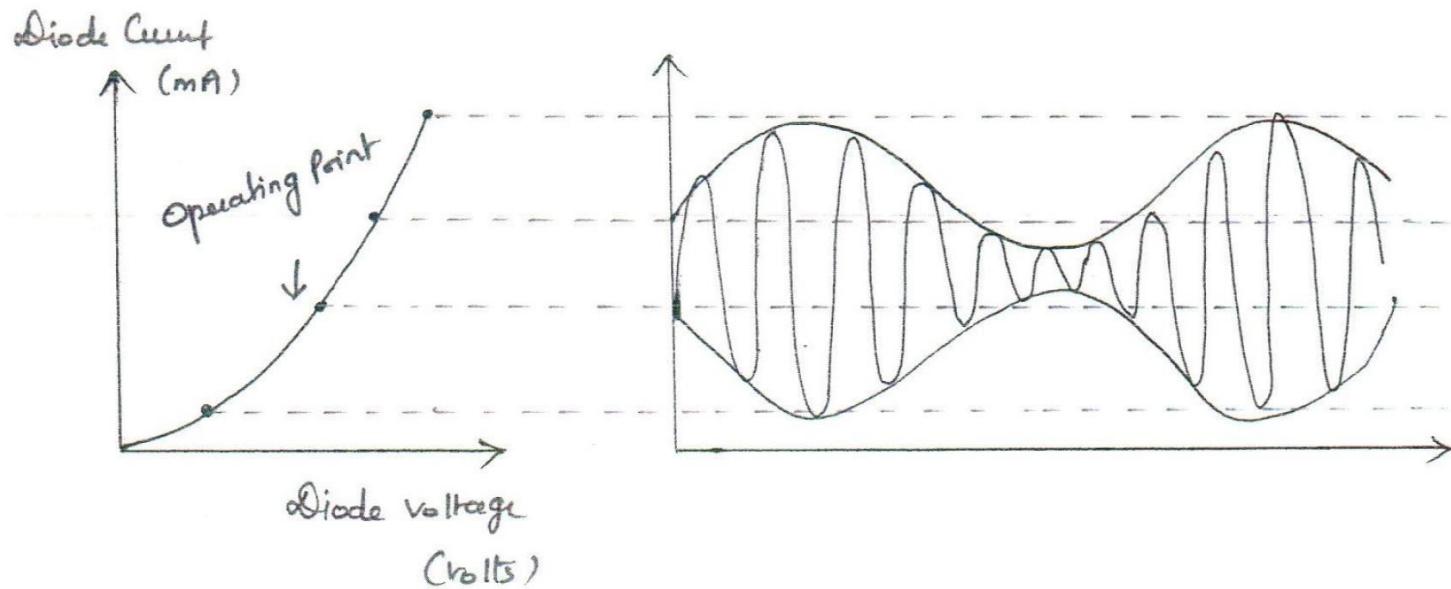
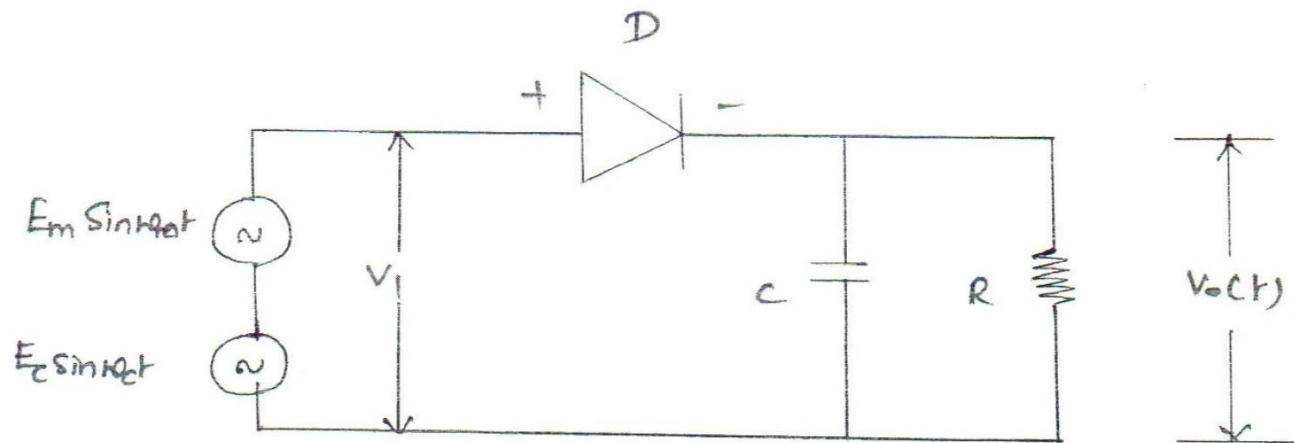
$$V_o = 2ka E_c \sin \omega_c t + 4kb E_m E_c \sin \omega_m t \sin \omega_c t$$

$$V_o = 2ka E_c \left[ 1 + \frac{2b E_m}{a} \sin \omega_m t \right] \sin \omega_c t.$$

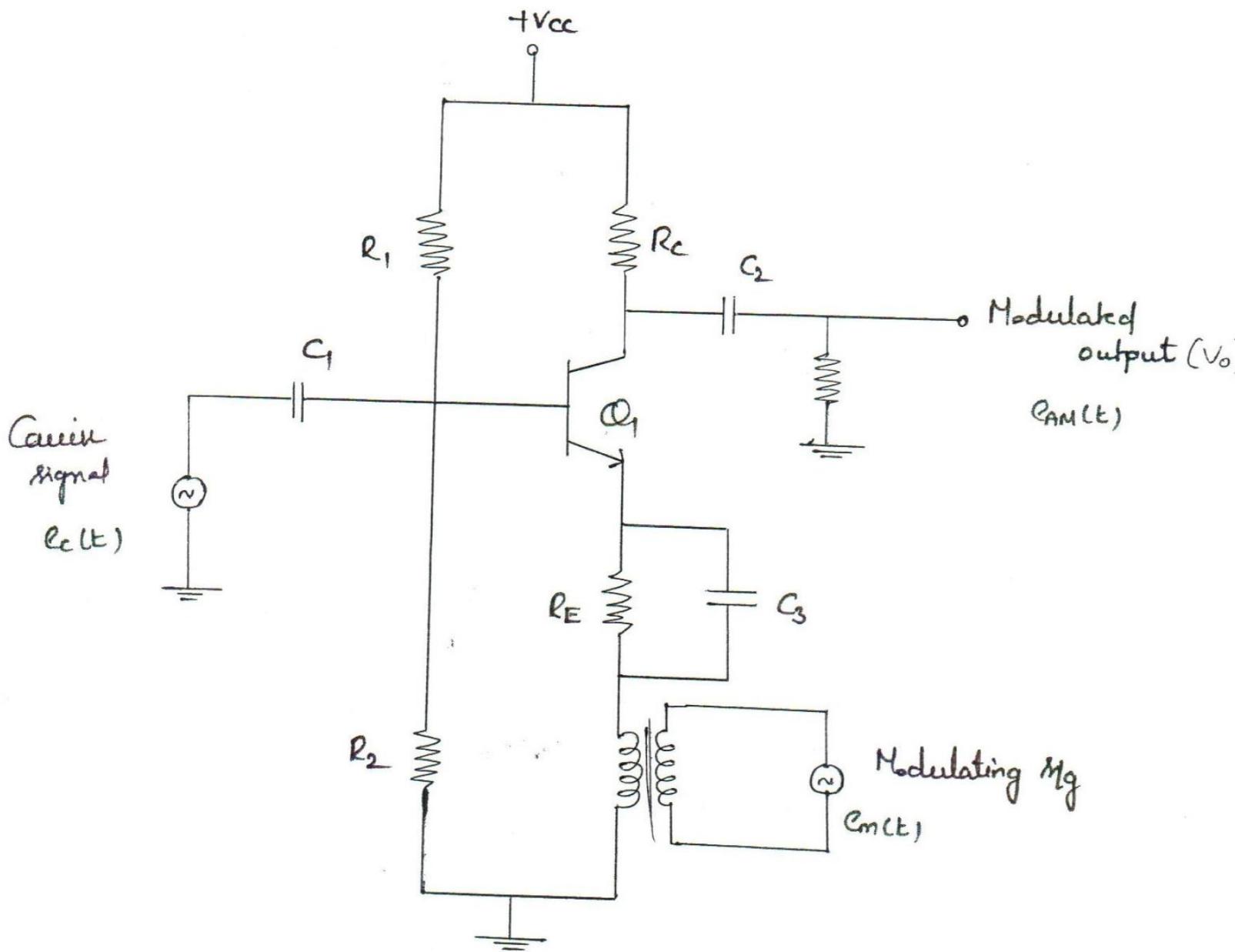
$m_a = \frac{2b E_m}{a}$  is the modulation index of AM.

$$\therefore V_o = 2ka E_c \left[ 1 + m_a \sin \omega_m t \right] \sin \omega_c t$$

# Square Law modulator using Diode



# Low Level AM modulator / Emitter Modulator



# Low Level AM modulator / Emitter Modulator

Gain of the transistor stage is depends upon the emitter current at quiescent point.

$$i_e = I_e + E_m \sin \omega_m t \quad \text{--- (1)}$$

Voltage gain of the transistor is proportional to the emitter current.

$$A_v \propto i_e$$

$$A_v = k i_e \quad \text{--- (2)} \quad k \rightarrow \text{Constant}$$

$$A_v = \frac{V_o}{V_i} \quad \text{--- (3)}$$

$$A_v V_i = V_o \quad \text{--- (4)}$$

# Low Level AM modulator / Emitter Modulator

$$V_i = E_c \sin \omega_c t \quad \text{--- (5)}$$

Sub (1) in (2)

$$Av = k ( I_e + E_m \sin \omega_m t ) \quad \text{--- (6)}$$

Sub (5) & (6) in (4)

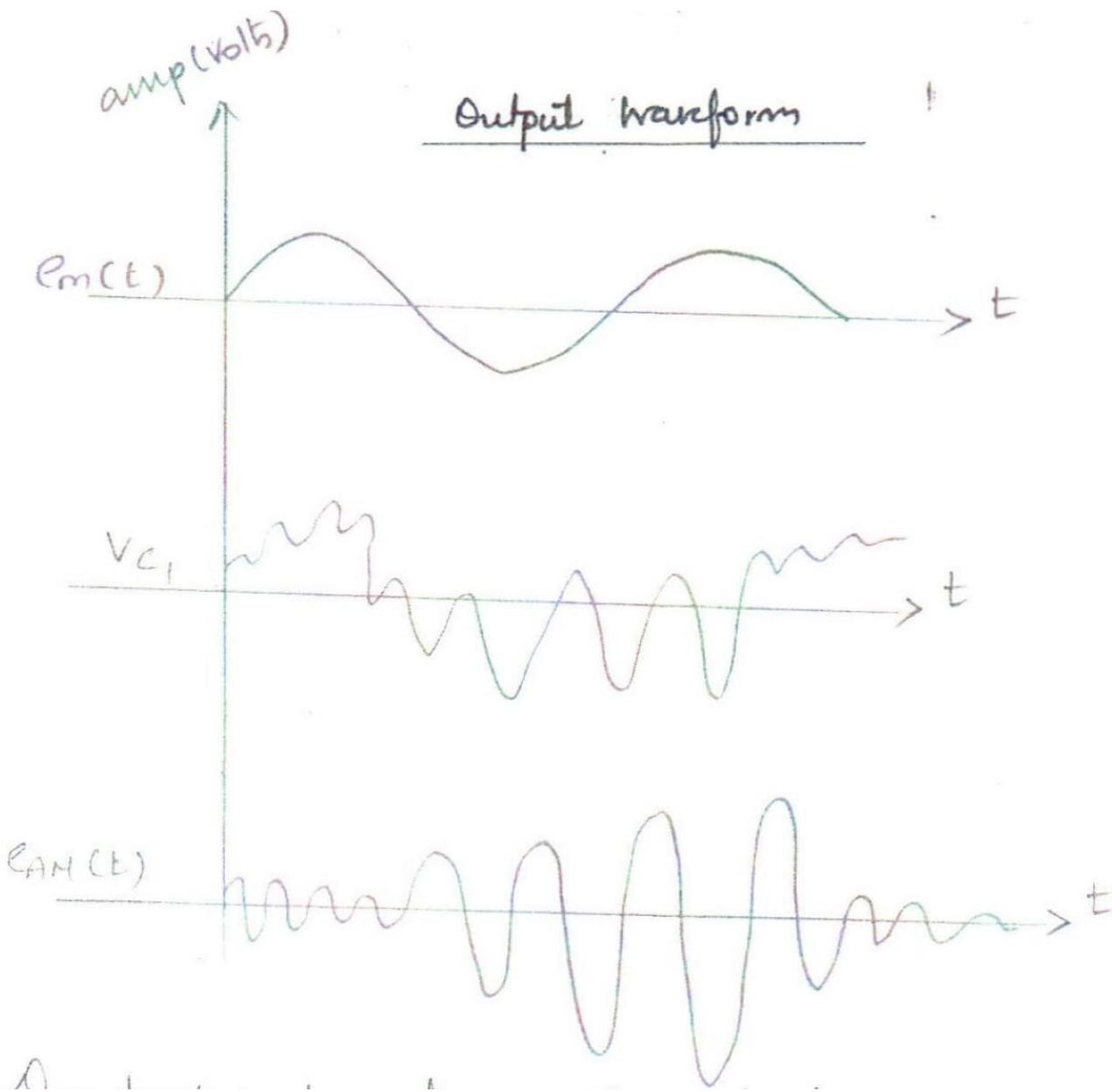
$$V_o = k ( I_e + E_m \sin \omega_m t ) E_c \sin \omega_c t$$

$$= k I_e E_c \sin \omega_c t + k E_m E_c \sin \omega_m t \sin \omega_c t$$

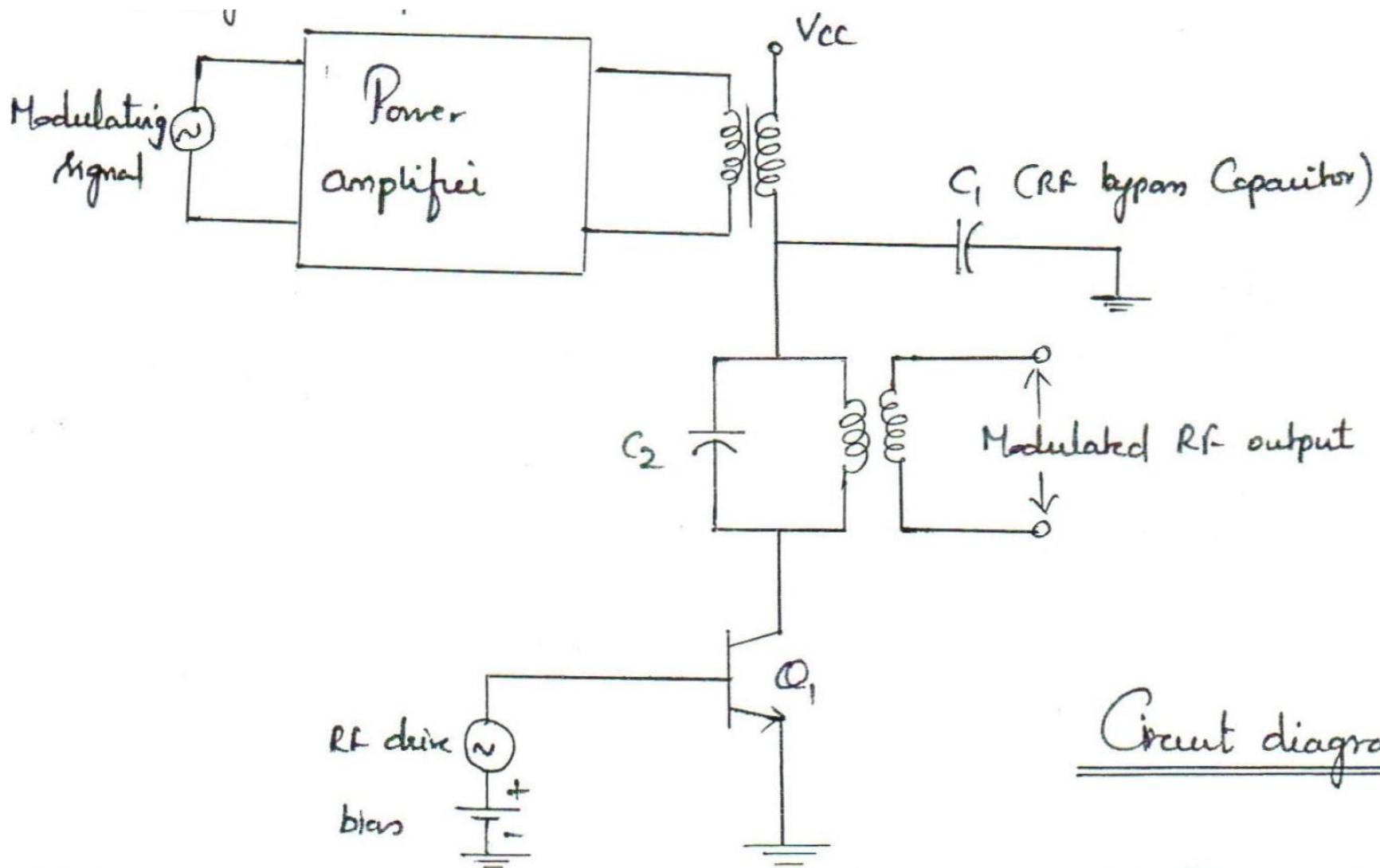
$$V_o = k I_e E_c \sin \omega_c t + k \frac{E_m E_c}{2} \left[ \cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t \right]$$

Thus AM wave is generated.

