1. The equation for a FM wave is $S(t) = 10 \cos[5.7 \times 10^8 t + 5 \sin(12 \times 10^3)t]$.

   Calculate. i. Carrier frequency ii. Modulating frequency

   iii. Modulation index iv. Frequency deviation

   v. Power dissipated in a 100Ω resistor load.

Sol:-

$$S(t) = 10 \cos[5.7 \times 10^8 t + 5 \sin(12 \times 10^3) t] \rightarrow (1)$$

Compare eq (1) with Standard equation for FM

$$S(t) = A_c \cos[\omega_c t + \beta \sin \omega_m t] \longrightarrow (2)$$

$A_c = 10V$, $\omega_c = 5.7 \times 10^8$, $\beta = 5$ & $\omega_m = 12 \times 10^3$

i) Carrier frequency $f_c = \frac{\omega_c}{2\pi} = \frac{5.7 \times 10^8}{2\pi}$

$$\boxed{f_c = 90.7183 \text{ MHz}}$$

ii) Modulating frequency $f_m = \frac{\omega_m}{2\pi} = \frac{12 \times 10^3}{2\pi}$

$$\boxed{f_m = 1.909 \text{ KHz}}$$

iii) Modulation Index

$$\boxed{\beta = 5}$$

iv) Frequency deviation $\Delta f = \beta f_m = 5 \times 1.909\,\text{KHz}$

$$\boxed{\Delta f = 9.545\ \text{KHz}}$$

v) power dissipated in a $100\,\Omega$ resistor load

$$P = \frac{A_c^2}{2R} = \frac{10^2}{2 \times 100}$$

$$\boxed{P = 0.5\,\text{W}}$$

2. A FM signal has sinusoidal modulation with f_m=15KHz and modulation index β=2. Using carson's rule, find the transmission bandwidth and deviation ratio. Assume Δf =75 KHz.

Given :-

$f_m = 15\,KHz$, $\beta = 2$, $\Delta f = 75\,KHz$

$BW = ?$ & Deviation ratio 'D' = ?

$* \quad BW = 2(\Delta f + f_m) = 2(75\,KHz + 15\,KHz) = \underline{180\,KHz.}$

$* \quad D = \dfrac{\Delta f}{f_m} = \dfrac{75\,KHz}{15\,KHz} = \underline{5}$

3. A sinusoidal modulating voltage of amplitude 5V and frequency 1 KHz is applied to frequency modulator. The frequency sensitivity of modulator is 40 Hz/V. The carrier frequency is 100KHz. Calculate i. Frequency deviator ii. Modulation index

Given :- $A_m = 5V$, $f_m = 1KHz$, $K_f = 40\ Hz/V$ & $f_c = 100KHz$.

i) Frequency deviation $\Delta f = K_f A_m = 40 \times 5 = \underline{200 Hz}$

ii) Modulation Index 'β' $= \dfrac{\Delta f}{f_m} = \dfrac{200}{1000} = \underline{0.2}$

A carrier wave of 100 MHz is frequency modulated by a 100 KHz sinewave of amplitude 20V, the sensitivity of the modulator is 25 KHz/V.

i. **Determine the frequency deviation and bandwidth of the modulated signal using Carson's rule.**

ii. **Repeat your calculation for PM wave, assume $k_p=k_f$**

Given: $f_c = 100MHz$, $f_m = 100KHz$, $A_m = 20V$, $K_f = 25KHz/V$.

i) $BW = 2[\Delta f + f_m]$

$\Delta f = K_f A_m = 25KHz \times 20 = \underline{500KHz}$

$BW = 2[500KHz + 100KHz]$

$$\boxed{BW = 1200KHz}$$

OR

$BW = 2f_m(1+\beta)$

$\beta = \frac{500KHz}{100KHz} = \underline{5}$

$BW = 2 \times 100KHz(1+5)$

$$\boxed{BW = 1200 KHz}$$

ii) Assuming that $K_p = K_f$ for PM wave

$\Delta f = K_p A_m f_m = 25KHz \times 20 \times 100KHz$

$$\boxed{\Delta f = 50000 KHz}$$

In the block diagram shown in Fig. find out the carrier frequency, frequency deviation and modulation index at the points A and B. Assume that at the output of the mixer, the additive frequency component is being selected.

