The Cartwheel Galaxy: Simulating the Collision that Led to its Formation

Girish Krishnan,* Ishaan H. Kavoori,† and Gokul Swaminathan‡
University of California San Diego

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The Cartwheel galaxy was formed in 1963, from the head-on perpendicular collision of a spiral disk galaxy and a smaller intruder galaxy. This mechanism was proposed by Lynds and Toomre (1976) [1] Our previous work involved simulating this collision as a restricted three-body problem, and this approximation led to fast and memory-efficient computations. In this paper, we aim to simulate the same collision using an N-body simulation instead, using the disk-halo-bulge model for the target galaxy described by Athanassoula et al., 1997 [2]. We aim to analyze the effectiveness of these simulations, and how they compare to the restricted three-body problem we studied earlier.

I. INTRODUCTION

The Cartwheel galaxy, situated 500 million light years away in the Sculptor constellation, has an unusual appearance consisting of a bright core and an outer ring dominated by supernovas and star formation. The particles in the galaxy's rings have an unusual distribution of velocities and electromagnetic emission spectra. Analyzing and understanding the cause of these distributions requires an explanation of how the Cartwheel galaxy was formed.

FIG. 1. An image of the Cartwheel Galaxy taken by The James Webb Space Telescope.[3]

One possible mechanism proposed by Lynds and Toomre (1976) [1] was that there was a high-speed head-on collision between a small intruder galaxy and a disk-shaped spiral galaxy that caused a gravitational ripple effect that moved compressed gas and dust outwards in a ring.

Our midterm project involved simulating this mechanism as a restricted three-body problem, by neglecting gravitational effects between particles in the target spiral galaxy (other than the nucleus). In this project, we aim to simulate the mechanism as an N-body problem using the disk-halo-bulge model described by Athanassoula et al., 1997 [2]. This is a more realistic model of the Cartwheel galaxy, compared to the restricted three-body problem solved for the midterm project.

This report will explain our procedures and highlight the results of our simulations, and compare them against those of Athanassoula et al. and our previous simulation results (using a simpler, restricted three-body problem).

II. METHODS

The general N-body equation of motion involves gravitational attractions between every pair of bodies. Let $\mathbf{r}_i$ represent the position vector of body i of mass m_i in the simulation. Let ϵ represent the softening parameter, a small value that is added to the expression for gravitational force to prevent the magnitude from exploding to infinity (and potentially causing computational overflow) for small displacements. Let G be the universal gravitational constant. For a simulation involving N bodies, we have:

$$\ddot{\mathbf{r}}_i = -G \sum_{j=1;\, j\neq i}^{N} \frac{m_j\,(\mathbf{r}_i - \mathbf{r}_j)}{(r_{ij}^2 + \epsilon^2)^{3/2}} \tag{1}$$

where r_{ij} is the distance between body i and body j.

Since there are N bodies, each particle has a gravitational interaction with $(N-1)$ other bodies. At each step of the simulation, calculating all the gravitational forces on each body is an operation of the order $\mathcal{O}(N^2)$.

To conduct the N-body simulation, we used gyrfalcON [4] which performs force computation with a cost of $\mathcal{O}(N)$. Note that the preparation phase in gyrfalcON (which involves building a hierarchical tree of cubic cells) has a cost of $\mathcal{O}(N \log N)$. To form the

* gikrishnan@ucsd.edu
† ikavoori@ucsd.edu
‡ gswamina@ucsd.edu

disk-halo-bulge model of the galaxy, we used the *magalie* [5] tool which allows one to specify the masses and number of points in the disk, halo, and bulge. We used the *mkplummer* [6] tool to generate a Plummer sphere, which represents the intruder galaxy (referred to as a "companion" galaxy in Athanassoula et al., 1997).

The link to our GitHub repository can be found here [7]. Our repository contains bash scripts invoking the following commands:

- Creating the companion galaxy and the target disk-shaped galaxy
- Visualizing the initial positions of the companion and target galaxy using *glnemo2* [8].
- Setting initial conditions for the companion and target galaxy, and running the simulations using gyrfalcON.
- Visualizing the resulting simulations, via animations and MPEG movies.

