SASTRA DEEMED UNIVERSITY

(A University under section 3 of the UGC Act, 1956)

End Semester Examinations

July 2022

Course Code: **MAT301R01**

Course: **ENGINEERING MATHEMATICS - IV**

Question Paper No. :**U0555**   Duration: **3** hours

Max. Marks:**100**

PART – A

Answer all the questions   **10 x 2 = 20 Marks**

1. Find the Partial differential equation of all planes cutting equal intercepts from the x and y axes.

2. Write the complementary function of $4z_{xy} - z_{yy} = \sin x . \cos y$.

3. Find the general solution of $x = y + e^{p+q}$.

4. How are Fourier & Laplace transforms related?

5. Find the Sine transform of x^{-1}.

6. Under what condition that the Fourier transform of second derivative function exists?

7. Derive the Newton-Raphson formula to evaluate the reciprocal of number N.

8. Write the condition for convergence of Gauss-Seidel method.

9. In what way Improved Euler method differs from Euler method?

10. Express u_{yy} in terms of difference quotient.

PART – B

Answer all the questions   **4 x 15 = 60 Marks**

11. (a) Find the singular integral of $2(z - px - qy) = \log_e(p^2 + q^2)$.
                                                                      (8)

(b) Solve: $(x + pz)^2 + (y + qz)^2 = 1$.   (7)

(OR)

12. (a) Find the general integral of $z(x - y) = px^2 - qy^2$.   (8)

(b) Solve: $(4D^2 - 4DD' + D'^2)z = 16\log(x + 2y)$.   (7)

13. (a) Find the Fourier transform of $e^{-a^2x^2}$ and hence show that $e^{-x^2/2}$ is self-reciprocal under Fourier transform.   (8)

(b) Using Fourier transform evaluate $\int\limits_0^\infty \dfrac{x^2\,dx}{(x^2 + 9)(x^2 + 16)}$.   (7)

(OR)

14. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & if\ |x| \le 1 \\ 0, & Otherwise \end{cases}$ & hence

evaluate $\int\limits_0^\infty \dfrac{Sin^4 t\,dt}{t^4}$.   (8)

(b) Show that $\dfrac{1}{\sqrt{x}}$ is self-reciprocal under Cosine transform.   (7)

15. (a)

Date	10.4.2021	12.4.2021	14.4.2021	16.4.2021	18.42021
y	5441	6618	6984	7987	9344

Where y denotes the number of persons affected by corona virus.

From the above data,
(i) Find the number of persons affected by corona virus on 11.4.2021.
(ii) Find the rate of growth of the number of persons affected by corona virus on 11.4.2021.   (8)

(b) Find a positive root of $10^{2(x-3)} = x$ by Newton-Raphson method. (correct to three decimals).   (7)

(OR)

16. (a) The table below gives the velocity V of a moving particle at time t seconds. Find the distance covered by the particle in 12 seconds and also, the acceleration at t = 2 seconds.   (8)

t	0	2	4	6	8	10	12
V	4	6	16	34	60	94	136

(b) In an Electrical network, the equations for the currents I_1, I_2, I_3 are given by

$28I_2I_3 + 4I_1I_3 - I_1I_2 = 32I_1I_2I_3$,

$I_2I_3 + 3I_1I_3 + 10I_1I_2 = 24I_1I_2I_3$,

$2I_2I_3 + 17I_1I_3 + 4I_1I_2 = 35I_1I_2I_3$

Find the currents by Gauss-Seidel method.   (7)

17. (a) Using Runge-Kutta method of fourth order, find y at x = 0.2 & x = 0.4 if $(y^2 + x^2)dy = (y^2 - x^2)dx$ given that y (0) = 1.   (8)

(b) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides x = 0, y = 0, x = 3 & y = 3 with u = 0 on the boundary and mesh length is one unit.   (7)

(OR)

18. (a) Find y & z at x = 0.1 using Runge-Kutta method fourth order if $\dfrac{dy}{dx} = x + z, \dfrac{dz}{dx} = x - y^2$ given that y(0) = 2 & z(0) = 1.   (8)

(b) Solve numerically, $4u_{xx} = u_{tt}$ with the boundary conditions u (0, t) = 0 = u(4, t) and the initial conditions u(x, 0) = x(4 − x) & $u_t(x,0) = 0$, taking h = 1 for 4 time steps. (7)

PART – C

Answer the following   **1 x 20 = 20 Marks**

19. (a) Solve the Boundary Value Problem by Fourier transform.

$\dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2}$ subject to the conditions u(0, t) = 0 = u(8, t) &

$u(x,0) = 4x - \dfrac{1}{2}x^2, 0 < x < 8.$ (10)

(b) Solve the above Boundary Value Problem numerically by taking

$u(x,0) = 4x - \dfrac{1}{2}x^2$ at the points $x = i, i = 0,1,2,...,7$ &

$t = \dfrac{1}{8}j, j = 0,1,2,...,5.$ (10)

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