# Low Level AM modulator / Emitter Modulator



# High Level AM modulator / Collector Modulator



Circuit diagram

# High Level AM modulator / Collector Modulator

Case (i). without message signal:

→ Circuit will act as a class-'c'-tuned amplifier

$$V_o(t) = V_{cc} E_c(t)$$

$$V_o(t) = V_{cc} E_c \sin \omega_c t$$

Case (ii). with message signal:

→ Supply voltage will change with respect to message signal

$$V'_{cc} = V_{cc} + V_m \sin \omega_m t$$

$$V_o(t) = V'_{cc} V_c \sin \omega_c t$$

# High Level AM modulator / Collector Modulator

$$= (V_{cc} + V_m \sin \omega_m t) V_c \sin \omega_c t$$

$$V_o(t) = V_{cc} V_c \sin \omega_c t + V_m V_c \sin \omega_c t \sin \omega_m t$$

$$V_o(t) = V_{cc} V_c \sin \omega_c t + \frac{V_m V_c}{2} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

Thus the AM wave is generated.

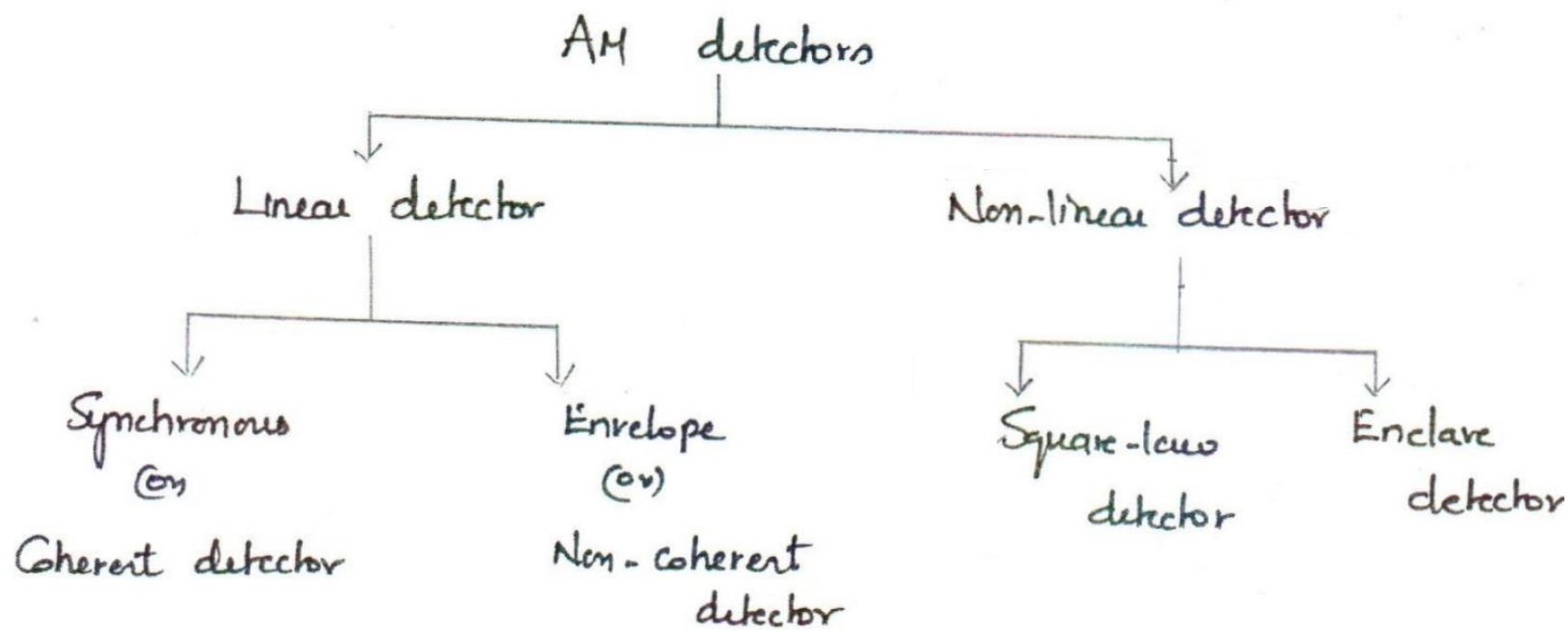
Adv.: Efficiency is high because class 'c' amp.

High o/p Power

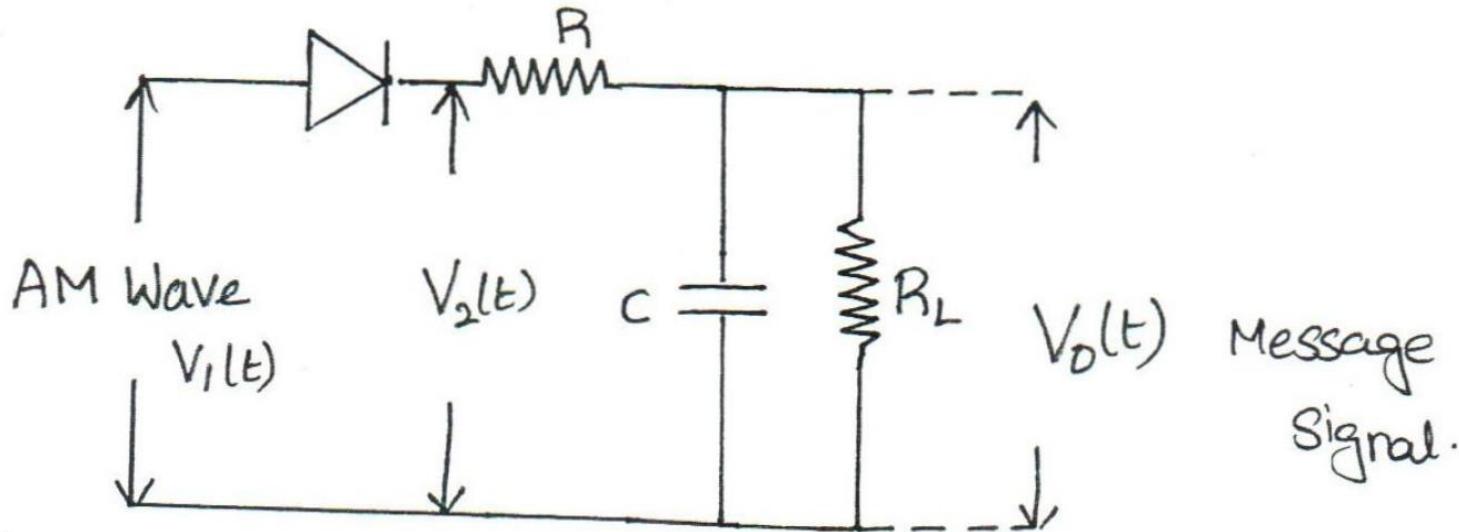
# Detection / Demodulation of AM Signal

## Demodulation / detection:

If it is the process in which the modulating voltage is recovered back from the modulated signal. This is the reverse process of modulation taking place in the receiver.



# Square Law Detector



$V - I \text{ (OR)}$   
Non Linear Relationship Between Input and Output,

$$V_2(t) = aV_1(t) + bV_1^2(t) \quad \text{--- (1)}$$

$V_1(t) \rightarrow \text{I/P of diode (AM Wave)}$

$V_2(t) \rightarrow \text{O/P of diode}$

## Square Law Detector

$$V_1(t) = E_c [1 + m_a \sin \omega_m t] \sin \omega_c t$$

Sub ② in ①

→ ②

$$\begin{aligned} V_2(t) &= a [E_c (1 + m_a \sin \omega_m t)] \sin \omega_c t + \\ &\quad b (E_c [1 + m_a \sin \omega_m t] \sin \omega_c t)^2 \end{aligned}$$

$$= a E_c \sin \omega_c t + a m_a E_c \sin \omega_m t \sin \omega_c t$$

$$+ b (E_c [1 + m_a \sin \omega_m t])^2 \sin^2 \omega_c t$$

## Square Law Detector

$$= aE_c \sin \omega_c t + a m_a E_c \sin \omega_m t$$

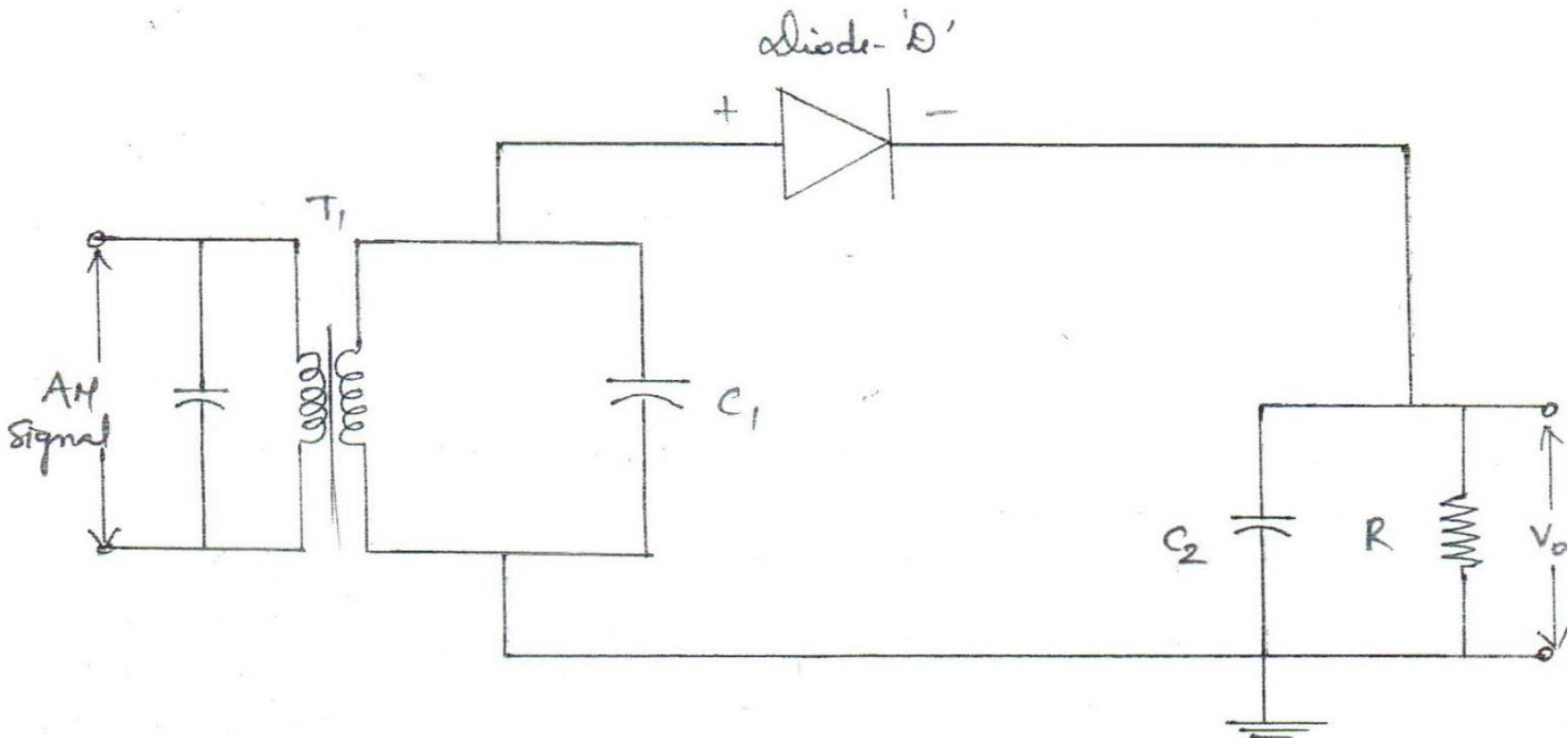
$$+ [b E_c^2 + \cancel{E_c^2 b^2 m_a} \sin \omega_m t + b E_c^2 m_a^2 \sin^2 \omega_m t] \left( 1 - \frac{\cos 4\pi f_c t}{2} \right)$$

out of these terms only  
desired term is extracted by using Low  
pass filter.

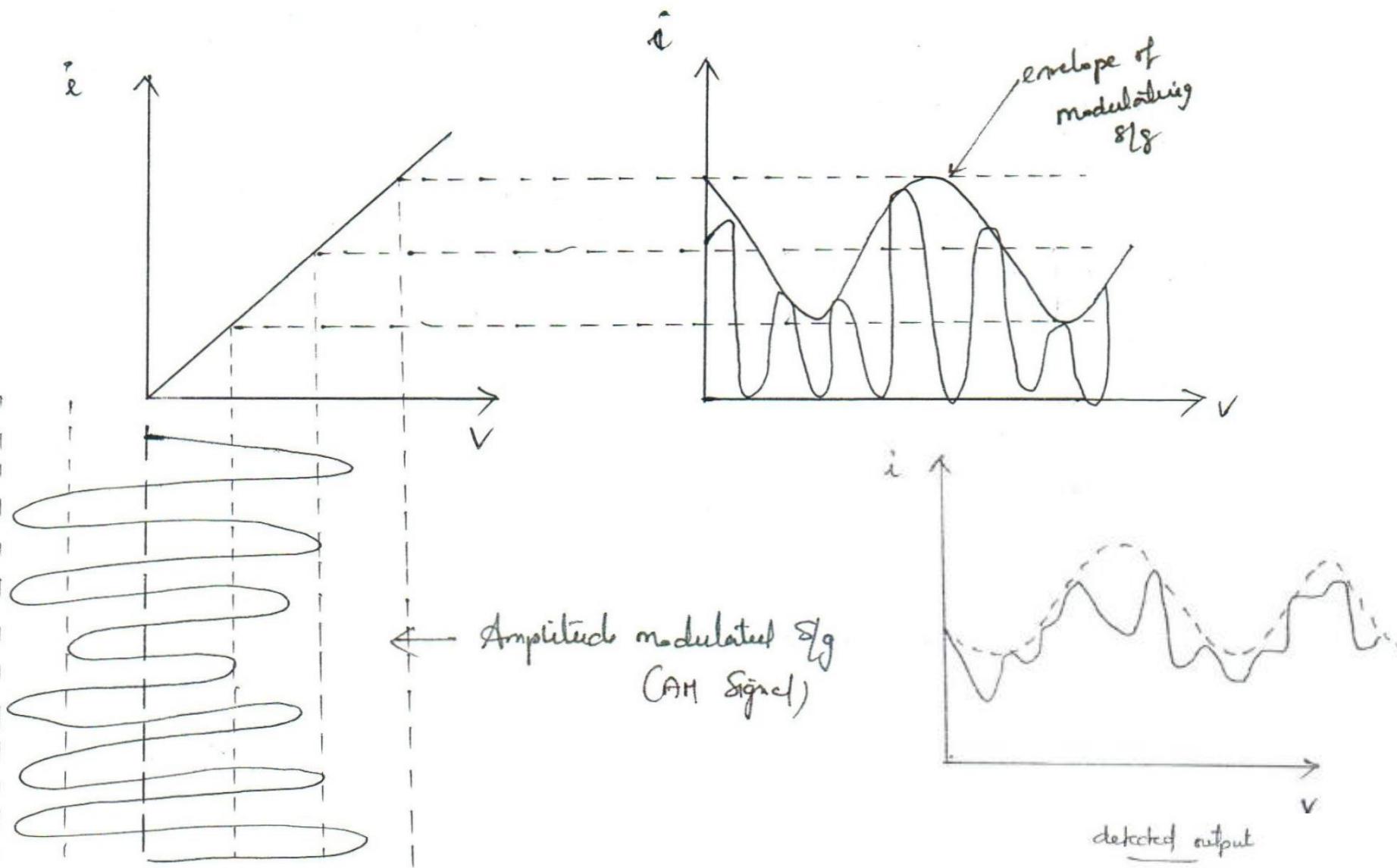
$$V_o(t) = b E_c^2 m_a \sin \omega_m t$$

Thus message signal recovered at the  
output of detector.