A. Units

While it is possible to use conventional SI units to perform all calculations, the astronomical scale of the different physical quantities involved in this galaxy collision make it difficult to use appropriate data structures to capture them precisely. We take the units used by Athanassoula et al., 1997 [2]:

- G = 1
- 1 length unit = 3 kpc
- 1 time unit = 1×10^7 years

When we define these as our length units, time units, and value of G, we subsequently get these units:

- 1 mass unit = $6 \times 10^{10} M_\odot$
- 1 velocity unit = 293 km/s
- 1 volume density unit = 2.22 $M_\odot/\mathrm{pc}^3$

B. Galaxy Preparation

Athanassoula et al. aimed to investigate the variation of target mass distribution and velocity dispersion, as well as the mass and orbit of the companion galaxy. For both barred and non-barred galaxies, Athanassoula et al. aimed to analyze the amplitude, shape, lifetime, and expansion velocity of the resulting rings after the galaxy collision. Furthermore, the formation of spokes in the cartwheel shape of the resulting galaxy was another interesting phenomenon studied. This section explains how the simulated galaxies were prepared to satisfy the above goals.

1. Companion Galaxy - Plummer Sphere

The companion galaxy was modeled as a Plummer sphere, whose initial conditions can easily be created using the *mkplummer* tool. The galaxy had 4000 particles, each having a mass of 4.9657×10^{-5} mass units. The scale length was 0.195 length units and the cutoff radius was 3.0 length units. A galaxy meeting these criteria is said to be a companion galaxy of standard mass. Following the directions in Athanassoula et al., 1997, we also simulated similar galaxies with double and half the standard mass.

2. Target Galaxy - Disk, Halo, Bulge

The bulge and halo of the target galaxy have a Truncated Plummer (PL) profile, and are modeled by the radial mass density function

$$\rho(r) = \frac{3M}{4\pi b^3}\left(1 + \frac{r^2}{b^2}\right)^{-\frac{5}{2}}$$

, where M is the mass of the bulge (or halo) and b is the scale length.

The disk was modeled by a Kuzmin/Toomre (KT) projected radial density profile, given by the function

$$\Sigma(r) = \frac{M_D}{2\pi b_D^2}\left(1 + \frac{r^2}{b_D^2}\right)^{-\frac{3}{2}}$$

, where b_D is the scale length and M_D is the mass of the disk.

C. Simulation Setup

when setting up the simulation, the halo and the bulge of the target galaxy are first constructed, prior to introducing the disk. The halo and bulge particles are allowed to relax until they approach equilibrium. Then, the positions of the disc particles are chosen based on the mass distribution mentioned previously. This was the same setup procedure followed in Athanassoula et al., 1997.

D. Initial Conditions

To decide the initial position and velocity of the companion galaxy relative to the target galaxy, we used the same initial conditions specified in Table 3 of Athanassoula et al., 1997 (see Figure 2). These initial conditions were decided by first determining the impact point and impact velocity for the model and companion. Then, the orbit required to produce such an impact point can be back-calculated. This is done while keeping the positions of the target galaxy particles fixed (as a simplifying approximation).

	POSITION			VELOCITY			$\|V/V_{esc}\|$	
IMPACT	x	y	z	V_x	V_y	V_z	S runs	R or C runs
PF	−10.02	10.04	20.30	1.04	−1.05	−2.31	7.88	
PS	0.0	10.0	10.0	−0.04	0.0	0.0	0.08	
CF	−8.98	9.39	18.08	1.03	−1.08	−2.07	6.96	5.67
CS	−10.01	19.07	17.39	0.33	−0.63	−0.56	2.75	2.35
CSC	−10.11	19.17	17.39	0.33	−0.63	−0.56		2.38
CVS	0.0	0.0	27.68	0.0	0.0	−0.90		2.36
PSB	−4.74	17.25	15.19	0.10	−0.59	−0.51		1.91
C01	0.0	0.0	12.0	0.0	0.0	0.0		0.0
C02	0.0	6.0	10.39	0.0	0.0	0.0		0.0

FIG. 2. Table from Athanassoula et al., 1997 showing the initial conditions used different types of impacts (such as oblique and head-on).

