CV 2 exercise 1

Summer Semester 2019
Gruop 8

CV2 exercise 1 | Group 8
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Problem 1 Probabilities and Statistics (13 Points)

1. 1 Point

x	y	example x	example y
discrete	discrete	exam grade	exam duration
continuous	discrete	location	number of bathtubducks in sight
continuous	continuous	velocity while driving	acceleration while driving

2. 2 Points

with the chainrule we can extend as follows:

$$p(v,w,x,y,z) = p(w,x,y,z)p(v|w,x,y,z) \tag{1}$$
$$p(v,w,x,y,z) = p(x,y,z)p(w|x,y,z)p(v|w,x,y,z) \tag{2}$$
$$p(v,w,x,y,z) = p(y,z)p(x|y,z)p(w|x,y,z)p(v|w,x,y,z) \tag{3}$$
$$p(v,w,x,y,z) = p(z)p(y|z)p(y,z)p(x|y,z)p(w|x,y,z)p(v|w,x,y,z) \tag{4}$$
$$\tag{5}$$

as equation (2) is already the answer we are looking for.

3. 3 Points

$$\begin{aligned}
p(w,x) &= \int\int p(w,x,y,z)dydz \\
&= \int\int p(w)p(y|z)p(z|x,w)p(x)dydz \\
&= p(x)p(w)\int\int \frac{p(y,z)}{p(z)}\frac{p(z,x,w)}{p(x,w)}dydz \\
&= p(x)p(w)\int(\int \frac{p(y,z)}{p(z)}dy)\frac{p(z,x,w)}{p(x,w)}dz \\
&= p(x)p(w)\int \frac{p(z)}{p(z)}\frac{p(z,x,w)}{p(x,w)}dz \\
&= p(x)p(w)\int \frac{p(z,x,w)}{p(x,w)}dz \\
&= p(x)p(w)\frac{p(x,w)}{p(x,w)} \\
&= p(x)p(w)
\end{aligned}$$

4. 3 Points

$$\begin{aligned}
E[(x-\mu)^2] &= E[x^2 - 2x\mu + \mu^2] \\
&= E[x^2] + E[-2x\mu] + E[\mu^2] \\
&= E[x^2] - 2\mu E[x] + \mu\mu && |E[x] = \mu \\
&= E[x^2] - 2E[x]E[x] + E[x]E[x] \\
&= E[x^2] - E[x]E[x]
\end{aligned}$$

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5. 4 Points

Lets call the choice of door 1 D, therefore the probability that the prize is behind door 1 is $p(D)) = \frac{1}{3}$ and the probality that the prize is not behind door 1 is $p(notD) = 1 - p(D) = \frac{2}{3}$.
Now the event R takes place, in which the game host reveals one Door with no prize behind it. Therefore we have the conditional probalities $p(R|D) = 1$ and $p(R|notD) = 1$.
We now want to know what the probability that the prize is behind door 1 is $p(D|R)$.
Given Bayes rule and that $p(R)$ can be factorized as $p(R|D)p(D) + p(R|notD)p(notD)$:

$$p(D|R) = \frac{p(R|D)p(D)}{p(R)} = \frac{p(R|D)p(D)}{p(R|D)p(D) + p(R|notD)p(notD)}$$

We plug in our probalities for each case:

$$p(D|R) = \frac{1 \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3}} = \frac{1}{3}$$

Therefore the probability behind the door $p(D_2)$, which the game show host didn't chose is

$$p(D_2) = 1 - p(D|R) = \frac{2}{3}$$

, which shows us that we should switch to the door 2.

Problem 2 Modeling (5 Points)

1. 1 Point

2. 1 Point

3. 2 Points

4. 1 Point

5. 1 Point

Problem 4 Stereo Likelihood (10 Points)

1. 1