P THINK FAST S

Competitive Programming

From Problem 2 Solution in O(1)

String Processing Algorithms

Suffix Arrays - O(nlogn)

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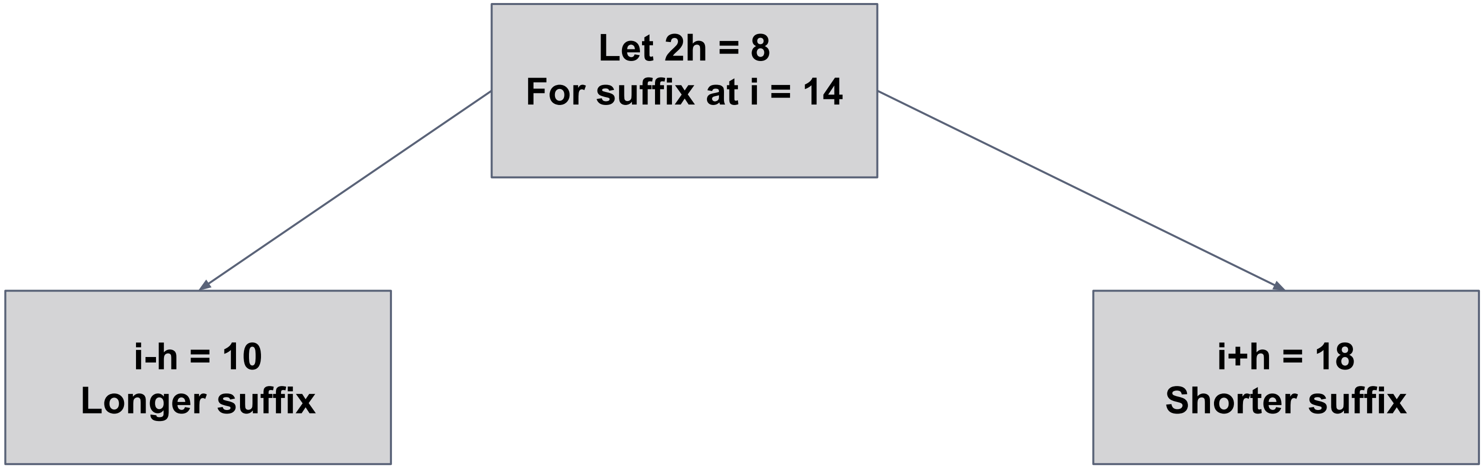
i+h vs i-h observations



Let 2h = 8
For suffix at i = 14

i-h = 10
Longer suffix

i+h = 18
Shorter suffix

Bottom up perspective

Suffix i (already sorted) is part of a longer suffix (to be sorted)

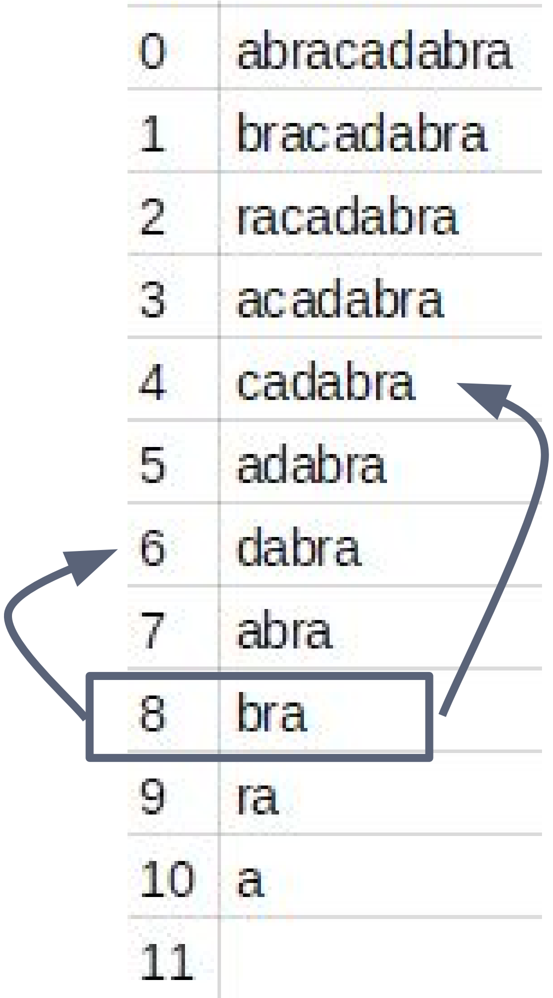
Observation on the **longer suffix** leads to O(nlogn)

Top down perspective

Suffix i (to be sorted) includes a shorter suffix (already sorted)

