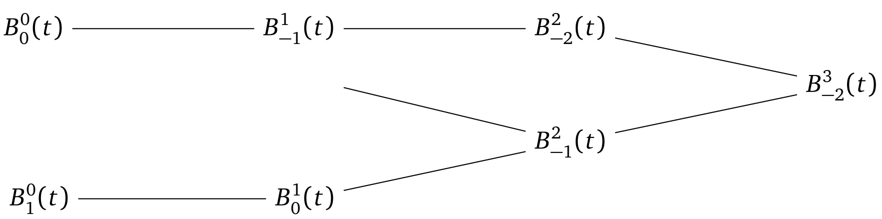
GDV 2 – Theorie Übung 3

Sommer Semester 2019
Übungsgruppe F

Aufgabe 1 Kubische B-Splines und de Boor Algorithmus (6 Punkte)

a) 3 Punkte



$$B_0^0(t) = \begin{cases} 1, & t \in [0,1) \\ 0, & \text{sonst} \end{cases}$$

$$B_1^0(t) = \begin{cases} 1, & t \in [1,2) \\ 0, & \text{sonst} \end{cases}$$

$$B_{-1}^1(t) = \frac{x_1 - t}{x_1 - x_0} B_0^0(t) = \begin{cases} 1-t, & t \in [0,1) \\ 0, & \text{sonst} \end{cases}$$

$$B_0^1(t) = \frac{t - x_0}{x_1 - x_0} B_0^0(t) + \frac{x_2 - t}{x_2 - x_1} B_1^0(t) = \begin{cases} t, & t \in [0,1) \\ 2-t, & t \in [1,2) \\ 0, & \text{sonst} \end{cases}$$

$$B_{-1}^2(t) = \frac{t - x_{-1}}{x_1 - x_{-1}} B_{-1}^1(t) + \frac{x_2 - t}{x_2 - x_0} B_0^1(t) = \begin{cases} 2t - \frac{3}{2}t^2, & t \in [0,1) \\ 2 - 2t + \frac{1}{2}t^2 & t \in [1,2) \\ 0, & \text{sonst} \end{cases}$$

$$B_{-2}^2(t) = \frac{x_{-2+2+1} - t}{x_{-2+2+1} - x_{-2+1}} * B_{-1}^1(t) = 1 - t * B_{-1}^1(t) = \begin{cases} t^2 - 2t + 1 & \text{if } t \in [0,1) \\ 0 & \text{sonst} \end{cases}$$

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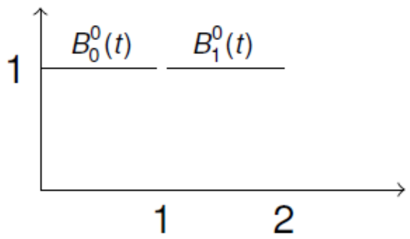
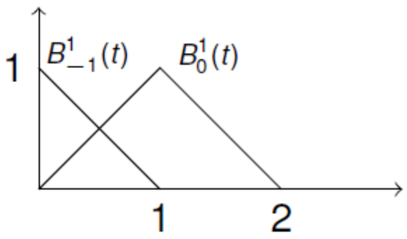
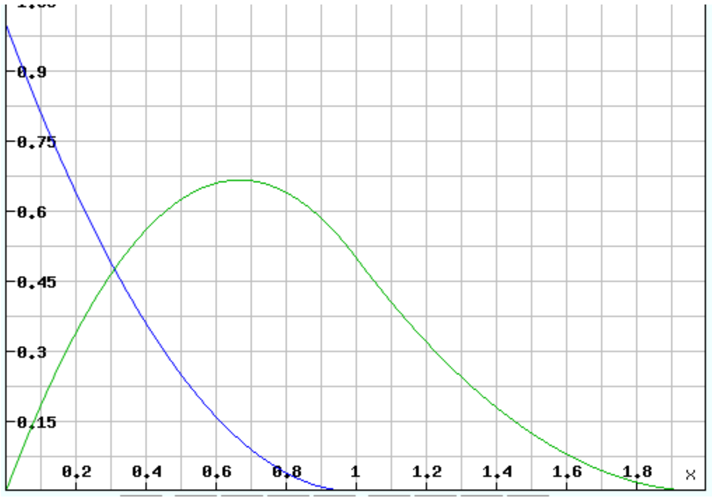


