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Competitive Programming

From Problem 2 Solution in O(1)

Algebra

Number Bases and Polynomials

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Number Bases

- Base 10 (Decimal) is the usual base in our life
- Base 2 (Binary) is in computer world (on/off)
- Base 8 (Octal): Christmas is Halloween?
- Base 16 (Hexadecimal) with A=10...F=15
- Base K has K digits: 0, 1, 2….K-1
- We can represent any base!
  - Up to 36, you can use Alphanumeric (A-Z), 26 letters
  - X = Tn*b^n+…..+ T2*b^2 +T1*b^1+T0*b^0 (base b)
  - 5736 = 5*10^3 + 7*10^2 + 3*10^1 + 6 * 10^0 (base 10)

Number Bases

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Octal	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

Src: Discrete Mathematics and Its Applications - Kenneth Rosen

Decimal Base

- 5736 = 5*10^3 + 7*10^2 + 3*10^1 + 6 * 10^0
- 5736 = ((5*10+7)*10+3)*10+6
- 5736 % 10 = 6 (**get** last digit)
- 5736 / 10 = 573 (**remove** last digit)
- All properties we do in base 10, are same
  - 2AF3 = 2*16^3 + A * 16^2 + F * 16^1 + 3 * 16^0
  - 2AF3 = 2*16^3 + 10 * 16^2 + 15 * 16^1 + 3 * 16^0
  - 2AF3 = 10995
- To convert X in base A to Base B
  - Convert X to base 10, then from base to base B

Bases Conversion

- To convert any base to decimal, just evaluate it: 2AF3 =(((2*16 + 10) * 16) + 15) * 16 + 3
  - This evaluation is **faster** than computing 16^i for each term. Its ideas based on **Horner's** method
- To convert decimal to any base
  - Remember: X = Tn*b^n+…..+ T2*b^2 +T1*b^1+T0*b^0
  - 522 = a*16^2 + b*16 + c (in base 16)
  - Then 522 % 16 = 0 + 0 + c
  - Then 522 / 16 = a*16 + b + 0
  - Fact: Get digit by %base, remove it by /base

Bases Conversion

```
string letters = "0123456789ABCDEF";
int toInt(char c) {     return letters.find(c);     }

int FromAnyBasetoDecimal(string in, int base) {
    int res = 0;
    for (size_t i = 0; i < in.size(); ++i)
        res *= base, res += toInt(in[i]);
    return res;
}

string FromDecimaltoAnyBase(int number, int base) {
    if (number == 0)
        return "0";

    string res = "";

    for (; number; number /= base)
        res = letters[number % base] + res;
    return res;
}
```

Hex - Binary Conversions

- You can use previous way always. Just as fact
- Hex is 16 base. Each digit can be converted to 4 binary bits
  - E.g. 6 => 0100, F => 1111
- From Hex to Binary: replace each digit with 4 bits
- From Binary to Hex: replace each 4 bits (from left) to 1 Hex
  - 1011110101101010010 = 101 1110 1011 0101 0010
  - 1011110101101010010 = 5 E B 5 2 = **5EB52**