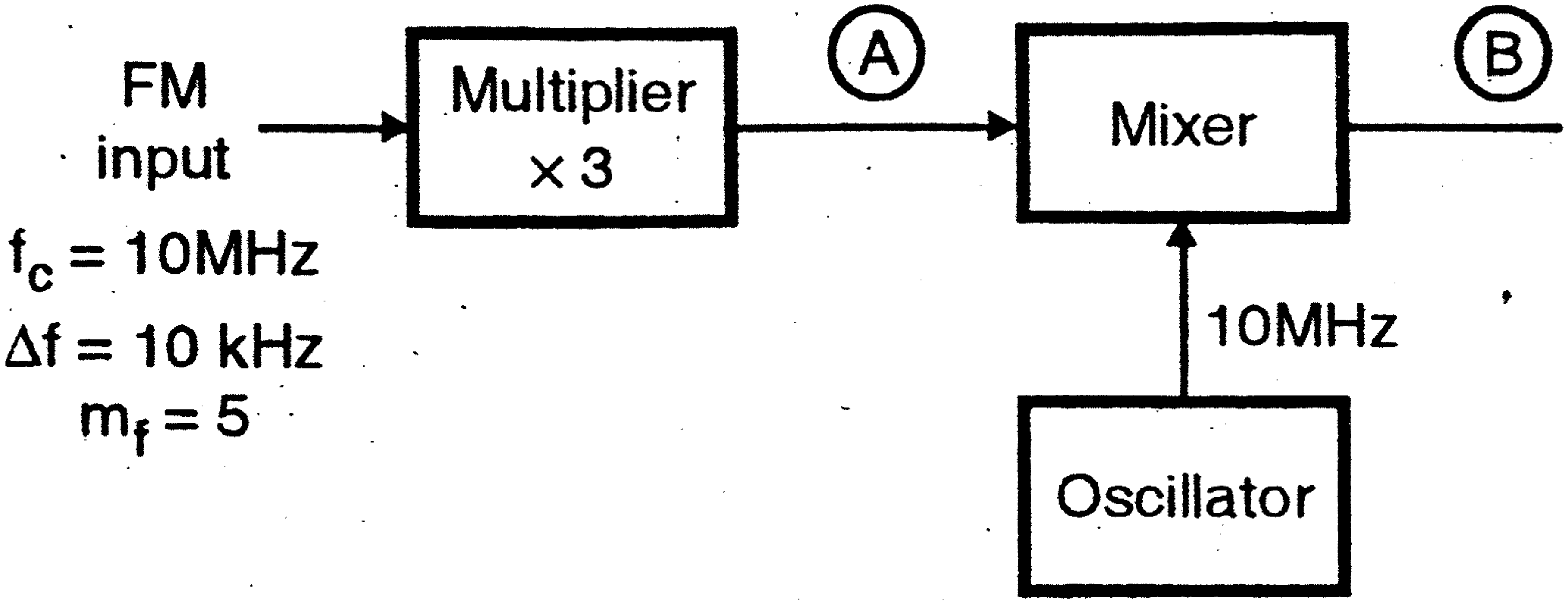


Fig.

Soln. :

(i) At point (A) :

$$\text{The carrier } f_c = 3 \times 10\ \text{MHz} = 30\ \text{MHz.}$$

$$\text{The frequency deviation } \delta = 3 \times 10\ \text{kHz} = 30\ \text{kHz and modulation index } m_f = 3 \times 5 = 15.$$

$$\text{The minimum frequency } f_{min} = 30\ \text{MHz} - 30\ \text{kHz} = 29.970\ \text{MHz}$$

$$\text{The maximum frequency } f_{max} = 30\ \text{MHz} + 30\ \text{kHz} = 30.030\ \text{MHz.}$$

(ii) At point (B) :

$$\text{Carrier frequency } f_c = 30\ \text{MHz} + 10\ \text{MHz} = 40\ \text{MHz.}$$

$$\text{Maximum frequency } f_{max} = 30.03 + 10 = 40.03\ \text{MHz}$$

$$\text{Minimum frequency } f_{min} = 29.970 + 10 = 39.970\ \text{MHz.}$$

As there is no change in deviation due to mixing, the modulation index will remain same i.e. $m_f = 15$.

