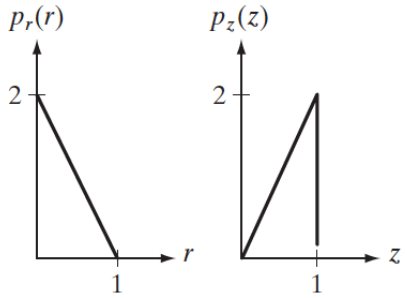
ECE558 Homework 03

Due 11/06/2022

How to submit your solutions: put your report (word or pdf) and results images (.png) in a folder named [your_unityid]_hw03 (e.g., twu19_hw03), and then compress it as a zip file (e.g., twu19_hw03.zip). Submit the zip file through **moodle**.

Problem 1 (20 points): An image with intensities in the range [0, 1] has the PDF shown in the following diagram. It is desired to transform the intensity levels of this image so that they will have the specified $p_z(z)$ shown. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this.



Problem 2 (15 points): Show that the Laplacian $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ is isotropic (invariant to rotation). You will need the following equations relating coordinates for axis rotation by an angle:

$$x = x'cos\theta - y'sin\theta$$

$$y = x'sin\theta + y'cos\theta$$

where (x, y) are the unrotated and (x', y') are the rotated coordinates.

Problem 3 (10 points): Show that the Laplacian of a continuous function $f(t, z)$ of two continuous variables, t *and* z, satisfies the following Fourier transform pair,

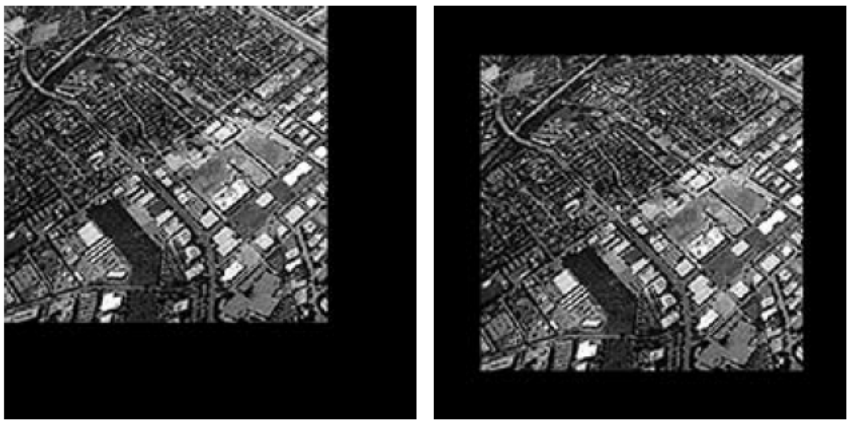
$$\nabla^2 f(t, z) \;\Leftrightarrow\; -4\pi^2(\mu^2 + v^2)F(\mu, v)$$

Where $F(\mu, v)$ is the FT of $f(t, z)$.

Problem 4 (15 points) Show that subtracting the Laplacian from an image is proportional to unsharp masking.

Problem 5 (20 points) Prove that both DFT and inverse DFT are infinitely periodic, with period M.That is, $F(\mu) = F(\mu + kM)$ and $f(x) = f(x + kM)$.

Problem 6 (10 points) The need for image padding when filtering in the frequency domain was discussed lecture note 13. We showed that images needed to be padded by appending zeros to the ends of rows and columns in the image (see the following image on the left). Do you think it would make a difference if we centered the image and surrounded it by a border of zeros instead (see image on the right), but without changing the total number of zeros used? Explain.



Problem 7 (10 points) The two Fourier spectra shown are of the same image.The spectrum on the left corresponds to the original image, and the spectrum on the right was obtained after the image was padded with zeros. Explain the significant increase in signal strength along the vertical and horizontal axes of the spectrum shown on the right.