ECE202 Analog Communication

Unit – 4
Noise

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1. Introduction

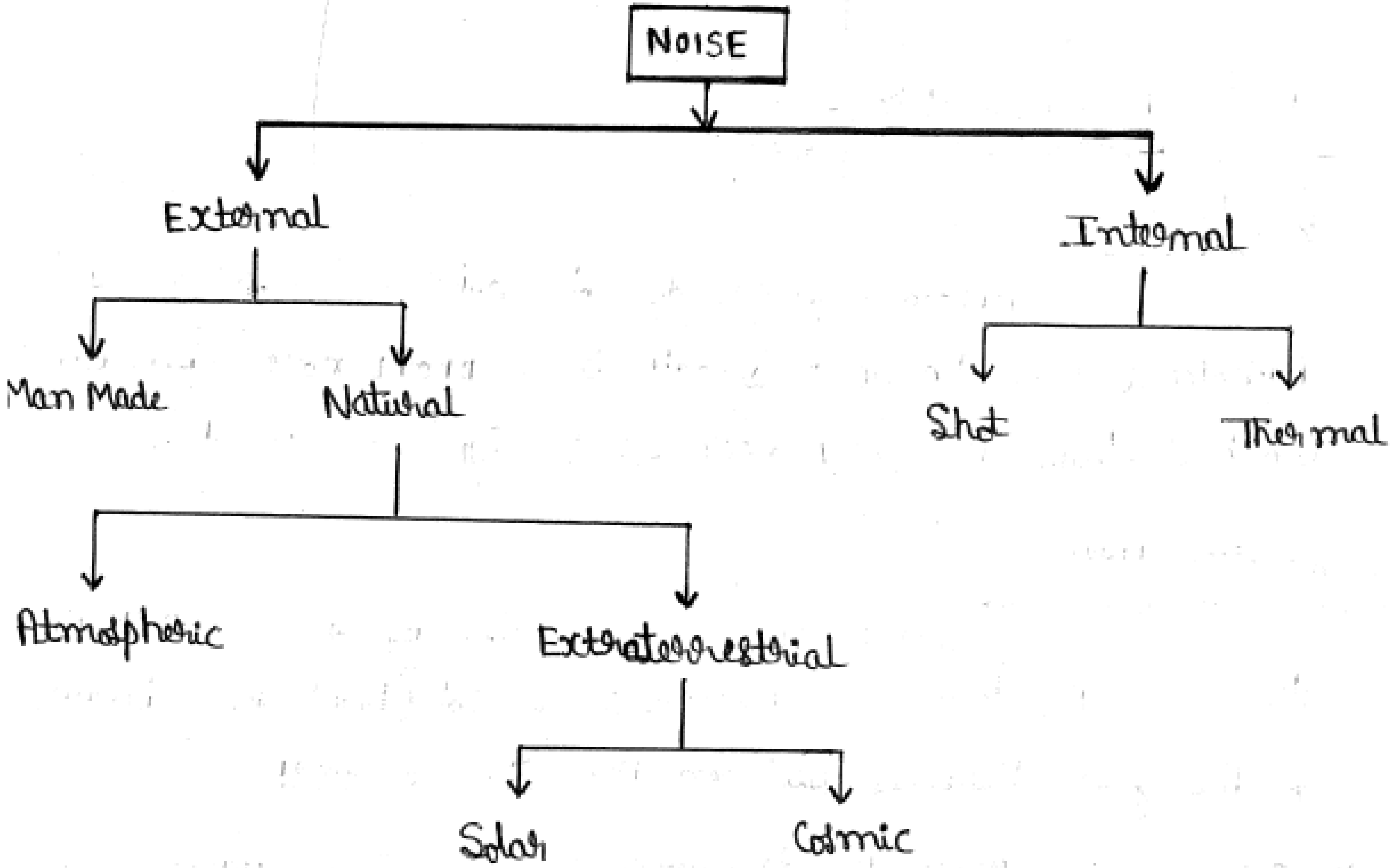
- Noise is a general term which is used to describe an unwanted signal which affects a wanted signal. These unwanted signals arise from a variety of sources which may be considered in one of two main categories:-

•*Interference, usually from a human source (man made)*
•*Naturally occurring random noise*

Interference

- Interference arises for example, from other communication systems (cross talk), 50 Hz supplies (hum) and harmonics, switched mode power supplies, thyristor circuits, ignition (car spark plugs) motors … etc.

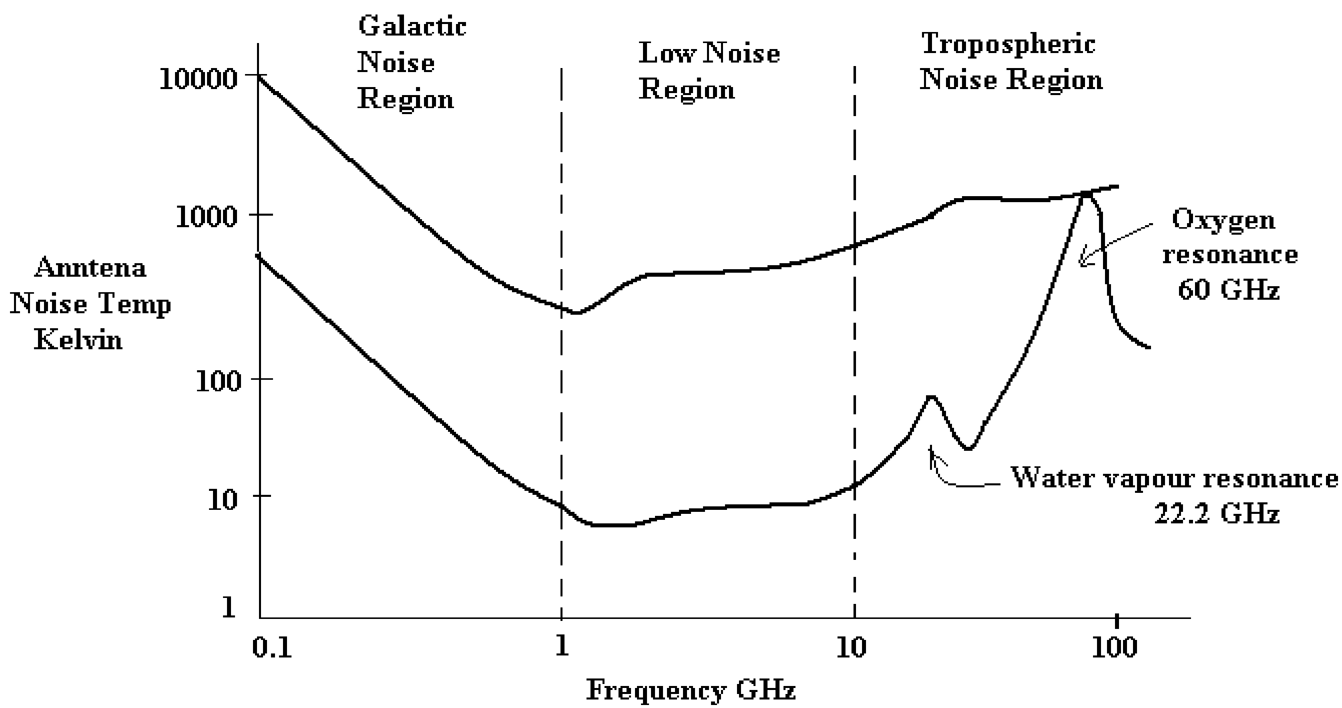
Noise Classification



Introduction (Cont'd)

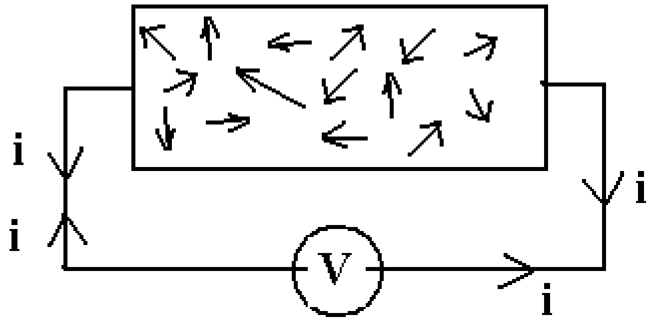
Natural Noise

Naturally occurring external noise sources include atmosphere disturbance (e.g. electric storms, lighting, ionospheric effect etc), so called '*Sky Noise*' or Cosmic noise which includes noise from galaxy, solar noise and 'hot spot' due to oxygen and water vapour resonance in the earth's atmosphere.



