OPERATION RESEARCH

Course Code: 21ME753

MODULE 1:

Introduction, Formulation of OR, Definition of OR, Scope of Or, Phases in OR study, Characteristics and limitations of OR, Models used in OR, Linear Programming Problem(LPP), Generalized LPP, Formulation of problems as LPP, Solutions to LPP by graphical method (Two variables).

Formulating an Operations Research problem involves:

- **Defining objectives**: Identify and quantify the objectives that need to be met.
- **Defining courses of action**: Define the possible courses of action, including the controlled variables and their constraints.
- **Identifying uncontrolled variables**: Identify any relevant uncontrolled variables.
- **Devising a performance measure**: Create a suitable measure of performance.
- **Understanding the system**: Gain a detailed understanding of how the system operates and its environment.
- **Selecting a decision criterion**: Choose an appropriate decision criterion based on the knowledge of possible outcomes.

Definition of OR

An operation research (OR) is a discipline that uses analytical methods to improve decision-making. It often uses techniques from other mathematical sciences, such as modelling, optimization, and statistics. The process of operations research can be broken down into the following steps:

1. Identify a problem
2. Construct a model of the problem
3. Use the model to derive solutions

Scope of Operation Research

In recent years of organized development, OR has entered successfully in many different areas of research. It is useful in the following various important fields

In agriculture

With the sudden increase of population and resulting shortage of food, every country is facing the problem of

- Optimum allocation of land to a variety of crops as per the climatic conditions
- Optimum distribution of water from numerous resources like canal for irrigation purposes

Hence there is a requirement of determining best policies under the given restrictions. Therefore a good quantity of work can be done in this direction.

In finance

In these recent times of economic crisis, it has become very essential for every government to do a careful planning for the economic progress of the country. OR techniques can be productively applied

- To determine the profit plan for the company
- To maximize the per capita income with least amount of resources
- To decide on the best replacement policies, etc

In industry

If the industry manager makes his policies simply on the basis of his past experience and a day approaches when he gets retirement, then a serious loss is encounter ahead of the industry. This heavy loss can be right away compensated through appointing a young specialist of OR techniques in business management. Thus OR is helpful for the industry director in deciding optimum distribution of several limited resources like men, machines, material, etc. to reach at the optimum decision.

In marketing

With the assistance of OR techniques a marketing administrator can decide upon

- Where to allocate the products for sale so that the total cost of transportation is set to be minimum
- The minimum per unit sale price
- The size of the stock to come across with the future demand
- How to choose the best advertising media with respect to cost, time etc.?
- How, when and what to buy at the minimum likely cost?

In personnel management

A personnel manager can utilize OR techniques

- To appoint the highly suitable person on minimum salary
- To know the best age of retirement for the employees
- To find out the number of persons appointed in full time basis when the workload is seasonal

In production management

A production manager can utilize OR techniques

- To calculate the number and size of the items to be produced
- In scheduling and sequencing the production machines
- In computing the optimum product mix
- To choose, locate and design the sites for the production plans

In L.I.C

OR approach is also applicable to facilitate the L.I.C offices to decide

- What should be the premium rates for a range of policies?
- How well the profits could be allocated in the cases of with profit policies?

The phases of operations research are:

- Problem formulation: Define the objective, identify the factors that affect it, and specify the constraints on the course of action
- Model construction: Create a model to reduce the complexities of the decision-making situation to a logical framework
- Deriving solutions: Use simulation or optimum seeking techniques to derive solutions from the model
- Testing the model: Compare the model's values to actual values under different conditions to see if the model needs to be updated
- Controlling the solution: Control the solution
- Implementation: Make implementation part of the research plan

❖ Main Characteristics of Operation Research

Operations research (OR) are a collection of techniques used to analyse and solve problems related to complex systems. Some of its main characteristics include:

- Quantitative analysis: OR uses quantitative analysis to solve problems.
- Interdisciplinary approach: OR is an interdisciplinary approach that uses a variety of techniques to solve problems.
- Decision support: OR provides decision support by helping decision-makers evaluate the consequences of different options.
- Model building: OR uses models to represent complex decision problems, including uncertainties, interdependencies, and multiple variables.
- Optimization: OR uses optimization to solve decision problems.
- Problem-solving: OR uses a variety of techniques to solve problems related to the design and operation of complex systems.
- Experimentation: OR uses experimentation as a characteristic
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Advantages and Disadvantages of Operation Research