A carrier wave of amplitude 5V & frequency 90MHz is frequency modulated by a sinusoidal voltage of amplitude 5V & frequency - 15KHz. The frequency deviation constant is 1KHz/V. Sketch the spectrum of the modulated FM wave.

From the table of Bessel functions, for $\beta = 0.333$

use approximate values for J_0, J_1 & J_2.

i) For carrier : $J_0 = 0.96$

ii) 1st side frequency : $J_1 = 0.18$

iii) 2nd side frequency : $J_2 = 0.02$

Sol :- Given : $A_c = 5V$, $f_c = 90 MHz$, $A_m = 5V$, $f_m = 15 KHz$. $K_f = 1 KHz/V$.

* Frequency deviation $\Delta f = K_f A_m = 1 KHz/V \times 5V = 5 KHz$.

* $\beta = \dfrac{\Delta f}{f_m} = \dfrac{5 KHz}{15 KHz} = 0.333$

From the table of Bessel functions, for $\beta = 0.333$

Use approximate values for J_0, J_1 & J_2.

i) For carrier : $J_0 = 0.96$

ii) 1^{st} Side frequency : $J_1 = 0.18$

iii) 2^{nd} Side frequency : $J_2 = 0.02$

Higher order Side frequencies are negligible since β is small.

i) Amplitude Spectrum of the Carrier : $A_c J_0(\beta) = 5V \times 0.96 = 4.8V$

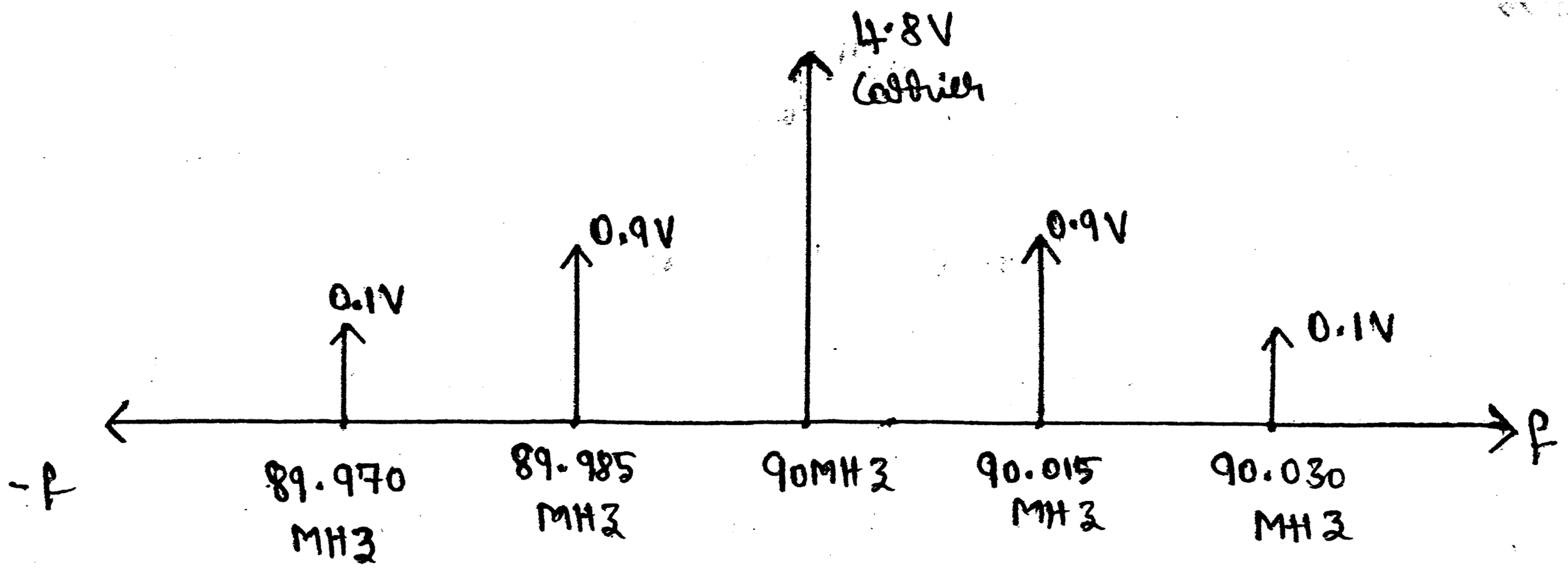
Carrier Frequency $f_c = 90MHz$

ii) Amplitude Spectrum of the 1st Side Frequency: $A_c J_1(\beta) = 5 \times 0.18V = 0.9V$

