SASTRA DEEMED UNIVERSITY

(A University under section 3 of the UGC Act, 1956)

End Semester Examinations

May 2024

Course Code: **MAT301R01**

Course: **ENGINEERING MATHEMATICS - IV**

QP No. :**U176-4**

Duration: 3 hours

Max. Marks:100

PART – A

Answer all the questions **10 x 2 = 20 Marks**

1. Obtain the partial differential equation of all spheres whose centers lie on the z axis.

2. Eliminate the arbitrary function f from $z = f(\sin x + \cos y)$ and give the partial differential equation.

3. Find the complete integral of $pq = xy$.

4. Using $F(e^{-|x|}) = \sqrt{\frac{2}{\pi}} \frac{1}{1+s^2}$, find $F(e^{-|x|} \cos 2x)$.

5. State convolution theorem in Fourier transform.

6. Find the finite Fourier sine transform of $f(x) = x$ in $(0, l)$.

7. The molar heat capacity of a solid compound is given by $c = a + bT$. When $c = 52$, $T = 100$ and when $c = 172$, $T = 400$. Find the values of a and b, by Gauss Jordan method.

8. Evaluate $\int_{\frac{1}{2}}^{1} \frac{1}{x}\, dx$ by Trapezoidal rule, dividing the range into 4 equal parts.

9. Find $y(0.2)$ for the equation $y' = y + e^x$, given that $y(0) = 0$ by using Euler's method.

10. What is the value of k to solve $u_t = \frac{1}{2} u_{xx}$ by Bender-Schmidt method with $h = 1$ if h and k are increments of x and t respectively?

PART – B

Answer all the questions **4 x 15 = 60 Marks**

11. (a) Solve $(D^2 + D'^2)z = \sin 2x \sin 3y + 2\sin^2(x + y)$. (8)

(b) Find the complete integral of the partial differential equation $z = px + qy + \frac{pq}{pq-p-q}$. Also obtain the envelope of the two parameter surfaces represented by the complete integral of the given partial differential equation. (7)

(OR)

12. (a) A thin layer of liquid drains down the side of a vessel whose motion is governed by $\frac{\partial h}{\partial t} + ah^2 \frac{\partial h}{\partial x} = 0$ where $h(x,t)$ is the thickness of the layer and a is the constant that depends on the viscosity, density and gravity constant. Find the solution for $h(x,t)$ given the initial condition $h(x,0) = \alpha\sqrt{x}$ where α is a constant. (8)

(b) Find the complete solution of the partial differential equation $z^2(p^2 + q^2 + 1) = c^2$, where c is constant. (7)

13. (a) Find the Fourier transform of $e^{-a^2x^2}$, $a > 0$. Are these functions $e^{\frac{-x^2}{2}}$ and e^{-x^2} self-reciprocal under Fourier transforms or not? Justify your answer. (8)

(b) Find Fourier sine and cosine transforms of e^{-ax}, where $a > 0$ and hence evaluate (i) $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$ and (ii) $\int_0^\infty \frac{x^2dx}{(a^2+x^2)^2}$. (7)

(OR)

14. Find the Fourier transform of $f(x) = \begin{cases} a - |x| \; for \; |x| < a \\ 0 \;\; for \; |x| > a > 0 \end{cases}$ and hence evaluate (i) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$ (ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$.

15. (a) Determine by Lagrange's method the percentage number of patients over 40 years, using the following data: (8)

Age over (x) years	30	35	45	55
% number (y) of patients	148	96	68	34

(b) The velocity v of a particle at distance s from a point on its linear path is given in the following data:

$s(m)$	0	2.5	5	7.5	10	12.5	15	17.5	20
$v(m/sec)$	16	19	21	22	20	17	13	11	9
	0	1	2	3	4	5	6	7	8

Estimate the time taken by the particle to traverse the distance of 20 meters, using Simpson's one-third rule. (7)

(OR)

16. (a) Find the values of $\cos 55^\circ$ and $\cos 45^\circ$ from the following table, using numerical differentiation based on Newton's backward interpolation formula:

x°	35	40	45	50	55
$\tan x^\circ$	0.7002	0.8391	1.0000	1.1918	1.4281

(8)

(b) Find the real positive root of $3x - \cos x - 1 = 0$, correct to 4 places of decimals, using Newton-Raphson method. (7)

17. (a) Solve the equation $\frac{d^2y}{dx^2} = xy^2$, $y(0) = 1, y'(0) = 0$ for $y(0.2)$ and $y(0.4)$ by Runge-Kutta method of the fourth order. (8)

(b) Solve $u_{xx} = u_t$ given that $u(0,t) = u(4,t) = 0$, $u(x,0) = x(4-x)$ assuming $h = k = 1$. Find the values of u upto $t = 5$. (7)

(OR)

18. Given that $u(x, y)$ satisfies the equation $\nabla^2 u = 0$ and the boundary conditions $u(x, 0) = 0$, $u(x, 4) = 8 + 2x$, $u(0, y) = \frac{y^2}{2}$ and $u(4, y) = y^2$, find the values of $u(i, j)$, $i = 1, 2, 3$; $j = 1, 2, 3$, correct to two places of decimals, by Liebmann's iteration method.

PART – C

Answer the following **1 x 20 = 20 Marks**

19. (a) Using transform method, solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 10$, given that $u(0, t) = 0$, $u(10, t) = 0$ for $t > 0$ and $u(x, 0) = 10x - x^2$ for $0 < x < 10$. (10)

(b) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, y = 0, x = 3, y = 3$ with $u = 0$ on the boundary and mesh length 1 unit. (10)

**