# Envelope / Diode / Linear Detector



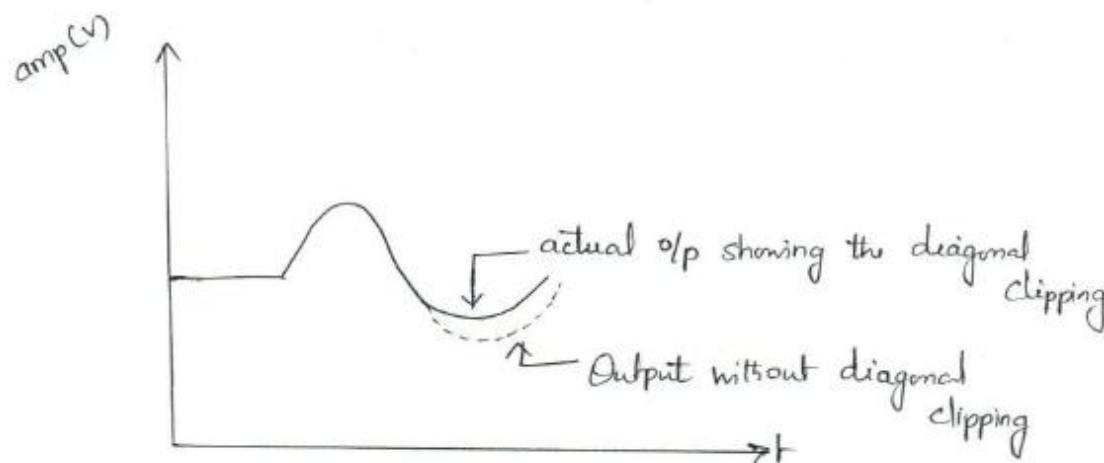
# Envelope / Diode / Linear Detector



# Distortion in Envelope Detector

## 1). Diagonal clipping:

- \* This type of distortion occurs when the RC time constant of load circuit is too long large
- \* If RC time constant is too large, it can't follow the fast changes in the modulating envelope
- \* As a result of distortion <sup>in</sup> diagonal of detected signal is clipped out



# Distortion in Envelope Detector

## 2) Negative-Peak clipping:

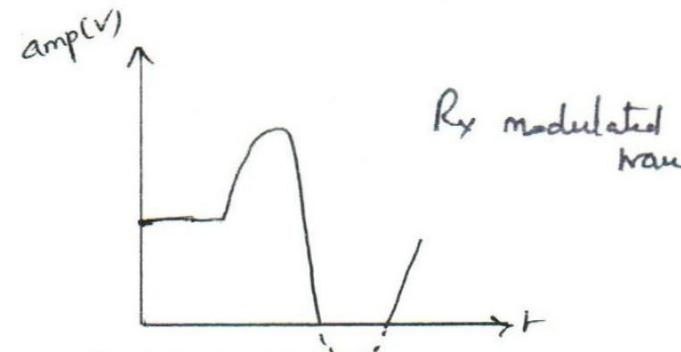
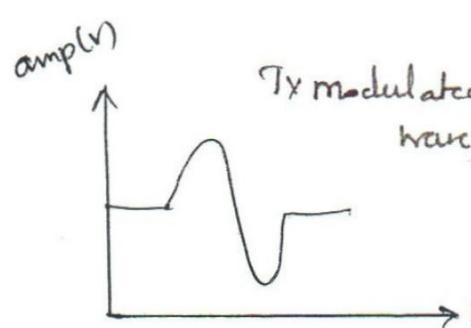
Modulation index of detected message is greater than modulation index of Input AM.

$$\text{I/p AM } (m_a) < \text{Detected message } (m_a)$$

If modulation index ( $m_a$ ) of message is high

$$E_m > E_c \quad m_a > 1, \text{ (i.e.) Over modulation.}$$

Hence negative peak clipping will take place as a result of Over modulation.



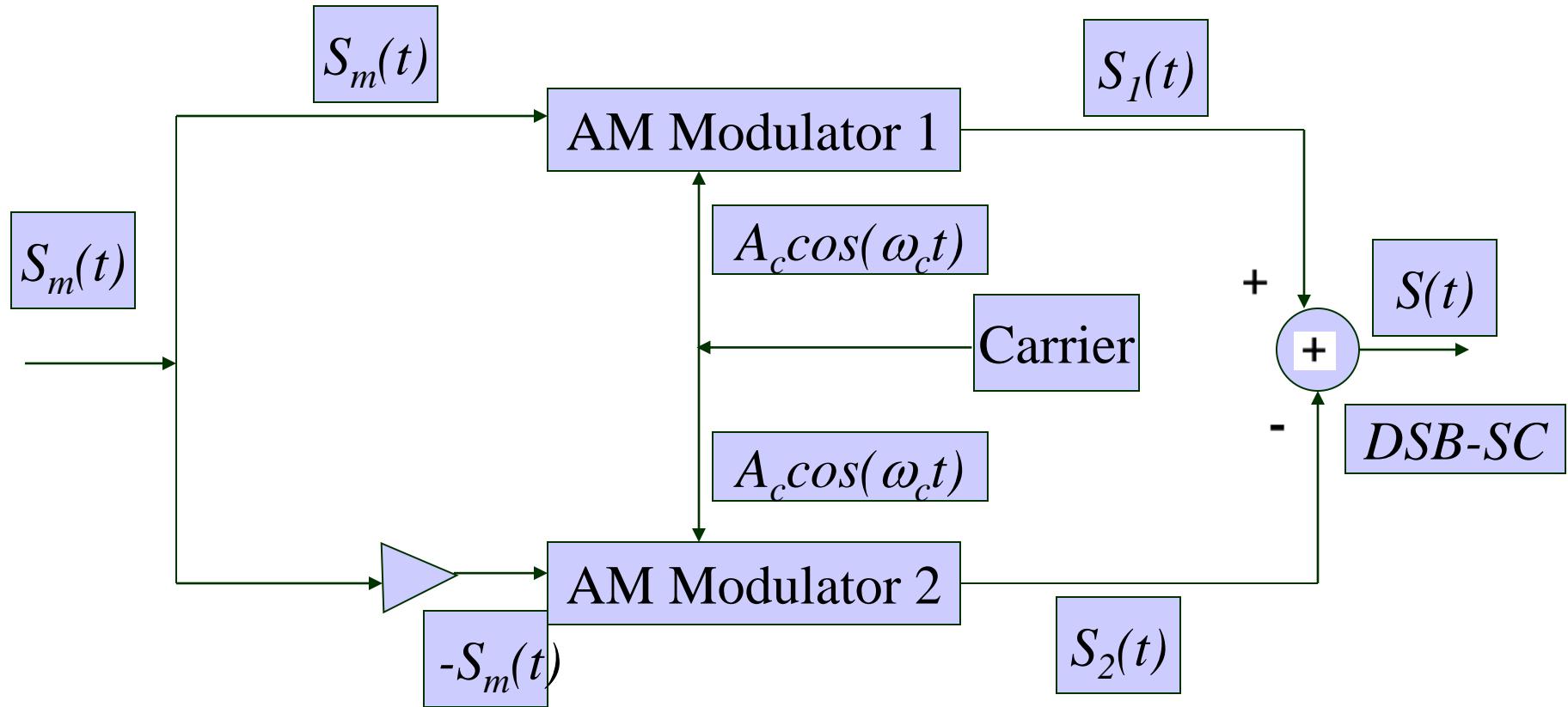
## Selection of RC time Constant:

$$\frac{1}{f_m} \ll RC \ll \frac{1}{f_c}$$

To avoid distortion print in envelope detector, RC time Constant should be as high as  $\frac{1}{f_c}$  & as low as  $\frac{1}{f_m}$ .

# DSB-SC Generation

## BALANCED MODULATOR



# DSB-SC Generation

- The two modulators are identical except for the sign reversal of the input to one of them. Thus,

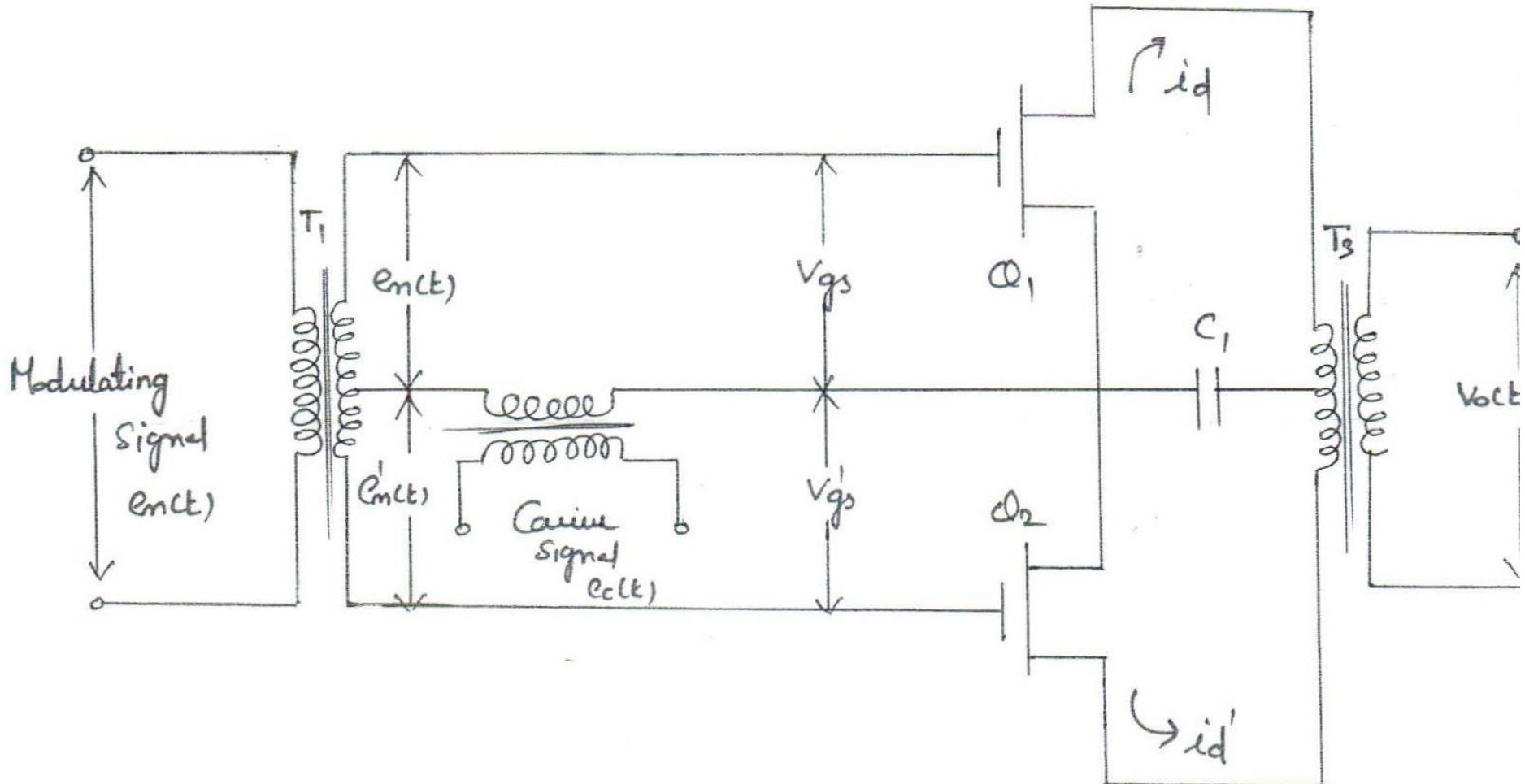
$$s_1(t) = A_c(1 + m \cos(\omega_m t)) \cos(\omega_c t)$$

$$s_2(t) = A_c(1 - m \cos(\omega_m t)) \cos(\omega_c t)$$

$$\begin{aligned} s(t) &= s_1(t) - s_2(t) \\ &= 2mA_c \cos(\omega_m t) \cos(\omega_c t) \end{aligned}$$

# DSB-SC Generation

*Balanced Modulator using FET*



# DSB-SC Generation

## Balanced Modulator using FET

The Voltage across the FET gates can be written as,

$$V_{gs} = E_c(t) + E_m(t)$$
$$= E_c \sin \omega_c t + E_m \sin \omega_m t \quad \text{--- } ①$$

$$V'_{gs} = E_c \sin \omega_c t - E_m \sin \omega_m t \quad \text{--- } ②$$

The drain Current of FET can be written as,

$$i_d = a_1 V_{gs} + a_2 V_{gs}^2 \quad \text{--- } ③$$

$$\dot{i}_d = a_1 V'_{gs} + a_2 V_{gs}^2 \quad \text{--- } ④$$

# DSB-SC Generation

## Balanced Modulator using FET

Now, the o/p of DSBSC signal  $v_o(t)$  is,

$$v_o(t) = k(i_q - i'_q) \quad \text{--- (5)}$$

Sub eqn (3) & (4) in eqn (5)

we get,  $v_o(t) = k [ (a_1 v_{qp} + a_2 v_{qp}^2) - (a_1 v'_{qp} + a_2 v'^2_{qp}) ]$

$$v_o(t) = k [ a_1 (v_{qp} - v'_{qp}) + a_2 (v_{qp}^2 - v'^2_{qp}) ]$$

$$= k [ a_1 (E_c \sin \omega_c t + E_m \sin \omega_m t - E_c \sin \omega_c t - E_m \sin \omega_m t) ]$$

$$+ a_2 [ (E_c \sin \omega_c t + E_m \sin \omega_m t)^2 - (E_c \sin \omega_c t - E_m \sin \omega_m t)^2 ]$$

# DSB-SC Generation

## Balanced Modulator using FET

$$= k \left[ 2a_1 E_m \sin \omega_m t + a_2 (4 E_m E_c \sin \omega_m t \sin \omega_c t) \right]$$

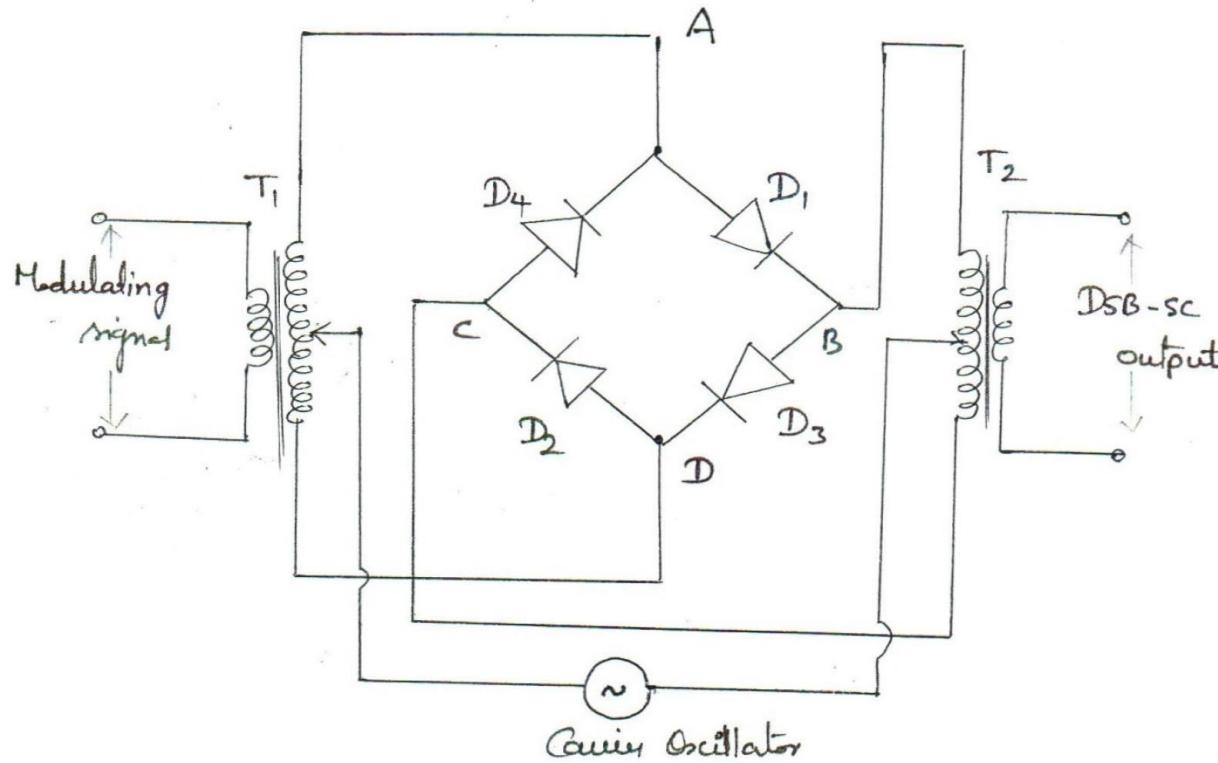
$$= 2ka_1 E_m \sin \omega_m t \left[ 1 + \frac{2a_2 E_c}{a_1} \cdot \frac{\sin \omega_c t}{\sin \omega_m t} \cdot \sin \omega_m t \right]$$

$$\therefore V_o(t) = 2ka_1 E_m \sin \omega_m t \left[ 1 + \frac{2a_2 E_c}{a_1} \sin \omega_c t \right] \quad \textcircled{b}$$

This is output,  $V_o(t)$  is applied to the Bandpass filter whose centre frequency is  $\pm \omega_c$ . Then we will get the output as two sideband signals.

# DSB-SC Generation

## *Ring Modulator / Diode bridge modulator*



A ring modulator using four diodes are shown in fig.

In a ring modulator Circuit four diodes are Connected in the form of a ring in which all four diodes point in the same manner. All the four diodes in the ring are Controlled by a square wave Carrier signal, applied through a Center-tapped transformer.

# DSB-SC Generation

## *Ring Modulator / Diode bridge modulator*

Circuit Operation : The Circuit Operation can be explained for two different conditions,

- 1) With only the Carrier signal applied to the Circuit without the message signal
- 2) With a Sinusoidal message signal applied to the Circuit along with the Carrier signal.