1st Side Frequency : $f_c + f_m = 90MHz + 15KHz = 90.015 MHz$

iii) Amplitude Spectrum of the 2nd Side Frequency: $A_c J_2(\beta) = 5 \times 0.02V = 0.1V$

2nd Side Frequency : $f_c + 2f_m = 90MHz + 2(15KHz) = 90.030MHz$



$$f_c - f_m = 90\,MHz - 15\,KHz = 89.985\,MHz$$

$$f_c - 2f_m = 90\,MHz - 2(15\,KHz) = 89.970\,MHz.$$

A carrier wave of amplitude 10V & frequency 100MHz is frequency modulated by a Sinusoidal voltage. The modulating voltage has an amplitude of 5V & frequency $f_m = 20KHz$. The frequency deviation constant is 2KHz/V. Draw the frequency Spectrum of FM wave.

NOTE :-

1) Carrier Signal $\rightarrow$ $A_c J_0(\beta) \cos 2\pi f_c t$

2) 1^{st} pair of Side frequencies $\rightarrow$ $A_c J_1(\beta) \cos 2\pi(f_c \pm f_m)t$

3) 2^{nd} pair of Side frequencies $\rightarrow$ $A_c J_2(\beta) \cos 2\pi(f_c \pm 2f_m)t$

$\vdots$

n) n^{th} pair of Side frequencies $\rightarrow$ $A_c J_n(\beta) \cos 2\pi(f_c \pm nf_m)t$

* From the table of Bessel functions, for $\beta = 0.5$ use approximate values of J-Coefficients are:

$$\boxed{J_0 = 0.94 \;,\; J_1 = 0.24 \;,\; J_2 = 0.03}$$

Sol :- Given : $A_c = 10V$, $f_c = 100MHz$, $K_f = 2KHz/V$.

$A_m = 5V$, $f_m = 20KHz$.

* $\Delta f = K_f A_m = 2KHz/\cancel{V} \times 5\cancel{V} = 10KHz$

* $\beta = \dfrac{\Delta f}{f_m} = \dfrac{10KHz}{20KHz} = \underline{0.5}$

* From the table of Bessel functions, for $\beta = 0.5$ we approximate values of J-coefficients are :

$$\boxed{J_0 = 0.94, \quad J_1 = 0.24, \quad J_2 = 0.03}$$

* The amplitude, frequencies of the carrier & sidebands are as follows :