Observation on the **shorter suffix** leads to O(nlogn^2)

i-h



0	abracadabra
1	bracadabra
2	racadabra
3	acadabra
4	cadabra
5	adabra
6	dabra
7	abra
8	bra
9	ra
10	a
11	

i = 8
h = 2
i-h = 6
bra part of dabra

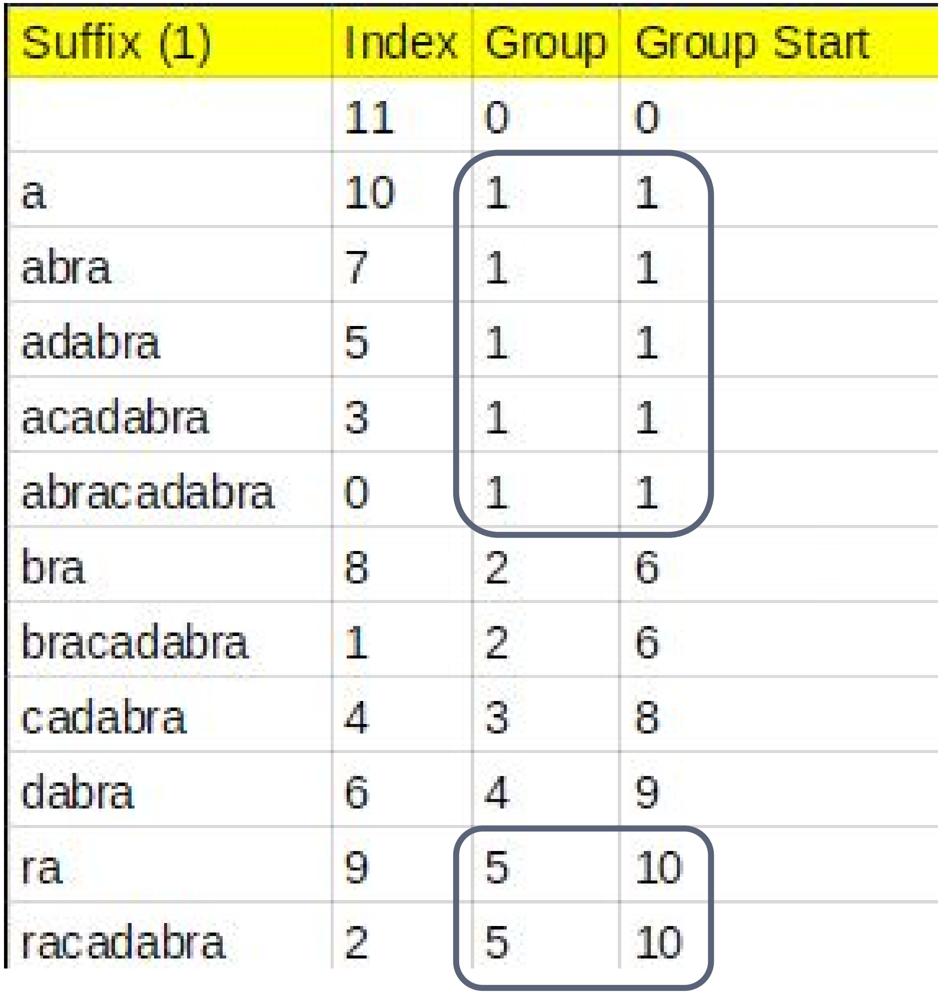
i = 8
h = 4
i-h = 4
bra part of cadabra

Let’s introduce: Group Start

The index of the **first time** for the group to appear

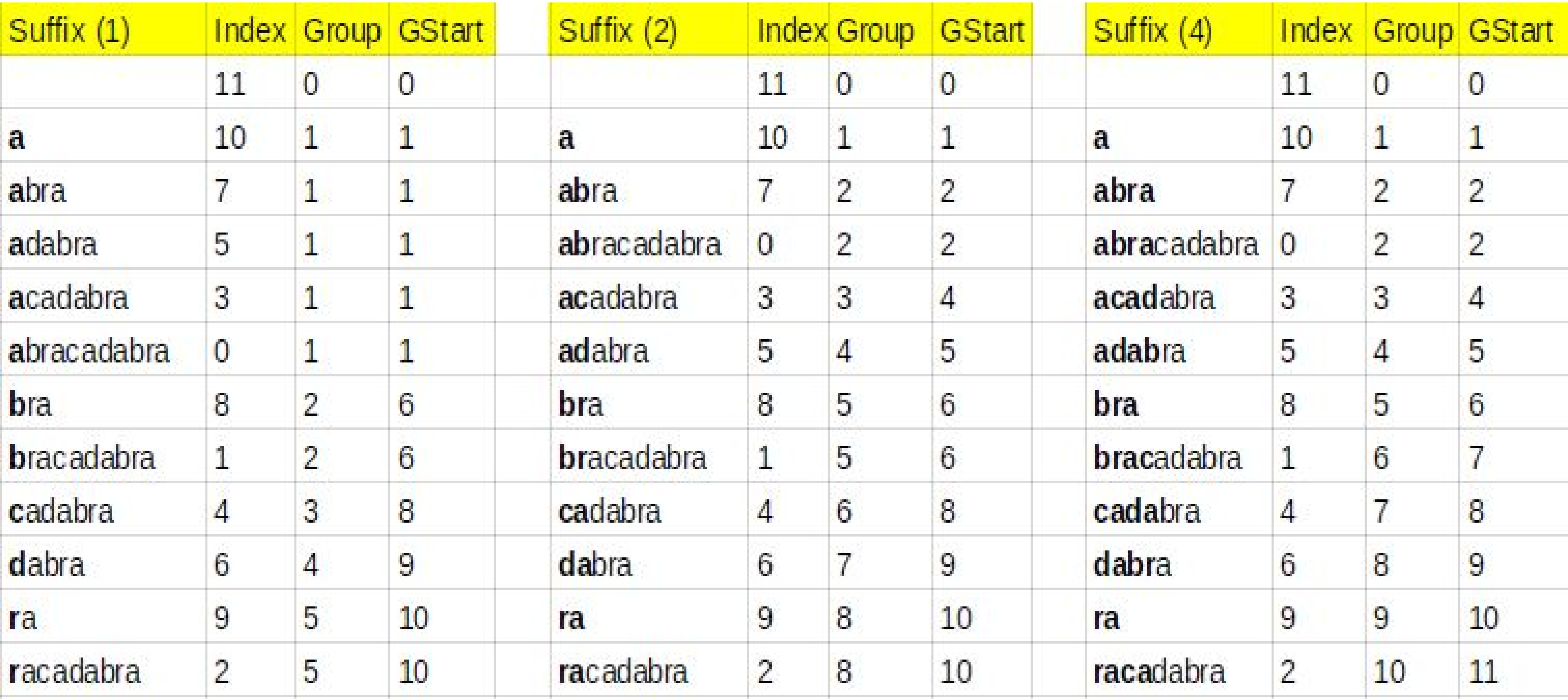
i = 6
suf[6] = 8 = bra
group[8] = 2
groupStart[2] = 6