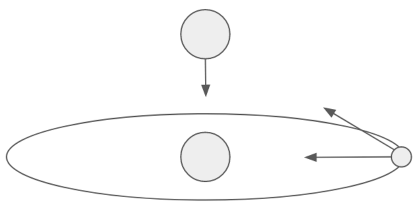

FIG. 3. A drawing of another star approaching a solar system with a single planet

E. Simulation using gyrfalcON

To generate the simulation of the galaxies, we used gyrfalcON [4], an efficient N-body simulator that is part of glnemo2. We used magalie to generate the disk, halo, and bulge of the target galaxy and mkplummer to generate the plummer sphere of the intruding galaxy. We then used snapstack to move the positions of the galaxies to match the initial conditions as used in Athanassoula et al., 1977. We scaled the masses to ensure the units were consistent, and ran our simulations for 80 time units with timestep = 0.05 and kmax = 6.

F. Visualization of Results

To visualize the results, we used *glnemo2* to render the simulation and FFMPEG to save the rendered simulation as a .mp4 video. These videos can be found in our GitHub repository.

III. RESULTS

To get a better conceptual understanding of the problem, it is helpful to consider a smaller scale version of the problem. Consider a star approaching the center of the solar system perpendicular to the plane of planetary orbits (Figure 3).

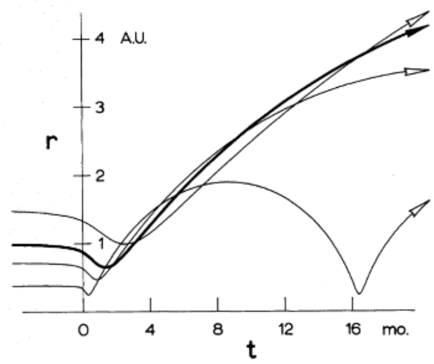

FIG. 4. Distance of perihelion of our solar system's inner planets over time as a hypothetical intruding star passes through the solar system

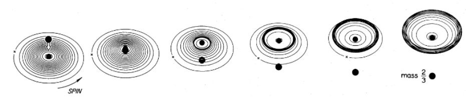

FIG. 5. The rings of a galaxy when the nucleus of another galaxy passes through the center.

As the intruding star approaches, planets orbiting the host star would experience an increasing force towards the intruding star. This would cause the planets' distance from their star to decrease while the intruding star is near. Then as the intruding star leaves and the force it exerts diminishes, the planets boost out to a larger orbit than before due to the increased velocity (Figure 4).

We can imagine this scaled up to rings of stars in a galaxy with the nucleus of another galaxy approaching at the right angle. Rings would contract and then be "pushed" away to form denser rings (Figure 5).

Figure 6 shows a visualization of the companion galaxy, which is modeled as a Plummer sphere, as described earlier. Figure 7 shows a visualization of the target galaxy, according to the disk-halo-bulge model described by Athanassoula et al., 1997 [2].

Figure 9 illustrates a head-on collision simulated by us

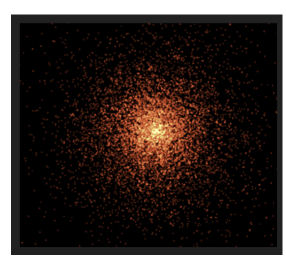

FIG. 6. A visualization of the companion galaxy (Plummer sphere) in glnemo2.

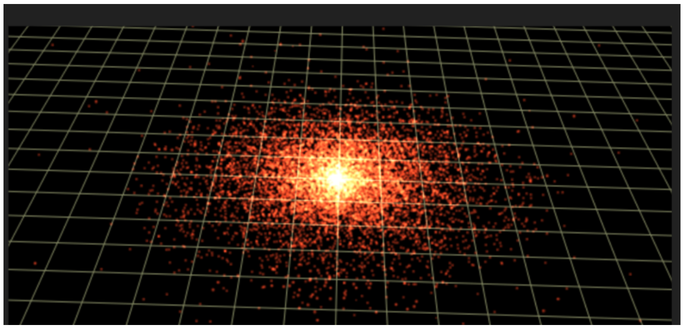

FIG. 7. A visualization of the target galaxy, modeled as a combination of a disk, a halo, and a bulge.