Operations Research (OR) is a problem-solving and decision-making analytical approach beneficial in organizational management. In operations research, issues are deconstructed into their simplest forms before being mathematically analysed and resolved in a set of processes. Operational research aims to achieve the greatest performance possible given the available conditions. Any firm that uses scarce resources has to allocate them as effectively as possible; operations research helps in this process.
Operations research has many advantages, but it also has its disadvantages. Before using operations research in a business decision, entrepreneurs must become familiar with all sides of the domain.

Advantages of Operation Research

- **Better Systems**
  An O.R. technique is frequently adopted to examine a specific decision-making issue, such as the best factory site or whether to construct a new warehouse. Additionally, it helps select cost-effective transportation methods, work sequencing, production planning, replacement of out-dated machinery, etc.
- **Enhanced Productivity**
  Operations research contributes to boosting an organization's production. Before making a critical choice, operations controls provide managers with valuable information. It helps a business make small decisions that build up to big ones. Many businesses employ simulation operations research techniques to increase productivity by combining them in various ways.
  Daily duties like inventory management, personnel planning, business development, installation and technological upgrades are also made easier with its aid. Making correct and effective decisions aids in increasing an organization's productivity.
- **Best Control**
  The management of big firms understands how hard and expensive it is to supervise every regular task continuously. An O.R. method could give the executive a quantitative and analytical foundation to pinpoint the issue. Applications in this area that are implemented most commonly deal with inventory replenishment and production schedule.

- **Provides many alternatives**
  Management is in charge of making critical organizational choices. Operations research offers several solutions to a single issue, assisting management in selecting the best option and putting it into practice to achieve the desired result.
  Managers can assess the risks connected to each option and then choose whether or not to implement it.
- **Improved Coordination**
  The coordination between a company's many departments and employees is improved by operations research because operations research concentrates not on a single department but the entire organization.
  As a result, managers in each area know what must be done to advance the organization's overall goal. As a result, when solutions are implemented across all departments, managers of various departments may collaborate better.
- **Better Decision making**
  O.R. models increase decision-making and lower the possibility of making bad decisions. The O.R. technique improves the executive's understanding of his decision-making process.
- **Reduce chances of failure**
  Operations and research managers discover all potential solutions for a given problem with the aid of operations, reducing the risk of failure.
  Before adopting a solution, all potential hazards are examined. As a result, until an unforeseen event occurs, the probability of failure decreases.

Disadvantages of Operation Research

- **Costly**
  The main drawback of operations research is its high price. Operations research uses mathematical equations that are created using pricey technologies. In addition, simulations require the assistance of professionals.
  Even though the cost would be quite expensive, all of this may offer useful answers. Companies with limited resources can use operations research due to the high implementation costs.
- **Dependence on a computerized system**
  O.R. methods look for the best solution while considering all the variables. Due to the size of these components in contemporary society, describing them quantitatively and developing links between them calls for extensive computations that computers can only perform.
- **Unquantifiable Elements**
  OR only gives a solution when all aspects of an issue can be quantified. Not all pertinent variables are amenable to quantification. In OR studies, factors that cannot be measured have no place. Models in OR do not consider emotional or qualitative elements, which may be quite significant.
- **Distance between Manager & Operations Researcher**
  O.R. specialists have to be mathematicians or statisticians who may not be familiar with business issues. Similarly, a manager cannot comprehend the intricate functioning of O.R. There is a gap t between the two.
- **Implementation**
  Any decision's implementation is a sensitive task. It must account for the intricacies of human relationships and behaviour. Sometimes resistance is presented due to psychological issues that have nothing to do with the problem or its solution.

Introduction: Evolution of OR, Definitions of OR, Scope of OR, Applications of OR, Phases in OR study. Characteristics and limitations of OR, models used in OR, Linear Programming Problem (LPP), Generalized LPP- Formulation of problems as L.P.P. Solutions to LPP by graphical method (Two Variables).

