ECE202 Analog Communication

Unit – 1
Amplitude Modulation

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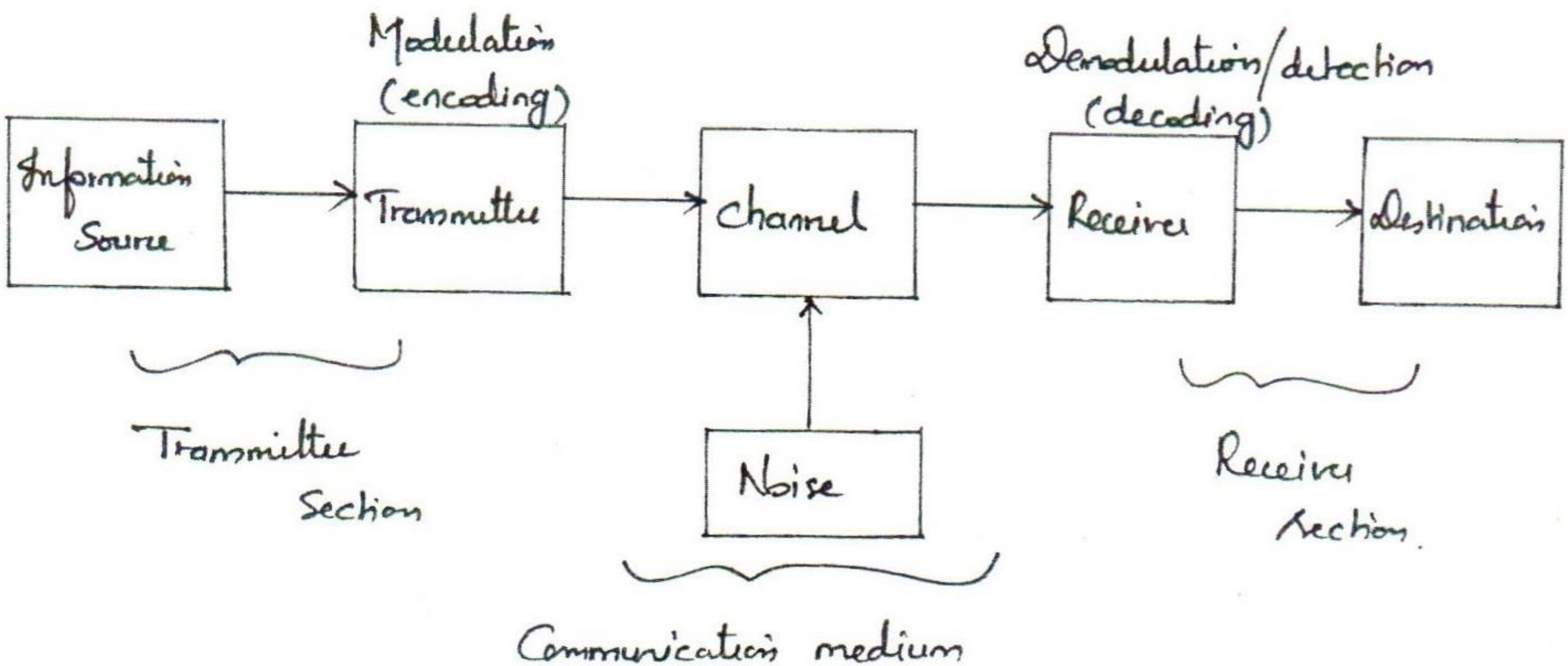
Basic Communication System



Modulation

Modulation is the process of changing the characteristics of high frequency Carrier Signal in accordance with the instantaneous Value of a modulating Signal (or) message signal.

The three characteristics of the Carrier signal are,

i). amplitude, ii). frequency, (iii). phase.

The Result of modulation process produces the modulated signal (or) Passband Signal.

Need for Modulation

i). Reduction in height of antenna

ii). Avoids mixing of signals.

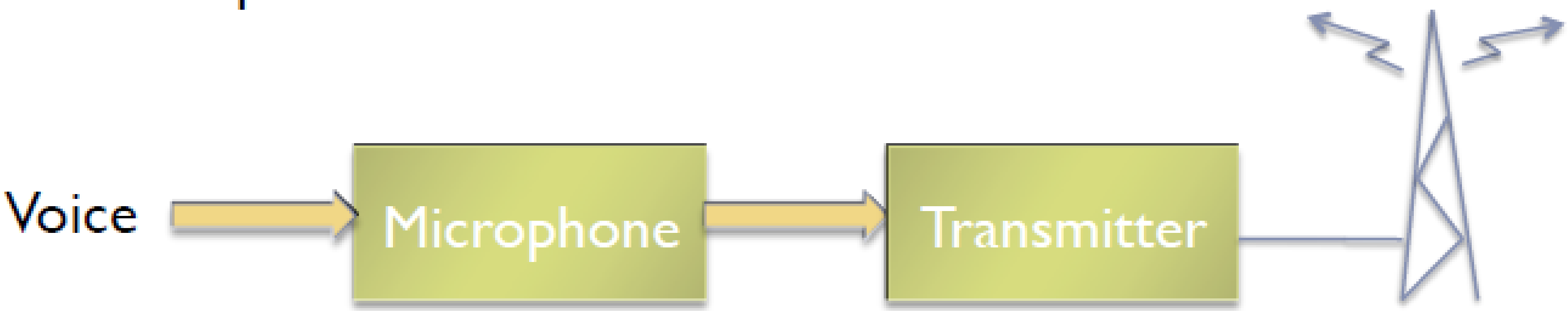
iii). Multiplexing is possible

iv) To Overcome equipment limitations

v). Improved quality.

Why do we need Modulation/Demodulation?

- Example: Radio transmission



Electric signal,
20 Hz – 20 KHz

Antenna:
Size requirement
> 1/10 wavelength

At 3 KHz: $\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^3} = 10^5 = 100km$

$\Rightarrow .1\lambda = 10km$

Antenna too large! Use modulation to transfer information to a higher frequency

$$Minimum\ antenna\ height = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times10\times10^3} = 7500\ meters\ i.e.7.5\ km$$

$$Minimum\ antenna\ height = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times10\times10^6} = 75\ meters$$

Modulation - Classification

- Modulation
  - Analog modulation
    - Continuous wave modulation (Carrier is Continuous)
      - AM
      - FM
      - PM
    - Pulse modulation (Carrier is pulses)
      - PAM
      - PWM
      - PPM
  - Digital modulation
    - Digital pulse modulation
      - PCM
      - DM
      - DPCM
    - keying techniques
      - ASK
      - FSK
      - PSK

i). Line (or) Wired Communication:

* The medium of transmission is pair of Conductors called Txion line.
* Txer & Rxer are Connected through a wire (or) lines.

Drawbacks:-

* Installation & maintenance of txion line is Costly and it Over Crowds Open Space.

ii). Wireless (or) Radio Communication:

* Message is txed through Open Space by EM waves called as radio waves.

Advantages:-

* Cost effectiveness
* Possible long distance Comm.
* Simplicity.

Amplitude Modulation or DSB-FC

Amplitude modulation may be defined as the process by which the amplitude of the high frequency Carrier signal varies in accordance with the instantaneous amplitude of the modulating (or message signal.

Let, $e_m(t)$ be the modulating signal

$$e_m(t) = E_m \sin 2\pi f_m t . \quad \text{——} \quad (1)$$

$e_c(t)$ be the Carrier signal,

$$e_c(t) = E_c \sin 2\pi f_c t . \quad \text{——} \quad (2)$$

$E_m \rightarrow$ maximum amplitude of the modulating signal.

$E_c \rightarrow$ maximum amplitude of the Carrier signal.

$f_m \rightarrow$ frequency of the modulating signal.

$f_c \rightarrow$ frequency of the Carrier signal.