### Case 1

i) For (+ve) half-cycle of the { Carrier signal      } diodes D<sub>1</sub> & D<sub>2</sub> are forward biased & they conduct produces the o/p current.

# DSB-SC Generation

## *Ring Modulator / Diode bridge modulator*

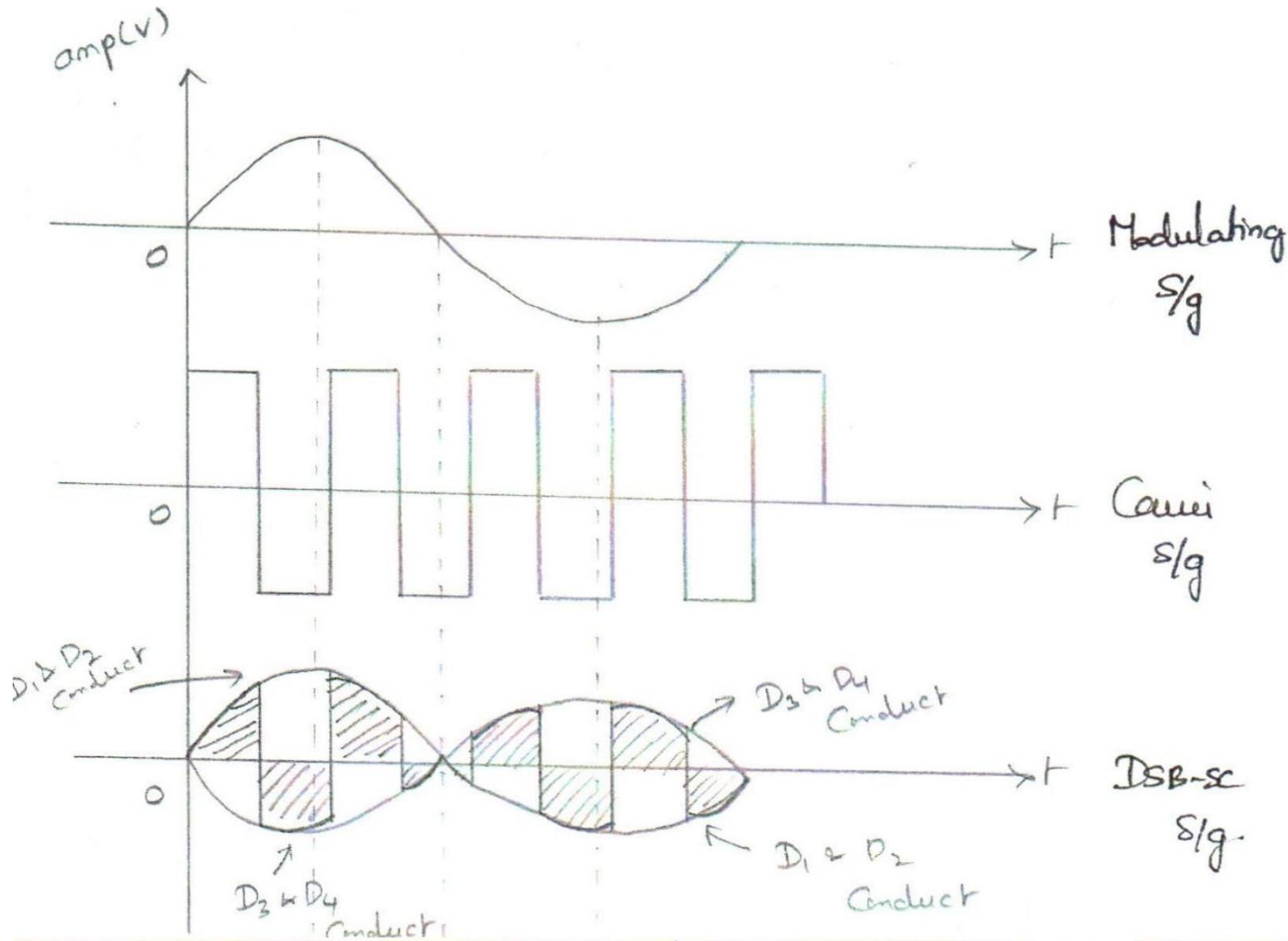
(ii) For +ve-half Cycle of the carrier signal }  
Diodes  $D_1 \& D_2$  are reverse biased & the diodes  $D_3 \& D_4$  are fwd biased.

### Case 2

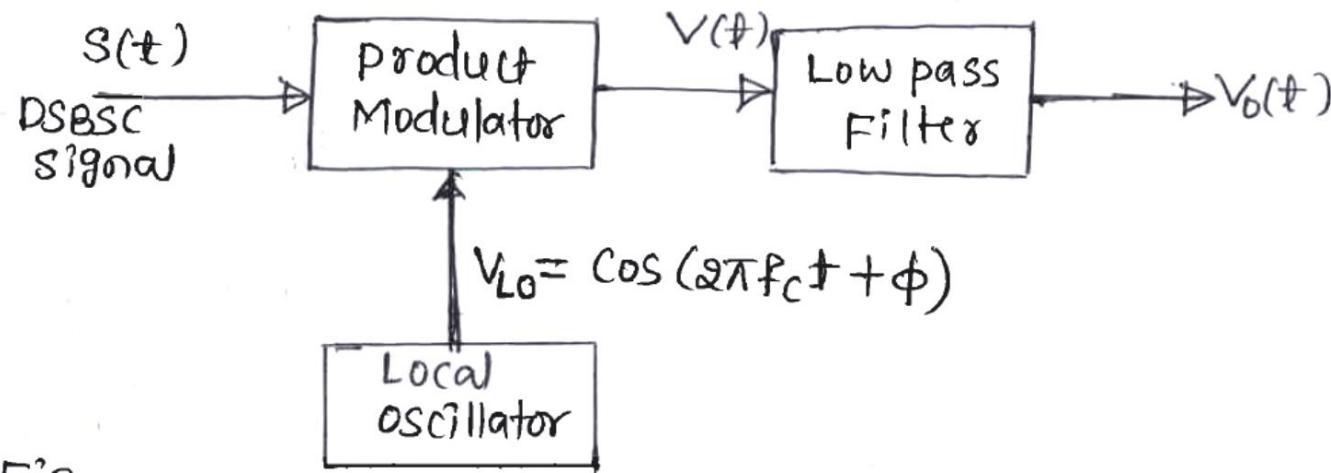
- i). For (+ve)-half cycle of the modulating signal }  
Diodes  $D_1 \& D_2$  are fwd biased  
& they will connect the secondary of  $T_1$  to the primary of  $T_2$ .
- ii). During negative half-cycle  $\Rightarrow$  Diodes  $D_3 \& D_4$  are fwd biased  
and they will connect the secondary of  $T_1$  to the primary of  $T_2$  with reverse connection.

# DSB-SC Generation

*Ring Modulator / Diode bridge modulator*



# Coherent Detection / Synchronous Detection



\*\* If Local oscillator output is not synchronized with that of carrier wave  $c(t)$  used to generate DSBSC-Signal, Coherent Detector produces either 'phase error' or 'frequency error' as follows.

Let the output voltage of the local oscillator is

$$v_L(t) = \cos(2\pi f_c t + \phi) \quad (1)$$

where  $\phi$  = phase error

# Coherent Detection

Then the product modulator output,

$$V(t) = s(t) \times V_L(t) \quad \rightarrow (2)$$

W.K.T, DSBSC-Signal  $s(t) = A_c m(t) \cos 2\pi f_c t$

$$\therefore V(t) = A_c m(t) \underbrace{\cos 2\pi f_c t}_B \times \underbrace{\cos(2\pi f_c t + \phi)}_A$$

$$\therefore V(t) = \frac{A_c m(t)}{2} \left[ \cos(2\pi f_c t + \phi - 2\pi b_c t) + \cos(2\pi f_c t + \phi + 2\pi b_c t) \right]$$

$$\therefore V(t) = \frac{A_c}{2} \cdot m(t) \left[ \cos \phi + \cos(4\pi f_c t + \phi) \right]$$

When  $V(t)$  is passed through LPF having  $\xrightarrow{(3)}$  Bandwidth " $\pm f_m$ "  
We get

$$V_o(t) = \frac{A_c}{2} \cdot \cos \phi \cdot m(t)$$

Output of LPF

# Coherent Detection

∴ The Demodulated signal  $V_0(t)$  is proportional to  $m(t)$ .  
and  $\cos \phi \Rightarrow$  phase error.

\* When  $\phi = 0^\circ \therefore V_0(t) = \frac{A_c}{2} \cdot m(t) \Rightarrow$  output voltage is Maximum

\* When  $\phi = 90^\circ \therefore V_0(t) = 0$  ( $\because \cos 90^\circ = 0$ ). Then the output voltage  $V_0(t)$  is minimum (zero). This effect is called Quadrature Null Effect of the Coherent Detector.

Note:-

↳ Similarly if frequencies are not synchronized, i.e., Local oscillator frequency  $f'_c \neq$  carrier frequency ' $f_c$ '.

then the frequency difference is 
$$(f'_c - f_c) = \Delta f$$

↳ The presence of frequency difference 
$$\Delta f = f'_c - f_c$$
, results in shift in the modulating signal frequency.

i.e., frequency of the output voltage signal of Coherent Detector is :  $f_o = f_m + \Delta f \rightarrow$  upward shift ( $f'_c < f_c$ )

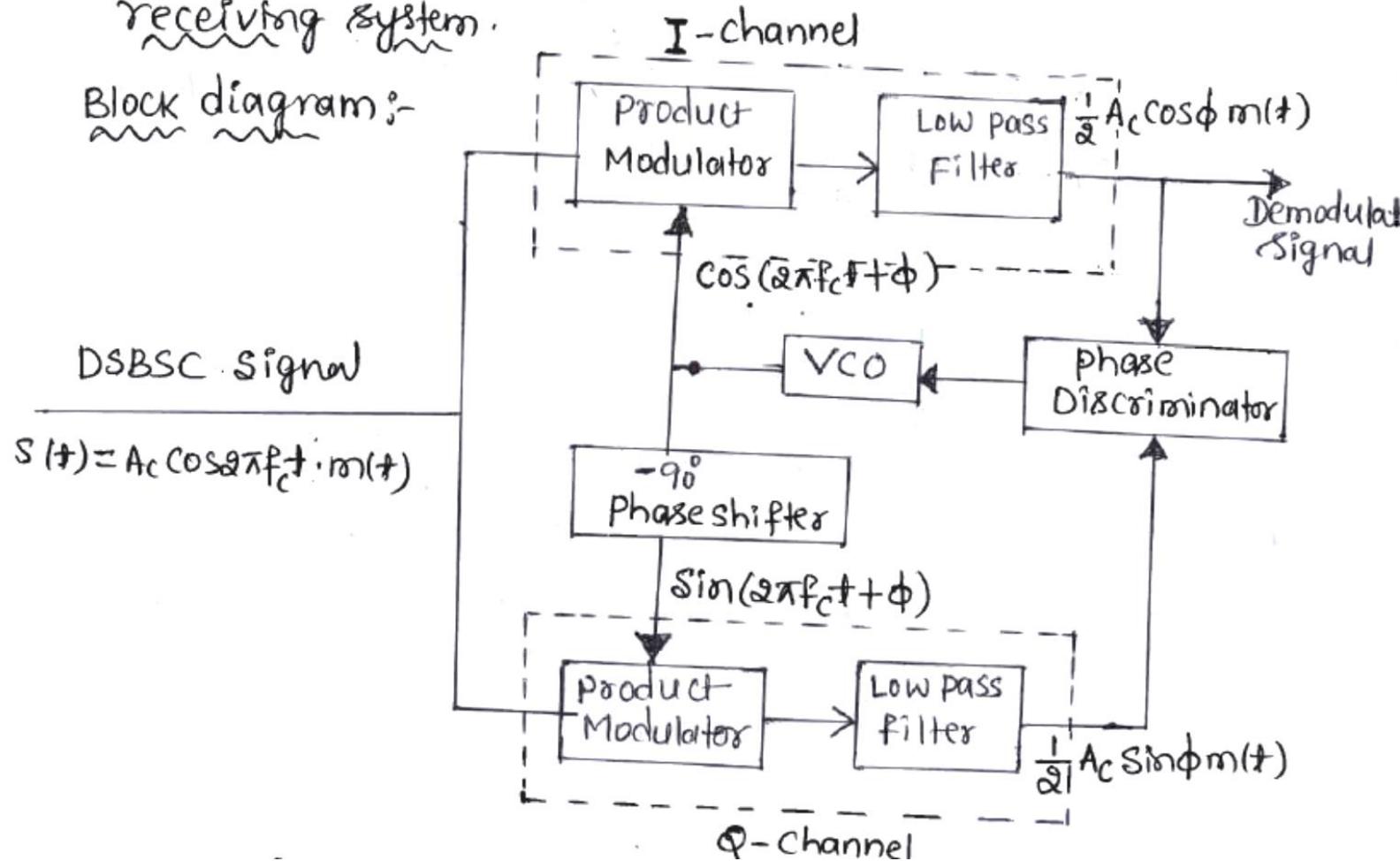
if  $\Delta f$  is negative

$$f_o = f_m - \Delta f \rightarrow$$
 downward shift ( $f'_c > f_c$ )

# Costas Receiver or Costas Loop

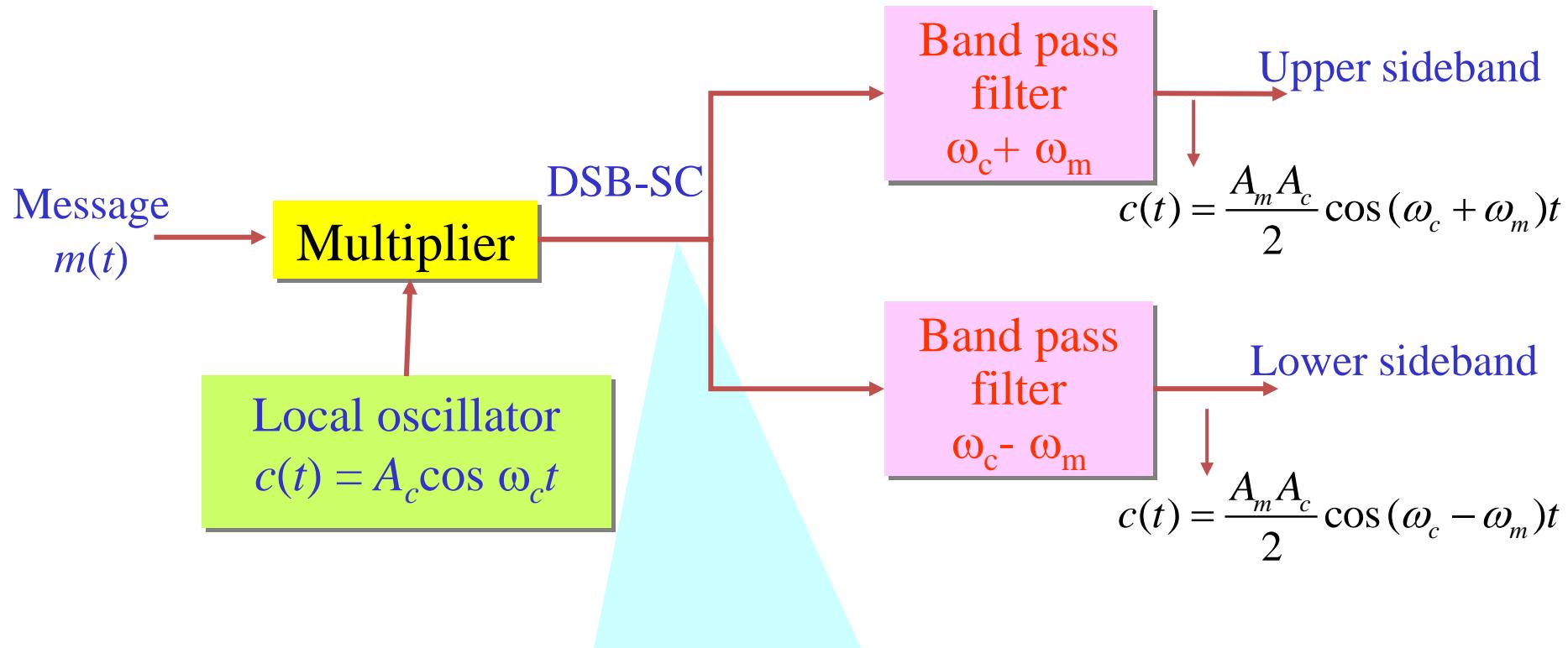
→ The Costas receiver is a practical synchronous receiver system, suitable for demodulating DSBSC-Waves. It is also named as Costas loop Ⓡ practical synchronous receiving system.