Phases of OR

1. **Defining the Problem and Gathering Data**
   - Analyze the system, define objectives, constraints, and alternatives.
   - Collect relevant data using methods like data mining for an accurate understanding.
2. **Formulating a Mathematical Model**
   - Represent the problem using mathematical symbols:
     - **Decision Variables:** Quantifiable decisions (e.g., x_1, x_2).
     - **Objective Function:** Performance measure (e.g., $P = 3x_1 + 5x_2$).
     - **Constraints:** Restrictions (e.g., $x_1 + 2x_2 \geq 20$).
     - **Parameters:** Constants in constraints.
   - Models simplify and clarify problems, reveal relationships, and facilitate computer analysis.
3. **Deriving Solutions from the Model**
   - Aim for optimal solutions that balance cost, time, and profit.
   - Techniques include heuristic procedures, metaheuristics, and post-optimality (what-if) analysis.
4. **Testing the Model and its Solutions**
   - Validate the model using retrospective tests (historical data comparison).
   - Ensure reliability and identify errors with the involvement of independent reviewers.
5. **Preparing to Apply the Model**
   - Establish a computer-based system with databases and Decision Support Systems (DSS).
   - Provide a managerial report for application insights.

6. **Implementation**
   - Collaborate with management to explain, monitor, and execute the new system.
   - Collect feedback and document processes for refinement.

Assumptions made in Linear Programming Problems (LPP):

1. **Proportionality (Linearity)**
   - The contribution of each decision variable to the objective function and constraints is directly proportional to its value. For example, doubling the variable's value doubles its contribution.
2. **Additivity**
   - The total value of the objective function or a constraint is the sum of the individual contributions of the decision variables. Interactions or combinations of variables are not considered.
3. **Multiplicativity**
   - There is no interaction between decision variables. Each variable's contribution is independent, and the model does not consider any product terms (e.g., x1×x2x_1 \times x_2x1×x2).
4. **Divisibility**
   - Decision variables can take fractional values, meaning that solutions are continuous. (If integer solutions are required, it becomes an Integer Linear Programming problem.)
5. **Deterministic**
   - All coefficients in the objective function and constraints are known with certainty and remain constant during the problem-solving process. There is no uncertainty in data.

LPP: Simplex method, Canonical and Standard form of LP problem, slack, surplus and artificial variables, Solutions to LPP by Simplex method, Big-M Method and two-phase Simplex Method, Degeneracy in LPP. Concept of Duality, writing Dual of given LPP. Solutions to L.P.P by Dual Simplex Method.

Explain the Following

I. **Slack variable**
II. **Surplus Variable**

III. **Artificial variable**

I. Slack Variable

A slack variable is introduced in a **less-than-or-equal-to (≤)** constraint to convert it into an equality constraint. It represents the unused resources in a system and is always non-negative.

Where It Is Used:

Slack variables are used in **Linear Programming (LP)** problems to make constraints suitable for the simplex method.

Example: For the constraint $2x + y \leq 8$, a slack variable $s \geq 0$ is added:

$2x + y + s = 8$

Here, s indicates the unused capacity.

II. Surplus Variable

A surplus variable is introduced in a **greater-than-or-equal-to (≥)** constraint to convert it into an equality constraint. It represents the excess amount or resources beyond the required limit.

Where It Is Used:

Surplus variables are used in **LP problems** where constraints involve excess resources, helping to simplify the problem for the simplex method.

Example: For the constraint $3x + 2y \geq 10$, a surplus variable $s \geq 0$ is subtracted:

$3x + 2y - s = 10$

Here, s represents the extra value exceeding the minimum requirement.

III. Artificial Variable

An artificial variable is introduced in equality constraints or **greater-than-or-equal-to** (≥) constraints to help find an initial feasible solution in the simplex method. It is used temporarily and eliminated later through techniques like the **Big-M Method** or **Two-Phase Method**.

Where It Is Used:

Artificial variables are used in the **simplex method** when a basic feasible solution is not readily available, particularly in **≥ constraints** or equality constraints.