Amplitude Modulation – Mathematical Model

Let E_{AM} be the amplitude of the modulated signal,

$$E_{AM} = E_c + e_m(t)$$

$$= E_c + E_m \sin 2\pi f_m t \quad \text{—— (3)}$$

$$= E_c \left[1 + \frac{E_m}{E_c} \sin 2\pi f_m t \right]$$

$$E_{AM} = E_c \left[1 + m_a \sin 2\pi f_m t \right] \quad \text{—— (4)}$$

Where $m_a = \frac{E_m}{E_c} \Rightarrow$ modulation index (or) depth of modulation

Therefore the instantaneous value of the amplitude modulated wave can be written as,

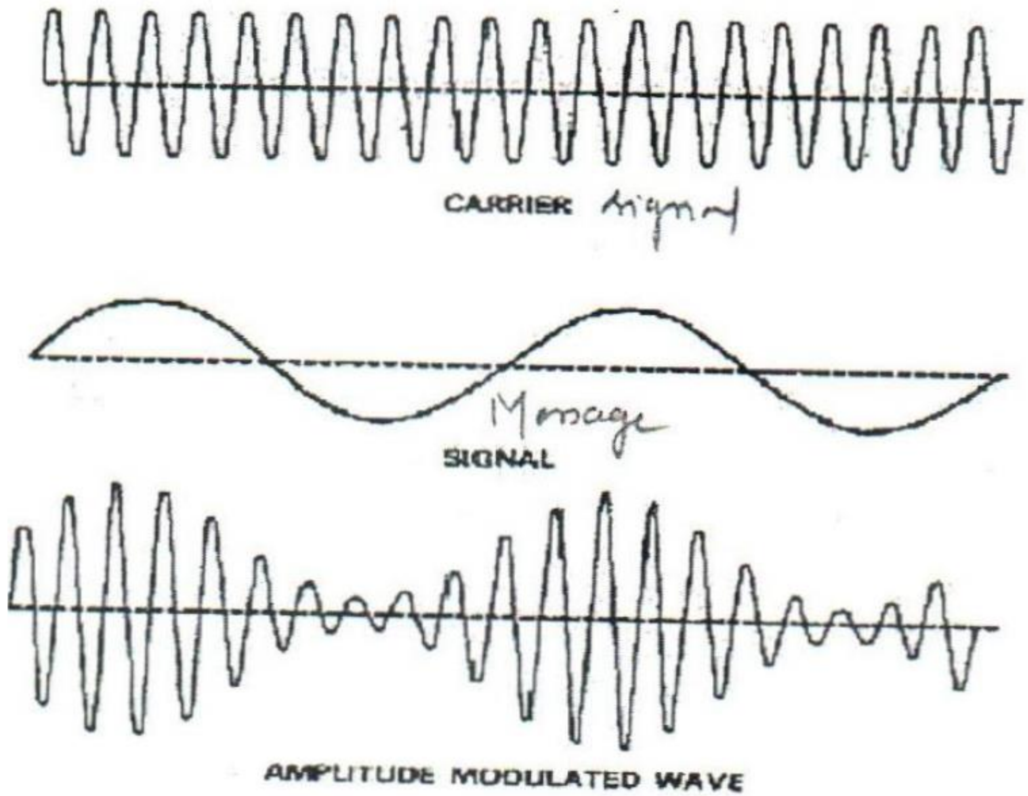
$$e_{AM}(t) = E_{AM} \sin 2\pi f_c t \quad \text{—— (5)}$$

Sub. equ (4) in (5)

$$\therefore \; e_{AM}(t) = E_c \left[1 + m_a \sin 2\pi f_m t \right] \cdot \sin 2\pi f_c t$$

$$e_{AM}(t) = E_c \sin 2\pi f_c t + E_c m_a \sin 2\pi f_m t \cdot \sin 2\pi f_c t$$

AM – Waveform Representation (Time Domain)



AM wave has a time-varying amplitude called as the envelope of the AM wave. The unique property of AM wave is that the envelope of the modulated carrier has the same shape as the message signal.

AM - Modulation Index (m_a)

The ratio of maximum amplitude of the modulating signal to the maximum amplitude of the carrier signal is called modulation index (or) depth of modulation (or) Co-efficient of modulation.

It's represented by ma.

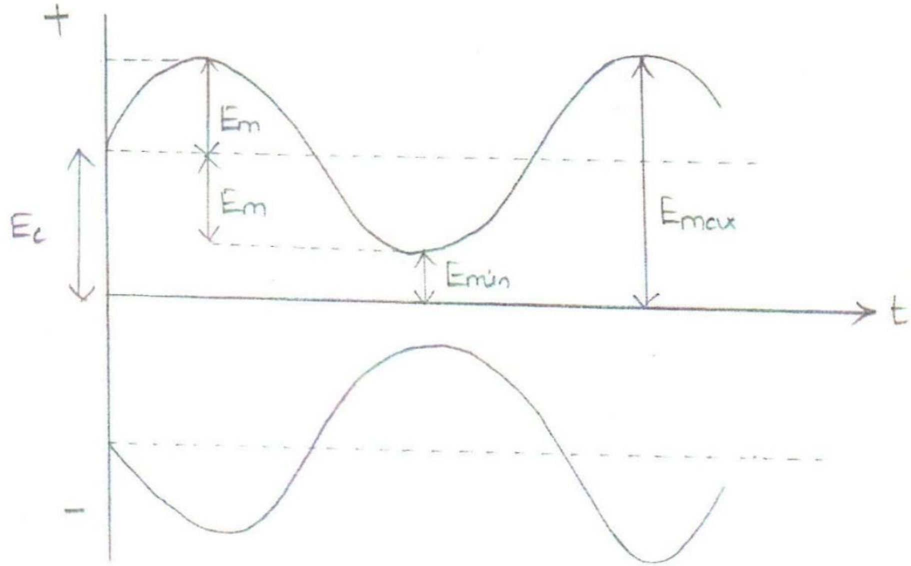
$$\boxed{m_a = \frac{E_m}{E_c}}$$

Modulation index is also known as depth of modulation, degree of modulation (or) modulation factor. The higher the degree of modulation, the greater the sideband power and the percentage of modulation the stronger and more intelligible the transmitted and received signal.

When the modulation index is expressed in percentage, it is called as the percentage of modulation & is denoted by 'M'

$$M = \frac{E_m}{E_c} \times 100$$

Calculation of Modulation Index



Graphical Representation of AM Signal

From the Fig.

$$2E_m = E_{max} - E_{min}$$

$$E_m = \frac{1}{2}\left[E_{max} - E_{min}\right] \quad \text{--- (1)}$$

$$E_c = E_{max} - E_m \quad \text{--- (2)}$$

Substituting the Value of Em in above eqn.

$$E_c = E_{max} - \left[\frac{E_{max} - E_{min}}{2}\right]$$

$$E_c = \frac{1}{2}\left[E_{max} + E_{min}\right] \quad \text{--- (3)}$$

We know that,

$$m_a = \frac{E_m}{E_c}$$

Sub eqn (2) & (3) in above exp^n

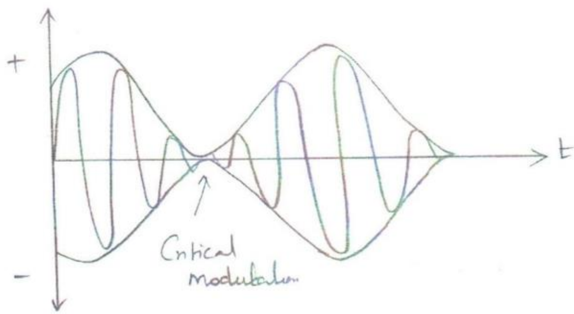
$$\boxed{m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}} \quad \text{--- (4)}$$

Equation (4) gives the modulation index interms of maximum and minimum amplitude of AM wave.

Amplitude Modulation – Degrees of Modulation

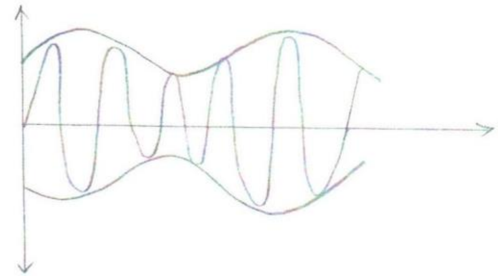
i). Critical Modulation :- [$m_a = 1$, $E_m = E_c$]

When $E_m = E_c$ modulation goes to 100% this situation is known as Critical modulation. The envelope of the modulated signal just reaches the zero amplitude axis. The message signal remaim reserved as shown below.



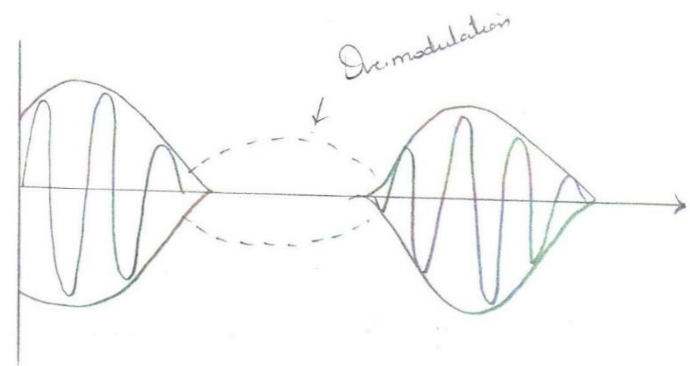
ii). Under Modulation :- [$m_a < 1$, $E_m < E_c$]

The envelope of the AM signal doesn't reach the zero amplitude axis. Therefore the message signal is fully Preserved in the AM envelope.



iii). Over Modulation :- [$m > 1$, $E_m > E_c$]

The amplitude of the modulating signal is greater than the amplitude of the Carrier signal. Therefore portion of the envelope of the modulated signal Crosses the zero amplitude axis, due to this envelope distortion occurs as shown below,



Amplitude Modulation – Frequency Spectrum

Frequency Spectrum and Bandwidth of AM.

