

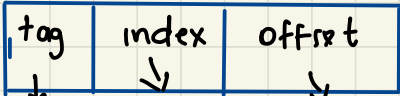
## Caching 1

### Cache Lines



- **data block**: cached data (i.e., copy of bytes from memory)
- **tag**: uniquely identifies which data is stored in the cache line
- **valid bit**: indicates whether or not the line contains meaningful information

address  $\rightarrow$  data (after hex to bin conversion):



The rest  $\rightarrow$  bits  
 $\log(n)$  bits  
 $\log(\text{block size})$  bits

Spacial locality: close items in memory  
 Temporal locality: referenced items tend to be referenced again soon

k-way set Associative Cache  
 k cache lines per set  
 index becomes  $\log(n/k)$

On a Cache miss, if we're not using a direct mapped cache, we have to choose the line that gets evicted. Most caches evict the least recently used line.

### Caching Organization Summarized

- A cache consists of lines
- A line contains
  - A block of bytes, the data values from memory
  - A tag, indicating where in memory the values are from
  - A valid bit, indicating if the data are valid
- Lines are organized into sets
  - Direct-mapped cache: one line per set
  - k-way associative cache: k lines per set
  - Fully associative cache: all lines in one set  $\rightarrow$  no index, check all lines
- Caches handle both reads and writes
  - write-through: write to both cache and memory
  - write-back: write only to cache, write to memory on evict
  - write-allocate: alloc on any miss
  - no-write allocate: alloc only on read miss

Cache size: total  $n$  bytes that can be stored in a cache

Avg access time = hit time + miss rate \* miss-penalty

### Category 0 misses

- Compulsory: 1st reference to address
- Capacity: Cache is too small
- Conflict: collisions in a specific set

# Caching and Writes

## What to do on a write-hit?

- Write-through:** write immediately to memory *(update both copies: in cache & in memory)*
- Write-back:** defer write to memory until replacement of line *→ only update cache*
  - Need a dirty bit (line different from memory or not)

## What to do on a write-miss?

- Write-allocate:** load into cache, update line in cache *only update cache*
  - Good if more writes to the location follow
- No-write-allocate:** writes straight to memory, does not load into cache *only update cache*

## Typical

- Write-through + No-write-allocate
- Write-back + Write-allocate** → if it's there you only update cache, otherwise, bring data into cache & then only update copy in cache.

Optimization

## Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}

for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    x = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += x * b[k][j];
  }
}

for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    x = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * x;
  }
}
```

ijk (& jik):

- 2 memory accesses (2 reads, 0 write)
- misses/iter = 1.25

kij (& jki):

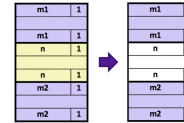
- 3 memory accesses (2 reads, 1 write)
- misses/iter = 0.5

iki (& jki):

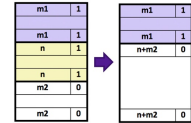
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## Constant-Time Coalescing

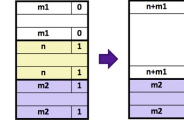
Case 1: Prev and next block allocated



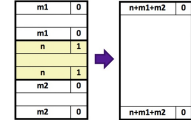
Case 2: Prev block free, next block allocated



Case 3: Prev block allocated, next block free



Case 4: Prev and next block free



## Machine Independent Optimization

### Compilers optimize assembly code

- Dead code elimination
- Code motion
- Factoring out common subexpressions
- Loop elimination
- Reduction in Strength

### Optimization blockers:

- Aliasing
  - Use local variables
- Procedure calls
  - Move them yourself

## Cache Performance Metrics

### Miss Rate

- Fraction of memory references not found in cache (misses / accesses)
- Typically 3-10% for L1
- can be quite small (e.g., < 1%) for L2, depending on size, etc.

### Hit Time

- Time to deliver a line in the cache to the processor
  - includes time to determine whether the line is in the cache
- Typically 4 clock cycles for L1, 10 clock cycles for L2

### Miss Penalty

- Additional time required because of a miss
  - typically 50-200 cycles for main memory (Trend: increasing!)

## Allocator Goals

### Throughput: number of requests completed per time unit

- Make allocator efficient
- Example: if your allocator processes 5,000 `malloc` calls and 5,000 `free` calls in 10 seconds then throughput is 1,000 operations/second

### Memory Utilization: fraction of heap memory allocated

- Minimize wasted space
- Peak Memory Utilization  $U_t = \frac{\max \text{space allocated at time } t}{\text{size of heap at time } t}$

### These goals are often conflicting

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for (k=0; k<n; k++) {
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for (j=0; j<n; j++) {
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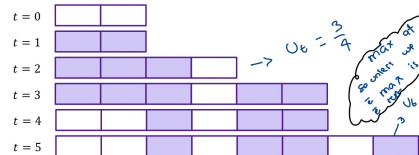
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## Exercise: Memory Utilization

- Recall that Peak Memory Utilization  $U_t = \frac{\max \text{space allocated at time } t}{\text{size of heap at time } t}$



- What is the Peak Memory Utilization at time  $t = 2$ ?
- What is the Peak Memory Utilization at time  $t = 5$ ?