1 Linear Equations in Two Variables

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Points to Remember:

- **Linear Equations in two variables:**

  An equation which contains two variables the degree of each term containing variable is one is called a linear equation in two variables.

  An equation of the from $ax + by + c = 0$ where $a \neq 0$ and $b \neq 0$ is called the standard form of linear equation in two variables x and y where $a, b, c \in R$.

  Here c may or may not be zero

  For example, The standard form of linear equation

  $3x = 4y - 12$ is $3x - 4y + 12 = 0$

  Activity : Complete the following table:
  (Textbook Page No. 1)

Sr. No.	Equation	Is the equation a linear equation in two variables or not
(1)	$4m + 3n = 12$	Yes
(2)	$3x^2 - 7y = 13$	No
(3)	$\sqrt{2}\,x - \sqrt{5}\,y = 16$	Yes
(4)	$0x + 6y - 3 = 0$	No
(5)	$0.3x + 0y - 36 = 0$	No
(6)	$\frac{4}{x} + \frac{5}{y} = 4$	No
(7)	$4xy - 5y - 8 = 0$	No

- **Simultaneous linear Equations:**

(1) When we consider two linear equations in two variables at the same time and we get unique common solution, then such set of equations is known as **simultaneous equations.**

(2) In the previous standard we have studied two methods of solving simultaneous equations
(a) Elimination by equating the coefficients
(b) Elimination by substitution.

- **Solve the following simultaneous equations:**

(1) $5x - 3y = 8$; $3x + y = 2$.

Solution:

Method I – Elimination by equating the coefficients

$5x - 3y = 8$ ...(i)

$3x + y = 2$ ...(ii)

Multiplying both sides of equation (ii) by 3,

$9x + 3y = 6$ ...(iii)

Adding (i) and (iii)

$$\begin{array}{rrcrcl} & 5x & - & 3y & = & 8 \\ + & 9x & + & 3y & = & 6 \\ \hline & 14x & & & = & 14 \end{array}$$

$\therefore$ $\mathbf{x = 1}$

Substituting $x = 1$ in (ii).

$3x + y = 2$

$\therefore$ $3 \times 1 + y = 2$

$\therefore$ $3 + y = 2$

$\therefore$ $y = 2 - 3$

$\therefore$ $\mathbf{y = -1}$

$\therefore$ **$(x, y) = (1, -1)$ is the solution of the given simultaneous equations.**

Method II : Elimination by Substitution

$5x - 3y = 8$ ...(i)

$3x + y = 2$ ...(ii)

Writing y in terms of x from (ii)

$y = 2 - 3x$ ...(iii)

Substituting the value of y in (i)

$5x - 3y = 8$

$\therefore$ $5x - 3(2 - 3x) = 8$

$\therefore$ $5x - 6 + 9x = 8$

$\therefore$ $14x - 6 = 8$

$\therefore$ $14x = 8 + 6$

$\therefore$ $14x = 14$

$\therefore$ $\mathbf{x = 1}$

Substituting the value of x in (iii)

$y = 2 - 3x$

$\therefore$ $y = 2 - 3 \times 1$

$\therefore$ $y = 2 - 3$

$\therefore$ $\mathbf{y = -1}$

$\therefore$ **$x = 1$, $y = -1$ is the solution of the given simultaneous equations.**

(2) Solve: $3x + 2y = 29$; $5x - y = 18$

Solution:

$3x + 2y = 29$ ...(i)

$5x - y = 18$ ...(ii)

To solve the equations, fill in the boxes below.

Multiplying (ii) by 2

$5x \times \boxed{2} - y \times \boxed{2} = 18 \times \boxed{2}$

$\therefore$ $10x - 2y = \boxed{36}$ ...(iii)

Adding (i) and (iii)

$$\begin{array}{cccc} 3x & + & 2y & = & 29 \\ \boxed{10x} & - & \boxed{2y} & = & \boxed{36} \\ \hline \boxed{13x} & & & = & \boxed{65} \end{array}$$

$\therefore$ $x = \boxed{5}$

Substituting $x = 5$ in (i)

$3x + 2y = 29$

$\therefore$ $3 \times \boxed{5} + 2y = 29$

$\therefore$ $\boxed{15} + 2y = 29$

$\therefore$ $2y = 29 - \boxed{15}$

$\therefore$ $2y = \boxed{14}$

$\therefore$ $y = \boxed{7}$

$\therefore$ **$(x, y) = (\boxed{5}, \boxed{7})$ is the solution of the given equations.**

MASTER KEY QUESTION SET - 1

Practice Set - 1.1 *(Textbook Page No. 4)*

(1) Complete the following activity to solve the simultaneous equations.

$5x + 3y = 9$; $2x - 3y = 12$ *(2 marks)*

Solution:

$5x + 3y = 9$ ...(i)

$2x - 3y = 12$ ...(ii)

Adding (i) and (ii)

$$\begin{array}{ccccc} & 5x & + & 3y & = & 9 \\ + & 2x & - & 3y & = & 12 \\ \hline & \boxed{7}\,x & & & = & \boxed{21} \end{array}$$

$\therefore$ $x = \dfrac{\boxed{21}}{\boxed{7}}$

$\therefore$ $x = \boxed{3}$

Place $x = 3$ in (i),

$5 \times \boxed{3} + 3y = 9$

$\therefore$ $3y = 9 - \boxed{15}$

$\therefore$ $3y = \boxed{-6}$

$\therefore$ $y = \dfrac{\boxed{-6}}{3}$

$\therefore$ $y = \boxed{-2}$

$\therefore$ **Solution is $(x, y) = (\boxed{3}, \boxed{-2})$.**

(2) Solve the following simultaneous equations.

(i) $3a + 5b = 26$; $a + 5b = 22$ *(2 marks)*

Solution:

$3a + 5b = 26$ ...(i)

$a + 5b = 22$ ...(ii)

Subtracting (ii) from (i) we get,

$$\begin{array}{ccccc} 3a & + & 5b & = & 26 \\ a & + & 5b & = & 22 \\ (-) & & (-) & & (-) \\ \hline 2a & & & = & 4 \end{array}$$

$\therefore$ $a = \dfrac{4}{2}$

$\therefore$ $\mathbf{a = 2}$

Substituting $a = 2$ in (ii), we get

$2 + 5b = 22$

$\therefore\ 5b = 22 - 2$

$\therefore\ 5b = 20$

$\therefore\ b = \frac{20}{5}$

$\therefore\ \mathbf{b = 4}$

$\therefore$ **(a, b) = (2, 4) is the solution of the given simultaneous equations.**

(ii) $\mathbf{x + 7y = 10\ ;\ 3x - 2y = 7}$ *(2 marks)*

Solution:

$x + 7y = 10$ ...(i)

$3x - 2y = 7$ ...(ii)

(i) can be written as

$x = 10 - 7y$ ...(iii)

Substituting the value of x in (ii),

$3(10 - 7y) - 2y = 7$

$\therefore\ 30 - 21y - 2y = 7$

$\therefore\ -23y = 7 - 30$

$\therefore\ -23y = -23$

$\therefore\ y = \frac{-23}{-23}$

$\therefore\ \mathbf{y = 1}$

Substituting $y = 1$ in (iii),

$\therefore\ x = 10 - 7y$

$\therefore\ x = 10 - 7 \times 1$

$\therefore\ x = 10 - 7$

$\therefore\ \mathbf{x = 3}$

$\therefore$ **(x , y) = (3, 1) is the solution of the given simultaneous equations.**

(iii) $\mathbf{2x - 3y = 9\ ;\ 2x + y = 13}$ *(2 marks)*

Solution:

$2x - 3y = 9$ ...(i)

$2x + y = 13$ ...(ii)

(ii) can be written as

$y = 13 - 2x$ ...(iii)

Substituting (iii) in (i),

$2x - 3(13 - 2x) = 9$

$\therefore\ 2x - 39 + 6x = 9$

$\therefore\ 8x - 39 = 9$

$\therefore\ 8x = 9 + 39$

$\therefore\ 8x = 48$

$\therefore\ x = \frac{48}{8}$

$\therefore\ \mathbf{x = 6}$

Substituting $x = 6$ in (iii),

$y = 13 - 2x$

$\therefore\ y = 13 - 2 \times 6$

$\therefore\ y = 13 - 12$

$\therefore\ \mathbf{y = 1}$

$\therefore$ **(x , y) = (6, 1) is the solution of the given simultaneous equations.**

(iv) $\mathbf{5m - 3n = 19;\ m - 6n = -7}$ *(2 marks)*

Solution:

$5m - 3n = 19$ ...(i)

$m - 6n = -7$ ...(ii)

Multiplying (i) by 2,

$10m - 6n = 38$ ...(iii)

Subtracting (iii) from (ii)

$$\begin{array}{rcrcr} m & - & 6n & = & -7 \\ 10\,m & - & 6n & = & 38 \\ (-) & & (+) & & (-) \\ \hline -9m & & & = & -45 \end{array}$$

$\therefore\ m = \frac{-45}{-9}$

$\therefore\ \mathbf{m = 5}$

Substituting $m = 5$ in (i)

$\therefore\ 5(5) - 3n = 19$

$\therefore\ 25 - 3n = 19$

$\therefore\ -3n = 19 - 25$

$\therefore\ -3n = -6$

$\therefore\ n = \frac{-6}{-3}$

$\therefore\ \mathbf{n = 2}$

$\therefore$ **(m , n) = (5, 2) is the solution of the given simultaneous equations.**

(v) $\mathbf{5x + 2y = -3\ ;\ x + 5y = 4}$ *(2 marks)*

Solution:

$5x + 2y = -3$ ...(i)

$x + 5y = 4$ ...(ii)

Multiplying (ii) by 5

$5x + 25y = 20$ ...(iii)

Subtracting (iii) from (i)

$$\begin{array}{rcrcr} 5x & + & 2y & = & -3 \\ 5x & + & 25y & = & 20 \\ (-) & & (-) & & (-) \\ \hline & & -23y & = & -23 \end{array}$$

$y = \frac{-23}{-23}$

$\therefore\ \mathbf{y = 1}$

Substituting $y = 1$ in (ii),

$\therefore \quad x + 5\,(1) = 4$

$\therefore \quad x = 4 - 5$

$\therefore \quad \mathbf{x = -1}$

$\therefore$ **$(x, y) = (-1, 1)$ is the solution of the given simultaneous equations.**

(vi) $\mathbf{\frac{1}{3}x + y = \frac{10}{3}\ ;\ 2x + \frac{1}{4}y = \frac{11}{4}}$ *(3 marks)*

Solution:

$\frac{1}{3}x + y = \frac{10}{3}$ ...(given)

Multiplying both sides by 3

$x + 3y = 10$ ...(i)

$2x + \frac{1}{4}y = \frac{11}{4}$ ... (given)

Multiplying both sides by 4

$8x + y = 11$ ...(ii)

Multiplying (ii) by 3,

$24x + 3y = 33$ ...(iii)

Subtracting (iii) from (i)

$$\begin{array}{rcrcr} x & + & 3y & = & 10 \\ 24x & + & 3y & = & 33 \\ (-) & & (-) & & (-) \\ \hline -23x & & & = & -23 \end{array}$$

$\therefore \quad x = \frac{-23}{-23}$

$\therefore \quad \mathbf{x = 1}$

Substituting $x = 1$ in (i)

$1 + 3y = 10$

$\therefore \quad 3y = 10 - 1$

$\therefore \quad 3y = 9$

$\therefore \quad y = \frac{9}{3}$

$\therefore \quad \mathbf{y = 3}$

$\therefore$ **$(x, y) = (1, 3)$ is the solution of the given simultaneous equations.**

Points to Remember:

- **Solving special types of Equations:**

Let us now understand how to solve equations of type $a_1x + b_1y = c_1$ and $b_1x + a_1y = c_2$.

We shall follow the three steps.

(i) Add (ii) Subtract (iii) Add

Let us see an example

Example Solve :

$5x + 10y = 35$; $10x + 5y = 40$

Solution:

$5x + 10y = 35$ ...(i)

$10x + 5y = 40$ ...(ii)

Adding (i) and (ii)

$$\begin{array}{rcrcr} 5x & + & 10y & = & 35 \\ 10x & + & 5y & = & 40 \\ \hline 15x & + & 15y & = & 75 \end{array}$$

$\therefore \quad x + y = 5$ ...(iii)

Subtracting (i) form (ii)

$$\begin{array}{rcrcr} 10x & + & 5y & = & 40 \\ 5x & + & 10y & = & 35 \\ (-) & & (-) & & (-) \\ \hline 5x & - & 5y & = & 5 \end{array}$$

Dividing both sides by 5.

$\therefore \quad x - y = 1$ ...(iv)

Adding (iii) and (iv)

$$\begin{array}{rcrcr} x & + & y & = & 5 \\ x & - & y & = & 1 \\ \hline 2x & & & = & 6 \end{array}$$

$\therefore \quad \mathbf{x = 3}$

Substituting $x = 3$ in (iii),

$3 + y = 5$

$y = 5 - 3$

$\therefore \quad \mathbf{y = 2}$

$\therefore$ **$x = 3$, $y = 2$ is the solution of the given simultaneous equations.**

(vii) **$99x + 101y = 499$; $101x + 99y = 501$** *(3 marks)*

Solution:

$99x + 101y = 499$ ...(i)

$101x + 99y = 501$ ...(ii)

Adding (i) and (ii)

$$\begin{array}{rcrcr} 99x & + & 101y & = & 499 \\ 101x & + & 99y & = & 501 \\ \hline 200x & + & 200y & = & 1000 \end{array}$$

$\therefore$ Dividing both sides by 200,

$\therefore \quad x + y = \frac{1000}{200}$

$\therefore \quad x + y = 5$ ..(iii)

Subtracting (ii) from (i)

$$\begin{array}{rcrcr} 99x & + & 101y & = & 499 \\ 101x & + & 99y & = & 501 \\ (-) & & (-) & & (-) \\ \hline -2x & + & 2y & = & -2 \end{array}$$

Dividing both sides by –2,

$\therefore \quad x - y = 1$ ...(iv)

Adding (iii) and (iv)

$$\begin{array}{rcrcr} x & + & y & = & 5 \\ x & - & y & = & 1 \\ \hline 2x & & & = & 6 \end{array}$$

$\therefore \quad x = \dfrac{6}{2}$

$\therefore \quad \mathbf{x = 3}$

Substituting $x = 3$ in (iii),

$3 + y = 5$

$\therefore \quad y = 5 - 3$

$\therefore \quad \mathbf{y = 2}$

$\therefore$ **(x, y) = (3, 2) is the solution of the given simultaneous equations.**

(viii) $49x - 57y = 172$; $57x - 49y = 252$ *(3 marks)*

Solution:

$49x - 57y = 172$ ...(i)

$57x - 49y = 252$ ...(ii)

Adding (i) and (ii)

$$\begin{array}{rcrcr} 49x & - & 57y & = & 172 \\ 57x & - & 49y & = & 252 \\ \hline 106x & - & 106y & = & 424 \end{array}$$

Dividing both sides by 106,

$\therefore \quad x - y = \dfrac{424}{106}$

$\therefore \quad x - y = 4$ ...(iii)

Subtracting (iii) form (i)

$$\begin{array}{rcrcr} 49x & - & 57y & = & 172 \\ 57x & - & 49y & = & 252 \\ (-) & & (+) & & (-) \\ \hline -8x & - & 8y & = & -80 \end{array}$$

Dividing both sides by – 8.

$x + y = \dfrac{80}{8}$

$\therefore \quad x + y = 10$ ...(iv)

Adding (iii) and (iv)

$$\begin{array}{rcrcr} x & - & y & = & 4 \\ x & + & y & = & 10 \\ \hline 2x & & & = & 14 \end{array}$$

$\therefore \quad x = \dfrac{14}{2}$

$\therefore \quad \mathbf{x = 7}$

Substituting $x = 7$ in (iv),

$7 + y = 10$

$y = 10 - 7$

$\therefore \quad \mathbf{y = 3}$

$\therefore$ **(x, y) = (7, 3) is the solution of the given simultaneous equations.**

Points to Remember:

- **Graph of a linear equation in two variables:**

  In previous standard we have studied that a linear equation in two variables represents a line. The ordered pair which will satisfy the given equation is the solution of that equation and also it is one of the points which lies on that graph of the given equation.

  Note : A linear equation such as $y = a$ (constants) is also write as $ox + y = a$. The graph of this line is parallel to x-axis. Similarly, equation such as $x = 2$ (constants) is written as $x + oy = 2$ and its graph is parallel to Y-axis.

- **The graphical method of solving simultaneous equations involves the following steps:**

  Step 1: To draw the graphs of the two given equations, first find three solutions of each equations and make the table for each equation.

  Step 2 : Now plot the points on same graph paper and join them to get the graph of each equation.

  Step 3: The lines of the two given simultaneous equations will intersect each other at a point. The co-ordinates of the point of intersection of the two lines is the solution of the given simultaneous equations.

  Note : Avoid two digit coordinates and using decimal coordinates while finding ordered pairs so that plotting points becomes easy.

- **Solution of simultaneous equations by Graphical method:**

  Ex. : Solve the following simultaneous equations by using Graphical method.

  $x + y = 4$; $2x - y = 2$

  Sol. $x + y = 4$ i.e. $y = 4 - x$

x	0	1	2
y	4	3	2
(x , y)	(0 , 4)	(1 , 3)	(2, 2)

when $x = 0$
$\therefore\ y = 4 - (0)$
$\therefore\ y = 4$

when $x = 1$
$\therefore\ y = 4 - 1$
$\therefore\ y = 3$

when $x = 2$
$\therefore\ y = 4 - 2$
$\therefore\ y = 2$

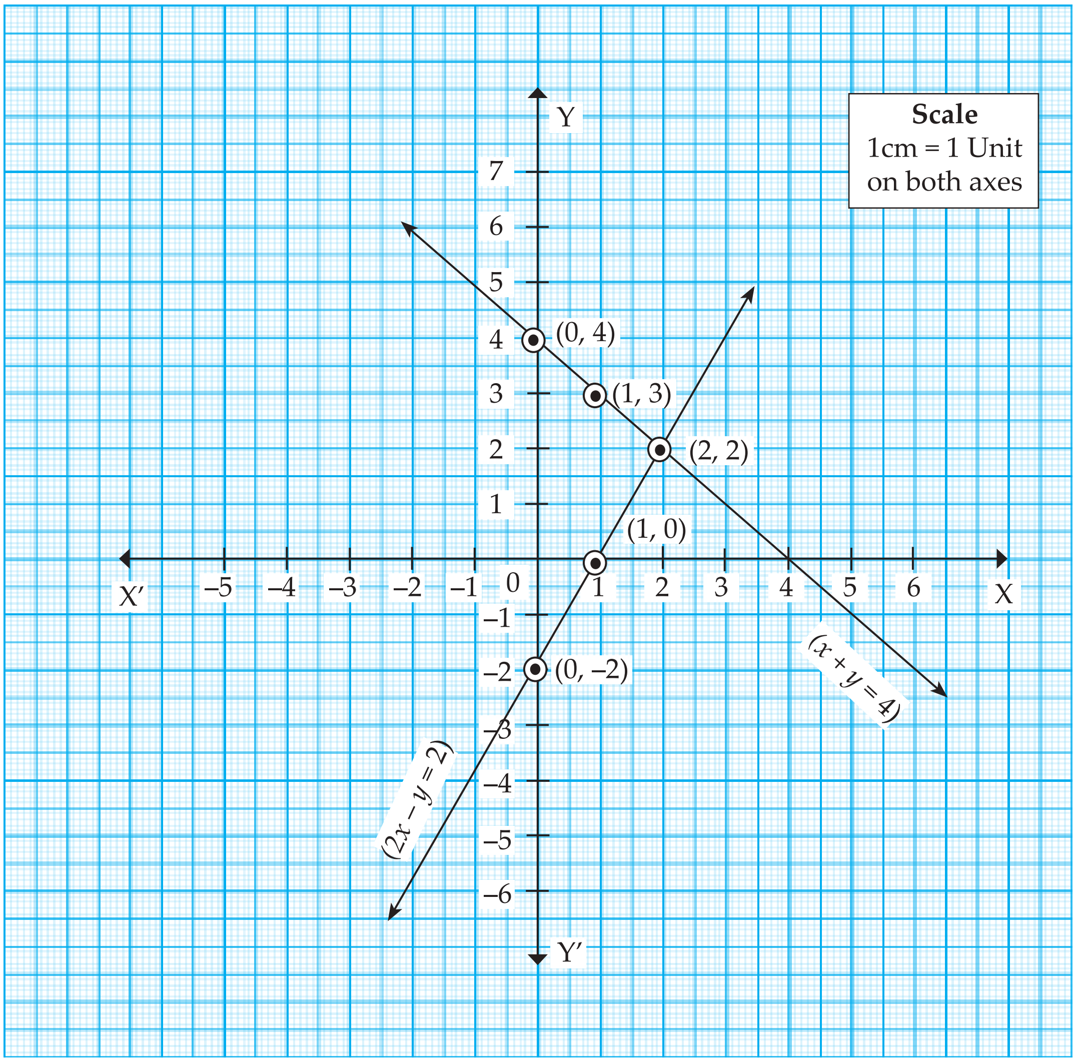
$2x - y = 2$ i.e. $y = 2x - 2$

x	0	1	2
y	–2	0	2
(x , y)	(0 , –2)	(1 ,0)	(2, 2)

when $x = 0$
$\therefore\ y = 2(0) - 2$
$\therefore\ y = 0 - 2$
$\therefore\ y = -2$

when $x = 1$
$\therefore\ y = 2(1) - 2$
$\therefore\ y = 2 - 2$
$\therefore\ y = 0$

when $x = 2$
$\therefore\ y = 2(2) - 2$
$\therefore\ y = 4 - 2$
$\therefore\ y = 2$



The lines of the two given simultaneous equations intersect each other at (2 , 2)

$\therefore$ The solution of the given simultaneous equations is (2, 2) i.e. $x = 2, y = 2$

Activity (I) : Solve the following simultaneous equations by graphical method.

- Complete the following tables to get ordered pairs. *(Textbook Page No. 8)*

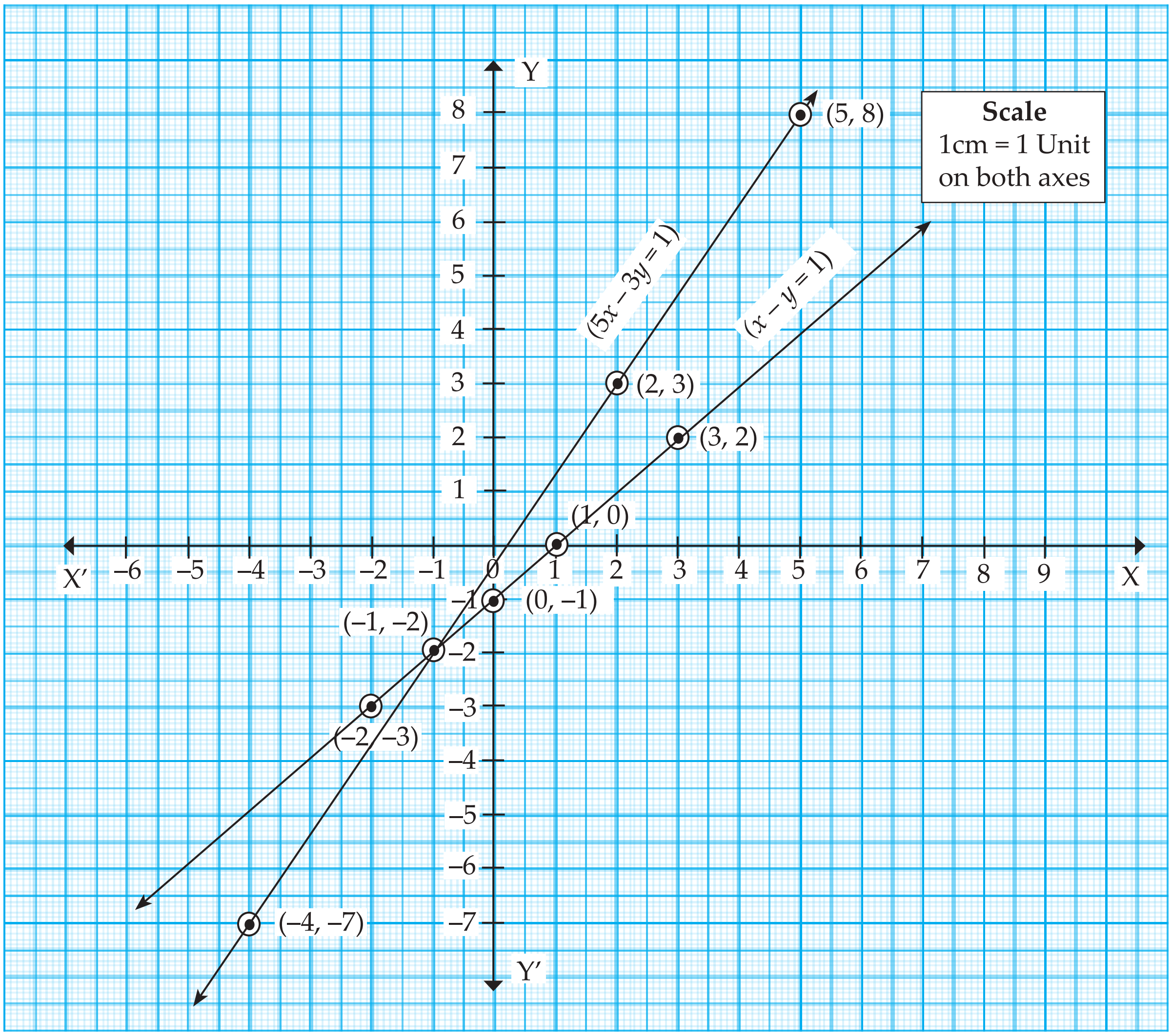
$x - y = 1$

x	0	$\boxed{1}$	3	$\boxed{-2}$
y	$\boxed{-1}$	0	$\boxed{2}$	–3
(x , y)	(0 , –1)	(1 , 0)	(3 , 2)	(–2, –3)

$5x - 3y = 1$

x	2	$\boxed{5}$	$\boxed{-1}$	– 4
y	$\boxed{3}$	8	–2	$\boxed{-7}$
(x , y)	(2 , 3)	(5 , 8)	(–1 , –2)	(–4 , –7)

- Plot the above ordered pair on the same co-ordinate plane.



The lines intersect each other at (–1 , –2)

∴ The solution of the given simultaneous equations is (–1, –2) i.e. $x = -1$, $y = -2$

Activity (I) : Solve the above equations by method of elimination. Check your solution with the solution obtained by graphical method.

(Textbook Page No. 8)

Solution:

$x - y = 1$ ...(i)

$5x - 3y = 1$ ...(ii)

Multiplying (i) by 3 we get,

$3x - 3y = 3$ ...(iii)

Subtracting (iii) from (ii) we get,

$$\begin{array}{rcrcr} & 5x & - 3y & = & 1 \\ - & 3x & - 3y & = & 3 \\ \hline & (-) & (+) & & (-) \\ \hline & 2x & & = & -2 \end{array}$$

$\therefore \ x = \dfrac{-2}{2}$

$\therefore \ x = -1$

Substituting x = – 1 in (i) we get,

$-1 - y = 1$

$\therefore \ y = -1 - 1$

$\therefore \ y = -2$

$\therefore \ x = -1, \ y = -2$ is the solution.