groupStart[group[suf[i]]] = 6



Suffix (1)	Index	Group	Group Start
	11	0	0
a	10	1	1
abra	7	1	1
adabra	5	1	1
acadabra	3	1	1
abracadabra	0	1	1
bra	8	2	6
bracadabra	1	2	6
cadabra	4	3	8
dabra	6	4	9
ra	9	5	10
racadabra	2	5	10

Let’s introduce: Group Start



Suffix (1)	Index	Group	GStart
	11	0	0
a	10	1	1
abra	7	1	1
adabra	5	1	1
acadabra	3	1	1
abracadabra	0	1	1
bra	8	2	6
bracadabra	1	2	6
cadabra	4	3	8
dabra	6	4	9
ra	9	5	10
racadabra	2	5	10

Suffix (2)	Index	Group	GStart
	11	0	0
a	10	1	1
abra	7	2	2
abracadabra	0	2	2
acadabra	3	3	4
adabra	5	4	5
bra	8	5	6
bracadabra	1	5	6
cadabra	4	6	8
dabra	6	7	9
ra	9	8	10
racadabra	2	8	10

Suffix (4)	Index	Group	GStart
	11	0	0
a	10	1	1
abra	7	2	2
abracadabra	0	2	2
acadabra	3	3	4
adabra	5	4	5
bra	8	5	6
bracadabra	1	6	7
cadabra	4	7	8
dabra	6	8	9
ra	9	9	10
racadabra	2	10	11