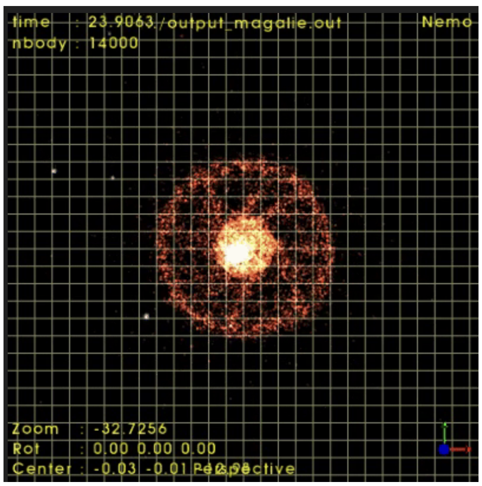

FIG. 8. A closer look at the formation of spokes in the simulated cartwheel galaxy, as seen in a head-on collision. There is also a second ring emerging.

(visualized in glnemo2) alongside an analogous collision simulation result shown by Athanassoula et. al. The initial conditions used here are shown in the "CVS" row of figure 2. A closer look at the resulting target galaxy is shown in Figure 8. This figure shows the formation of a second ring after the collision, as well as the formation of spokes (that appear as thick lines going from the center to the outer ring).

Figure 10 demonstrates an example simulation of an oblique collision, where the intruder galaxy approaches the target at a different angle (instead of the head-on collision investigated in the midterm project). This time, we can still see the formation of spokes near the end of the impact. Furthermore, the overall shape of the cartwheel galaxy is more distorted and irregular.

Figure 11 shows a plot of the potential, kinetic, and total energies over time for the oblique collision. The pattern and trends in the graph are identical for head-on collisions as well. The total energy is positive, and the total energy is shown to be conserved as it stays constant throughout the simulation. The kinetic energy increases (and the potential energy decreases) when the companion gets close to the target galaxy, due to the gravitational acceleration. There is an interesting "spike" observed at the very beginning of the simulation, but this is mainly due to a few points on the Plummer sphere shooting out of the sphere due to having a very large velocity. This spike was found to reduce when using a larger softening parameter.

IV. CONCLUSION

We were able to make a successful N-body simulation of the formation of the Cartwheel galaxy using NEMO tools. Unlike the very simplified restricted three-body simulation we created for the midterm project, our N-body simulation was more realistic and accounted for gravitational interaction between each of the particles. Our model also gave a deeper insight into the behavior of particles in the companion galaxy during the simulation, as some particles were seen to merge with the target galaxy (which is typically expected in a such a galaxy collision) while other particles bounced back further away.

Furthermore, our simulations involve analyzing collisions at varying angles and varying orbits of the companion galaxy.

For further improvements on this project, we can investigate the spike in kinetic energy at the start of the energy graph. If we can figure out why the model is doing that, we can address the error in the model's creation

Another possible extension to this project would be to continue implementing the various initial conditions described in the paper Athanassoula et al., 1997. The authors present many different models for the disk, halo, bulge, and companion that can be used as well as adjustments to the mass, radii, number of particles, and other initial conditions. It could give us more insights into the formation of the Cartwheel galaxy to experiment with the various combinations of conditions and models from the paper.

V. CONTRIBUTIONS

All three authors contributed equally towards writing this manuscript.