The solution is same as obtained by graphical method.

Practice Set - 1.2 *(Textbook Page No. 8)*

(1) Complete the following table to draw graph of the equations. *(1 mark)*

$x + y = 3$; $x - y = 4$

$x + y = 3$

x	3	$\square$	$\square$
y	$\square$	5	3
(x, y)	(3, 0)	$\square$	(0, 3)

$x - y = 4$

x	$\square$	–1	0
y	0	$\square$	–4
(x, y)	$\square$	$\square$	(0, –4)

Solution:

$x + y = 3$

x	3	$\boxed{-2}$	$\boxed{0}$
y	$\boxed{0}$	5	3
(x, y)	(3, 0)	$\boxed{(-2, 5)}$	(0, 3)

when $x = 3$
$\therefore\ 3 + y = 3$
$\therefore\ y = 3 - 3$
$\therefore\ y = 0$

when $y = 5$
$\therefore\ x + y = 3$
$\therefore\ x + 5 = 3$
$\therefore\ x = 3 - 5$
$\therefore\ x = -2$

when $y = 3$
$\therefore\ x + 3 = 3$
$\therefore\ x = 3 - 3$
$\therefore\ x = 0$

$x - y = 4$

x	$\boxed{4}$	–1	0
y	0	$\boxed{-5}$	– 4
(x, y)	$\boxed{(4, 0)}$	$\boxed{(-1, -5)}$	(0, –4)

when $y = 0$
$\therefore\ x - 0 = 4$
$\therefore\ x = 4$

when $x = -1$
$\therefore\ -1 - y = 4$
$\therefore\ -y = 4 + 1$
$\therefore\ y = -5$

Problem Set - 1 *(Textbook Page No. 27)*

(2) Complete the following table to draw the graph of $2x - 6y = 3$ *(2 marks)*

x	– 5	$\square$
y	$\square$	0
(x, y)	$\square$	$\square$

Solution:

When $x = -5$
$\therefore\ 2x - 6y = 3$
$\therefore\ 2(-5) - 6y = 3$
$\therefore\ -10 - 6y = 3$
$\therefore\ -6y = 3 + 10$
$\therefore\ -6y = 13$
$\therefore\ y = \dfrac{-13}{6}$

When $y = 0$
$\therefore\ 2x - 6y = 3$
$\therefore\ 2x - 6(0) = 3$
$\therefore\ 2x = 3$
$\therefore\ x = \dfrac{3}{2}$

x	– 5	$\boxed{\frac{3}{2}}$
y	$\boxed{\frac{-13}{6}}$	0
(x, y)	$\boxed{\left(-5, \frac{-13}{6}\right)}$	$\boxed{\left(\frac{3}{2}, 0\right)}$

Practice Set - 1.2 *(Textbook Page No. 8)*

(2) Solve the following simultaneous equations graphically. *(4 marks)*

(i) $x + y = 6$; $x - y = 4$

Solution:

$x + y = 6$ i.e. $y = 6 - x$

x	0	1	2
y	6	5	4
(x, y)	(0, 6)	(1, 5)	(2, 4)

when $x = 0$
$\therefore\ y = 6 - 0$
$\therefore\ y = 6$

when $x = 1$
$\therefore\ y = 6 - 1$
$\therefore\ y = 5$

when $x = 2$
$\therefore\ y = 6 - (2)$
$\therefore\ y = 4$

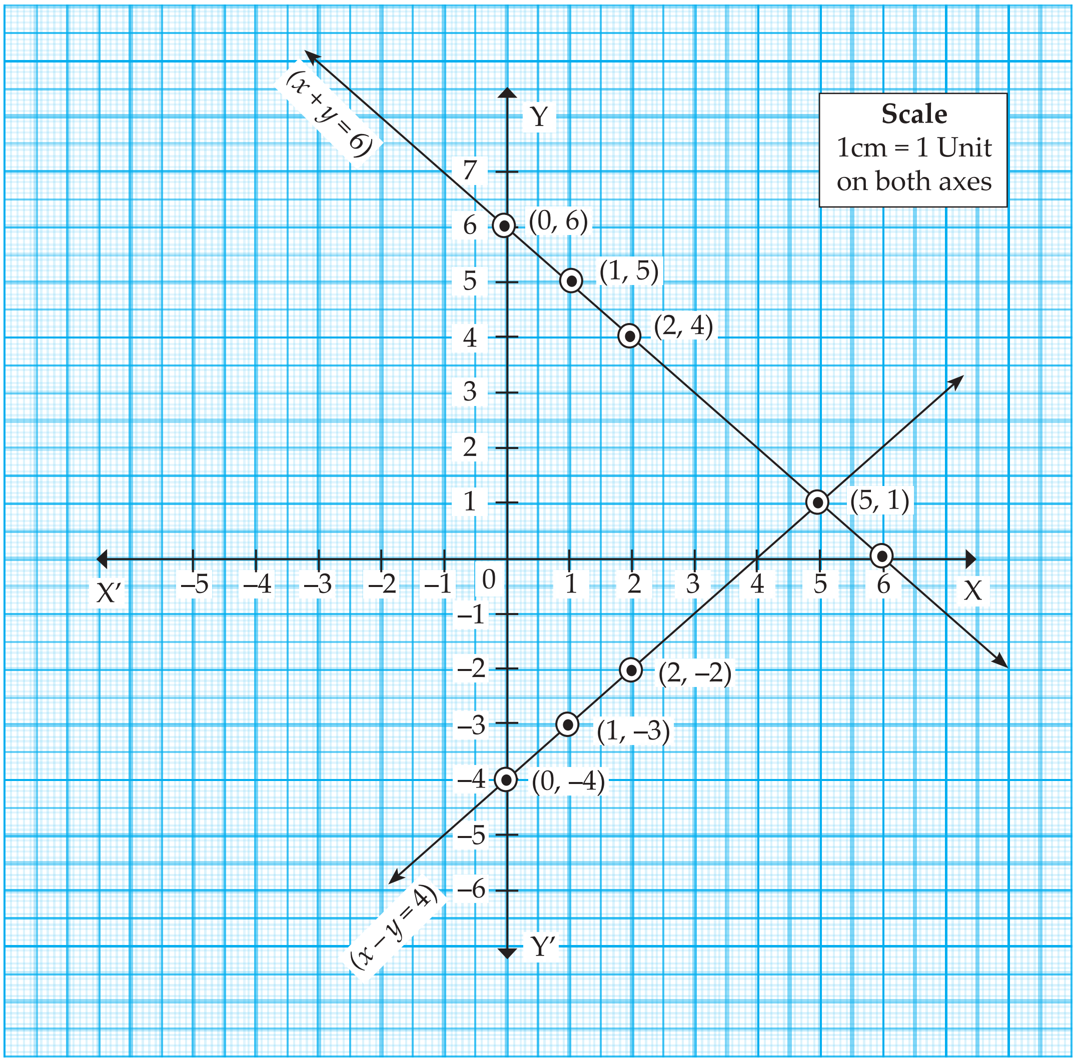
$x - y = 4$ i.e. $y = x - 4$

x	0	1	2
y	–4	–3	–2
(x, y)	(0, –4)	(1, –3)	(2, –2)

when $x = 0$
$\therefore\ y = 0 - 4$
$\therefore\ y = -4$

when $x = 1$
$\therefore\ y = 1 - 4$
$\therefore\ y = -3$

when $x = 2$
$\therefore\ y = 2 - 4$
$\therefore\ y = -2$



The lines of the two given simultaneous equations intersect each other at (5 , 1)

∴ **The solution of given the simultaneous equations is (5, 1) i.e. $x = 5, y = 1$**

(ii) $x + y = 5$; $x - y = 3$ *(4 marks)*

Solution:

$x + y = 5$ i.e. $y = 5 - x$

x	0	1	2
y	5	4	3
(x, y)	(0, 5)	(1, 4)	(2 , 3)

when $x = 0$

∴ $y = 5 - 0$

∴ $y = 5$

when $x = 1$

∴ $y = 5 - (1)$

∴ $y = 5 - 1$

∴ $y = 4$

when $x = 2$

∴ $y = 5 - 2$

∴ $y = 3$

$x - y = 3$ i.e. $y = x - 3$

x	0	1	2
y	–3	–2	–1
(x, y)	(0, –3)	(1, –2)	(2, –1)

when $x = 0$

∴ $y = 0 - 3$

∴ $y = -3$

when $x = 1$

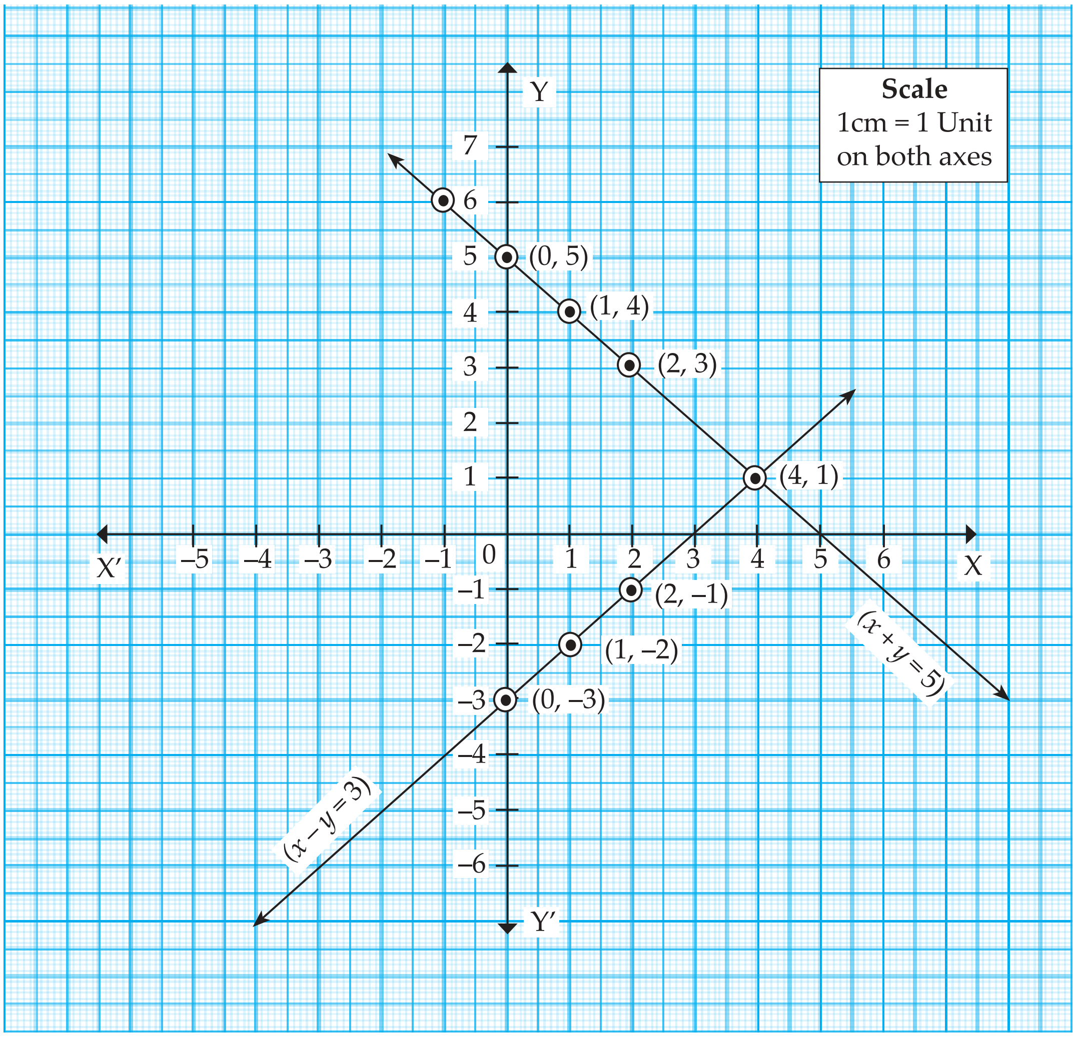
∴ $y = 1 - 3$

∴ $y = -2$

when $x = 2$

∴ $y = 2 - 3$

∴ $y = -1$



The lines of the two given simultaneous equations intersect each other at (4 , 1)

$\therefore$ **The solution of the given simultaneous equations is (4, 1) i.e. $x = 4, y = 1$**

(iii) $x + y = 0$; $2x - y = 9$ *(4 marks)*

Solution:

$x + y = 0$ i.e. $y = -x$

x	0	1	2
y	0	– 1	– 2
(x, y)	(0, 0)	(1, –1)	(2, –2)

when $x = 0$

$\therefore y = (-0)$

$\therefore y = 0$

when $x = 1$

$\therefore y = -(1)$

$\therefore y = -1$

when $x = 2$

$\therefore y = (-2)$

$\therefore y = -2$

$2x - y = 9$ i.e. $y = 2x - 9$

x	0	1	2
y	–9	–7	–5
(x, y)	(0, –9)	(1, –7)	(2, –5)

when $x = 0$

$\therefore y = 2(0) - 9$

$\therefore y = 0 - 9$

$\therefore y = -9$

when $x = 1$

$\therefore y = 2(1) - 9$

$\therefore y = 2 - 9$

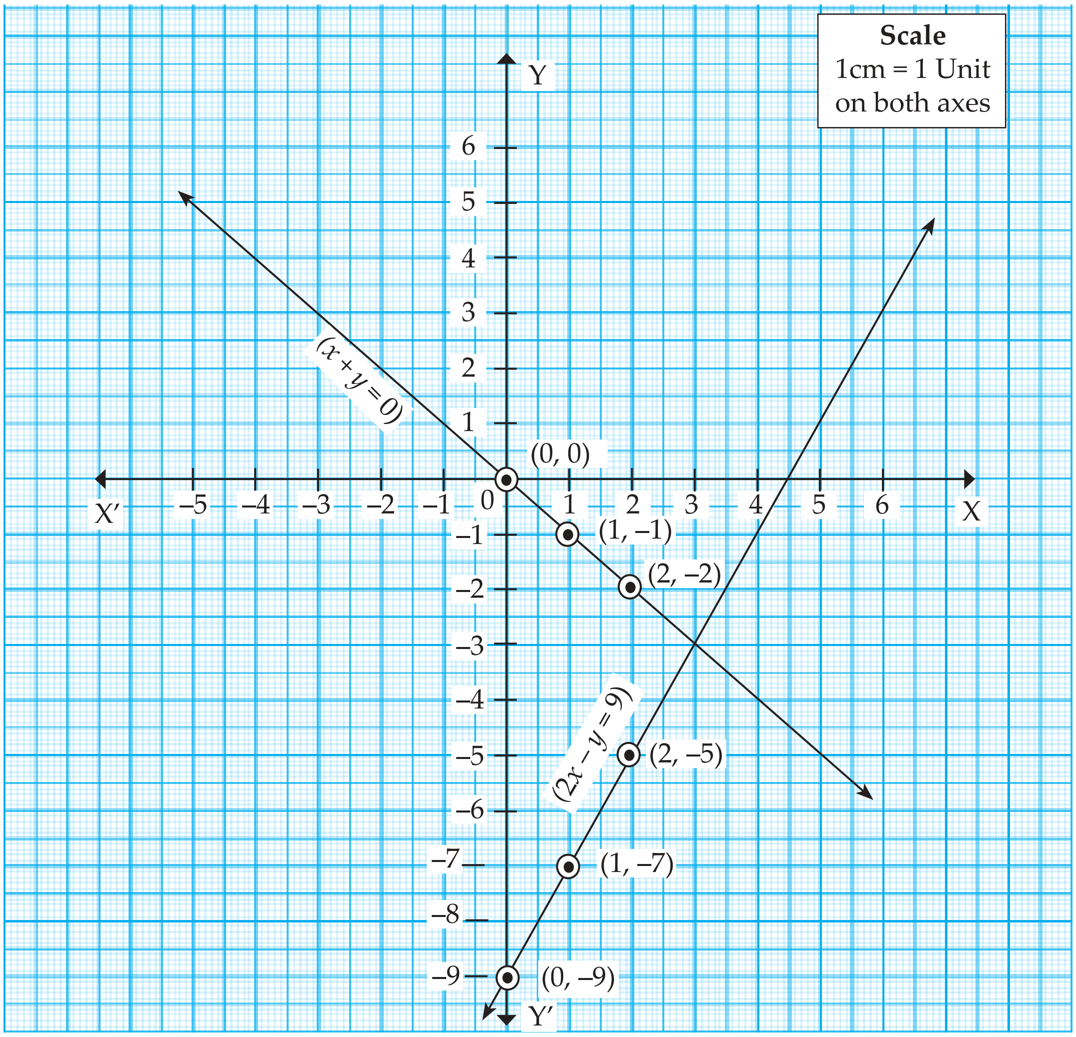
$y = -7$

when $x = 2$

$\therefore y = 2(2) - 9$

$\therefore y = 4 - 9$

$\therefore y = -5$



The lines of the two given simultaneous equations intersect each other at (3 , –3)

$\therefore$ **The solution of given simultaneous equations is (3, –3) i.e. $x = 3, y = -3$**

(iv) $3x - y = 2$; $2x - y = 3$ *(4 marks)*

Solution:

$3x - y = 2$ i.e. $y = 3x - 2$

x	0	1	2
y	–2	1	4
(x, y)	(0, –2)	(1, 1)	(2, 4)

when $x = 0$

$\therefore \ y = 3(0) - 2$

$\therefore \ y = 0 - 2$

$\therefore \ y = -2$

when $x = 1$

$\therefore \ y = 3(1) - 2$

$\therefore \ y = 3 - 2$

$\therefore \ y = 1$

when $x = 2$

$\therefore \ y = 3(2) - 2$

$\therefore \ y = 6 - 2$

$\therefore \ y = 4$

$2x - y = 3$ i.e. $y = 2x - 3$

x	0	1	2
y	–3	–1	1
(x, y)	(0, –3)	(1, –1)	(2, 1)

when $x = 0$

$\therefore \ y = 2(0) - 3$

$\therefore \ y = 0 - 3$

$\therefore \ y = -3$

when $x = 1$

$\therefore \ y = 2(1) - 3$

$\therefore \ y = 2 - 3$

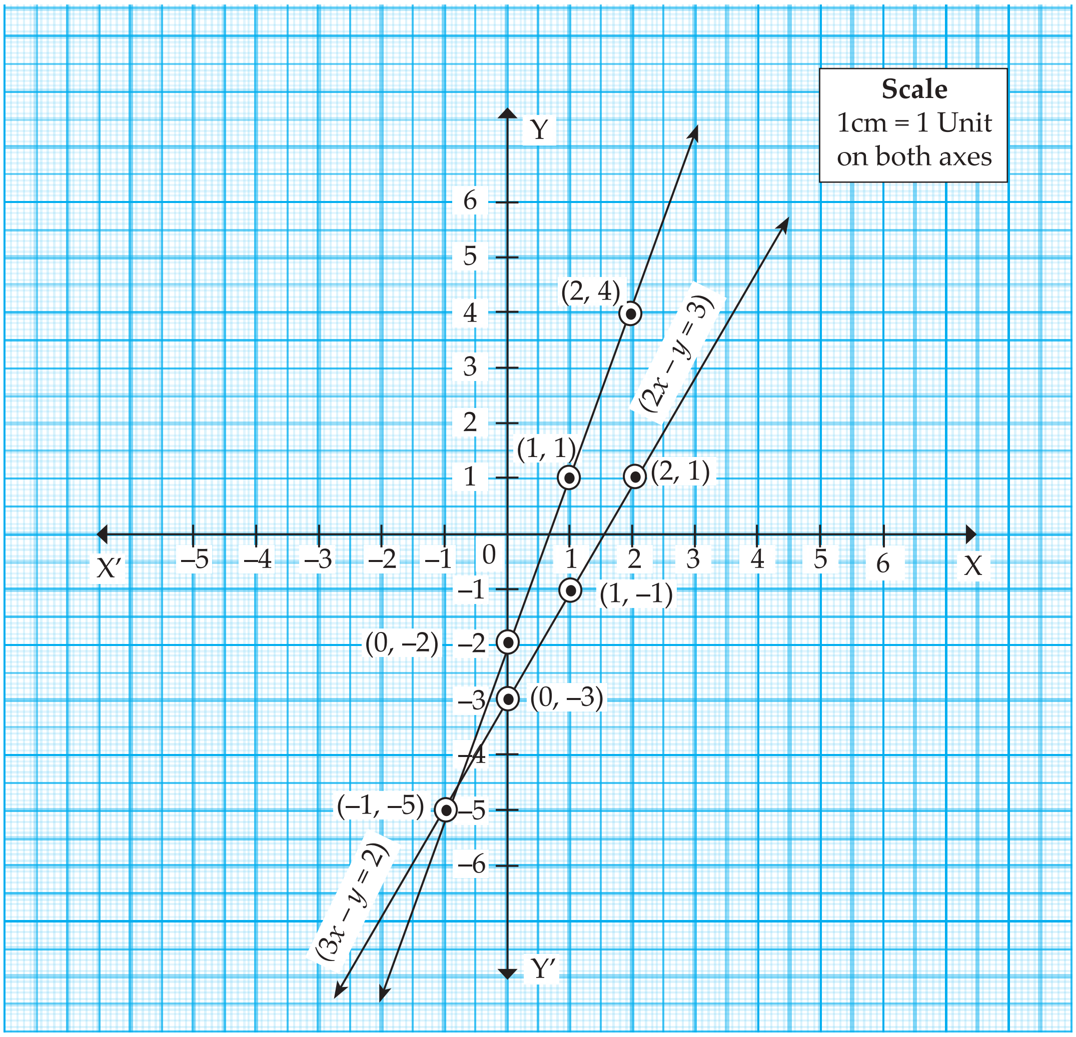
$\therefore \ y = -1$

when $x = 2$

$\therefore \ y = 2(2) - 3$

$\therefore \ y = 4 - 3$

$\therefore \ y = 1$



The lines of the two given simultaneous equations intersect each other at (–1 , –5)

∴ **The solution of given simultaneous equations is (–1, –5) i.e. $x = -1, y = -5$**

(v) **$3x - 4y = -7$; $5x - 2y = 0$** *(4 marks)*

Solution:

$3x - 4y = -7$ i.e. $y = \dfrac{3x+7}{4}$

x	–1	3	–5
y	1	4	–2
(x, y)	(–1, 1)	(3, 4)	(–5, –2)

when $x = -1$

$\therefore\ y = \dfrac{3(-1)+7}{4}$

$\therefore\ y = \dfrac{-3+7}{4}$

$\therefore\ y = \dfrac{4}{4}$

$\therefore\ y = 1$

when $x = 3$

$\therefore\ y = \dfrac{3(3)+7}{4}$

$\therefore\ y = \dfrac{9+7}{4}$

$\therefore\ y = \dfrac{16}{4}$

$\therefore\ y = 4$

when $x = -5$

$\therefore\ y = \dfrac{3(-5)+7}{4}$

$\therefore\ y = \dfrac{-15+7}{4}$

$\therefore\ y = \dfrac{-8}{4}$

$\therefore\ y = -2$

$5x - 2y = 0$ i.e. $y = \dfrac{5}{2}x$

x	0	2	–2
y	0	5	–5
(x, y)	(0, 0)	(2, 5)	(–2, –5)

when $x = 0$

$\therefore\ y = \dfrac{5}{2} \times 0$

$\therefore\ y = 0$

when $x = 2$

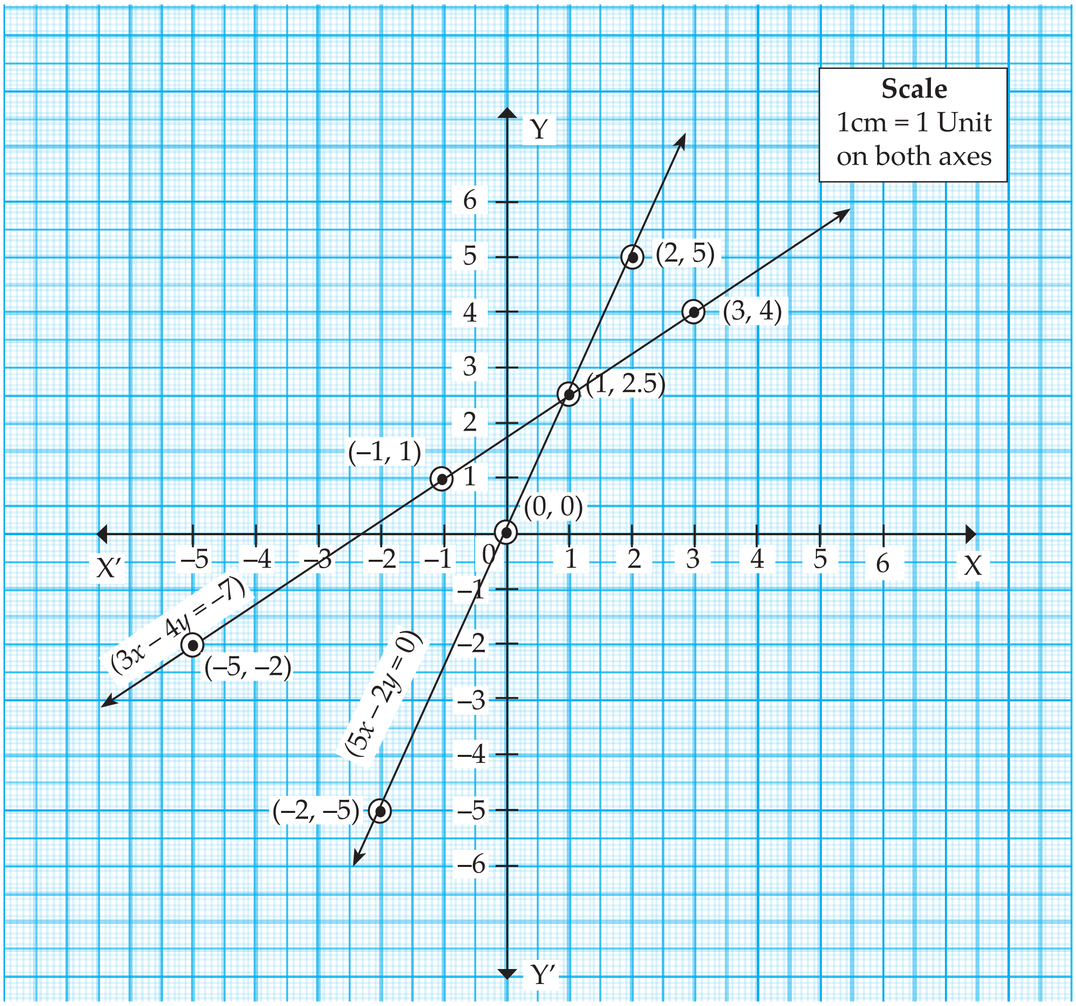
$\therefore\ y = \dfrac{5}{2} \times 2$

$\therefore\ y = 5$

when $x = -2$

$\therefore\ y = \dfrac{5}{2}(-2)$

$\therefore\ y = -5$



The lines of the two given simultaneous equations intersect each other at (1 , 2.5)

∴ **The solution of given simultaneous equations is (1, 2.5) i.e. $x = 1, y = 2.5$**

(vi) $2x - 3y = 4\ ;\ 3y - x = 4$ *(4 marks)*

Solution:

$2x - 3y = 4$

∴ $2x = 4 + 3y$

∴ $x = \dfrac{4+3y}{2}$

x	2	5	–1
y	0	2	–2
(x, y)	(2, 0)	(5 , 2)	(–1, –2)

when $y = 0$

$x = \dfrac{4+3(0)}{2}$

∴ $x = \dfrac{4+0}{2}$

∴ $x = \dfrac{4}{2}$

∴ $x = 2$

when $y = 2$

$x = \dfrac{4+3(2)}{2}$

∴ $x = \dfrac{4+6}{2}$

∴ $x = \dfrac{10}{2}$

∴ $x = 5$

when $y = -2$

$x = \dfrac{4+3(-2)}{2}$

∴ $x = \dfrac{4-6}{2}$

∴ $x = \dfrac{-2}{2}$

∴ $x = -1$

$3y - x = 4$ i.e. $x = 3y - 4$

x	–4	–1	2
y	0	1	2
(x, y)	(–4, 0)	(–1, 1)	(2, 2)

when $y = 0$

∴ $x = 3(0) - 4$

∴ $x = 0 - 4$

∴ $x = -4$

when $y = 1$

∴ $x = 3(1) - 4$

∴ $x = 3 - 4$

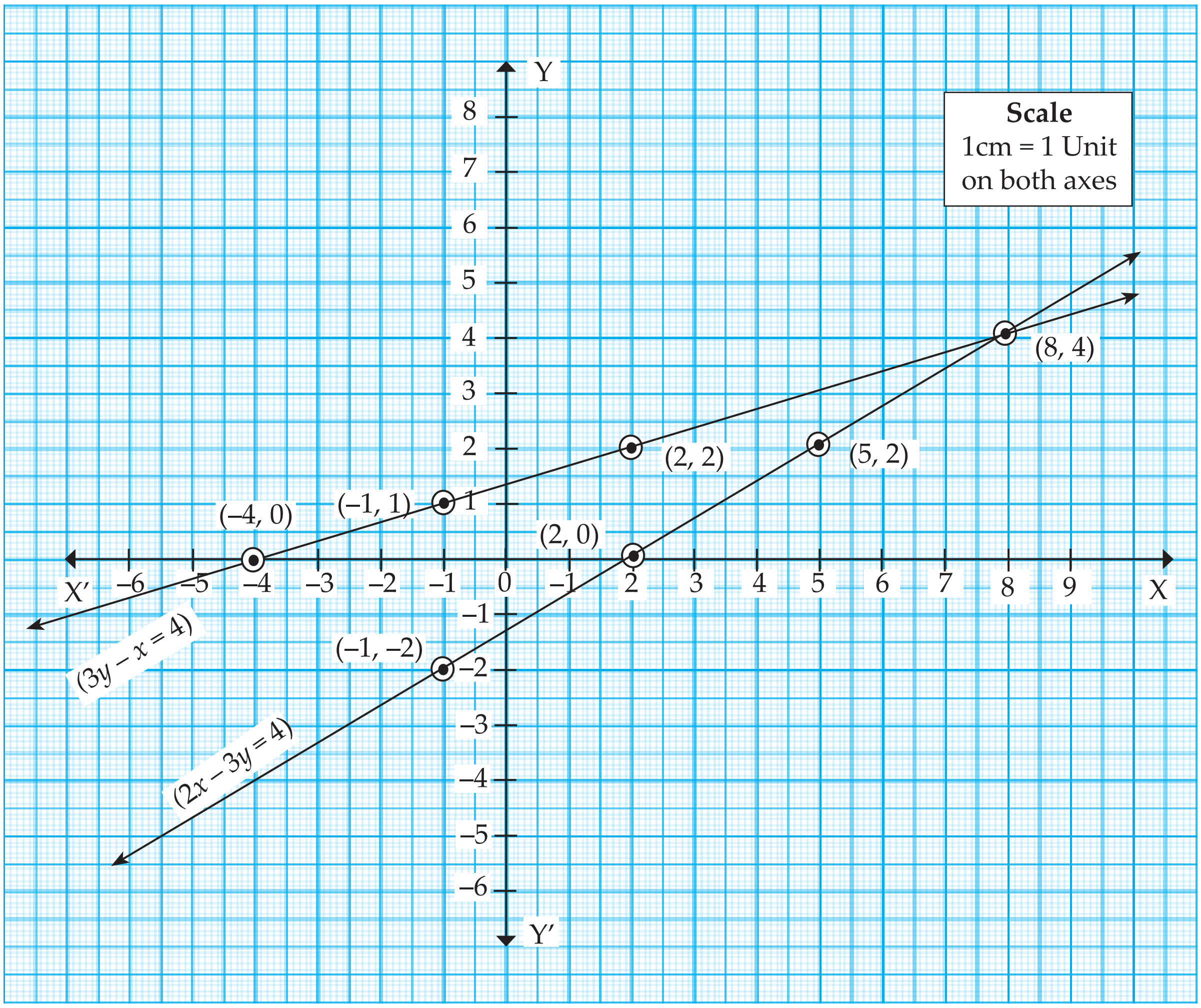
∴ $x = -1$

when $y = 2$

∴ $x = 3(2) - 4$

∴ $x = 6 - 4$

∴ $x = 2$



The lines of the two given simultaneous equations intersect each other at (8, 4)

∴ $\boxed{\textbf{The solution of given simultaneous equations is (8,4) i.e. } x = 8, y = 4}$

Problem Set - 1 *(Textbook Page No. 27)*

(3) Solve the following simultaneous equations by using Graphical method. *(4 marks)*

(i) $2x + 3y = 12$; $x - y = 1$

Solution:

$2x + 3y = 12$, $2x = 12 - 3y$ ∴ $x = \dfrac{12 - 3y}{2}$

x	6	3	0
y	0	2	4
(x, y)	(6, 0)	(3, 2)	(0, 4)

When $y = 0$

∴ $x = \dfrac{12 - 3(0)}{2}$

∴ $x = \dfrac{12 - 0}{2}$

∴ $x = \dfrac{12}{2}$

∴ $x = 6$

When $y = 2$

∴ $x = \dfrac{12 - 3(2)}{2}$

∴ $x = \dfrac{12 - 6}{2}$

∴ $x = \dfrac{6}{2}$

∴ $x = 3$

When $y = 4$

∴ $x = \dfrac{12 - 3(4)}{2}$

∴ $x = \dfrac{12 - 12}{2}$

∴ $x = \dfrac{0}{2}$

∴ $x = 0$

$x - y = 1$ i.e. $y = x - 1$

x	0	1	2
y	–1	0	1
(x, y)	(0, –1)	(1, 0)	(2, 1)

When $x = 0$

∴ $y = x - 1$

∴ $y = 0 - 1$

∴ $y = -1$

When $x = 1$

∴ $y = x - 1$

∴ $y = 1 - 1$

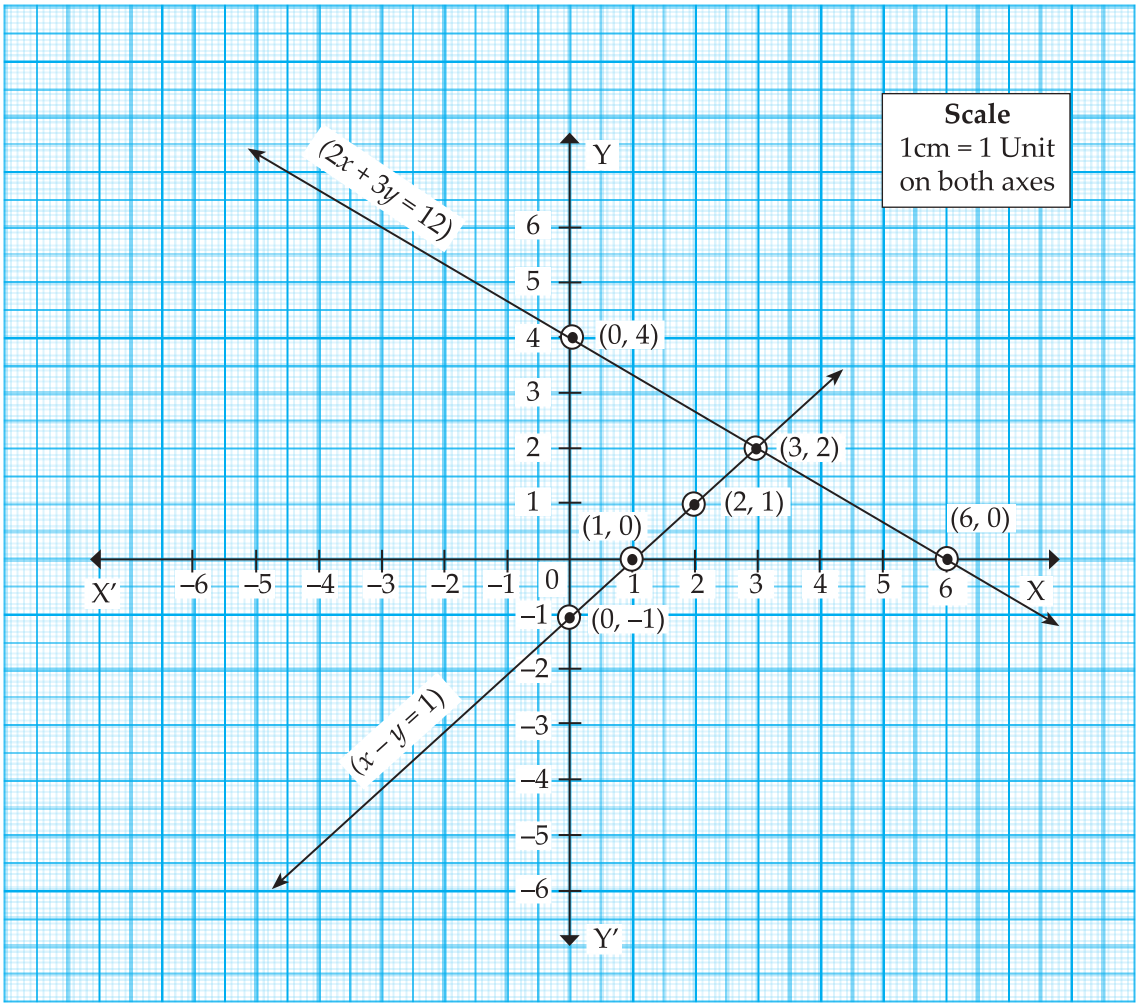
∴ $y = 0$

When $x = 2$

∴ $y = x - 1$

∴ $y = 2 - 1$

∴ $y = 1$



The lines of the two given simultaneous equations intersect each other at (3 ,2)

$\therefore$ $\boxed{\textbf{x = 3, y = 2 is the solution of the given simultaneous equations.}}$

(ii) $\quad$ $\boldsymbol{x - 3y = 1 \;;\; 3x - 2y + 4 = 0}$

Solution:

$x - 3y = 1 \;;\; x = 1 + 3y$

x	1	4	7
y	0	1	2
(x, y)	(1, 0)	(4, 1)	(7, 2)

When $y = 0$
$x = 1 + 3y$
$\therefore x = 1 + 3(0)$
$\therefore x = 1 + 0$
$\therefore x = 1$

When $y = 1$
$x = 1 + 3y$
$\therefore x = 1 + 3(1)$
$\therefore x = 1 + 3$
$\therefore x = 4$

When $y = 2$
$x = 1 + 3y$
$\therefore x = 1 + 3(2)$
$\therefore x = 1 + 6$
$\therefore x = 7$

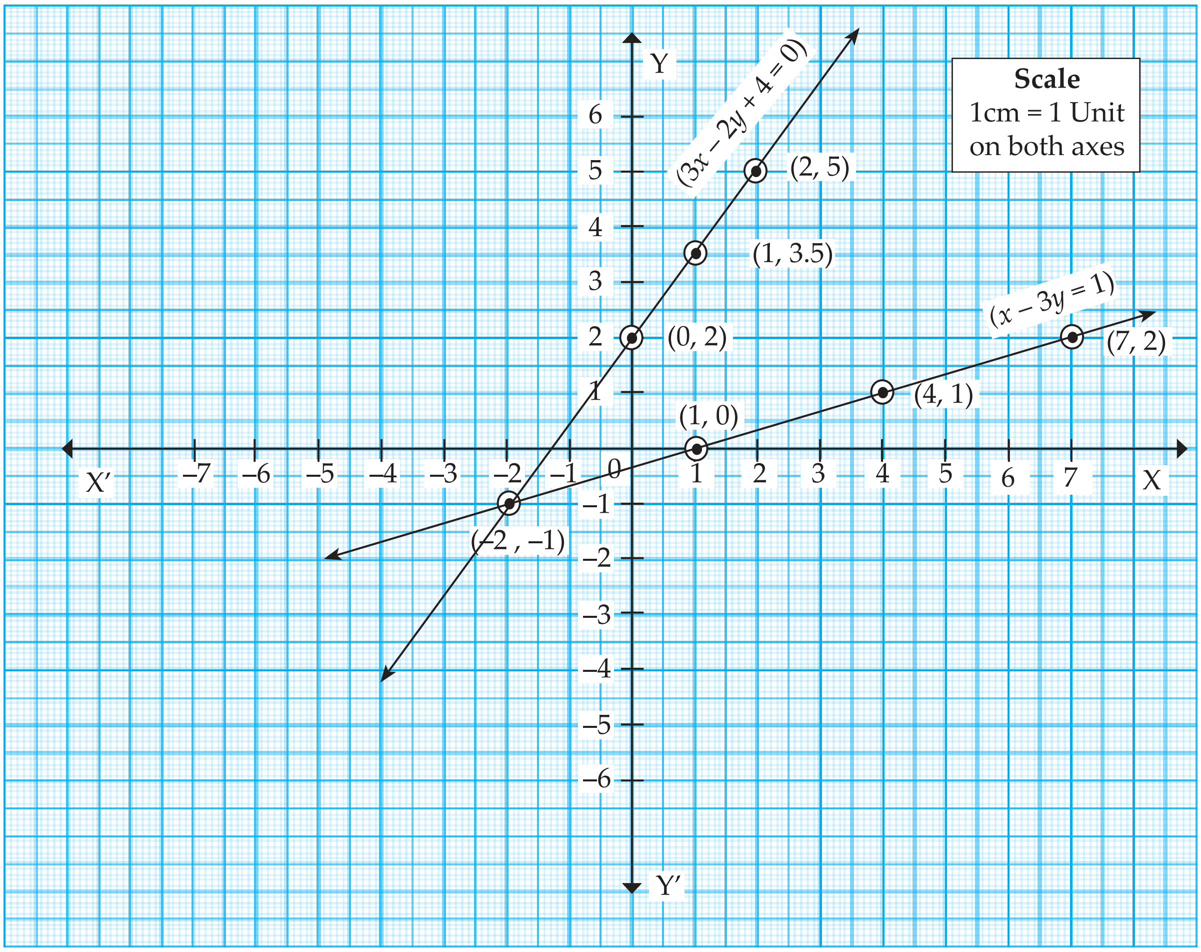
$3x - 2y + 4 = 0$ $\quad$ i.e. $\quad 2y = 3x + 4 \therefore y = \dfrac{3x+4}{2}$

x	0	1	2
y	2	3.5	5
(x, y)	(0, 2)	(1, 3.5)	(2, 5)

When $x = 0$
$y = \dfrac{3x+4}{2}$
$\therefore y = \dfrac{3(0)+4}{2}$
$\therefore y = \dfrac{0+4}{2}$
$\therefore y = \dfrac{4}{2}$
$\therefore y = 2$

When $x = 1$
$y = \dfrac{3x+4}{2}$
$\therefore y = \dfrac{3(1)+4}{2}$
$\therefore y = \dfrac{3+4}{2}$
$\therefore y = \dfrac{7}{2}$
$\therefore y = 3.5$

When $x = 2$
$y = \dfrac{3x+4}{2}$
$\therefore y = \dfrac{3(2)+4}{2}$
$\therefore y = \dfrac{6+4}{2}$
$\therefore y = \dfrac{10}{2}$
$\therefore y = 5$



The lines of the two given simultaneous equations intersect each other at (–2 ,–1)

$\therefore$ $\boxed{x = -2 \text{ and } y = -1 \text{ is the solution of given simultaneous equations.}}$

(iii) **$5x - 6y + 30 = 0$; $5x + 4y - 20 = 0$** *(4 marks)*

Solution:

$5x - 6y + 30 = 0$ i.e. $5x = 6y - 30$ $\therefore x = \dfrac{6y - 30}{5}$

x	-6	0	6
y	0	5	10
(x, y)	$(-6, 0)$	$(0, 5)$	$(6, 10)$

When $y = 0$

$x = \dfrac{6(0) - 30}{5}$

$\therefore x = \dfrac{0 - 30}{5}$

$\therefore x = \dfrac{-30}{5}$

$\therefore x = -6$

When $y = 5$

$x = \dfrac{6(5) - 30}{5}$

$\therefore x = \dfrac{30 - 30}{5}$

$\therefore x = \dfrac{0}{5}$

$\therefore x = 0$

When $y = 10$

$x = \dfrac{6(10) - 30}{5}$

$\therefore x = \dfrac{60 - 30}{5}$

$\therefore x = \dfrac{30}{5}$

$\therefore x = 6$

$5x + 4y - 20 = 0$ i.e. $y = \dfrac{20 - 5x}{4}$

x	0	4	-4
y	5	0	10
(x, y)	$(0, 5)$	$(4, 0)$	$(-4, 10)$

When $x = 0$

$y = \dfrac{20 - 5x}{4}$

$\therefore y = \dfrac{20 - 5(0)}{4}$

$\therefore y = \dfrac{20}{4}$

$\therefore y = 5$

When $x = 4$

$y = \dfrac{20 - 5x}{4}$

$\therefore y = \dfrac{20 - 5(4)}{4}$

$\therefore y = \dfrac{20 - 20}{4}$

$\therefore y = \dfrac{0}{4}$

$\therefore y = 0$

When $x = -4$

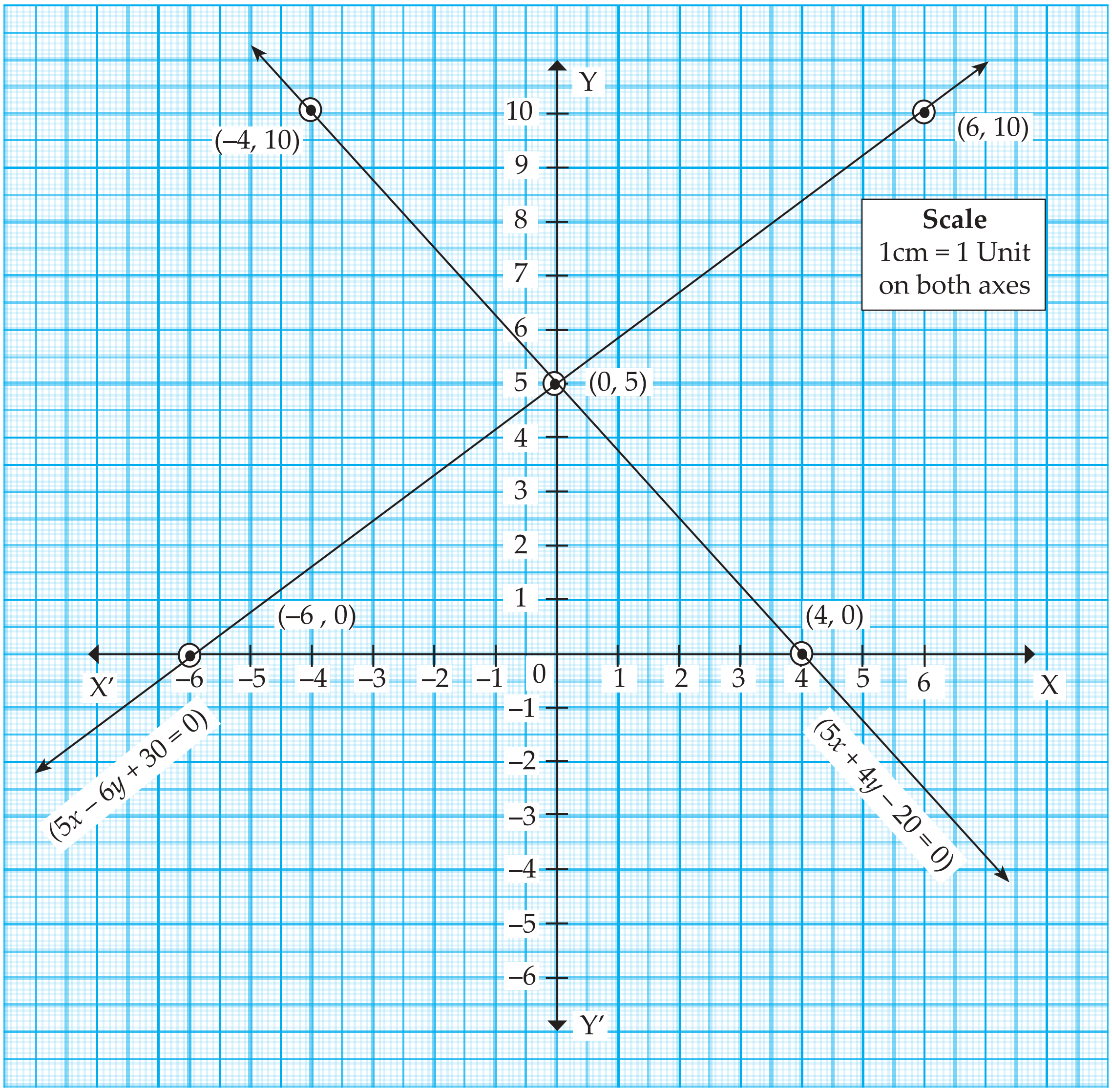
$y = \dfrac{20 - 5x}{4}$

$\therefore y = \dfrac{20 - 5(-4)}{4}$

$\therefore y = \dfrac{20 + 20}{4}$

$\therefore y = \dfrac{40}{4}$

$\therefore y = 10$



The lines of the two given simultaneous equations intersect each other at (0 ,5)

$\therefore$ **$x = 0$ and $y = 5$ is the solution of given simultaneous equations.**

(iv) **$3x - y - 2 = 0;\ 2x + y = 8$** *(4 marks)*

Solution:

$3x - y - 2 = 0$ i.e. $y = 3x - 2$

x	0	1	2
y	–2	1	4
(x, y)	(0, –2)	(1, 1)	(2, 4)

When $x = 0$
$y = 3x - 2$
$\therefore\ y = 3(0) - 2$
$\therefore\ y = 0 - 2$
$\therefore\ y = -2$

When $x = 1$
$\therefore\ y = 3x - 2$
$\therefore\ y = 3(1) - 2$
$\therefore\ y = 3 - 2$
$\therefore\ y = 1$

When $x = 2$
$y = 3x - 2$
$\therefore\ y = 3(2) - 2$
$\therefore\ y = 6 - 2$
$\therefore\ y = 4$

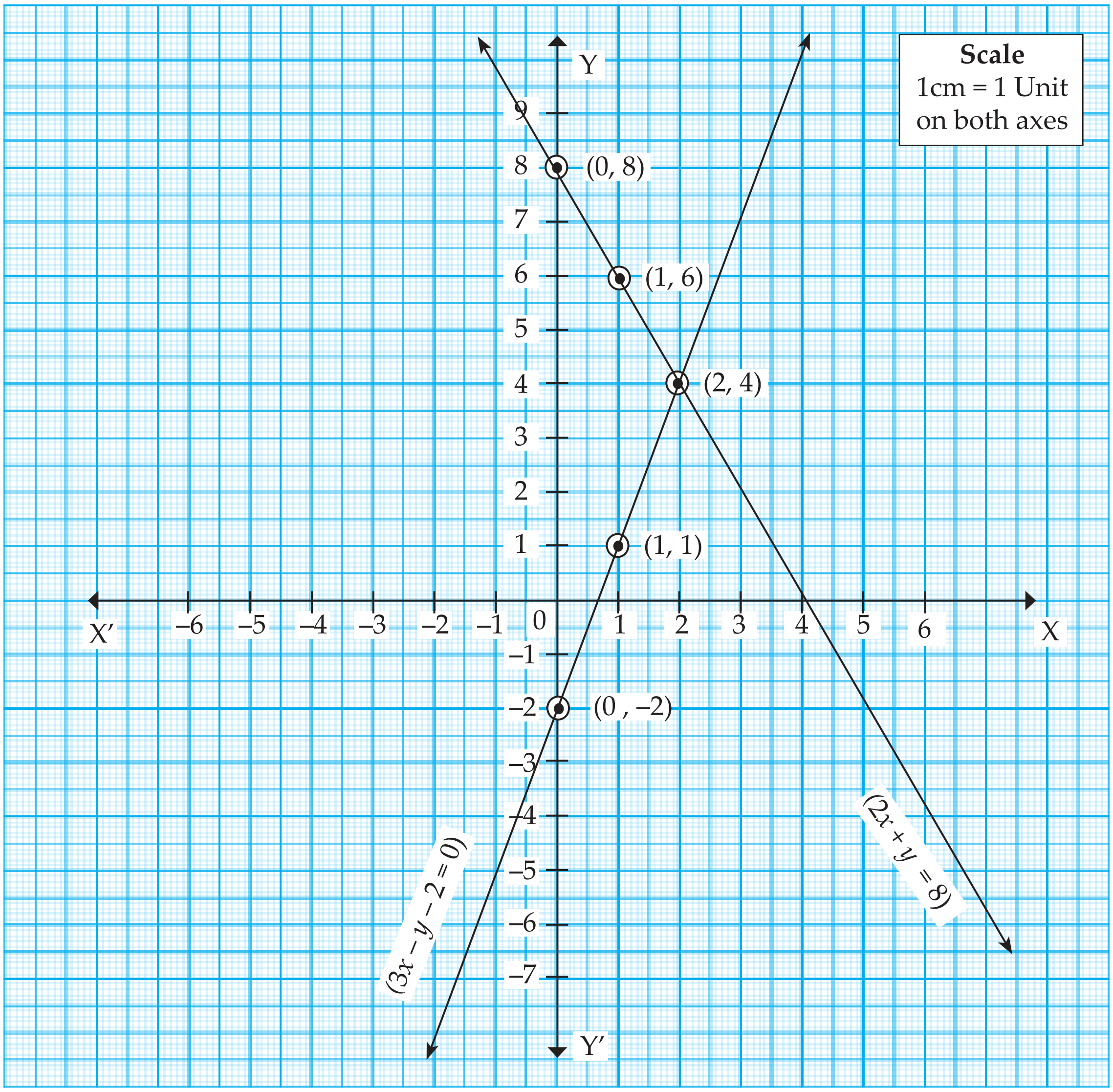
$2x + y = 8$ i.e. $y = 8 - 2x$

x	0	1	2
y	8	6	4
(x, y)	(0, 8)	(1, 6)	(2, 4)

When $x = 0$
$y = 8 - 2x$
$\therefore\ y = 8 - 2(0)$
$\therefore\ y = 8 - 0$
$\therefore\ y = 8$

When $x = 1$
$y = 8 - 2x$
$\therefore\ y = 8 - 2(1)$
$\therefore\ y = 8 - 2$
$\therefore\ y = 6$

When $x = 2$
$y = 8 - 2x$
$\therefore\ y = 8 - 2(2)$
$\therefore\ y = 8 - 4$
$\therefore\ y = 4$



The lines of the two given simultaneous equations intersect each other at (2 ,4)