2. Thermal Noise (Johnson Noise)

This type of noise is generated by all resistances (e.g. a resistor, semiconductor, the resistance of a resonant circuit, i.e. the real part of the impedance, cable etc).



Experimental results (by Johnson) and theoretical studies (by Nyquist) give the mean square noise voltage as

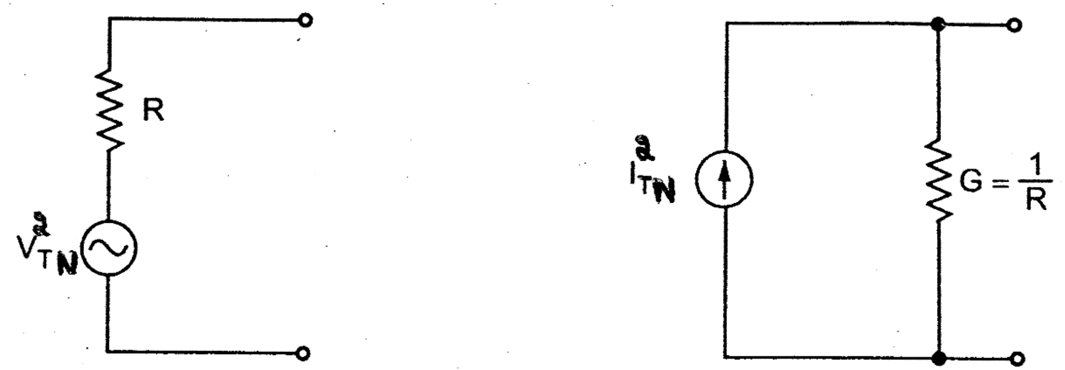
$$\overline{V}^2 = 4kTB_N R \ (volt^2)$$

Where k = Boltzmann's constant = 1.38 x 10^{-23} Joules per K
T = absolute temperature
B_N = Noise bandwidth measured in (Hz)
R = resistance (ohms)

Equivalent Noise Source for Thermal Noise

Figure

Equivalent noise sources for thermal noise



(a) Thevenin equivalent circuit **(b) Norton equivalent circuit**

* Fig (a) Shows a model of a Noisy resistor.

The Thevenin equivalent ckt consisting of a Noise voltage generator with a mean-Square values of V_{TN}^2 in Series with a noiseless resistor.

Equivalent Noise Source for Thermal Noise

* Similarly Fig (b) shows Norton equivalent ckt consisting of a Noise current generator in parallel with a Noiseless conductance.

The mean-square value of the Noise current generator is :

$$I_{TN}^2 = \frac{V_{TN}^2}{R^2} = \frac{4KTB_N \cancel{R}}{R^{\cancel{2}}} = 4KTB_N \frac{1}{R}$$

$$\boxed{I_{TN}^2 = 4KTB_N G} \quad \text{amps}^2$$

Where, $G = \frac{1}{R}$ is the conductance.

Available Noise power :-

* The Root-mean square value of the voltage V_{RMS} across the matched load R_L is

$$\boxed{V_{RMS} = \frac{\sqrt{V_{TN}^2}}{2}}$$

Equivalent Noise Source for Thermal Noise

* The maximum average Noise power delivered to the load is :

$$P_n = \frac{V_{RMS}^2}{R} = \frac{V_{IN}^2}{4R} = \frac{\cancel{4}KTB_N\cancel{R}}{\cancel{4R}}$$

$$\boxed{P_n = KTB_N}$$

Thus, the available Noise power 'P_a' is equal to 'KTB_N' & is independent of 'R'.

FORMULAE :

1) RMS Noise voltage : $V_{IN}^2 = 4KTB_NR$

$$\boxed{V_{IN} = \sqrt{4KTB_NR}}$$

2) Thermal Noise power

$$\boxed{P_n = KTB_N}$$

Equivalent Noise Source for Thermal Noise

1) Calculate the rms noise voltage and thermal noise power appearing across a 20 kΩ resistor at 25°c temperature with an effective noise bandwidth of 10 KHz.

2) A receiver has a noise power bandwidth of 12 KHz. A resistor which matches with the receiver I/p Impedance is connected across the antenna terminals. What is the noise power contributed by this resistor in the receiver bandwidth? Assume temperature to be 30°c.

Equivalent Noise Source for Thermal Noise

Sol :- Given : $R = 20\,K\Omega$, $T = 273 + 25 = 298°K$, $B_N = 10\,KHz$, $K = 1.38 \times 10^{-23}$

* $V_{TN} = \sqrt{4KTB_NR} = \sqrt{4 \times 1.38 \times 10^{-23} \times 298 \times 10 \times 10^{3} \times 20 \times 10^{3}}$

$$\boxed{V_{TN} = 1.81\,\mu V}$$

* $P_n = KTB_N = 1.38 \times 10^{-23} \times 298 \times 10 \times 10^{3}$

$$\boxed{P_n = 4.11 \times 10^{-17}}$$

Sol :- Given : $B_N = 12\,KHz$, $T = 30°c + 273 = 303°K$, $K = 1.38 \times 10^{-23}$

$P_n = KTB_N = 1.38 \times 10^{-23} \times 303 \times 12 \times 10^{3}$

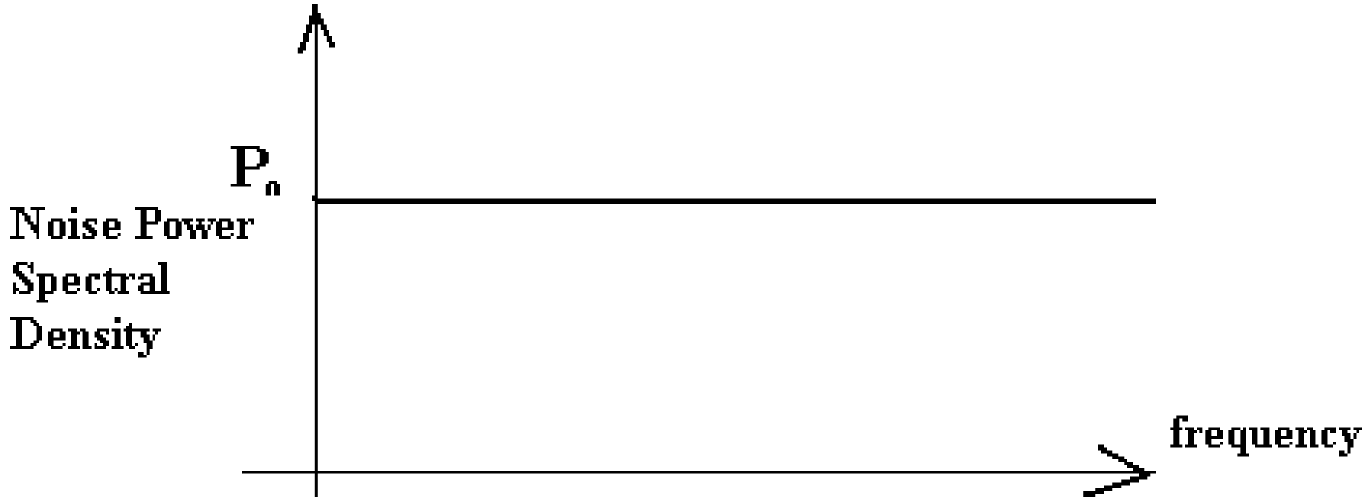
$$\boxed{P_n = 5.01768 \times 10^{-17}\,W}$$

2. Thermal Noise (Johnson Noise) (Cont'd)

The law relating noise power, P_N, to the temperature and bandwidth is

$$P_N = \mathbf{k\ TB\ watts}$$

Thermal noise is often referred to as 'white noise' because it has a uniform 'spectral density'.