The expⁿ for an AM wave is given by,

$$e_{AM}(t) = \left[E_c + E_m 2\pi f_m t\right] \sin 2\pi f_c t \quad \text{--- (1)}$$

We know that,

$$m_a = \frac{E_m}{E_c} \quad \text{--- (2)}$$

$$E_m = m_a E_c \quad \text{--- (3)}$$

Sub equ. (3) in equ (1)

$$e_{AM}(t) = \left[E_c + m_a E_c \sin 2\pi f_m t\right] \sin 2\pi f_c t$$

Amplitude Modulation – Frequency Spectrum

$$= E_c \left[1 + m_a \sin 2\pi f_m t \right] \sin 2\pi f_c t.$$

$$e_{AM}(t) = E_c \sin 2\pi f_c t + m_a E_c \sin 2\pi f_c t \sin 2\pi f_m t \quad \text{—— (4)}$$

Expanding the eqn. (4) using the trigonometrical relation,

$$\sin A \sin B = \frac{1}{2} \left[\cos(A-B) - \cos(A+B) \right]$$

we get,

$$e_{AM}(t) = E_c \sin 2\pi f_c t + \frac{E_c m_a}{2} \left[\cos 2\pi (f_c - f_m) t - \cos 2\pi (f_c + f_m) t \right]$$

Amplitude Modulation – Frequency Spectrum

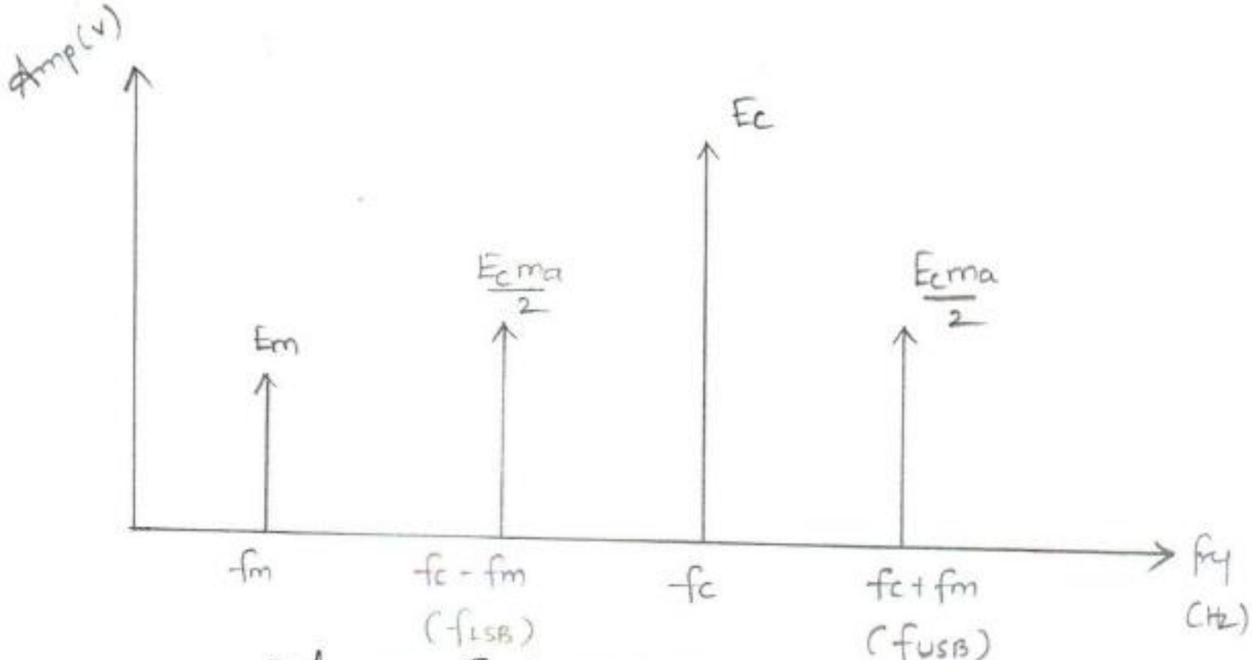
$$e_{AM}(t) = E_c \sin 2\pi f_c t + \frac{m_a E_c}{2} \cos 2\pi (f_c - f_m) t - \frac{m_a E_c}{2} \cos 2\pi (f_c + f_m$$

(5)

The above eq^n (5) Contains three frequency Components as follo

1. Unmodulated Carrier signal.
2. Lower side band $(f_c - f_m)$ having amplitude $\frac{m_a E_c}{2}$.
3. Upper side band $(f_c + f_m)$ having amplitude $\frac{m_a E_c}{2}$.

Amplitude Modulation – Frequency Spectrum



Frequency Spectrum of AM wave

Eqn ⑤, Can be also be written with the Corresponding frequency term,

(ie) $f_{LSB} = f_c - f_m$ & $f_{USB} = f_c + f_m$

$$e_{AM}(t) = E_c \sin 2\pi f_c t + \frac{mE_c}{2} \cos 2\pi f_{LSB} t - \frac{mE_c}{2} \cos 2\pi f_{USB} t \quad —(\cdot$$

Amplitude Modulation – Bandwidth

Bandwidth of AM:

The bandwidth of AM wave is equal to the difference between the highest upper side frequency and lowest lower Side frequency. This is the frequency range over which the information signal is transmitted.

$$BW = f_{USB} - f_{LSB}$$

$$f_{LSB} = f_c - f_m \quad \& \quad f_{USB} = f_c + f_m$$

$$BW = (f_c + f_m) - (f_c - f_m)$$

$$= \cancel{f_c} + f_m - \cancel{f_c} + f_m$$

$$\boxed{BW = 2f_m} \text{ Hz}$$

Amplitude Modulation – Power Distribution

The total power in the modulated wave will be,

$$P_{t\,(AM)} = P_c + P_{USB} + P_{LSB} \quad \text{——} \quad (1)$$

To find Carrier power (P_c):

The average power of the unmodulated Carrier is equal to the rms Carrier Voltage squared divided by the load resistance (generally antenna Resistance).

Amplitude Modulation – Power Distribution

Mathematically, Power in an unmodulated Carrier is,

$$P_c = \frac{\left(\frac{E_c}{\sqrt{2}}\right)^2}{R}$$

$$P_c = \frac{E_c^2}{2R} \quad \text{———} \quad (2)$$

where, $P_c \rightarrow$ Carrier power in watts

$E_c \rightarrow$ Peak Carrier Voltage in Volts

$R \rightarrow$ load resistance in ohm.

Amplitude Modulation – Power Distribution

AM wave equation is given as,

$$e_{AM}(t) = E_c \sin 2\pi f_c t + \frac{m_a E_c}{2} \cos(f_c - f_m) 2\pi t$$

$$- \frac{m_a E_c}{2} \cos(f_c + f_m) 2\pi t.$$

Hence the upper and lower sideband powers are mathematically expressed as,

$$P_{LSB} = P_{USB} = \frac{\left[\dfrac{\frac{m_a E_c}{2}}{\sqrt{2}}\right]^2}{R}$$

$$P_{LSB} = P_{USB} = \frac{m_a^2 E_c^2}{8R}.$$

$P_{LSB} \rightarrow$ Lower Side band power (watts)

$P_{USB} \rightarrow$ Upper Side band power (watts)

Amplitude Modulation – Power Distribution

$\therefore$ Total power of the AM Wave $\Big\}$ $P_{t(AM)} = P_c + P_{LSB} + P_{USB}$

$$= \frac{E_c^2}{2R} + \frac{m_a E_c^2}{8R} + \frac{m_a E_c^2}{8R}$$

$$\therefore P_{t(AM)} = \frac{E_c^2}{2R}\left[1 + \frac{m_a^2}{4} + \frac{m_a^2}{4}\right]$$

$$= \frac{E_c^2}{2R}\left[1 + \frac{m_a^2}{2}\right]$$

$$\therefore \boxed{P_{t(AM)} = P_c\left[1 + \frac{m_a^2}{2}\right]}$$

If $m_a = 1$ (ie) 100% modulation

$$P_t = P_c \times \left[1 + \frac{1}{2}\right] \Rightarrow \boxed{P_t = 1.5 P_c}$$