Example: For the equality constraint $x + y = 5$, an artificial variable $a \geq 0$ is introduced:

$x + y + a = 5$

Artificial variables are used only to ensure a starting solution and have no physical significance in the final model.

Transportation Problem: Formulation of transportation problem, types, initial basic feasible solution using North-West Corner rule, Vogel's Approximation method. Optimality in Transportation problem by Modified Distribution (MODI) method. Unbalanced T.P. Maximization T.P. Degeneracy in transportation problems, application of transportation problem. Assignment Problem-Formulation, Solutions to assignment problems by Hungarian method, Special cases in assignment problems, unbalanced, Maximization assignment problems. Travelling Salesman Problem (TSP). Difference between assignment and T.S.P, Finding best route by Little's method. Numerical Problems.

5	a.	With the aid of simple examples, describe the procedure to be adopted to balance, (i) Transportation problem. (ii) Assignment problem (08 Mark

1. Balancing a Transportation Problem

The transportation problem deals with distributing goods from multiple suppliers to multiple consumers at the lowest transportation cost. A transportation problem is said to be balanced if the total supply equals the total demand. If not, balancing is required.

Procedure:

1. **Identify Imbalance**: Check if the total supply equals total demand.

2. **Add Dummy Supply/Demand**:
   - If total supply > total demand, add a dummy demand with the deficit amount.
   - If total demand > total supply, add a dummy supply with the deficit amount.
3. **Assign Costs**: Assign zero transportation cost to all routes associated with the dummy row or column.
4. **Solve the Problem**: Proceed with solving using methods like the Northwest Corner Rule, Least Cost Method, or Vogel's Approximation Method.

Example:

Supply/Demand	D1	D2	D3	Supply
S1	4	6	8	50
S2	2	5	3	40
Demand	30	30	20	80

- Total supply = 50 + 40 = 90
- Total demand = 30 + 30 + 20 = 80
- Imbalance: Total supply > Total demand.
- Add a dummy demand (D4) with demand = 90 - 80 = 10.

Supply/Demand	D1	D2	D3	D4 (Dummy)	Supply
S1	4	6	8	0	50
S2	2	5	3	0	40
Demand	30	30	20	10	90

2. Balancing an Assignment Problem

The assignment problem deals with assigning tasks to agents such that the total cost/time is minimized. It is said to be balanced if the number of tasks equals the number of agents. If not, balancing is required.

Procedure:

1. **Identify Imbalance**: Check if the number of agents equals the number of tasks.
2. **Add Dummy Row/Column**:
   - If agents > tasks, add a dummy column (task) with zero cost.
   - If tasks > agents, add a dummy row (agent) with zero cost.
3. **Solve the Problem**: Proceed using methods like the Hungarian Algorithm.

Example:

Agent/Task	**T1**	**T2**	**T3**	**Cost**
A1	5	9	6	
A2	4	7	8	
Cost				

- 3 tasks, 2 agents → Imbalance.
- Add a dummy agent (A3) with zero costs.

Agent/Task	**T1**	**T2**	**T3**	**Cost**
A1	5	9	6	
A2	4	7	8	
A3 (Dummy)	0	0	0	

6 a. What do you understand by a balanced and unbalanced transportation problem? How an unbalanced problem is tackled? **(06 Marks)**

Balanced vs. Unbalanced Transportation Problem

- **Balanced Transportation Problem**: A transportation problem where the total supply equals the total demand. It can be solved directly without any modifications.
- **Unbalanced Transportation Problem**: A transportation problem where the total supply does not equal the total demand. It requires adjustments to balance before solving.

How to Handle an Unbalanced Problem

1. **Identify the Imbalance**:
   - Calculate the total supply and total demand.
   - Check whether total supply exceeds total demand or vice versa.
2. **Add a Dummy Node**:
   - If total supply > total demand, add a **dummy demand** with a value equal to the excess supply.
   - If total demand > total supply, add a **dummy supply** with a value equal to the excess demand.
   - Assign a transportation cost of zero for all routes involving the dummy node.
3. **Solve the Balanced Problem**:
   - Use methods like the **Northwest Corner Rule**, **Least Cost Method**, or **Vogel's Approximation Method** to obtain an initial feasible solution.
   - Optimize the solution using the **Stepping Stone Method** or **MODI Method**.

