ECE 514 Random Process - Project I Report Part I

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Problem 1: Simulating Random Variables:

1. Creating Random Variables:
   We wanted to simulate random variables with help of both MATLAB routine and rejection method for the given conditions:
   - Normal with mean =2 and variance =2
   - Uniform [2,4]
   - Exponential with parameter 2

   To execute these simulations of random variables we can make use of n built functions in MATLAB such as randn, rand and expnd. Out of these variables randn gives us normal variates, rand gives uniform variates and expnd gives us the exponential variates for simulation. These random variables are generated for T = 100, 1000 and 10000 to compute the required parameters. For all three values of T we have to plot the histograms of all three distributions which are given in the figures below :

   A. Normal with mean =2 and variance = 4

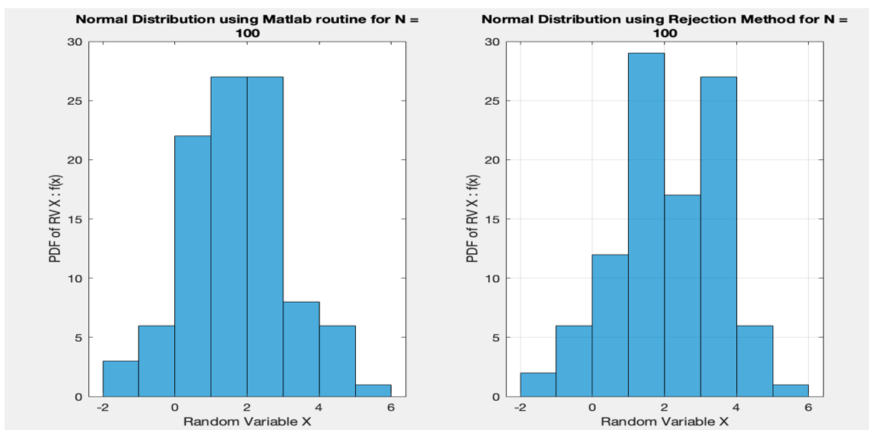

Fig 1. Histogram of Normal Distribution for T=100

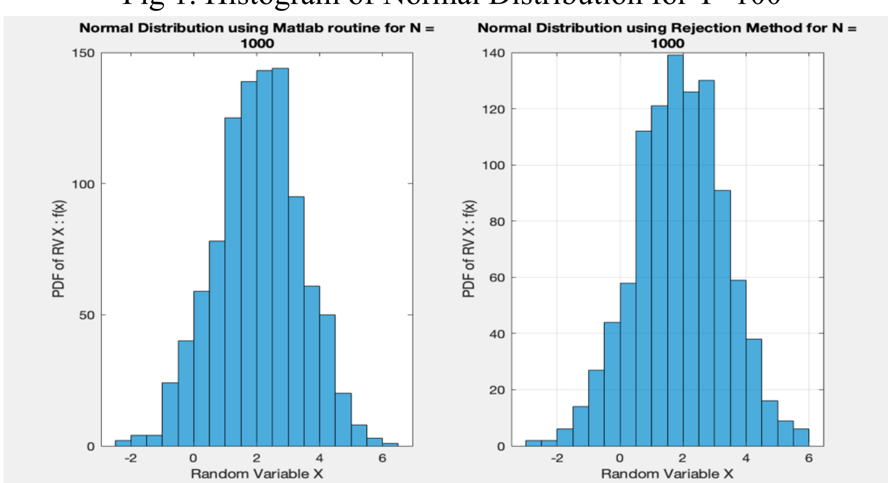

Fig 2. Histogram of Normal Distribution for T =1000

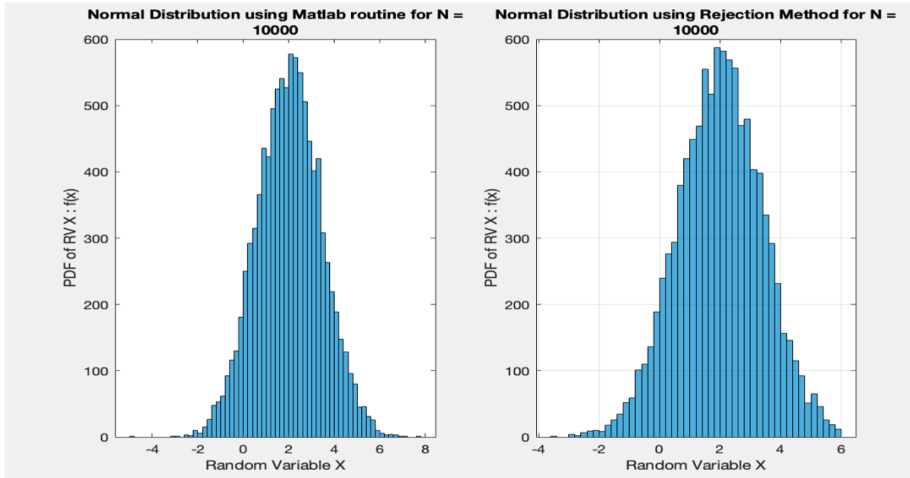

Fig 3. Histogram of Normal Distribution for T =10000

Table 1 : Mean of Normal Distribution

Variate Count (T)	MATLAB Routine Result	Rejection Method Result
100	1.7109	2.1347
1000	2.0577	1.9089
10000	2.0169	1.9991