Block diagram:-



# SSB Generation

## Filter Method

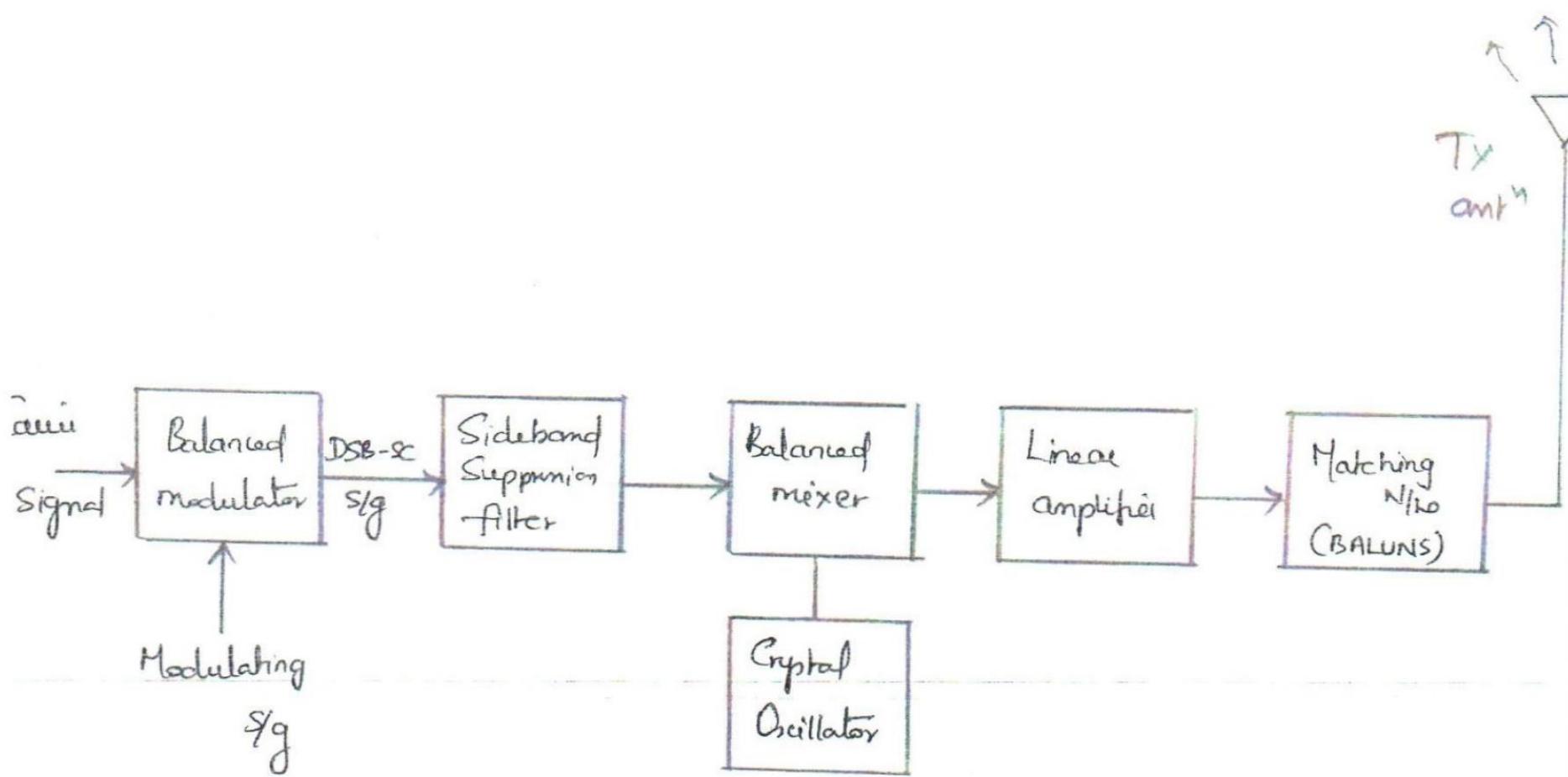


$$c(t) = A_m \cos \omega_m t \ A_c \cos \omega_c t$$

$$= \frac{A_m A_c}{2} \cos(\omega_c + \omega_m)t + \frac{A_m A_c}{2} \cos(\omega_c - \omega_m)t$$

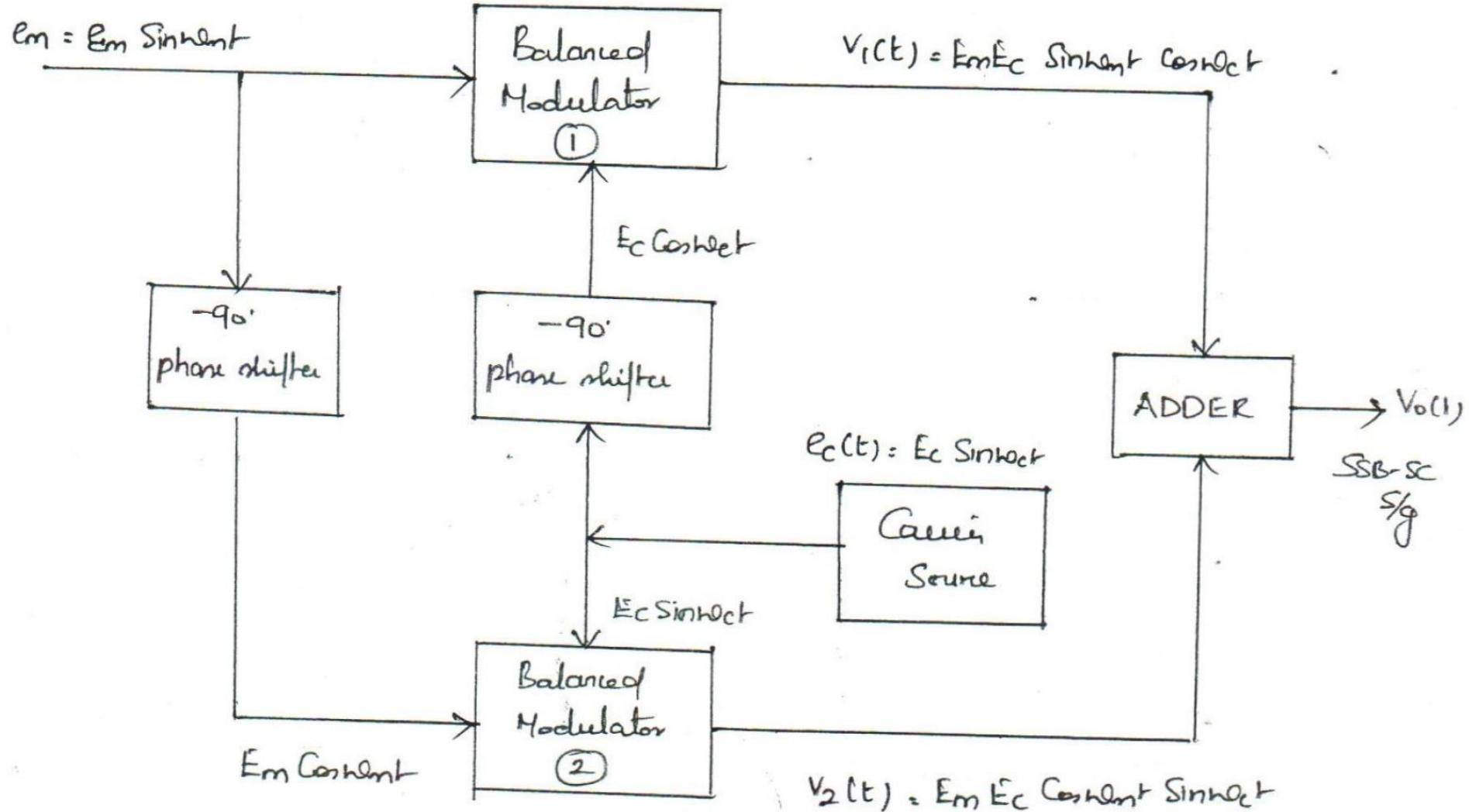
# SSB Generation

## Filter Method



# SSB Generation

## Phase Shift Method / Hartley Method



# SSB Generation

## Phase Shift Method / Hartley Method

Output of adder is,

$$V_o(t) = V_1(t) + V_2(t)$$

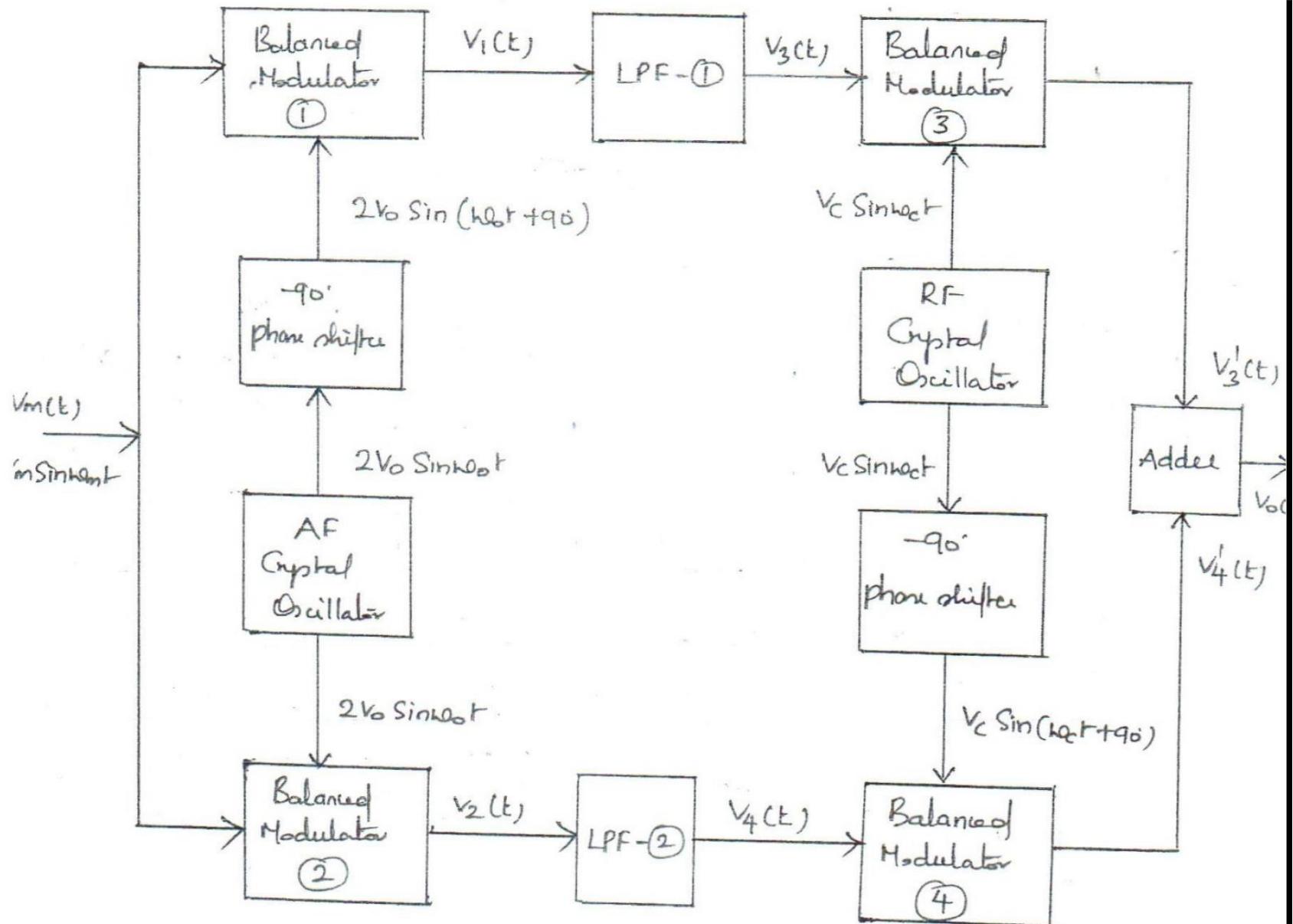
$$V_o(t) = E_m E_c \sin \omega_m t \cos \omega_c t + E_m E_c \cos \omega_m t \sin \omega_c t$$

$$= E_m E_c \left[ \sin \omega_m t \cos \omega_c t + \cos \omega_m t \sin \omega_c t \right]$$

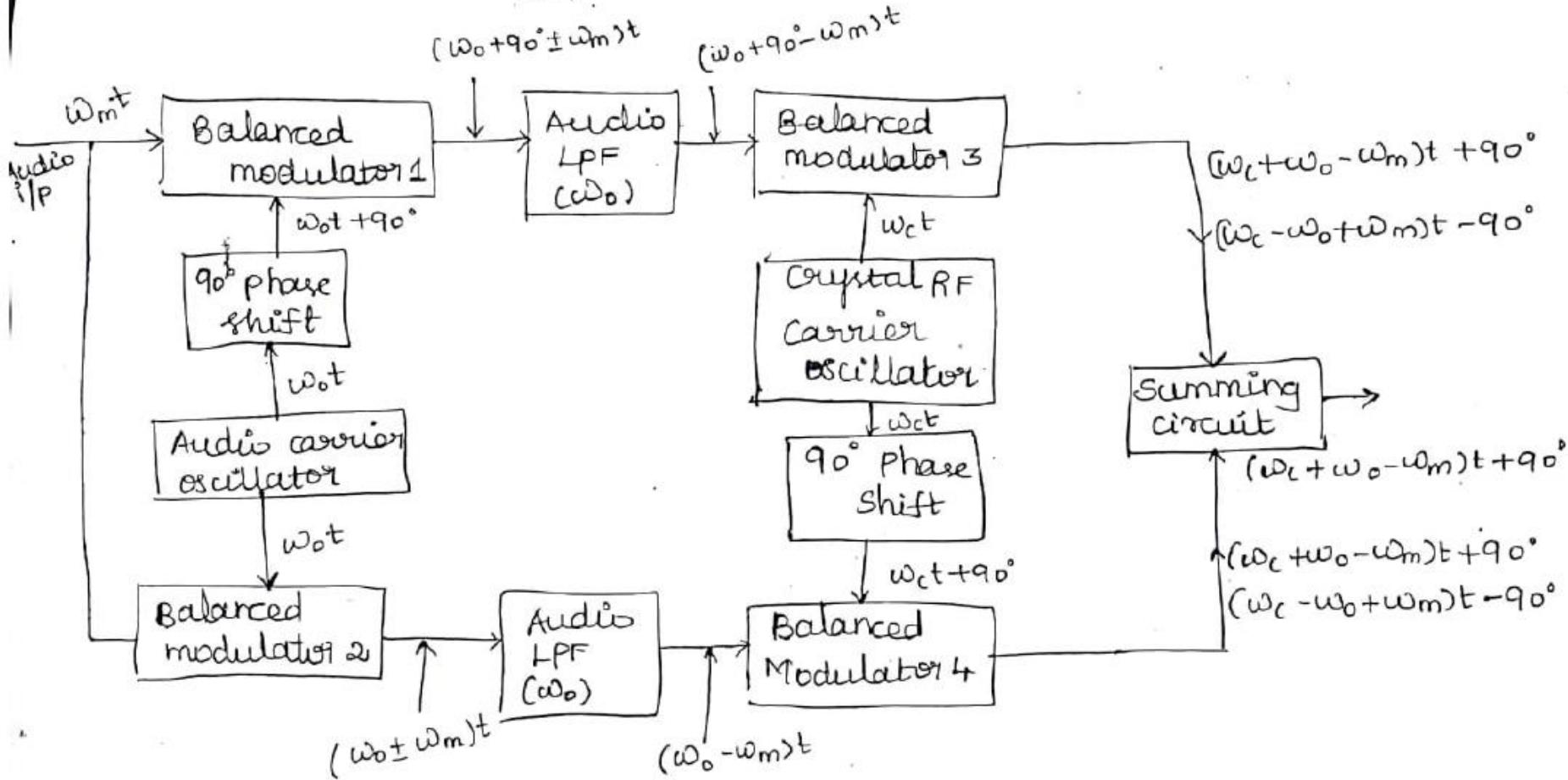
$$V_o(t) = E_m E_c \left[ \sin (\omega_c + \omega_m) t \right] //$$

# SSB Generation

Weavers Method or Modified Phase Shift Method or Third Method



# SSB Generation



Output of SSB signal :

The modulating sig  
AF carrier sig  
R.F carrier sig

$$V_m(t) = V_m \sin \omega_m t \rightarrow ①$$

$$V_o(t) = 2V_o \sin \omega_0 t \rightarrow ②$$

$$V_c(t) = 2V_c \sin \omega_c t \rightarrow ③$$

# SSB Generation

Output of Balanced modulator 1 :-

$$= 2 V_o \sin(\omega_0 t + q_0) V_m \sin \omega_m t$$

$$= V_m V_o [\cos(\omega_0 t - \omega_m t) + q_0] - \cos[(\omega_0 t + \omega_m t) + q_0] \rightarrow ④$$

Output of Balanced modulator 2 :-

$$= 2 V_o \sin \omega_0 t V_m \sin \omega_m t$$

$$= V_m V_o [\cos(\omega_0 t - \omega_m t) - \cos(\omega_0 t + \omega_m t)] \rightarrow ⑤$$

The low pass filters in the BM1 & BM2 eliminates the upper sidebands of modulator.

Output of LPF<sub>1</sub> is  $V_m V_o \cos(\omega_0 t - \omega_m t) + q_0$

Output of LPF<sub>2</sub> is  $V_m V_o \cos(\omega_0 - \omega_m)t$ .