Amplitude Modulation – Power Distribution

$$\Rightarrow \quad P_c = \frac{P_t}{1+\frac{m_a^2}{2}}$$

$$P_c\left(1+\frac{m_a^2}{2}\right) = P_t$$

$$\frac{P_t}{P_c} = 1+\frac{m_a^2}{2}$$

$$\frac{P_t}{P_c} - 1 = \frac{m_a^2}{2}$$

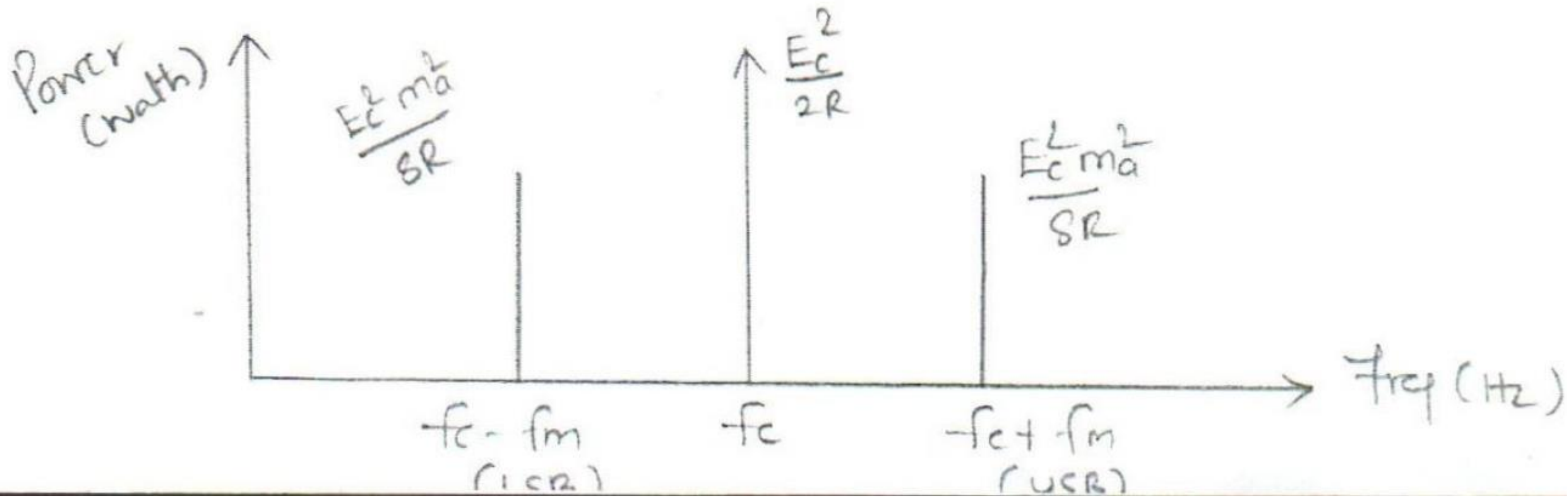
$$m_a^2 = 2\left[\frac{P_t}{P_c} - 1\right]$$

$$\boxed{m_a = \sqrt{2\left(\frac{P_t}{P_c} - 1\right)}}$$

Amplitude Modulation – Power Spectrum

$$P_t(AM) = P_c + P_{LSB} + P_{USB}$$

$$= \frac{E_c^2}{2R} + \frac{E_c^2 m_a^2}{8R} + \frac{E_c^2 m_a^2}{8R}$$



Amplitude Modulation – Current Distribution

In general the power and current in the antenna are related by $P = I^2 R$.

We know that,

$$P_{t(AM)} = P_c \left[1 + \frac{m_a^2}{2}\right]$$

$$I_t^2 R = I_c^2 R \left[1 + \frac{m_a^2}{2}\right]$$

$$I_t^2 = I_c^2 \left[1 + \frac{m_a^2}{2}\right]$$

$$\boxed{I_t = I_c \sqrt{1 + \frac{m_a^2}{2}}}$$

Amplitude Modulation – Current Distribution

to find modulation index (m_a) :-

$$\frac{I_t}{I_c} = \sqrt{1 + \frac{m_a^2}{2}}$$

$$\frac{I_t^2}{I_c^2} = 1 + \frac{m_a^2}{2}$$

$$\frac{m_a^2}{2} = \frac{I_t^2}{I_c^2} - 1$$

$$m_a^2 = 2\left[\frac{I_t^2}{I_c^2} - 1\right]$$

$$\boxed{m_a = \sqrt{2\left[\frac{I_t^2}{I_c^2} - 1\right]}}$$ ⟸ Modulation index (m_a) interms of Current.

Amplitude Modulation – Transmission Efficiency

It Can be defined as the ratio of power in sidebands to total power because side bands only Contain the useful information.

$$\% \eta = \frac{\text{Power in Sidebands}}{\text{total power}} \times 100$$

$$= \frac{P_{LSB} + P_{USB}}{P_{t}(AM)} \times 100$$

$$= \frac{\frac{m_a^2 E_c^2}{8R} + \frac{m_a^2 E_c^2}{8R}}{\frac{E_c^2}{2R}\left[1 + \frac{m_a^2}{2}\right]} \times 100$$

$\because$

$$P_t(AM) = P_c\left[1 + \frac{m_a^2}{2}\right]$$

$$P_c = \frac{E_c^2}{2R}$$

Amplitude Modulation – Transmission Efficiency

$$= \frac{\dfrac{m_a^2 E_c^2}{4R}}{\dfrac{E_c^2}{2R}\left[1+\dfrac{m_a^2}{2}\right]}$$

$$= \frac{\cancel{\dfrac{E_c^2}{2R}} \cdot \dfrac{m_a^2}{2}}{\cancel{\dfrac{E_c^2}{2R}}\left[1+\dfrac{m_a^2}{2}\right]} = \frac{m_a^2}{2+m_a^2}$$

if $m_a = 100\%$

$m_a = 1$

$$\therefore \ \% \eta = \frac{1}{2+1} \times 100 = \frac{1}{3} \times 100$$

$$\boxed{\% \eta = 33.33 \%}$$

Modifications of AM

1. ***Double sideband-suppressed carrier* (*DSB-SC*) *modulation***

- The transmitted wave consists of only the upper and lower sidebands.
- But the channel bandwidth requirement is the same as before.

***2. Single sideband (SSB) modulation*.**

- The modulation wave consists only of the upper sideband or the lower sideband.
- To translate the spectrum of the modulating signal to a new location in the frequency domain.

3. Vestigial sideband (VSB) modulation

- One sideband is passed almost completely and just a trace of the other sideband is retained.
- The required channel bandwidth is slightly in excess of the message bandwidth by an amount equal to the width of the vestigial sideband.

DSB-SC (Double Side Band - Suppressed Carrier)

The important parameters of a Comm. Sty are transmitting Power and the bandwidth. Hence saving of power and bandwidth are highly desirable in a Comm. Sty.

Disadvantages of AM (DSB-FC):

* Power wastage takes place in DSBFC
* Bandwidth inefficient
* AM wave gets affected due to noise

Applications of AM:

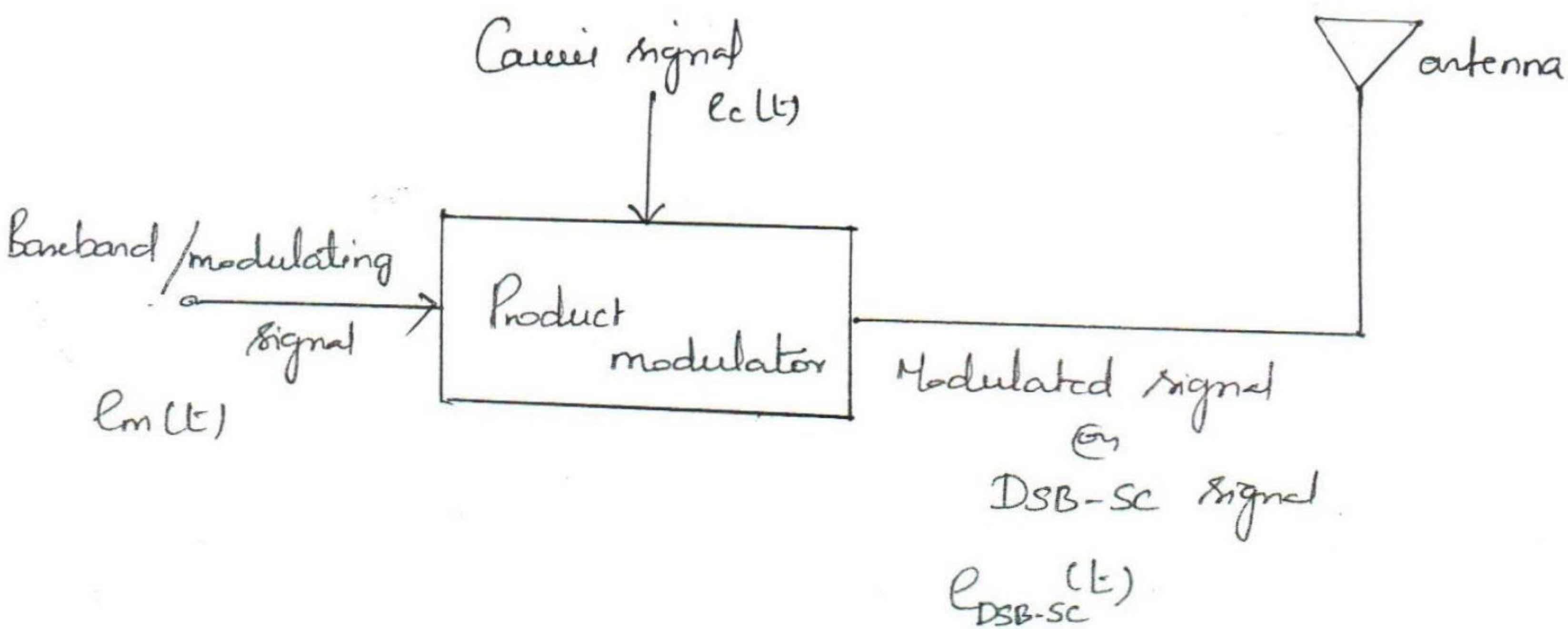
* Radio broadcasting
* Picture transmission in TV System

DSB-SC (Double Side Band - Suppressed Carrier)

Let,

modulating signal, $e_m(t) = E_m \sin \omega_m t$

Carrier signal, $e_c(t) = E_c \sin \omega_c t$



DSB-SC (Double Side Band - Suppressed Carrier)

When multiplying both the Carrier and message signal, the Resulting signal is the DSB-SC-AM signal.