Table 2: Variance of Normal Distribution

Variate Count (T)	MATLAB Routine Result	Rejection Method Result
100	2.0843	2.0625
1000	1.8589	2.0392
10000	1.9978	1.9505

B. Uniform [2,4]

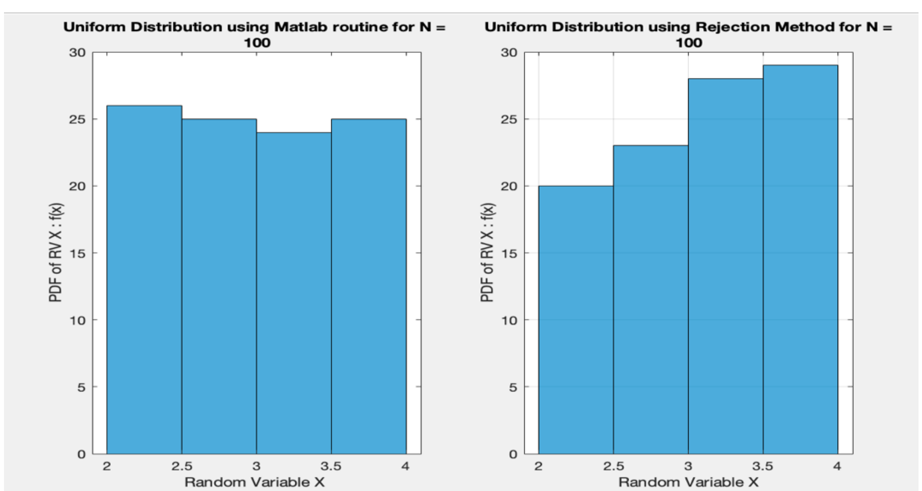

Fig 4. Histogram for Uniform Distribution for T = 100

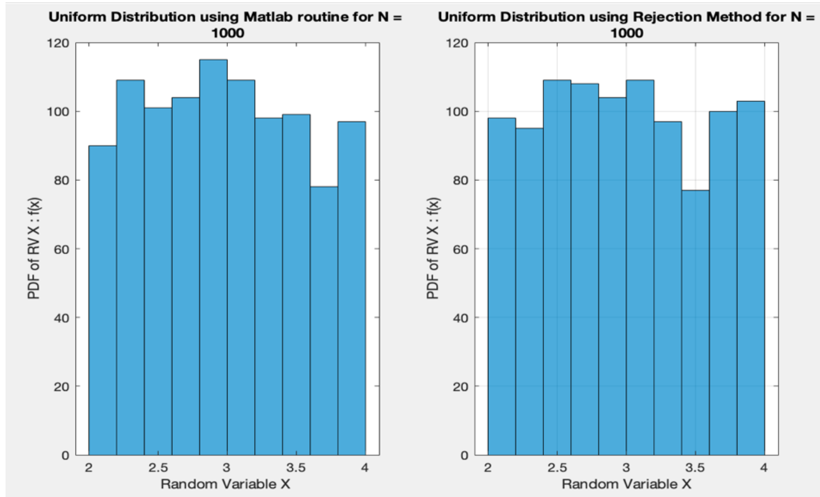

Fig 5. Histogram for Uniform Distribution for T = 1000

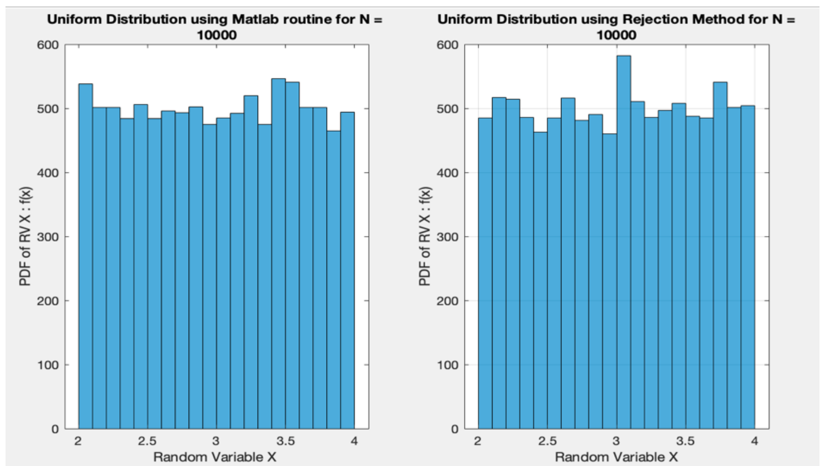

Fig 6. Histogram for Uniform Distribution for T = 10000

Table 3 : Mean of Uniform Distribution

Variate Count (T)	MATLAB Routine Result	Rejection Method Result
100	2.9764	3.0867
1000	2.9856	2.9921
10000	2.9987	3.0053