$\therefore$ **$x = 2, y = 4$ is the solution of given simultaneous equations.**

(v) **$3x + y = 10$; $x - y = 2$** *(4 marks)*

Solution:

$3x + y = 10$ i.e. $y = 10 - 3x$

x	1	2	3
y	7	4	1
(x, y)	(1, 7)	(2, 4)	(3, 1)

When $x = 1$

$y = 10 - 3x$

$\therefore$ $y = 10 - 3(1)$

$\therefore$ $y = 10 - 3$

$\therefore$ $y = 7$

When $x = 2$

$y = 10 - 3x$

$\therefore$ $y = 10 - 3(2)$

$\therefore$ $y = 10 - 6$

$\therefore$ $y = 4$

When $x = 3$

$y = 10 - 3x$

$\therefore$ $y = 10 - 3(3)$

$\therefore$ $y = 10 - 9$

$\therefore$ $y = 1$

$x - y = 2$ i.e. $y = x - 2$

x	0	1	2
y	–2	–1	0
(x, y)	(0, –2)	(1, –1)	(2, 0)

When $x = 0$

$y = x - 2$

$\therefore$ $y = 0 - 2$

$\therefore$ $y = -2$

When $x = 1$

$y = x - 2$

$\therefore\ y = 1 - 2$

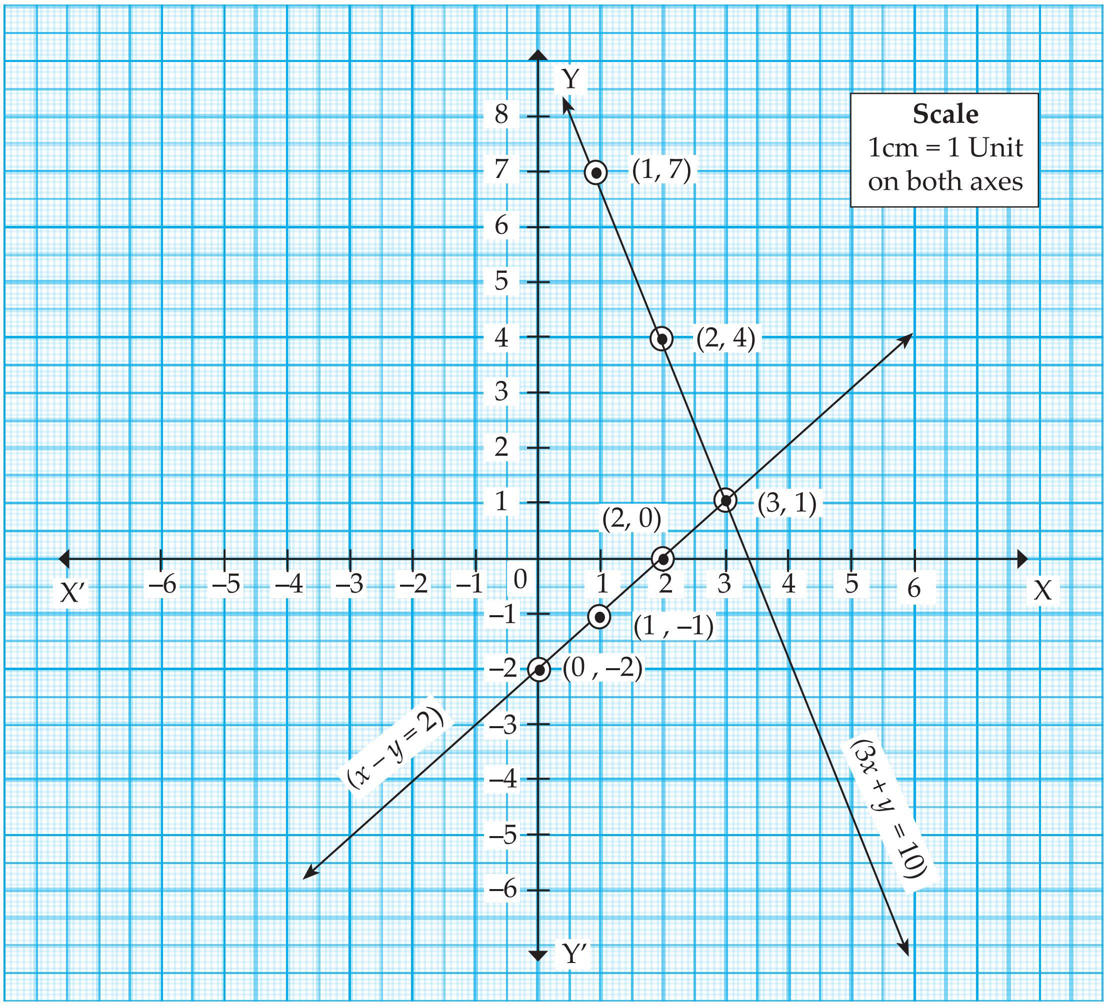
$\therefore\ y = -1$

When $x = 2$

$y = x - 2$

$\therefore\ y = 2 - 2$

$\therefore\ y = 0$



The lines of the two given simultaneous equations intersect each other at (3 ,1)

$\therefore$ **x = 3 and y = 1 is the solution of given simultaneous equations.**

Points to Remember:

(1) To solve simultaneous equations $x + 2y = 4$; $3x + 6y = 12$ graphically, following are the ordered pairs. Plotting the above ordered pairs, graph is drawn. Observe it and find answers of the following questions:

(Textbook Page No. 9)

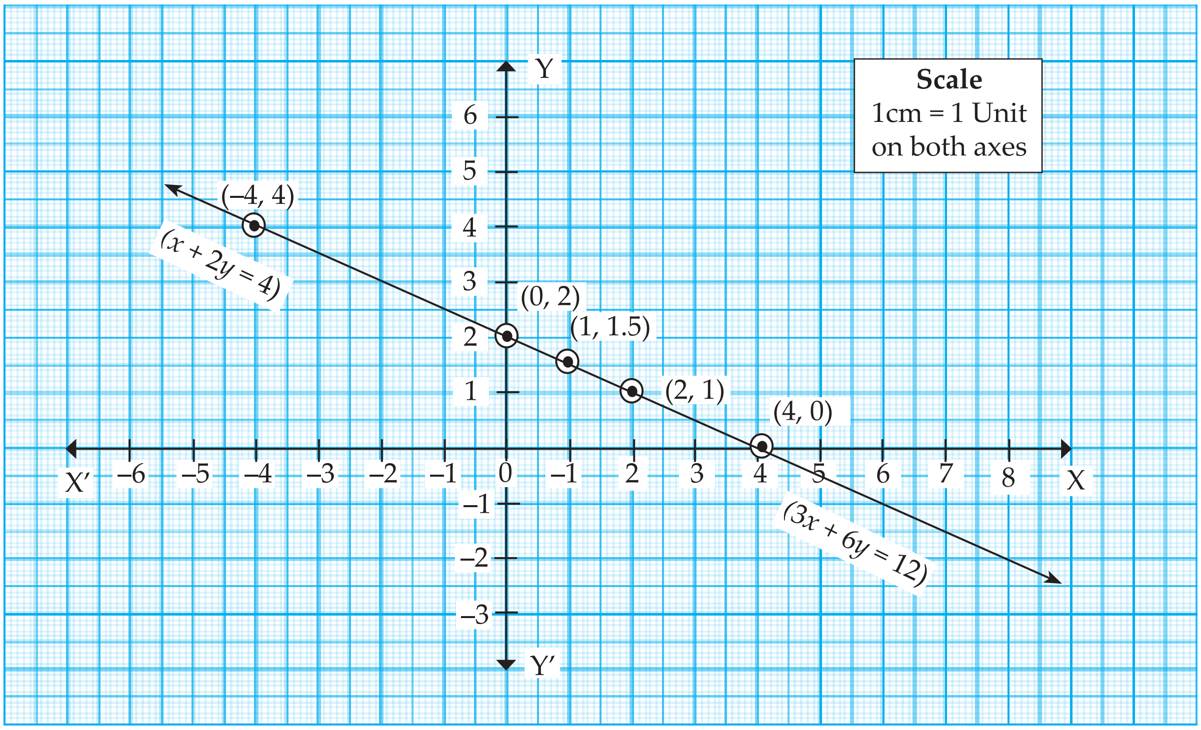
$x + 2y = 4$; $x = 4 - 2y$

x	4	2	0
y	0	1	2
(x , y)	(4, 0)	(2 , 1)	(0 , 2)

$3x + 6y = 12$; $6y = 12 - 3x$

$\therefore\ y = \dfrac{12 - 3x}{6}$

x	–4	1	4
y	4	1.5	0
(x , y)	(–4 , 4)	(1 , 1.5)	(4 , 0)



(i) Are the graphs of both the equations different or same?

Ans. The graphs of both the equations are same.

(ii) What are the solutions of the two equations $x + 2y = 4$ and $3x + 6y = 12$? How many solutions are possible?

Ans. Infinite solutions are possible.

(iii) What are the relations between coefficients of x, coefficients of y and constant terms in both the equations?

Ans. When equation (i) is multiplied by 3 we get equation (ii).

(iv) What conclusion can you draw when two equations are given but the graph is only one line?

Ans. The simultaneous equations of above type are consistent and will have infinity many solutions..

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(2) Draw graphs of $x - 2y = 4$, $2x - 4y = 12$ on the same co-ordinate plane. Observe it. Think of the solutions of the given equations. *(Textbook page no. 10)*

$x - 2y = 4$;

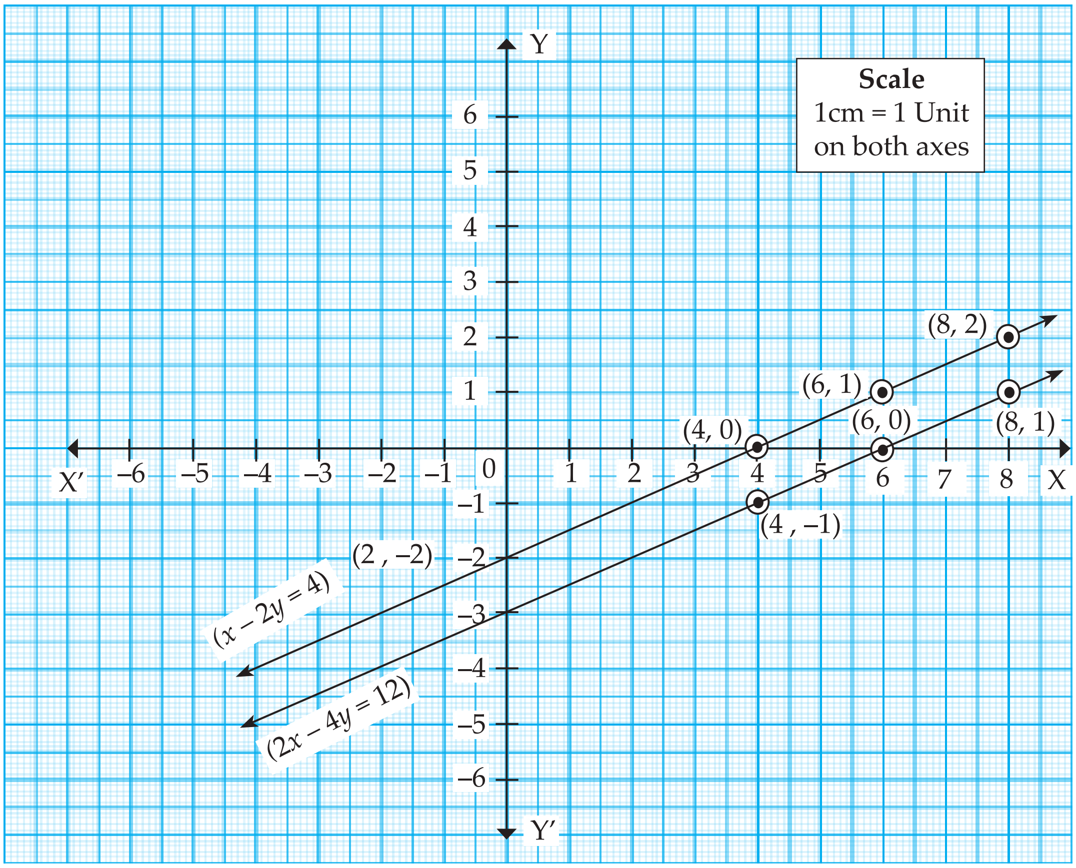
$$2x - 4y = 12$$

$$\therefore\ 2x = 12 + 4y$$

$$\therefore\ x = \frac{12 + 4y}{2}$$

x	4	6	8
y	0	1	2
(x, y)	(4 , 0)	(6 , 1)	(8 , 2)

x	6	8	4
y	0	1	–1
(x, y)	(6 , 0)	(8 , 1)	(4 , –1)



The lines represented by given simultaneous equations are parallel. Thus the given simultaneous equations will not have a common solution.

- **Determinant:**

If a, b, c and d are any four numbers, the value $ad - bc$ is represented as $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

This type of representation of the numbers is called as determinant of order two as it has two rows and two columns.

The value of the determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

To name determinants, generally English capital letters such as A, B, C, D … are used.

- **Determinant method (Cramer's rule) :**

Let the given simultaneous equations be:

$a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$,

Where a_1, a_2, b_1 , b_2, c_1 and c_2 are the real numbers such that $a_1 b_2 - a_2 b_1 \neq 0$ and x and y are variables. Let us solve these equations by equating the coefficients:

$a_1 x + b_1 y = c_1$ ...(i)

$a_2 x + b_2 y = c_2$ ...(ii)

Multiplying (i) by b_2 and (ii) by b_1 ,

$a_1 b_2 x + b_1 b_2 y = c_1 b_2$ ...(iii)

$a_2 b_1 x + b_1 b_2 y = c_2 b_1$ ...(iv)

Subtracting (iv) from (iii),

$$\begin{array}{rcrcl} a_1 b_2 x & + & b_1 b_2 y & = & c_1 b_2 \\ a_2 b_1 x & + & b_1 b_2 y & = & c_2 b_1 \\ - & & - & & - \\ \hline (a_1 b_2 - a_2 b_1)\, x & & & = & c_1 b_2 - c_2 b_1 \end{array}$$

$$\therefore \quad x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} \quad \text{... (v)}$$

Similarly we get,

$$y = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} \quad \text{...(vi)}$$

From (v) and (vi), we can write the value of x and y in determinant form as follows:

$$x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$y = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} , \ (a_1 b_2 - a_2 b_1) \neq 0$$

We denote,

$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $Dx = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ and $Dy = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

$\therefore \; x = \dfrac{Dx}{D}$ and $y = \dfrac{Dy}{D}$

This method of obtaining solution of simultaneous equations by using determinants is known as **Cramer's rule:**

- **Steps for solving simultaneous equations using cramer's rule are as follows:**

Step I: Write the given equations in the form of $ax + by = c$

Step II: Find the value of D, Dx and Dy

Step III : Now apply cramer's rule to get the value of x and y.

$x = \dfrac{Dx}{D}$ and $y = \dfrac{Dy}{D}$

- **Activity 1 : Fill in the blanks to solve simultaneous equations using Cramer's rule.**

(1) $\mathbf{y + 2x - 19 = 0 \; ; \; 2x - 3y + 3 = 0}$ *(Textbook Pg. No. 14)*

Solution:

By writing the equation in the form of $ax + by = c$

$2x + y = 19$

$2x - 3y = -3$

$D = \begin{vmatrix} \boxed{2} & \boxed{1} \\ 2 & -3 \end{vmatrix}$

$= [\boxed{2} \times (-3)] - [2 \times \boxed{1}]$

$= \boxed{-6} - \boxed{2}$

$= \boxed{-8}$

$Dx = \begin{vmatrix} 19 & \boxed{1} \\ \boxed{-3} & -3 \end{vmatrix}$

$= [19 \times \boxed{-3}] - [\boxed{-3} \times \boxed{1}]$

$= \boxed{-57} - \boxed{-3}$

$= \boxed{-54}$

$Dy = \begin{vmatrix} \boxed{2} & 19 \\ 2 & \boxed{-3} \end{vmatrix}$

$= \boxed{2} \times \boxed{-3} - \boxed{2} \times \boxed{19}$

$= \boxed{-6} - \boxed{38}$

$= \boxed{-44}$

By Cramer's rule

$x = \dfrac{Dx}{D}$

$x = \dfrac{\boxed{-54}}{\boxed{-8}}$

$\therefore \; x = \boxed{6.75}$

$y = \dfrac{Dy}{D}$

$y = \dfrac{\boxed{-44}}{\boxed{-8}}$

$\therefore \; y = \boxed{5.5}$

$\therefore$ **$(x, y) = (\boxed{6.75}, \boxed{5.5})$ is the solution of given simultaneous equations.**

- **Activity 2 : Complete the following activity.**

(Textbook Page No. 15)

Solution:

$3x - 2y = 3$ $\qquad$ $2x + y = 16$

Value of Determinants

$D = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = \boxed{7}$ $\qquad$ $Dx = \begin{vmatrix} 3 & -2 \\ 16 & 1 \end{vmatrix} = \boxed{35}$ $\qquad$ $Dy = \begin{vmatrix} 3 & 3 \\ 2 & 16 \end{vmatrix} = \boxed{42}$

According to Cramer's rule

$x = \dfrac{\boxed{35}}{\boxed{7}}$

$\therefore \; x = \boxed{5}$

$y = \dfrac{\boxed{42}}{\boxed{7}}$

$\therefore \; y = \boxed{6}$

$\therefore$ **$(x, y) = (\boxed{5}, \boxed{6})$ is the solution.**

- **Let's think :** *(Textbook Page No. 16)*

(i) What is the nature of solution if D = 0 ?

(ii) What can you say about lines if common solution is not possible?

Ans. If D = 0, then there is no common solution possible.

D = 0 means $a_1b_2 - a_2b_1 = 0$ i.e. $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$, for this condition. There are two possibilities.

(i) If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ then the given simultaneous equations will have infinitely, many solutions. The lines represented by then are overlapping.

(iii) If $\frac{a_1}{b_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the given simultaneous equations will have no solution. The lines represented by them are parallel.

Practice Set - 1.3 *(Textbook Page No. 16)*

(1) Fill in the blanks with correct number. *(1 mark)*

$$\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 3 \times \boxed{} - \boxed{} \times 4$$
$$= \boxed{} - 8$$
$$= \boxed{}$$

Solution:

$$\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 3 \times \boxed{5} - \boxed{2} \times 4$$
$$= \boxed{15} - 8$$
$$= \boxed{7}$$

(2) Find the values of the following determinants.

(i) $\begin{vmatrix} -1 & 7 \\ 2 & 4 \end{vmatrix}$ *(1 mark)*

Solution:

$$\begin{vmatrix} -1 & 7 \\ 2 & 4 \end{vmatrix}$$
$$= (-1 \times 4) - (7 \times 2)$$
$$= -4 - 14$$
$$= \boxed{-18}$$

(ii) $\begin{vmatrix} 5 & 3 \\ -7 & 0 \end{vmatrix}$ *(1 mark)*

Solution:

$$\begin{vmatrix} 5 & 3 \\ -7 & 0 \end{vmatrix}$$
$$= (-5 \times 0) - (3 \times -7)$$
$$= 0 - (-21)$$
$$= 0 + 21$$
$$= \boxed{21}$$

(iii) $\begin{vmatrix} \frac{7}{3} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix}$ *(1 mark)*

Solution:

$$\begin{vmatrix} \frac{7}{3} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix}$$
$$= \left(\frac{7}{3} \times \frac{1}{2}\right) - \left(\frac{5}{3} \times \frac{3}{2}\right)$$
$$= \frac{7}{6} - \frac{15}{6}$$
$$= \frac{7-15}{6}$$
$$= \frac{-8}{6}$$
$$= \boxed{\frac{-4}{3}}$$

Problem Set- 1 *(Textbook Page No. 27)*

(4) Find the value of the following determinants.

(i) $\begin{vmatrix} 4 & 3 \\ 2 & 7 \end{vmatrix}$ *(1 mark)*

Solution:

$$\begin{vmatrix} 4 & 3 \\ 2 & 7 \end{vmatrix}$$
$$= (4 \times 7) - (3 \times 2)$$
$$= 28 - 6$$
$$= \boxed{22}$$

(ii) $\begin{vmatrix} 5 & -2 \\ -3 & 1 \end{vmatrix}$ *(1 mark)*

Solution:

$$\begin{vmatrix} 5 & -2 \\ -3 & 1 \end{vmatrix}$$
$$= (5 \times 1) - [(-2 \times -3)]$$
$$= 5 - 6$$
$$= \boxed{-1}$$

(iii) $\begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix}$ *(1 mark)*

Solution:

$$\begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix}$$
$$= (3 \times 4) - (-1 \times 1)$$
$$= 12 - (-1)$$
$$= 12 + 1$$
$$= \boxed{13}$$

Practice Set - 1.3 *(Textbook Page No. 16)*

(3) Solve the following simultaneous equations using Cramer's rule.

(i) $3x - 4y = 10$; $4x + 3y = 5$ *(3 marks)*

Solution:

$3x - 4y = 10$ ...(i)

$4x + 3y = 5$ ...(ii)

$$D = \begin{vmatrix} 3 & -4 \\ 4 & 3 \end{vmatrix}$$

$= (3 \times 3) - (-4 \times 4)$

$= 9 - (-16)$

$= 9 + 16$

$\therefore$ **D = 25**

$$D_x = \begin{vmatrix} 10 & -4 \\ 5 & 3 \end{vmatrix}$$

$= (10 \times 3) - (-4 \times 5)$

$= 30 - (-20)$

$= 30 + 20$

$\therefore$ $\mathbf{D_x = 50}$

$$D_y = \begin{vmatrix} 3 & 10 \\ 4 & 5 \end{vmatrix}$$

$= (3 \times 5) - (10 \times 4)$

$= 15 - 40$

$\therefore$ $\mathbf{D_y = -25}$

By Cramer's rule

$$x = \frac{D_x}{D} = \frac{50}{25} = 2 \text{ and}$$

$$y = \frac{D_y}{D} = \frac{-25}{25} = -1$$

$\therefore$ **$x = 2$ and $y = -1$ is the solution of the given simultaneous equations.**

(ii) $4x + 3y - 4 = 0$; $6x = 8 - 5y$ *(3 marks)*

Solution:

$4x + 3y - 4 = 0$

$\therefore$ $4x + 3y = 4$ ...(i)

$6x = 8 - 5y$

$\therefore$ $6x + 5y = 8$ ...(ii)

$$D = \begin{vmatrix} 4 & 3 \\ 6 & 5 \end{vmatrix}$$

$= (4 \times 5) - (3 \times 6)$

$= 20 - 18$

$\therefore$ **D = 2**

$$D_x = \begin{vmatrix} 4 & 3 \\ 8 & 5 \end{vmatrix}$$

$= (4 \times 5) - (3 \times 8)$

$= 20 - 24$

$\therefore$ $\mathbf{D_x = -4}$

$$Dy = \begin{vmatrix} 4 & 4 \\ 6 & 8 \end{vmatrix}$$

$= (4 \times 8) - (4 \times 6)$

$= 32 - 24$

$\therefore$ $\mathbf{D_y = 8}$

By Cramer's rule

$$x = \frac{D_x}{D} = \frac{-4}{2} = -2 \text{ and}$$

$$y = \frac{D_y}{D} = \frac{8}{2} = 4$$

$\therefore$ **$x = -2$ and $y = 4$ is the solution of the given simultaneous equations.**

(iii) $x + 2y = -1$; $2x - 3y = 12$ *(3 marks)*

Solution:

$x + 2y = -1$ ...(i)

$2x - 3y = 12$ ...(ii)

$$D = \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$= (1 \times -3) - (2 \times 2)$

$= -3 - 4$

$\therefore$ **D = – 7**

$$D_x = \begin{vmatrix} -1 & 2 \\ 12 & -3 \end{vmatrix}$$

$= (-1 \times -3) - (2 \times 12)$

$= 3 - 24$

$\therefore$ $\mathbf{D_x = -21}$

$$D_y = \begin{vmatrix} 1 & -1 \\ 2 & 12 \end{vmatrix}$$

$= (1 \times 12) - (-1 \times 2)$

$= 12 - (-2)$

$= 12 + 2$

$\therefore$ $\mathbf{D_y = 14}$

By Cramer's rule

$$x = \frac{D_x}{D} = \frac{-21}{-7} = 3 \text{ and}$$

$$y = \frac{D_y}{D} = \frac{14}{-7} = -2$$

$\therefore$ **$x = 3$ and $y = -2$ is the solution of the given simultaneous equations.**

Problem Set - 1 *(Textbook Page No. 28)*

(5) Solve the following simultaneous equations using Cramer's method.

(i) $6x - 3y = -10$; $3x + 5y - 8 = 0$ *(4 marks)*

Solution:

$6x - 3y = -10$ ... (i)

Expressing the given equation $3x + 5y - 8 = 0$ in the form $ax + by = c$ we get,

$3x + 5y = 8$ ... (ii)

$$D = \begin{vmatrix} 6 & -3 \\ 3 & 5 \end{vmatrix}$$

$\therefore = (6 \times 5) - (-3 \times 3)$

$\therefore = 30 - (-9)$

$\therefore = 30 + 9$

$\therefore$ **D = 39**

$$D_x = \begin{vmatrix} -10 & -3 \\ 8 & 5 \end{vmatrix}$$

$= (-10 \times 5) - (-3 \times 8)$

$= -50 - (-24)$

$= -50 + 24$

$\therefore$ $\mathbf{D_x = -26}$

$$D_y = \begin{vmatrix} 6 & -10 \\ 3 & 8 \end{vmatrix}$$

$= (6 \times 8) - (-10 \times 3)$

$= 48 - (-30)$

$= 48 + 30$

$\therefore$ $\mathbf{D_y = 78}$

By Cramer's rule

$x = \dfrac{D_x}{D} = \dfrac{-26}{39} = \dfrac{-2}{3}$ and

$y = \dfrac{D_y}{D} = \dfrac{78}{39} = 2$

$\therefore$ $\boxed{\textbf{$x = \frac{-2}{3}$ and $y = 2$ is the solution of the given simultaneous equations.}}$

(ii) $4m - 2n = -4$; $4m + 3n = 16$ *(4 marks)*

Solution:

$4m - 2n = -4$; $4m + 3n = 16$

$$D = \begin{vmatrix} 4 & -2 \\ 4 & 3 \end{vmatrix}$$

$= (4 \times 3) - (-2 \times 4)$

$= 12 - (-8)$

$= 12 + 8$

$\therefore$ **D = 20**

$$D_m = \begin{vmatrix} -4 & -2 \\ 16 & 3 \end{vmatrix}$$

$= (-4 \times 3) - (-2 \times 16)$

$= -12 - (-32)$

$= -12 + 32$

$\therefore$ $\mathbf{D_m = 20}$

$$D_n = \begin{vmatrix} 4 & -4 \\ 4 & 16 \end{vmatrix}$$

$= (4 \times 16) - (-4 \times 4)$

$= 64 - (-16)$

$= 64 + 16$

$\therefore$ $\mathbf{D_n = 80}$

By Cramer's rule

$m = \dfrac{D_m}{D} = \dfrac{20}{20} = 1$ and

$n = \dfrac{D_n}{D} = \dfrac{80}{20} = 4$

$\therefore$ $\boxed{\textbf{$m = 1$ and $n = 4$ is the solution of given simultaneous equations.}}$

(iv) $7x + 3y = 15$; $12y - 5x = 39$ *(4 marks)*

Solution:

$7x + 3y = 15$ ... (i)

Expressing the equation $12y - 5x = 39$ in the form of $ax + by = c$, we get

$-5x + 12y = 39$ ... (ii)

$$D = \begin{vmatrix} 7 & 3 \\ -5 & 12 \end{vmatrix}$$

$= (7 \times 12) - (3 \times -5)$

$= 84 - (-15)$

$= 84 + 15$

$\therefore$ **D = 99**

$$D_x = \begin{vmatrix} 15 & 3 \\ 39 & 12 \end{vmatrix}$$

$= (15 \times 12) - (3 \times 39)$

$= 180 - 117$

$\therefore$ $\mathbf{D_x = 63}$

$$D_y = \begin{vmatrix} 7 & 15 \\ -5 & 39 \end{vmatrix}$$

$= (7 \times 39) - (15 \times -5)$

$= 273 - (-75)$

$= 273 + 75$

$\therefore$ $\mathbf{D_y = 348}$

By Cramer's rule

$x = \dfrac{D_x}{D} = \dfrac{63}{99} = \dfrac{7}{11}$ and

$y = \dfrac{D_y}{D} = \dfrac{348}{99} = \dfrac{116}{33}$

$\therefore$ $\boxed{\mathbf{x = \dfrac{7}{11} \text{ and } y = \dfrac{116}{33} \text{ is the solution of the given simultaneous equations.}}}$

Practice Set - 1.3 *(Textbook Page No. 16)*

(3) Solve the following simultaneous equations using Cramer's rule.