Uniform Noise Power Spectral Density

2. Thermal Noise (Johnson Noise) (Cont'd)

Noise power in decibels:

Signal power is often measured in dBm (decibels relative to 1 milliwatt, assuming a 50 ohm load). From the equation above, noise power in a resistor at room temperature, in dBm, is then:

$$P_{\mathrm{dBm}} = 10\ \log_{10}(k_B T \Delta f \times 1000)$$

where the factor of 1000 is present because the power is given in milliwatts, rather than watts. This equation can be simplified by separating the constant parts from the bandwidth:

$$P_{\mathrm{dBm}} = 10\ \log_{10}(k_B T \times 1000) + 10\ \log(\Delta f)$$

which is more commonly seen approximated as:

$$P_{\mathrm{dBm}} = -174 + 10\ \log_{10}(\Delta f)$$

2. Thermal Noise (Johnson Noise) (Cont'd)

Noise power at different bandwidths is then simple to calculate:

Bandwidth (Δf)	Thermal noise power	Notes
1 Hz	−174 dBm	
10 Hz	−164 dBm	
100 Hz	−154 dBm	
1 kHz	−144 dBm	
10 kHz	−134 dBm	FM channel of 2-way radio
100 kHz	−124 dBm	
180 kHz	−121.45 dBm	One LTE resource block
200 kHz	−120.98 dBm	One GSM channel (ARFCN)
1 MHz	−114 dBm	
2 MHz	−111 dBm	Commercial GPS channel
6 MHz	−106 dBm	Analog television channel
20 MHz	−101 dBm	WLAN 802.11 channel

3. Shot Noise

• Shot noise was originally used to describe noise due to random fluctuations in electron emission from cathodes in vacuum tubes (called shot noise by analogy with lead shot).
• Shot noise also occurs in semiconductors due to the liberation of charge carriers.
• For *pn* junctions the mean square shot noise current is

$$I_n^2 = 2\left(I_{DC} + 2I_o\right) q_e B \qquad (amps)^2$$

Where

T_{DC} is the direct current as the *pn* junction (amps)
I_o is the reverse saturation current (amps)
q_e is the electron charge = 1.6 x 10^{-19} coulombs
B is the effective noise bandwidth (Hz)

• Shot noise is found to have a uniform spectral density as for thermal noise

4. Low Frequency or Flicker Noise

Active devices, integrated circuit, diodes, transistors etc also exhibits a low frequency noise, which is frequency dependent (i.e. non uniform) known as flicker noise or ‘one – over – f’ noise.

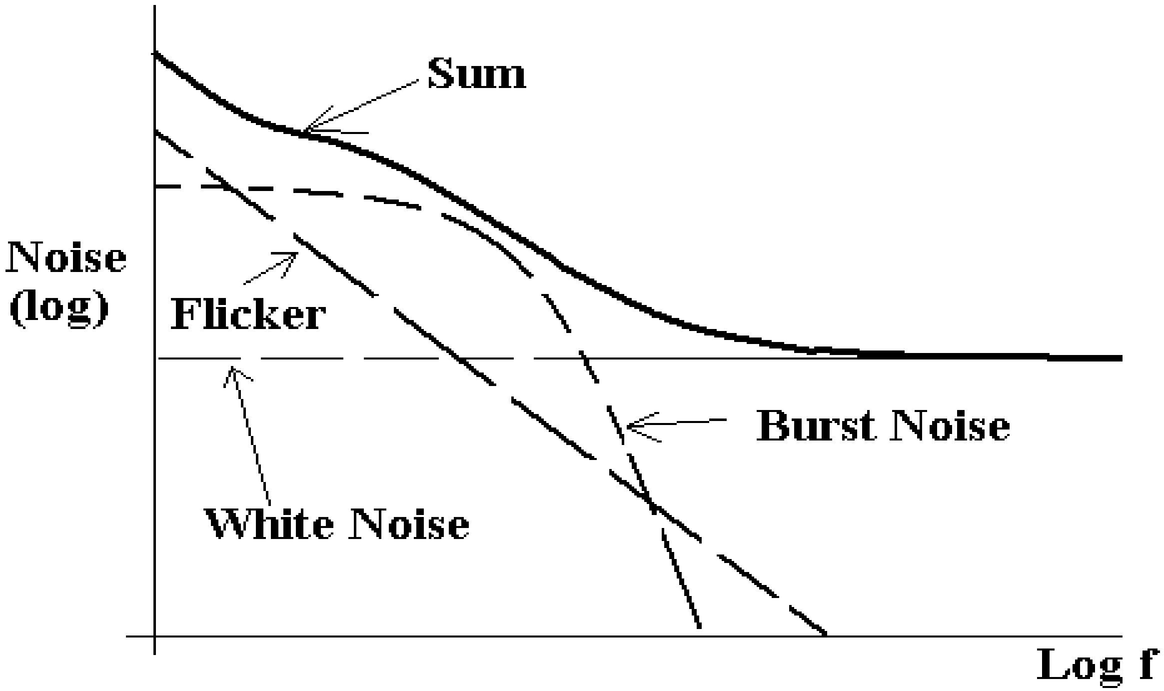
5. Excess Resistor Noise

Thermal noise in resistors does not vary with frequency, as previously noted, by many resistors also generates as additional frequency dependent noise referred to as excess noise.

6. Burst Noise or Popcorn Noise

Some semiconductors also produce burst or popcorn noise with a spectral density which is proportional to $\left(\frac{1}{f}\right)^2$

7. General Comments



For frequencies below a few KHz (low frequency systems), flicker and popcorn noise are the most significant, but these may be ignored at higher frequencies where 'white' noise predominates.

8. Noise Evaluation

The essence of calculations and measurements is to determine the signal power to Noise power ratio, i.e. the (S/N) ratio or (S/N) expression in dB.

$$\left(\frac{S}{N}\right)_{ratio} = \frac{S}{N}$$

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

Also recall that

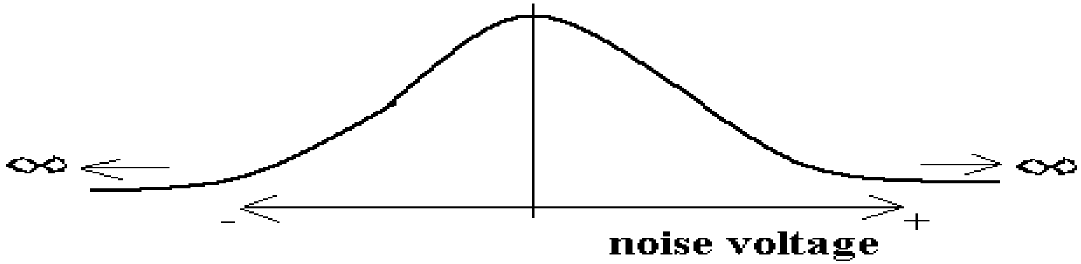
$$S_{dBm} = 10\log_{10}\left(\frac{S(mW)}{1mW}\right)$$

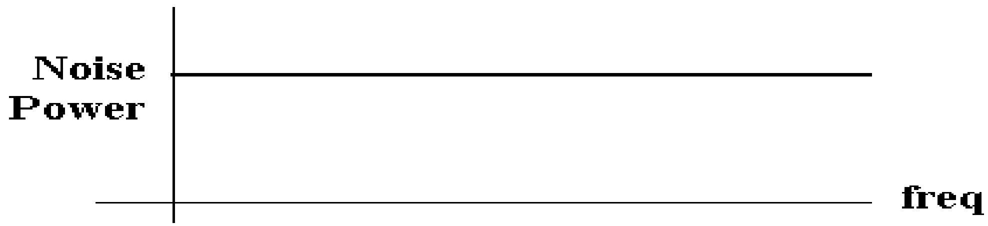
$$and\ \ N_{dBm} = 10\log_{10}\left(\frac{N(mW)}{1mW}\right)$$

$$i.e.\left(\frac{S}{N}\right)_{dB} = 10\log_{10} S - 10\log_{10} N$$

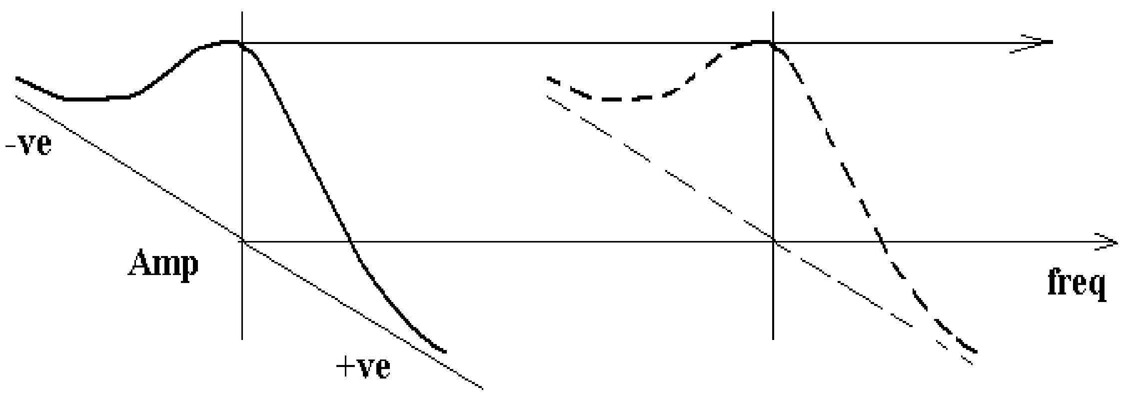
$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$