Table 4: Variance of Uniform Distribution

Variate Count (T)	MATLAB Routine Result	Rejection Method Result
100	0.3512	0.3492
1000	0.3137	0.3294
10000	0.3348	0.3336

C. Exponential with parameter 2

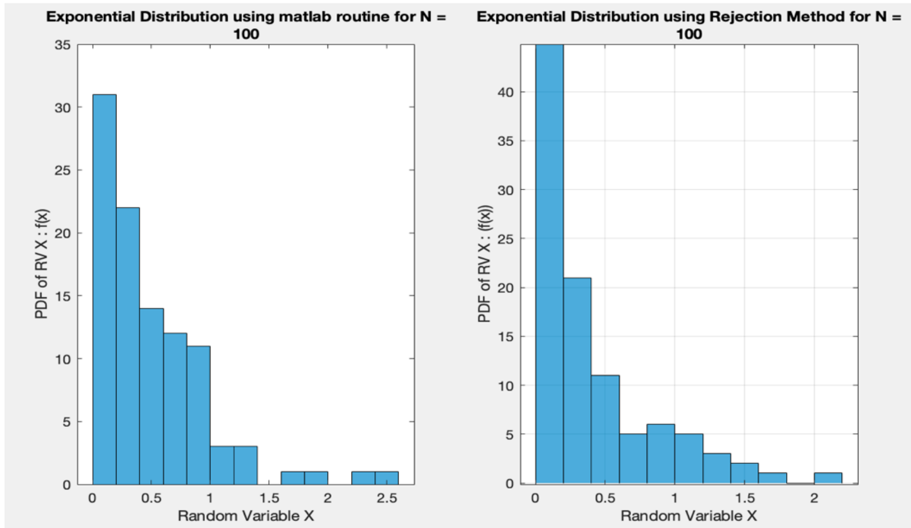

Fig 7. Histogram for Exponential Distribution at T = 100

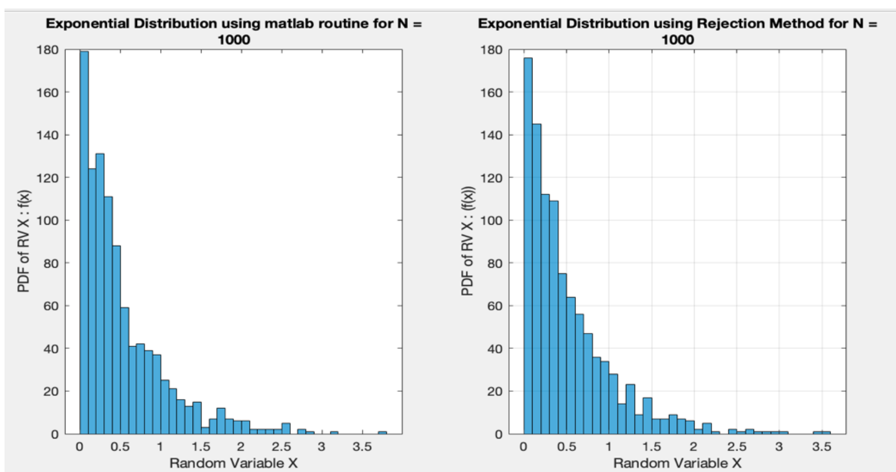

Fig 8. Histogram for Exponential Distribution at T = 1000

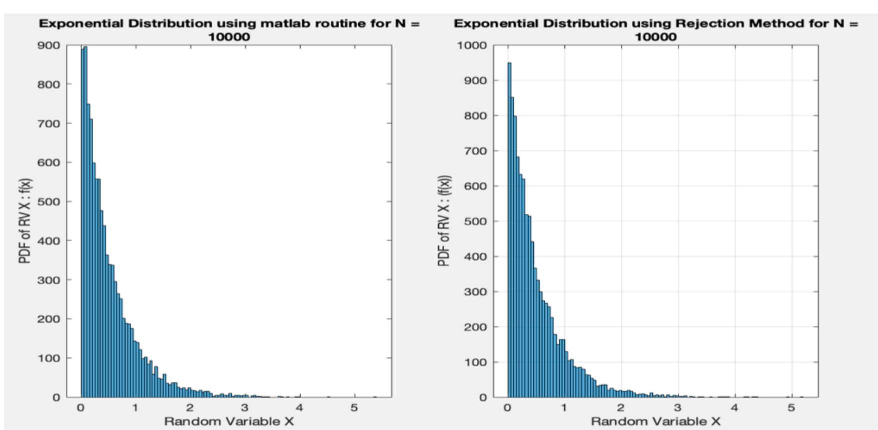

Fig 9. Histogram for Exponential Distribution at T = 10000

Table 5 : Mean of Exponential Distribution

Variate Count (T)	MATLAB Routine Result	Rejection Method Result
100	0.4997	0.4079
1000	0.5258	0.5213
10000	0.5081	0.4963

