

GDV 2 – Theorie Übung 3



TECHNISCHE
UNIVERSITÄT
DARMSTADT

Sommer Semester 2019
Übungsgruppe F

Aufgabe 1 Kubische B-Splines und de Boor Algorithmus (6 Punkte)

a) 3 Punkte

$$B_0^0(t) \xrightarrow{} B_{-1}^1(t) \xrightarrow{} B_{-2}^2(t) \xrightarrow{} B_{-2}^3(t)$$

$$B_1^0(t) \xrightarrow{} B_0^1(t) \xrightarrow{} B_{-1}^2(t) \xrightarrow{} B_{-2}^3(t)$$

$$B_0^0(t) = \begin{cases} 1, & t \in [0, 1) \\ 0, & \text{sonst} \end{cases}$$

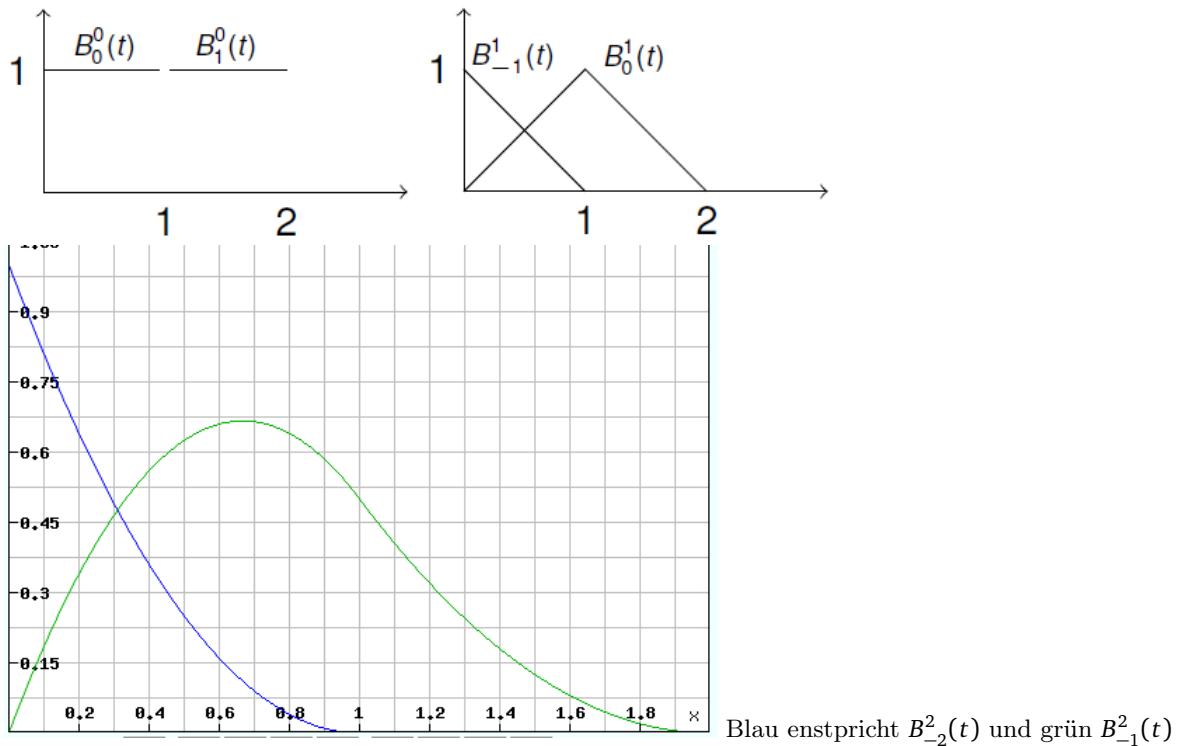
$$B_1^0(t) = \begin{cases} 1, & t \in [1, 2) \\ 0, & \text{sonst} \end{cases}$$

$$B_{-1}^1(t) = \frac{x_1 - t}{x_1 - x_0} B_0^0(t) = \begin{cases} 1 - t, & t \in [0, 1) \\ 0, & \text{sonst} \end{cases}$$

$$B_0^1(t) = \frac{t - x_0}{x_1 - x_0} B_0^0(t) + \frac{x_2 - t}{x_2 - x_1} B_1^0(t) = \begin{cases} t, & t \in [0, 1) \\ 2 - t, & t \in [1, 2) \\ 0, & \text{sonst} \end{cases}$$

