Computer Vision 2
Assignment 2

Summer Semester 2019
Group 8
Moritz Fuchs & Diedon Xhiha

Problem 1 Graphical models (20 Points)

1. 1 Point

Two main ingredients the set of random variables describing the entities involved the problem, and the second ingredient is the set of conditional probabilities, that tell us the relation between a certain variable and another variable or variables from the set.

2. 1 Point

As we know that directed graphs are versatile, but not always appropriate. So they are not always be convenient to provide conditional distributions, and some of certain conditional independence structures that a directed graph can not represent. So for example Loopy graph, it is not possible to express the same conditional independence statements using the a directed graph model.

3. 2 Points

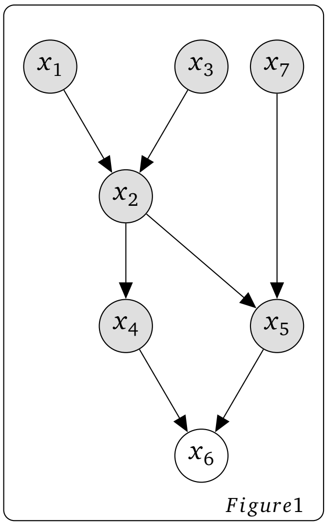


Figure1

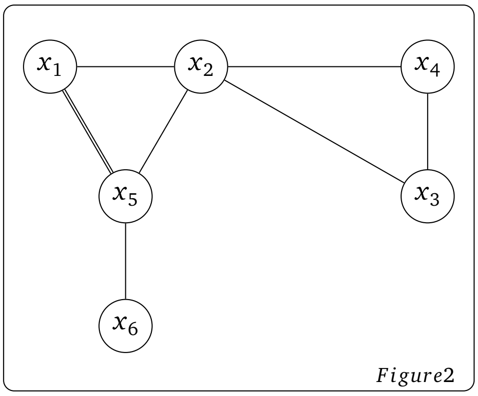
The Markov Blanket of variable x_2 is formed from the variables that are filled with gray color.

4. 2 Points

The factorization of the directed graphical model is as follows
$p(x_1, x_2, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}) = p(x_1)p(x_2)p(x_3)p(x_6)p(x_7)p(x_4|x_1, x_2)p(x_5|x_2, x_3)p(x_{10}|x_7)$
$p(x_{11}|x_{10})p(x_8|x_4, x_5)p(x_9|x_3, x_5, x_6)p(x_14|x_{11})p(x_12|x_8, x_9)p(x_{13}|x_9)p(x_{15}|x_{12})$

The factorization of the undirected graphical model is as follows
$p(x_1, x_2, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}) = \frac{1}{Z}\phi_0(x_3)\phi_1(x_1, x_4)\phi_2(x_2, x_4, x_5, x_8)\phi_3(x_7, x_{10})\phi_4(x_7, x_{11})$
$\phi_5(x_{11}, x_{14})\phi_6(x_5, x_9)\phi_7(x_8, x_{12})\phi_8(x_9, x_{12}, x_{13}, x_{15})\phi_9(x_6, x_9)$

5. 1 Point



—- excluded—-

6. 2 Points

x_1	x_2	x_3	x_4	$p(x_1, x_2, x_3, x_4)$	$\frac{1}{Z}p(x_1, x_2, x_3, x_4)$
0	0	0	0	1*1*1*1=1	0.1077
0	0	0	1	1*1*0.1*0.1=0.01	0.001077
0	0	1	0	1*0.1*0.1*1=0.01	0.001077
0	0	1	1	1*0.1*2*0.1=0.02	0.002155
0	1	0	0	0.1*0.1*1*1=0.01	0.001077
0	1	0	1	0.1*0.1*0.1*0.1=0.0001	0.00001077
0	1	1	0	0.1*2*0.1*1=0.02	0.002155
0	1	1	1	0.1*2*2*0.1=0.04	0.00431
1	0	0	0	0.1*1*1*0.1=0.01	0.001077
1	0	0	1	0.1*1*0.1*2=0.02	0.002155
1	0	1	0	0.1*0.1*0.1*0.1=0.0001	0.00001077
1	0	1	1	0.1*0.1*2*2=0.04	0.00431
1	1	0	0	2*0.1*1+0.1=0.02	0.002155
1	1	0	1	2*0.1*0.1*2=0.04	0.00431
1	1	1	0	2*2*0.1*0.1=0.04	0.00431
1	1	1	1	2*2*2*2=8	0.86206

We have computed Z as the sum of all the probabilities above, and after the calculation we found Z=9.28.