Table 6: Variance of Exponential Distribution

Variate Count (T)	MATLAB Routine Result	Rejection Method Result
100	0.2226	0.1905
1000	0.2784	0.2673
10000	0.2549	0.2484

2. Computing Mean and Variance:
   For all values of T given by 100,1000 and 10000 we are finding the mean and variance for Normal, Exponential and Uniform distributions. The above tables represent the mean and variance calculated for random variates.
   Theoretical values for the same can be calculated as follows:
   - Normal Distribution : Mean = 2 & Variance = 2
   - Uniform Distribution : a+b/2 =0.5 & Variance = (b-a)^2/12
   - Exponential Distribution : Mean = 0.5 & Variance = ¼

3. Comparison with theoretical results:
   Matlab routine and rejection method give results for each type of distribution and variate generation for different sample numbers of 100,1000 and 10000. Based on these results and calculated theoretical values we can compare the mean and variance values.

   Comparison for Mean:
   As we can see from the tables above mean values obtained from matlab routine function and rejection method can be compared based on the their deviation with respect to theoretically calculated values. As seen in table 1, for normal distribution the largest deviation for mean in matlab routine is 0.29 and rejection method is 0.13 which is observed at T=100 value. As we can observe in the table, with increasing number of samples the deviation of mean values from theoretical mean keeps decreasing and values keeps getting close to the actual value. This is implication of law of large numbers.
   Same form of results is observed in case of uniform and exponential distribution as seen in table 3 and table 5.

   Comparison of Variance:
   Similar to the result of mean, we can see from the results of table 2 that the variance values follow the similar pattern. We can see that variance deviation also keeps getting smaller as the number of variates is increased. For T= 10000 we observe the least deviation from the theoretical values in case of both MATLAB routine and rejection method. Analogous results are observed for uniform and exponential distribution as observed in table 4 and table 6.

4. Reasons for difference in mean and variance values from theoretically calculated value:
   As seen through the results the variation in mean and variance values compared to theoretical values is larger for small number of samples and it is decreasing as the number of sample values increases and becomes very large. From this result we can conclude that one of the reasons for the difference in the values compared could be based on number of samples taken into account. As we take infinite number of samples we will observe lesser difference in values.
   Also in rejection method the sample generation is based on random number generation which can accept the wrong samples leading to difference in values. For small number of samples the contribution of wrongly accepted samples is larger and sample number increases this contribution decreases giving us closer values to that of theoretical one.

Problem 2. Transforming Random variables

We are given a random variable Y such that Yi=_$\Sigma XiiT$ _i=_1_…T. We are plotting for all three distribution of normal, uniform and exponential for all three values of T as 100,1000 and 10000,we get 9 historgams for all these sample sizes and distributions.
From the histograms shown below we can see the results similar to that of Central Limit Theorem according to which as sample size increases mean of transform Y becomes equal to men of X which we used to generate samples and variance becomes equal to $\sigma\sqrt{n}$; in this form σ is standard deviation of X. From the observed histograms for large sample number distribution becomes similar to that of Gaussian Distribution.

