Quantum Entanglement Distillation for Network Edge Claiming

MIT iQuHack 2026 — Team Report

1 Problem Formulation

We consider a quantum network graph $G(V,E)$ where edges represent noisy entanglement links. Each edge holds N identical Werner-like Bell pairs with initial fidelity $F_0 < F_{\mathrm{th}}$, where F_{th} is the threshold for claiming. The fidelity is measured with respect to the maximally entangled state:

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right), \quad F = \left|\left\langle \Phi^+|\psi\middle|\Phi^+|\psi\right\rangle\right|^2 \tag{1}$$

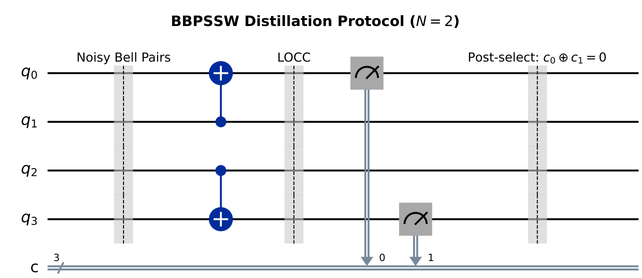
A noisy Bell pair under depolarizing channel can be modeled as a Werner state:

$$\rho_W = F\left|\Phi^+\right\rangle\left\langle\Phi^+\right| + \frac{1-F}{3}\sum_{i\neq\Phi^+}|B_i\rangle\langle B_i| \tag{2}$$

where $\{B_i\} = \{|\Phi^\pm\rangle, |\Psi^\pm\rangle\}$ is the Bell basis. Our objective is to design LOCC (Local Operations and Classical Communication) circuits that distill high-fidelity pairs from noisy inputs.

2 BBPSSW Protocol Implementation

We implement the Bennett-Brassard-Popescu-Schumacher-Smolin-Wootters (BBPSSW) protocol for $N = 2$ Bell pairs. The qubit pairing follows the outside-in convention: pair 1 uses qubits (q_0, q_3) as ancilla, and pair 2 uses (q_1, q_2) as the data pair.



Circuit Operations: Apply bilateral CNOT gates locally—Alice performs $\mathrm{CNOT}_{1\to0}$ and Bob performs $\mathrm{CNOT}_{2\to3}$:

$$U_{\mathrm{LOCC}} = (I_B \otimes \mathrm{CX}_{2\to3}) \cdot (\mathrm{CX}_{1\to0} \otimes I_B) \tag{3}$$

After measuring ancilla qubits q_0 and q_3, we apply **post-selection**: keep only outcomes where $c_0 \oplus c_1 = 0$ (parity check):

$$\mathrm{flag} = c_0 \oplus c_1 = 0 \implies \text{accept} \tag{4}$$

For input Werner states with fidelity F_0, the output fidelity is:

$$F_{\mathrm{out}} = \frac{F_0^2 + \frac{1}{9}(1-F_0)^2}{F_0^2 + \frac{2}{3}F_0(1-F_0) + \frac{5}{9}(1-F_0)^2} \tag{5}$$

The success probability is $P_{\mathrm{succ}} = F_0^2 + \frac{2}{3}F_0(1-F_0) + \frac{5}{9}(1-F_0)^2$. Distillation succeeds when $F_0 > 0.5$, yielding $F_{\mathrm{out}} > F_0$.

3 DEJMPS Variant

We also implement a Deutsch-Ekert-Jozsa-Macchiavello-Popescu-Sanpera (DEJMPS) inspired variant that handles phase errors more effectively. The modification applies Hadamard gates to ancilla qubits before the bilateral CNOTs:

$$U_{\mathrm{DEJMPS}} = U_{\mathrm{LOCC}} \cdot (H_0 \otimes I \otimes I \otimes H_3) \tag{6}$$

This transforms the noise basis, enabling the protocol to correct both bit-flip (X) and phase-flip (Z) errors simultaneously. For asymmetric noise channels where $p_X \neq p_Z$, this variant can outperform standard BBPSSW.

4 Competitive Strategy

The claim strength for a vertex is computed as:

$$S = \sum_{i=1}^{n} \frac{F_i \cdot P_i}{\sqrt{r_i}} \tag{7}$$

where F_i is the achieved fidelity, P_i is success probability, and r_i is the rank of edge i (sorted by fidelity). Our automated solver prioritizes edges by difficulty rating and threshold, iteratively applying both protocols until $F \geq F_{\mathrm{th}}$. Failed attempts do not consume budget, enabling aggressive exploration of the solution space while preserving resources for successful claims.