(iv) $6x - 4y = -12$; $8x - 3y = -2$ *(3 marks)*

Solution:

$6x - 4y = -12$ ...(i)

$8x - 3y = -2$ ...(ii)

$$D = \begin{vmatrix} 6 & -4 \\ 8 & -3 \end{vmatrix}$$

$= (6 \times -3) - [(-4) \times 8]$

$= -18 - (-32)$

$= -18 + 32$

$\therefore$ $\mathbf{D = 14}$

$$D_x = \begin{vmatrix} -12 & -4 \\ -2 & -3 \end{vmatrix}$$

$= (-12 \times -3) - (-4 \times -2)$

$= 36 - 8$

$\therefore$ $\mathbf{D_x = 28}$

$$D_y = \begin{vmatrix} 6 & -12 \\ 8 & -2 \end{vmatrix}$$

$= (6 \times -2) - (-12 \times 8)$

$= -12 - (-96)$

$= -12 + 96$

$\therefore$ $\mathbf{D_y = 84}$

By Cramer's value

$x = \dfrac{D_x}{D} = \dfrac{28}{14} = 2$ and

$y = \dfrac{D_y}{D} = \dfrac{84}{14} = 6$

$\therefore$ $\boxed{\mathbf{x = 2 \text{ and } y = 6 \text{ is the solution of the given simultaneous equations.}}}$

(v) $4m + 6n = 54$; $3m + 2n = 28$. *(3 marks)*

Solution:

$4m + 6n = 54$ ...(i)

$3m + 2n = 28$ ...(ii)

$$D = \begin{vmatrix} 4 & 6 \\ 3 & 2 \end{vmatrix}$$

$= (4 \times 2) - (6 \times 3)$

$= 8 - 18$

$\therefore$ $\mathbf{D = -10}$

$$D_m = \begin{vmatrix} 54 & 6 \\ 28 & 2 \end{vmatrix}$$

$= (54 \times 2) - (6 \times 28)$

$= 108 - 168$

$\therefore$ $\mathbf{D_m = -60}$

$$D_n = \begin{vmatrix} 4 & 54 \\ 3 & 28 \end{vmatrix}$$

$= (4 \times 28) - (54 \times 3)$

$= 112 - 162$

$\therefore$ $\mathbf{D_n = -50}$

By Cramer's rule

$m = \dfrac{D_m}{D} = \dfrac{-60}{-10} = 6$ and

$n = \dfrac{D_n}{D} = \dfrac{-50}{-10} = 5$

$\therefore$ $\boxed{\mathbf{m = 6 \text{ and } n = 5 \text{ is the solution of the given simultaneous equations.}}}$

(vi) $2x + 3y = 2$; $x - \dfrac{y}{2} = \dfrac{1}{2}$. *(3 marks)*

Solution:

Method - I

$2x + 3y = 2$ ...(i)

$x - \dfrac{y}{2} = \dfrac{1}{2}$ ...(ii)

$$D = \begin{vmatrix} 2 & 3 \\ 1 & -\dfrac{1}{2} \end{vmatrix}$$

$= \left(2 \times \dfrac{-1}{2}\right) - (3 \times 1)$

$= -1 - 3$

$\therefore$ $\mathbf{D = -4}$

$$D_x = \begin{vmatrix} 2 & 3 \\ \dfrac{1}{2} & -\dfrac{1}{2} \end{vmatrix}$$

$$= \left(2 \times \frac{-1}{2}\right) - \left(3 \times \frac{1}{2}\right)$$

$$= -1 - \frac{3}{2}$$

$$= \frac{-2-3}{2}$$

$$\therefore \quad \mathbf{D_x = \frac{-5}{2}}$$

$$D_y = \begin{vmatrix} 2 & 2 \\ 1 & \frac{1}{2} \end{vmatrix}$$

$$= \left(2 \times \frac{1}{2}\right) - (2 \times 1)$$

$$= 1 - 2$$

$$\therefore \quad \mathbf{D_y = -1}$$

By Cramer's rule

$$x = \frac{D_x}{D} = \frac{-\frac{5}{2}}{-4} = \frac{-5}{2} \times \frac{1}{-4} = \frac{5}{8}$$

$$y = \frac{D_y}{D} = \frac{-1}{-4} = \frac{1}{4}$$

$\therefore$ **$x = \frac{5}{8}$ and $y = \frac{1}{4}$ is the solution of the given simultaneous equations.**

Method - II

$2x + 3y = 2$ ...(i)

$x - \frac{y}{2} = \frac{1}{2}$ ...(ii)

Multiply both the sides of (ii) by 2, we get

$\therefore \quad 2x - y = 1$ ...(iii)

Now consider $2x + 3y = 2$; $2x - y = 1$

$$D = \begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix}$$

$$= (2 \times -1) - (3 \times 2)$$

$$= -2 - 6$$

$$\therefore \quad \mathbf{D = -8}$$

$$D_x = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$$

$$= (2 \times -1) - (3 \times 1)$$

$$= -2 - 3$$

$$\therefore \quad \mathbf{D_x = -5}$$

$$D_y = \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (2 \times 1) - (2 \times 2)$$

$$= 2 - 4$$

$$\therefore \quad \mathbf{D_y = -2}$$

By cramer's rule

$$x = \frac{D_x}{D} = \frac{-5}{-8} = \frac{5}{8} \quad \text{and}$$

$$y = \frac{D_y}{D} = \frac{-2}{-8} = \frac{1}{4}$$

$\therefore$ **$x = \frac{5}{8}$ and $y = \frac{1}{4}$ is the solution of the given simultaneous equations.**

Problem Set - 1 *(Textbook Page No. 28)*

(5) Solve the following simultaneous equations using Cramer's rule.

(iii) $\mathbf{3x - 2y = \frac{5}{2}\ ;\ \frac{1}{3}x + 3y = \frac{-4}{3}}$ *(3 marks)*

Solution:

$$3x - 2y = \frac{5}{2} \quad ; \quad \frac{1}{3}x + 3y = \frac{-4}{3}$$

$$D = \begin{vmatrix} 3 & -2 \\ \frac{1}{3} & 3 \end{vmatrix}$$

$$= (3 \times 3) - \left(-2 \times \frac{1}{3}\right)$$

$$= 9 - \left(-\frac{2}{3}\right)$$

$$= 9 + \frac{2}{3}$$

$$= \frac{27+2}{3}$$

$$\therefore \quad \mathbf{D = \frac{29}{3}}$$

$$D_x = \begin{vmatrix} \frac{5}{2} & -2 \\ \frac{-4}{3} & 3 \end{vmatrix}$$

$$= \left(\frac{5}{2} \times 3\right) - \left[\left(-2 \times \frac{-4}{3}\right)\right]$$

$$= \frac{15}{2} - \left(\frac{8}{3}\right)$$

$$= \frac{45-16}{6}$$

$$\therefore \quad \mathbf{D_x = \frac{29}{6}}$$

$$D_y = \begin{vmatrix} 3 & \frac{5}{2} \\ \frac{1}{3} & -\frac{4}{3} \end{vmatrix}$$

$$= \left(3 \times -\frac{4}{3}\right) - \left(\frac{5}{2} \times \frac{1}{3}\right)$$

$$= -4 - \left(\frac{5}{6}\right)$$

$$= \frac{-24-5}{6}$$

$$= \frac{-29}{6}$$

$\therefore$ $\mathbf{D_y = \frac{-29}{6}}$

By Cramer's rule,

$$x = \frac{D_x}{D} = \frac{\frac{29}{6}}{\frac{29}{3}} = \frac{29}{6} \times \frac{3}{29} = \frac{1}{2} \quad \text{and}$$

$$y = \frac{D_y}{D} = \frac{\frac{-29}{6}}{\frac{29}{3}} = \frac{-29}{6} \times \frac{3}{29} = \frac{-1}{2}$$

$\therefore$ **$x = \frac{1}{2}$ and $y = \frac{-1}{2}$ is the solution of given simultaneous equations.**

(v) $\dfrac{x+y-8}{2} = \dfrac{x+2y-14}{3} = \dfrac{3x-y}{4}$ *(4 marks)*

Solution:

$$\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x-y}{4}$$

From the above statement we get,

$$\frac{x+y-8}{2} = \frac{x+2y-14}{3}$$

$\therefore$ $3(x+y-8) = 2(x+2y-14)$

$\therefore$ $3x + 3y - 24 = 2x + 4y - 28$

$\therefore$ $3x - 2x + 3y - 4y = -28 + 24$

$\therefore$ $x - y = -4$ ....(i)

Similarly $\dfrac{x+2y-14}{3} = \dfrac{3x-y}{4}$

$\therefore$ $4(x + 2y - 14) = 3(3x - y)$

$\therefore$ $4x + 8y - 56 = 9x - 3y$

$\therefore$ $4x - 9x + 8y + 3y = 56$

$\therefore$ $-5x + 11y = 56$ ...(ii)

$$D = \begin{vmatrix} 1 & -1 \\ -5 & 11 \end{vmatrix}$$

$= (1 \times 11) - [(-1 \times -5)]$

$= 11 - (5)$

$\therefore$ $\mathbf{D = 6}$

$$D_x = \begin{vmatrix} -4 & -1 \\ 56 & 11 \end{vmatrix}$$

$= (-4 \times 11) - (-1 \times 56)$

$= -44 - (-56)$

$= -44 + 56$

$\therefore$ $\mathbf{D_x = 12}$

$$D_y = \begin{vmatrix} 1 & -4 \\ -5 & 56 \end{vmatrix}$$

$= (1 \times 56) - [(-4 \times -5)]$

$= 56 - (20)$

$\therefore$ $\mathbf{D_y = 36}$

By Cramer's rule

$$x = \frac{D_x}{D} = \frac{12}{6} = 2 \quad \text{and}$$

$$y = \frac{D_y}{D} = \frac{36}{6} = 6$$

$\therefore$ **$x = 2$ and $y = 6$ is the solution of given simultaneous equations.**

Points to Remember:

- **Equation reducible to a pair of linear equations in two variables:**

  Consider the simultaneous equations

  $$\frac{4}{x} + \frac{3}{y} = 1 \;;\; \frac{8}{x} - \frac{9}{y} = 7$$

  $(x \neq 0, y \neq 0)$ which are not linear. Such equation can be reduced to pair of linear equations by making suitable substitutions

Activity: To solve given equations fill the boxes below suitably. *(Textbook Page No. 19)*

$$\frac{5}{x-1} + \frac{1}{y-2} = 2\ ;\quad \frac{6}{x-1} + \frac{3}{y-2} = 1$$

Replacing $\left(\frac{1}{x-1}\right)$ by m and $\left(\frac{1}{y-2}\right)$ by n

$5m + n = 2\ ;$ $\qquad$ $6m - 3n = 1$

New equation on solving

$m = \boxed{\frac{1}{3}}$ and $n = \boxed{\frac{1}{3}}$

Replacing m, n by their original valves.

$\left(\frac{1}{x-1}\right) = \frac{1}{3}$ $\qquad$ an $\qquad$ $\left(\frac{1}{y-2}\right) = \frac{1}{3}$

solving

$x = \boxed{4}$ and $y = \boxed{5}$

$\therefore$ **$(x, y) = (4, 5)$ is the solution of the given simultaneous equations.**

Practice Set - 1.4 *(Textbook Page No. 19)*

(1) Solve the following simultaneous equations.

(i) $\frac{2}{x} - \frac{3}{y} = 15\ ;\ \frac{8}{x} + \frac{5}{y} = 77$ *(4 marks)*

Solution:

$\frac{2}{x} - \frac{3}{y} = 15$ ...(i)

$\frac{8}{x} + \frac{5}{y} = 77$ ...(ii)

Substituting $\frac{1}{x} = m$ and $\frac{1}{y} = n$ in the above equations, we get

$2m - 3n = 15$ ...(iii)

$8m + 5n = 77$ ...(iv)

Multiplying (iii) by 5 and (iv) by 3, we get,

$10m - 15n = 75$ ...(v)

$24m + 15n = 231$ ... (vi)

Adding (v) and (vi), we get,

$$\begin{array}{rcl} 10m - 15n &=& 75 \\ 24m + 15n &=& 231 \\ \hline 34m &=& 306 \end{array}$$

$\therefore\ m = \frac{306}{34}$

$\therefore\ m = 9$

Substituting $m = 9$ in (iii) we get

$2 \times 9 - 3n = 15$

$\therefore\ 18 - 3n = 15$

$\therefore\ -3n = 15 - 18$

$\therefore\ 3n = -3$

$\therefore \quad n = \frac{-3}{-3}$

$\therefore \quad n = 1$

Resubstituting the value of m and n we get,

$\frac{1}{x} = 9$ and $\frac{1}{y} = 1$

$\therefore \quad x = \frac{1}{9}$ and $y = 1$

$\therefore$ **$x = \frac{1}{9}$ and $y = 1$ is the solution of given simultaneous equations.**

Problem Set - 1 *(Textbook Page No. 28)*

(6) Solve the following simultaneous equations.

(i) $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$; $\frac{3}{x} + \frac{2}{y} = 0$ *(4 marks)*

Solution:

$\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$ ...(i)

$\frac{3}{x} + \frac{2}{y} = 0$ ...(ii)

Substituting $\frac{1}{x} = m$ and $\frac{1}{y} = n$ in equation (i) and equation (ii) we get

$\therefore \quad 2m + \frac{2}{3}n = \frac{1}{6}$

Multiplying both sides by 6 we get,

$\therefore \quad 12m + 4n = 1$ ...(iii)

$3m + 2n = 0$ ...(iv)

Multiplying both sides of (iv) by 2 we get,

$6m + 4n = 0$ ...(v)

Subtracting (v) from (iii)

$$\begin{array}{rcrcl} 12m & + & 4n & = & 1 \\ 6m & + & 4n & = & 0 \\ - & & - & & - \\ \hline 6m & & & = & 1 \end{array}$$

$\therefore \quad m = \frac{1}{6}$

Substituting $m = \frac{1}{6}$ in (iii) we get,

$12 \times \frac{1}{6} + 4n = 1$

$\therefore \quad 2 + 4n = 1$

$\therefore \quad 4n = 1 - 2$

$\therefore \quad 4n = -1$

$\therefore \quad n = \frac{-1}{4}$

Resubstituting the values of m and n we get,

$\frac{1}{x} = \frac{1}{6}$ and $\frac{1}{y} = \frac{-1}{4}$

$\therefore \quad x = 6$ and $y = -4$

$\therefore$ **$x = 6, y = -4$ is the solution of the given simultaneous equations.**

(iii) $\frac{148}{x} + \frac{231}{y} = \frac{527}{xy}$; $\frac{231}{x} + \frac{148}{y} = \frac{610}{xy}$

(4 marks)

Solution:

$\frac{148}{x} + \frac{231}{y} = \frac{527}{xy}$ ...(i)

$\frac{231}{x} + \frac{148}{y} = \frac{610}{xy}$ ...(ii)

Multiplying both the sides of equation (i) and equation (ii) by xy we get,

$148y + 231x = 527$

i.e. $231x + 148y = 527$ ...(iii)

$231y + 148x = 610$

i.e. $148x + 231y = 610$ ...(iv)

Adding (iii) and (iv) we get

$$\begin{array}{rcl} 231x + 148y & = & 527 \\ 148x + 231y & = & 610 \\ \hline 379x + 379y & = & 1137 \end{array}$$

Dividing throughout by 379,

$\therefore \quad x + y = \frac{1137}{379}$

$\therefore \quad x + y = 3$ ...(v)

Subtracting (iv) from (iii) we get,

$$\begin{array}{rcrcl} 231x & + & 148y & = & 527 \\ 148x & + & 231y & = & 610 \\ - & & - & & - \\ \hline 83x & - & 83y & = & -83 \end{array}$$

Dividing throughout by 83

$\therefore \quad x - y = -1$ ...(vi)

Adding (v) and (vi) we get,

$$\begin{array}{rcrcl} x & + & y & = & 3 \\ x & - & y & = & -1 \\ \hline 2x & & & = & 2 \end{array}$$

$\therefore$ **$x = 1$**

Substituting $x = 1$ in equation (v) we get

$x + y = 3$

$\therefore \quad 1 + y = 3$

$\therefore \quad y = 3 - 1$

$\therefore \quad y = 2$

$\therefore$ **$x = 1$ and $y = 2$ is the solution of the given simultaneous equations.**

(iv) $\dfrac{7x-2y}{xy} = 5 \;;\; \dfrac{8x+7y}{xy} = 15$ *(4 marks)*

Solution:

$\dfrac{7x-2y}{xy} = 5$ (given)

$\therefore \quad \dfrac{7x}{xy} - \dfrac{2y}{xy} = 5$

$\therefore \quad \dfrac{7}{y} - \dfrac{2}{x} = 5$

i.e. $\dfrac{-2}{x} + \dfrac{7}{y} = 5$ ...(i)

Also $\dfrac{8x+7y}{xy} = 15$ (Given)

$\therefore \quad \dfrac{8x}{xy} + \dfrac{7y}{xy} = 15$

i.e. $\dfrac{7}{x} + \dfrac{8}{y} = 15$ ...(ii)

Substituting $\dfrac{1}{x} = m$ and $\dfrac{1}{y} = n$ in (i) and (ii) we get,

$-2m + 7n = 5$ ...(iii)

$7m + 8n = 15$ ...(iv)

Multiplying (iii) by 7 and (iv) by 2 we get,

$-14m + 49n = 35$ ...(v)

$14m + 16n = 30$ ...(vi)

Adding (v) and (vi) we get,

$$\begin{array}{rcrcr} -14m & + & 49n & = & 35 \\ 14m & + & 16n & = & 30 \\ \hline & & 65n & = & 65 \end{array}$$

$\therefore \quad n = \dfrac{65}{65}$

$\therefore \quad n = 1$

Substituting $n = 1$ in (iv) we get,

$7m + 8n = 15$

$\therefore \quad 7m + 8(1) = 15$

$\therefore \quad 7m + 8 = 15$

$\therefore \quad 7m = 15 - 8$

$\therefore \quad 7m = 7$

$\therefore \quad m = \dfrac{7}{7}$

$\therefore \quad m = 1$

Resubstituting the values of m and n we get,

$\dfrac{1}{x} = 1$ and $\dfrac{1}{y} = 1$

$x = 1$ and $y = 1$

$\therefore$ **$x = 1$ and $y = 1$ is the solution of the given simultaneous equations.**

Practice Set - 1.4 *(Textbook Page No. 19)*

(1) Solve the following simultaneous equations.

(iii) $\dfrac{27}{x-2} + \dfrac{31}{y+3} = 85 \;;\; \dfrac{31}{x-2} + \dfrac{27}{y+3} = 89$

(4 marks)

Solution:

$\dfrac{27}{x-2} + \dfrac{31}{y+3} = 85$ ...(i)

$\dfrac{31}{x-2} + \dfrac{27}{y+3} = 89$ ...(ii)

Substituting $\dfrac{1}{x-2} = m$ and $\dfrac{1}{y+3} = n$

in (i) and (ii) we get,

$27m + 31n = 85$ ...(iii)

$31m + 27n = 89$ ...(iv)

Adding (iii) and (iv) we get,

$$\begin{array}{rcrcr} 27m & + & 31n & = & 85 \\ 31m & + & 27n & = & 89 \\ \hline 58m & + & 58n & = & 174 \end{array}$$

Dividing throughout by 58

$\therefore \quad m + n = \dfrac{174}{58}$

$m + n = 3$ ...(v)

Subtracting (iv) from (iii)

$$\begin{array}{rcrcr} 27m & + & 31n & = & 85 \\ 31m & + & 27n & = & 89 \\ (-) & & (-) & & (-) \\ \hline -4m & + & 4n & = & -4 \end{array}$$

Dividing throughout by -4

$m - n = \dfrac{-4}{-4}$

$\therefore \quad m - n = 1$ ...(vi)

Adding (v) and (vi), we get

$$\begin{array}{rcrcr} m & + & n & = & 3 \\ m & - & n & = & 1 \\ \hline 2m & & & = & 4 \end{array}$$

$\therefore \quad m = \dfrac{4}{2}$

$\therefore \quad m = 2$

Substituting $m = 2$ in (v) we get,

$m + n = 3$

$\therefore\ 2 + n = 3$

$\therefore\ n = 3 - 2$

$\therefore\ n = 1$

Resubstituting the values of m and n we get,

$\dfrac{1}{x-2} = 2$ and $\dfrac{1}{y+3} = 1$

$\therefore\ x - 2 = \dfrac{1}{2}$ and $y + 3 = 1$

$\therefore\ x = \dfrac{1}{2} + 2$ and $y = 1 - 3$

$\therefore\ x = \dfrac{1+4}{2}$ and $y = -2$

$\therefore\ x = \dfrac{5}{2}$ and $y = -2$

$\therefore$ **$x = \dfrac{5}{2}$, $y = -2$ is the solution of the given simultaneous equations.**

Problem Set - 1 *(Textbook Page No. 28)*

(6) Solve the following simultaneous equations.

(ii) $\dfrac{7}{2x+1} + \dfrac{13}{y+2} = 27$; $\dfrac{13}{2x+1} + \dfrac{7}{y+2} = 33$

(4 marks)

Solution:

$\dfrac{7}{2x+1} + \dfrac{13}{y+2} = 27$ ...(i)

$\dfrac{13}{2x+1} + \dfrac{7}{y+2} = 33$ ...(ii)

Substituting $\dfrac{1}{2x+1} = m$ and $\dfrac{1}{y+2} = n$

in (i) and (ii) respectively, we get,

$7m + 13n = 27$ ...(iii)

$13m + 7n = 33$ ...(iv)

Adding (iii) and (iv) we get,

$$\begin{array}{rcrcr} 7m & + & 13n & = & 27 \\ 13m & + & 7n & = & 33 \\ \hline 20m & + & 20n & = & 60 \end{array}$$

Dividing throughout by 20,

$\therefore\ m + n = \dfrac{60}{20}$

$\therefore\ m + n = 3$ ...(v)

Subtracting (iv) from (iii), we get

$$\begin{array}{rcrcr} 7m & + & 13n & = & 27 \\ 13m & + & 7n & = & 33 \\ (-) & & (-) & & (-) \\ \hline -6m & + & 6n & = & -6 \end{array}$$

Dividing throughout by -6

$m - n = 1$ ...(vi)

Adding (v) and (vi) we get,

$$\begin{array}{rcrcr} m & + & n & = & 3 \\ m & - & n & = & 1 \\ \hline 2m & & & = & 4 \end{array}$$

$\therefore\ m = \dfrac{4}{2}$

$\therefore\ m = 2$

Substituting $m = 2$ in (v) we get,

$m + n = 3$

$\therefore\ 2 + n = 3$

$\therefore\ n = 3 - 2$

$\therefore\ n = 1$

Resubstituting the values of m and n we get,

$\dfrac{1}{2x+1} = 2$ and $\dfrac{1}{y+2} = 1$

$\therefore\ 2x + 1 = \dfrac{1}{2}$ and $y + 2 = 1$

$\therefore\ 4x + 2 = 1$ and $y = 1 - 2$

$\therefore\ 4x = 1 - 2$ and $y = -1$

$\therefore\ 4x = -1$

$\therefore\ x = \dfrac{-1}{4}$

$\therefore$ **$x = \dfrac{-1}{4}$ and $y = -1$ is the solution of the given simultaneous equations.**

Practice Set - 1.4 *(Textbook Page No. 19)*

(1) Solve the following simultaneous equations.

