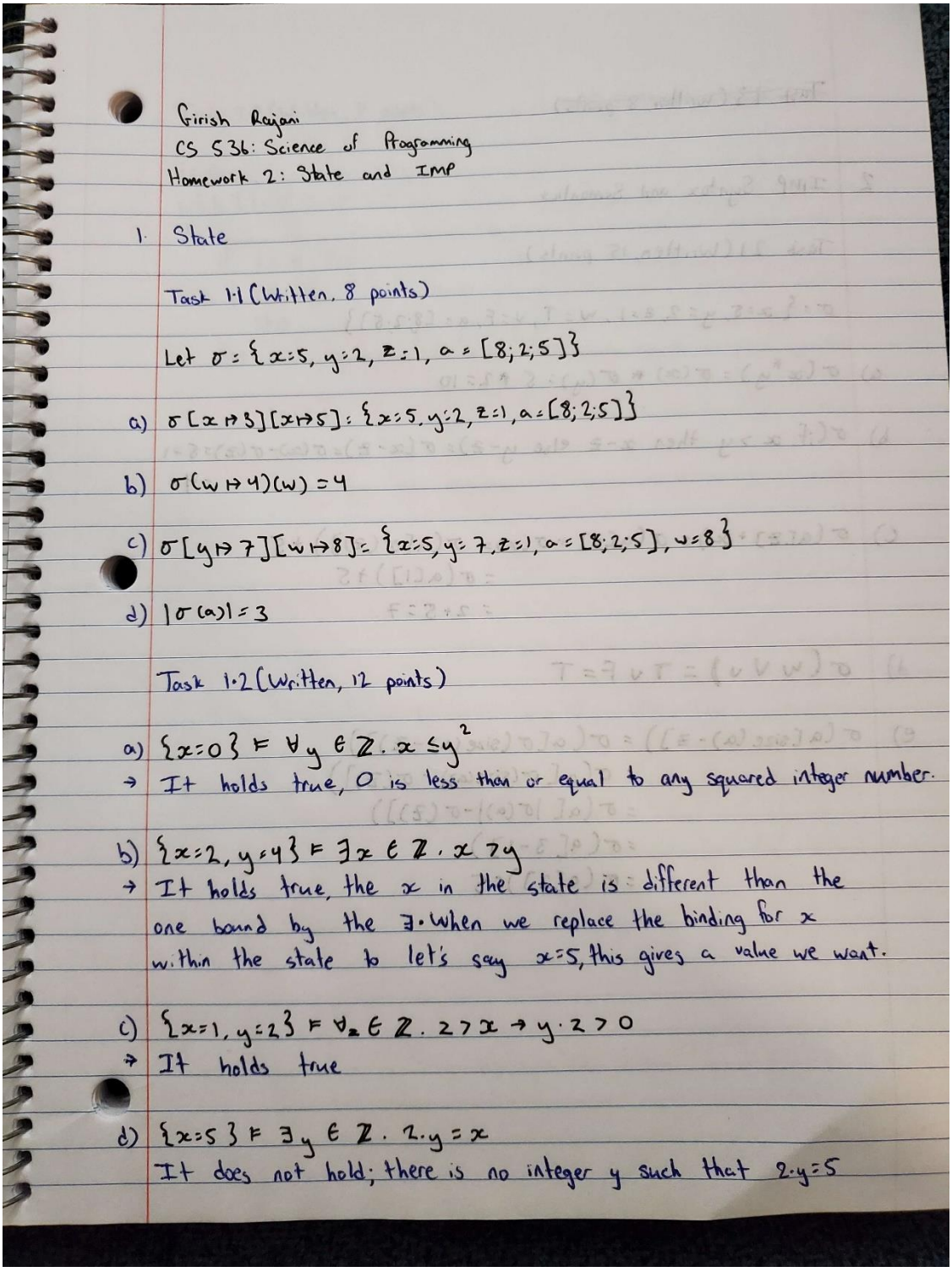
CS 536 – Science of Programming

Homework 2 – State and IMP

Girish Rajani

A20503736



Girish Rajani
CS 536: Science of Programming
Homework 2: State and IMP

1. State

Task 1.1 (Written, 8 points)

Let $\sigma = \{x=5, y=2, z=1, a=[8;2;5]\}$

a) $\sigma[x \mapsto 3][x \mapsto 5] = \{x=5, y=2, z=1, a=[8;2;5]\}$

b) $\sigma(w \mapsto 4)(w) = 4$

c) $\sigma[y \mapsto 7][w \mapsto 8] = \{x=5, y=7, z=1, a=[8;2;5], w=8\}$

d) $|\sigma(a)| = 3$

Task 1.2 (Written, 12 points)

a) $\{x=0\} \models \forall y \in \mathbb{Z}. x \leq y^2$
→ It holds true, 0 is less than or equal to any squared integer number.

b) $\{x=2, y=4\} \models \exists x \in \mathbb{Z}. x > y$
→ It holds true, the x in the state is different than the one bound by the $\exists$. When we replace the binding for x within the state to let's say $x=5$, this gives a value we want.

c) $\{x=1, y=2\} \models \forall_z \in \mathbb{Z}. z > x \rightarrow y \cdot z > 0$
→ It holds true

d) $\{x=5\} \models \exists y \in \mathbb{Z}. 2 \cdot y = x$
It does not hold; there is no integer y such that $2 \cdot y = 5$

Task 1.3 (Written, 8 points).

a) $\models \exists x \in \mathbb{Z}. \forall y \in \mathbb{Z}. p$ if for **all** states σ, it is true that $\sigma[x \mapsto \alpha_1][y \mapsto \alpha_2] \models p$ for **some** $\alpha_1 \in \mathbb{Z}$ and **all** $\alpha_2 \in \mathbb{Z}$.

b) $\models \neg(\forall x \in \mathbb{Z}. \exists y \in \mathbb{Z}. q)$ if for **all** states σ, it is true that $\sigma[x \mapsto \alpha_1][y \mapsto \alpha_2] \models p$ for **some** $\alpha_1 \in \mathbb{Z}$ and **all** $\alpha_2 \in \mathbb{Z}$.