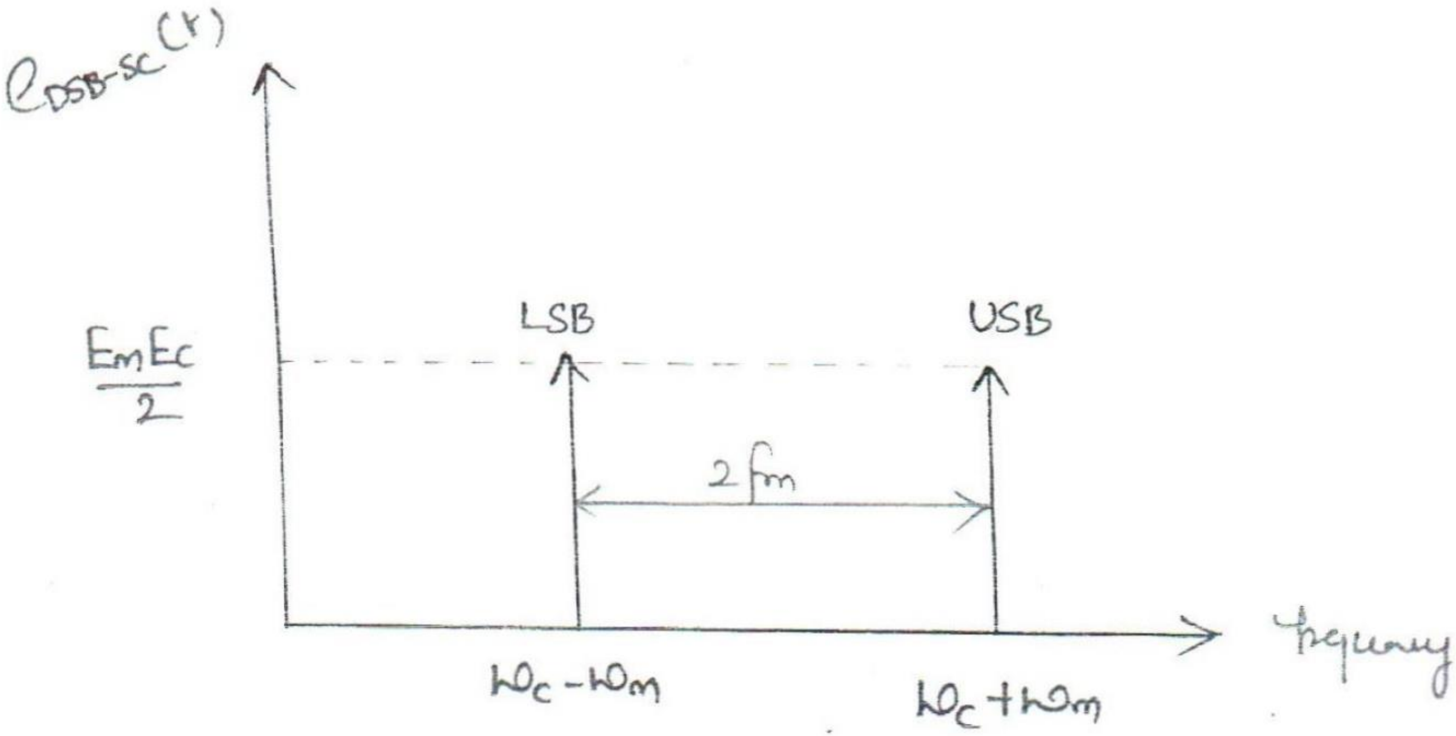
$$e_{DSBSC-AM}(t) = e_m(t) \cdot e_c(t).$$

$$\therefore \quad e_{DSB-SC}(t) = E_m \sin \omega_m t \cdot E_c \sin \omega_c t$$

$$= E_m E_c \sin \omega_m t \sin \omega_c t$$

$$= \frac{E_m E_c}{2} \left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t \right]$$

DSB-SC - Spectrum



DSB-SC – Power Relation

Total power of DSB-SC $= P_t = P_{USB} + P_{LSB}$

W.K.T, $P_{USB} = P_{LSB} = \frac{m_a^2 E_c^2}{8R}$

$$\therefore P_t = \frac{m_a^2 E_c^2}{8R} + \frac{m_a^2 E_c^2}{8R}$$

$$P_t = \frac{m_a^2 E_c^2}{4R}$$

if $R = 1\Omega$

$$P_t = \frac{m_a^2 E_c^2}{4} \Rightarrow \frac{E_c^2}{2} \cdot \frac{m_a^2}{2}.$$

$$P_c = \frac{E_c^2}{2}$$

Hence, $P_t = \frac{m_a^2}{2} \cdot P_c$

for 100% modulation $m_a = 1$

$$\therefore \boxed{P_{t(DSB\text{-}SC)} = \frac{P_c}{2}}$$

DSB-SC – Transmission Efficiency / Power Savings

$$\text{Power Saving} = \frac{P_{AM} - P_{DSBSC}}{P_{AM}}$$

$$P_{AM} = P_c\left(1 + \frac{m_a^2}{2}\right)$$

$$P_{DSB\text{-}SC} = P_c \cdot \frac{m_a^2}{2}.$$

$$\therefore \text{Power Saving} = \frac{P_c\left(1+\frac{m_a^2}{2}\right) - P_c \cdot \frac{m_a^2}{2}}{P_c\left(1+\frac{m_a^2}{2}\right)}$$

$$= \frac{P_c + \cancel{\frac{P_c m_a^2}{2}} - \cancel{\frac{P_c m_a^2}{2}}}{P_c\left(1+\frac{m_a^2}{2}\right)}$$

$$= \frac{\cancel{P_c}}{\cancel{P_c}\left(1+\frac{m_a^2}{2}\right)} = \frac{2}{2+m_a^2}$$

For 100% modulation, $m_a = 1$,

$$\text{Power Saving} = \frac{2}{2+1} = \frac{2}{3} = 0.66$$

$$\boxed{\%\ \text{of power Saving} = 66.7\%}$$

SSB-SC (Single Side Band - Suppressed Carrier)

Both the sidebands in the DSBSC are carrying the same information hence only one sideband is sufficient to convey the message. So we can suppress one sideband & transmit the other. This is called as Single sideband with suppressed carrier (or) Single sideband (SSB) system.

In this case, the power saving increases by eliminating any one sideband in addition with the carrier component.

Advantages:

i). Efficiency is increased (83.33%)

ii). Bandwidth is reduced (f_m)

iii). Fading effect which arises because of the interference of carrier and two sidebands is removed in SSB.