5	a.	What is degeneracy in transportation problems and how it is resolved?	(06 Marks)

Degeneracy in Transportation Problem

Definition:

Degeneracy in a transportation problem occurs when the number of basic variables (occupied cells) in a feasible solution is less than m+n−1, where mmm is the number of rows (supply points) and n is the number of columns (demand points) in the transportation table.

- Basic variables are the variables assigned a value during the initial solution process.
- Degeneracy leads to difficulty in applying optimization methods (e.g., MODI method) as the solution becomes non-unique or invalid.

Causes of Degeneracy

1. Supply and demand values result in fewer occupied cells.
2. Dummy rows/columns added for balancing the problem still lead to insufficient allocations.
3. A tie in minimum cost allocation prevents filling all required cells.

Resolving Degeneracy

To resolve degeneracy, introduce **artificial allocations** (fictitious allocations) to empty cells to ensure the number of occupied cells equals m+n−1.

Steps to Resolve:

1. **Identify Missing Basic Variables**:
    - Count the current number of occupied cells.
    - If it's less than m+n−1 ,degeneracy exists.

2. **Add Artificial Allocations**:
   - Select empty cells arbitrarily (preferably with the least cost).
   - Assign a very small value (e.g., ϵ) to these cells to make them occupied. This value is treated as zero for calculations.
3. **Proceed with the Solution**:
   - Solve the modified problem using methods like the Stepping Stone or MODI method.

6	a.	What is assignment problem? How does it differ from a transportation problem?	(06 Marks)

Assignment Problem

The **assignment problem** is a special type of optimization problem where tasks (or jobs) are assigned to agents (or resources) in a way that minimizes the total cost (or maximizes the total profit).

- Each task must be assigned to exactly one agent, and each agent can handle exactly one task.
- The problem is represented in a cost matrix, where each element represents the cost of assigning a task to an agent.

Key Characteristics:

1. Square cost matrix (same number of tasks and agents).
2. Objective: Minimize cost or maximize profit.
3. Solved using methods like the **Hungarian Algorithm**.

Transportation Problem

The **transportation problem** involves finding the most cost-effective way to transport goods from multiple supply points (sources) to multiple demand points (destinations) while satisfying supply and demand constraints.

- The problem is represented in a cost matrix where each element represents the cost of transporting goods from a source to a destination.

Key Characteristics:

1. Typically involves a rectangular cost matrix (number of sources ≠ number of destinations).
2. Objective: Minimize total transportation cost.
3. Solved using methods like the **Northwest Corner Rule**, **Least Cost Method**, or **Vogel's Approximation Method**.

Differences Between Assignment and Transportation Problems

Aspect	Assignment Problem	Transportation Problem
Nature	Tasks are assigned to agents.	Goods are transported from supply points to demand points.
Matrix Size	Square matrix (equal tasks and agents).	Rectangular matrix (sources and destinations may differ).
Objective	Minimize assignment cost or maximize profit.	Minimize transportation cost.
Constraints	Each task assigned to one agent; each agent performs one task.	Total supply and demand constraints.
Methods for Solution	Hungarian Algorithm.	Northwest Corner Rule, Least Cost Method, Vogel's Method.

Balancing Required?	Typically balanced.	May require balancing (dummy rows/columns).

Network analysis: Introduction, Construction of networks, Fulkerson's rule for numbering the nodes, AON and AOA diagrams; Critical path method to find the expected completion time of a project, determination of floats in networks, PERT networks, determining the probability of completing a project, predicting the completion time of project; Cost analysis in networks. Crashing of networks- Problems. Queuing Theory: Queuing systems and their characteristics, Pure-birth and Pure-death models (only equations), Kendall & Lee's notation of Queuing, empirical queuing models – Numerical on M/M/1 and M/M/C Queuing models.

7	a.	What is critical path? How does it help project manager?	**(04 Marks)**

Critical Path

The **critical path** is the longest sequence of dependent tasks in a project that determines the shortest time in which the project can be completed. Any delay in the tasks on this path directly delays the entire project.