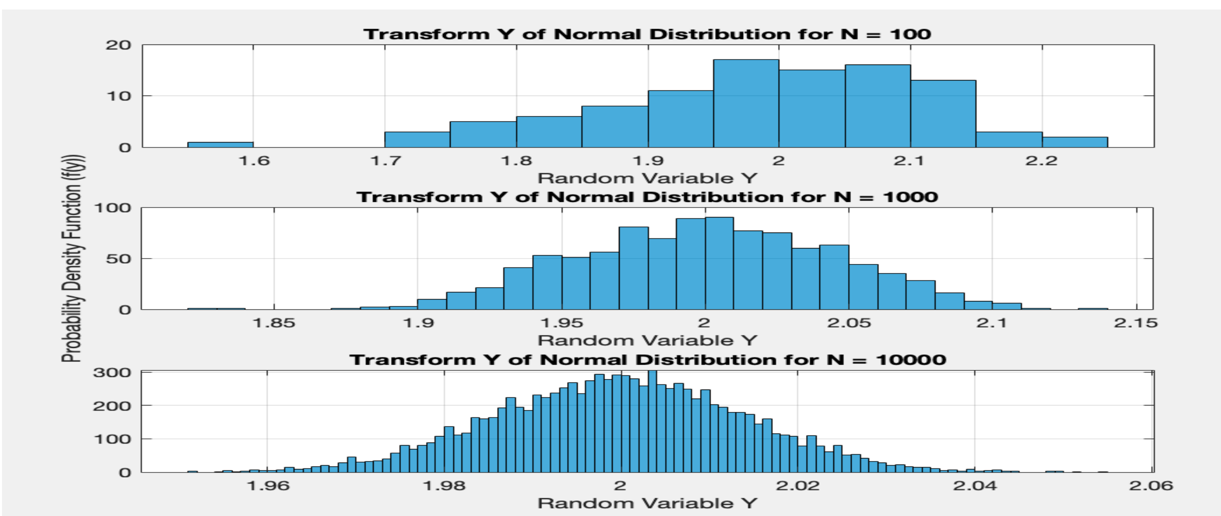

Fig 10. Histogram for Normal distribution of Y

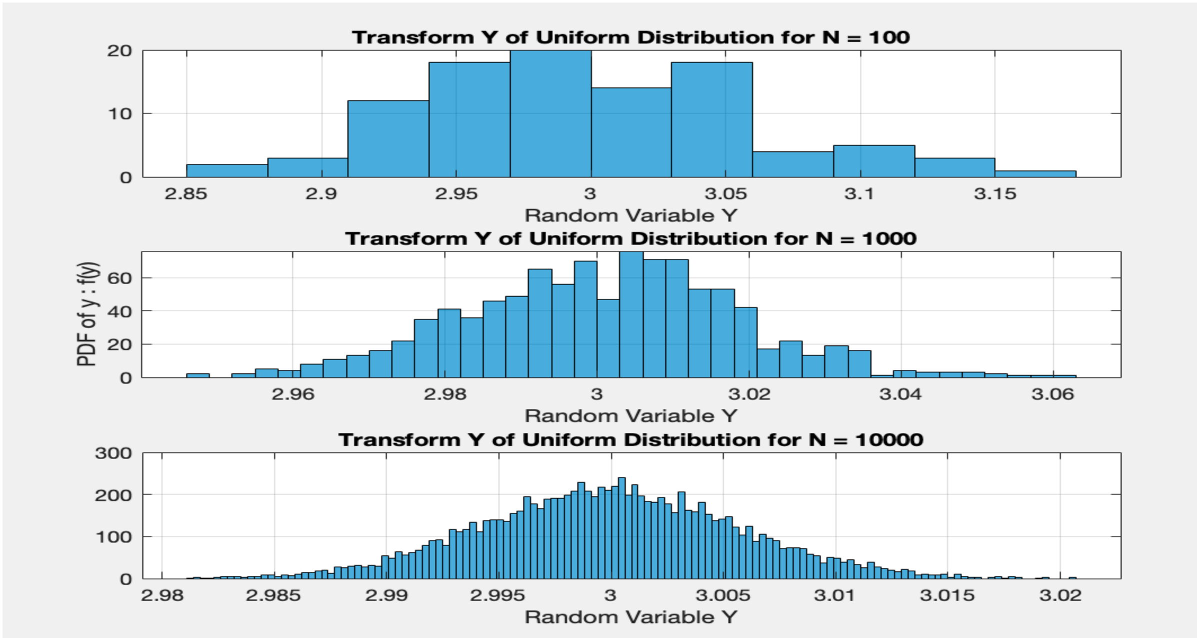

Fig 11. Histogram for Uniform distribution of Y

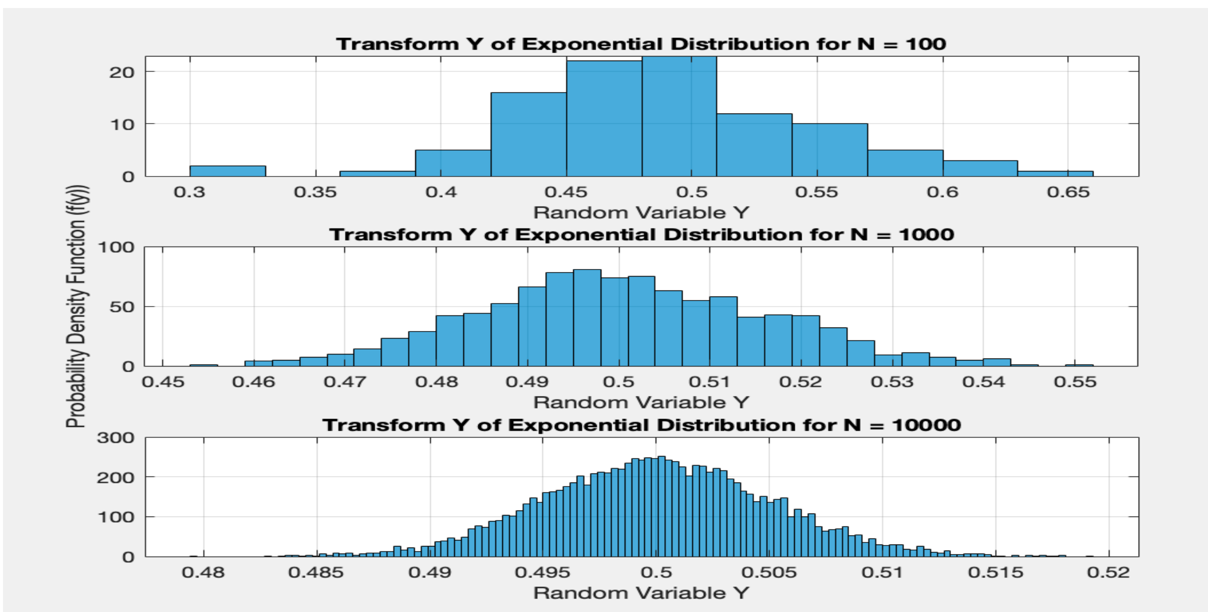

Fig 12. Histogram for Exponential distribution of Y

Problem 3 : Convergence of Random Variables:

We need to generate the MATLAB demo to demonstrate the following conditions :

1. $Y_T \xrightarrow{P} 0$
2. $Y_T \xrightarrow{A.S} 0$
3. $Y_T \xrightarrow{M.S} 0$
4. $Y_T \xrightarrow{L} X$

Generating the convergence of Y for different distributions such as Normal, Exponential and Uniform; referring to the paper – "Understanding Convergence Concepts: A Visual-Minded and Graphical Simulation-Based Approach", we can decide that sample size T is chosen to be 2000 whereas M = 500 which is number of realizations. We observe from the graphs from demo, Y has distribution which is in consistent with law of large numbers stating that for large sample values, normalized mean takes value close to the mean of distribution from which you sample with high probability.

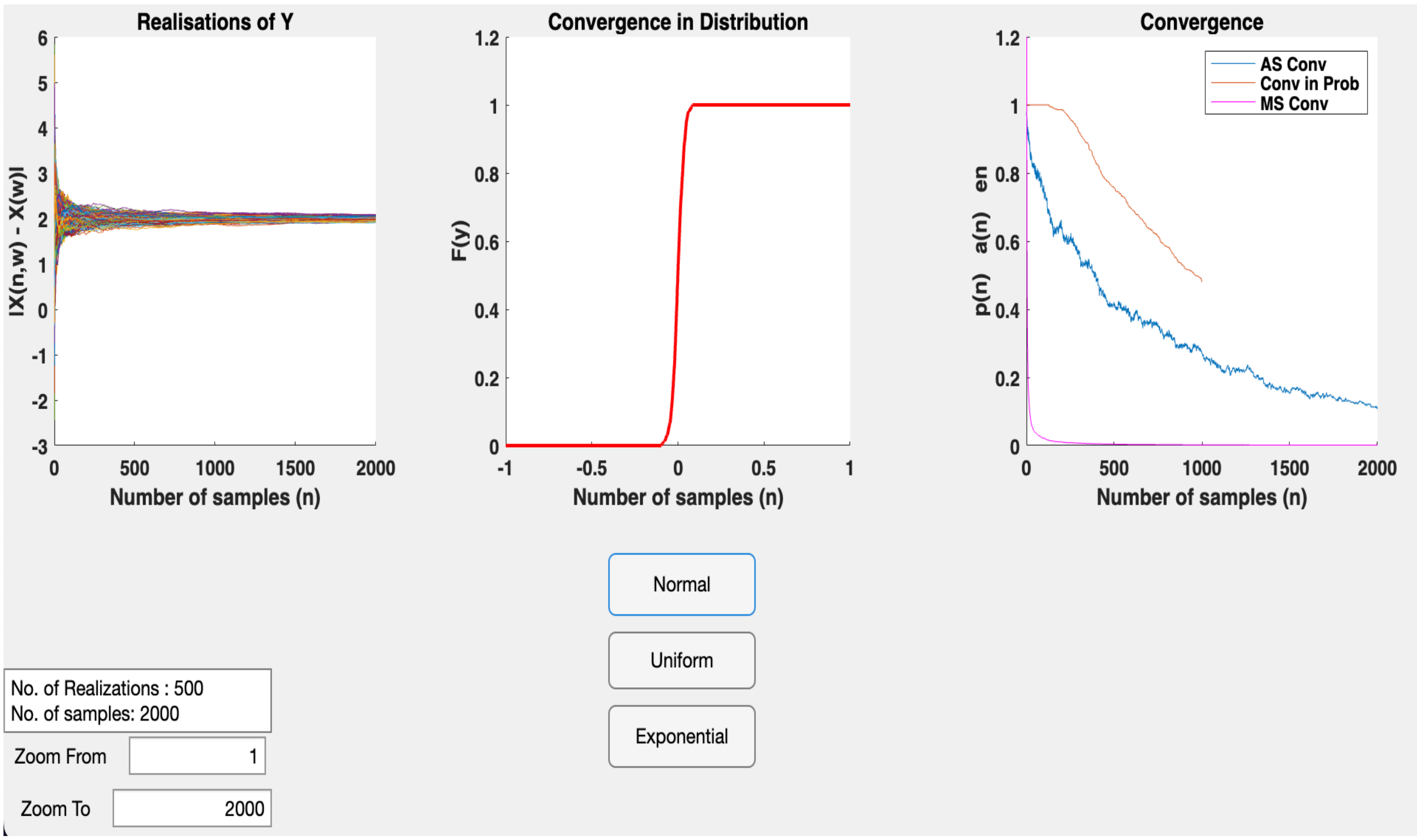

Fig 13. GUI demo for Normal Distribution representing different convergence modes of Y