SSB-SC – Power Distribution

$$P_{t\,(SSB-SC)} = P_{USB} = \frac{E_c^2 m_a^2}{8R}$$

Power Saving of SSB-SC $= P_{tAM} - P_{t\,DSB-SC}$

$$P_{t\,(SSB-SC)} = \frac{E_c^2}{2R} \cdot \frac{m_a^2}{4}$$

if $R = 1$

$$= \frac{E_c^2}{2} \cdot \frac{m_a^2}{4}$$

$$P_{t\,(SSB-SC)} = P_c \cdot \frac{m_a^2}{4}.$$

for 100% modulation, $m_a = 1$.

$$P_{t\,(SSB-SC)} = \frac{P_c}{4}.$$

SSB-SC – Power Saving w.r.t DSB-FC

$$\text{Power Saving} = \frac{P_{t(AM)} - P_{t(SSB-SC)}}{P_{t(AM)}}$$

$$= \frac{P_c\left(1+\frac{m_a^2}{2}\right) - P_c \cdot \frac{m_a^2}{4}}{P_c\left(1+\frac{m_a^2}{2}\right)}$$

$$= \frac{P_c + \cancel{\frac{P_c m_a^2}{2}} - \cancel{\frac{P_c m_a^2}{4}}}{P_c\left(1+\frac{m_a^2}{2}\right)}$$

$$= \frac{P_c + \frac{P_c m_a^2}{2}}{P_c\left(1+\frac{m_a^2}{2}\right)} = \frac{4\cancel{P_c}\,(4+m_a^2)}{\cancel{P_c}\left(1+\frac{m_a^2}{2}\right)}$$

$$= \frac{4+m_a^2}{4} \times \frac{2}{2+m_a^2}$$

$$= \frac{4+m_a^2}{2(2+m_a^2)}$$

for 100% $m_a = 1$

$$\%\eta = \frac{5}{6} = 0.8333$$

% of Power Saving = 83.33%

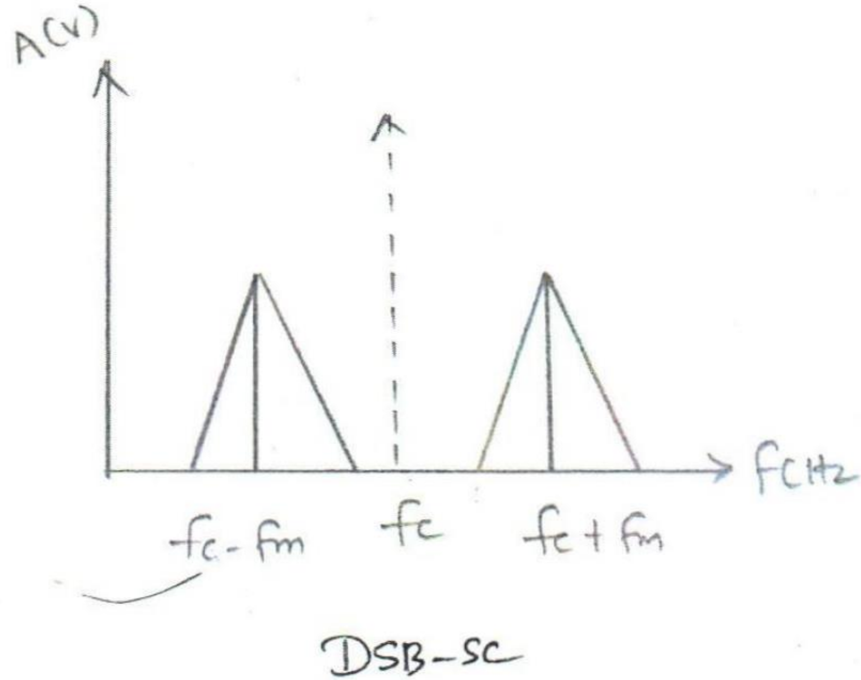
SSB-SC – Power Saving w.r.t DSB-SC

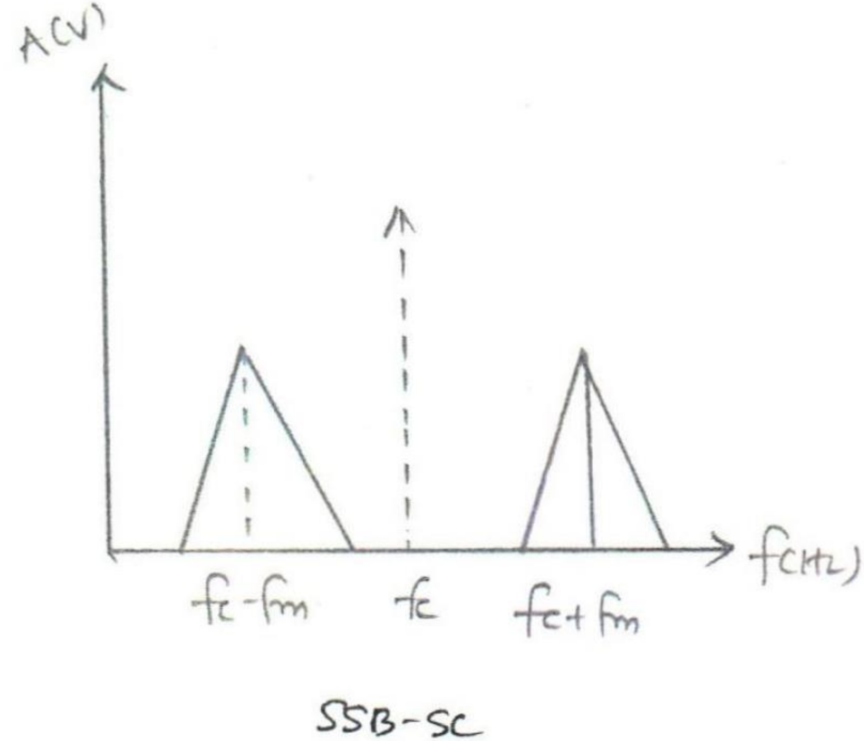
$$\eta = \frac{P_{t(DSB-SC)} - P_{t(SSB-SC)}}{P_{t(DSB-SC)}} = \frac{\frac{P_c m_a^2}{2} - \frac{P_c m_a^2}{4}}{\frac{P_c m_a^2}{2}}.$$

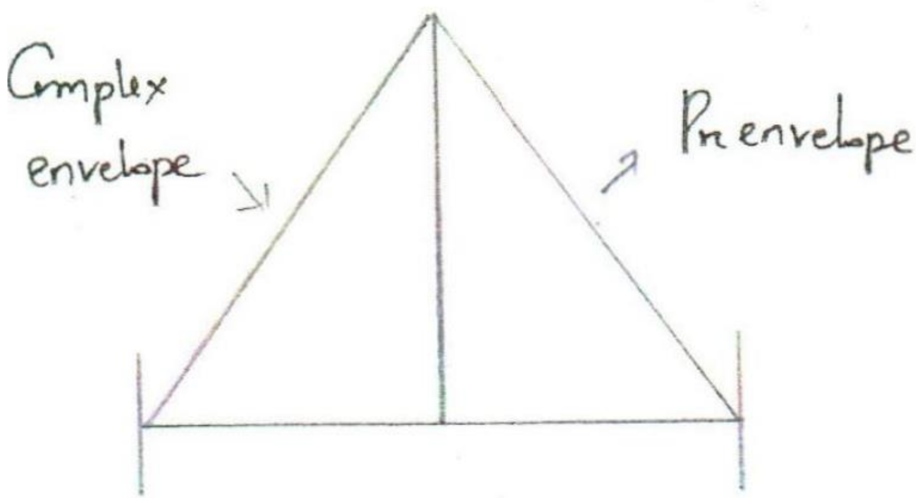
$$\eta = \frac{P_c m_a^2}{4} \cdot \frac{2}{P_c m_a^2} = \frac{1}{2} = 0.5$$

$$\% \text{ of } \eta = 50\%$$

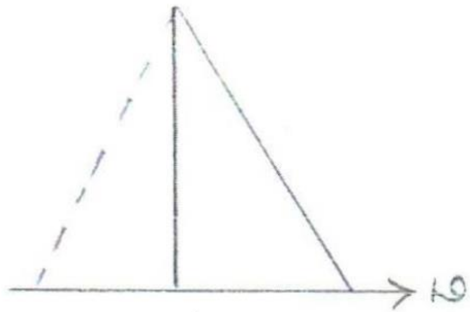
SSB-SC – Time Domain



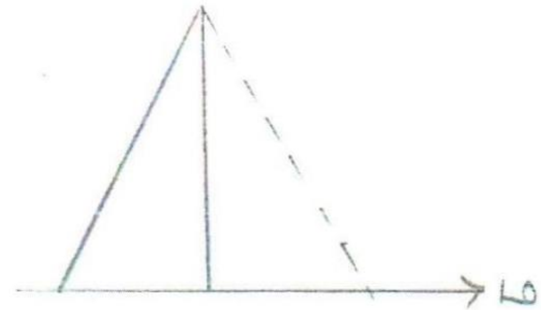




SSB-SC – Time Domain



$x_p^*(t)$



$x_p(t)$

$$s(t) = x_p(\omega) + x_p^*(\omega) \quad \text{—— (1)}$$

$$x_p(\omega) = \frac{1}{4}\, x_p(t) \cdot e^{-j\omega_c t} \quad \text{—— (2)}$$

$$x_p^*(\omega) = \frac{1}{4}\, x_p^*(t)\, e^{j\omega_c t} \quad \text{—— (3)}$$

$$x_p(t) = x(t) + j\, x_h(t) \quad \text{—— (4)}$$

$$x_p^*(t) = x(t) - j\, x_h(t) \quad \text{—— (5)}$$

Sub equ (4) in (2), we get

$$x_p(\omega) = \frac{1}{4}\left[x(t) + j\, x_h(t)\right] e^{-j\omega_c t} \quad \text{—— (6)}$$