Assume  $V_m = V_o = V_C = 1$

Output of Balanced modulator 3 :-

$$= 2 \sin \omega_0 t \cos(\omega_0 t - \omega_m t + q_0) \rightarrow ⑥$$

It is in the form of  $\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$

$$\therefore \sin[(\omega_C + \omega_0 - \omega_m)t + q_0] + \sin[(\omega_C - \omega_0 + \omega_m)t + q_0] \rightarrow ⑦$$

# SSB Generation

Output of Balanced modulator :-

$$= \& \sin(\omega_c t + 90^\circ) \cos(\omega_0 - \omega_m)t$$

$$= \sin[(\omega_c + \omega_0 - \omega_m)t + 90^\circ] + \sin[(\omega_c - \omega_0 + \omega_m)t + 90^\circ]$$

⑧

From eq ⑦ & ⑧, the output of summer circuit is,

$$V_o = \sin[(\omega_c + \omega_0 - \omega_m)t + 90^\circ] + \sin[(\omega_c - \omega_0 + \omega_m)t - 90^\circ] +$$

$$\sin[(\omega_c + \omega_0 - \omega_m)t + 90^\circ] + \sin[(\omega_c - \omega_0 + \omega_m)t + 90^\circ]$$

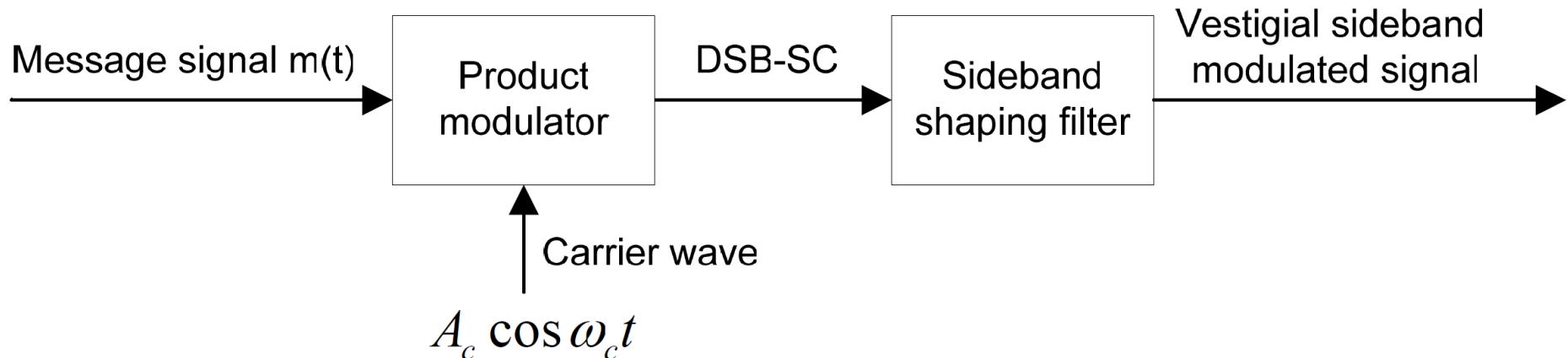
$$= 2 \sin[(\omega_c + \omega_0 - \omega_m)t + 90^\circ] \rightarrow ⑨$$

$$V_o = 2 \cos(\omega_c + \omega_0 - \omega_m)t$$

The final RF output frequency is  $f_c + f_0 - f_m$  which is essentially the lower sideband of RF carrier  $f_c + f_0$ .

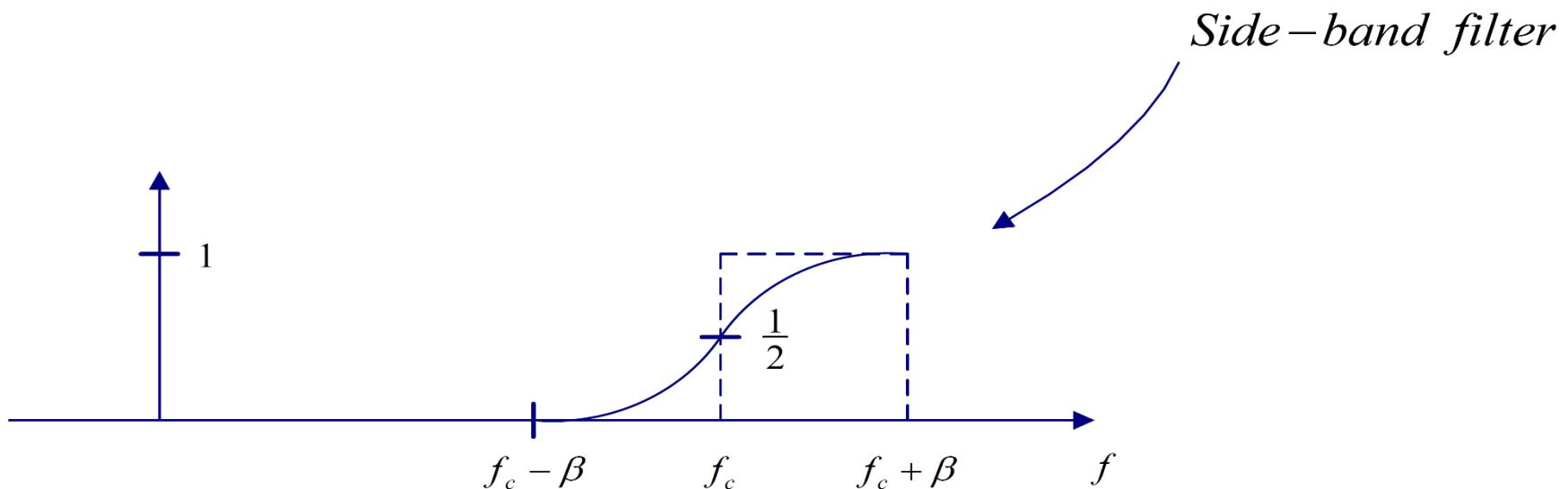
## Filter Method

# VSB Generation

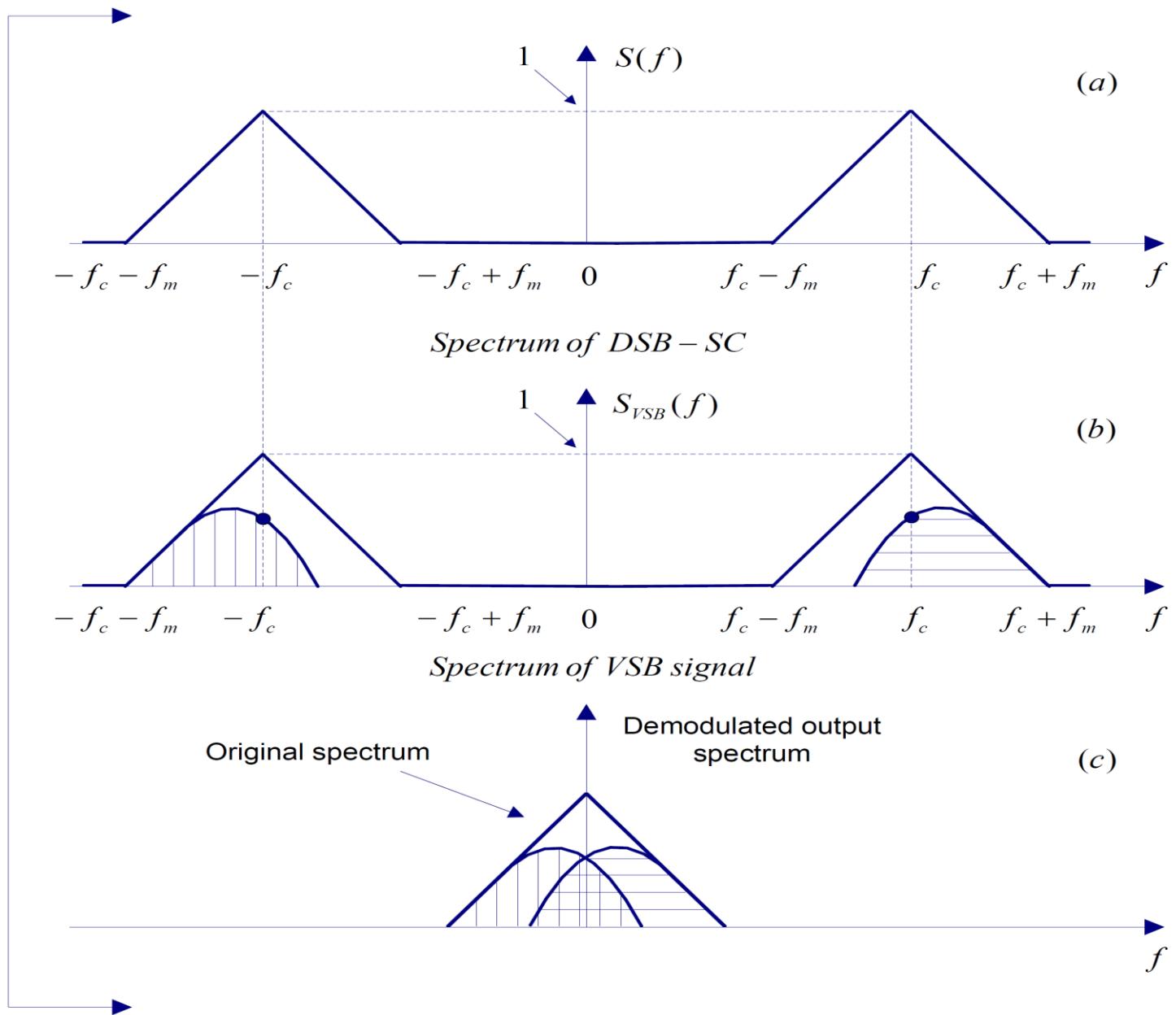


In VSB modulation, one sideband is passed almost completely whereas just a **trace** or vestige, of the other sideband is retained.

The key to VSB is the sideband filter, a typical transfer function being shown below:



# VSB



# VSB

In this method, first we will generate DSBSC wave with the help of the product modulator. Then, apply this DSBSC wave as an input of sideband shaping filter. This filter produces an output, which is VSBSC wave.

The modulating signal  $m(t)$  and carrier signal  $A_c \cos(2\pi f_c t)$  are applied as inputs to the product modulator. Hence, the product modulator produces an output, which is the product of these two inputs.

Therefore, the output of the product modulator is

$$p(t) = A_c \cos(2\pi f_c t)m(t)$$

Apply Fourier transform on both sides

$$P(f) = \frac{A_c}{2} [M(f - f_c) + M(f + f_c)]$$

The above equation represents the equation of DSBSC frequency spectrum.

# VSB

Let the transfer function of the sideband shaping filter be  $H(f)$ . This filter has the input  $p(t)$  and the output is VSBSC modulated wave  $s(t)$ . The Fourier transforms of  $p(t)$  and  $s(t)$  are  $P(f)$  and  $S(f)$  respectively.

Mathematically, we can write  $S(f)$  as

$$S(f) = P(f) H(f)$$

Substitute  $P(f)$  value in the above equation.

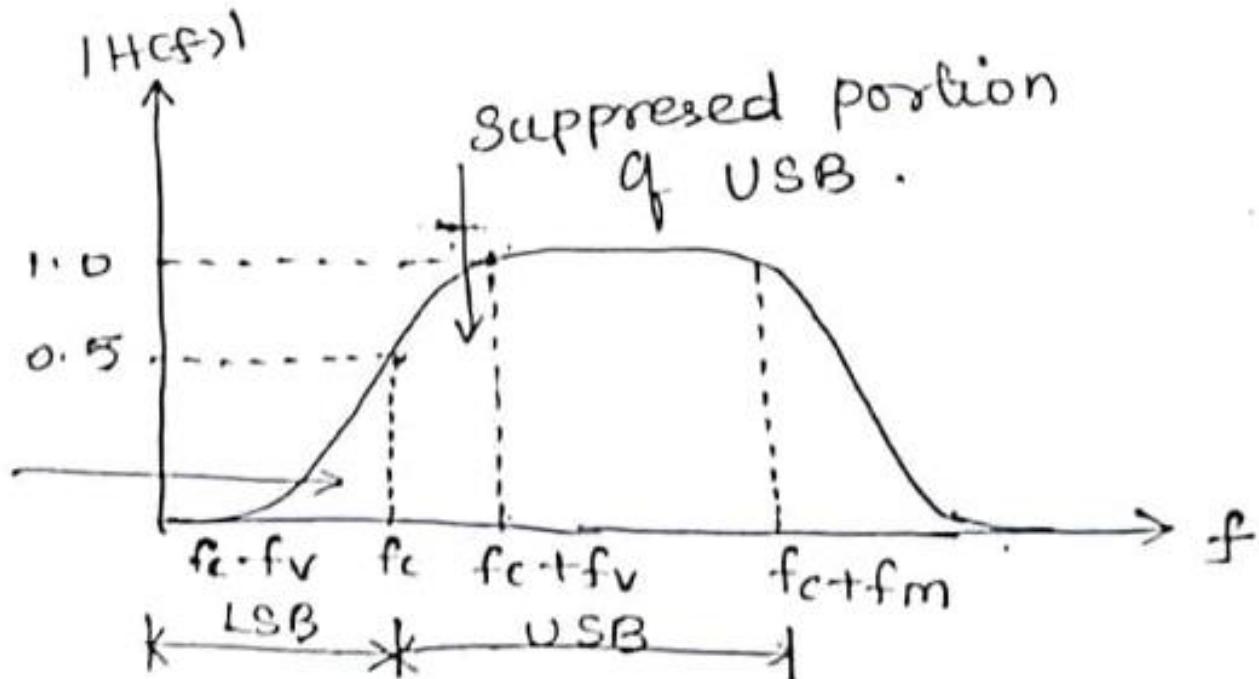
$$S(f) = \frac{A_c}{2} [M(f - f_c) + M(f + f_c)] H(f)$$

The above equation represents the equation of VSBSC frequency spectrum.

# VSB Generation

## Filter Method

Vestige of LSB .



The portion from  $fc$  to  $fc + fm$  is USB . The portion from  $fc$  to  $fc + fv$  is suppressed partially .

The portion from  $fc$  to  $fc - fm$  is LSB . Its portion from  $fc - fv$  to  $fc$  is to be transmitted as vestige . .

Transmission Bandwidth :-

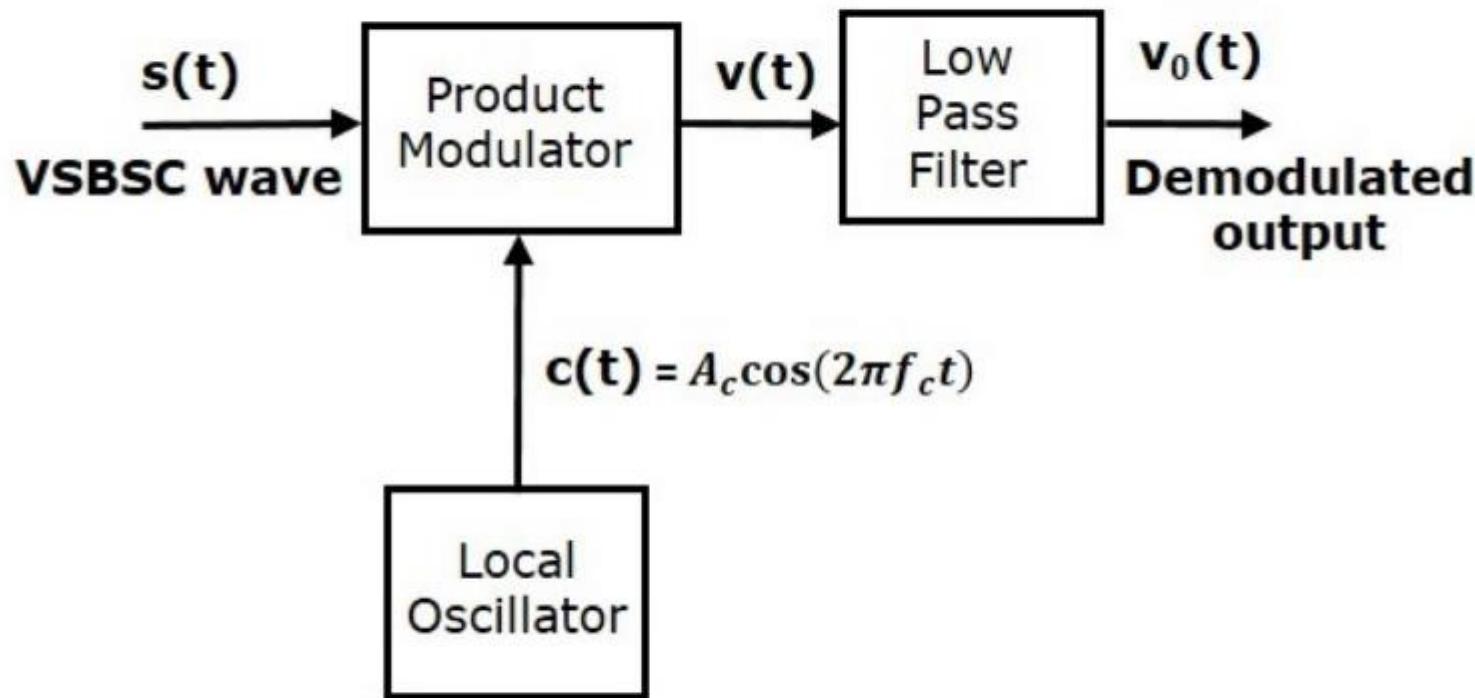
The transmission bandwidth of VSB modulation is

$$B-T = fm + fv \rightarrow ①$$

width of vestigial sideband  
message bandwidth .