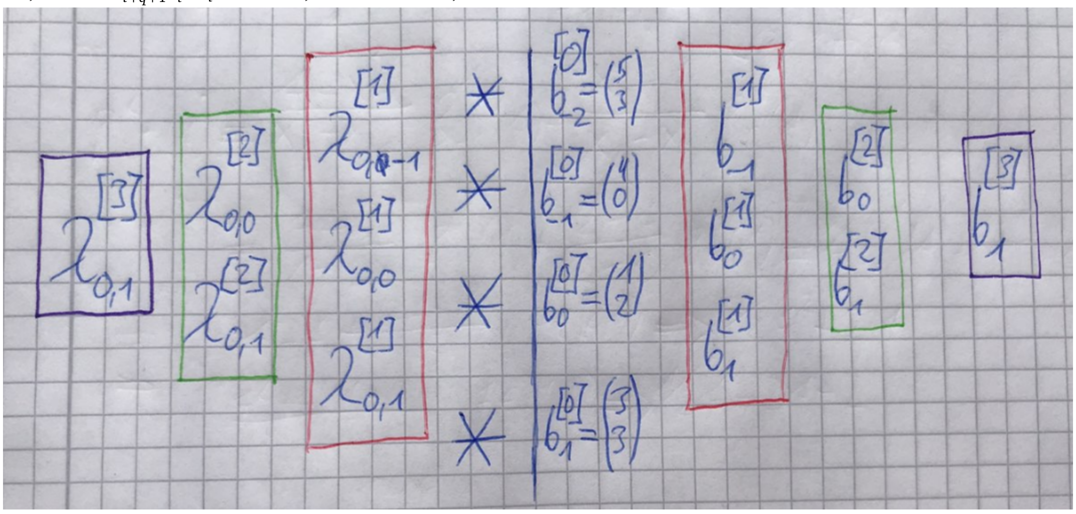




Blau enstpricht $B_{-2}^2(t)$ und grün $B_{-1}^2(t)$

b) 3 Punkte

1) $1.5 \in [x_j, x_{j+1}) \implies 1.5 \in [1,2) \implies j = 1$
2) $i = j - q, ..., j \implies i = -2, ..., 1$
3) $\lambda_{1,i}^{[l]}(\xi) = \frac{x_{i+q+1-l} - \xi}{x_{i+q+1-l} - x_i}$ und $\lambda_{2,i}^{[l]}(\xi) = 1 - \lambda_{1,i}^{[l]}(\xi)$

4) 

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$$\tau_{0,-1}^{[1]} = \frac{x_{-1+3+1-1} - 1{,}5}{2-0} = \frac{2-1{,}5}{2} = 1/4 \qquad \tau_{1,-1} = 3/4$$

$$\tau_{0,0}^{[1]} = \frac{3-1{,}5}{3} = \frac{1{,}5}{3} = 1/2 \qquad \tau_{1,0} = 1/2$$

$$\tau_{0,1}^{[1]} = \frac{3-1{,}5}{3-1} = \frac{1{,}5}{2} = 3/4 \qquad \tau_{1,1} = 1/4$$

+

$$b_{-1}^{[1]} = \tau_{0,-1}(1{,}5)\cdot b_{-1-1}^{[1-1]} + \tau_{1,-1}(1{,}5)\cdot b_{-1}^{[1-1]} = 1/4\cdot\begin{pmatrix}5\\3\end{pmatrix} + 3/4\cdot\begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}5/4\\3/4\end{pmatrix} = \begin{pmatrix}17/4\\3/4\end{pmatrix}$$

$$b_0^{[1]} = 1/2\cdot\begin{pmatrix}4\\0\end{pmatrix} + 1/2\cdot\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}1/2\\1\end{pmatrix} = \begin{pmatrix}5/2\\1\end{pmatrix}$$

$$b_1^{[1]} = 3/4\cdot\begin{pmatrix}1\\2\end{pmatrix} + 1/4\cdot\begin{pmatrix}3\\3\end{pmatrix} = \begin{pmatrix}3/2\\9/4\end{pmatrix}$$

O

$$\tau_{0,0}^{[2]} = \frac{x_{0+3+1-2} - 1{,}5}{2-0} = 1/4 \qquad \tau_{1,0}^{[2]} = 3/4$$

$$\tau_{0,1}^{[2]} = \frac{3-1{,}5}{3-1} = \frac{1{,}5}{2} = 3/4 \qquad \tau_{1,1}^{[2]} = 1/4$$

$$b_0^{[2]} = 1/4\cdot b_{-1}^{[1]} + 3/4\cdot b_0^{[1]} = 1/4\cdot\begin{pmatrix}17/4\\3/4\end{pmatrix} + 3/4\cdot\begin{pmatrix}5/2\\1\end{pmatrix} = \begin{pmatrix}17/16\\3/16\end{pmatrix} + \begin{pmatrix}15/8\\3/4\end{pmatrix} = \begin{pmatrix}47/16\\15/16\end{pmatrix}$$

$$b_1^{[2]} = 3/4\cdot\begin{pmatrix}5/2\\1\end{pmatrix} + 1/4\cdot\begin{pmatrix}3/2\\9/4\end{pmatrix} = \begin{pmatrix}15/8\\3/4\end{pmatrix} + \begin{pmatrix}3/8\\9/16\end{pmatrix} = \begin{pmatrix}18/8\\21/16\end{pmatrix}$$