As we can see in above figure Y converges to mean value of X which was 2 i.e. E[X]=2. For random variable ZT=$YT-E[X]$ and study the convergence in probability of ZT to the constant 0. As we can see for different convergence methods, the convergence in probability, mean square convergence and almost sure convergence are converging to zero . The centre graph shows convergence in distribution having mean value at zero because of Zt function we are subtracting mean from Yt. These graphs demonstrate we are getting the consistent results with expected values.

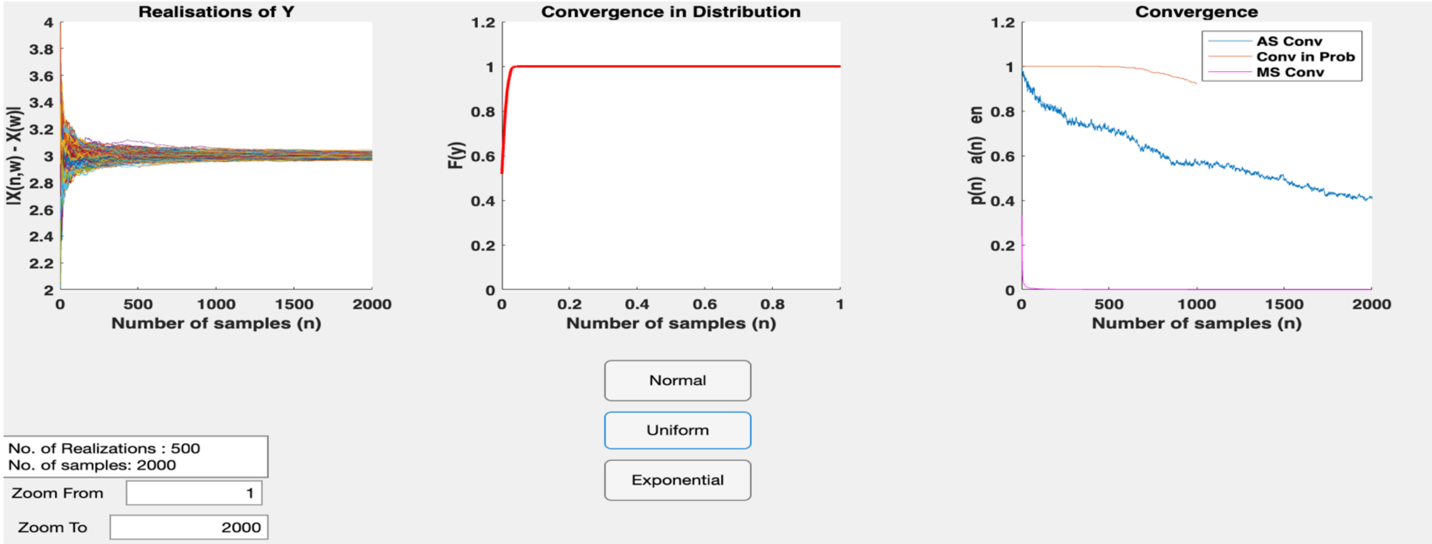

Fig 14. GUI demo for Uniform Distribution representing different convergence modes of Y

For Uniform distribution realization of Y has mean of 3 which is same as that of mean of X as E[X]=3 from calculated results. As seen the graph of Almost sure, convergence in probability and mean square convergence these values are converging to zero and as seen in the centre graph convergence in distribution is converging to 0 on x axis as we subtract the mean of X as seen in $ZT{=}YT{-}\ E[X]$, hence we get the mean at zero giving us the results as expected.

