
PURE MATHEMATICS ADVANCED LEVEL

“ONCE YOUR SOUL HAS BEEN ENLARGED BY A TRUTH, IT CAN NEVER RETURN TO ITS ORIGINAL SIZE.”
-BLAISE PASCAL

NOTES BY

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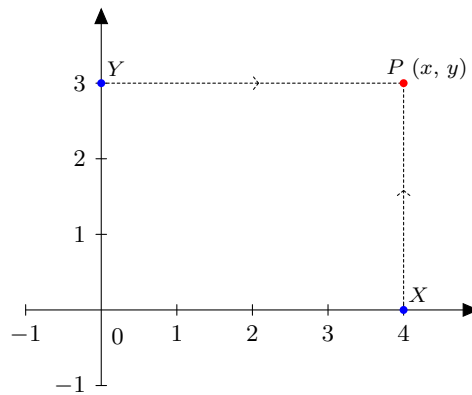
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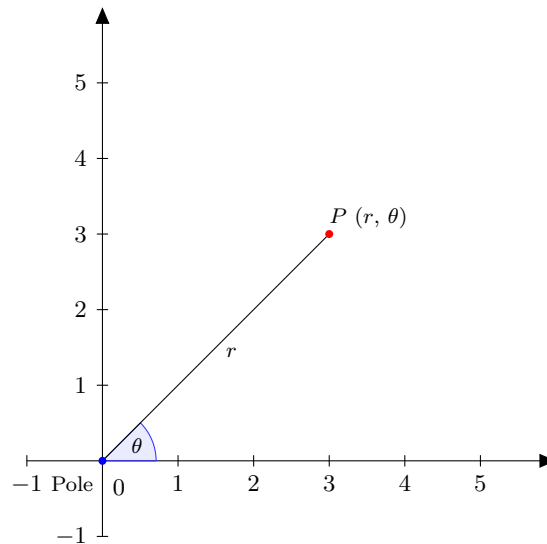
1 | Polar Curves

Introduction

The position of a point on a plane can be described in several ways. With respect to some origin O we can locate a point P on a plane by noting a horizontal distance followed by a vertical distance.



However the point P can be located on a plane with respect to the origin O , a horizontal line and the distance of P from O .



This is the polar coordinate system where we refer to the origin as the pole and the horizontal line as the initial line. The anti clockwise angle is usually measured in the principal range $-\pi < \theta \leq \pi$.

Relationship between Polar and Cartesian Coordinates

Consider the following diagram showing the point P on the plane, both in Cartesian and Polar coordinates
The above relationship can be used to convert from one form to another.

Ex. 1. Find the polar coordinates of the curve given by $y = x^2 + y^2 = 2x$

$$\begin{aligned}x^2 + y^2 &= 2x \\ \implies 2r \cos \theta &= r^2 \\ \implies 2 \cos \theta &= r\end{aligned}$$

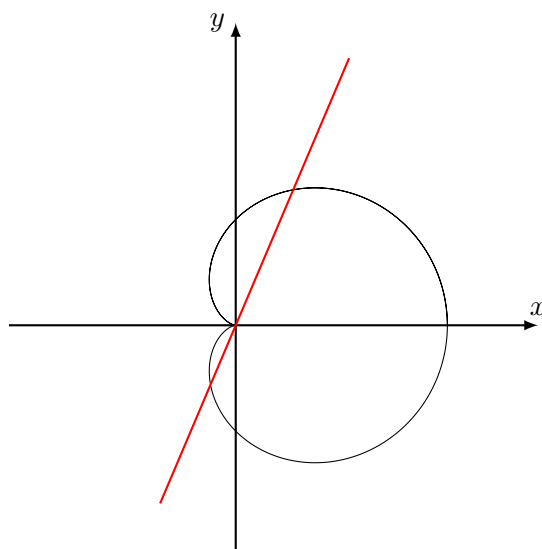
Ex. 2. Find the Cartesian equation corresponding to the curves **a)** $r = 4(1 + \cos \theta)$ and **b)** $3 = r \sin 2\theta$.

$$\begin{aligned}3 &= r \sin 2\theta \\ r &= 4(1 + \cos \theta) & 3 &= 2r \sin \theta \cos \theta \\ 3 &= 2x \sin \frac{y}{r} \\ i^2 + 1^2 &= 0\end{aligned}$$

Sketching Polar Curves

Ex. 1. Sketch the graph of $2 + 2 \cos \theta$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	4	3.7	3.4	3	2	1	0.6	0.3	0



Ex. 2. Sketch the graph with polar equation $= 3 - 2 \cos \theta$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	1	1.3	1.6	2	3	4	4.4	4.7	5