[redacted]

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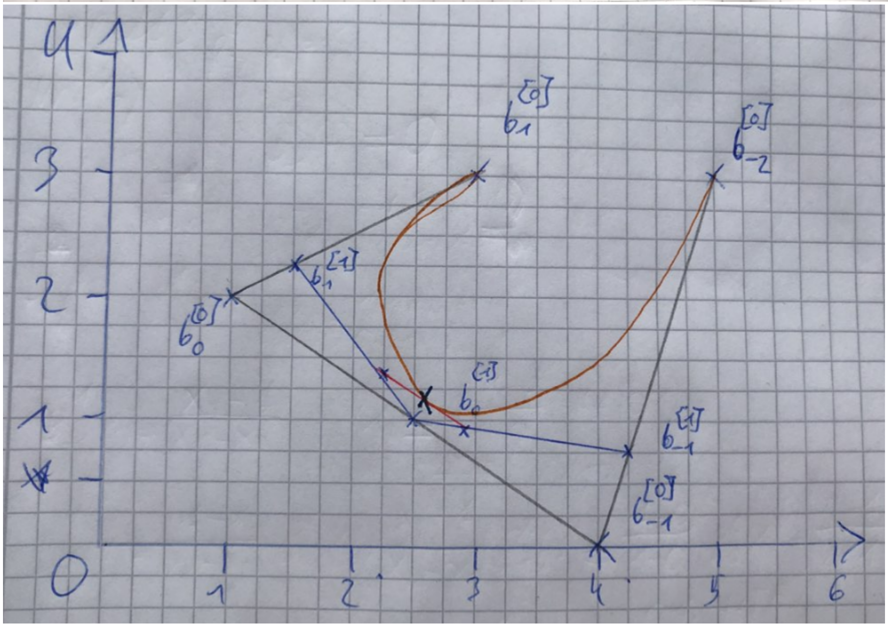
$X^{[3]}$ [redacted]

$$\lambda_{0,1} = \frac{2-1,5}{2-1} = 1/2$$

$$b_1^{[3]} = 1/2 \cdot \begin{pmatrix} 47/16 \\ 15/16 \end{pmatrix} + 1/2 \cdot \begin{pmatrix} 19/8 \\ 21/16 \end{pmatrix} = \begin{pmatrix} 47/32 \\ 15/32 \end{pmatrix} + \begin{pmatrix} 19/16 \\ 21/32 \end{pmatrix} = \begin{pmatrix} 83/32 \\ 36/32 \end{pmatrix}$$



Aufgabe 2 Bernstein-Bézier-Tensorprodukte (7 Punkte)

a) 3 Punkte

Wir wenden de Casteljau an mit $\upsilon = \frac{3}{4}$ und $\lambda = \frac{1}{4}$:

$$\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} \quad\to\quad \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -2.5 \\ 2.75 \\ 6.25 \end{pmatrix} \quad\to\quad \begin{pmatrix} -2.125 \\ 2.5625 \\ 5.4375 \end{pmatrix}$$

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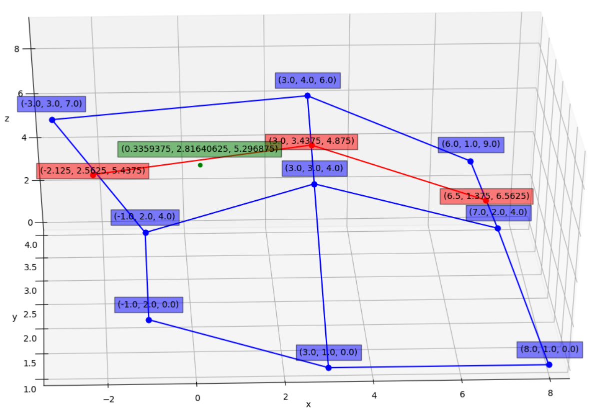
$$\begin{array}{ccc} \begin{pmatrix}3\\1\\0\end{pmatrix} & & \\ & \begin{pmatrix}3\\2.5\\3\end{pmatrix} & \\ \begin{pmatrix}3\\3\\4\end{pmatrix} & & \begin{pmatrix}3\\3.4375\\4.875\end{pmatrix} \\ & \begin{pmatrix}3\\3.75\\5.5\end{pmatrix} & \\ \begin{pmatrix}3\\4\\6\end{pmatrix} & & \end{array}$$

