# SASTRA DEEMED UNIVERSITY

(A University under section 3 of the UGC Act, 1956)

**End Semester Examinations**

**May 2023**

Course Code: **MAT301R01**

Course: **ENGINEERING MATHEMATICS - IV**

QP No. :**U157-4**

Duration: **3** hours

Max. Marks:**100**

## PART-A

**Answer all the questions** **10 x 2 = 20 Marks**

1. Find the PDE of all planes cutting equal intercepts from the $x$ *and* $y$ axis.

2. Solve the equation $py = 2yx + \log q$.

3. Find the particular integral of the one-dimensional wave equation for the vibration of string model $\frac{\partial^2 y}{\partial t^2} - a^2 \frac{\partial^2 y}{\partial x^2} = E \sin pt$.

4. State and prove the modulation theorem on Fourier transform.

5. Find the signal $f(t)$ if its sine transform is $e^{-as}$.

6. If the approximate value of the root of equation $x^x = 1000$ is 4.5, find a better approximation of the root by the Newton-Raphson method.

7. Find the second-degree polynomial fitting the following data:

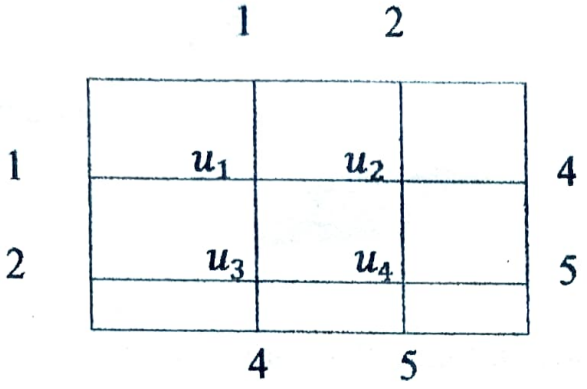
| x | 1 | 2 | 4 |
|---|---|---|---|
| y | 4 | 5 | 13 |

8. The velocity of a particle that starts from rest is given in the following table.

| t(sec.) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| v ( ft/sec.) | 0 | 16 | 29 | 40 | 46 | 51 | 32 | 18 | 8 | 3 | 0 |

Estimate the total distance traveled in 20 seconds using Trapezoidal rule.

9. For the following mesh in solving $\nabla^2 u = 0$ find one set of rough values of u at interior mesh points.




10. Express $a^2 u_{xx} = u_{tt}$ in terms of difference quotients.

## PART-B

**Answer all the questions** **4 x 15 = 60 Marks**

11. a) Solve $z^2(p^2 + q^2) = x^2 + y^2$. (7)

b) Find the equation of the curve satisfying the quasi-linear equation $px + qy = z$ and passing through the circle $x^2 + y^2 + z^2 = 4,\ x + y + z = 2$. (8)

**(OR)**

12. a) Form the PDE by eliminating the arbitrary functions $f$ and $\emptyset$ from $z = xf\left(\frac{y}{x}\right) + y\emptyset(x)$. (7)

b) Solve $(D^2 - 2DD' + D'^2)z = e^x(x + 2y)$. (8)

13. a) Find the Fourier transform of $\frac{\sin ax}{x}$ and hence prove that $\int_{-\infty}^{\infty} \frac{\sin^2 ax}{x^2} dx = a\pi$. (7)

b) Find Fourier cosine transform of $e^{-a^2x^2}$ and hence find $F_s(x\, e^{-a^2x^2})$. (8)

**(OR)**

14. Solve the diffusion equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 10$, given $u(0,t) = u(10,t) = 0$ *for* $t > 0$ and $u(x,0) = 10x - x^2$, for $0 < x < 10$ by the finite Fourier transform method.

15. a) Solve the following system of equations by the Gauss-Seidel method: $28x + 4y - z = 32$; $x + 3y + 10z = 24$; $2x + 17y + 4z = 35$. (7)

b) The following data gives the melting point of an alloy of zinc and lead, $\theta$ is the temperature and $x$ is the percentage load. Using Newton's interpolation formula, find (i) $\theta$ *when* $x = 48$ (ii) $\theta$ *when* $x = 84$. (8)

| $x$ | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|
| $\theta$ | 184 | 204 | 226 | 250 | 276 | 304 |

**(OR)**

16. a) Find the root of the equation $\sin x = 1 + x^3$ between $(-2, -1)$ to 3 decimal places by Newton-Raphson method. (7)

b) A curve passes through the points as given in the table. Find (i) the area bounded by the curve, the x-axis, $x = 1$ *and* $x = 9$. (ii) the volume of the solid generated by revolving this area about the x-axis. (8)

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 0.2 | 0.7 | 1.0 | 1.3 | 1.5 | 1.7 | 1.9 | 2.1 | 2.3 |

17. a) Solve the damping equation $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0,\ y(0) = 1\ and\ y'(0) = 0$ by using Runge-Kutta method of 4th order for $x = 0.2$ correct to 4 decimal places. (7)

b) Solve $16u_{xx} = u_{tt}$ given that $u(0,t) = u(5,t) = 0$ and $u(x,0) = x^2(x-5)$ *and* $u_t(x,0) = 0$ by taking $h = 1$ and up to 5 times steps. (8)

**(OR)**

18. Solve $u_{xx} + u_{yy} = 0\ in\ 0 \le x \le 4,\ 0 \le y \le 4,$ given that $u(0,y) = 0, u(4,y) = 8 + 2y,\ u(x,0) = \frac{x^2}{2}$ and $u(x,4) = x^2$. Take $h = k = 1$ and obtain the results correct to one decimal.

## PART-C

**Answer the following** **1 x 20 = 20 Marks**

19. a) Find the Fourier cosine transform of $e^{-4x}$. Deduce that $\int_0^\infty \frac{cos2x}{x^2+16}\ dx = \frac{\pi}{8}e^{-8}$ and $\int_0^\infty \frac{xsin\,2x}{x^2+16}\ dx = \frac{\pi}{2}e^{-8}$. (10)

b) A rod is rotating in a plane. The following table gives the angle $\theta$ (radians) through which the rod has turned for various values of time $t$ (seconds). (10)

| $t$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|---|
| $\theta$ | 0 | 0.12 | 0.49 | 1.12 | 2.02 | 3.20 |

Calculate the angular velocity and the angular acceleration of the rod when $t = 0.6$ seconds.

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