i-h

aacdzz
aacdee
aacdxx
aacdwz

group 3

ssmnehabxx
ssmnaacdzz
ssmnaacdxx

group 9

ehabhz
ehabtn
ehabxx
aacdab

group 5

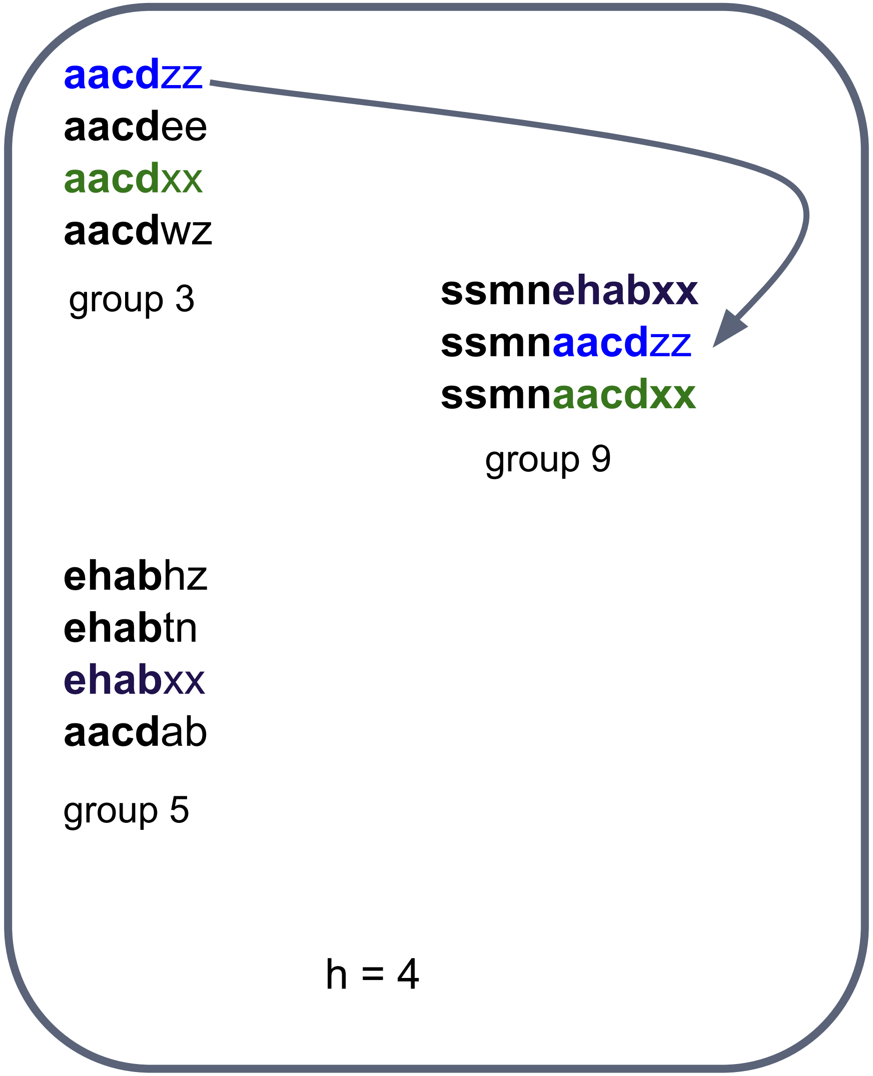
h = 4

- Note: These are invalid suffixes

- Assume we sorted suffixes on h = 4
- We have grouped the suffixes over h = 4

- In group 9, suffixes of length 8+
- We know they are sorted on first 4
- What about the remaining letters?
- We know they are sorted on their suffixes on their first 4 letters

i-h



aacdzz
aacdee
aacdxx
aacdwz

group 3

ssmnehabxx
ssmnaacdzz
ssmnaacdxx

group 9

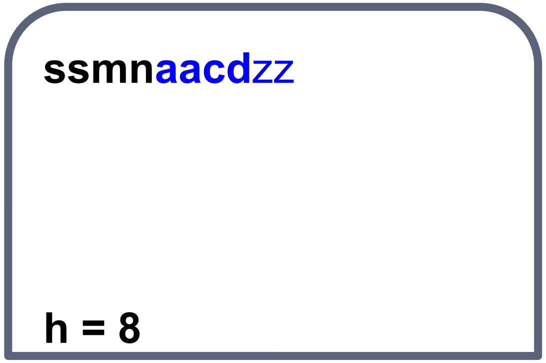
ehabhz
ehabtn
ehabxx
aacdab

group 5

h = 4

- **aacd**zz is sorted on 4 letters
- It belong to the **smallest** group
- It is *2nd 4+ letters* from **ssmnaacd**zz

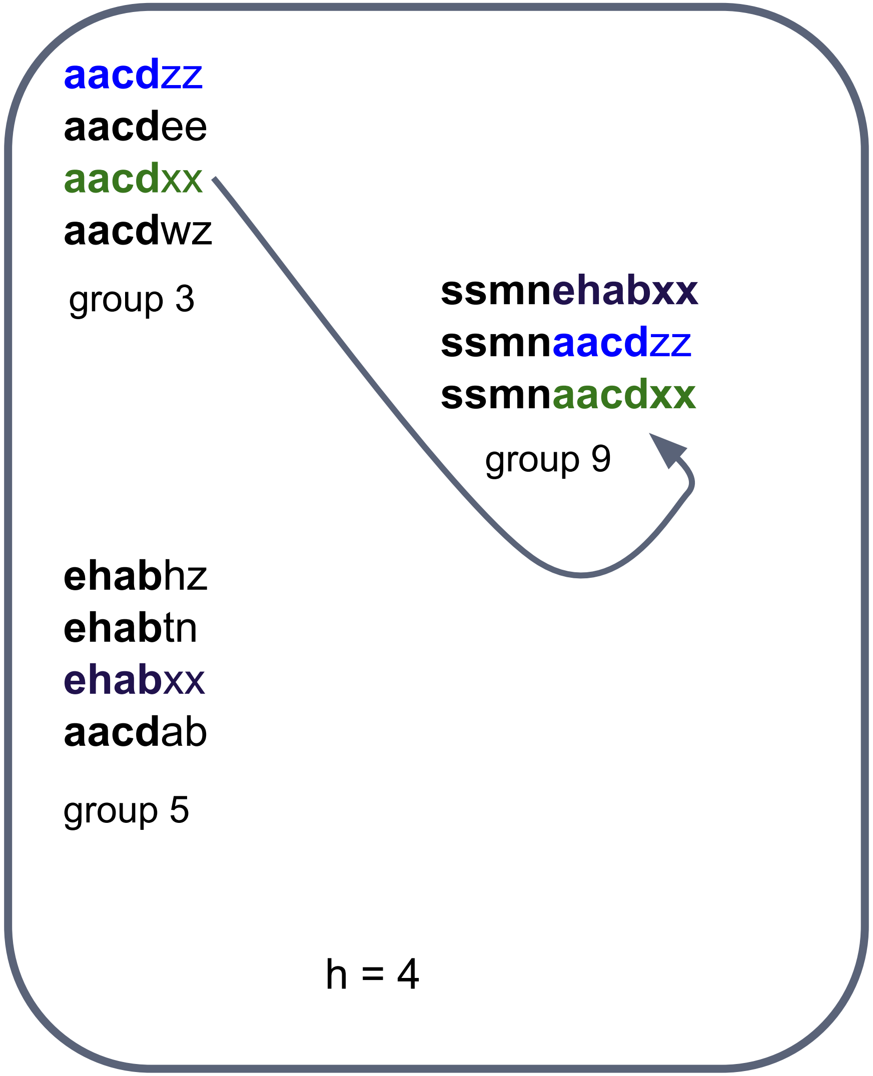
- Then **ssmnaacd**zz must be in the top of its new group for h = 8
- As it has same first 4 letters in its group
- And its 2nd 4 letters has smaller group