i) Carrier amplitude $\rightarrow A_c J_0(\beta) = 10V \times 0.94 = 9.4V$

Carrier frequency $\rightarrow$ 100MHz

ii) Frequency of 1st Sideband → $f_c + f_m = 100.2\,MHz$

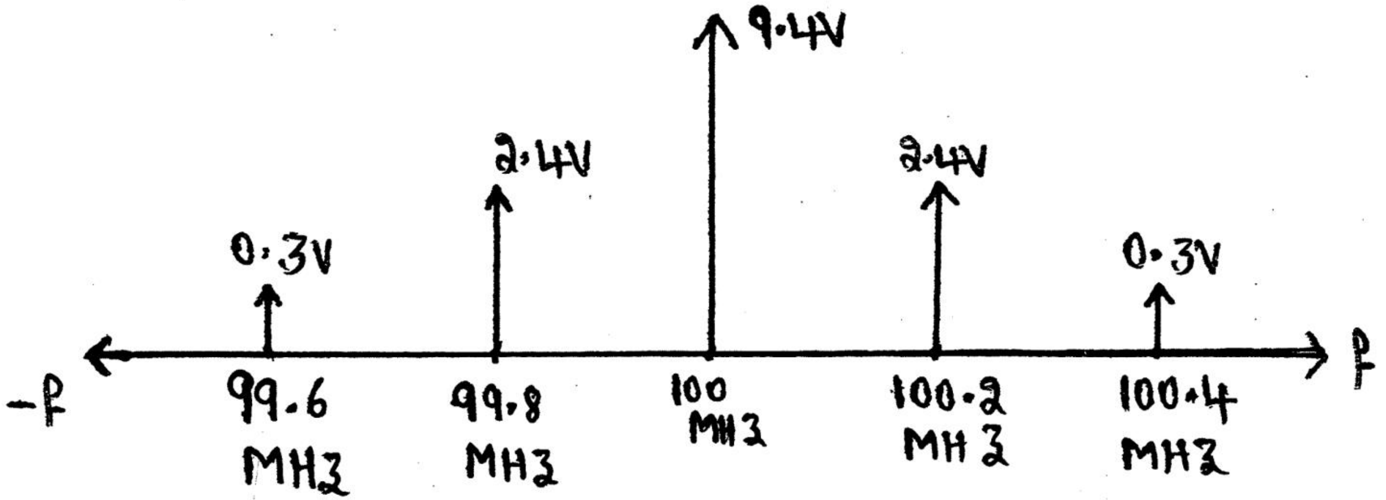
$f_c - f_m = 99.8\,MHz$

Amplitude of 1st Sideband → $A_c J_1(\beta) = 10V \times 0.24 = 2.4V$

iii) Frequency of 2nd Sideband → $f_c + 2f_m = 100.4\,MHz$

$f_c - 2f_m = 99.6\,MHz$

Amplitude of 2nd Sideband → $A_c J_2(\beta) = 10V \times 0.03 = 0.3V$.



Spectrum of FM wave.

Problem 1

Consider the frequency multiplier of Fig. 4-4 and an NBFM signal

$$x_{\mathrm{NBFM}}(t) = A\cos(\omega_c t + \beta \sin \omega_m t)$$

with $\beta < 0.5$ and $f_c = 200$ kHz. Let f_m range from 50 Hz to 15 kHz, and let the maximum frequency deviation Δf at the output be 75 kHz. Find the required frequency multiplication n and the maximum allowed frequency deviation at the input.

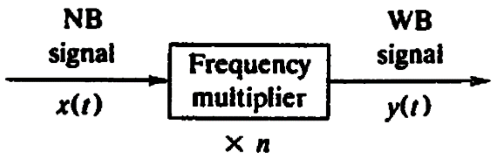


Fig. 4-4 Frequency multiplier

Solution: From Eq. (4.22), $\beta = \Delta f / f_m$. Thus,

$$\beta_{\min} = \frac{75(10^3)}{15(10^3)} = 5 \qquad \beta_{\max} = \frac{75(10^3)}{50} = 1500$$

If $\beta_1 = 0.5$, where β_1 is the input β, then the required frequency multiplication is

$$n = \frac{\beta_{\max}}{\beta_1} = \frac{1500}{0.5} = 3000$$

The maximum allowed frequency deviation at the input, denoted Δf_1, is

$$\Delta f_1 = \frac{\Delta f}{n} = \frac{75(10^3)}{3000} = 25\,\mathrm{Hz}$$

Problem 2

A block diagram of an indirect (Armstrong) FM transmitter is shown in Fig. 4-9. Compute the maximum frequency deviation Δf of the output of the FM transmitter and the carrier frequency f_c if $f_1 = 200$ kHz, $f_{LO} = 10.8$ MHz, $\Delta f_1 = 25$ Hz, $n_1 = 64$, and $n_2 = 48$.

