SASTRA DEEMED UNIVERSITY

(A University under section 3 of the UGC Act, 1956)

End Semester Examinations

Dec 2023

Course Code: **MAT301R01**

Course: **ENGINEERING MATHEMATICS - IV**

QP No. :**UD137-A**

Duration: **3 hours**

Max. Marks:**100**

PART - A

Answer all the questions **10 x 2 = 20 Marks**

1. Find the partial differential equation of all spheres whose centres lie on the z axis.

2. Eliminate f from $z = f(x^2 + y^2 + z^2)$.

3. Find the complete integral of $pq = xy$.

4. Find Fourier sine transform of $\frac{1}{x}$.

5. Find the complex Fourier transform of Dirac delta function $\delta(t-a)$.

6. Find the finite Fourier sine transform of $f(x) = x$ in $(0, l)$.

7. Write the condition of convergence for Gauss-Seidel method.

8. Evaluate $\int_1^2 \frac{dx}{1+x^2}$ taking $h = 0.2$ using Trapezoidal rule.

9. Given $y' = 2xy$ and $y(0) = 1$, determine $y(0.25)$ by Modified Euler method.

10. State Crank-Nicholson difference scheme or method.

PART - B

Answer all the questions **4 x 15 = 60 Marks**

11. a) Solve the equation $z = px + qy + c\sqrt{1 + p^2 + q^2}$ (7)

b) Solve the equation $(x^2 - y^2 - z^2)p + 2xyq = 2zx$ (8)

(OR)

12. a) Solve the equation

$(D^2 + 4DD'^2 - 5D'^2)z = xy + Sin(2x + 3y)$ (7)

b) Solve the equation $\frac{x^2}{p} + \frac{y^2}{q} = z.$ (8)

13. a) Evaluate $\int_0^\infty \frac{\mu^2 d\mu}{(a^2+\mu^2)(b^2+\mu^2)}$ using transforms. (7)

b) Find Fourier sine and cosine transform of x^{n-1}. Deduce that $\frac{1}{\sqrt{x}}$ is self-reciprocal under Fourier sine and cosine transforms. (8)

(OR)

14. Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, for & |x| < a \\ 0, \ for & |x| > a > 0 \end{cases}$

Hence evaluate (i) $\int_0^\infty \frac{\sin t - t\cos t}{t^3} dt$ and (ii) $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt$

15. a) Find the positive root of $x = \cos x$ using Newton-Raphson method. (7)

b) Find the values of $Sin\ 18^\circ$ and $Sin\ 45^\circ$ from the following table, using numerical differentiation based on Newton's forward interpolation formula:

x°	0	10	20	30	40
$Cos\ x^\circ$	1.0000	0.9848	0.9397	0.8660	0.7660

(8)

(OR)

16\. a) Evaluate $I = \int_0^6 \frac{dx}{1+x}$ using (i) Trapezoidal rule (ii) Simpson's rule (both). (7)

b) Find the age corresponding to the annuity value 13.6 given the table. (8)

Age (x)	30	35	40	45	50
Annuity value(y)	15.9	14.9	14.1	13.3	12.5

17\. a) Compute $y(0.2)$ given $\frac{dy}{dx} + y + xy^2 = 0, y(0) = 1$ by taking $h = 0.1$ using R.K method of fourth order (correct to 4 decimals). (7)

b) Solve $\nabla^2 u = 8x^2y^2$ for square mesh given $u = 0$ on the 4 boundaries dividing the square $u = 0$ on the 4 boundaries dividing the square into 16 sub-squares of length 1 unit. (8)

(OR)

18\. a) Using Crank-Nicholson's scheme, solve $u_{xx} = 16u_t$, $0 < x < 1$, $t > 0$ given $u(x,0) = 0$, $u(0,t) = 0$, $u(1,t) = 100\,t$. Compute for one step in t direction taking $h = \frac{1}{4}$. (7)

b) Evaluate the pivotal values of the following equation taking $h = 1$ and upto one half of the period of the oscillation $16u_{xx} = u_{tt}$ given $u(0,t) = u(5,t) = 0$, $u(x,0) = x^2(5-x)$ and $u_t(x,0) = 0$. (8)

PART - C

Answer the following **1 x 20 = 20 Marks**

19\. a) Using Transform method solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi, t > 0$ given $u(0,t) = u(x,t) = 0$ for $t > 0$ and $u(x,0) = Sin^3 x$. (10)

b) Solve $u_{xx} + u_{yy} = 0$ over the square mesh of side 4 units; satisfying the following boundary conditions:

i) $u(0, y) = 0$ for $0 \leq y \leq 4$

ii) $u(4, y) = 12 + y$ for $0 \leq y \leq 4$

iii) $u(x, 0) = 3x$ for $0 \leq x \leq 4$

iv) $u(x, 4) = x^2$ for $0 \leq x \leq 4$ (10)

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