$$\begin{array}{ccc} \begin{pmatrix}8\\1\\0\end{pmatrix} & & \\ & \begin{pmatrix}7.25\\1.75\\3\end{pmatrix} & \\ \begin{pmatrix}7\\2\\4\end{pmatrix} & & \begin{pmatrix}6.5\\1.375\\6.5625\end{pmatrix} \\ & \begin{pmatrix}6.25\\1.25\\7.75\end{pmatrix} & \\ \begin{pmatrix}6\\1\\9\end{pmatrix} & & \end{array}$$

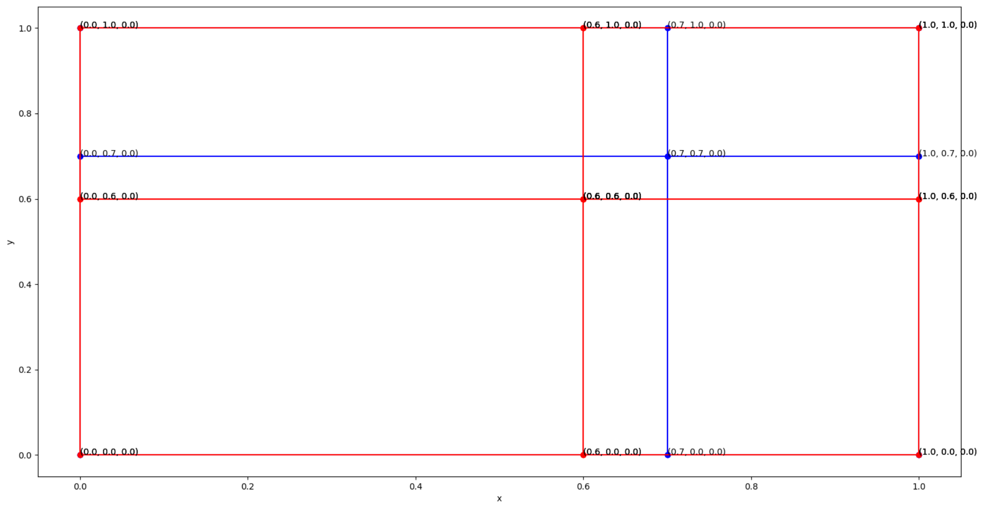
Wir wenden de Casteljau an mit $u = \frac{1}{4}$ und $\lambda = \frac{3}{4}$:

$$\begin{array}{ccc} \begin{pmatrix}-2.125\\2.5625\\5.4375\end{pmatrix} & & \\ & \begin{pmatrix}-0.84375\\2.78125\\5.296875\end{pmatrix} & \\ \begin{pmatrix}3\\3.4375\\4.875\end{pmatrix} & & \begin{pmatrix}0.3359375\\2.81640625\\5.296875\end{pmatrix} \\ & \begin{pmatrix}3.875\\2.921875\\5.296875\end{pmatrix} & \\ \begin{pmatrix}6.5\\1.375\\6.5625\end{pmatrix} & & \end{array}$$

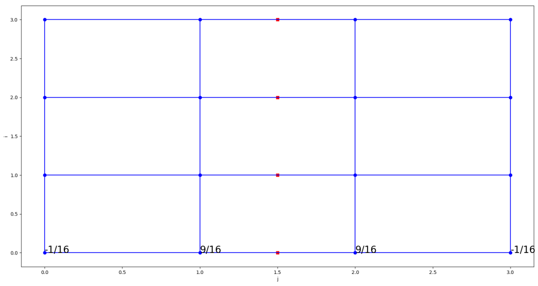


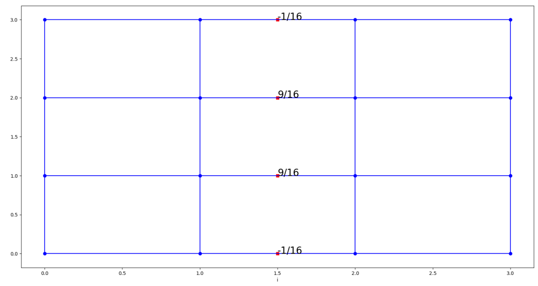
b) 2 Punkte

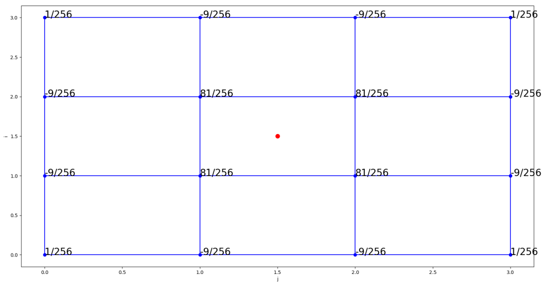
hierzu bestimmen wir die kontrollpunkte an den Intervallsgrenzen, wie in Aufgabeteil a. in der grafik sind ist die Neue untereilung eingezeichnet.