7. 2 Points

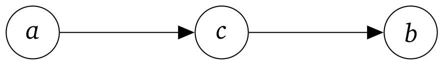
Based on those four records

a	b	c	p(a,b,c)
0	1	0	0.048
1	0	0	0.192
0	1	1	0.216
1	0	1	0.064

The first and second row c remain the same but a and b change and with them the distribution too, the same happens in the third and fourth row. So we can conclude that those variable a and b are marginally dependent, when b=1 and a=0 then the distribution ia 0.048 but in the other hand, when b=0 and a=1 then the distribution is 0.192, so this means knowing event b does help in value of event a and vise versa.

8. 3 Points

Based on the given distributions $p(a)$ $p(b|c)$ $p(c|a)$ we can draw the directed graph as follows :



$Figure 3$

And from the figur above we can write:
$p(a,b,c) = p(a)p(a|c)p(b|a,c) = p(a)p(a|c)p(b|c)$
As we know from directed graph, because 'c' tell us about 'a', we can take away 'a' from $p(b|a,c)$, and therefore we get the desired probability.

9. 2 Points

Markov blanket of variables in undirected graph:
$x_1 -> x_2, x_3$
$x_2 -> x_1, x_4$
$x_3 -> x_1, x_4$
$x_4 -> x_2, x_3$

Markov blanket of variables in directed graph:
$x_1 -> x_2, x_3$
$x_2 -> x_1, x_3, x_4$
$x_3 -> x_1 x_2, x_4$
$x_4 -> x_2, x_3$

10. 4 Points

Based also in the previous question about for Markov blanket for each of the variables, we can conclude that in the figure 2, undirected graph, the first one that says x_1 is independent of x_4 and that x_2 and x_3 are the Markov blanket of x_1, is completely true based also in the definition that, a node is conditionally independent of all other nodes given its Markov blanket.
Further mathematical proof. We have the factorization of the undirected graph: $p(x_1,x_2,x_3,x_4) = \frac{1}{z}\phi(x_1,x_2)\phi(x_2,x_4)\phi(x_4,x_3)\phi($

$$p(x_4|x_1,x_2,x_3) = \frac{p(x_1,x_2,x_3,x_4)}{p(x_1,x_2,x_3)}$$

$$= \frac{p(x_1,x_2,x_3,x_4)}{\sum_{x_4} p(x_1,x_2,x_3,x_4)}$$

$$= \frac{\frac{1}{z}\phi(x_1,x_2)\phi(x_2,x_4)\phi(x_4,x_3)\phi(x_1,x_3)}{\sum_{x_4}\frac{1}{z}\phi(x_1,x_2)\phi(x_2,x_4)\phi(x_4,x_3)\phi(x_1,x_3)}$$

$$= \frac{\phi(x_1,x_2)\phi(x_2,x_4)\phi(x_4,x_3)\phi(x_1,x_3)}{\phi(x_1,x_2)\phi(x_1,x_3)\sum_{x_4}\phi(x_2,x_4)\phi(x_4,x_3)}$$

$$= \frac{\phi(x_2,x_4)\phi(x_4,x_3)}{\sum_{x_4}\phi(x_2,x_4)\phi(x_4,x_3)}$$

$$= \frac{\phi(x_2,x_4)\phi(x_4,x_3)\sum_{x_1}\frac{1}{z}\phi(x_1,x_3)\phi(x_1,x_2)}{\sum_{x_4}\phi(x_2,x_4)\phi(x_4,x_3)\sum_{x_1}\frac{1}{z}\phi(x_1,x_3)\phi(x_1,x_2)}$$

$$= \frac{\sum_{x_1} \frac{1}{z} \phi(x_2, x_4)\phi(x_4, x_3)\phi(x_1, x_3)\phi(x_1, x_2)}{\sum_{x_4} \sum_{x_1} \frac{1}{z} \phi(x_2, x_4)\phi(x_4, x_3)\phi(x_1, x_3)\phi(x_1, x_2)}$$