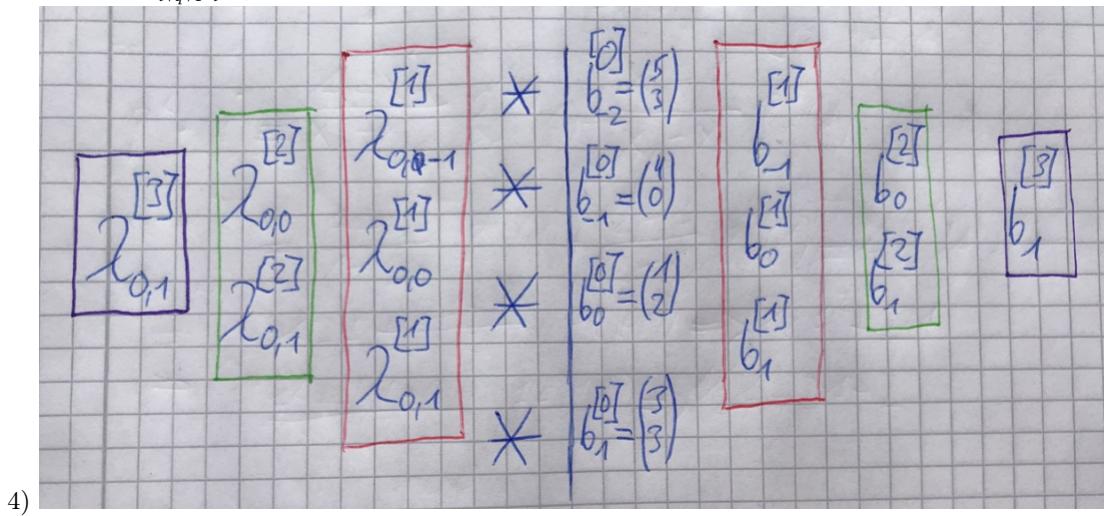
$$B_{-1}^2(t) = \frac{t - x_{-1}}{x_1 - x_{-1}} B_{-1}^1(t) + \frac{x_2 - t}{x_2 - x_0} B_0^1(t) = \begin{cases} 2t - \frac{3}{2}t^2, & t \in [0, 1) \\ 2 - 2t + \frac{1}{2}t^2 & t \in [1, 2) \\ 0, & \text{sonst} \end{cases}$$

$$B_{-2}^3(t) = \frac{x_{-2+2+1}-t}{x_{-2+2+1}-x_{-2+1}} * B_{-1}^2(t) = 1 - t * B_{-1}^2(t) = \begin{cases} t^2 - 2t + 1 & \text{if } t \in [0, 1) \\ 0 & \text{sonst} \end{cases}$$



b) 3 Punkte

- 1) $1.5 \in [x_j, x_{j+1}] \implies 1.5 \in [1, 2] \implies j = 1$
- 2) $i = j - q, \dots, j \implies i = -2, \dots, 1$
- 3) $\lambda_{1,i}^{[l]}(\xi) = \frac{x_{i+q+1-l}-\xi}{x_{i+q+1-l}-x_i}$ und $\lambda_{2,i}^{[l]}(\xi) = 1 - \lambda_{1,i}^{[l]}(\xi)$



$$\begin{aligned}
 x_{0,-1}^{[1]} &= \frac{x_{-1+3+1-1} - 1,5}{2-0} = \frac{2-1,5}{2} = \frac{1}{4} & x_{1,-1} &= \frac{3}{4} \\
 x_{0,0}^{[1]} &= \frac{3-1,5}{3} = \frac{1,5}{3} = \frac{1}{2} & x_{1,0} &= \frac{1}{2} \\
 x_{0,1}^{[1]} &= \frac{3-1,5}{3-1} = \frac{1,5}{2} = \frac{3}{4} & x_{1,1} &= \frac{1}{4}
 \end{aligned}$$

