School of Chemical and Biotechnology
SASTRA Deemed to Be University
CIA – II
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Degree: B.Tech. /M.Tech.(Int.) Branch: BT/BE/BIN/CHEM/MBT/MNT
Year: II Sem: IV
Course code: MAT309
Course name: APPLIED MATHEMATICAL METHODS
Max Marks: 50 Duration: 1 ½ hr

Part A: Answer all the questions [5 x 2 =10 Marks]

1. State the change of scale property of Fourier transform.
2. Define self-reciprocal function with respect to Fourier cosine transform and give an example.
3. Find the Fourier sine transform of $f(x) = x$ in $(0, 2)$.
4. Use Taylor's series method to find y(0.1) given that $y' = 1 - y,\ y(0) = 0$.
5. Write the convergence condition of Picard's iteration method.

Part B: Answer any FOUR questions [4 x 10 = 40 Marks]

6. Apply the Fourier transform to convert a signal expressed in terms of the parabolic function $f(x) = \begin{cases} 4 - x^2; & |x| \le 2 \\ 0; & otherwise \end{cases}$ to a function of frequency $F(s)$. Also find the energy of the signal.

7. Find the Fourier sine transform of Exponential decay function f(x) = $e^{-ax}, a > 0$. Hence find (i) $\int_0^\infty \frac{s}{s^2+1} \sin sx\, ds$ (ii) $\int_{-\infty}^\infty \frac{x^2}{(x^2+4)(x^2+1)}\, dx$.

8. The one-dimensional heat equation that is used to study the heat flow in a rod of length 5cm is given by $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$. Find the temperature distribution *u(x,t)*, if the temperatures at the ends are u(0,t) = 0 and u(5,t) = 0, and the initial temperature of the rod is u(x,0) =$x^2(25 - x^2),\ 0 < x < 5$.

9. Consider a prey-predator model, where y and z denote Rabbits (prey) and Foxes (predator) population quantities, respectively. The initial populations of rabbits and foxes are 1000 and 200, respectively. The birth rate of rabbits is considered to be constant (a = 2), and the death rate is therefore dependent on the size of the fox population (c =0.001). The death rate of the foxes is constant (b = 3). The birth rate, however, depends on the quantity of rabbits that the foxes have at their disposal (d = 0.006). Hence the mathematical model is given by

$$\frac{dy}{dx} = ay - c\,yz$$

$$\frac{dz}{dx} = -bz + d\,yz$$

subject to $y(0) = 1000$ and $z(0) = 200$.

Find the population quantity of rabbits and foxes at $x = 0.05$ by using R-K 4th method.

10. Let B = 10 (in Lakhs) represent the carrying capacity for white-tailed Deer in a given area, and let r = 0.1341 be the growth rate of deer. The function *y(x)* represents the population of deer as a function of time *x*, and the constant y (0) = 3 (in Lakhs) represents the initial population. Then the logistic differential equation is

$$\frac{dy}{dx} = r\; y\left(1 - \frac{y}{B}\right),$$

and $y(0) = 3$.

Find the population of deer at (i) $x = 0.1$ by improved Euler method (ii) $x = 0.2$ by Modified Euler method.

====== End of Question Paper ======