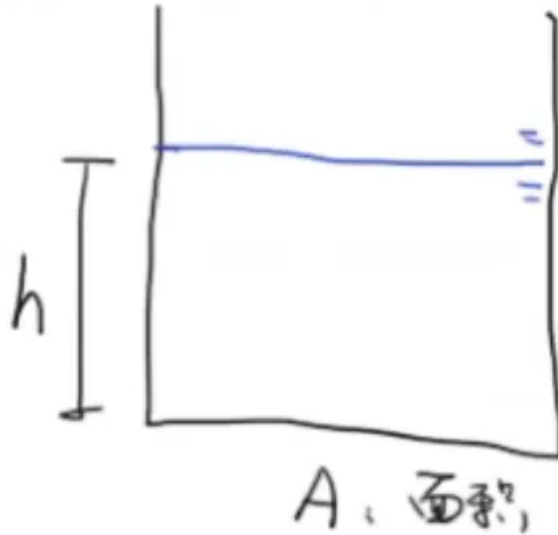


3. 流体系统建模

基本概念

模型推导

基本概念



首先明确几个概念：

Q 流量 (Flow rate) : 单位 m^3/s ;

V 体积 (Volume) : 单位 m^3 ;

h 高度 (Height) : 单位 m ;

P 压强 (Pressure) : 单位 N/m^2 (Pascal) ;

静压 (Static Pressure / Hydrostatic Pressure) :

$$P_{Hydro} = \frac{F_{Hydro}}{A} = \frac{mg}{A} = \rho gh \quad (1)$$

$$m = \rho V = \rho Ah \quad (2)$$

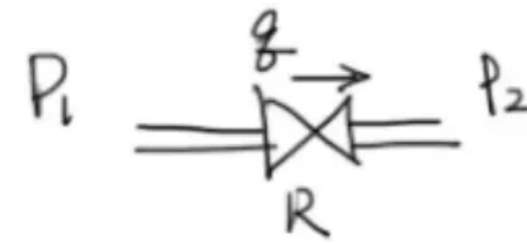
绝对压强 (Absolute Pressure) : P_a 为大气压强

$$P_{abs} = P_a + P_{Hydro} = P_a + \rho gh \quad (3)$$

表压 (Gauge Pressure) : 一般为相对于大气压强的压力测量值

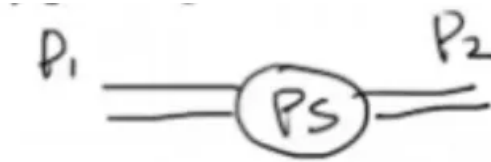
$$P_{gauge} = P_{abs} - P_a \quad (4)$$

R 流阻 (Flow Resistance) :



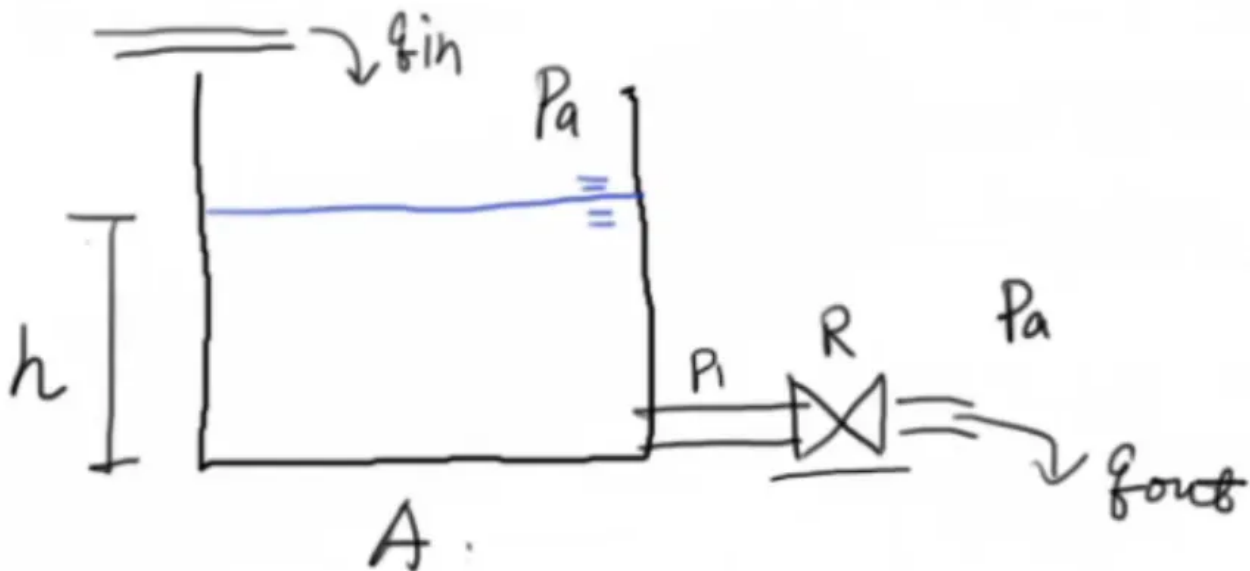
$$\Delta P = P_1 - P_2 = \rho Q R \quad (5)$$

理想压源:



$$P_2 = P_1 + P_S \quad (6)$$

模型推导



由质量守恒定律:

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$$

$$Q = \frac{dV}{dt} = \frac{dm}{dt} / \rho = \dot{m} / \rho$$

$$\frac{dm}{dt} / \rho = \frac{\dot{m}_{in}}{\rho} - \frac{\dot{m}_{out}}{\rho}$$

$$\frac{dV}{dt} = Q_{in} - Q_{out}$$

$$V = Ah$$

$$A \frac{dh}{dt} = Q_{in} - Q_{out} \Rightarrow \frac{dh}{dt} = \frac{1}{A} (Q_{in} - Q_{out})$$

$$P_{abs} = P_a + \rho gh$$

$$\frac{dP_{abs}}{dt} = \rho g \frac{dh}{dt} = \frac{\rho g}{A} (Q_{in} - Q_{out})$$

经过上面的推导我们得到了公式(7):

$$\frac{dh}{dt} = \frac{1}{A} (Q_{in} - Q_{out}) \quad (7)$$

由Eq. (5): $P_1 - P_a = \rho Q_{out} R$

那么:

$$Q_{out} = \frac{P_1 - P_a}{\rho R} = \frac{P_a + \rho gh - P_a}{\rho R} = \frac{gh}{R} \quad (8)$$

将Eq. (8) 代入Eq. (7):

$$\frac{dh}{dt} = \frac{Q_{in}}{A} - \frac{gh}{AR} \quad (9)$$

最终我们得到了流量输入 Q_{in} 和液面高度变化的关系。