$$= \frac{\sum_{x_1} p(x_1, x_2, x_3, x_4)}{\sum_{x_4} \sum_{x_1} p(x_1, x_2, x_3, x_4)}$$

$$= \frac{p(x_2, x_3, x_4)}{p(x_2, x_3)}$$

$$= p(x_4 | x_2, x_3)$$

The second one that says x_2 is independent of x_3, because the x_3 is not in the Markov blanket of x_2, and from definition a node is conditionally independent of all other nodes given its Markov blanket, and that x_1 and x_4 are the Markov blanket of x_2, because they are direct node of x_2.
Further mathematical proof (As the first one):

$$p(x_3 | x_1, x_2, x_4) = \frac{p(x_1, x_2, x_3, x_4)}{p(x_1, x_2, x_4)}$$

$$= \frac{p(x_1, x_2, x_3, x_4)}{\sum_{x_3} p(x_1, x_2, x_3, x_4)}$$

$$= \frac{\frac{1}{z} \phi(x_1, x_2)\phi(x_2, x_4)\phi(x_4, x_3)\phi(x_1, x_3)}{\sum_{x_3} \frac{1}{z} \phi(x_1, x_2)\phi(x_2, x_4)\phi(x_4, x_3)\phi(x_1, x_3)}$$

$$= \frac{\phi(x_1, x_2)\phi(x_2, x_4)\phi(x_4, x_3)\phi(x_1, x_3)}{\phi(x_1, x_2)\phi(x_2, x_4) \sum_{x_3} \phi(x_4, x_3)\phi(x_1, x_3)}$$

$$= \frac{\phi(x_4, x_3)\phi(x_1, x_3)}{\sum_{x_3} \phi(x_4, x_3)\phi(x_1, x_3)}$$

$$= \frac{\phi(x_4, x_3)\phi(x_1, x_3) \sum_{x_2} \frac{1}{z} \phi(x_1, x_2)\phi(x_2, x_4)}{\sum_{x_3} \phi(x_4, x_3)\phi(x_1, x_3) \sum_{x_2} \frac{1}{z} \phi(x_1, x_2)\phi(x_2, x_4)}$$

$$= \frac{\sum_{x_2} \frac{1}{z} \phi(x_4, x_3)\phi(x_1, x_3)\phi(x_1, x_2)\phi(x_2, x_4)}{\sum_{x_3} \sum_{x_2} \frac{1}{z} \phi(x_4, x_3)\phi(x_1, x_3)\phi(x_1, x_2)\phi(x_2, x_4)}$$

$$= \frac{\sum_{x_2} p(x_1, x_2, x_3, x_4)}{\sum_{x_3} \sum_{x_2} p(x_1, x_2, x_3, x_4)}$$

$$= \frac{p(x_1, x_3, x_4)}{p(x_1, x_4)}$$

$$= p(x_3 | x_1, x_4)$$

And for the directed Graph, first one that says x_1 independent of x_4, exactly as the definition from above, and that x_2 and x_3 are the Markov blanket of x_1, because x_1 has 2 children and no other parents and also those children have just one parent and its x_1. Further mathematical proof. We have the factorization of the directed graph: $p(x_1, x_2, x_3, x_4) = p(x_1)p(x_2|x_1)p(x_3|x_1)p(x_4|x_3, x_2)$.

$$p(x_4|x_1, x_2, x_3) = \frac{p(x_1, x_2, x_3, x_4)}{p(x_1, x_2, x_3)}$$

$$= \frac{p(x_1, x_2, x_3, x_4)}{\sum_{x_4} p(x_1, x_2, x_3, x_4)}$$

$$= \frac{p(x_1)p(x_2|x_1)p(x_3|x_1)p(x_4|x_3, x_2)}{\sum_{x_4} p(x_1)p(x_2|x_1)p(x_3|x_1)p(x_4|x_3, x_2)}$$

$$= \frac{p(x_1)p(x_2|x_1)p(x_3|x_1)p(x_4|x_3, x_2)}{p(x_1)p(x_2|x_1)p(x_3|x_1)\sum_{x_4} p(x_4|x_3, x_2)}$$

$$= \frac{p(x_4|x_3, x_2)}{\sum_{x_4} p(x_4|x_3, x_2)}$$

$$= \frac{p(x_4|x_3, x_2)}{1} = p(x_4|x_3, x_2)$$

Also $p(x_1|x_2, x_3, x_4) = p(x_1|x_2.x_3)$ can be proofen.