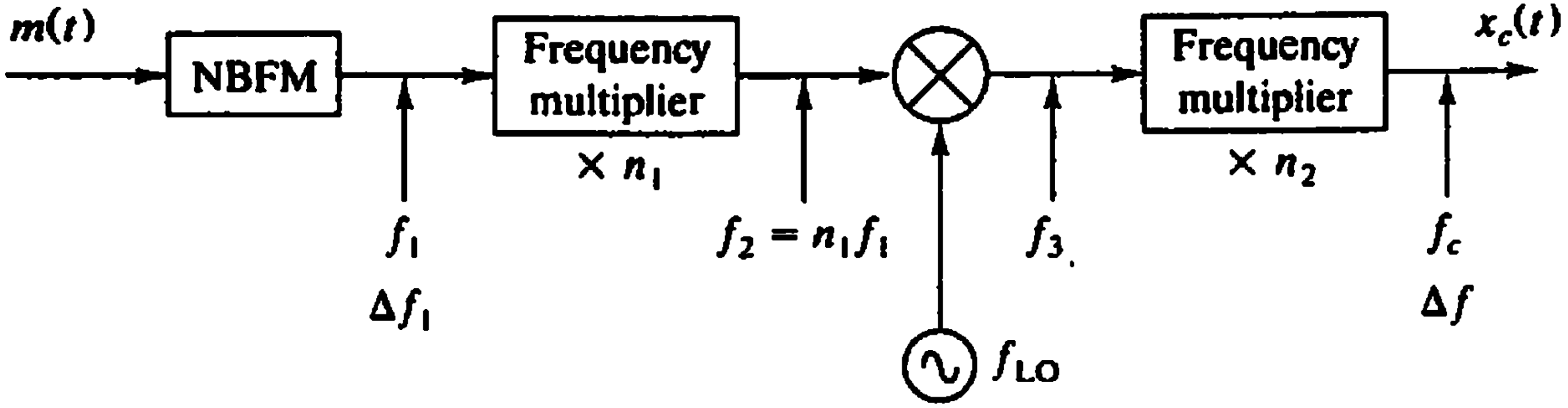


Fig. 4-9 Block diagram of an indirect FM transmitter

Solution

$$\Delta f = (\Delta f_1)(n_1)(n_2) = (25)(64)(48)\,\text{Hz} = 76.8\,\text{kHz}$$

$$f_2 = n_1 f_1 = (64)(200)(10^3) = 12.8(10^6)\,\text{Hz} = 12.8\,\text{MHz}$$

$$f_3 = f_2 \pm f_{LO} = (12.8 \pm 10.8)(10^6)\,\text{Hz} = \begin{cases} 23.6 & \text{MHz} \\ 2.0 & \text{MHz} \end{cases}$$

Thus, when $f_3 = 23.6\,\text{MHz}$, then

$$f_c = n_2 f_3 = (48)(23.6) = 1132.8\,\text{MHz}$$

When $f_3 = 2\,\text{MHz}$, then

$$f_c = n_2 f_3 = (48)(2) = 96\,\text{MHz}$$

Problem 3

In an Armstrong-type FM generator of Fig. 4-9 (Prob. *4.16*), the crystal oscillator frequency is 200 kHz. The maximum phase deviation is limited to 0.2 to avoid distortion. Let f_m range from 50 Hz to 15 kHz. The carrier frequency at the output is 108 MHz, and the maximum frequency deviation is 75 kHz. Select multiplier and mixer oscillator frequencies.