ssmnaacdzz

h = 8

i-h



aacdzz
aacdee
aacdxx
aacdwz

group 3

ssmnehabxx
ssmnaacdzz
ssmnaacdxx

group 9

ehabhz
ehabtn
ehabxx
aacdab

group 5

h = 4

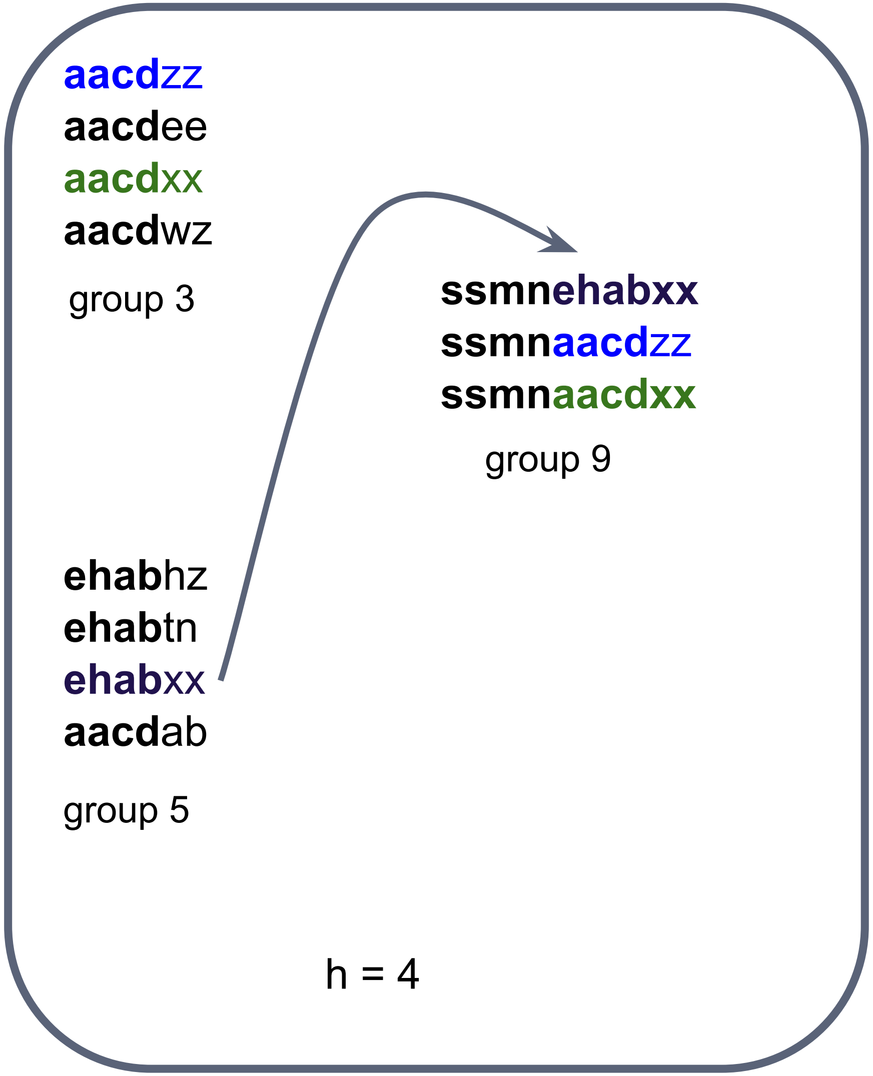
- Similarly for **aacd**xx
- It belong to the **smallest** group
- It is *2nd 4+ letters* from **ssmnaacdxx**