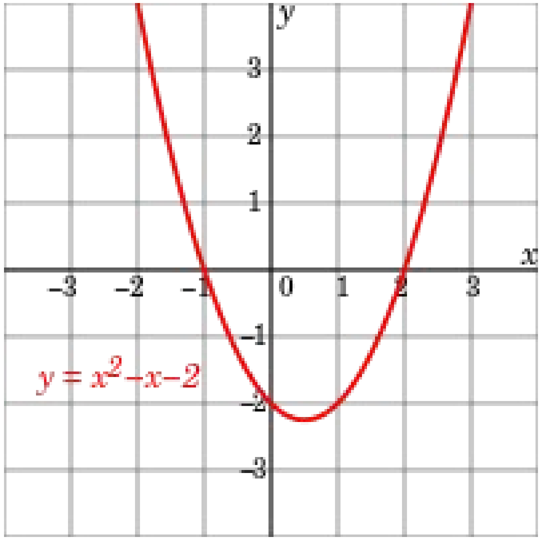
Polynomial

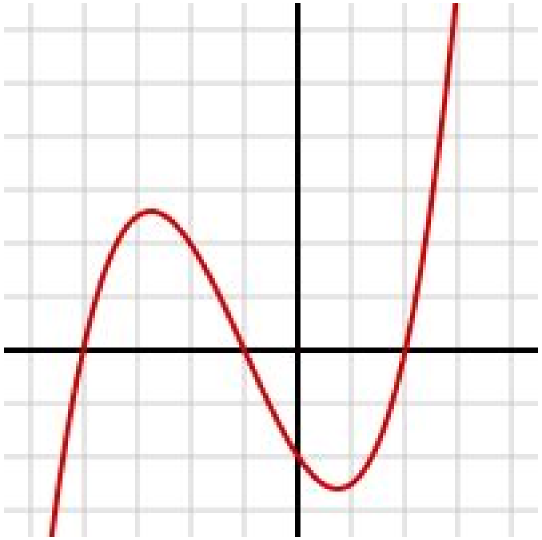
- Expression of variables and coefficients with the +, -, *, and non-negative integer exponents
- x, $x^2 - 4x + 7$, $x^3 + 2xyz^2 + yz + 1$
- $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,$ $\sum_{i=0}^{n} a_i x^i$
- To evaluate polynomial, just evaluate terms
- $f(x) = x^3 - x$ f(2) = 8 - 2 = 6
- $f(x, y) = 2x^3 + 4x^2 y + xy^5 + y^2 - 7.$ f(3, 2) = ?
- Polynomial degree is the highest term
  - Degree zero f(x) = 0 is just axis.
  - Degree 1 is line equation: f(x) = 2x+3

Graphs of polynomial functions

(*x* + 1)(*x* − 2)

2nd degree (parabola)



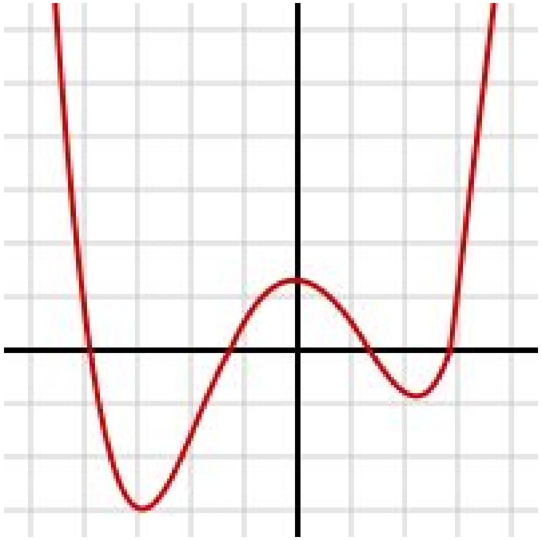


1/4 (*x* + 4)(*x* + 1)(*x* − 2)

3rd degree

1/14 (*x* + 4)(*x* + 1)(*x* − 1)
(*x* − 3) + 0.5

4th degree



1/20 (*x* + 4)(*x* + 2)(*x* + 1)
(*x* − 1)(*x* − 3)+ 2

5th degree

Arithmetic of polynomials

$$P = 3x^2 - 2x + 5xy - 2$$
$$Q = -3x^2 + 3x + 4y^2 + 8$$

$$P + Q = 3x^2 - 2x + 5xy - 2 - 3x^2 + 3x + 4y^2 + 8$$
$$P + Q = x + 5xy + 4y^2 + 6$$

$$P = 2x + 3y + 5$$
$$Q = 2x + 5y + xy + 1$$

$$\begin{aligned} PQ = \; & (2x \cdot 2x) + (2x \cdot 5y) + (2x \cdot xy) + (2x \cdot 1) \\ + \; & (3y \cdot 2x) + (3y \cdot 5y) + (3y \cdot xy) + (3y \cdot 1) \\ + \; & (5 \cdot 2x) + (5 \cdot 5y) + (5 \cdot xy) + (5 \cdot 1) \end{aligned}$$

$$PQ = 4x^2 + 21xy + 2x^2y + 12x + 15y^2 + 3xy^2 + 28y + 5$$

Solving polynomial equations

- To solve it (find roots) let f(x) = 0
  - E.g. find values that makes evaluation is zero
  - In other words, f meets the x-axis
- Linear case is trivial
  - f(x)=2x - 6 = 0 => 2x = 6 => x = 6 / 2 = 3
  - f(x)=a(x-3)-a=0 => a(x-3) = a => x-3 = 1 (Wrong a = 0 ?)
- 2nd degree ([Quadratic] equation)
  - $ax^2 + bx + c = 0$ $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
  - a != 0. a > 0 graph opens up. a < 0 opens down
  - Min/Max point at x = -b/2a
- [Practice] for on solving equations

Polynomials: Misc

- Monomial is 1 term (3xy), Binomial is 2 terms (4x+3), Trinomial is 3 terms ($x+y^2-5$)
- $3x^2-6X-24$ => Factorize => (x-4)(3x+6)
  - Roots: x = 4, x = -2
  - x^2-5x => x(x-5) => roots x = 0, x = 5
- Derivative $aX^b = a*b*X^{(b-1)}$
  - Derivative ($3x^2-6X-24$) = 3*2*X - 6 = 6X-6
- Homework - Write code that:
  - represents and evaluates a polynomial in O(n)
  - multiply 2 polynomials / computes polynomial derivative
  - solves the quadratic equation

Polynomials: More later to study

- Some hard problems need more knowledge
- Learn how to solve roots of polynomial generally (integer roots, real roots)
- Some problems can be solved by Lagrange Interpolation

تم بحمد الله

علمكم الله ما ينفعكم

ونفعكم بما تعلمتم

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