SASTRA DEEMED UNIVERSITY

(A University under section 3 of the UGC Act, 1956)

End Semester Examinations

May 2024

Course Code: **MAT309**

Course: **APPLIED MATHEMATICAL METHODS**

QP No. :**U027-4**

Duration: **3** hours

Max. Marks:**100**

PART – A

Answer all the questions **10 x 2 = 20 Marks**

1. Form the PDE by eliminating the arbitrary constants a and b from $z = ae^y + b\, log x$.

2. Solve $\frac{p}{x^2} + \frac{q}{y^2} = 1$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

3. Write any two solutions of the Laplace's equation $u_{xx} + u_{yy} = 0$ involving exponential terms in x or y by the method of separation of variables.

4. Find $f(x)$ if its sine transform is e^{-as}.

5. Find the finite Fourier sine transform of $f(x) = x$ in $(0, \pi)$.

6. Find the solution of linear population growth model $\frac{dy}{dx} = 1 - y$, $y(0) = 0$ for $x = 0.1$ by Euler's method.

7. Which is better – Taylor's series method or Runge-Kutta method? Why?

8. Classify the equation $x^2 f_{xx} + (1 - y^2) f_{yy} = 0$,

$\ldots, -1 < y < 1$.

9. Write the finite difference scheme for the **Poisson's equation** $\nabla^2 u = f(x, y)$.

10. Write the normal equations to fit a quadratic **curve by least square** method.

PART – B

Answer all the questions **4 x 15 = 60 Marks**

11. a) Form the PDE by eliminating the arbitrary functions f and g from $z = f(3x - y) + g(3x + y)$. (7)

b) Find the integral surface of the linear PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0,\ z = 1$. (8)

(OR)

12. a) Solve $z = px + qy + p^2 + pq + q^2$. (7)
b) Solve $(D^2 - DD' - 2D'^2) = (y - 1)e^x$. (8)

13. a) Find the Fourier transform of $e^{-a^2x^2}$. Hence prove that $e^{-\frac{x^2}{2}}$ is self-reciprocal with respect to Fourier transforms. (7)

b) Use transforms method to evaluate $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$. (8)

(OR)

14. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi,\ t > 0$ given that $u(0, t) = u(\pi, t) = 0$ for $t > 0$ and $u(x, 0) = sin^3 x$ by the finite Fourier transform method.

15. Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$.

Find,

a) $y(0.1)$ by Taylor's method

b) $y(0.2)$ by Euler's method

c) $y(0.3)$ by fourth order R-K method

d) $y(0.4)$ by Milne's method.

(OR)

16. Using the finite difference method, find $y(0.25), y(0.5)$ and $y(0.75)$ satisfying the differential equation $\frac{d^2y}{dx^2} + y = x$ subject to the boundary conditions $y(0) = 0$, $y(1) = 2$.

17. Solve $u_{xx} + u_{yy} = 0$ over the square mesh of side 4 units satisfying the following boundary conditions using Liebmann's iteration process.

a) $u(0, y) = 0$ for $0 \leq y \leq 4$

b) $u(4, y) = 12 + y$ for $0 \leq y \leq 4$

c) $u(x, 0) = 3x$ for $0 \leq x \leq 4$

d) $u(x, 4) = x^2$ for $0 \leq x \leq 4$.

(OR)

18. a) Solve $25u_{xx} - u_{tt} = 0$ for u at the pivotal points, given $u(0, t) = u(5, t) = 0, u_t(x, 0) = 0$

$$and\ u(x,0) = \begin{cases} 2x & for\ \ 0 \leq x \leq 2.5 \\ 10 - 2x & for\ \ 2.5 \leq x \leq 5 \end{cases}$$

for one half period of vibration by the finite difference method. (7)

b) The pressure and volume of a gas are related by the equation $pv^{\lambda} = k$, (λ and k are constants). Fit this equation for the following data using the principle of least squares. (8)

p	0.5	1.0	1.5	2.0	2.5	3.0
v	1.62	1.00	0.75	0.62	0.52	0.46

PART – C

Answer the following **1 x 20 = 20 Marks**

19. a) Derive the various possible solutions of one-dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. (10)

b) Find $F_c[x^{n-1}]$ and $F_s[x^{n-1}]$, $0 < n < 1$. Also, prove that $\frac{1}{\sqrt{x}}$ is self-reciprocal under both the transforms. (10)

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