Last Equation:

$$p(x_3, x_2|x_1) = \frac{p(x_1, x_2, x_3)}{p(x_1)}$$

$$= \frac{\sum_{x_4} p(x_1, x_2, x_3, x_4)}{p(x_1)}$$

$$= \frac{\sum_{x_4} p(x_1)p(x_2|x_1)p(x_3|x_1)p(x_4|x_3, x_2)}{p(x_1)}$$

$$= \frac{p(x_1)p(x_2|x_1)p(x_3|x_1)\sum_{x_4} p(x_4|x_3, x_2)}{p(x_1)}$$

$$= p(x_2|x_1)p(x_3|x_1)\sum_{x_4} p(x_4|x_3, x_2)$$

$$= p(x_2|x_1)p(x_3|x_1) * 1$$

Problem 2 Markov random fields with Student-t potentials (10 Points)

Tasks 1-4 within code

2. 1 Point

As printed out: gt log prior: -9928.589720178397

3. 2 Points

As printed out: random disparity log prior: -478661.4937203116

4. 2 Points

As printed out: constant disparity log prior: 0.0

5. 3 Points

As we use MRF model, the potentialfunction expresses the compatibility between neighour pixels.
Therefore our random map has the lowest score as most neighbours don't fit each ohter.
The log probability of constant map ist the lowest, because neigbouring pixels are as close together as they can get and we want them to be localy smooth.
The ground truth lies between those log- priors as its not constant, therefore not maximally locally smooth and may have some small artifacts.

Problem 3 Stereo with gradient-based optimization (17 Points)

Everything else is in code.

1. 4 Points

When initailized with a constant disparity map the struggels to find another map which does not increase the function value as the prior increases in absolut value. It therefore only detects the edges and textures. The rest stays almost constant. It looks there a bit grainy.
The random disparity does not converge to something meaningful as its to far away from such a solution, as neighbouring pixels are incompatible. Therefore we area left with a grainy disparity in area, where the disparities ar too incompatible to each other.
The ground truth converges to something smoother than the orignal, but is left with some artifacts(face).

Problem 4 Stereo with a generalized robust function (10 Points)

Everything else is in code.

5. Based on your experiments, what recommendations can you give for choosing the parameters α and c? How does each parameter influence the result? How did you go about fi
nding good parameters (?? Point/s)

In the code we wrote some parameter set for each input disparity image, which works more or less.
c does manly influences how much influence an outliner has and therefore the results is smoothed. This limits the amount of detail we can e.g. from the writing on the white board. As it is small values(e.g. 1) smooth more and bigger value (e.g. 10) for sharper edges
α efects seems quite small. Result with smaller values seem to have less depth to it, but also less grainyness.

We just started playing around with alpha and keep c fixed and then switch it. repeat sometimes but didnt have anything bigger strategy than our intuition.

6. In general, using the generalized robust function and its gradients will not allow us to learn the shape parameters which is why we tune them manually. Why is it that gradient-based optimization using Eqs. (8) and (9) is not applicable here to obtain good shape parameters?(?? Point/s)

The proposed loss function approximates many different loss function and interpolates inbetween them. This means we would need to fix α to find the perfect c, and in next turn fix c and do the same for α. which means we would have the perfect c anymore. This result is because its a non convex function and we may only get stuck in a local optima. But The bigger problem is that every input image has its best parameter set, which we can optimize with an Algorithm because we have nothing to compare to besides our ground truth(which is not quite optimal and as we can generate is tby another way we rather just us this).
It is better to use the different properties of the incorpated loss functions by change the α between iteration and do as they describe: we can initialize α such that our loss is convex and then gradually reduce α (and therefore reduce convexity and increase robustness) during optimization, thereby enabling robust estimation that (often) avoids local minima.