c) 2 Punkte







Aufgabe 3 Implizite Oberflächen (7 Punkte)

a) 3 Punkte

$f(x,y,z) := x^2 + 2y^2 + 4z^2 - 10 = 0$

$r(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t * \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} t \\ t \\ t \end{pmatrix}$

$f(r(t)) = 7t^2 - 10 = f(t)$

$f([-4,2]) = 7 * [-4,2]^2 - 10 = 7 * [0,16] - 10 = [0,112] - 10 = [-10,102]$

$f'(t) = 14t$

$f'([-4,2]) = 14[-4,2] = [-56,28]$

Wird gesplittet in $[-4,-1]$ und $[-1,2]$.

$f([-4,-1]) = 7[-4,-1]^2 - 10 = 7[1,16] - 10 = [7,112] - 10 = [-3,102]$

$f'([-4,-1]) = 14[-4,-1] = [-56,-14]$

$f(-4) * f(-1) = 102 * -3 = -306 \rightarrow$ genau eine Nullstelle in [-4,-1].

$f([-1,2]) = 7[-1,2]^2 - 10 = 7[0,4] - 10 = [0,28] - 10 = [-10,18]$

$f'([-1,2]) = 14[-1,2] = [-14,28]$

Wird gesplittet in $[-1, 0.5]$ und $[0.5, 2]$.

$f([-1, 0.5]) = 7[-1, 0.5]^2 - 10 = 7[0, 1] - 10 = [-10, -3] \rightarrow$ keine Nullstelle.

$f([0.5, 2]) = 7[0.5, 2]^2 - 10 = 7[0.25, 4] - 10 = [-8.25, 18]$
$f'([0.5, 2]) = 14[0.5, 2] = [7, 28]$
$f(0.5) * f(2) = -8.25 * 18 = -148.5 \rightarrow$ genau eine Nullstelle in [0.5,2].

b) 2 Punkte

i) Beschreibt eine Fläche entlang der Achsen z und -y.
$N = \bigtriangledown f(P_0) \neq 0$ ist Normalenvektor.

$$\bigtriangledown f = \begin{pmatrix} \frac{\delta f}{\delta x} \\ \frac{\delta f}{\delta y} \\ \frac{\delta f}{\delta z} \end{pmatrix}$$

$$\bigtriangledown f_1(P_0) = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \neq 0 \rightarrow Normalenvektor$$

ii) Elliptisches Paraboloid um -4 in z Richtung verschoben.

$$\bigtriangledown f_2(P_0) = \begin{pmatrix} 2x_0 \\ 2y_0 \\ 0 \end{pmatrix} \neq 0$$

iii) Kugel mit dem Radius 2 und dem Mittelpunkt = (2, 3, 2).

$$\bigtriangledown f_3(P_0) = \begin{pmatrix} 2x_0 - 4 \\ 2y_0 - 6 \\ 2z_0 - 4 \end{pmatrix} \neq 0$$

iiii) Ellipsoid

$$\bigtriangledown f_4(P_0) = \begin{pmatrix} 2x_0 \\ y_0 \\ 4z_0 \end{pmatrix} \neq 0$$

c) 2 Punkte

i)
$Kugel = f_k(x, y, z) := (x-4)^2 + (y-3)^2 + (z-5)^2 - (2^2) = 0$
$Box = f_b(x, y, z) := y \geq 3$
$\cap(f_k, f_b) = max(f_k, f_b)$
ii)
$Kugel = f_{k5}(x, y, z) := (x-4)^2 + (y-3)^2 + (z-5)^2 - (5^2) = 0$
$Kugel = f_{k4}(x, y, z) := (x-4)^2 + (y-3)^2 + (z-5)^2 - (4^2) = 0$
$(f_{k5}(x, y, z) \setminus f_{k4}(x, y, z)) \cap f_b(x, y, z) = max(max(f_{k5}(x, y, z), -f_{k4}(x, y, z)), f_b(x, y, z))$