- Then **ssmnaacdxx** must be in the next available position its new group for h = 8

ssmnaacdzz
ssmnaacdxx

h = 8

i-h



aacdzz
aacdee
aacdxx
aacdwz

group 3

ssmnehabxx
ssmnaacdzz
ssmnaacdxx

group 9

ehabhz
ehabtn
ehabxx
aacdab

group 5

h = 4

- Moving to the next smallest group = 5
- It has **ehab**xx
- It is part of **ssmnehabxx**
- So add it in next possible position in its group h = 8

- Now this new group has 3 suffixes sorted on first 8 letters

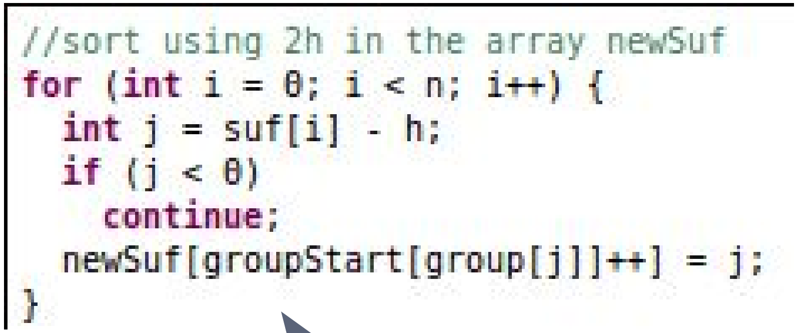
ssmnaacdzz
ssmnaacdxx
ssmnehabxx

h = 8

See notes **below** for a written text in case needed

Assume sorted on h

- We don’t sort in-place like sort function
- So create a new array (newSuf) for sorted suffixes

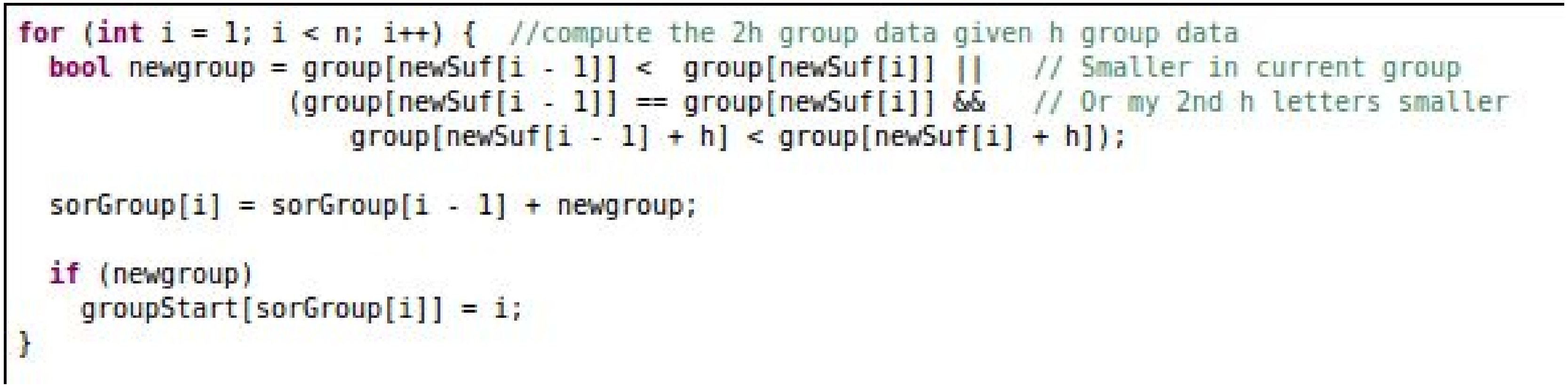


```
//sort using 2h in the array newSuf
for (int i = 0; i < n; i++) {
  int j = suf[i] - h;
  if (j < 0)
    continue;
  newSuf[groupStart[group[j]]++] = j;
}
```

- j is a 4 letters sorted suffix in its group
- suf[i] is part of suffix at j
- suf[i] is the smallest in current order
- then put suf[j] in the next available position in its **group**

- groupStart[group[j]] is the next available position in its group
- Add suffix at j in ths position
- Increment for next possible one