2. IMP Syntax and Semantics

Task 2.1 (Written, 15 points).

$\sigma = \{x:5, y:2, z:1, w:T, v:F, a:[8;2;5]\}$

a) $\sigma(x * y) = \sigma(x) * \sigma(y) = 5 * 2 = 10$

b) $\sigma(\text{if } x > y \text{ then } x - z \text{ else } y - z) = \sigma(x - z) = \sigma(x) - \sigma(z) = 5 - 1 = 4$

c) $\sigma(a[z] + x) = \sigma(a[z] + \sigma(x) = \sigma(a[\sigma(z)]) + 5$
$= \sigma(a[1]) + 5$
$= 2 + 5 = 7$

d) $\sigma(w \vee v) = T \vee F = T$

e) $\sigma(a[\text{size}(a) - z]) = \sigma(a[\sigma(\text{size}(a) - z)])$
$= \sigma(a[\sigma(\text{size}(a)) - \sigma(z)])$
$= \sigma(a[|\sigma(a)| - \sigma(z)])$
$= \sigma(a[3 - 1])$
$= \sigma(a[2]) = 5$

Task 2.2 (Written, 8 points)

```
i=0;
while (i<size(a))
do
    if i<x then
        i=i+1;
        skip
    else
        a[i]=0
            i=i+1
od
```

The program starts at initial index and iterates through the array. If the index in array a is less than length x, don't do anything then go to the next index. Once the index is equal to x, change the value of that index and all elements after that to 0. This satisfies the condition if $x \geq |a|$, the array doesn't change because of the while loop.

Task 2.3 (Written, 14 points)

S = while $x > y$ do $x := y$ od $\quad \sigma = \{x=3, y=2\}$

a) Using rule 5 from class, change while to if with while:

$\langle S, \{x=3, y=2\} \rangle \rightarrow \langle$ if $x > y$ then $x := y$; S else skip, $\sigma \rangle$

Using rule 3 from class to check if condition:

$\rightarrow \langle x := y; S, \sigma \rangle$

Using rule 6 from class to perform the sequences:

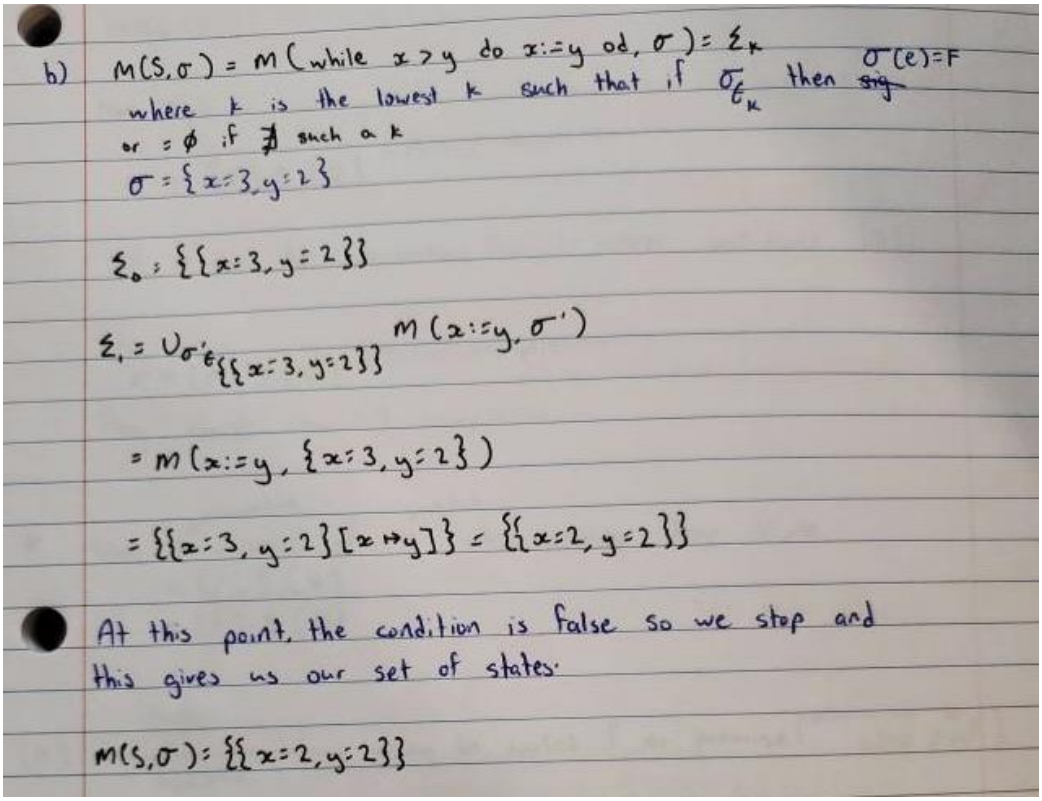
$\rightarrow \langle$ skip; S, $\sigma[x \mapsto 2] \rangle$
$\rightarrow \langle S, \{x=2, y=2\} \rangle$

Check if condition with new state:

$\rightarrow \langle$ if $x > y$ then $x := y$; S else skip, $\{x=2, y=2\} \rangle$

Using rule 4 from class since condition is false: $\sigma(e) = F$

$\rightarrow \langle$ skip, $\{x=2, y=2\} \rangle$



Task 3.1

Took appx 4-5 hours