8. Noise Evaluation (Cont'd)





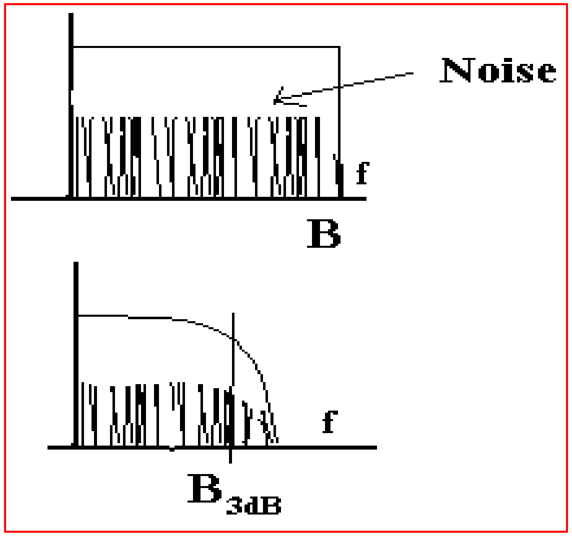
The probability of amplitude of noise at any frequency or in any band of frequencies (e.g. 1 Hz, 10Hz… 100 KHz .etc) is a Gaussian distribution.



8. Noise Bandwidth

Noise may be quantified in terms of noise power spectral density, p_o watts per Hz, from which Noise power N may be expressed as

$$N = p_o B_n \text{ watts}$$



Ideal low pass filter

Bandwidth B Hz = B_n

$N = p_o B_n$ watts

Practical LPF

3 dB bandwidth shown, but noise does not suddenly cease at B_{3dB}

Therefore, $B_n > B_{3dB}$, B_n depends on actual filter.

$N = p_0 B_n$

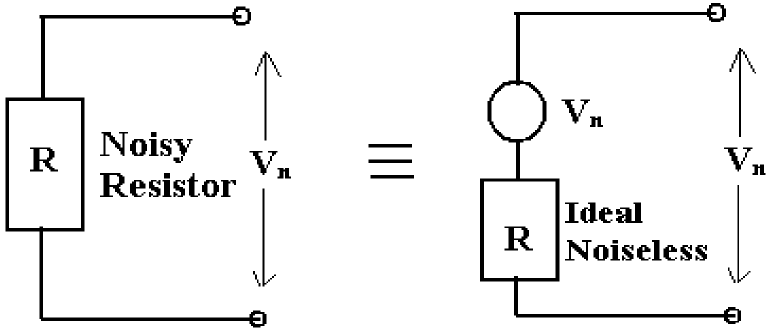
In general the equivalent noise bandwidth is $> B_{3dB}$.

$$For\ \mathrm{RC}\ LPF,\ B_n = \frac{\pi f_c}{2}$$

9. Analysis of Noise In Communication Systems

Thermal Noise (Johnson noise)

This thermal noise may be represented by an equivalent circuit as shown below

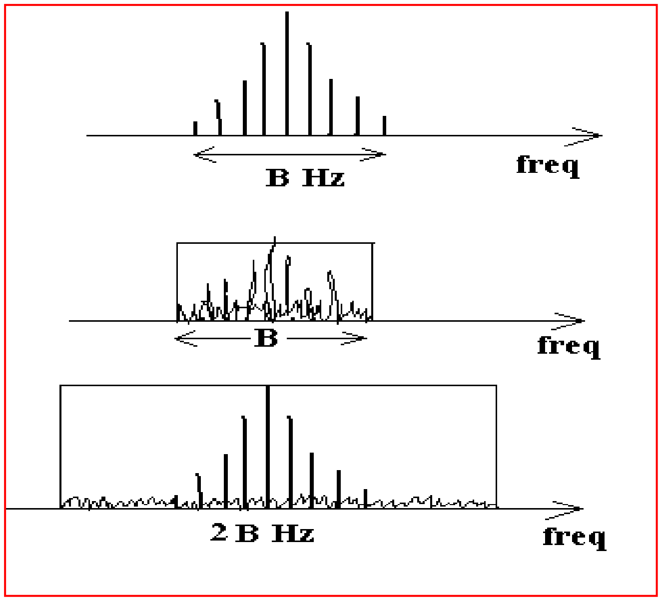


$$\overline{V^2} = 4kTBR\ (volt^2)$$

(mean square value , power)

then $V_{RMS} = \sqrt{\overline{V^2}} = 2\sqrt{kTBR} = V_n$

i.e. V_n is the RMS noise voltage.



A) System BW = B Hz

N= Constant B (watts) = KB

B) System BW

N= Constant 2B (watts) = K2B

For A, $\frac{S}{N} = \frac{S}{KB}$

For B, $\frac{S}{N} = \frac{S}{K2B}$

9. Analysis of Noise In Communication Systems (Cont'd)

Resistors in Series

Assume that R_1 at temperature T_1 and R_2 at temperature T_2, then

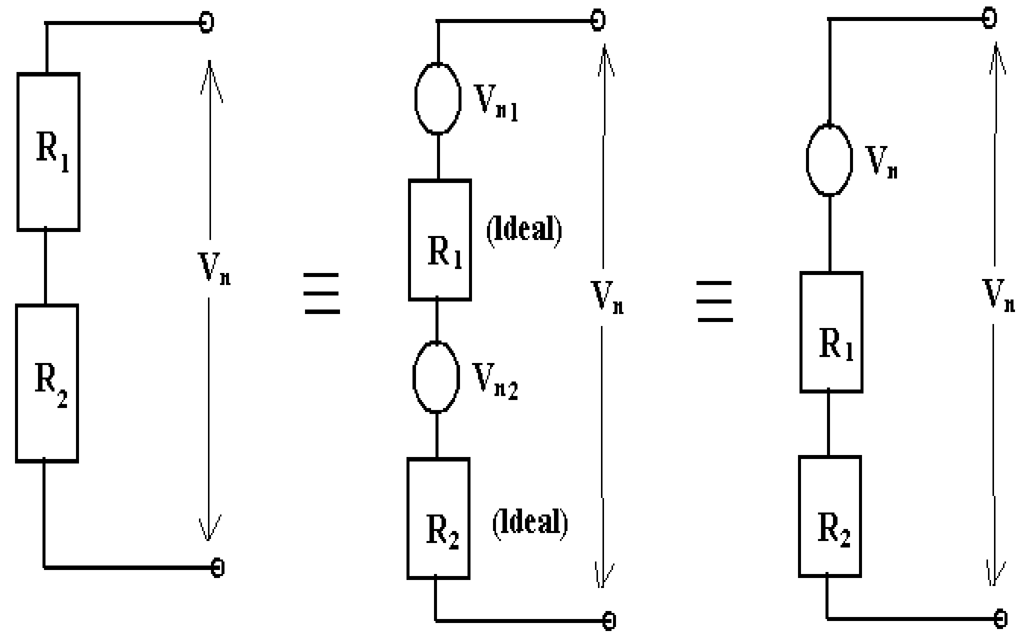
$$\overline{V_n^{\,2}} = \overline{V_{n1}}^{\,2} + \overline{V_{n2}}^{\,2}$$