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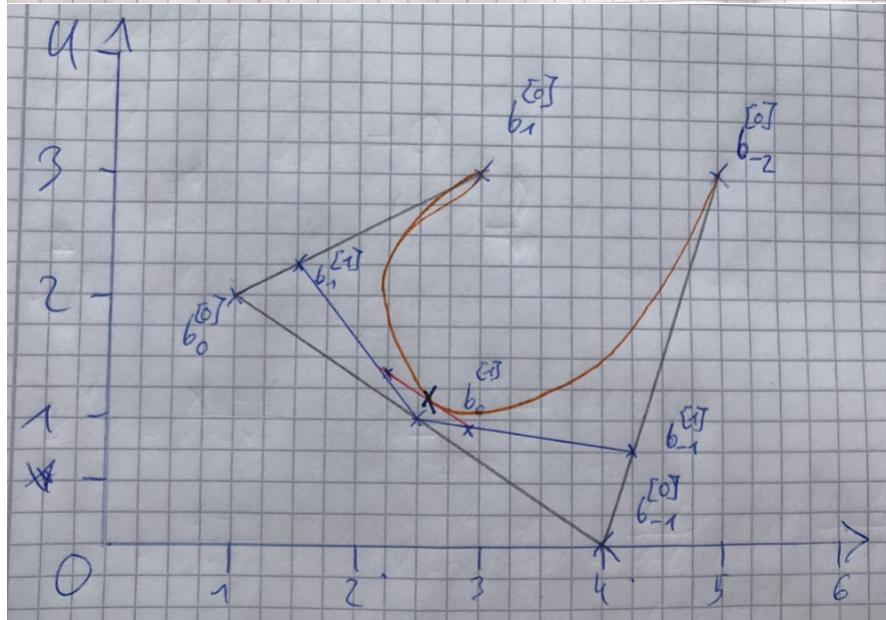
$$\begin{aligned}
 + \quad b_{-1}^{[1]} &= x_{0,-1}^{[1]} \cdot b_{-1}^{[1]} + x_{1,-1}^{[1]} \cdot b_{-1}^{[1]} = \frac{1}{4} \cdot \binom{1}{3} + \frac{3}{4} \cdot \binom{4}{0} = \binom{3}{0} + \frac{3}{4} \binom{4}{0} \\
 b_0^{[1]} &= \frac{1}{2} \cdot \binom{4}{0} + \frac{1}{2} \cdot \binom{1}{2} = \binom{2}{0} + \binom{1}{1} = \binom{5/2}{1} \\
 b_1^{[1]} &= \frac{3}{4} \cdot \binom{1}{2} + \frac{1}{4} \cdot \binom{3}{3} = \binom{3/2}{3/4}
 \end{aligned}$$

$$\begin{aligned}
 0 \quad x_{0,0}^{[2]} &= \frac{x_{0+3+1-2} - 1,5}{2-0} = \frac{1}{4} & x_{1,0}^{[2]} &= \frac{3}{4} \\
 x_{0,1}^{[2]} &= \frac{3-1,5}{3-1} = \frac{1,5}{2} = \frac{3}{4} & x_{1,1}^{[2]} &= \frac{1}{4}
 \end{aligned}$$

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$$\begin{aligned}
 b_0^{[2]} &= \frac{1}{4} \cdot b_{-1}^{[1]} + \frac{3}{4} \cdot b_0^{[1]} = \frac{1}{4} \cdot \binom{17/4}{3/4} + \frac{3}{4} \cdot \binom{5/2}{1} = \binom{17/16}{3/16} + \binom{15/8}{3/4} \\
 b_1^{[2]} &= \frac{3}{4} \cdot \binom{5/2}{1} + \frac{1}{4} \cdot \binom{3/2}{3/4} = \binom{15/8}{3/4} + \binom{3/8}{3/4} = \binom{47/16}{15/16} \\
 &= \binom{18/8}{2/16}
 \end{aligned}$$

$$\begin{aligned} & \boxed{\begin{array}{l} X_{23} \\ 20,1 = \frac{2-15}{2-1} = 12 \end{array}} \quad \boxed{\text{[REDACTED]}} \\ & \boxed{\begin{array}{l} b_1 \\ b_1 = 12 \cdot \left(\frac{47}{16} \right) + 12 \cdot \left(\frac{19}{16} \right) = \left(\frac{47}{16} \right) + \left(\frac{19}{16} \right) = \left(\frac{83}{32} \right) \end{array}} \end{aligned}$$