Once sorted over 2h



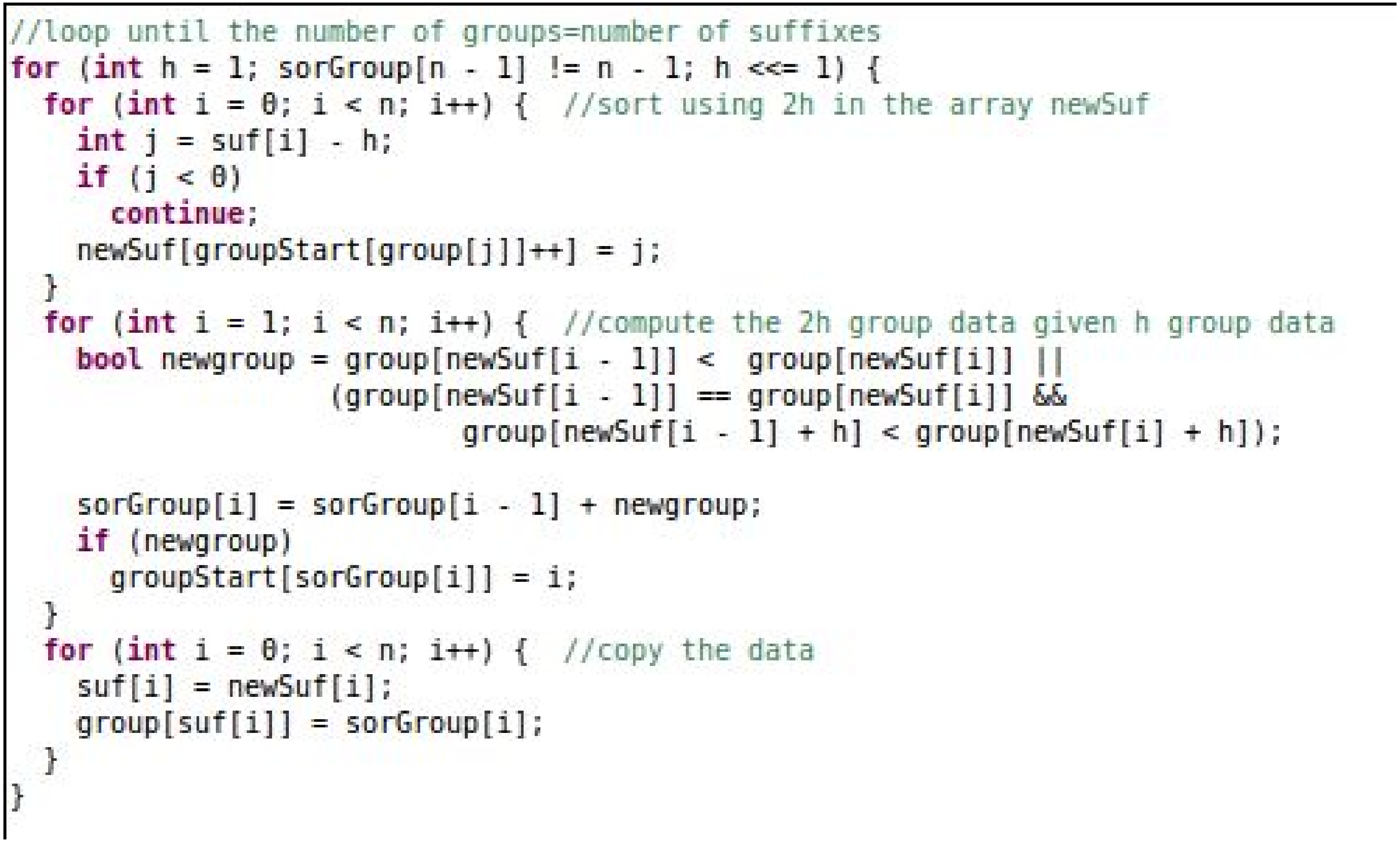
```
for (int i = 1; i < n; i++) {  //compute the 2h group data given h group data
  bool newgroup = group[newSuf[i - 1]] <  group[newSuf[i]] ||    // Smaller in current group
                 (group[newSuf[i - 1]] == group[newSuf[i]] &&    // Or my 2nd h letters smaller
                     group[newSuf[i - 1] + h] < group[newSuf[i] + h]);

  sorGroup[i] = sorGroup[i - 1] + newgroup;

  if (newgroup)
    groupStart[sorGroup[i]] = i;
}
```

- Build new sorted groups and group start
- Very similar to old one
- Either we already in different groups, so we are still different
- Are we were in same group, and out **shorter suffix** is different