- **Girish Krishnan**: Girish worked on the units, initial conditions, and wrote bash scripts to run multiple simulations and visualize the results shown on this report.
- **Ishaan Kavoori**: Ishaan worked on setting up simulations and generating galaxies using the

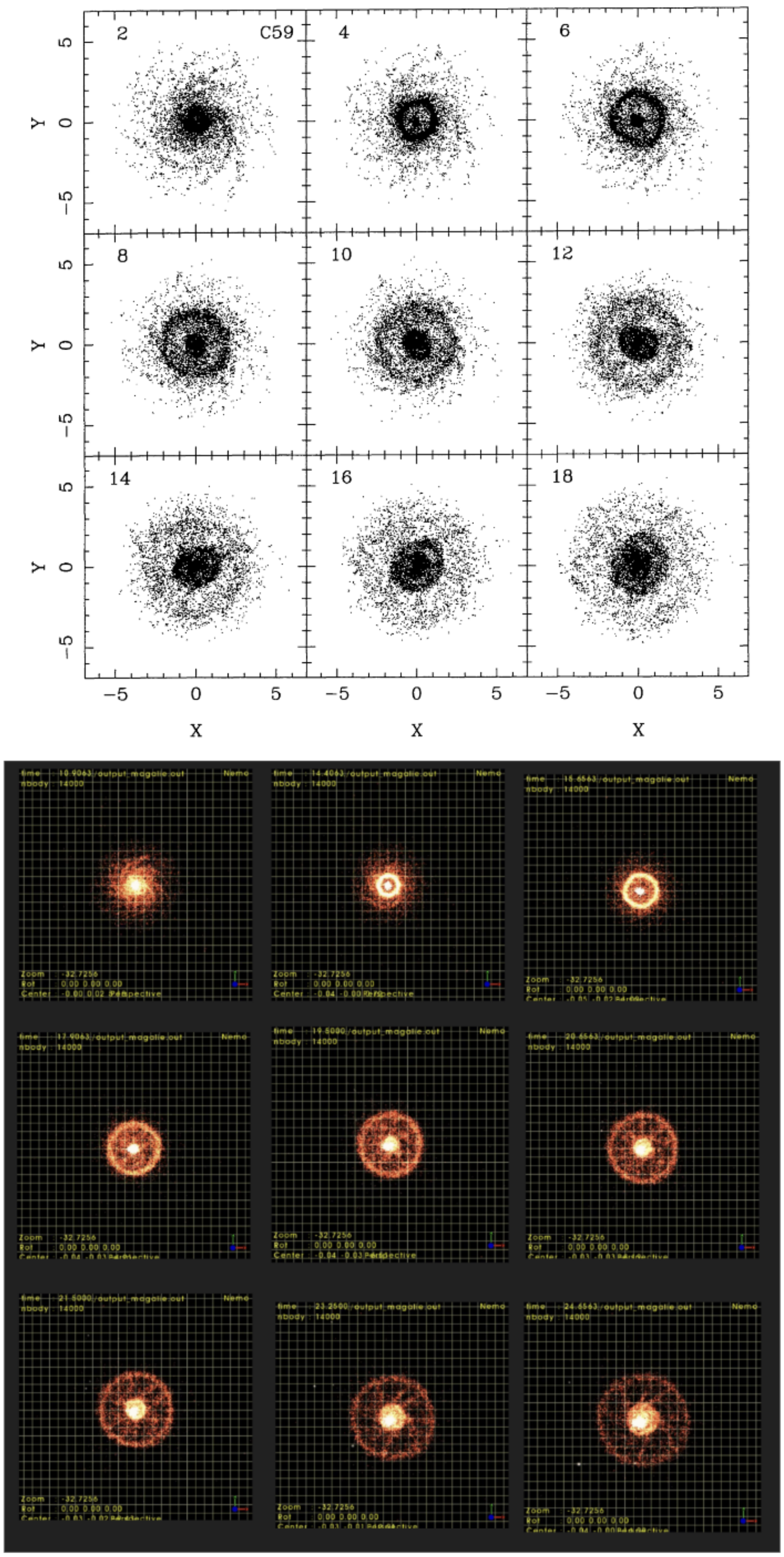

FIG. 9. Comparison between our simulation for a head-on collision between the companion and the target galaxy, against the results shown by Athanassoula et. al.