(ii) $\dfrac{10}{x+y} + \dfrac{2}{x-y} = 4$; $\dfrac{15}{x+y} - \dfrac{5}{x-y} = -2$

(4 marks)

Solution:

$\dfrac{10}{x+y} + \dfrac{2}{x-y} = 4$ ...(i)

$\dfrac{15}{x+y} - \dfrac{5}{x-y} = -2$ ...(ii)

Substituting $\dfrac{1}{x+y} = a$ and $\dfrac{1}{x-y} = b$

in (i) and (ii) we get,

$10a + 2b = 4$ ...(iii)

$15a - 5b = -2$ ...(iv)

Multiplying (iii) by 5 and (iv) by 2 we get,

$50a + 10b = 20$ ...(v)

$30a - 10b = -4$ ...(vi)

Adding (v) and (vi), we get

$$\begin{array}{rcrcr} 50a & + & 10b & = & 20 \\ 30a & - & 10b & = & -4 \\ \hline 80a & & & = & 16 \end{array}$$

$\therefore \quad a = \dfrac{16}{80}$

$\therefore \quad a = \dfrac{1}{5}$

Substituting $a = \dfrac{1}{5}$ in (iii), we get,

$\therefore \quad 10a + 2b = 4$

$\therefore \quad 10 \times \dfrac{1}{5} + 2b = 4$

$\therefore \quad 2 + 2b = 4$

$\therefore \quad 2b = 4 - 2$

$\therefore \quad 2b = 2$

$\therefore \quad b = \dfrac{2}{2}$

$\therefore \quad b = 1$

Resubstituting the values of a and b we get,

$\dfrac{1}{x+y} = \dfrac{1}{5}$ and $\dfrac{1}{x-y} = 1$

$\therefore \quad x + y = 5$ ....(vii)

$\therefore \quad x - y = 1$ ...(viii)

Adding (vii) and (viii), we get,

$$\begin{array}{rl} & x + y = 5 \\ + & x - y = 1 \\ \hline & 2x \;= 6 \end{array}$$

$\therefore \quad x = \dfrac{6}{2}$

$\mathbf{x = 3}$

Substituting $x = 3$ in equation (vii) we get,

$x + y = 5$

$\therefore \quad 3 + y = 5$

$y = 5 - 3$

$\therefore \quad \mathbf{y = 2}$

$\therefore$ **$x = 3$, $y = 2$ is the solution of the given simultaneous equations.**

(iv) $\dfrac{1}{3x+y} + \dfrac{1}{3x-y} = \dfrac{3}{4}; \quad \dfrac{1}{2(3x+y)} - \dfrac{1}{2(3x-y)} = -\dfrac{1}{8}$

(4 marks)

Solution:

$\dfrac{1}{3x+y} + \dfrac{1}{3x-y} = \dfrac{3}{4}$ ...(i)

$\dfrac{1}{2(3x+y)} - \dfrac{1}{2(3x-y)} = -\dfrac{1}{8}$

Multiplying both sides by 2 we get,

$\dfrac{1}{3x+y} - \dfrac{1}{3x-y} = -\dfrac{1}{4}$ ...(ii)

Substituting $\dfrac{1}{3x+y} = m$ and $\dfrac{1}{3x-y} = n$

in (i) and (ii) we get,

$m + n = \dfrac{3}{4}$

Multiplying both sides by 4

$4m + 4n = 3$ ... (iii)

$m - n = \dfrac{-1}{4}$

Multiplying both sides by 4

$4m - 4n = -1$ ... (iv)

Adding (iii) and (iv)

$$\begin{array}{rcl} 4m + 4n & = & 3 \\ 4m - 4n & = & -1 \\ \hline 8m & = & 2 \end{array}$$

$\therefore \quad m = \dfrac{2}{8}$

$\therefore \quad m = \dfrac{1}{4}$

Substituting $m = \dfrac{1}{4}$ in (iii) we get,

$4m + 4n = 3$

$\therefore \quad 4\left(\dfrac{1}{4}\right) + 4n = 3$

$\therefore \quad 1 + 4n = 3$

$\therefore \quad 4n = 2$

$\therefore \quad n = \dfrac{2}{4}$

$\therefore \quad n = \dfrac{1}{2}$

Resubstituting the values of m and n we get,

$\dfrac{1}{3x+y} = \dfrac{1}{4}$ and $\dfrac{1}{3x-y} = \dfrac{1}{2}$

$\therefore \quad 3x + y = 4$ and $3x - y = 2$

$\therefore \quad 3x + y = 4$ ...(v) and

$\therefore \quad 3x - y = 2$ ...(vi)

Adding (v) and (vi) we get,

$$\begin{array}{rrcl} & 3x + y & = & 4 \\ + & 3x - y & = & 2 \\ \hline & 6x & = & 6 \end{array}$$

$\therefore \quad x = \dfrac{6}{6}$

$\therefore \quad \mathbf{x = 1}$

Substituting $x = 1$ in (v) we get,

$3(1) + y = 4$

$\therefore \ 3 + y = 4$

$\therefore \ y = 4 - 3$

$\therefore \ \mathbf{y = 1}$

$\therefore$ **$x = 1, y = 1$ is the solution of the given simultaneous equations.**

Problem Set - 1 *(Textbook Page No. 28)*

(6) Solve the following simultaneous equations.

(v) $$\frac{1}{2(3x+4y)} + \frac{1}{5(2x-3y)} = \frac{1}{4};$$

$$\frac{5}{(3x+4y)} - \frac{2}{(2x-3y)} = -\frac{3}{2}$$ *(4 marks)*

Solution:

$$\frac{1}{2(3x+4y)} + \frac{1}{5(2x-3y)} = \frac{1}{4}$$

Multiplying both sides of above equation by 20 we get,

$$\frac{10}{(3x+4y)} + \frac{4}{(2x-3y)} = 5 \quad \text{... (i)}$$

$$\frac{5}{(3x+4y)} - \frac{2}{(2x-3y)} = \frac{-3}{2} \quad \text{...(ii)}$$

Substituting $\frac{1}{3x+4y} = m$ and $\frac{1}{2x-3y} = n$

in (i) and (ii) we get,

$\therefore \ 10m + 4n = 5$ ...(iii)

$5m - 2n = \frac{-3}{2}$

Multiplying by 2 we get,

$10m - 4n = -3$ ...(iv)

Adding (iii) and (iv) we get

$$\begin{array}{rcrcl} 10m & + & 4n & = & 5 \\ 10m & - & 4n & = & -3 \\ \hline 20m & & & = & 2 \end{array}$$

$\therefore \ m = \frac{2}{20}$

$\therefore \ m = \frac{1}{10}$

Substituting $m = \frac{1}{10}$ in (iii) we get,

$10m + 4n = 5$

$\therefore \ 10\left(\frac{1}{10}\right) + 4n = 5$

$\therefore \ 1 + 4n = 5$

$\therefore \ 4n = 5 - 1$

$\therefore \ 4n = 4$

$\therefore \ n = \frac{4}{4}$

$\therefore \ n = 1$

$$\frac{1}{3x+4y} = \frac{1}{10} \quad \text{and} \quad \frac{1}{2x-3y} = 1$$

$\therefore \ 3x + 4y = 10$ and $2x - 3y = 1$

$3x + 4y = 10$ ...(v)

$2x - 3y = 1$ ...(vi)

Multiplying (v) by 3 and (vi) by 4, we get,

$9x + 12y = 30$ (vii)

$8x - 12y = 4$ (viii)

Adding (vii) and (viii) we get

$$\begin{array}{rcrcl} 9x & + & 12y & = & 30 \\ +\ 8x & - & 12y & = & 4 \\ \hline 17x & & & = & 34 \end{array}$$

$\therefore \ x = \frac{34}{17}$

$\therefore \ \mathbf{x = 2}$

Substituting $x = 2$ in equation (vi) we get,

$3x + 4y = 10$

$\therefore \ 3(2) + 4y = 10$

$\therefore \ 6 + 4y = 10$

$\therefore \ 4y = 10 - 6$

$\therefore \ 4y = 4$

$\therefore \ y = \frac{4}{4}$

$\therefore \ \mathbf{y = 1}$

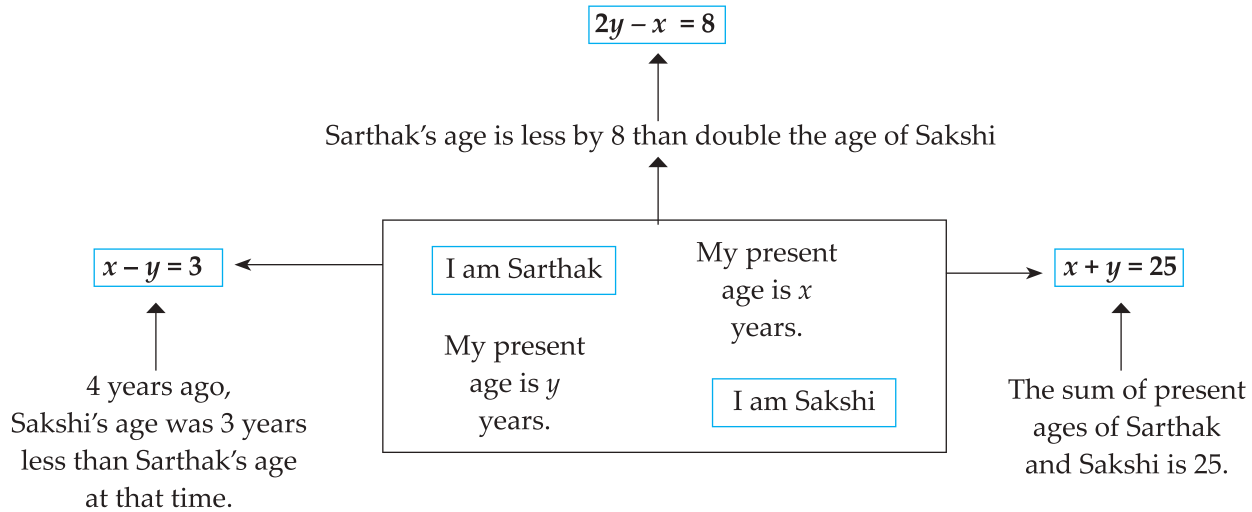
$\therefore$ **$x = 2$ and $y = 1$ is the solution of given simultaneous equations.**

Points to Remember:

- **Application of simultaneous equations:**

  Use of simultaneous equations is common in science and mathematics. Thus, the study of linear equations, system of linear equations and related concepts has its own importance. Let us see how the simultaneous equations are used in solving word problems.

  Activity: There are some instructions given below. Frame the equations from the information and write them in the blank boxes shown by arrows. *(Textbook Page No. 20)*



Practice Set - 1.5 *(Textbook Page No. 26)*

(1) Two numbers differ by 3. The sum of twice the smaller number and thrice the greater number is 19. Find the numbers. *(3 marks)*

Solution:

Let the greater number be x and the smaller number be y.

According to the first condition

$x - y = 3$ ...(i)

Three times the greater number = $3x$ and twice the smaller number = $2y$

According to the second condition

$3x + 2y = 19$ ...(ii)

Multiplying (i) by 2 we get,

$2x - 2y = 6$ ...(iii)

Adding (ii) and (iii)

$$\begin{array}{rcrcr} 3x & + & 2y & = & 19 \\ 2x & - & 2y & = & 6 \\ \hline 5x & & & = & 25 \end{array}$$

$\therefore\ x = \dfrac{25}{5}$

$\therefore\ \mathbf{x = 5}$

Substituting the value of x in equation (i)

$x - y = 3$

$\therefore\ 5 - y = 3$

$\therefore\ -y = 3 - 5$

$\therefore\ -y = -2$

$\therefore\ \mathbf{y = 2}$

$\therefore$ **The greater number is 5 and smaller number is 2.**

(4) The denominator of a fraction is 4 more than twice the numerator. Denominator becomes 12 times the numerator, if both the numerator and denominator are reduced by 6. Find the fraction. *(3 marks)*

Solution:

Let the numerator of the fraction be x and the denominator be y.

$\therefore$ The fraction is $\dfrac{x}{y}$

According to first condition,

$y = 2x + 4$

$\therefore\ -2x + y = 4$ ...(i)

According to second condition,

$(y - 6) = 12\,(x - 6)$

$\therefore\ y - 6 = 12x - 72$

$\therefore\ -6 + 72 = 12x - y$

$\therefore\ 66 = 12x - y$

$\therefore\ 12x - y = 66$ ...(ii)

Adding (i) and (ii) we get,

$$\begin{array}{rcrcr} -2x & + & y & = & 4 \\ 12x & - & y & = & 66 \\ \hline 10x & & & = & 70 \end{array}$$

$\therefore\ x = \dfrac{70}{10}$

$\therefore\ \mathbf{x = 7}$

Substituting $x = 7$ in equation (i) we get

$-2x + y = 4$

$\therefore\ -2(7) + y = 4$

$\therefore\ -14 + y = 4$

$\therefore\ y = 4 + 14$

$\therefore\ \mathbf{y = 18}$

$\therefore \quad \frac{x}{y} = \frac{7}{18}$

$\therefore$ **The required fraction is $\frac{7}{18}$**

Problem Set - 1 *(Textbook Page No. 28)*

(7) Solve the following problems using two variables.

(i) A two digit number and the number with digits interchanged add up to 143. In the given number the digit in unit's place is 3 more than the digit in the ten's place. Find the original number. *(4 marks)*

Solution:

Let the digit in units place is x and that in tens place is y.

$\therefore$ The number $= \boxed{10}\, y + x$

$\therefore$ The number obtained by interchanging the digits is $= \boxed{10}\, x + y$

According to first condition,

Two digit number + Number obtained by interchanging the digits = 143

$10y + x + \boxed{10x + y} = 143$

$\boxed{11}\, x + \boxed{11}\, y = 143$

$\therefore \quad x + y = \boxed{13}$ ...(i)

According to second condition,

Digit in units place = Digit in ten's place + 3

$\therefore \quad x = \boxed{y} + 3$

$\therefore \quad x - y = 3$ ...(ii)

Adding (i) and (ii) we get

$2x = \boxed{16}$

$x = 8$

Putting this value at x in (ii) we get,

$\therefore \quad x + y = 13$

$\therefore \quad 8 + \boxed{y} = 13$

$\therefore \quad y = 5$

$\therefore$ **The Original number is $10y + x$**

$= \boxed{50} + 8 = 58$

(iii) To find number of notes that Anushka had, complete the following activity. *(3 marks)*

Suppose that Anushka had $\boxed{x}$ notes of ₹ 100 and $\boxed{y}$ notes of ₹ 50 each

Anushka got ₹ 2500/- from Anand as denominations mentioned above

$\therefore$ $100x + 50y = 2500$ ...(i)

If Anand would have given her the amount by interchanging number of notes, Anushka would have received ₹ 500 less than the previous amount

$\therefore$ $50x + 100y = 2000$ ...(ii)

Solution:

$100x + 50y = 2500$ ...(i)

$50x + 100y = 2000$ ...(ii)

Multiplying (ii) by 2 we get,

$100x + 200y = 4000$ ...(iii)

Subtracting equation (iii) from equation (i)

$$\begin{array}{rcrcr} 100x & + & 50y & = & 2500 \\ 100x & + & 200y & = & 4000 \\ - & & - & & - \\ \hline & & -150y & = & -1500 \\ & & 150y & = & 1500 \end{array}$$

$\therefore \quad y = \frac{1500}{150}$

$\therefore \quad \mathbf{y = 10}$

Substituting $y = 10$ in equation (i) we get,

$100x + 50y = 2500$

$\therefore \quad 100x + 50(10) = 2500$

$\therefore \quad 100x + 500 = 2500$

$\therefore \quad 100\,x = 2500 - 500$

$\therefore \quad 100\,x = 2000$

$\therefore \quad x = \frac{2000}{100}$

$\therefore$ $x = 20$

$\therefore$ **Number of 100 rupees notes is 20 and number of 50 rupees notes is 10**

(ii) Kantabai bought $1\frac{1}{2}$ kg tea and 5 kg sugar from a shop. She paid ₹ 50 as return fare for rickshaw. Total expense was ₹ 700. Then she realised that by ordering online the goods can be bought with free home delivery at the same price. So next month she placed the order online for 2 kg tea and 7 kg sugar. She paid ₹ 880 for that. Find the rate of sugar and tea per kg. *(4 marks)*

Solution:

Let the cost of 1 kg. of tea be ₹ x and 1 kg. of Sugar be ₹ y

Amount paid as Rickshaw fare = ₹ 50

$\therefore$ Total Amount spent by Kantabai = ₹ 700

$\therefore$ Total cost of tea and sugar = ₹ 700 – ₹ 50

= ₹ 650

According to first condition,

$\frac{3}{2}x + 5y = 650$

Multiplying both sides by 2

$\therefore$ $3x + 10y = 1300$ ...(i)

According to second condition,

$2x + 7y = 880$ ...(ii)

Multiplying (i) by 2 and (ii) by 3 we get,

$6x + 20y = 2600$ ...(iii)

$6x + 21y = 2640$ ...(iv)

Subtracting equation (iv) from equation (iii) we get,

$$\begin{array}{rcrcr} 6x & + & 20y & = & 2600 \\ 6x & + & 21y & = & 2640 \\ - & & - & & - \\ \hline & & -y & = & -40 \end{array}$$

$\therefore$ $y = 40$

Substituting $y = 40$ in (ii) we get,

$2x + 7y = 880$

$\therefore$ $2x + 7(40) = 880$

$\therefore$ $2x + 280 = 880$

$\therefore$ $2x = 880 - 280$

$\therefore$ $2x = 600$

$\therefore$ $x = \frac{600}{2}$

$\therefore$ $\mathbf{x = 300}$

$\therefore$ **The cost of 1 Kg. tea is ₹ 300 and 1 Kg. Sugar is ₹ 40**

(v) In a factory the ratio of salary of skilled and unskilled workers is 5 : 3. Total salary of one day of both of them is ₹ 720. Find daily wages of skilled and unskilled workers. *(4 marks)*

Solution:

Let the per day salary of skilled workers be ₹ x and per day salary of unskilled workers be ₹ y

According to the first condition

$\frac{x}{y} = \frac{5}{3}$

$3x = 5y$

$3x - 5y = 0$ ...(i)

According to the second condition

$x + y = 720$ ...(ii)

Multiplying (ii) by 5 we get

$5x + 5y = 3600$ ...(iii)

Adding (i) and (iii) we get,

$$\begin{array}{rcrcr} 3x & - & 5y & = & 0 \\ 5x & + & 5y & = & 3600 \\ \hline 8x & & & = & 3600 \end{array}$$

$\therefore$ $x = \frac{3600}{8}$

$\therefore$ $\mathbf{x = 450}$

Substituting $x = 450$ equation (ii) we get,

$x + y = 720$

$\therefore$ $450 + y = 720$

$\therefore$ $y = 720 - 450$

$\therefore$ $\mathbf{y = 270}$

$\therefore$ **Daily wages of skilled workers is ₹ 450 and that of unskilled worker is ₹ 270**

Practice Set - 1.5 *(Textbook Page No. 26)*

(5) Two types of boxes A, B are to be placed in a truck having capacity of 10 tons. When 150 boxes of type A and 100 boxes of type B are loaded in the truck, it weighes 10 tons. But when 260 boxes of type A are loaded in the truck, it can still accommodate 40 boxes of type B, so that it is fully loaded. Find the weight of each type of box? *(4 marks)*

Solution:

1 ton = 1000 kg.

$\therefore$ 10 ton = 10,000 kg.

$\therefore$ Capacity of truck = 10 ton = 10,000 kg.

Let the weight of type A box be x kg. and the weight of each of type B box be y kg.

According to the first condition,

$$150x + 100y = 10{,}000$$

$\therefore$ $15x + 10y = 1000$...(i) [Dividing both sides by 10]

According to the second condition,

$$260x + 40y = 10{,}000$$

Dividing throughout by 4,

$\therefore$ $65x + 10y = 2500$ ...(ii)

Subtracting (i) from (ii),

$$\begin{array}{cccccc} & 65x & + & 10y & = & 2500 \\ & 15x & + & 10y & = & 1000 \\ (-) & & (-) & & (-) & \\ \hline & 50x & & & = & 1500 \end{array}$$

$\therefore$ $x = \dfrac{1500}{50}$

$\therefore$ $x = 30$

Substituting $x = 30$ in (i) we get,

$15x + 10y = 1$

$\therefore$ $15\,(30) + 10y = 1000$

$\therefore$ $450 + 10y = 1000$

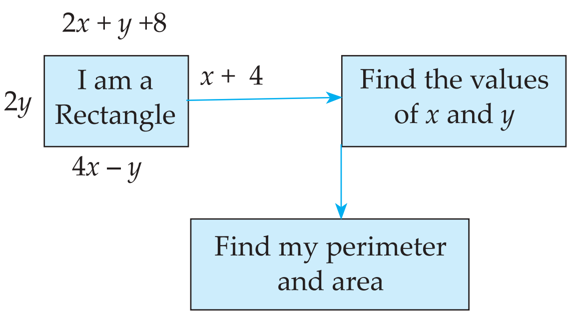
$\therefore$ $10y = 1000 - 450$

$\therefore$ $10y = 550$

$\therefore$ $y = 55$

$\therefore$ **The weight of type A box is 30 kg and the weight of type B box is 55 kg**

(2) Complete the following activity. *(4 marks)*



Solution:

In a rectangle, opposite sides are equal in length.

$\therefore$ $2x + y + 8 = 4x - y$

$\therefore$ $2x - 4x + y + y = -8$

$\therefore$ $-2x + 2y = -8$

$\therefore$ $2(-x + y) = -8$

$\therefore$ $-x + y = \dfrac{-8}{2}$

$\therefore$ $-x + y = -4$ ...(i)

Also, $2y = x + 4$

$\therefore$ $-4 = x - 2y$

i.e. $x - 2y = -4$ ...(ii)

Adding (i) and (ii) we get,

$$\begin{array}{cccccc} - & x & + & y & = & -4 \\ + & x & - & 2y & = & -4 \\ \hline & & - & y & = & -8 \end{array}$$

$\therefore$ $\boldsymbol{y = 8}$

Substituting $y = 8$ in (i) we get,

$-x + 8 = -4$

$\therefore$ $-x = -4 - 8$

$\therefore$ $-x = -12$

$\therefore$ $\boldsymbol{x = 12}$

$\therefore$ The length of rectangle $= 2x + y + 8$

$= 2(12) + 8 + 8$

$= 24 + 16$

$= 40$ units

The breadth of rectangle $= 2y$

$= 2 \times 8$

$= 16$ units

$\therefore$ Perimeter of rectangle = 2 [length + breadth]

$= 2\,[\,40 + 16\,]$

$= 2 \times 56$

$\therefore$ **Perimeter of rectangle = 112 units**

Area of rectangle = length $\times$ breadth

$= 40 \times 16$

$\therefore$ **Area of rectangle = 640 Sq. units**

(3) The sum of father's age and twice the age of his son is 70. If we double the age of the father and add it to the age of his son the sum is 95. Find their present ages. *(3 marks)*

Solution:

Let the age of son be x years and the age of father be y years.

According to the first condition,

$2x + y = 70$ ...(i)

According to the second condition,

$x + 2y = 95$ ...(ii)

Multiplying (i) by 2 we get,

$4x + 2y = 140$ ...(iii)

Subtracting (iii) from (ii)

$$\begin{array}{cccccc} & x & + & 2y & = & 95 \\ & 4x & + & 2y & = & 140 \\ (-) & & (-) & & (-) & \\ \hline & -3x & & & = & -45 \end{array}$$

$\therefore \quad x = \frac{-45}{-3}$

$\therefore \quad \mathbf{x = 15}$

Substituting $x = 15$ in (i), we get,

$2x + y = 70$

$\therefore \quad 2(15) + y = 70$

$\therefore \quad 30 + y = 70$

$\therefore \quad y = 70 - 30$

$\therefore \quad \mathbf{y = 40}$

$\therefore$ **The age of son is 15 years and the age of father is 40 years.**

Problem Set - 1 *(Textbook Page No. 29)*

(7) Solve the following problems using two variables.

(iv) Sum of the present ages of Manish and Savita is 31. Manish's age 3 years ago was 4 times the age of Savita. Find their present ages. *(3 marks)*

Solution:

Let the he present age of Manish be x years and age of Savita y years.

According to the first condition

$x + y = 31 \qquad \text{...(i)}$

Three years ago,

Manish's Age = $(x - 3)$ years and

Savita's age = $(y - 3)$ years

According to second condition,

$x - 3 = 4\,(y - 3)$

$\therefore \quad x - 3 = 4y - 12$

$\therefore \quad x - 4y = -12 + 3$

$\therefore \quad x - 4y = -9 \qquad \text{...(ii)}$

Subtracting equation (ii) from equation (i) we get,

$$\begin{array}{rcrcr} x & + & y & = & 31 \\ x & - & 4y & = & -9 \\ (-) & & (+) & & (+) \\ \hline & & 5y & = & 40 \end{array}$$

$\therefore \quad y = \frac{40}{5}$

$\therefore \quad \mathbf{y = 8}$

Substituting $y = 8$ in equation (i) we get,

$\therefore \quad x + y = 31$

$\therefore \quad x + 8 = 31$

$\therefore \quad x = 31 - 8$

$\therefore \quad \mathbf{x = 23}$

$\therefore$ **The present age of Manisha is 23 years and the present age of Savita is 8 years.**

(vi) Places A and B are 30 km apart and they are on a straight road. Hamid travels from A to B on bike. At the same time Joseph starts from B on bike, travels towards A. They meet each other after 20 minutes. If Joseph would have started from B at the same time but in the opposite direction (instead of towards A) Hamid would have caught up with him after 3 hours. Find the speed of Hamid and Joseph. *(4 marks)*

Solution:

Let the speed at which Hamid is driving his motocycle be x km/hr. and the speed at which Joseph is driving his motocycle be y km/hr. $[x > y]$

Case I : When they move towards each other.

In this case, both motorcycles are moving towards each other and they meet at C after 20

minutes i.e. $\frac{20}{60} = \frac{1}{3}$ hours

Then AC + BC = AB

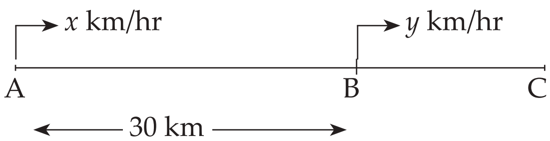
i.e. distance covered by

Hamid in $\frac{1}{3}$ hours + distance covered by Joseph in $\frac{1}{3}$ hours = 30 km.

$\therefore \quad \frac{1}{3}x + \frac{1}{3}y = 30 \quad$ [Distance = speed × time]

$\therefore \quad x + y = 90 \qquad \text{...(i)}$

Case II : When Joseph move in opposite direction. (Instead of towards A)



As shown in figure, both are moving in the same direction and they meet at C after 3 hours

Then AC – BC = AB

i.e. Distance covered by Hamid in 3 hours – distance covered by Joseph in 3 hours = 30km.

$3x - 3y = 30 \qquad$ [Distance = speed × time]

$x - y = 10 \qquad \text{...(ii)}$

Adding (i) and (ii) we get,

$$\begin{array}{rcrcr} x & + & y & = & 90 \\ x & - & y & = & 10 \\ \hline 2x & & & = & 100 \end{array}$$

$\therefore \quad x = \dfrac{100}{2}$

$\therefore \quad \mathbf{x = 50}$

Substituting $x = 50$ in (i) we get

$\therefore \quad x + y = 90$

$\therefore \quad 50 + y = 90$

$\therefore \quad y = 90 - 50$

$\therefore \quad \mathbf{y = 40}$

$\therefore$ **Speed of Hamids motorcycle = 50 km/hr. and Speed of Joseph's Motorcycle is 40 Km/hr.**

Practice Set - 1.5 *(Textbook Page No. 26)*

(6) Out of 1900 km, Vishal travelled some distance by bus and some by aeroplane. Bus travels with average speed 60 km/hr and the average speed of aeroplane is 700 km/hr. It takes 5 hours to complete the journey. Find the distance, Vishal travelled by bus. *(4 marks)*

Solution:

Let the distance travelled by bus be x km and the distance travelled by aeroplane be y km.

$\therefore$ According to the first condition.

$x + y = 1900$ ...(i)

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time taken by bus to cover x km at 60 km/hr

$= \dfrac{x}{60}$ hours

Time taken by aeroplane to cover y km at 700 km/hr.

$= \dfrac{y}{700}$ hours

The journey was completed in 5 hours

$\therefore$ According to second condition,

$$\frac{x}{60} + \frac{y}{700} = 5$$

Multiplying both the sides of above equation by 2100 i.e. LCM of 60 and 70,

$\therefore \quad 35x + 3y = 10500$ ...(ii)

Multiplying equation (i) by 3 we get,

$\therefore \quad 3x + 3y = 5700$ ...(iii)

Subtracting (iii) from (ii)

$$\begin{array}{rcrcr} 35x & + & 3y & = & 10500 \\ 3x & + & 3y & = & 5700 \\ (-) & & (-) & & (-) \\ \hline 32x & & & = & 4800 \end{array}$$

$\therefore \quad x = \dfrac{4800}{32}$

$\therefore \quad \mathbf{x = 150}$

$\therefore$ **Vishal travelled 150 km by bus.**

Problem Set - 1 *(Textbook Page No. 29)*

MCQ's

Choose correct alternative for each of the following questions: *(1 mark each)*

(1) To draw graph of $4x + 5y = 19$, Find y when $x = 1$.

(A) 4 (B) 3 (C) 2 (D) -3

(2) For simultaneous equations in variables x and y, $D_x = 49$, $D_y = -63$, $D = 7$ then what is x ?