Sub equ (5) in (3), we get

$$x_p^*(\omega) = \frac{1}{4}\left[x(t) - j\, x_h(t)\right] e^{j\omega_c t} \quad \text{—— (7)}$$

SSB-SC – Time Domain

Sub equ (6) & (7) in (1)

$$S(t) = \frac{1}{4}\left[x(t) + j\,x_h(t)\right] e^{-j\omega_c t} + \frac{1}{4}\left[x(t) - j\,x_h(t)\right] e^{j\omega_c t}.$$

$$S(t) = \frac{1}{4}\left[x(t)\,e^{-j\omega_c t} + j\,x_h(t)\,e^{-j\omega_c t} + x(t)\,e^{j\omega_c t} - j\,x_h(t)\,e^{j\omega_c t}\right]$$

$$S(t) = \frac{1}{4}\left[x(t)\left[e^{+j\omega_c t} + e^{-j\omega_c t}\right] + j\,x_h(t)\left[e^{-j\omega_c t} - e^{j\omega_c t}\right]\right]$$

$$= \frac{1}{4}\left[x(t) \cdot 2\cos\omega_c t \mp j\,(2j\sin\omega_c t)\,x_h(t)\right]$$

SSB-SC – Time Domain

$$s(t) = \frac{1}{2}\left[x(t)\cos w_c t + x_h(t)\sin w_c t\right]$$

Sub, $x(t) = \sin w_m t$, $x_h(t) = \cos w_m t$

$$s(t) = \frac{1}{2}\left[\sin w_m t \cos w_c t + \cos w_m t \sin w_c t\right]$$

$$\boxed{s(t) = \frac{1}{2}\left[\sin(w_c + w_m)t\right]}$$

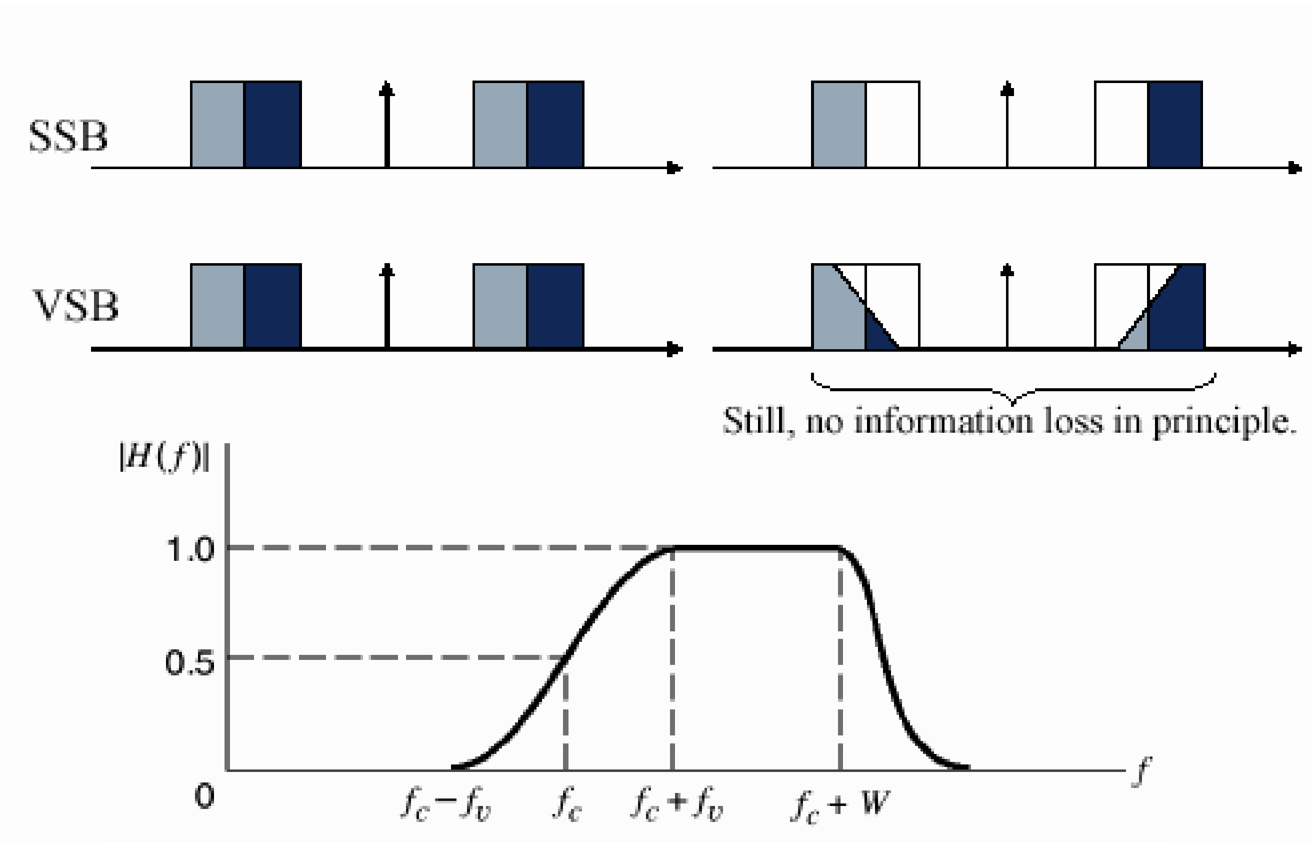
Sub $x(t) = \cos w_m t$, $x_h(t) = \sin w_m t$

$$s(t) = \frac{1}{2}\left[\cos w_m t \cos w_c t + \sin w_m t \sin w_c t\right]$$

$$\boxed{s(t) = \frac{1}{2}\left[\cos(w_c - w_m)t\right]}$$

Vestigial Sideband Modulation (VSB)

- Instead of transmitting only one sideband as SSB, VSB modulation transmits a partially suppressed sideband and a vestige of the other sideband.



Vestigial Side Band

- The DSB-SC signal is given by

$$v_{DSB-SC} = \frac{mV_c}{2}\cos(\omega_c - \omega_m)t - \frac{mV_c}{2}\cos(\omega_c + \omega_m)t$$

- If LSB is wanted sideband in case of VSB, the instantaneous voltage of the VSB signal may be expressed as

$$v_{VSB} = \frac{mV_c}{2}\cos(\omega_c - \omega_m)t + F\left[-\frac{mV_c}{2}\cos(\omega_c + \omega_m)t\right]$$

- Alternatively, if USB is wanted sideband, the instantaneous voltage of VSB may be given by

$$v_{VSB} = -\frac{mV_c}{2}\cos(\omega_c + \omega_m)t + F\left[\frac{mV_c}{2}\cos(\omega_c - \omega_m)t\right]$$

where F represents the fraction.

- The power and bandwidth requirements of VSB is slightly more than SSB, but less than DSB.