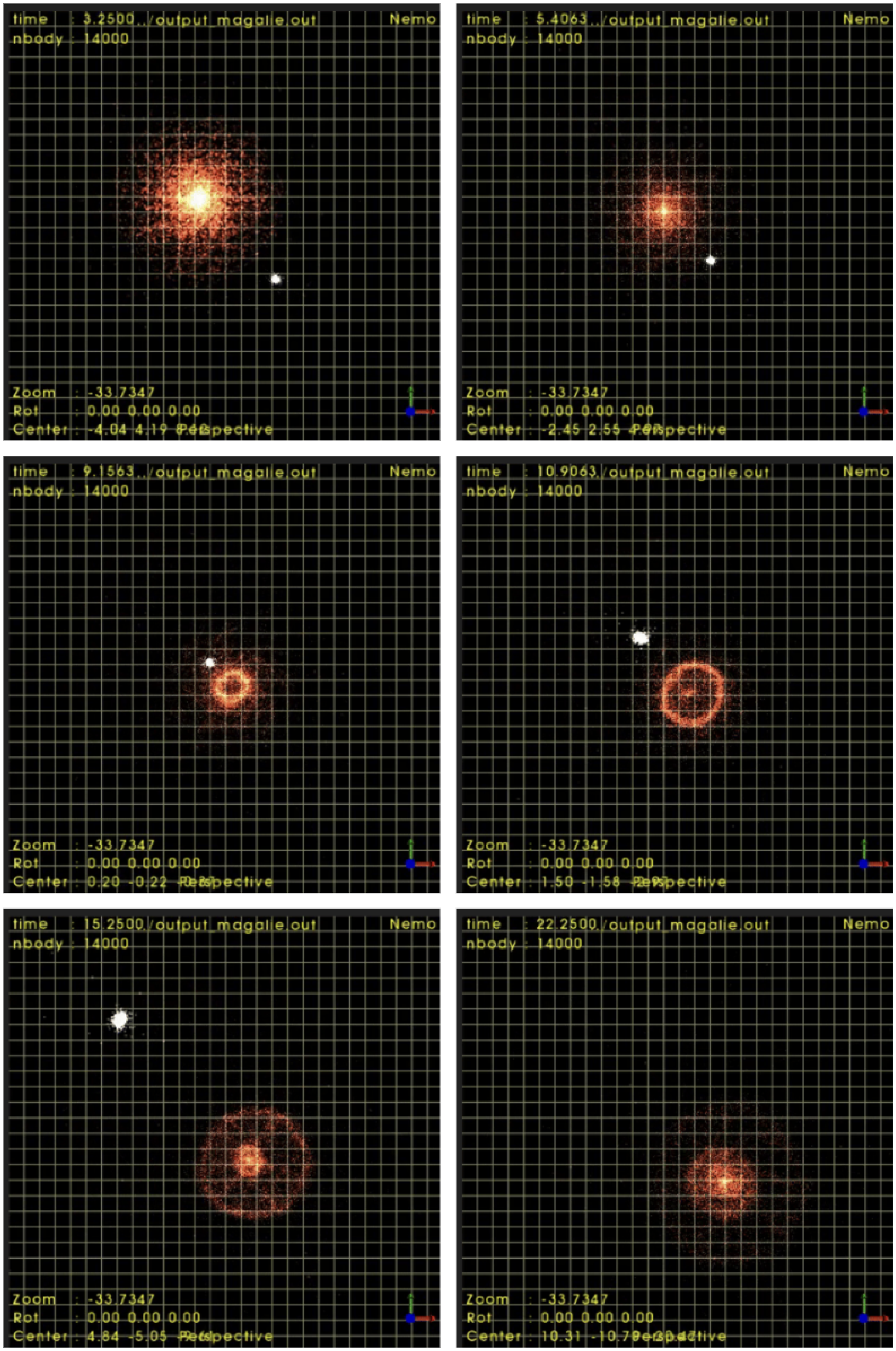

FIG. 10. Simulation of an oblique collision, where the companion galaxy strikes the target galaxy at a different angle. The companion galaxy hits the target galaxy at roughly the center of the disc.