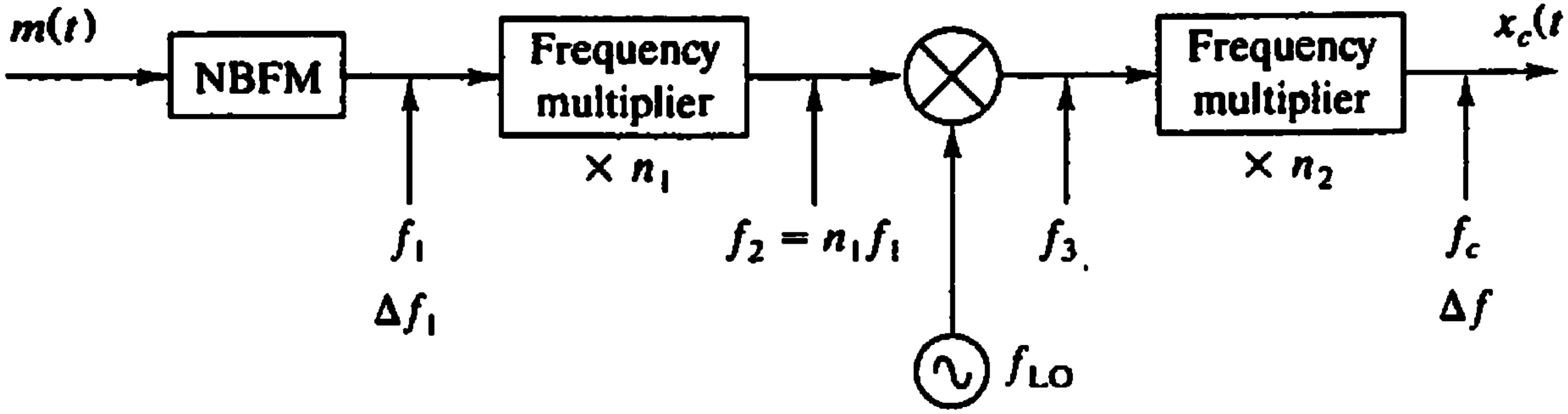


Fig. 4-9 Block diagram of an indirect FM transmitter

Solution

$$\Delta f_1 = \beta f_m = (0.2)(50) = 10\,\text{Hz}$$

$$\frac{\Delta f}{\Delta f_1} = \frac{75(10^3)}{10} = 7500 = n_1 n_2$$

$$f_2 = n_1 f_1 = n_1(2)(10^5)\,\text{Hz}$$

Assuming down conversion, we have

$$f_2 - f_{LO} = \frac{f_c}{n_2}$$

Thus,

$$f_{LO} = n_1 f_1 - \frac{f_c}{n_2} = \frac{7500(2)(10^5) - 108(10^6)}{n_2} = \frac{1392}{n_2}(10^6)\ \text{Hz}$$

Letting $n_2 = 150$, we obtain

$$n_1 = 50 \qquad \text{and} \qquad f_{LO} = 9.28\,\text{MHz}$$

Problem 4

Design an Armstrong indirect FM modulator to generate an FM carrier with a carrier frequency of 96 MHz and Δf = 20 kHz. A narrowband FM generator with f_c = 200 kHz and adjustable Δf in the range of 9 to 10 Hz is available. We also have an oscillator with adjustable frequency in the range of 9 to 10 MHz and there is a bandpass filter with any centre frequency and only frequency doublers are available.

Solution

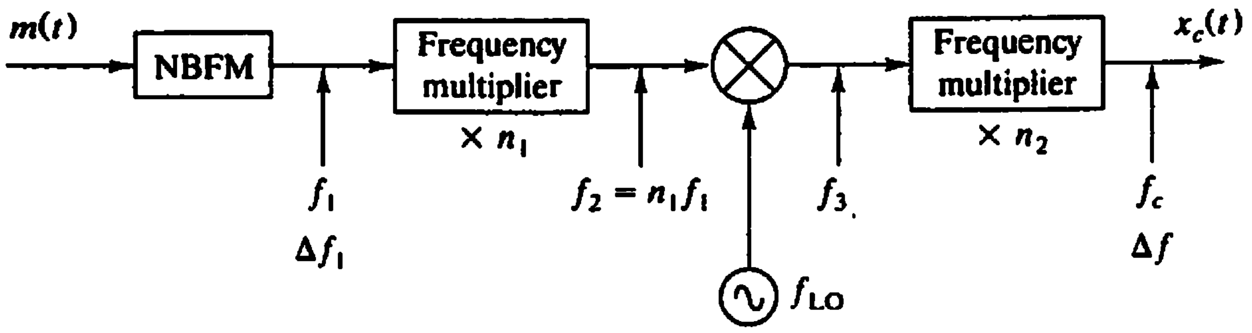


Fig. 4-9 Block diagram of an indirect FM transmitter

$9 < \Delta f_1 < 10$ W. K. T $\Delta f = n_1\, n_2\, \Delta f_1$

$$9 < \frac{20000}{n_1 n_2} < 10$$

$\Delta f = 20$ kHz

Since only doublers are available, therefore $n_1\, n_2$ has to be power of 2.

By trail and error, $n_1\, n_2 = 2018$

Hence, $n_1 = 64$, $n_2 = 32$

The output of the first multiplier is 64 x 200 kHz = 12.8 MHz

Input to the second multiplier has to be $\frac{96 \times 10^6}{32} = 3MHz$

The local oscillator frequency = 12.8 -3 = 9.8 MHz which is in the given range.

Modulation index	Sideband amplitude																
	Carrier	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.00	1.00																
0.25	0.98	0.12															
0.5	0.94	0.24	0.03														
1.0	0.77	0.44	0.11	0.02													
1.5	0.51	0.56	0.23	0.06	0.01												
2.0	0.22	0.58	0.35	0.13	0.03												
2.41	0	0.52	0.43	0.20	0.06	0.02											
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01										
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01										
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02									
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02								
5.53	0	−0.34	−0.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01							
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02							
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02						
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03					
8.65	0	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02				
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01			
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01		
12.0	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01