Probability

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Lecture 1

The Bigamy problem.

Suppose a group of n males is to be paired up with n females. Suppose further that a female is to be paired with only 1 male.

With each matchup, we eliminate one female from the possible sample and thus reducing $n - 1$

Lecture 2: Geometric Probability

Step 1: Determine the set that contains all possible points:

$$\Omega = \{(x, y) \in \mathbb{R}^2 \colon x^2 + y^2 \leq r^2\}$$
$$\mathbb{P}[s_1] = \frac{A_{s_1}}{A_\Omega} = \frac{\pi r^2 - B}{8\pi r^2}$$

Problem. The rod of length L.
A rod of length l is split randomly into 3 parts by making two cuts at random distances x and y from one end.

Step 1: Determine the set that contains all possible point:

$$\Omega = \{(x, y) \colon 0 \leq x \leq l \text{ and } 0 \leq y \leq l\}$$
$$A_\Omega = l^2$$

Step 2: Identify the region for which our condition holds, namely the values of x and y for which *a triangle can be formed.*

Case 1: $x < y$

$$\frac{l}{2} < y - x$$
$$\Rightarrow y > x + \frac{l}{2}$$

Case 2: $y < x$

$$\frac{l}{2} > x - y$$
$$\Rightarrow y > x - \frac{l}{2}$$

$$\mathbb{P}[\text{triangle is formed}] = \frac{l^2/4}{l^2} = \frac{1}{4}$$

Problem. Buffon's needle.
A needle of lenght l is thrown randomly on a plane which is ruled by infinite parallel lines equally seperated by a distande d, where $l < d$. We wish to find the probability that the needle intersects one of them.

Let x be the distance of the mid=point of the needle to the closest line. There are three extreme cases.

- One where the needle does not touch any of the rules (lands in between),
- one in which one end of the needle just touches one of the rules,
- and the final where the midpoint of the needle intersects one of the rules.

Step 1: Identify the set of all possible outcomes.

$$\Omega = \{(x, \theta) : 0 \leq x \leq \frac{l}{2}; 0 \leq \theta \leq \pi\}$$