(A) 7 (B) -7 (C) $\dfrac{1}{7}$ (D) $\dfrac{-1}{7}$

(3) Find the value of $\begin{vmatrix} 5 & 3 \\ -7 & -4 \end{vmatrix}$

(A) -1 (B) -41 (C) 41 (D) 1

(4) To solve $x + y = 3$; $3x - 2y - 4 = 0$ by determinant method find D.

(A) 5 (B) 1 (C) -5 (D) -1

(5) $ax + by = c$ and $mx + ny = d$ and an $\neq$ bm then these simultaneous equations have -

(A) Only one common solution.

(B) No solution

(C) Infinite number of solutions.

(D) Only two Solutions

Additional MCQ's

(6) The general form of linear equation in two variables is ....

(A) $ax + b = 0$ (B) $ax + by = c$

(C) $ax^2 + bx + c = 0$ (D) None of these

(7) .... is one of the solution of equation $3x - 5y = 10$.

(A) (0, 2) (B) (2, 0) (C) (–2, 0) (D) (0, –2)

(8) If $12x + 13y = 29$ and $13x + 12y = 21$ then the value of $x + y$ is

(A) 1 (B) 25 (C) 2 (D) 50

(9) Express the following information in mathematical form using x and y variables: one number is 5 more than seven times the other number .

(A) $x - 5y = 7$ (B) $x - 7y = 5$

(C) $x + 7y = 5$ (D) $x - 7y = -5$

(10) Write D_x for the following simultaneous equations. $3x + 4y = 8$; $x - 2y = 5$

(A) $\begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix}$ (B) $\begin{vmatrix} 8 & 4 \\ 5 & -2 \end{vmatrix}$

(C) $\begin{vmatrix} 4 & 8 \\ -2 & 5 \end{vmatrix}$ (D) $\begin{vmatrix} 3 & 8 \\ 1 & 5 \end{vmatrix}$

(11) .... is the solution of given simultaneous equations. $x - y = 7$, $x + y = 11$.

(A) (–3, –8) (B) (–9, –2)

(C) (9, 2) (D) (6, 5)

(12) When we consider two linear equations in two variables, such equations are called as .... .

(A) simultaneous equations

(B) linear equations

(C) quadratic equations

(D) non-linear equations.

(13) Which is not a linear equation in two variables ....

(A) $x + 7y = 1$ (B) $3x + 4y - xy = 0$

(C) $3x + 9 = 4y - 1$ (D) $3x = 4y$

(14) Which is linear equation in two variables ....

(A) $3x + 9 = \sqrt{2}\,y + 2$ (B) $3x - 4y + xy = 0$

(C) $2m - 8 = 4m$ (D) $3x - 14 = 9$

(15) Select pair of simultaneous equations from the following ....

(1) $2x + 2y = 7$ (2) $4x + 3z = 9$

(3) $3y + 4z = 8$ (4) $3z + 9x = 18$

(A) 1 and 2 (B) 2 and 3

(C) 3 and 4 (D) 2 and 4

(16) Equation of X axis is ....

(A) $x = 0$ (B) $x = b$ (C) $y = 0$ (D) $y = a$

(17) The co-ordinates of the point of origin are ....

(A) (0, 0) (B) (1, 0) (C) (0, 1) (D) (1, 1)

(18) If the value of the determinant $\begin{vmatrix} m & -2 \\ 2 & 1 \end{vmatrix}$ is 7, then value of m is ....

(A) – 3 (B) 3 (C) – 7 (D) 7

(19) The perimeter of a rectangle is 64, is expressed in the mathematical equation form as ....

(A) $\frac{1}{2}(x+y) = 64$ (B) $2(x + y) = 64$

(C) $x \times y = 64$ (D) $\frac{xy}{2} = 64$

(20) The value of determinant $\begin{vmatrix} 5 & 2 \\ 7 & 4 \end{vmatrix}$ is ....

(A) 6 (B) – 6 (C) 34 (D) – 34

(21) If $D_x = -18$ and $D = 3$ are values of determinant for certain simultaneous equation in x and y then value of x is ....

(A) + 6 (B) – 15 (C) – 6 (D) + 15

(22) Find m if value of determinant $\begin{vmatrix} m & 2 \\ -5 & 7 \end{vmatrix}$ is 31.

(A) 14 (B) 3 (C) 28 (D) 21

(23) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the two simultaneous equations have ....

(A) one solution (B) no solution

(C) infinitely many solutions (D) two solutions

(24) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the two simultaneous equations have ....

(A) one solution (B) no solution

(C) infinitely many solutions (D) two solutions

(25) If $(a, 3)$ is point lying on graph of equation $5x + 2y = -4$ then a = ...

(A) 2 (B) – 2 (C) $\frac{2}{5}$ (D) $\frac{-2}{5}$

(26) Without solving simultaneous equation decide $3x + 5y = 16$ and $4x - y = 6$ have ....

(A) unique solution (B) no solution

(C) infinitely many solutions (D) none of these

(27) If simultaneous equations do not have any solution then their graphical interpretation contains ....

(A) parallel lines (B) coincident lines

(C) intersecting lines (D) can't determine

(28) By Cramer's rule the value of x is ....

(A) $\frac{D}{D_x}$ (B) $\frac{D_x}{D}$ (C) $\frac{D_y}{D}$ (D) $\frac{D}{D_y}$

ANSWERS

(1) (B) 3 **(2)** (A) 7 **(3)** (D) 1 **(4)** (C) – 5

(5) (A) Only one common solution **(6)** (B) $ax + by = c$

(7) (D) (0, –2) **(8)** (C) 2 **(9)** (B) $x - 7y = 5$

(10) (B) $\begin{vmatrix} 8 & 4 \\ 5 & -2 \end{vmatrix}$ **(11)** (C) (9, 2)

(12) (A) simultaneous equations **(13)** (B) $3x + 4y - xy = 0$

(14) (A) $3x + 9 = \sqrt{2}\,y + 2$ **(15)** (D) 2 and 4

(16) (C) $y = 0$ **(17)** (A) (0, 0) **(18)** (B) 3

(19) (B) $2(x + y) = 64$ **(20)** (A) 6 **(21)** (C) –6

(22) (B) 3 **(23)** (C) infinitely many solutions

(24) (B) no solution **(25)** (B) -2

(26) (A) unique solution **(27)** (A) parallel lines

(28) (B) $\dfrac{D_x}{D}$

PROBLEMS FOR PRACTICE

Based on Practice Set 1.1

(1) Solve the following simultaneous equations:

(i) $3x - y = 2 ;\ 5x - 2y = 1$ *(2 marks)*

(ii) $47x + 31y = 63 ; 31x + 47y = 15$ *(3 marks)*

(iii) $4m + 3n = 18 ; 3m - 2n = 5$ *(2 marks)*

(iv) $2x - 3y = 14 ; 5x + 2y = 16$ *(2 marks)*

(v) $\frac{1}{3}x + 5y = 13 \ ; \ 2x + \frac{1}{2}y = 19$ *(3 marks)*

(vi) $\frac{1}{3}x + \frac{1}{4}y = 4 \ ; \ \frac{5}{6}x - \frac{1}{8}y = 4$ *(3 marks)*

(vii) $64m - 45n = 289 ; 45m - 64n = 365$ *(3 marks)*

Based on Practice Set 1.2

(2) Solve the following simultaneous equations using Graphical method: *(4 marks each)*

(i) $x + y = 8 ;\ x - y = 2$

(ii) $3x + 4y = -5 ; x - y = -4$

(iii) $x + 3y = 7 ; 2x + y = -1$

(iv) $x + 2y = 5 ; 2x + y = -2$

(v) $4x - y = -5 ; 2x - y = -1$

Based on Practice Set 1.3

(3) Find the value of following determinants: *(1 mark each)*

(i) $\begin{vmatrix} 5 & -2 \\ -3 & 1 \end{vmatrix}$ **(ii)** $\begin{vmatrix} -3 & 8 \\ 6 & 0 \end{vmatrix}$ **(iii)** $\begin{vmatrix} \frac{1}{2} & \frac{-2}{3} \\ \frac{3}{4} & \frac{-4}{5} \end{vmatrix}$

(4) Solve the following simultaneous equations using Cramer's method: *(4 marks each)*

(i) $3x - 2y = 3; 2x + y = 16$ **(ii)** $x + 2y + 4 = 0; 3x = -4y - 16$

(iii) $3x - y = 7;\ x + 4y = 11$ **(iv)** $3x + y = 1;\ 2x = 11y + 3$

(v) $4x + 3y = 4 \ ; \ 6x + 5y = 8$

Based on Practice Set 1.4

(5) Solve the following simultaneous equations: *(4 marks each)*

(i) $\frac{4}{x} + \frac{3}{y} = 1 ; \frac{8}{x} - \frac{9}{y} = 7$

(ii) $\frac{7}{2x+1} + \frac{13}{y+2} = 27 ; \frac{13}{2x+1} + \frac{7}{y+2} = 33$

(iii) $\frac{14}{x+y} + \frac{3}{x-y} = 5 ; \frac{21}{x+y} - \frac{2}{x-y} = 1$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2 ; \frac{6}{x-1} - \frac{3}{y-2} = 1$

Based on Practice Set 1.5

(6) Solve the following simultaneous equations:

(i) Shabana's age 10 years hence, will be twice juhi's present age. 6 years back shabana's age was $\frac{5}{3}$ times Juhi's at that time find their present ages. *(3 marks)*

(ii) If 1 is added to the numerator of a certain fraction its value becomes $\frac{1}{2}$ and if 1 is added to its denometer $\frac{1}{3}$. Find the original fraction. *(3 marks)*

(iii) Sum of two number is 45 and the greater number is twice the smaller number. Find the numbers. *(3 marks)*

(iv) A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But, If he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car. *(4 marks)*

ANSWERS

(1) (i) $x = 3 ,\ y = 7$ (ii) $x = 2,\ y = -1$ (iii) $m = 3 ,\ n = 2$ (iv) $x = 4,\ y = -2$ (v) $x = 9 ,\ y = 2$ (vi) $x = 6,\ y = 8$ (vii) $m = 1 ,\ n = -5$

(2) (i) $x = 5 ,\ y = 3$ (ii) $x = -3,\ y = 1$ (iii) $x = -2, y = 3$ (iv) $x = -3,\ y = 4$ (v) $x = -2 ,\ y = -3$

(3) (i) -1 (ii) -48 (iii) $\frac{1}{10}$

(4) (i) $x = 5, y = 6$ (ii) $x = -8, y = 2$ (iii) $x = 3 ,\ y = 2$ (iv) $x = \frac{2}{5},\ y = \frac{-1}{5}$, (v) $x = -2, y = 4$

(5) (i) $x = 2,\ y = -3$ (ii) $x = \frac{-1}{4}, y = -1$ (iii) $x = 4, y = 3$ (iv) $x = 4\ \ y = 5$

(6) (i) 26 years, 18 years (ii) $\frac{3}{8}$ (iii) 30, 15 (iv) 100 km/hr and 80 km/hr.

❖ ❖ ❖

ASSIGNMENT – 1

Time : 1 Hr. **Marks : 20**

Q.1. (A) Choose the correct alternative answer and fill in the blanks: **(2)**

(1) Select pair of simultaneous equations from the following ....

(1) $2x + 2y = 7$ (2) $4x + 3z = 9$ (3) $3y + 4z = 8$ (4) $3z + 9x = 18$

(2) $ax + by = c$ and $mx + ny = d$ and an $\neq$ bm then these simultaneous equations have ....

(A) Only one common solution. (B) No Solution

(C) Infinite number of solutions (D) Only two Solutions

(B) Perform the following question: **(3)**

(1) Write D_x for the following simultaneous questions: $5x + 2y = 10$; $-3x + y = -11$

Solution : $5x + 2y = 10$; $-3x + y = -11$ $\therefore$ $D_x = \left|\quad\right|$

(2) Find the value of the following determinant: $\begin{vmatrix} 5 & 7 \\ 2 & 4 \end{vmatrix}$

Solution: $\begin{vmatrix} 5 & 7 \\ 2 & 4 \end{vmatrix}$

$= 5 \times \square - \square \times 7 = \square - 14 = \square$

(3) If $3x + 4y = -10$; $4x + 3y = 3$; then find the value of x + y.

Solution:

$3x + 4y = -10$ ... (i) $4x + 3y = 3$ ...(ii)

Adding (i) and (ii)

$$\begin{array}{rrrrrr} & 3x & + & 4y & = & 10 \\ + & 4x & + & 3y & = & 3 \\ \hline & 7x & + & 7y & = & \square \end{array}$$

$\therefore$ $x + y = \square$

Q.2. Perform the following activities: (Any 2) **(4)**

(1) Complete the following table to draw graph for equation $x + 2y = 5$

x	0	$\square$
y		-2
(x, y)	$\square$	$\square$

(2) Solve the following simultaneous equations.

$x + y = 8$, $x - y = 2$

$x + y = 8$ ...(i)

$x - y = 2$ ...(ii)

Adding (i) and (ii)

$$\begin{array}{rrrrr} & x & +y & = & 8 \\ + & x & -y & = & 2 \\ \hline \therefore & \square & x & = & \square \end{array}$$

$\therefore$ $x = \square$

Substituting the value of x in (i)

$\therefore$ $\square + y = 8$

$\therefore$ $y = \square$

(3) Find the value of the following determinats:

(i) $\begin{vmatrix} \frac{7}{8} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix}$ (ii) $\begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix}$

Q.3. Perform the following activities: (Any 1) **(3)**

(1) The perimeter of rectangle is 40 cm. The length of rectangle is 2cm more than twice its breadth then find the length and the breadth of rectangle.

Complete the following activity.

Rectangle y cm

x cm

Perimeter of rectangle = 40 cm $\therefore x + y = \square$ ...(i)

According to 2nd condition $\square - \square = \square$ ...(ii)

After solving (i) and (ii) $x = \square$ and $y = \square$

$\therefore$ Length = ... cm and Breadth = ... cm

(2) Solve: $15x + 17y = 21$; $17x + 15y = 11$

Complete the following activity.

$15x + 17y = 21$ ....(i)

$17x + 15y = 11$ ...(ii)

Adding (i) and (ii) we get,

$\square + \square = \square$ ...(iii)

Subtracting (ii) from (i) we get,

$\square - \square = \square$ ...(iv)

After solving (iii) and (iv) $x = \square$ and $y = \square$

$\therefore (x, y) = (\quad)$ is the solution of given simultaneous equations.

Q.4. Attempt the following: (Any 2) **(8)**

(1) A boat takes 6 hours to travel 8 km up stream and 32 km down stream and it takes 7 hours to travel 20 km upstream and 16 km downstream. Find the speed of the boat in still water and the speed of the stream.

(2) Solve the following simultaneous equations using Graphical method: $2x + 3y = 12$; $x - y = 1$

(3) The sum of two digit number and the number obtained by reversing the order of its digits is 143. The digit at ten's place is greater than digit of units place by 3 then find the original number.

❖ ❖ ❖

2 Quadratic Equations

... INDEX ...

Points to Remember:

- **Standard form of the quadratic equation:**

  The equation $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$ is called **quadratic equation** in variable x.

  The equation $ax^2 + bx + c = 0$ is called standard form of quadratic equation.

  The highest index of variable is 2. So, it is a second degree equation in one variable.

Activity:

Complete the following table:

Quadratic equation	Standard form	a	b	c
$x^2 - 4 = 0$	$x^2 + 0x - 4 = 0$	1	0	-4
$y^2 = 2y - 7$	$\mathbf{y^2 - 2y + 7 = 0}$	**1**	**-2**	**7**
$x^2 + 2x = 0$	$\mathbf{x^2 + 2x + 0 = 0}$	**1**	**2**	**0**

- **Which of the following are quadratic equations?** *(Textbook page no. 31)*

(i) $3x^2 - 5x + 3 = 0$

Solution:

$3x^2 - 5x + 3 = 0$

Here $\boldsymbol{x}$ is the only variable and maximum index of variable x is **2**.

$\therefore$ **It is** a quadratic equation.

(ii) $9y^2 + 5 = 0$

Solution:

$9y^2 + 5 = 0$

Here $\boxed{y}$ is the only variable and maximum index of variable y is $\boxed{2}$.

$\therefore$ **It** $\boxed{\textbf{is}}$ a quadratic equation.

(iii) $m^3 - 5m^2 + 4 = 0$

Solution:

$m^3 - 5m^2 + 4 = 0$

Here, $\boxed{m}$ is the only variable but maximum index of variable m is not **2**.

$\therefore$ **It** $\boxed{\textbf{is not}}$ a quadratic equation.

(iv) $(l + 2)(l - 5) = 0$

Solution:

$(l + 2)(l - 5) = 0$

$\therefore$ $l(l - 5) + 2(l - 5) = 0$

$\therefore$ $l^2 - 5l + 2l - 10 = 0$

$\therefore$ $l^2 - 3l - 10 = 0$

Here, $\boxed{l}$ is the only variable and maximum index of variable l is $\boxed{2}$.

$\therefore$ **It** $\boxed{\textbf{is}}$ a quadratic equation.

MASTER KEY QUESTION SET - 2

Practice Set - 2.1 *(Textbook Page No. 34)*

(1) Write any two quadratic equations. *(1 mark)*

Solution:

$x^2 + 3x + 4 = 0$ and $2x^2 - 5x + 7 = 0$ are quadratic equations in variable x.

(2) Decide which of the following are quadratic equations.

(i) $x^2 + 5x - 2 = 0$ *(1 mark)*

Solution:

$x^2 + 5x - 2 = 0$

Here, x is the only variable and maximum index of variable x is 2.

$\therefore$ **It is a quadratic equation in variable x.**

(ii) $y^2 = 5y - 10$ *(1 mark)*

Solution:

$y^2 = 5y - 10$

$\therefore$ $y^2 - 5y + 10 = 0$

Here, y is the only variable and maximum index of variable y is 2.

$\therefore$ **It is a quadratic equation in variable y.**

(iii) $y^2 + \frac{1}{y} = 2$ *(1 mark)*

Solution:

$y^2 + \frac{1}{y} = 2$

Multiplying both sides by y we get,

$y^3 + 1 = 2y$

i.e. $y^3 - 2y + 1 = 0$

Here maximum index of the variable y is 3 and not 2

$\therefore$ **It is not a quadratic equation.**

(iv) $x + \frac{1}{x} = -2$ *(1 mark)*

Solution:

$x + \frac{1}{x} = -2$

Multiplying both sides by x we get,

$x^2 + 1 = -2x$

i.e. $x^2 + 2x + 1 = 0$

Here, x is the only variable and maximum index of variable x is 2.

$\therefore$ **It is a quadratic equation in variable x.**

(v) $(m + 2)(m - 5) = 0$ *(1 mark)*

Solution:

$(m + 2)(m - 5) = 0$

$\therefore$ $m(m - 5) + 2(m - 5) = 0$

$\therefore$ $m^2 - 5m + 2m - 10 = 0$

$\therefore$ $m^2 - 3m - 10 = 0$

Here, m is the only variable and maximum index of variable m is 2.

$\therefore$ **It is a quadratic equation in variable m.**

(vi) $m^3 + 3m^2 - 2 = 3m^3$ *(1 mark)*

Solution:

$m^3 + 3m^2 - 2 = 3m^3$

$\therefore$ $3m^3 - m^3 - 3m^2 + 2 = 0$

$\therefore$ $2m^3 - 3m^2 + 2 = 0$

Here, maximum index of the variable m is 3 and not 2

$\therefore$ **It is not a quadratic equation in variable m.**

Problem Set - 2 *(Textbook Page No. 54)*

(2) Which of the following are quadratic equations?

(i) $x^2 + 2x + 11 = 0$ *(1 mark)*

Solution:

$x^2 + 2x + 11 = 0$

Here, x is the only variable and maximum index of variable x is 2.

$\therefore$ **It is a quadratic equation in variable x.**

(ii) $x^2 - 2x + 5 = x^2$ *(1 mark)*

Solution:

$x^2 - 2x + 5 = x^2$

$\therefore$ $x^2 - x^2 - 2x + 5 = 0$

$\therefore$ $-2x + 5 = 0$

Here, maximum index of variable x is 1 and not 2.

$\therefore$ **It is not a quadratic equation.**

(iii) $(x + 2)^2 = 2x^2$ *(1 mark)*

Solution:

$(x + 2)^2 = 2x^2$ [Using $(a + b)^2 = a^2 + 2ab + b^2$]

$\therefore$ $x^2 + 4x + 4 = 2x^2$

$\therefore$ $2x^2 - x^2 - 4x - 4 = 0$

$\therefore$ $x^2 - 4x - 4 = 0$

Here, x is the only variable and maximum index of variable x is 2.

$\therefore$ **It is a quadratic equation in variable x.**

Practice Set - 2.1 *(Textbook Page No. 34)*

(3) Write the following equations in the form of $ax^2 + bx + c = 0$, then write the values of a, b, c for each equation.

(i) $2y = 10 - y^2$ *(1 mark)*

Solution:

$2y = 10 - y^2$

$\therefore$ $y^2 + 2y - 10 = 0$

Comparing with $ay^2 + by + c = 0$ we get,

$a = 1, b = 2, c = -10$

(ii) $(x-1)^2 = 2x + 3$ *(1 mark)*

Solution:

$(x-1)^2 = 2x + 3$

$\therefore$ $x^2 - 2x + 1 = 2x + 3$ ...[Using $(a-b)^2 = a^2 - 2ab + b^2$]

$\therefore$ $x^2 - 2x - 2x + 1 - 3 = 0$

$\therefore$ $x^2 - 4x - 2 = 0$

Comparing with $ax^2 + bx + c = 0$ we get,

$a = 1, b = -4, c = -2$

(iii) $x^2 + 5x = -(3-x)$ *(1 mark)*

Solution:

$x^2 + 5x = -(3-x)$

$\therefore$ $x^2 + 5x = -3 + x$

$\therefore$ $x^2 + 5x - x + 3 = 0$

$\therefore$ $x^2 + 4x + 3 = 0$

Comparing with $ax^2 + bx + c = 0$ we get,

$a = 1, b = 4, c = 3$

(iv) $3m^2 = 2m^2 - 9$ *(1 mark)*

Solution:

$3m^2 = 2m^2 - 9$

$\therefore$ $3m^2 - 2m^2 + 9 = 0$

$\therefore$ $m^2 + 9 = 0$

$\therefore$ $m^2 + 0m + 9 = 0$

Comparing with $am^2 + bm + c = 0$ we get,

$a = 1, b = 0, c = 9$

(v) $p(3 + 6p) = -5$ *(1 mark)*

Solution:

$p(3 + 6p) = -5$

$\therefore$ $3p + 6p^2 = -5$

$\therefore$ $6p^2 + 3p + 5 = 0$

Comparing with $ap^2 + bp + c = 0$ we get,

$a = 6, b = 3, c = 5$

(vi) $x^2 - 9 = 13$ *(1 mark)*

Solution:

$x^2 - 9 = 13$

$\therefore$ $x^2 - 9 - 13 = 0$

$\therefore$ $x^2 - 22 = 0$

$\therefore$ $x^2 + 0x - 22 = 0$

Comparing with $ax^2 + bx + c = 0$ we get,

$a = 1, b = 0, c = -22$

Points to Remember:

Roots of a quadratic equation:

The values of the variable which satisfy the equation [or the value for which both the sides of equation are equal] are called solutions or the **roots of the equations.**

- **Determine whether**

(i) $x = \frac{3}{2}$ and

(ii) x = –2 are the roots of the quadratic equation $2x^2 - 7x + 6 = 0$

Solution:

(i) By substituting $x = \frac{3}{2}$ in LHS we get,

$$\text{LHS} = 2\left(\frac{3}{2}\right)^2 - 7\left(\frac{3}{2}\right) + 6$$

$$= 2 \times \frac{9}{4} - \frac{21}{2} + 6$$

$$= \frac{9}{2} - \frac{21}{2} + 6$$

$$= \frac{-12}{2} + 6$$

$$= -6 + 6$$

$$= 0$$

$\therefore$ LHS = RHS

$\therefore$ The given equation is satisfied.

$\therefore$ $x = \frac{3}{2}$ is the root of the given quadratic equation.

(ii) By substituting $x = -2$ in LHS, we get,

$$\text{LHS} = 2(-2)^2 - 7(-2) + 6$$

$$= 2 \times 4 + 14 + 6$$

$$= 8 + 20$$

$$= 28$$

$\therefore$ LHS $\neq$ RHS

$\therefore$ $x = -2$ is not the root of the given quadratic equation.

Activity: *(Textbook page no. 33)*

If $x = 5$ is a root of the quadratic equation $kx^2 - 14x - 5 = 0$, then find the value of k by completing the following activities.

Solution:

One of the roots of equation $kx^2 - 14x - 5 = 0$ is $\boxed{5}$

Substituting $x = \boxed{5}$ in given equation.

$\therefore$ $k\boxed{5}^2 - 14\boxed{5} - 5 = 0.$

$\therefore$ $25k - 70 - 5 = 0.$

$\therefore$ $25k - \boxed{75} = 0.$

$\therefore$ $25k = \boxed{75}$.

$\therefore$ $k = \frac{\boxed{75}}{\boxed{25}} = 3.$

Practice Set - 2.1 *(Textbook Page No. 34)*

(4) Determine whether the values given against each of the quadratic equation are the roots of the equation.

(i) $x^2 + 4x - 5 = 0, x = 1, -1$ *(2 marks)*

Solution:

$x^2 + 4x - 5 = 0$

(a) By substituting $x = 1$ in L.H.S., we get,

$$\begin{aligned} \text{LHS} &= (1)^2 + 4(1) - 5 \\ &= 1 + 4 - 5 \\ &= 5 - 5 \\ &= 0 \end{aligned}$$

∴ LHS = RHS

∴ The given equation is satisfied.

∴ $\boxed{x = 1 \text{ is the root of the given quadratic equation.}}$

(b) By substituting $x = -1$ in L.H.S., we get,

$$\begin{aligned} \text{LHS} &= (-1)^2 + 4(-1) - 5 \\ &= 1 - 4 - 5 \\ &= -8 \end{aligned}$$

∴ LHS ≠ RHS

∴ $\boxed{x = -1 \text{ is not the root of the given quadratic equation.}}$

(ii) $2m^2 - 5m = 0, m = 2, \frac{5}{2}$ *(2 marks)*

Solution:

$2m^2 - 5m = 0$

(a) By substituting $m = 2$ in L.H.S., we get,

$$\begin{aligned} \text{LHS} &= 2(2)^2 - 5(2) \\ &= 2 \times 4 - 10 \\ &= 8 - 10 \\ &= -2 \end{aligned}$$

∴ LHS ≠ RHS

∴ $\boxed{m = 2 \text{ is not the root of the given quadratic equation.}}$

(b) By substituting $m = \frac{5}{2}$ in LHS we get,

$$\begin{aligned} \text{LHS} &= 2\left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) \\ &= 2 \times \frac{25}{4} - \frac{25}{2} \\ &= \frac{25}{2} - \frac{25}{2} \\ &= 0 \end{aligned}$$

∴ LHS = RHS

∴ The given equation is satisfied.

∴ $\boxed{m = \frac{5}{2} \text{ is the root of the given quadratic equation.}}$

(5) Find k if $x = 3$ is a root of equation $kx^2 - 10x + 3 = 0$. *(2 marks)*

Solution:

x = **3** is one of the roots of the given quadratic equation.

∴ It satisfies the given equation.

Substituting $x = 3$ in given quadratic equation.