Frequency Spectrum of VSB

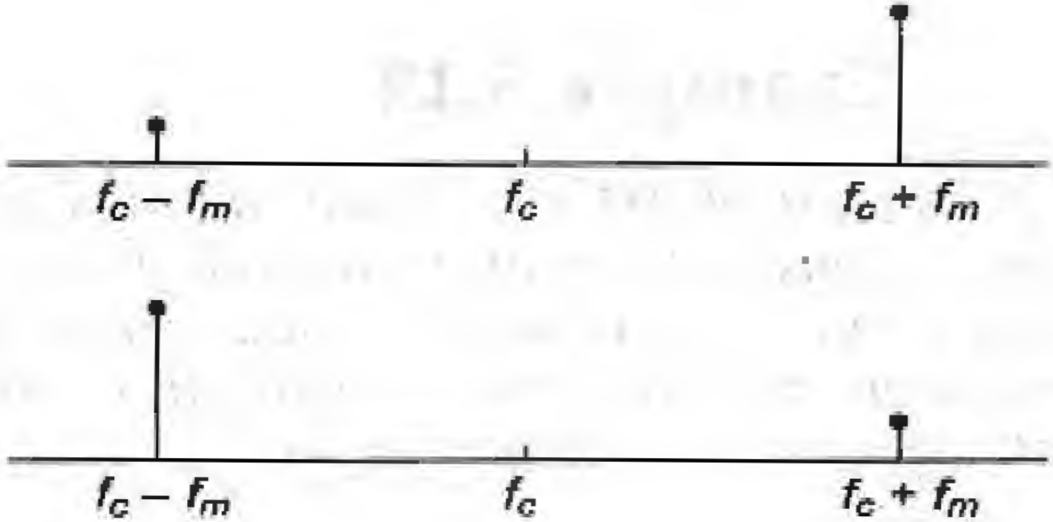
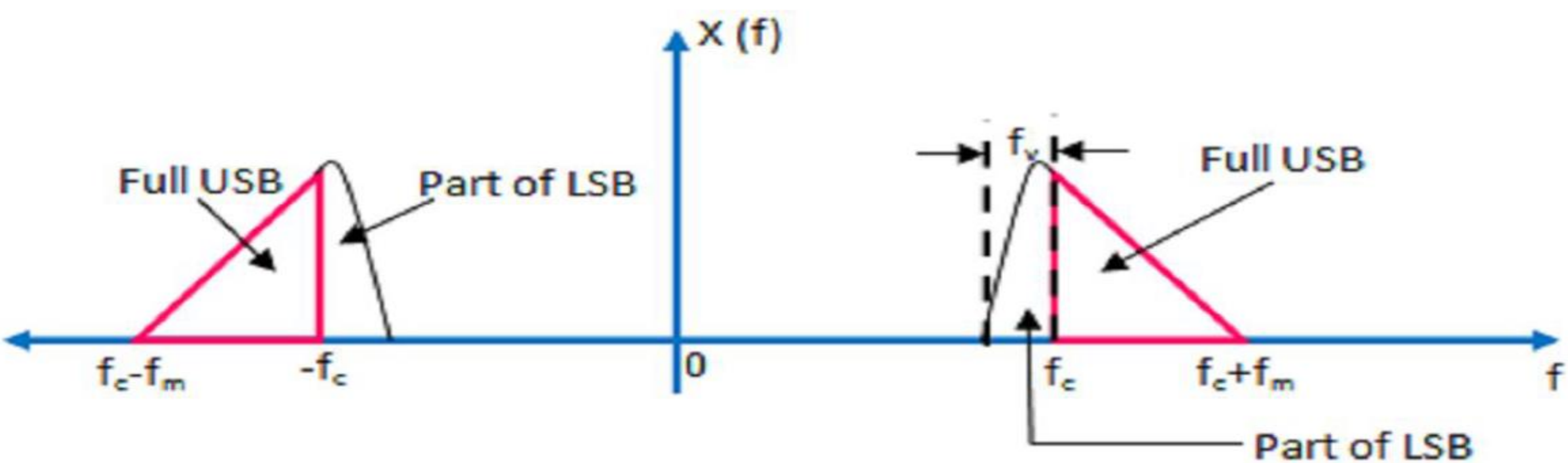


Fig. *Frequency spectrum of a VSB wave. Spectrum for (a) VSB = USB + vestige of LSB, and (b) VSB = LSB + vestige of USB.*



Power Relation in VSB

The total power in the DSBSC modulated wave will be

$$P_{DSBSC} = \frac{V_{LSB}^2}{2} + \frac{V_{USB}^2}{2}$$

where all the voltages are rms values and R is the resistance in which the power is dissipated

$$P_{LSB} = P_{USB} = \frac{V_{SB}^2}{R} = \left(\frac{mV_c/2}{\sqrt{2}}\right) \div R = \frac{m^2V_c^2}{8R} = \frac{m^2}{4}\frac{V_c^2}{2R}$$

Substituting these equations in the total power equation, we have

$$P_{DSBSC} = \frac{m^2}{4}\frac{V_c^2}{2R} + \frac{m^2}{4}\frac{V_c^2}{2R}$$

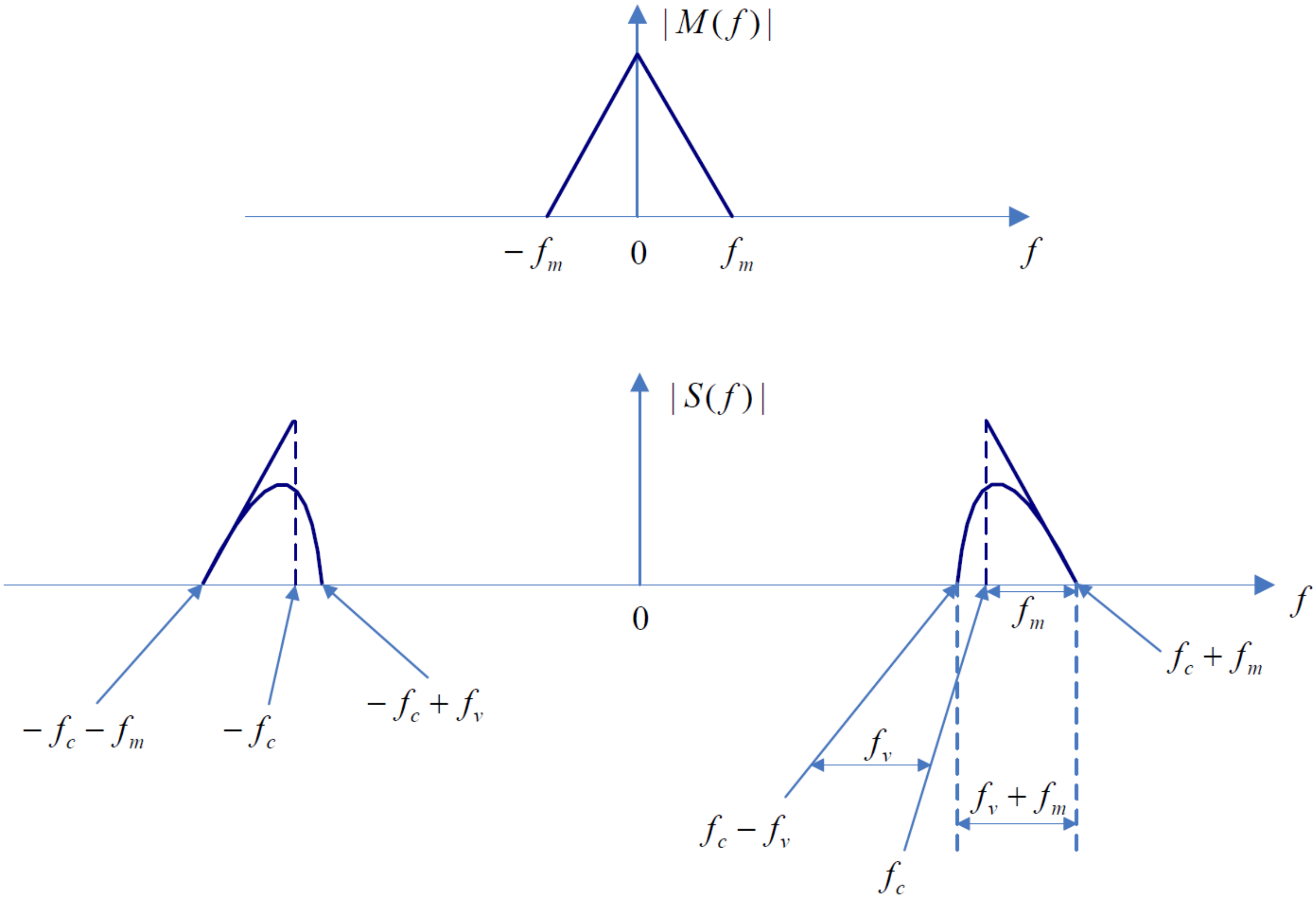
If LSB is wanted sideband in VSB, then

$$P_{VSB} = \frac{m^2}{4}P_c + F\left(\frac{m^2}{4}P_c\right)$$

Alternatively, if USB is wanted sideband in VSB, then

$$P_{VSB} = F\left(\frac{m^2}{4}P_c\right) + \frac{m^2}{4}P_c$$

Bandwidth in VSB



Specifically, the transmitted vestige of the lower sideband compensates for the amount removed from the upper sideband.

The transmission bandwidth (f_T) required by the VSB modulated wave is therefore

$$f_T = f_m + f_v$$

Comparisions of Amplitude modulation Techniques :-

SL No	parameter	DSB-FC Standard AM	DSB-SC	SSB	VSB
1)	Power	High	Medium	Less	Less than DS but greater than SSB
2)	Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW < 2$
3)	Carrier Suppression	NO	Yes	Yes	NO
4)	Sideband Transmission	NO	NO	one Sideband Completely	one Sideba Suppressed partly
5)	Transmission efficiency	Minimum	Moderate	Maximum	Moderate
6)	Receiver Complexity	Simple	Complex	complex	Simple
7)	Modulation type	Non-linear	Linear	Linear	Linear
8)	Applications	Radio Communication	Linear Radio Communication	Linear point to point mobile communication	Television

Frequency Range in Communication System

Types of Signal	Frequency Range	Application
VLF	3 to 30 kHz	Long distance, Point to point Communication
LF	30 to 300 kHz	Radio Navigation
MF	300 kHz to 3 MHz	Broadcasting, Marine application
HF	3 MHz to 30 MHz	Radio telephony
VHF	30 MHz to 300 MHz	FM broadcasting, TV, Mobile radio, radio navigation
UHF	300 MHz to 3000 MHz	,,
EHF (μWaves)	3 GHz to 30 GHz	Multichannel telephony links Radar, Satellite Communication