Key Characteristics:

1. It includes tasks with **zero float/slack** (i.e., tasks that cannot be delayed without affecting the project timeline).
2. Critical tasks are crucial for timely project completion.
3. Identified using **Critical Path Method (CPM)**, which involves:
   - Listing tasks, their dependencies, and durations.
   - Building a project network diagram.
   - Calculating early start (ES), early finish (EF), late start (LS), and late finish (LF) for each task.

How Does the Critical Path Help a Project Manager?

1. **Identifies Key Activities**:
   Helps the manager focus on tasks that directly impact project deadlines.
2. **Efficient Resource Allocation**:
   Ensures resources are prioritized for critical tasks to prevent delays.
3. **Improved Scheduling**:
   Helps plan task timelines and identify dependencies to avoid bottlenecks.
4. **Risk Management**:
   Highlights high-risk tasks where delays are unacceptable, enabling proactive measures.
5. **Progress Monitoring**:
   Allows tracking of critical tasks to ensure they stay on schedule.
6. **Impact Assessment**:
   If changes or delays occur, the manager can assess how the critical path is affected and make adjustments.

8	a.	Explain Kendall's notations for representing queuing models.	**(04 Marks)**
	b.	Describe the characteristics of queuing models.	**(08 Marks)**

Kendall's Notation

Kendall's notation is a shorthand used to describe the characteristics of queuing systems, presented in the format:
A/B/c/K/N/D

Key Components:

1. **A (Arrival distribution):**
   Describes how customers arrive.
   - **M**: Markovian (random arrivals, exponential inter-arrival times).
   - **D**: Deterministic (fixed arrival intervals).

- **E_k**: Erlang distribution (generalized form for less randomness).
- **G**: General distribution (any unspecified distribution).

2. **B (Service distribution):**

   Describes how customers are served.

   - Uses the same notations as A (M, D, E_k, G).

3. **c (Number of servers):**

   Represents the number of service channels available (e.g., 1, 2, or more).

4. **K (System capacity):**

   Maximum number of customers allowed in the system (waiting + in service).

5. **N (Calling population):**

   Total number of potential customers (finite or infinite).

6. **D (Queue discipline):**

   Rule for selecting the next customer to be served:

   - **FIFO (FCFS):** First-In, First-Out.
   - **LIFO (LCFS):** Last-In, First-Out.
   - **SIRO:** Service in Random Order.
   - **Priority:** Based on customer priority levels.

Examples:

1. **M/M/1:**
   - Exponential arrivals, exponential service times, and 1 server.
2. **M/D/3:**
   - Exponential arrivals, deterministic service times, and 3 servers.

(b) Characteristics of Queuing Models

Queuing models analyze systems where entities arrive, wait if necessary, get served, and then leave. Key characteristics include:

1. Arrival Process:

- **Arrival rate (λ):** Average number of customers arriving per unit of time.
- **Interarrival times:** Time between successive arrivals, following specific distributions.

2. Queue Characteristics:

- **Queue length:** Number of customers waiting at any given time.
- **Queue capacity:** Maximum customers that the queue can hold.
- **Queue discipline:** Rule for serving customers (FIFO, LIFO, etc.).

3. Service Process:

- **Service rate (μ):** Average number of customers served per unit of time per server.
- **Service time distribution:** Describes how long it takes to serve a customer.
- **Number of servers:** Indicates single or multiple service channels.

4. System Capacity:

Total number of customers that can be in the system (waiting + in service).

5. Calling Population:

- **Finite:** Limited potential customers.
- **Infinite:** Unlimited or very large customer pool.

6. Performance Metrics:

Used to evaluate queuing systems:

- **Average waiting time in the queue (Wq).**
- **Average waiting time in the system (Ws).**

- **Average queue length (Lq).**
- **Average system length (Ls).**
- **Server utilization (U):** Proportion of time servers are busy.