$$\overline{V_{n1}^{\,2}} = 4kT_1BR_1$$

$$\overline{V_{n2}^{\,2}} = 4kT_2BR_2$$

$$\therefore\ \overline{V_n^{\,2}} = 4kB(T_1R_1 + T_2R_2)$$

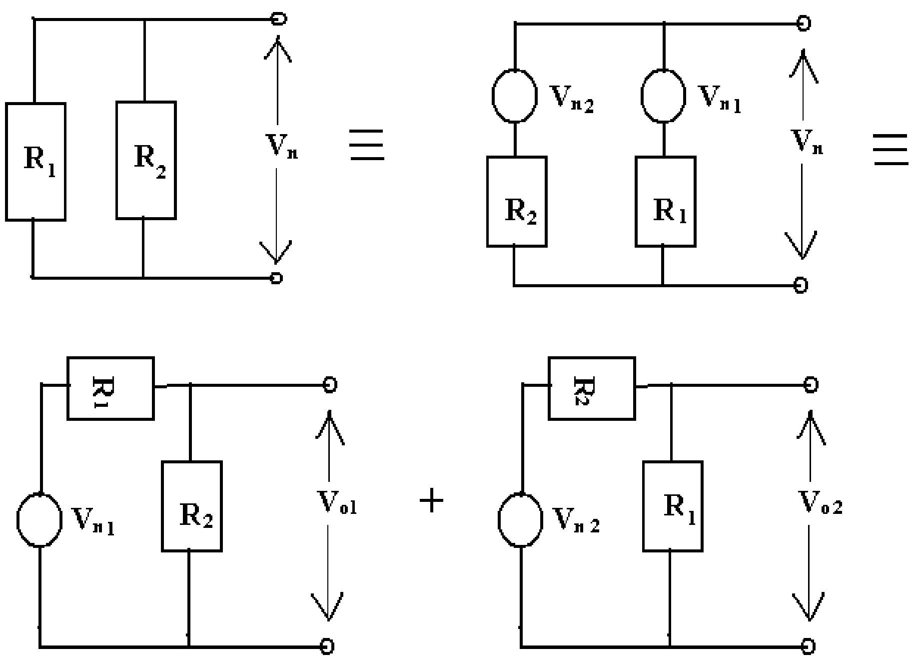
$$\overline{V_n^{\,2}} = 4kTB(R_1 + R_2)$$



i.e. The resistor in series at same temperature behave as a single resistor

9. Analysis of Noise In Communication Systems (Cont'd)

Resistance in Parallel



$$V_{o1} = V_{n1}\frac{R_2}{R_1+R_2} \qquad V_{o2} = V_{n2}\frac{R_1}{R_1+R_2}$$

$$\overline{V_n^{\,2}} = \overline{V_{o1}^{\,2}} + \overline{V_{o2}^{\,2}}$$

$$\overline{V_n^{\,2}} = \frac{4kB}{(R_1+R_2)^2}\left[R_2^2\,T_1R_1 + R_1^2\,T_2R_2\right]$$

$$\overline{V_n^{\,2}} = \frac{4kB\,R_1R_2(T_1R_1+T_2R_2)}{(R_1+R_2)^2}$$

$$\overline{V_n^{\,2}} = 4kTB\left(\frac{R_1R_2}{R_1+R_2}\right)$$

Noise in Communication Systems

3) A 600 Ω resistor is connected across the 600 Ω antenna input of a radio receiver. The bandwidth of the radio receiver is 20 KHZ & the resistor is at room temperature of 27°c. Calculate the noise power & the noise voltage applied at the I/p of the receiver.

Sol:- Given : $R_1 = 600\,\Omega$, $R_2 = 600\,\Omega$, $B_N = 20\times10^3$ Hz, $T = 27+273 = 300\,°K$.

$P_n = ?$, $V_{TN} = ?$, $K = 1.38\times10^{-23}$

* Noise power : $P_n = KTB_N = 1.38\times10^{-23} \times 300 \times 20\times10^3$

$$\boxed{P_n = 8.28\times10^{-17}\,W}$$

* Noise voltage at the receiver I/p :

Since the two resistors are in parallel

* $R = R_1 \| R_2 = 600\,\Omega \| 600\,\Omega = 300\,\Omega$.

$\therefore$ Noise voltage $V_{TN} = \sqrt{4KTB_N R} = \sqrt{4\times1.38\times10^{-23} \times 300\times20\times10^3 \times 300}$

$$\boxed{V_{TN} = 0.3152\,\mu V}$$

Noise in Communication Systems

1) Three 5 kΩ resistors are connected in series. For room temperature $KT = 4\times10^{-21}$ & an effective noise bandwidth of 1 MHz, determine

a) The noise voltage appearing across each resistor.

b) The noise voltage appearing across the series combination.

c) What is the rms noise voltage which appears across same three resistors connected in parallel under the same conditions?

Noise in Communication Systems

Sol :- Given : $R_1 = R_2 = R_3 = 5K\Omega$, $KT = 4 \times 10^{-21}$, $B_N = 1MHz$.

a) $V_{TN} = \sqrt{4KTB_N R} = \sqrt{4 \times 4 \times 10^{-21} \times 1 \times 10^6 \times 5 \times 10^3}$

$$\boxed{V_{TN} = 8.94\ \mu V}$$

b) $R_{ser} = R_1 + R_2 + R_3 = 5K\Omega + 5K\Omega + 5K\Omega = 15K\Omega$.

$V_{TN} = \sqrt{4KTB_N R_{ser}} = \sqrt{4 \times 4 \times 10^{-21} \times 1 \times 10^6 \times 5 \times 10^3}$

$$\boxed{V_{TN} = 15.5\ \mu V}$$

Noise in Communication Systems

c) $R_p = R_1 \parallel R_2 \parallel R_3$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{5k\Omega} + \frac{1}{5k\Omega} + \frac{1}{5k\Omega} = \frac{1}{6\times10^{-4}}$$

$$\boxed{R_p = 1.66\ k\Omega}$$

$$V_{TN} = \sqrt{4KTB_N R_p} = \sqrt{4\times4\times10^{-21}\times1\times10^{6}\times1.66\times10^{3}}$$

$$\boxed{V_{TN} = 5.15\ \mu V}$$

Signal to Noise Ratio

The signal to noise ratio is given by

$$\frac{S}{N} = \frac{Signal\,Power}{Noise\,Power}$$

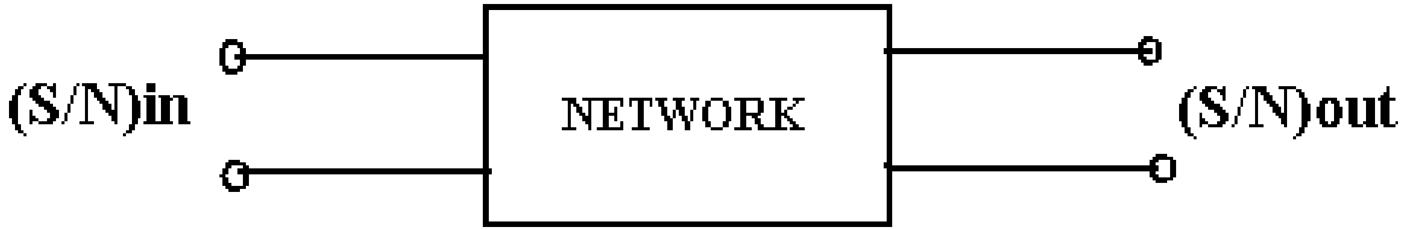
The signal to noise in dB is expressed by

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$ for S and N measured in mW.

Noise Factor- Noise Figure

Consider the network shown below,



Noise Factor- Noise Figure (Cont'd)

• The amount of noise added by the network is embodied in the Noise Factor F, which is defined by

$$\text{Noise factor F} = \frac{\left(S/N\right)_{IN}}{\left(S/N\right)_{OUT}}$$

• F equals to 1 for noiseless network and in general F > 1. The noise figure in the noise factor quoted in dB

i.e. Noise Figure F dB = 10 log10 F $\quad F \geq 0$ dB

• The noise figure / factor is the measure of how much a network degrades the $(S/N)_{IN}$, the lower the value of F, the better the network.

