# CV 2 exercise 1

TECHNISCHE UNIVERSITÄT DARMSTADT

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Group 8

CV2 exercise 1 | Group 8
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Problem 1 Probabilities and Statistics (13 Points)

1. 1 Point

| x | y | example x | example y |
|---|---|---|---|
| discrete | discrete | exam grade | exam duration |
| continuous | discrete | location | number of bathtubducks in sight |
| continuous | continuous | velocity while driving | acceleration while driving |

2. 2 Points

with the chainrule we can extend as follows:

$$p(v,w,x,y,z) = p(w,x,y,z)p(v|w,x,y,z) \tag{1}$$
$$p(v,w,x,y,z) = p(x,y,z)p(w|x,y,z)p(v|w,x,y,z) \tag{2}$$
$$p(v,w,x,y,z) = p(y,z)p(x|y,z)p(w|x,y,z)p(v|w,x,y,z) \tag{3}$$
$$p(v,w,x,y,z) = p(z)p(y|z)p(y,z)p(x|y,z)p(w|x,y,z)p(v|w,x,y,z) \tag{4}$$
$$\tag{5}$$

as equation (2) is already the answer we are looking for.

3. 3 Points

$$\begin{aligned}
p(w,x) &= \int\int p(w,x,y,z)dydz \\
&= \int\int p(w)p(y|z)p(z|x,w)p(x)dydz \\
&= p(x)p(w)\int\int \frac{p(y,z)}{p(z)}\frac{p(z,x,w)}{p(x,w)}dydz \\
&= p(x)p(w)\int(\int \frac{p(y,z)}{p(z)}dy)\frac{p(z,x,w)}{p(x,w)}dz \\
&= p(x)p(w)\int \frac{p(z)}{p(z)}\frac{p(z,x,w)}{p(x,w)}dz \\
&= p(x)p(w)\int \frac{p(z,x,w)}{p(x,w)}dz \\
&= p(x)p(w)\frac{p(x,w)}{p(x,w)} \\
&= p(x)p(w)
\end{aligned}$$

4. 3 Points

$$\begin{aligned}
E[(x-\mu)^2] &= E[x^2 - 2x\mu + \mu^2] \\
&= E[x^2] + E[-2x\mu] + E[\mu^2] \\
&= E[x^2] - 2\mu E[x] + \mu\mu && |E[x] = \mu \\
&= E[x^2] - 2E[x]E[x] + E[x]E[x] \\
&= E[x^2] - E[x]E[x]
\end{aligned}$$

5. 4 Points

Lets call the choice of door 1 $D$, therefore the probability that the prize is behind door 1 is $p(D)) = \frac{1}{3}$ and the probality that the prize is not behind door 1 is $p(notD) = 1 - p(D) = \frac{2}{3}$.
Now the event $R$ takes place, in which the game host reveals one Door with no prize behind it. Therefore we have the conditional probalities $p(R|D) = 1$ and $p(R|notD) = 1$.
We now want to know what the probability that the prize is behind door 1 is $p(D|R)$.
Given Bayes rule and that $p(R)$ can be factorized as $p(R|D)p(D) + p(R|notD)p(notD)$:

$$p(D|R) = \frac{p(R|D)p(D)}{p(R)} = \frac{p(R|D)p(D)}{p(R|D)p(D) + p(R|notD)p(notD)}$$

We plug in our probalities for each case:

$$p(D|R) = \frac{1 \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3}} = \frac{1}{3}$$

Therefore the probability behind the door $p(D_2)$, which the game show host didn't chose is

$$p(D_2) = 1 - p(D|R) = \frac{2}{3}$$

, which shows us that we should switch to the door 2. This is only true aslong the gameshow host choses the door the way he does.

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## Problem 2 Modeling (5 Points)

### 1. 1 Point

We assume conditional independence of the pixels in the Image $I^1$ with the pixels that are produced from $I^0$ with translation that has been done at each pixel by the d(disparity). Therefore we make an assumption that we have brightness or color constancy and no occlusions, if they are not fullfilled than this does not be valid.

### 2. 1 Point

Gaussian distributions are light tailed, due to this they are very sensitive to outliers, they give large penalty or really low probability to outliers.These outliers are caused by occlusions, shading, shadows, gain control from camera etc. In order to deal with those problems we should choose robust likelihood instead of Gaussian.
As we know that the influence is proportional to the derivative of the error function then we have by derivative of squared Gaussian a linear function, it means that the further away the outliers is the bigger is the influence on the Estimate. In the other hand we take absolute value function that corresponds the Laplacian distribution, by taking the derivative of the ablsolute value function scaled by $\sigma$. So Influence of a outlier does not increase the further it is away. The influence on any point on Estimate is the same no matter where the point is. It is more robust then Gaussian but of course with it's issues.

### 3. 2 Points

The pairwise MRF prior only which pairwise relations are more likely and which are not, therefore we can not directly express relations between three or more Disparity values. This indepence assumption of our pairwise model gives us a disparity value of all 4 direct neigbhours and is therefore not depenend on any other disparity value dispite those 4. We can not get more information about a disparity value than knowing the value of it's 4 neigbhours gives us. Therefore $d_1$ and $d_3$ are not independent given $d_2$ because connections in the MRF model a undirected dependency and knowing $d_3$ would give us information about $d_1$.

### 4. 1 Point

Kronecker delta used in Potts Model assign high compatibility when the disparities in the two neighbor pixels are the same and if they are different a lower compatibility, no matter how different they are. We could use a function which has it maximum at 1 when $a$ and $b$ are the same. Further we could penalize bigger difference more than smaller ones. A function which does this is:

$$\delta = \frac{1}{1 + |a - b|_2}$$

### 5. 1 Point

As we have learnt, in likelihood function are compared only single pixel values, and with help of MRF prior are modelled dependencies between nearby pixels based on the disparity values. This help us to conlcude that dependencies between pixels are not constraind to a region.

## Problem 4 Stereo Likelihood (10 Points)

Everything else is in code.

### 3. 1 Point

For the likelihood we need multiply the probability of each pixel. If we now assume a Gaussian and we are sampling for an almost optimal case. We could assume the following equation:

$$max \mathcal{N}(0,1)^{384 \cdot 288} \approx 0.4^{384 \cdot 288} \approx 0$$

For the negative log likelihood we would not multiply numbers $|n| < 1$ and herefore again something signifcant bigger than 0.

### 6. 1 Point

We calculate the negative log likelihoods for the Gaussian and Laplacian without the scaling term and compute the increasing factor from 0% noise. Both Models are not really robust, but the Laplacian seems to be less effected by outliers than the Gaussian.

| noise | Gaussian | Factor |
|---|---|---|
| 0% | 88.97 | 1 |
| 12% | 622.75 | 7 |
| 25% | 1208.51 | 13.58 |

| noise | Lapplacian | Factor |
|---|---|---|
| 0% | 1236.49 | 1 |
| 12% | 3882.1 | 3.14 |
| 25% | 6775.24 | 5.48 |