Aufgabe 2 Bernstein-Bézier-Tensorprodukte (7 Punkte)

a) 3 Punkte

Wir wenden de Casteljau an mit $\nu = \frac{3}{4}$ und $\lambda = \frac{1}{4}$:

$$\begin{array}{c} \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \\ \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} \\ \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} \end{array} \xrightarrow{\quad} \begin{array}{c} \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \\ \begin{pmatrix} -2.5 \\ 2.75 \\ 6.25 \end{pmatrix} \end{array} \xrightarrow{\quad} \begin{pmatrix} -2.125 \\ 2.5625 \\ 5.4375 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 3 \\ 2.5 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 3 \\ 3.4375 \\ 4.875 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \quad \begin{pmatrix} 3 \\ 3.75 \\ 5.5 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 7.25 \\ 1.75 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 6.5 \\ 1.375 \\ 6.5625 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix} \quad \begin{pmatrix} 6.25 \\ 1.25 \\ 7.75 \end{pmatrix}$$

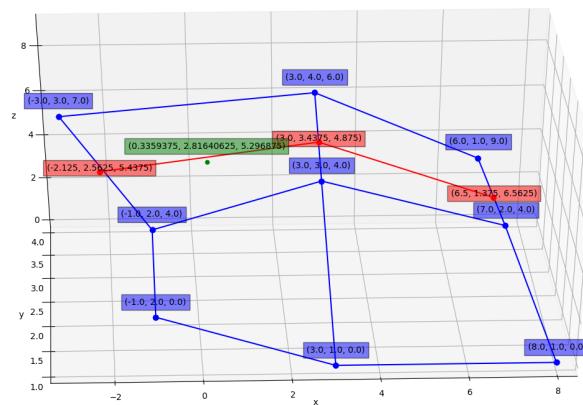
$$\begin{pmatrix} 6 \\ 1 \\ 9 \end{pmatrix}$$

Wir wenden die Casteljau an mit $u = \frac{1}{4}$ und $\lambda = \frac{3}{4}$:

$$\begin{pmatrix} -2.125 \\ 2.5625 \\ 5.4375 \end{pmatrix} \quad \begin{pmatrix} -0.84375 \\ 2.78125 \\ 5.296875 \end{pmatrix} \quad \begin{pmatrix} 0.3359375 \\ 2.81640625 \\ 5.296875 \end{pmatrix}$$

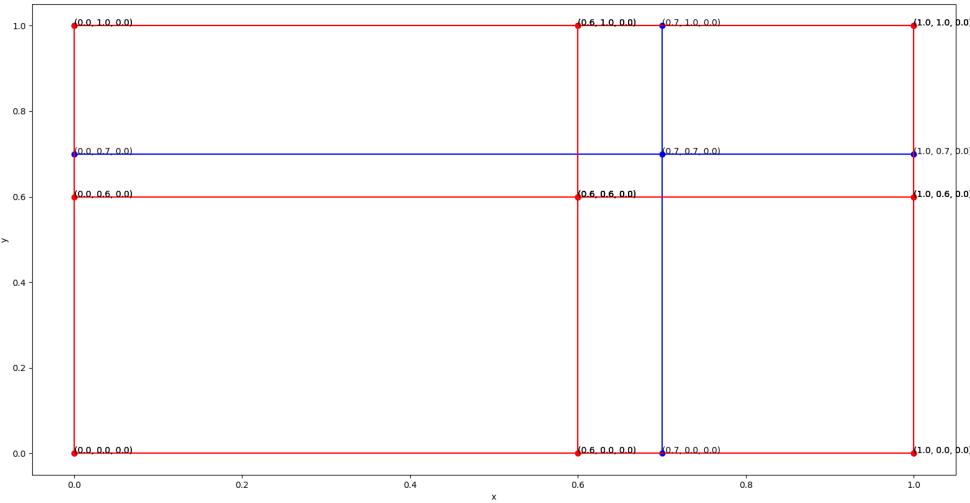
$$\begin{pmatrix} 3 \\ 3.4375 \\ 4.875 \end{pmatrix} \quad \begin{pmatrix} 3.875 \\ 2.921875 \\ 5.296875 \end{pmatrix}$$

$$\begin{pmatrix} 6.5 \\ 1.375 \\ 6.5625 \end{pmatrix}$$



b) 2 Punkte

hierzu bestimmen wir die kontrollpunkte an den Intervallsgrenzen, wie in Aufgabeteil a. in der grafik sind ist die Neue unterteilung eingezeichnet.



