Probability

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Lecture 3: Multi-event probability

$$\mathbb{P}\left[A_1 \cup A_2 \cup \cdots \cap A_3\right] = \bigcup_{i \in \mathbb{N}}^{n} A_i = \frac{1}{n!}$$

k balls are distributed among n cells, each ball being equally likely to be in any of the available cells. What is the probability that m cells are empty?

Let A_i be the probability that the i^{th} cell is empty.

The probability that no cell is empty is equivalent to the complement of the probability that at least one cell is empty.

$$1 - \mathbb{P}\left[\bigcup_{i \in \mathbb{N}}^{n} A_i\right]$$

$$\mathbb{P}[A_i] = \frac{(n-1)^k}{n^k} = \left(1 - \frac{1}{n}\right)^k$$

$$\mathbb{P}[A_i \cap A_j] = \frac{(n-2)^k}{n^k} = \left(1 - \frac{2}{n}\right)^k$$

$$\mathbb{P}[A_i \cap A_j \cap A_k] = \frac{(n-3)^k}{n^k} = \left(1 - \frac{3}{n}\right)^k$$

$$\mathbb{P}[A_i \cap A_j \cap A_k \cap \cdots \cap A_n] = \frac{(n-n)^k}{n^k} = \left(1 - \frac{n}{n}\right)^k = 0$$