Overall



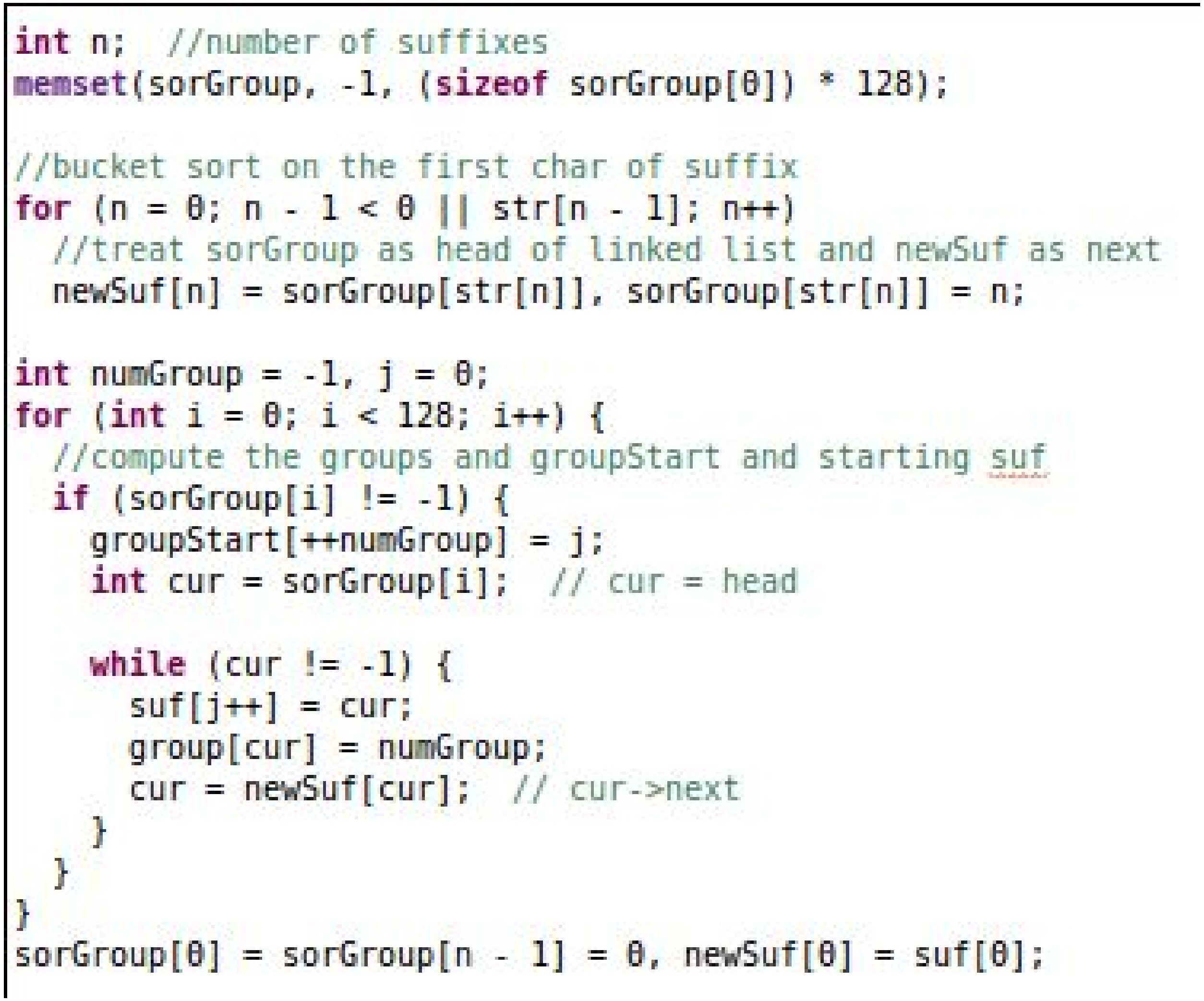
```
//loop until the number of groups=number of suffixes
for (int h = 1; sorGroup[n - 1] != n - 1; h <<= 1) {
  for (int i = 0; i < n; i++) {  //sort using 2h in the array newSuf
    int j = suf[i] - h;
    if (j < 0)
      continue;
    newSuf[groupStart[group[j]]++] = j;
  }
  for (int i = 1; i < n; i++) {  //compute the 2h group data given h group data
    bool newgroup = group[newSuf[i - 1]] <  group[newSuf[i]] ||
                   (group[newSuf[i - 1]] == group[newSuf[i]] &&
                           group[newSuf[i - 1] + h] < group[newSuf[i] + h]);

    sorGroup[i] = sorGroup[i - 1] + newgroup;
    if (newgroup)
      groupStart[sorGroup[i]] = i;
  }
  for (int i = 0; i < n; i++) {  //copy the data
    suf[i] = newSuf[i];
    group[suf[i]] = sorGroup[i];
  }
}
```

Initialization

- One can do that in several easy ways
- Quick sort
  - Sort in O(nlogn) based on the first letter
  - Iterate over strings and build group and groupStart arrays
- Bucket sort (linear)
  - Linked list (vector) of positions per letter
  - E.g. vector<vector<int>> lists;
  - Then iterate and build group and groupStart arrays
  - *Or one might use any 2 1D arrays to build these lists*
  - Use 1 array has head of list and the other as next (->)
  - Iterating on them = iterating on the linked lists

Initialization



```
int n;  //number of suffixes
memset(sorGroup, -1, (sizeof sorGroup[0]) * 128);

//bucket sort on the first char of suffix
for (n = 0; n - 1 < 0 || str[n - 1]; n++)
  //treat sorGroup as head of linked list and newSuf as next
  newSuf[n] = sorGroup[str[n]], sorGroup[str[n]] = n;

int numGroup = -1, j = 0;
for (int i = 0; i < 128; i++) {
  //compute the groups and groupStart and starting suf
  if (sorGroup[i] != -1) {
    groupStart[++numGroup] = j;
    int cur = sorGroup[i];  // cur = head

    while (cur != -1) {
      suf[j++] = cur;
      group[cur] = numGroup;
      cur = newSuf[cur];  // cur->next
    }
  }
}
sorGroup[0] = sorGroup[n - 1] = 0, newSuf[0] = suf[0];
```

تم بحمد الله

علمكم الله ما ينفعكم

ونفعكم بما تعلمتم

وزادكم علماً