Noise Factor - Noise Figure

Noise Factor :-

* The Noise factor 'F' of an amplifier or any Network is defined interms of Signal to Noise ratio is defined as:

$$\text{Noise factor, } F = \frac{\text{available S/N power ratio at the I/p}}{\text{available S/N power ratio at the O/p}} = \frac{(SNR)_i}{(SNR)_o}$$

$$F = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}}$$

$$\boxed{F = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}} \longrightarrow (1)$$

{ * In general any amplifier will add Noise to the I/p Signal, therefore the SNR at the O/p of the amplifier is less than the SNR at the I/p. Hence the Noise factor is a measure of degradation of the Signal to Noise ratio or the amount of noise added by the S/M }

Noise Factor - Noise Figure

* The available power gain 'G' is given by

$$G = \frac{\text{Signal power at the o/p}}{\text{Signal power at the I/p}}$$

$$\boxed{G = \frac{P_{so}}{P_{si}}} \longrightarrow (2)$$

From eq (1), we can re-arrange

$$F = \left(\frac{P_{si}}{P_{so}}\right) \times \frac{P_{no}}{P_{ni}} \longrightarrow (3)$$

Substituting eq (2) in eq (3), we get

$$F = \frac{1}{G} \cdot \frac{P_{no}}{P_{ni}}$$

$$F = \frac{P_{no}}{G P_{ni}}$$

$$P_{no} = F G P_{ni}$$

W.K.T the Noise power at I/p, $P_{ni} = KTB_N$

$$\boxed{P_{no} = FGKTB_N}$$

Thus with increase in the Noise factor 'F', the noise power at the o/p will increase.

Noise Factor - Noise Figure

NOISE Figure :-

* When noise factor is expressed in decibels, it is called Noise Figure.

$$\text{Noise Figure} = 10 \log_{10}(F)$$

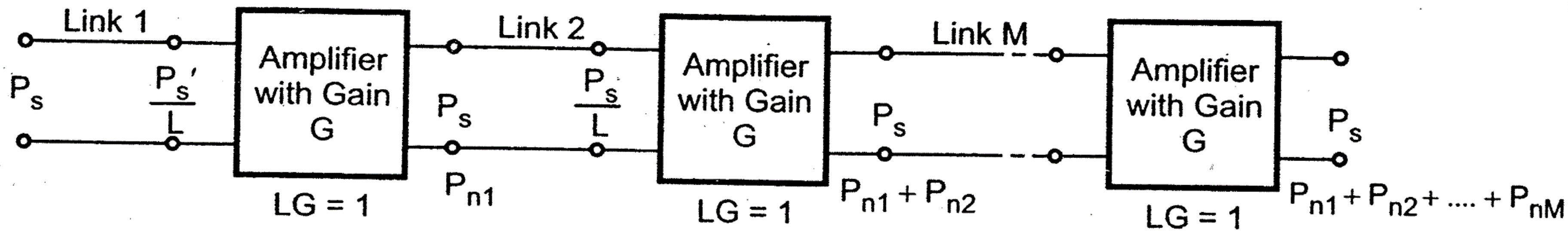
$$= 10 \log_{10}\left[\frac{\text{S/N at the I/p "}(S/N)_i\text{"}}{\text{S/N at the o/p "}(S/N)_o\text{"}}\right]$$

$$= 10 \log_{10}\left[\frac{(S/N)_i}{(S/N)_o}\right]$$

$$\boxed{\text{Noise Figure } (F)_{dB} = 10 \log_{10}(S/N)_i - 10 \log_{10}(S/N)_o}$$

* The Ideal value of Noise figure is 0 dB.

Signal to Noise ratio of Tandem Connection



* In telephone Systems, telephone cables are used as media to transmit Signals. The Signals gets attenuated as it travels through telephone cables due to power loss in the telephone cables. To make up this power loss the Signal is amplified Such that, if the power loss of a line Section is 'L', then the amplified power gain 'G' is chosen So that LG = 1.

* A long telephone line is divided into equal Sections called links.

* As Signals travel through these links, each amplifier adds its own noise to the System.

Therefore at the receiving end we get the accumulated Noise power as shown in fig above.

Signal to Noise ratio of Tandem Connection

* The total Noise power at the o/p of the M^{th} link is

$$P_n = P_{n1} + P_{n2} + P_{n3} + \dots + P_{nM}$$

Where,

P_{n1} = Noise power added at the end of 1^{st} link

P_{n2} = Noise power added at the end of 2^{nd} link.

P_{n3} = Noise power added at the end of 3^{rd} link.

⋮

P_{nM} = Noise power added at the end of M^{th} link.

* If links are identical such that each link adds Noise power 'P_n' then the total Noise power is given as:

$$\boxed{P_{n\,total} = M \times P_n}$$

Signal to Noise ratio of Tandem Connection

∴ The o/p Signal to Noise ratio is :

$$\left(\frac{S}{N}\right)_{M\,dB} = 10\log\left(\frac{P_s}{P_{n\text{-}total}}\right)$$

$$= 10\log\left(\frac{P_s}{MP_n}\right)$$

$$= 10\log\left(\frac{P_s}{P_n}\right) - 10\log(M)$$

$$\boxed{\left(\frac{S}{N}\right)_{M\,dB} = \left(\frac{S}{N}\right)_{1\,dB} - (M)_{dB}}$$

Where,

$(M)_{dB} \rightarrow$ Signal to Noise ratio at the end of M- links

$\left(\frac{S}{N}\right)_{1\,dB} \rightarrow$ Signal to Noise ratio at the end of 1^{st} link.

1) The Signal power & Noise power measured at the I/p of an amplifier are 150 μw & 1.5 μw respectively. If the Signal power at the o/p 1.5W & Noise power is 40 mW, calculate the amplifier noise factor & Noise figure.

Sol:- Given : $P_{Si} = 150\,\mu w$, $P_{ni} = 1.5\,\mu w$, $P_{So} = 1.5W$, $P_{no} = 40\,mw$.

* Noise Factor 'F' $= \dfrac{P_{Si}}{P_{ni}} \times \dfrac{P_{no}}{P_{So}}$

$$= \frac{150 \times 10^{-6}}{1.5 \times 10^{-6}} \times \frac{40 \times 10^{-3}}{1.5}$$

$$\boxed{F = 2.666}$$

* Noise Figure $(F)_{dB} = 10 \log_{10}(F) = 10 \log_{10}(2.666)$

$$\boxed{(F)_{dB} = 4.26\,dB}$$

2) The Signal to Noise ratio at the I/p of an amplifier is 40 dB. If the Noise Figure of an amplifier is 20 dB, calculate the Signal to Noise ratio in dB at the amplifier o/p.

Sol :- Given : $(S/N)_i = 40\,dB$, $(S/N)_o = ?$, $(S/N)_o = 20\,dB$

W.K.T

$$\text{Noise Figure } (F)_{dB} = (S/N)_{i\,dB} - (S/N)_{o\,dB}$$

$$(S/N)_{o\,dB} = (S/N)_{i\,dB} - (F)_{dB}$$

$$= 40\,dB - 20\,dB$$

$$\boxed{(S/N)_{o\,dB} = 20\,dB}$$

Equivalent Noise Temperature

N_{IN} is the 'external' noise from the source i.e. $N_{IN} = \mathrm{k}T_S\ B_n$

T_S is the equivalent noise temperature of the source (usually 290K).