7 a. Compare the PERT and CPM in network analysis. **(06 Marks)**

Aspect	**PERT**	**CPM**
Nature of Project	Best for research, development, and uncertain projects.	Best for construction, production, and routine projects.
Focus	Deals with **time uncertainty** and probabilistic estimates.	Focuses on **time-cost trade-offs** with deterministic estimates.
Time Estimates	Uses three-time estimates: Optimistic, Pessimistic, and Most Likely (O, P, M).	Uses a single-time estimate for activity durations.
Activity	Activities are not repetitive; they are unique and subject to variability.	Activities are repetitive and predictable.
Critical Path	Identifies critical path based on probabilistic data.	Identifies critical path using deterministic durations.
Emphasis	Time estimation and control.	Cost optimization and time-cost trade-off.
Uncertainty	Best for projects with high uncertainty.	Best for projects with well-defined and predictable tasks.
Application	Suitable for R&D, software development, and projects involving innovation.	Suitable for construction, production, and maintenance projects.
Complexity	More complex due to probabilistic calculations.	Less complex due to deterministic approach.

Slack/Float Calculation	Focuses on variability and flexibility of project timelines.	Focuses on identifying and reducing project costs.

8	a.	Explain various queuing disciplines and customer behaviours.	(08 Marks)

Queuing Disciplines

Queuing disciplines define the order in which customers in a queue are served. Common types include:

1. **First-Come, First-Served (FCFS)**
   - Customers are served in the order they arrive.
   - Common in banks, ticket counters, and supermarkets.
2. **Last-Come, First-Served (LCFS)**
   - The most recent customer is served first.
   - Common in emergency rooms or stack-like systems.
3. **Service in Random Order (SIRO)**
   - Customers are selected randomly for service.
   - Used in cases where fairness is emphasized, such as lotteries.
4. **Priority Queue**
   - Customers are served based on their assigned priority level.
   - High-priority tasks, such as medical emergencies, are served first.
5. **Shortest Job Next (SJN)**
   - Customers with the shortest service time are served first.
   - Common in computer processing for optimizing throughput.

6. **Round Robin (RR)**
   - Each customer gets a fixed time slot in a rotating order.
   - Frequently used in computer networks and operating systems.

7. **Earliest Due Date (EDD)**
   - Customers with the closest deadline are served first.
   - Useful in project scheduling and manufacturing.

Customer Behaviors in Queuing Systems

1. **Balking**
   - The customer decides not to join the queue after seeing its length or waiting time.
   - Example: A person avoids a long line at a grocery store.
2. **Reneging**
   - The customer joins the queue but leaves before being served due to impatience.
   - Example: A person leaving a restaurant waiting line after waiting too long.
3. **Jockeying**
   - The customer switches from one queue to another, hoping for faster service.
   - Example: Shifting between supermarket checkout lanes.
4. **Patient Behavior**
   - The customer remains in the queue until served, regardless of the waiting time.
   - Example: Waiting for a popular event ticket despite long delays.

5. **Collusion**
   - Customers cooperate or influence the system to gain unfair advantages.
   - Example: Letting someone jump ahead in the queue.

Game Theory: Definition, Pure Strategy problems, Saddle point, Max-Min and Min-Max criteria, Principle of Dominance, Solution of games with Saddle point. Mixed Strategy problems. Solution of 2X2 games by Arithmetic method, Solution of 2Xn m and mX2 games by graphical method. Formulation of games. Sequencing: Basic assumptions, Johnson's algorithm, sequencing 'n' jobs on single machine using priority rules, sequencing using Johnson's rule-'n' jobs on 2 machines, 'n' jobs on 3 machines, 'n' jobs on 'm' machines. Sequencing of 2 jobs on 'm' machines using graphical method.

Let us consider a **two-player zero-sum game** where Player 1 (the row player) and Player 2 (the column player) have strategies to choose from. The payoff matrix is as follows:

Player 1/Player 2	**Strategy A**	**Strategy B**
Strategy X	**3**	**1**
Strategy Y	**2**	**4**

1. Payoff Matrix

- Player 1's strategies: **Strategy X** and **Strategy Y** (rows).
- Player 2's strategies: **Strategy A** and **Strategy B** (columns).
- The values in the matrix are Player 1's payoffs (Player 2's payoffs are the negatives since it's a zero-sum game).
  Example: If Player 1 chooses Strategy X and Player 2 chooses Strategy A, Player 1 gets a payoff of 3, and Player 2 gets -3.