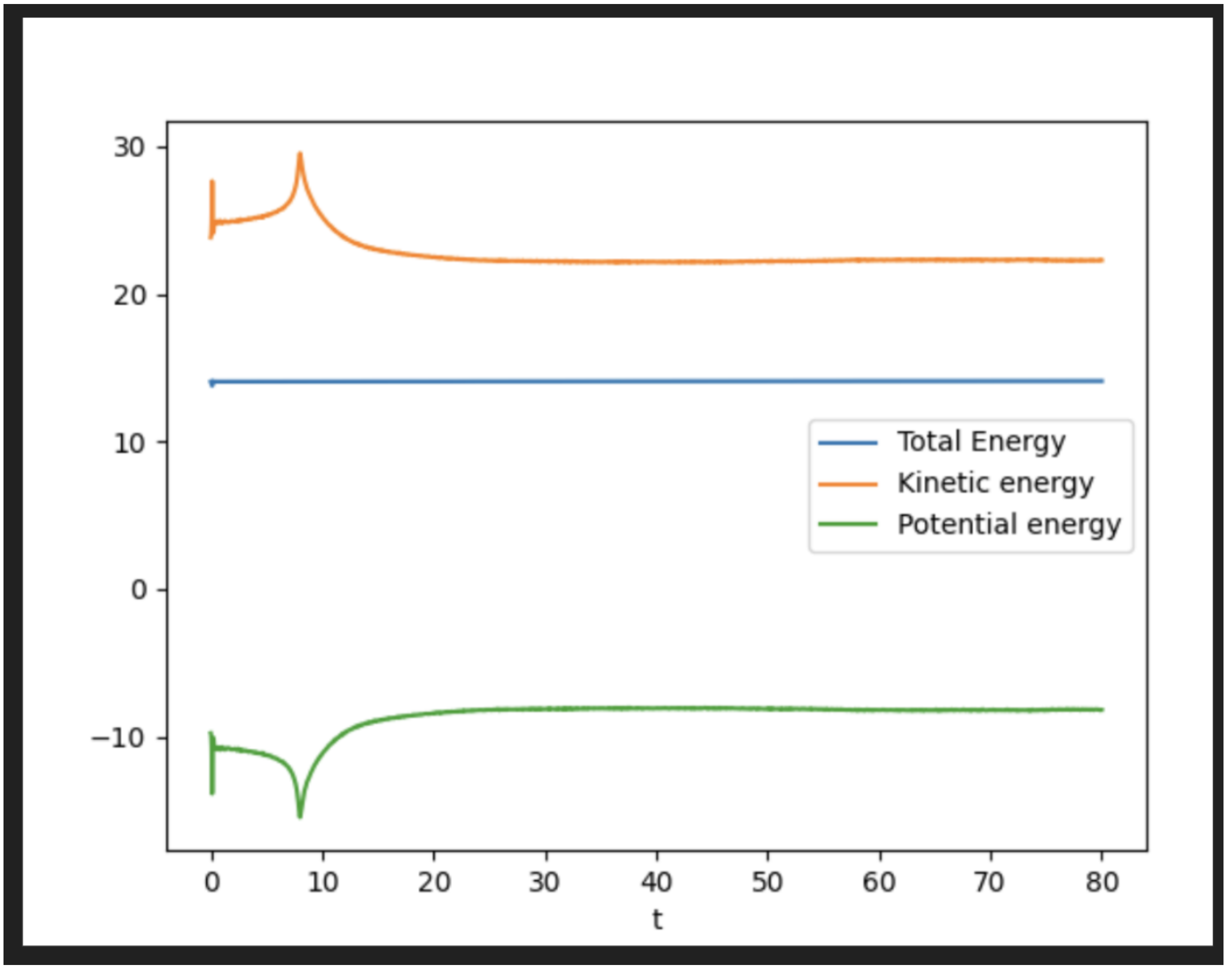

FIG. 11. Plot showing the potential, kinetic, and total energies over time.

model in Athanassoula et al., 1997.

- **Gokul Swaminathan**: Gokul improved our collaborative workflow by setting up a VNC that allows all team members to remote access a system to collaborate on code.

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