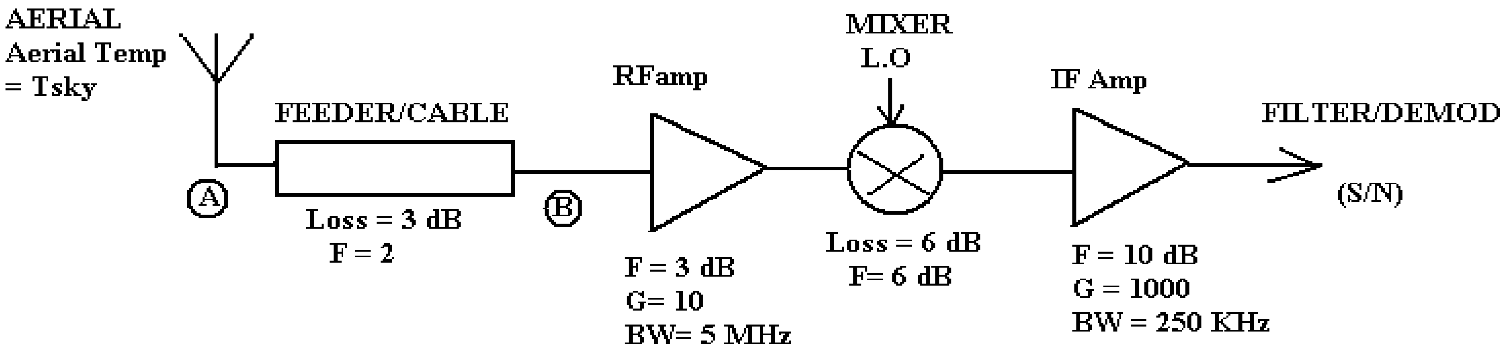
We may also write $N_e = \mathrm{k}T_e\ B_n$, where T_e is the equivalent noise temperature of the element i.e. with noise factor F and with source temperature T_S.

$$\text{i.e. } \mathrm{k}T_e\ B_n = (\mathrm{F}-1)\ \mathrm{k}T_S\ B_n$$

$$\text{or } T_e = (\mathrm{F}-1)T_S$$

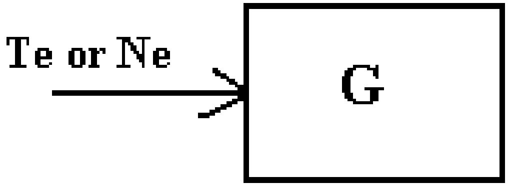
Cascaded Network

❖ A receiver systems usually consists of a number of passive or active elements connected in series. A typical receiver block diagram is shown below, with example



❖ In order to determine the (S/N) at the input, the overall receiver noise figure or noise temperature must be determined.

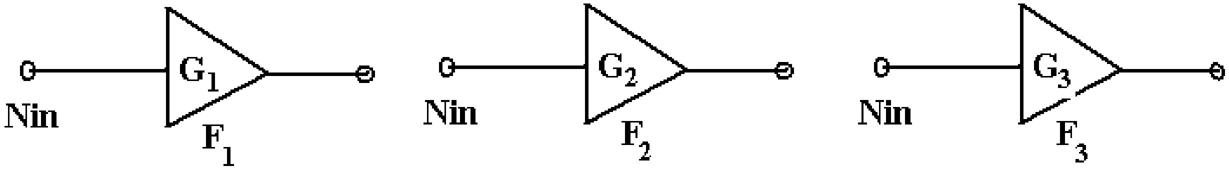
❖ In order to do this all the noise must be referred to the same point in the receiver, for example to A, the feeder input or B, the input to the first amplifier.



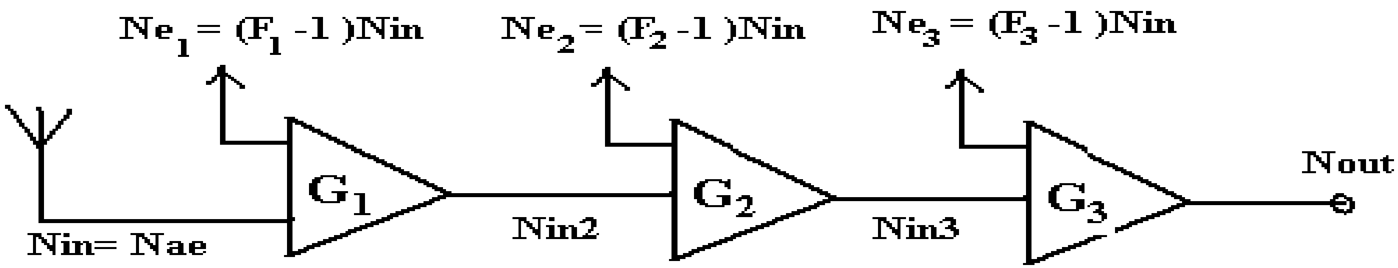
T_e or N_e is the noise referred to the input.

System Noise Figure

Assume that a system comprises the elements shown below,



Assume that these are now cascaded and connected to an aerial at the input, with $N_{IN} = N_{ae}$ from the aerial.



Now,
$$N_{OUT} = G_3\left(N_{IN3} + N_{e3}\right)$$
$$= G_3\left(N_{IN3} + \left(F_3 - 1\right)N_{IN}\right)$$

Since
$$N_{IN3} = G_2\left(N_{IN2} + N_{e2}\right) = G_2\left(N_{IN2} + \left(F_2 - 1\right)N_{IN}\right)$$

similarly
$$N_{IN2} = G_1\left(N_{ae} + \left(F_1 - 1\right)N_{IN}\right)$$

System Noise Figure (Cont'd)

$$N_{OUT} = G_3\left[G_2\left[G_1 N_{ae} + G_1\left(F_1 - 1\right)N_{IN}\right] + G_2\left(F_2 - 1\right)N_{IN}\right] + G_3\left(F_3 - 1\right)N_{IN}$$

The overall system Noise Factor is

$$F_{sys} = \frac{N_{OUT}}{GN_{IN}} = \frac{N_{OUT}}{G_1 G_2 G_3 N_{ae}}$$

$$= 1 + \left(F_1 - 1\right)\frac{N_{IN}}{N_{ae}} + \frac{\left(F_2 - 1\right)}{G_1}\frac{N_{IN}}{N_{ae}} + \frac{\left(F_3 - 1\right)}{G_1 G_2}\frac{N_{IN}}{N_{ae}}$$

If we assume N_{ae} is $\approx N_{IN}$, i.e. we would measure and specify F_{sys} under similar conditions as F_1, F_2 etc (i.e. at 290 K), then for n elements in cascade.

$$F_{sys} = F_1 + \frac{\left(F_2 - 1\right)}{G_1} + \frac{\left(F_3 - 1\right)}{G_1 G_2} + \frac{\left(F_4 - 1\right)}{G_1 G_2 G_3} + \ldots\ldots\ldots\ldots + \frac{\left(F_n - 1\right)}{G_1 G_2 \ldots\ldots\ldots G_{n-1}}$$

The equation is called FRIIS Formula.

System Noise Temperature

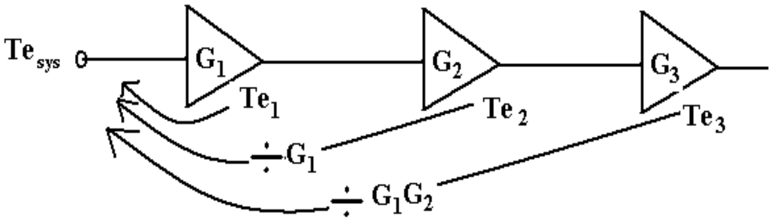
Since $T_e = (L-1)T_S$, i.e. $F = 1 + \frac{T_e}{T_s}$

L – Insertion loss of the Network

Then T$_e$ – Equivalent noise temperature

Since

F = L



$$F_{sys} = 1 + \frac{T_{e\,sys}}{T_s}$$

$$\begin{cases} where\, T_{e\ sys}\ is\ the\ equivalent\ Noise\ temperature\ of\ the\ system \\ and\ T_s\ is\ the\ noise\ temperature\ of\ the\ source \end{cases}$$

and

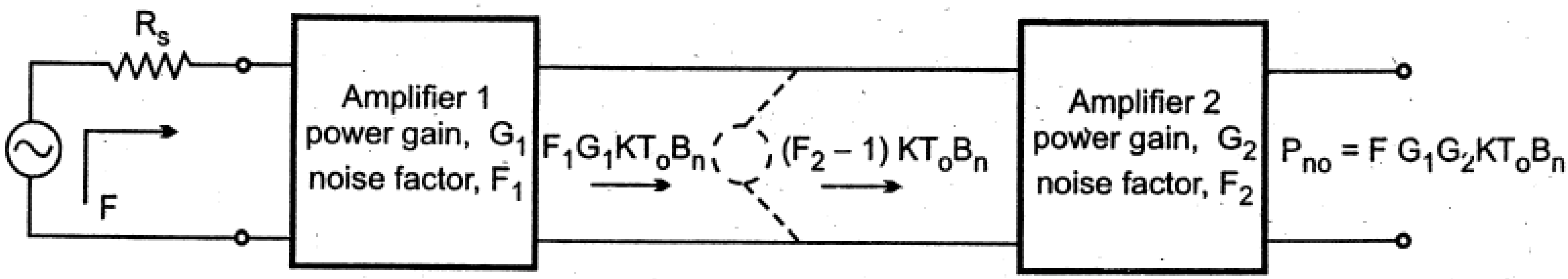
$$\left(1 + \frac{T_{e\,sys}}{T_s}\right) = \left(1 + \frac{T_{e1}}{T_s}\right) + \frac{\left(1 + \frac{T_{e2}}{T_s} - 1\right)}{G_1} + \ldots etc$$

$$i.e.\ from\, F_{sys} = F_1 + \frac{(F_2 - 1)}{G_1} + \ldots\ldots etc$$

$$which\ gives$$

$$T_{e\,sys} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \frac{T_{e4}}{G_1G_2G_3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Noise Factor of two Amplifier in Cascade