# Demodulation of VSB

Demodulation of VSBSC wave is similar to the demodulation of SSBSC wave. Here, the same carrier signal (which is used for generating VSBSC wave) is used to detect the message signal. Hence, this process of detection is called as **coherent** or **synchronous detection**. The VSBSC demodulator is shown in the following figure.



In this process, the message signal can be extracted from VSBSC wave by multiplying it with a carrier, which is having the same frequency and the phase of the carrier used in VSBSC modulation. The resulting signal is then passed through a Low Pass Filter. The output of this filter is the desired message signal.

# Demodulation of VSB

Let the VSBSC wave be  $s(t)$  and the carrier signal is  $A_c \cos(2\pi f_c t)$ .

From the figure, we can write the output of the product modulator as

$$v(t) = A_c \cos(2\pi f_c t) s(t)$$

Apply Fourier transform on both sides

$$V(f) = \frac{A_c}{2} [S(f - f_c) + S(f + f_c)]$$

We know that

$$S(f) = \frac{A_c}{2} [M(f - f_c) + M(f + f_c)] H(f)$$

From the above equation, let us find  $S(f - f_c)$  and  $S(f + f_c)$ .

$$S(f - f_c) = \frac{A_c}{2} [M(f - f_c - f_c) + M(f - f_c + f_c)] H(f - f_c)$$

# Demodulation of VSB

$$\Rightarrow S(f - f_c) = \frac{A_c}{2} [M(f - 2f_c) + M(f)] H(f - f_c)$$

$$S(f + f_c) = \frac{A_c}{2} [M(f + f_c - f_c) + M(f + f_c + f_c)] H(f + f_c)$$

$$\Rightarrow S(f + f_c) = \frac{A_c}{2} [M(f) + M(f + 2f_c)] H(f + f_c)$$

Substitute,  $S(f - f_c)$  and  $S(f + f_c)$  values in  $V(f)$ .

$$V(f) = \frac{A_c}{2} \left[ \frac{A_c}{2} [M(f - 2f_c) + M(f)] H(f - f_c) + \right.$$

$$\left. \frac{A_c}{2} [M(f) + M(f + 2f_c)] H(f + f_c) \right]$$

$$\Rightarrow V(f) = \frac{A_c^2}{4} M(f) [H(f - f_c) + H(f + f_c)]$$

$$+ \frac{A_c^2}{4} [M(f - 2f_c) H(f - f_c) + M(f + 2f_c) H(f + f_c)]$$

In the above equation, the first term represents the scaled version of the desired message signal frequency spectrum. It can be extracted by passing the above signal through a low pass filter.

$$V_0(f) = \frac{A_c^2}{4} M(f) [H(f - f_c) + H(f + f_c)]$$

# Example of VSB

**Example:** Let the message signal be the sum of two sinusoids.

$$m(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

The message signal is multiplied by a carrier  $\cos \omega_c t$  to form the DSB signal.

DSB-SC

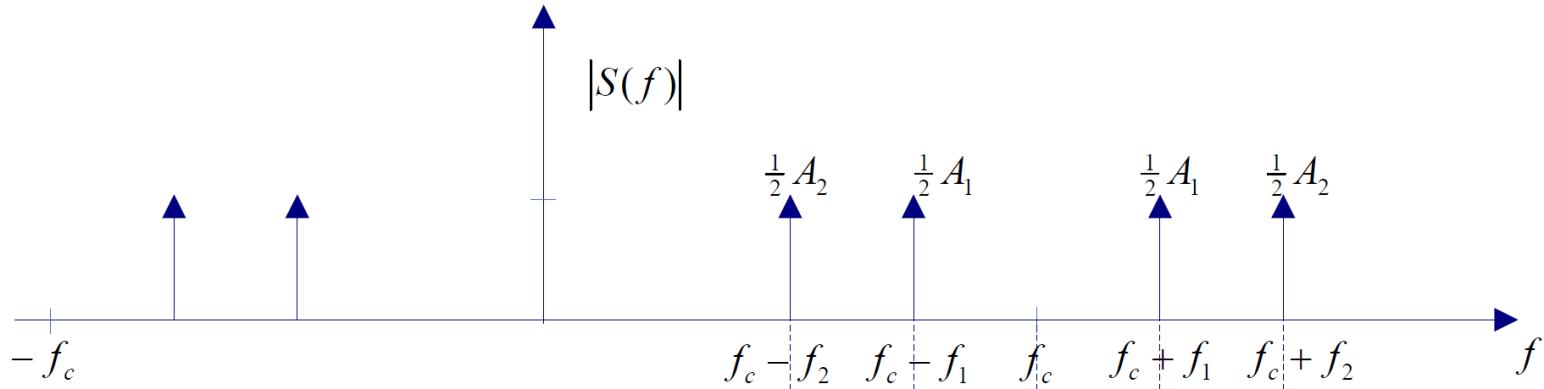


$$s(t) = (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) \cos \omega_c t$$

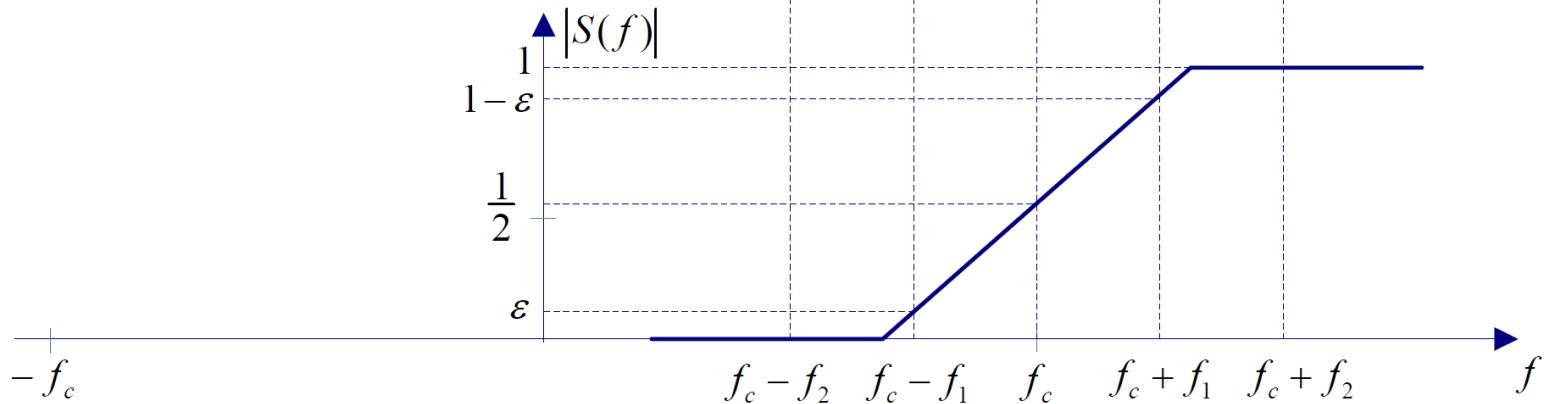
$$= \frac{1}{2} A_1 \cos(\omega_c - \omega_1)t + \frac{1}{2} A_2 \cos(\omega_c - \omega_2)t + \frac{1}{2} A_2 \cos(\omega_c + \omega_2)t + \frac{1}{2} A_1 \cos(\omega_c + \omega_1)t$$

Single sideband spectrum is shown below.

# Example of VSB

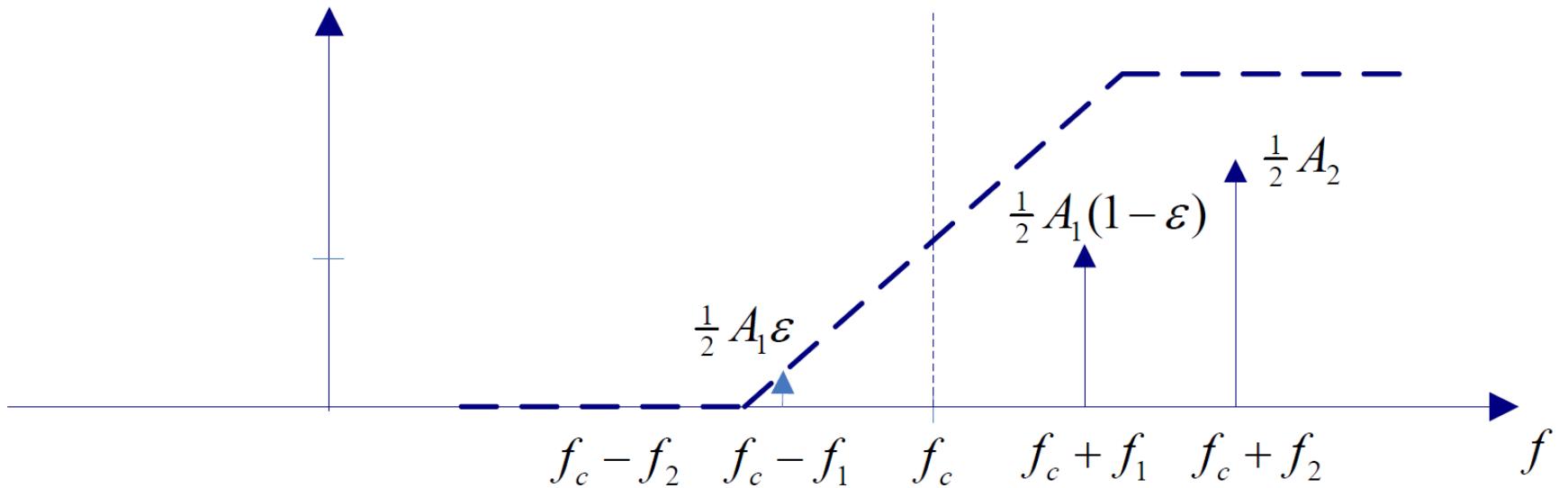


A vestigial sideband filter is then used to generate the VSB signal.



A vestigial sideband filter is then used to generate the VSB signal.

# Example of VSB



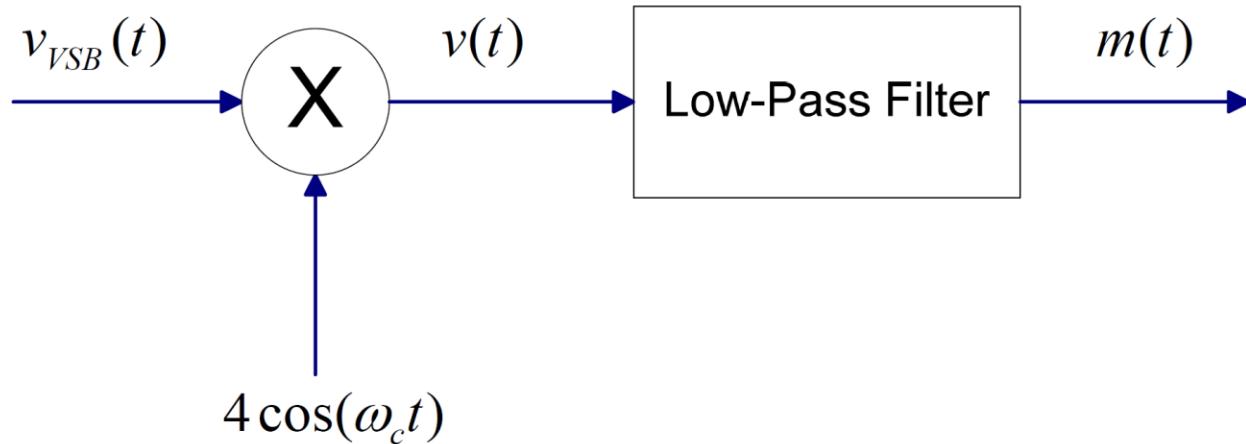
The spectrum of the VSB signal is

$$VSB_{spectrum} = |S(f)| \cdot |H(f)|$$

$$v_{VSB}(t) = \frac{1}{2}A_1\varepsilon \cos(\omega_c - \omega_1)t + \frac{1}{2}A_1(1-\varepsilon) \cos(\omega_c + \omega_1)t + \frac{1}{2}A_2 \cos(\omega_c + \omega_2)t$$

# Example of VSB

## Demodulation:



$$v(t) = v_{VSB}(t) \cdot 4 \cos \omega_c t =$$

$$2A_1\varepsilon \cos(\omega_c - \omega_1)t \cdot \cos \omega_c t + 2A_1(1 - \varepsilon) \cos(\omega_c + \omega_1)t \cdot \cos \omega_c t + 2A_2\varepsilon \cos(\omega_c + \omega_2)t \cdot \cos \omega_c t$$

$$= A_1\varepsilon [\cos(2\omega_c - \omega_1)t + \cos \omega_1 t] + A_1(1 - \varepsilon) [\cos(2\omega_c + \omega_1)t + \cos \omega_1 t] + A_2 [\cos(2\omega_c + \omega_2)t + \cos \omega_2 t]$$

After low-pass filtering,

$$m(t) = A_1\varepsilon \cos \omega_1 t + A_1(1 - \varepsilon) \cos \omega_1 t + A_2 \cos \omega_2 t$$

$$\Rightarrow m(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$
, which is the assumed message signal.

# Designations / Regulations of AM

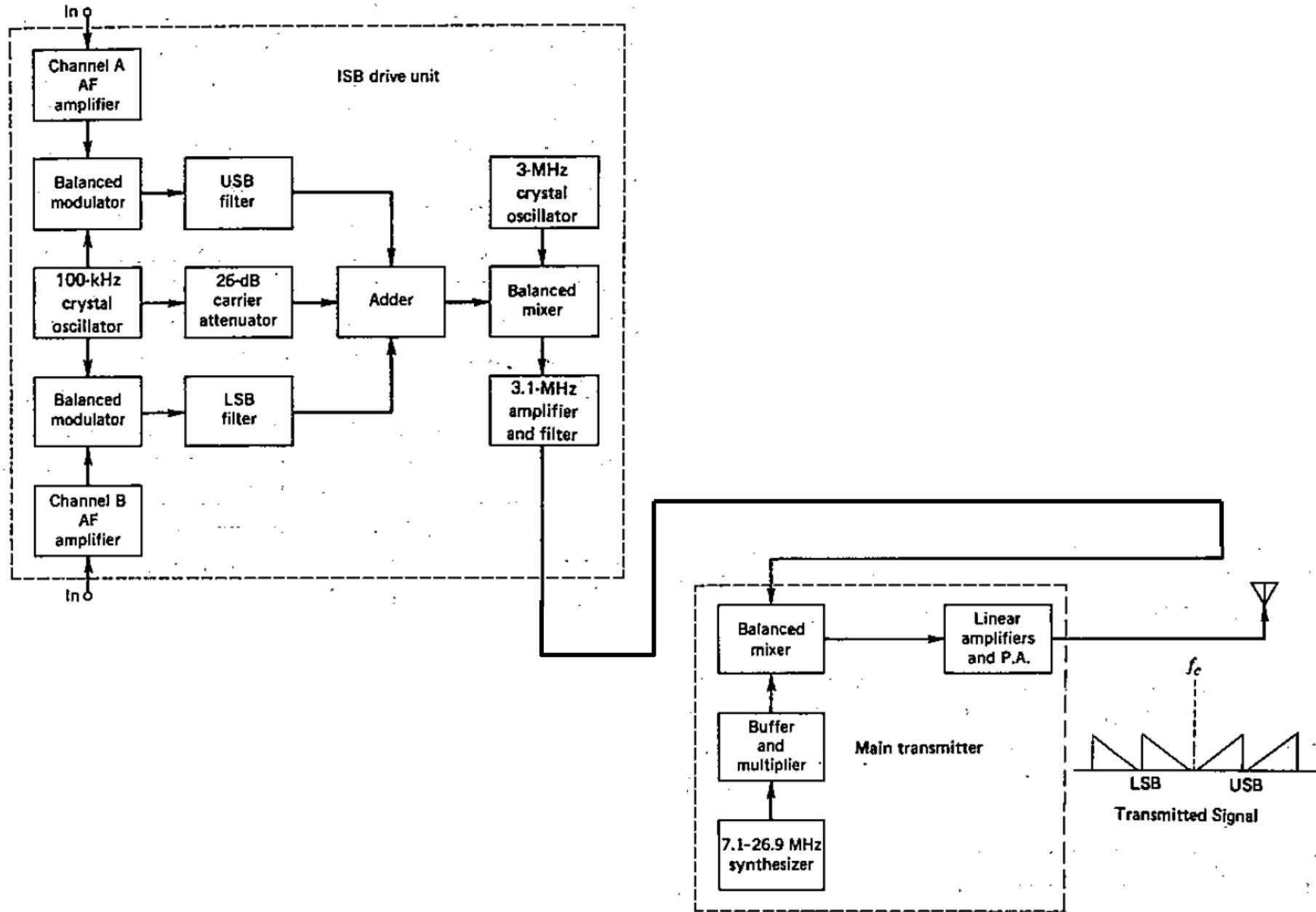
- A3E – DSB / FC
- R3E – DSB / RC
- H3E – SSB / FC
- J3E – SSB / SC
- B8E – ISB / SC
- C3F – VSB

FC – Full Carrier

RC – Reduced Carrier

SC – Suppressed Carrier

# Independent Sideband Systems (ISB) – R3E



# Hilbert Transform

Hilbert transform of a signal  $x(t)$  represented as,

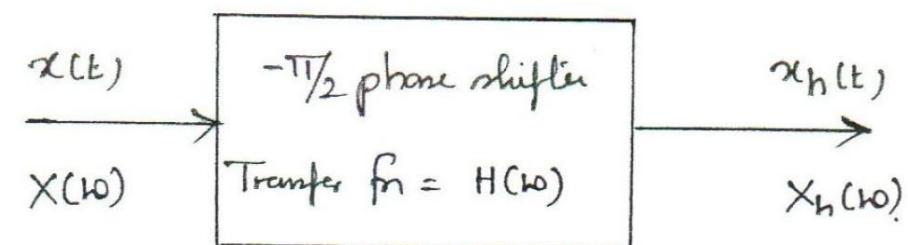
$$x_h(t) \text{ or } H(x(t)).$$

It's used to provide  $-\pi/2$  or  $-90^\circ$  phase shift for every component present in the signal  $x(t)$ .