c) 2 Punkte

Aufgabe 3 Implizite Oberflächen (7 Punkte)

a) 3 Punkte

$$f(x, y, z) := x^2 + 2y^2 + 4z^2 - 10 = 0$$

$$r(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t * \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} t \\ t \\ t \end{pmatrix}$$

$$f(r(t)) = 7t^2 - 10 = f(t)$$

$$f([-4, 2]) = 7 * [-4, 2]^2 - 10 = 7 * [0, 16] - 10 = [0, 112] - 10 = [-10, 102]$$

$$f'(t) = 14t$$

$$f'([-4, 2]) = 14[-4, 2] = [-56, 28]$$

Wird gesplittet in $[-4, -1]$ und $[-1, 2]$.

$$f([-4, -1]) = 7[-4, -1]^2 - 10 = 7[1, 16] - 10 = [7, 112] - 10 = [-3, 102]$$

$$f'([-4, -1]) = 14[-4, -1] = [-56, -14]$$

$$f(-4) * f(-1) = 102 * -3 = -306 \rightarrow \text{genau eine Nullstelle in } [-4, -1].$$

$$f([-1, 2]) = 7[-1, 2]^2 - 10 = 7[0, 4] - 10 = [0, 28] - 10 = [-10, 18]$$

$$f'([-1, 2]) = 14[-1, 2] = [-14, 28]$$

Wird gesplittet in $[-1, 0.5]$ und $[0.5, 2]$.

$$f([-1, 0.5]) = 7[-1, 0.5]^2 - 10 = 7[0, 1] - 10 = [-10, -3] \rightarrow \text{keine Nullstelle.}$$

$$f([0.5, 2]) = 7[0.5, 2]^2 - 10 = 7[0.25, 4] - 10 = [-8.25, 18]$$

$$f'([0.5, 2]) = 14[0.5, 2] = [7, 28]$$

$$f(0.5) * f(2) = -8.25 * 18 = -148.5 \rightarrow \text{genau eine Nullstelle in } [0.5, 2].$$

b) 2 Punkte

i) Beschreibt eine Fläche entlang der Achsen z und -y.

$N = \nabla f(P_0) \neq 0$ ist Normalenvektor.

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$$

$$\nabla f_1(P_0) = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \neq 0 \rightarrow \text{Normalenvektor}$$

ii) Elliptisches Paraboloid um -4 in z Richtung verschoben.

$$\nabla f_2(P_0) = \begin{pmatrix} 2x_0 \\ 2y_0 \\ 0 \end{pmatrix} \neq 0$$

iii) Kugel mit dem Radius 2 und dem Mittelpunkt = (2, 3, 2).

$$\nabla f_3(P_0) = \begin{pmatrix} 2x_0 - 4 \\ 2y_0 - 6 \\ 2z_0 - 4 \end{pmatrix} \neq 0$$

iv) Ellipsoid

$$\nabla f_4(P_0) = \begin{pmatrix} 2x_0 \\ y_0 \\ 4z_0 \end{pmatrix} \neq 0$$

c) 2 Punkte

i)

$$\text{Kugel } f_k(x, y, z) := (x - 4)^2 + (y - 3)^2 + (z - 5)^2 - (2^2) = 0$$

$$\text{Box } f_b(x, y, z) := y \geq 3$$

$$\cap(f_k, f_b) = \max(f_k, f_b)$$

ii)

$$\text{Kugel } f_{k5}(x, y, z) := (x - 4)^2 + (y - 3)^2 + (z - 5)^2 - (5^2) = 0$$

$$\text{Kugel } f_{k4}(x, y, z) := (x - 4)^2 + (y - 3)^2 + (z - 5)^2 - (4^2) = 0$$

$$(f_{k5}(x, y, z) \setminus f_{k4}(x, y, z)) \cap f_b(x, y, z) = \max(\max(f_{k5}(x, y, z), -f_{k4}(x, y, z)), f_b(x, y, z))$$