* Consider two amplifier connected in cascade as shown above. The available Noise power at the o/p of 1st amplifier is

$$\boxed{P_{no1} = F_1 G_1 K T_o B_N} \longrightarrow (1)$$

* This is available to the 2nd amplifier & 2nd amplifier has noise $(F_2-1)KTB_N$ of its own at its I/p of the 2nd amplifier is

$$\boxed{P_{ni2} = F_1 G_1 KTB_N + (F_2-1)\,KTB_N} \longrightarrow (2)$$

Noise Factor of two Amplifier in Cascade

* Consider 2^{nd} amplifier as a Noiseless amplifier with amplifier gain 'G_2'

We have

$$\boxed{P_{no2} = G_2 P_{ni2}} \longrightarrow (3)$$

Substituting eq (2) in eq (3), we get

$$P_{no2} = G_2 \left[F_1 G_1 KTB_N + (F_2 - 1) KTB_N \right] \longrightarrow (4)$$

* WKT, the overall voltage gain of the two amplifiers in cascade is

$$G = G_1 G_2 \quad \&$$

* From figure, the overall Noise power is

$$\boxed{P_{no} = F G_1 G_2 KTB_N} \longrightarrow (5)$$

Noise Factor of two Amplifier in Cascade

* Equating eq ④ & ⑤, we get

$$P_{no} = P_{no2}$$

$$F G_1 G_2 KTB_N = G_2 \left[F_1 G_1 KTB_N + (F_2 - 1) KTB_N \right]$$

$$F = \frac{F_1 G_1 G_2 KTB_N + (F_2 - 1) G_2 KTB_N}{G_1 G_2 KTB_N}$$

$$F = \frac{\cancel{F_1 G_1 G_2 KTB_N}}{\cancel{G_1 G_2 KTB_N}} + \frac{(F_2 - 1) \cancel{G_2 KTB_N}}{G_1 \cancel{G_2 KTB_N}}$$

$$\boxed{F = F_1 + \frac{(F_2 - 1)}{G_1}}$$

By having G_1 large, the noise contribution of the 2^{nd} stage can be made negligible.

Noise due to Several Amplifiers in Cascade

Example 4.3.1 *For the circuit shown in Fig. 4.3.2 calculate the equivalent input noise resistance.*

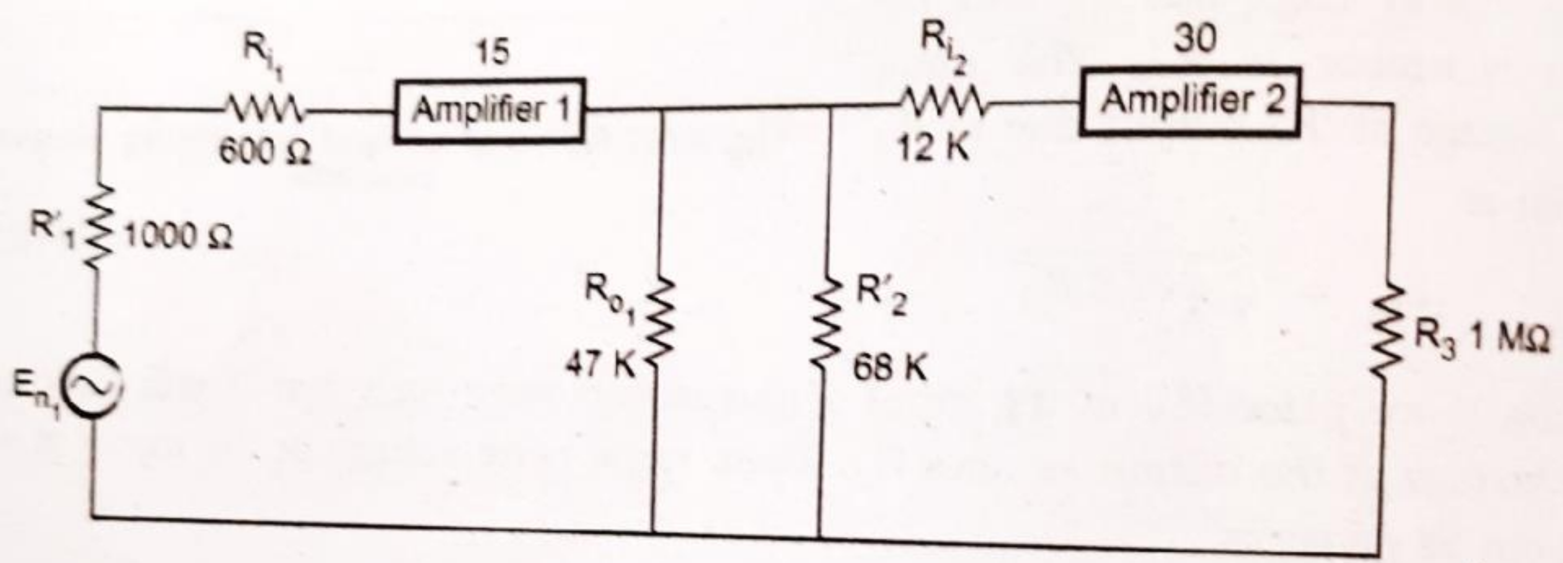


Fig. 4.3.2 Circuit for example 4.3.1

Noise due to Several Amplifiers in Cascade

Example 4.3.1 *For the circuit shown in Fig. 4.3.2 calculate the equivalent input noise resistance.*

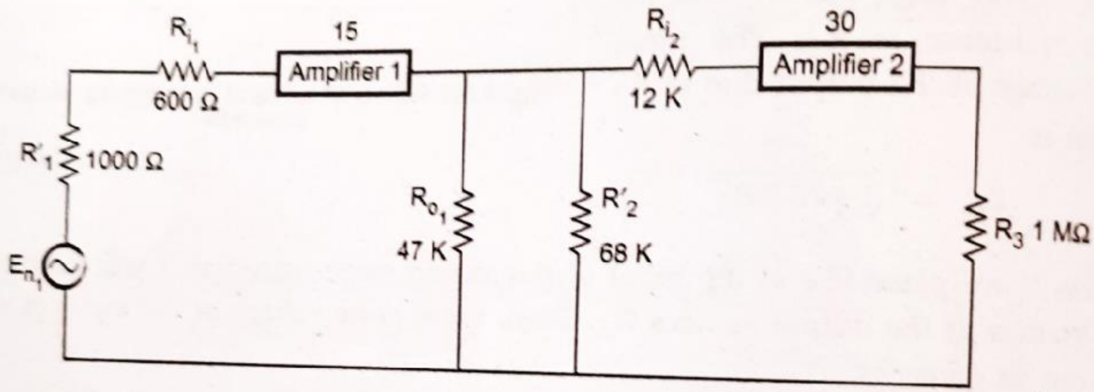


Fig. 4.3.2 Circuit for example 4.3.1

Solution :

$$R_1 = R'_1 + R_{i_1} = 1000 + 600 = 1600\ \Omega$$

$$R_2 = R'_2 \| R_{o_1} + R_{i_2} = \frac{47 \times 68}{47 + 68} + 12 = 27.79\ \text{k}\Omega$$

We know that,

$$R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2\, A_2^2} \qquad \text{from equation (4.3.1)}$$

$$= 1600 + \frac{27790}{(15)^2} + \frac{1 \times 10^6}{(15)^2\,(30)^2} = 1600 + 123.5 + 4.94$$

$$= 1728.44\ \Omega$$