# SASTRA DEEMED UNIVERSITY

(A University under section 3 of the UGC Act, 1956)

**End Semester Examinations**

**Nov 2024**

Course Code: **MAT301R01**

Course: **ENGINEERING MATHEMATICS - IV**

QP No. :**U1335-A**

Duration: 3 hours

Max. Marks:**100**

## PART – A

**Answer all the questions** **10 x 2 = 20 Marks**

1. Find the general solution of $z_x - z_y = \log(x+y)$.

2. Derive the singular integral of $z - xz_x - yz_y - z_x z_y = 0$.

3. Identify the particular integral of $z_{xx} - 2z_{xy} + z_{yy} = 2^{3y}$.

4. Write the Fourier transform of Dirac-delta function.

5. Solve the integral equation $\int_0^\infty f(x)Cos(\alpha x)dx = e^{-a\alpha}, a > 0$.

6. Point out the function whose finite Fourier sine transform is $\frac{2\pi}{p^3}(-1)^{p-1}, p = 1,2,3,\ldots \;\&\; 0 < x < \pi$.

7. Write the condition for convergence of Newton-Raphson method.

8. Write the Simpson's three-eighth formula.

9. Use Euler's method to approximate y when x = 0.1 given that $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$.

10. Write the suitable scheme to solve Poisson's equation numerically.

## PART – B

**Answer all the questions** **4 x 15 = 60 Marks**

11. a) Show that the integral surface of the partial differential equation $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$ which contains the straight Line $x+y=0, z=1$ is $x^2+y^2+2xyz-2z+2=0$. (8)

b) Solve: $p(z\sin x)^2 + q(z\cos y)^2 = 1$. (7)

**(OR)**

12. a) Find the general integral of

$$\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x+2y) + e^{2x+y}.$$ (8)

b) Solve: $(x^2+y^2)(p^2+q^2)=1$ by using $x = r\cos\theta, y = r\sin\theta$. (7)

13. a) A chip is a signal that sweeps in frequency and is used in radar by bats and humans to facilitate the sorting out of the emitted signal from the echo under conditions where the first echoes will be returning while the emission is still continuing. The chip signal f(x) is given by

$$\begin{cases} 1 - x^2, if |x| \le 1 \\ 0, Otherwise \end{cases}$$

Using Fourier transform, evaluate $\int_0^\infty \frac{(\sin t - t\cos t)^2 dt}{t^6}$. (8)

b) Find $F_c(e^{-3x})$ & $F_s(xe^{-3x})$. (7)

**(OR)**



14. a) Using Fourier transform, evaluate

$$\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)} \;\&\; \int_0^\infty \frac{dx}{(x^2+a^2)^2}.$$ (8)

b) In spectroscopy the equivalent which of a spectral line is defined as the width of a rectangular profile which has the same central intensity and the same area as the line. Using infinite Fourier sine & cosine transform find the value of width function f(x) equals to $x^{m-1}$. (7)

15. a) A slider in a machine moves along a fixed straight rod. Its Distance x cm along a rod is given below for various values of time t sec. Find the velocity & acceleration of the slider when t = 1.5sec. (8)

| Time t | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|
| Distance x | 3.375 | 7.0 | 13.625 | 24.0 | 38.875 | 59.0 |

b) Find a positive real root of $x\log_{10} x = 1.2$ by Newton-Raphson method. (correct to three decimals). (7)

**(OR)**

16. a) Solve the following linear simultaneous equations $8x - 3y + 2z = 20$, $6x + 3y + 12z = 35$, $4x + 11y - z = 33$ by Gauss-seidel method (correct to four decimals). (8)

b) The speed, v meters per second, of a car, t seconds after it starts, is shown in the following table:

| t | 0 | 12 | 24 | 36 | 48 | 60 | 72 |
|---|---|---|---|---|---|---|---|
| v | 0 | 3.60 | 10.08 | 18.90 | 21.60 | 18.54 | 10.26 |

| t | 84 | 96 | 108 | 120 |
|---|---|---|---|---|
| v | 5.40 | 4.50 | 5.40 | 9.00 |

Using Simpson's rule, find the distance travelled by the car in 2 minutes. (7)

17. a) Solve $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1$ in the range $0 \le x \le 0.2$ using the Modified Euler's method. Take h = 0.1. (8)

b) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides x = 0, y = 0, x = 3 & y = 3 with u = 0 on the boundary and mesh length is one unit. (7)

**(OR)**

18. a) Solve $\frac{d^2 y}{dx^2} = x\frac{dy}{dx} - y$, given that $y = 3, \frac{dy}{dx} = 0$ when x = 0 to approximate y (0.1) using Runge - kutta method of fourth order. (8)

b) Using Crank-Nicholson's method, solve
$u_{xx} = 16u_t, 0 < x < 1, t > 0$ given
$u(x,0) = 0, u(0,t) = 0$ & $u(1,t) = 50t$. Compute u for two steps in t direction taking h = ¼. (7)

## PART – C

**Answer the following** **1 x 20 = 20 Marks**

19. a) Solve the Partial differential equation.

$$4\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 16\log(x+2y).$$ (10)

b) Solve the one-dimensional heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ using Fourier transform subject to the conditions

$$u(x,0) = 2x, 0 < x < 4, t > 0 \;\&\; \frac{\partial u(0,t)}{\partial x} = 0 = \frac{\partial u(4,t)}{\partial x}.$$ (10)

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