2. Saddle Point

A **saddle point** is determined by:

- **Row minimums**: The smallest value in each row.
  - For Row 1 (Strategy X): Min = **1**.
  - For Row 2 (Strategy Y): Min = **2**.
- **Column maximums**: The largest value in each column.
  - For Column 1 (Strategy A): Max = **3**.
  - For Column 2 (Strategy B): Max = **4**.

The **saddle point** is where the **row minimum equals the column maximum**.

- In this matrix, the saddle point is **4** at (Row 2, Column 2).

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3. Value of the Game

- The **value of the game** is **4**, which is the payoff at the saddle point.
- This means if both players play optimally, Player 1 will win 4 units, and Player 2 will lose 4 units.

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4. Pure Strategy Example

- If both players use **pure strategies**, they stick to one choice:
  - Player 1 chooses **Strategy Y**.
  - Player 2 chooses **Strategy B**.
  - Result: Payoff is **4**.

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5. Mixed Strategy Example

If the saddle point does not exist (no row minimum equals column maximum), players use a **mixed strategy** to assign probabilities to their choices. For example:

- Player 1: Choose Strategy X with 60% probability and Strategy Y with 40%.
- Player 2: Choose Strategy A with 50% probability and Strategy B with 50%.
  The expected payoff is calculated based on these probabilities.

Fair Game

- A game is **fair** if the **value of the game** is **0**.
  - In such cases, neither player has an inherent advantage.
  - The total gain of one player is equal to the total loss of the other player, and the net payoff is **0**.

Example for a Fair Game:

Consider the following **payoff matrix**:

Player 1 / Player 2	**Strategy A**	**Strategy B**
Strategy X	2	-2
Strategy Y	-3	3

1. For any optimal strategy chosen by both players:
   - If **Player 1** wins 2 units, **Player 2** loses 2 units, and vice versa.
2. **Value of the Game**: 0 (Fair Game).

Two-Person Zero-Sum Game

- A **two-person zero-sum game** is a competitive situation involving two players where:
  - **One player's gain is exactly equal to the other player's loss**.

  - The sum of payoffs for both players is always **0** for any combination of strategies.

Key Features:

- The total payoff (Player 1's payoff + Player 2's payoff) = 0.
- Players act rationally to **maximize their own gain** or **minimize their loss**.

Example of Two-Person Zero-Sum Game:

For the given **payoff matrix**:

Player 1 / Player 2	**Strategy A**	**Strategy B**
Strategy X	3	1
Strategy Y	2	4

1. **Player 1's Payoffs**: The matrix values represent Player 1’s gain.
2. **Player 2's Payoffs**: These are the negatives of Player 1’s payoffs:
   - If Player 1 chooses Strategy X and Player 2 chooses Strategy A, Player 2’s payoff is -3.

Properties:

- Total payoff:
  - For (Row 1, Column 1): $3+(-3)=0$.
  - For (Row 2, Column 2): $4+(-4)=0$.
- **Two-person zero-sum nature**: The game’s payoff matrix reflects the zero-sum property.

Assumptions Made while solving sequencing problem

- **Processing Times are Constant**
  - The processing times for all jobs on all machines are known in advance and remain constant, irrespective of the sequence of operations.
- **Negligible Transfer and Setup Time**
  - The time taken to move a job from one machine to another or for setup is either negligible or already included in the processing time.
- **Non-Interruptible Processing**
  - Once a job begins processing on a machine, it must be completed without interruptions before starting the next job.
- **Immediate Job and Machine Availability**
  - Jobs and machines are always available as soon as both are idle, ensuring no delays in starting a job.
- **Single Job per Machine at a Time**
  - Each machine can process only one job at a time, and a job cannot be processed on more than one machine simultaneously.
- **Negligible Inventory Cost**
  - The cost of holding semi-finished jobs in inventory is either negligible or assumed to be the same for all jobs.
- **No Priority Between Jobs**
  - All jobs are treated equally with no priority, and their processing times are independent of the order in which they are executed.
- **Single Machine of Each Type**
  - There is only one machine available for each type of operation.