Mathematically,

$$H[x(t)] = x_h(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} \cdot d\tau$$

$$x_h(t) = \frac{1}{\pi} \cdot x(t) \cdot \frac{1}{t}$$



phase shifting only.

# Hilbert Transform

Characteristics of phase shifting S/y:

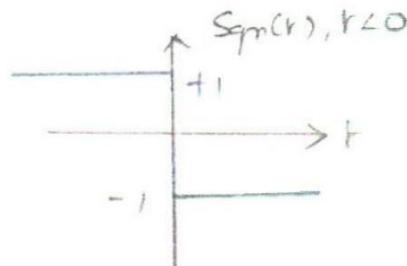
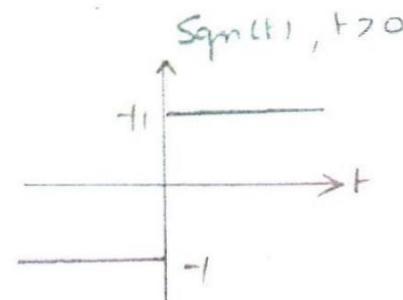
- ①. Magnitude of the  $\Phi_{\text{pp}}$ . Components present in  $x(t)$  remains unchange, when it is passed through the  $S/y$ . (ie)  $H(\omega_0) = 1$ .
- ②. The phase of the  $(t \times)$   $\Phi_{\text{pp}}$ . Comp? is shifted by  $-\pi/2$ . (or  $-90^\circ$ ).  
Hence,  $H(\omega_0) = -j X(\omega_0) \text{Sgn}(\omega_0)$ .

$$X_h(\omega_0) = X(\omega_0) \cdot H(\omega_0) \quad \text{--- } ①$$

$$H(\omega_0) = -j \text{Sgn}(\omega_0) \quad \text{--- } ②$$

Sub equ ② in ①

$$X_h(\omega_0) = -j X(\omega_0) \cdot \text{Sing}(\omega_0) \quad \text{--- } ③$$



# Hilbert Transform

Derivation of  $H(\omega)$ :

$$H(\omega) = e^{j\theta(\omega)} \quad \text{--- (4)} \quad \Leftrightarrow H(\omega) = |H(\omega)| \cdot e^{j\theta(\omega)} \\ = 1 \cdot e^{j\theta(\omega)}$$

$$\theta(\omega) = \begin{cases} \frac{\pi}{2} & \text{for } \omega < 0 \\ -\frac{\pi}{2} & \text{for } \omega > 0 \end{cases}$$

$$\text{hence, } H(\omega) = \begin{cases} e^{j\frac{\pi}{2}}, \omega < 0 \\ e^{-j\frac{\pi}{2}}, \omega > 0 \end{cases} \quad \text{--- (5)}$$

Sub eqn (5) in (4)

$$e^{j\frac{\pi}{2}} = \cos \frac{\pi}{2} + j \sin \frac{\pi}{2} = j$$

$$e^{-j\frac{\pi}{2}} = \cos \frac{\pi}{2} - j \sin \frac{\pi}{2} = -j$$

# Hilbert Transform

$$\therefore H(\omega) = \begin{cases} j, & \omega < 0 \\ -j, & \omega > 0 \end{cases}$$

$$\frac{H(\omega)}{j} = \begin{cases} 1, & \omega < 0 \\ -1, & \omega > 0 \end{cases}$$

$$\therefore H(\omega) = -j \operatorname{Sgn}(\omega)$$

$$\therefore X_h(\omega) = -j x(\omega) \cdot \operatorname{Sgn}(\omega).$$

Taking Inverse Fourier transform

$$\tilde{F}^{-1}[X_h(\omega)] = x_h(t) = F^{-1}[-j x(\omega) \operatorname{Sgn}(\omega)]$$

$$x_h(t) = x(t) \cdot \frac{1}{\pi t}$$

# Hilbert Transform - Properties

①. A s/g  $x(t)$  & its hilbert transform  $x_h(t)$  have the same amplitude Spectrum and same auto correlation fn.

②.  $x(t)$  &  $x_h(t)$  are Orthogonal

(ie)  $\int_{-\infty}^{\infty} x(t) x_h(t) dt = 0$ . [dot product of  
2/p w/o p in zr]

③. Hilbert transform of  $x_h(t)$  is  $-x(t)$

## Pre-envelope or Analytic Signal

\* Useful in deriving the general exp<sup>r</sup> of the SSB-SC S/g.

The preenvelope of real Valued S/g  $x(t)$  is defined as.

$$x_p(t) = x(t) + j x_h(t).$$

Obviously, the pre-envelope  $x_p(t)$  is a Complex Valued S/g

The real part of  $x_p(t)$  is  $x(t)$  and the imaginary part is its hilbert transform  $x_h(t)$ .

# Pre-envelope or Analytic Signal

The use of phasors simplifies manipulation of ac currents and voltages, by pre-envelope handling of bandpass signals and system. pre-envelope of the signal  $g(t)$  is defined as,

$$g_+(t) = g(t) + j\hat{g}(t)$$

H.T of  $g(t)$

real part of pre-envelope

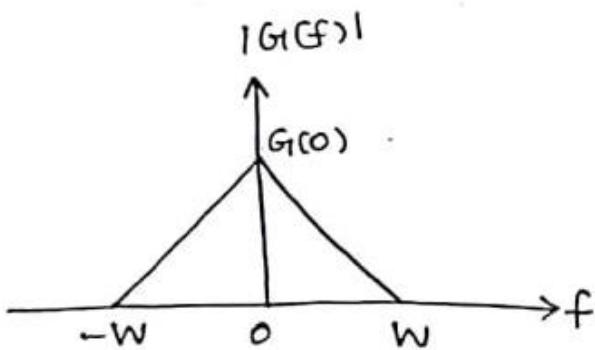
pre-envelope of  $\operatorname{sgn} g(t)$ .

# Pre-envelope or Analytic Signal

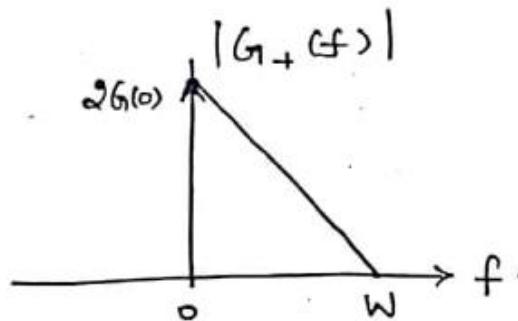
Consider the spectrum of pre-envelope,

$$G_{1+}(f) = G_1(f) + j[-j \operatorname{sgn}(f)] G_1(f)$$

$$= \begin{cases} 2G_1(f), & f > 0 \\ G_1(0), & f = 0 \\ 0, & f < 0 \end{cases}$$



Amplitude spectrum of  
low pass s/g  $g(t)$ .



Amplitude spectrum of  
pre-envelope  $g_{1+}(t)$ .

The pre-envelope of a low pass signal has no negative frequency component.

# Complex Envelope

The new quantity based on the analytic s/g, called the Complex envelope is defined as,

$$x(t) = x(t) e^{j2\pi f_c t}$$

Taking Fourier transform,

$$F[x(t)] = X(f)$$

$$X(f) = \begin{cases} 2 \times (f + f_c) & \text{for } f > 0 \\ H(0) & \text{for } f = 0 \\ 0 & \text{for } f < 0 \end{cases}$$

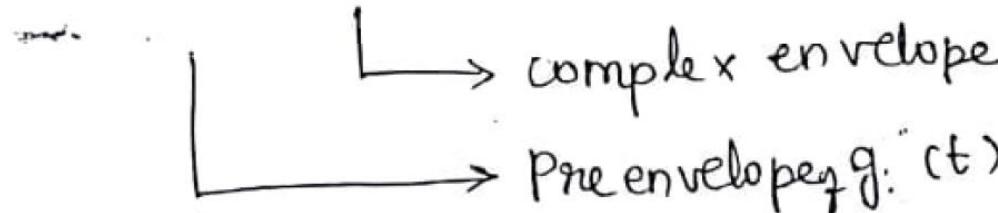
The Complex envelope is just the low pass S/g, part of the analytic s/g. The analytic low pass S/g has been multiplied by the Complex exponential at the carrier S/g.

# Complex Envelope

consider a bandpass s/g  $g(t)$ , the spectral components of  $g(t)$  are negligible outside the frequency band  $f_c - w$  to  $f_c + w$ . and their signal bandwidth is much less than the carrier frequency ( $\omega w \ll f_c$ ). Bandpass signals satisfying this constraint are called "narrowband bandpass s/g"

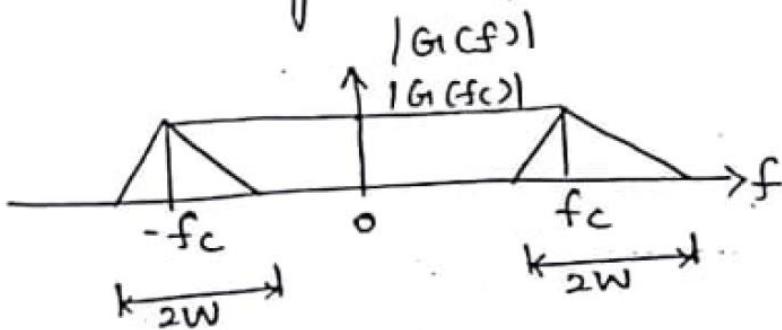
The complex envelope of  $\tilde{g}(t)$  of such a bandpass s/g  $g(t)$  is defined in terms of its pre-envelope.

$$g_+(t) = \tilde{g}(t) \exp(j2\pi f_c t)$$

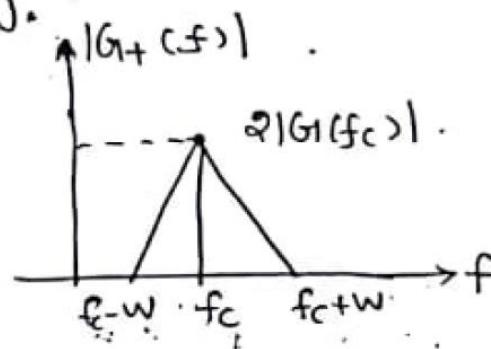


# Complex Envelope

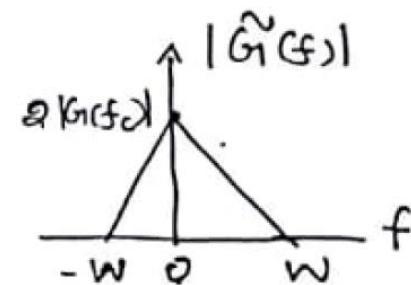
It is obvious that complex envelope spectrum  $\tilde{G}(f)$  should be identical to pre-envelope spectrum  $G_+(f)$  but shifted down the frequency axis by an amount  $f_c$ . So it is a spectrum centred around the zero frequency and extending from  $-W$  to  $+W$ .



Amplitude Spectrum of  
Bandpass sig  $g(t)$



Amplitude spectrum  
of pre-envelope  $g_+(t)$



Amplitude spectrum  
of complex  
envelope  $\tilde{g}(f)$

# Complex Envelope

$$g(t) = \operatorname{Re} [g_+(t)] .$$
$$= \operatorname{Re} [\tilde{g}(t) \exp(j2\pi f_c t)] .$$

In general,  $\tilde{g}(t) = g_i(t) + jg_q(t)$

Quadrature component  
In phase component

$$\therefore g(t) = \operatorname{Re} [ \{g_i(t) + jg_q(t)\} \exp(j2\pi f_c t) ]$$
$$= \operatorname{Re} [ \{g_i(t) + jg_q(t)\} [\cos(2\pi f_c t) + j \sin(2\pi f_c t)] ]$$

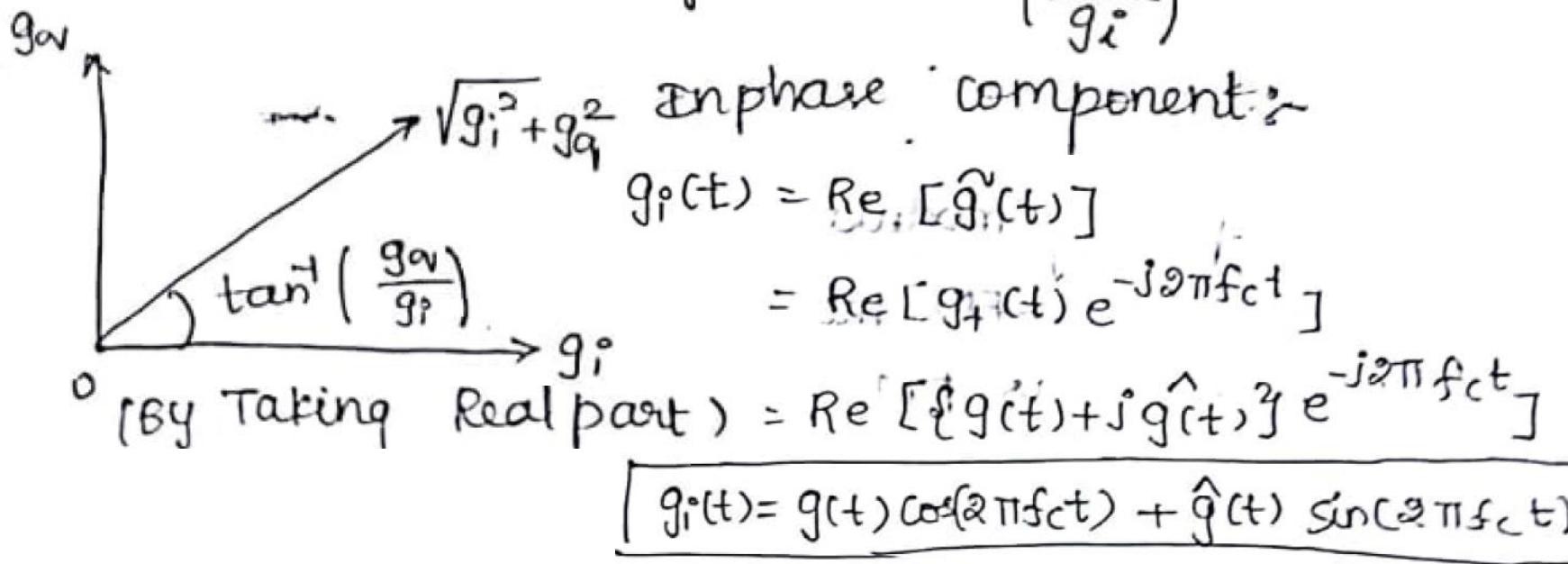
$$g(t) = g_i(t) \cos(2\pi f_c t) - g_q(t) \sin(2\pi f_c t) .$$

# Complex Envelope

Phasor Representation :-

$$\text{Amplitude} = \sqrt{g_i^2 + g_q^2}$$

$$\text{Phase angle} = \tan^{-1} \left( \frac{g_{av}}{g_i} \right)$$



Representation of Quadrature component :-

$$g_q(t) = \text{Img} [\hat{g}_i(t)]$$

(By taking Imag part)

$$\boxed{g_q(t) = \hat{g}_i(t) \cos(2\pi f_c t) - g_i(t) \sin(2\pi f_c t)}$$

Parameters / Description	AM with Carrier	DSB-SC- AM	SSB-SC- AM	VSB- AM
Method	Carrier and both Sidebands	Only Sidebands	Only one sideband	One Sideband and Part of other sideband
Bandwidth	2fm	2fm	fm	$fm < BW < 2fm$
Power transmitted	$P_t = P_c \left[ 1 + \frac{m_a^2}{2} \right]$	$P_t = P_c \left[ \frac{m_a^2}{2} \right]$	$P_t = \frac{P_c m_a^2}{4}$	$P_{tSSB} < P_{tVSB} < P_{tDSB}$
Power Saving (%)	33.33%	66.66%	83%	75%
Transmission efficiency				
Generation	Not difficult	Not difficult	Complex	Complex
Application	AM broadcasting	Carrier telephony	Police wireless, mobile etc,	Television and high speed data transmission