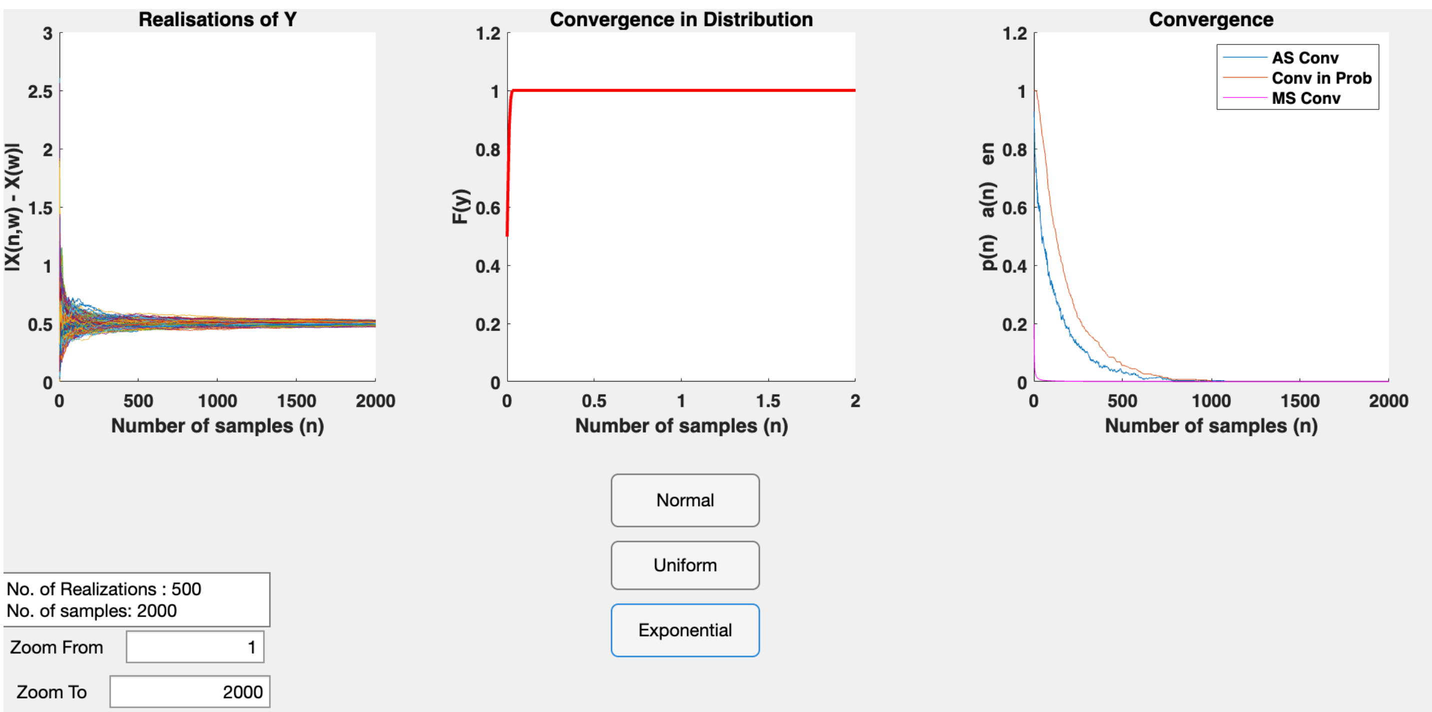

Fig 15. GUI demo for Exponential Distribution representing different convergence modes of Y

As we can see in above figure Y converges to mean value of X which was 0.5 i.e. E[X]=0.5. As we can see for different convergence methods, the convergence in probability, mean square convergence and almost sure convergence are converging to zero clearly. The mean in convergence in distribution is at 0 corresponding to x axis, since we have subtracted the mean of x i.e. E[X] from Yt we are getting the results with mean centred at zero. These graphs demonstrate we are getting the consistent results with expected values.

ECE 514 Random Process - Project I Report Part II

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Let X and Y be two jointly continuous random variables with the joint PDF:

$$f_{XY}(x,y) = \begin{cases} x + \frac{3}{2}y^2 & ,0 < x, y \leq 1 \\ 0 & ,Otherwise \end{cases}$$

We are given X and Y are two jointly continuous random variables with joint PDF as expressed above and Vector U is given in the form as shown below

$$\mathbf{U} = \begin{pmatrix} X \\ Y \end{pmatrix}$$

1. Correlation matrix and Covariance matrix of U :
   From the matlab file Q2.m, the executed code gives the required correlation matrix and covariance matrix as shown below
   Correlation Matrix :

```
correlation_matrix1 =

[0.417, 0.354]
[0.354, 0.467]
```

   Covariance Matrix :

```
Covxymat1 =

[0.076, -0.01]
[-0.01, 0.076]
```

2. 1000 sample vector series Xs with covariance matrix same as U.
   From the covariance matrix calculated above we can generate the sample vector series Xs as shown in code below

```
%generate 1000 sample Xs
L= chol(Covxymat,'nocheck');
n=500;
R=randn(2,n);
X=L'*R;
Xs=cov(X')
disp(Xs);
```

Here Cholesky factorization is used to generate autocorrelated sequences, using Chol function we are creating the sample series for Xs.

3. Covariance of Xs :
   From the code executed we can find the Covariance matrix for Xs which is of form as shown below:

```
Covar_Xs =

[  0.08, -0.013]
[-0.013,  0.076]
```

4. Comparison of Covariance matrix of U and Covariance matrix Xs :

```
Covar_Xs =

[  0.08, -0.013]
[-0.013,  0.076]
```

```
Covxymat1 =

[0.076, -0.01]
[-0.01, 0.076]
```

As we can see from above comparison covariance matrix of U and that of Xs is similar and their difference can be shown as matrix shown below

```
difference =

[-0.004, 0.003]
[ 0.003,     0]
```

This difference could be the result of approximation error in Cholesky factorization and as we increase the number of samples we get the lesser error.