∴ $k(3)^2 - 10(3) + 3 = 0.$

∴ $9k - 30 + 3 = 0.$

∴ $9k - 27 = 0.$

∴ $9k = 27.$

∴ $k = \frac{27}{9}.$

∴ $\boxed{k = 3.}$

(6) One of the roots of equation $5m^2 + 2m + k = 0$ is $-\frac{7}{5}$. Complete the following activity to find the value of 'k'. *(2 marks)*

Solution:

$\boxed{-\frac{7}{5}}$ is the root of the equation $5m^2 + 2m + k = 0$

∴ Put $m = \boxed{-\frac{7}{5}}$ in the equation,

∴ $5 \times \boxed{\left(-\frac{7}{5}\right)^2} + 2 \times \boxed{-\frac{7}{5}} + k = 0$

∴ $\boxed{\frac{49}{5}} + \boxed{\left(\frac{-14}{5}\right)} + k = 0$

∴ $\boxed{7} + k = 0$

∴ $k = \boxed{-7}$

Problem Set - 2 *(Textbook Page No. 54)*

(4) One of the roots of quadratic equation $2x^2 + kx - 2 = 0$ is -2, find k. *(2 marks)*

Solution:

$x = -2$ is the root of the given quadratic equation.

∴ It satisfies the given equation.

Substituting $x = -2$ in given equation,

∴ $2(-2)^2 + k(-2) - 2 = 0$

∴ $2 \times 4 - 2k - 2 = 0$

∴ $8 - 2k - 2 = 0$

∴ $6 - 2k = 0$

∴ $2k = 6$

∴ $k = \frac{6}{2}$

∴ $\boxed{k = 3.}$

Points to Remember:

Methods for solving quadratic equations:

(i) Factorisation method
(ii) Completing square method
(iii) Formula method

(I) Solution of a quadratic equation by factorisation:

- If the product of two numbers is zero, then at least one of them must be zero,

  i.e. If $a \times b = 0$, then either $a = 0$ or $b = 0$
- To solve a quadratic equation by factorisation, first write the equation in the form of $ax^2 + bx + c = 0$, then find the factors of it.

  If $(px + q)$ and $(rx + t)$ are the factors of $ax^2 + bx + c$, then the given quadratic equation will be

  $(px + q)(rx + t) = 0$

$\therefore$ $px + q = 0$ or $rx + t = 0$

$\therefore$ $x = \dfrac{-q}{p}$ or $x = \dfrac{-t}{r}$

$\therefore$ The roots of given quadratic equation are $\dfrac{-q}{p}$ and $\dfrac{-t}{r}$

Practice Set - 2.2 *(Textbook Page No. 36)*

(1) Solve the following quadratic equations by factorization.

(i) $x^2 - 15x + 54 = 0$ *(2 marks)*

Solution:

$x^2 - 15x + 54 = 0$

$\therefore$ $x^2 - 9x - 6x + 54 = 0$

$\therefore$ $x(x - 9) - 6(x - 9) = 0$

$\therefore$ $(x - 9)(x - 6) = 0$

$\therefore$ $x - 9 = 0$ or $x - 6 = 0$

$\therefore$ $x = 9$ or $x = 6$

$\therefore$ **The roots of given quadratic equation are 9 and 6.**

(ii) $x^2 + x - 20 = 0$ *(2 marks)*

Solution:

$x^2 + x - 20 = 0$

$\therefore$ $x^2 + 5x - 4x - 20 = 0$

$\therefore$ $x(x + 5) - 4(x + 5) = 0$

$\therefore$ $(x + 5)(x - 4) = 0$

$\therefore$ $x + 5 = 0$ or $x - 4 = 0$

$\therefore$ $x = -5$ or $x = 4$

$\therefore$ **The roots of given quadratic equation are -5 and 4.**

(iii) $2y^2 + 27y + 13 = 0$ *(2 marks)*

Solution:

$2y^2 + 27y + 13 = 0$

$\therefore$ $2y^2 + 26y + y + 13 = 0$

$\therefore$ $2y(y + 13) + 1(y + 13) = 0$

$\therefore$ $(y + 13)(2y + 1) = 0$

$\therefore$ $y + 13 = 0$ or $2y + 1 = 0$

$\therefore$ $y = -13$ or $2y = -1$

$\therefore$ $y = -13$ or $y = \dfrac{-1}{2}$

$\therefore$ **The roots of given quadratic equation are -13 and $\dfrac{-1}{2}$.**

(iv) $5m^2 = 22m + 15$ *(2 marks)*

Solution:

$5m^2 = 22m + 15$

$\therefore$ $5m^2 - 22m - 15 = 0$

$\therefore$ $5m^2 - 25m + 3m - 15 = 0$

$\therefore$ $5m(m - 5) + 3(m - 5) = 0$

$\therefore$ $(m - 5)(5m + 3) = 0$

$\therefore$ $m - 5 = 0$ or $5m + 3 = 0$

$\therefore$ $m = 5$ or $5m = -3$

$\therefore$ $m = 5$ or $m = \dfrac{-3}{5}$

$\therefore$ **The roots of given quadratic equation are 5 and $\dfrac{-3}{5}$.**

(v) $2x^2 - 2x + \dfrac{1}{2} = 0$ *(3 marks)*

Solution:

$2x^2 - 2x + \dfrac{1}{2} = 0$

Multiplying throughout by 2, we get

$4x^2 - 4x + 1 = 0$

$\therefore$ $4x^2 - 2x - 2x + 1 = 0$

$\therefore$ $2x(2x - 1) - 1(2x - 1) = 0$

$\therefore$ $(2x - 1)(2x - 1) = 0$

$\therefore$ $2x - 1 = 0$ or $2x - 1 = 0$

$\therefore$ $2x = 1$ or $2x = 1$

$\therefore$ $x = \dfrac{1}{2}$ or $x = \dfrac{1}{2}$.

$\therefore$ **The root of given quadratic equation are $\dfrac{1}{2}$ and $\dfrac{1}{2}$.**

(vi) $6x - \dfrac{2}{x} = 1$ *(2 marks)*

Solution:

$6x - \dfrac{2}{x} = 1$

Multiplying both the sides by x, we get

$6x^2 - 2 = x$

$\therefore$ $6x^2 - x - 2 = 0$

$\therefore$ $6x^2 - 4x + 3x - 2 = 0$

$\therefore$ $2x(3x - 2) + 1(3x - 2) = 0$

$\therefore$ $(3x - 2)(2x + 1) = 0$

$\therefore$ $3x - 2 = 0$ or $2x + 1 = 0$

$\therefore$ $3x = 2$ or $2x = -1$

$\therefore$ $x = \frac{2}{3}$ or $x = \frac{-1}{2}$

$\therefore$ **The roots of the given quadratic equation are $\frac{2}{3}$ and $\frac{-1}{2}$.**

(vii) **$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ to solve this quadratic equation by factorisation, complete the following activity.** *(3 marks)*

Solution:

$\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$

$\therefore$ $\sqrt{2}\,x^2 + \boxed{5x} + \boxed{2x} + 5\sqrt{2} = 0$

$\therefore$ $x(\sqrt{2}\,x + 5) + \sqrt{2}(\sqrt{2}\,x + 5) = 0$

$\therefore$ $(\sqrt{2}\,x + 5)(x + \sqrt{2}) = 0$

$\therefore$ $\sqrt{2}\,x + 5 = 0$ or $x + \sqrt{2} = 0$

$\therefore$ $x = \boxed{\frac{-5}{\sqrt{2}}}$ or $x = -\sqrt{2}$

$\therefore$ **$\boxed{\frac{-5}{\sqrt{2}}}$ and $-\sqrt{2}$ are roots of the equation.**

***(viii)** **$3x^2 - 2\sqrt{6}\,x + 2 = 0$** *(3 marks)*

Solution:

$3x^2 - 2\sqrt{6}\,x + 2 = 0$

$\therefore$ $3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$

$\therefore$ $\sqrt{3}\,x(\sqrt{3}\,x - \sqrt{2}) - \sqrt{2}(\sqrt{3}\,x - \sqrt{2}) = 0$

$\therefore$ $(\sqrt{3}\,x - \sqrt{2})(\sqrt{3}\,x - \sqrt{2}) = 0$

$\therefore$ $\sqrt{3}\,x - \sqrt{2} = 0$ or $\sqrt{3}\,x - \sqrt{2} = 0$

$\therefore$ $x = \frac{\sqrt{2}}{\sqrt{3}}$ or $x = \frac{\sqrt{2}}{\sqrt{3}}$

$\therefore$ **The root of given quadratic equation are $\frac{\sqrt{2}}{\sqrt{3}}$ and $\frac{\sqrt{2}}{\sqrt{3}}$.**

(ix) **$2m(m - 24) = 50$** *(3 marks)*

Solution:

$2m(m - 24) = 50$

$\therefore$ $2m^2 - 48m = 50$

$\therefore$ $2m^2 - 48m - 50 = 0$

$\therefore$ $m^2 - 24m - 25 = 0$ ....(Dividing both sides by 2)

$\therefore$ $m^2 - 25m + m - 25 = 0$

$\therefore$ $m(m - 25) + 1(m - 25) = 0$

$\therefore$ $(m - 25)(m + 1) = 0$

$\therefore$ $m - 25 = 0$ or $m + 1 = 0$

$\therefore$ $m = 25$ or $m = -1$

$\therefore$ **The roots of given quadratic equation are 25 and –1.**

(x) **$25m^2 = 9$** *(2 marks)*

Solution:

$25m^2 = 9$

$\therefore$ $25m^2 - 9 = 0$

$\therefore$ $(5m)^2 - (3)^2 = 0$

$\therefore$ $(5m + 3)(5m - 3) = 0$ ..$[\because a^2 - b^2 = (a + b)(a - b)]$

$\therefore$ $5m = -3$ or $5m = 3$

$\therefore$ $m = \frac{-3}{5}$ or $m = \frac{3}{5}$

$\therefore$ **The roots of given quadratic equation are $\frac{3}{5}$ and $\frac{-3}{5}$.**

(xii) **$m^2 - 11 = 0$** *(2 marks)*

Solution:

$m^2 - 11 = 0$

$\therefore$ $m^2 - \left(\sqrt{11}\right)^2 = 0$

$\therefore$ $(m + \sqrt{11})(m - \sqrt{11}) = 0$ ..$[\because a^2 - b^2 = (a + b)(a - b)]$

$\therefore$ $m + \sqrt{11} = 0$ or $m - \sqrt{11} = 0$

$\therefore$ $m = -\sqrt{11}$ or $m = \sqrt{11}$

$\therefore$ **The roots of given quadratic equation are $-\sqrt{11}$ and $\sqrt{11}$.**

(xi) **$7m^2 = 21m$** *(2 marks)*

Solution:

$7m^2 = 21m$

$\therefore$ $7m^2 - 21m = 0$

$\therefore$ $7m(m - 3) = 0$

$\therefore$ $7m = 0$ or $m - 3 = 0$

$\therefore$ $m = \frac{0}{7}$ or $m = 3$

$\therefore$ $m = 0$ or $m = 3$

$\therefore$ **The roots of given quadratic equation are 0 and 3.**

Problem Set - 2 *(Textbook Page No. 54)*

(7) **Solve the following quadratic equations:**

(iii) **$(2x + 3)^2 = 25$** *(3 marks)*

Solution:

$(2x + 3)^2 = 25$

$\therefore$ $(2x + 3)^2 - 25 = 0$

$\therefore$ $(2x + 3)^2 - (5)^2 = 0$

$\therefore$ $(2x + 3 + 5)(2x + 3 - 5) = 0$

.....$[\because a^2 - b^2 = (a + b)(a - b)]$

$\therefore$ $(2x + 8)(2x - 2) = 0$

$\therefore$ $2x + 8 = 0$ or $2x - 2 = 0$

$\therefore$ $2x = -8$ or $2x = 2$

$\therefore$ $x = \frac{-8}{2}$ or $x = \frac{2}{2}$

$\therefore$ $x = -4$ or $x = 1$

$\therefore$ **The roots of given quadratic equation are –4 and 1.**

(ii) $x^2 - \frac{3x}{10} - \frac{1}{10} = 0$

Solution:

$x^2 - \frac{3x}{10} - \frac{1}{10} = 0$

Multiplying both sides by 10 we get,

$10x^2 - 3x - 1 = 0$

$\therefore$ $10x^2 - 5x + 2x - 1 = 0$

$\therefore$ $5x(2x - 1) + 1(2x - 1) = 0$

$\therefore$ $(2x - 1)(5x + 1) = 0$

$\therefore$ $2x - 1 = 0$ or $5x + 1 = 0$

$\therefore$ $2x = 1$ or $5x = -1$

$\therefore$ $x = \frac{1}{2}$ or $x = \frac{-1}{5}$

$\therefore$ **The roots of given quadratic equation are $\frac{1}{2}$ and $\frac{-1}{5}$.**

Points to Remember:

(II) Solution of a quadratic equation by completing the square:

(Assuming the variable involved in equation to be x)

Step 1 : Check the coefficient of x^2, it has to be 1. If not, then make it 1 by dividing each term by the coefficient of x^2.

Step 2 : Find the term to be added by using the formula $= \left[\frac{1}{2} \times \textit{coefficient of } x\right]^2$

Step 3 : Add and subtract the term found in step 2, due to which the addition of first three terms becomes a complete square.

Step 4 : Express first three term as a complete square and send remaining two constant terms to R.H.S.

Step 5 : Take square roots on both sides.

Step 6 : Find the value of x and write the final answer.

Practice Set - 2.3 *(Textbook Page No. 39)*

Solve the following quadratic equations by completing the square method.

(1) $x^2 + x - 20 = 0$ *(3 marks)*

Solution:

$x^2 + x - 20 = 0$

Term to be added

$= \left[\frac{1}{2} \times \textit{coefficient of } x\right]^2$

$= \left(\frac{1}{2} \times 1\right)^2$

$= \left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$

$\therefore$ $x^2 + x + \frac{1}{4} - \frac{1}{4} - 20 = 0$

$\therefore$ $\left(x + \frac{1}{2}\right)^2 = \frac{1}{4} + 20$

$\therefore$ $\left(x + \frac{1}{2}\right)^2 = \frac{1 + 80}{4}$

$\therefore$ $\left(x + \frac{1}{2}\right)^2 = \frac{81}{4}$

$\therefore$ $x + \frac{1}{2} = \pm \frac{9}{2}$ ... (Taking square root)

$\therefore$ $x + \frac{1}{2} = \frac{9}{2}$ or $x + \frac{1}{2} = -\frac{9}{2}$

$\therefore$ $x = \frac{9}{2} - \frac{1}{2}$ or $x = -\frac{9}{2} - \frac{1}{2}$

$\therefore$ $x = \frac{9 - 1}{2}$ or $x = \frac{-9 - 1}{2}$

$\therefore$ $x = \frac{8}{2}$ or $x = \frac{-10}{2}$

$\therefore$ $x = 4$ or $x = -5$

$\therefore$ **The roots of given quadratic equation are 4 and –5.**

(2) $x^2 + 2x - 5 = 0$ *(3 marks)*

Solution:

$x^2 + 2x - 5 = 0$

Term to be added

$= \left[\frac{1}{2} \times \textit{coefficient of } x\right]^2$

$= \left(\frac{1}{2} \times 2\right)^2$

$= 1$

$\therefore$ $x^2 + 2x + 1 - 1 - 5 = 0$

$\therefore \quad (x+1)^2 = 1+5$

$\therefore \quad (x+1)^2 = 6$

$\therefore \quad x+1 = \pm\sqrt{6}$ ... (Taking square root)

$\therefore \quad x+1 = \sqrt{6}$ or $x+1 = -\sqrt{6}$

$\therefore \quad x = -1+\sqrt{6}$ or $x = -1-\sqrt{6}$

$\therefore$ **The roots of given quadratic equation are $-1+\sqrt{6}$ and $-1-\sqrt{6}$.**

(3) $m^2 - 5m = -3$ *(3 marks)*

Solution:

$m^2 - 5m = -3$

Term to be added

$= \left[\frac{1}{2} \times \textit{coefficient of } m\right]^2$

$= \left(\frac{1}{2} \times (-5)\right)^2$

$= \left(\frac{-5}{2}\right)^2$

$= \frac{25}{4}$

$\therefore \quad m^2 - 5m + 3 = 0$

$\therefore \quad m^2 - 5m + \frac{25}{4} - \frac{25}{4} + 3 = 0$

$\therefore \quad \left(m - \frac{5}{2}\right)^2 = \frac{25}{4} - 3$

$\therefore \quad \left(m - \frac{5}{2}\right)^2 = \frac{25-12}{4}$

$\therefore \quad \left(m - \frac{5}{2}\right)^2 = \frac{13}{4}$

$\therefore \quad m - \frac{5}{2} = \pm\frac{\sqrt{13}}{2}$ ... (Taking square root)

$\therefore \quad m - \frac{5}{2} = \frac{\sqrt{13}}{2}$ or $m - \frac{5}{2} = -\frac{\sqrt{13}}{2}$

$\therefore \quad m = \frac{5}{2} + \frac{\sqrt{13}}{2}$ or $m = \frac{5}{2} - \frac{\sqrt{13}}{2}$

$\therefore \quad m = \frac{5+\sqrt{13}}{2}$ or $m = \frac{5-\sqrt{13}}{2}$

$\therefore$ **The roots of given quadratic equation are $\frac{5+\sqrt{13}}{2}$ and $\frac{5-\sqrt{13}}{2}$.**

(4) $9y^2 - 12y + 2 = 0$ *(3 marks)*

Solution:

$9y^2 - 12y + 2 = 0$

Dividing both sides by 9, we get,

$y^2 - \frac{12}{9}y + \frac{2}{9} = 0$

$\therefore \quad y^2 - \frac{4}{3}y + \frac{2}{9} = 0$

Term to be added

$= \left[\frac{1}{2} \times \textit{coefficient of } y\right]^2$

$= \left(\frac{1}{2} \times \left(\frac{-4}{3}\right)\right)^2$

$= \left(\frac{-2}{3}\right)^2$

$= \frac{4}{9}$

$\therefore \quad y^2 - \frac{4}{3}y + \frac{4}{9} - \frac{4}{9} + \frac{2}{9} = 0$

$\therefore \quad \left(y - \frac{2}{3}\right)^2 = \frac{4}{9} - \frac{2}{9}$

$\therefore \quad \left(y - \frac{2}{3}\right)^2 = \frac{4-2}{9}$

$\therefore \quad \left(y - \frac{2}{3}\right)^2 = \frac{2}{9}$

$\therefore \quad y - \frac{2}{3} = \pm\frac{\sqrt{2}}{3}$ ... (Taking square root)

$\therefore \quad y - \frac{2}{3} = \frac{\sqrt{2}}{3}$ or $y - \frac{2}{3} = -\frac{\sqrt{2}}{3}$

$\therefore \quad y = \frac{2}{3} + \frac{\sqrt{2}}{3}$ or $y = \frac{2}{3} - \frac{\sqrt{2}}{3}$

$\therefore \quad y = \frac{2+\sqrt{2}}{3}$ or $y = \frac{2-\sqrt{2}}{3}$

$\therefore$ **The roots of given quadratic equation are $\frac{2+\sqrt{2}}{3}$ and $\frac{2-\sqrt{2}}{3}$.**

(5) $2y^2 + 9y + 10 = 0$ *(3 marks)*

Solution:

$2y^2 + 9y + 10 = 0$

Dividing both sides by 2 we get,

$y^2 + \frac{9}{2}y + \frac{10}{2} = 0$

$\therefore \quad y^2 + \frac{9}{2}y + 5 = 0$

Term to be added

$= \left[\frac{1}{2} \times \textit{coefficient of } y\right]^2$

$= \left(\frac{1}{2} \times \frac{9}{2}\right)^2$

$= \left(\frac{9}{4}\right)^2$

$= \frac{81}{16}$

$\therefore \quad y^2 + \frac{9}{2}y + \frac{81}{16} - \frac{81}{16} + 5 = 0$

$\therefore \quad \left(y+\frac{9}{4}\right)^2 = \frac{81}{16} - 5$

$\therefore \quad \left(y+\frac{9}{4}\right)^2 = \frac{81-80}{16}$

$\therefore \quad \left(y+\frac{9}{4}\right)^2 = \frac{1}{16}$

$\therefore \quad y + \frac{9}{4} = \pm\frac{1}{4}$ ... (Taking square root)

$\therefore \quad y + \frac{9}{4} = \frac{1}{4}$ or $y + \frac{9}{4} = -\frac{1}{4}$

$\therefore \quad y = -\frac{9}{4} + \frac{1}{4}$ or $y = -\frac{9}{4} - \frac{1}{4}$

$\therefore \quad y = \frac{-9+1}{4}$ or $y = \frac{-9-1}{4}$

$\therefore \quad y = \frac{-8}{4}$ or $y = \frac{-10}{4}$

$\therefore \quad y = -2$ or $y = \frac{-5}{2}$

$\therefore$ **The roots of given quadratic equation are -2 and $\frac{-5}{2}$.**

(6) $5x^2 = 4x + 7$ *(3 marks)*

Solution:

$5x^2 = 4x + 7$

$\therefore \quad 5x^2 - 4x - 7 = 0$

Dividing both sides by 5 we get,

$x^2 - \frac{4}{5}x - \frac{7}{5} = 0$

Term to be added

$= \left[\frac{1}{2} \times \textit{coefficient of } x\right]^2$

$= \left(\frac{1}{2} \times \frac{-4}{5}\right)^2$

$= \left(\frac{-2}{5}\right)^2$

$= \frac{4}{25}$

$\therefore \quad x^2 - \frac{4}{5}x + \frac{4}{25} - \frac{4}{25} - \frac{7}{5} = 0$

$\therefore \quad \left(x-\frac{2}{5}\right)^2 = \frac{4}{25} + \frac{7}{5}$

$\therefore \quad \left(x-\frac{2}{5}\right)^2 = \frac{4+35}{25}$

$\therefore \quad \left(x-\frac{2}{5}\right)^2 = \frac{39}{25}$

$\therefore \quad x - \frac{2}{5} = \pm\frac{\sqrt{39}}{5}$ ... [Taking square root]

$\therefore \quad x - \frac{2}{5} = \frac{\sqrt{39}}{5}$ or $x - \frac{2}{5} = -\frac{\sqrt{39}}{5}$

$\therefore \quad x = \frac{2}{5} + \frac{\sqrt{39}}{5}$ or $x = \frac{2}{5} - \frac{\sqrt{39}}{5}$

$\therefore \quad x = \frac{2+\sqrt{39}}{5}$ or $x = \frac{2-\sqrt{39}}{5}$

$\therefore$ **The roots of given quadratic equation are $\frac{2+\sqrt{39}}{5}$ and $\frac{2-\sqrt{39}}{5}$.**

Points to Remember:

Formula for solving a quadratic equation:

$ax^2 + bx + c = 0$, Here, $a \neq 0$.

Dividing both sides by a, we get,

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0.$

Term to be added

$= \left[\frac{1}{2} \times \textit{coefficient of } x\right]^2$

$= \left(\frac{1}{2} \times \frac{b}{a}\right)^2$

$= \left(\frac{b}{2a}\right)^2$

$= \frac{b^2}{4a^2}$

$\therefore \quad x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0.$

$\therefore \quad \left(x+\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

$\therefore \quad \left(x+\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$

$\therefore \quad \left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$\therefore \quad x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$

$\therefore \quad x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

$\therefore \quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

The roots of quadratic equation are denoted by α (Alpha) and β (Beta).

$\therefore \quad \alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

If we put value of a, b, c of equation $ax^2+bx+c=0$ in $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, then we get the roots of the equation.

$\therefore \quad x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ is the formula for solving quadratic equation.

Activity:

Solve the equation $2x^2 + 13x + 15 = 0$ by factorisation method, by completing the square method and by using the formula. Verify that you will get the same roots every time.

Solution:

(i) By Factorisation method,

$2x^2 + 13x + 15 = 0$

$2x^2 + 10x + 3x + 15 = 0$

$2x(x + 5) + 3(x + 5) = 0$

$(x + 5)(2x + 3) = 0$

$(x + 5) = 0$ or $(2x + 3) = 0$

$\therefore \quad x = -5$ or $x = \frac{-3}{2}$

(ii) By Completing Square Method,

$2x^2 + 13x + 15 = 0$

Dividing both sides by 2,

$x^2 + \frac{13}{2}x + \frac{15}{2} = 0$

Term to be added

$= \left[\frac{1}{2}\times \textit{coefficient of } x\right]^2$

$= \left[\frac{1}{2}\times\frac{13}{2}\right]^2$

$= \left(\frac{13}{4}\right)^2$

$= \frac{169}{16}$

$= \quad x^2 + \frac{13}{2}x + \frac{169}{16} - \frac{169}{16} + \frac{15}{2} = 0$

$\therefore \quad \left(x+\frac{13}{4}\right)^2 = \frac{169}{16} - \frac{15}{2}$

$\therefore \quad \left(x+\frac{13}{4}\right)^2 = \frac{49}{16}$

$\therefore \quad x + \frac{13}{4} = \pm\frac{7}{4}$ *(taking square root)*

$\therefore \quad x + \frac{13}{4} = \frac{7}{4}$ *or* $x + \frac{13}{4} = -\frac{7}{4}$

$\therefore \quad x = -\frac{13}{4} + \frac{7}{4}$ *or* $x = -\frac{7}{4} - \frac{13}{4}$

$\therefore \quad x = \frac{-6}{4}$ *or* $x = \frac{-20}{4}$

$\therefore \quad x = \frac{-3}{2}$ *or* $x = -5$

(iii) By Formula

$2x^2 + 13x + 15 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$\boldsymbol{a = 2, b = 13, c = 15}$

$\therefore \quad b^2 - 4ac = (13)^2 - 4\times 2\times 15$

$= 169 - 120$

$= 49$

$\therefore \quad x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-13\pm\sqrt{49}}{2\times 2}$

$= \frac{-13\pm 7}{4}$

$\therefore \quad x = \frac{-13+7}{4}$ *or* $x = \frac{-13-7}{4}$

$\therefore \quad x = \frac{-6}{4}$ *or* $x = \frac{-20}{4}$

$\therefore \quad x = \frac{-3}{2}$ *or* $x = -5$

Using above three methods, we get the same roots as -5 and $\frac{-3}{2}$

Practice Set - 2.4 *(Textbook Page No. 43)*

(1) Compare the given quadratic equation to the general form and write values of a, b, c.

(i) $x^2 - 7x + 5 = 0$ *(1 mark)*

Solution:

$x^2 - 7x + 5 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$\boxed{a = 1, b = -7, c = 5}$

(ii) $2m^2 = 5m - 5$ *(1 mark)*

Solution:

$2m^2 = 5m - 5$

$\therefore \quad 2m^2 - 5m + 5 = 0$

Comparing with $am^2 + bm + c = 0$, we get,

$\boxed{a = 2, b = -5, c = 5}$

(iii) $y^2 = 7y$ *(1 mark)*

Solution:

$y^2 = 7y$

$\therefore$ $y^2 - 7y = 0$

$\therefore$ $y^2 - 7y + 0 = 0$

Comparing with $ay^2 + by + c = 0$, we get,

$\boxed{a = 1, b = -7, c = 0}$

(2) Solve using formula.

(i) $x^2 + 6x + 5 = 0$ *(3 marks)*

Solution:

$x^2 + 6x + 5 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = 1, b = 6, c = 5$

$\therefore$ $b^2 - 4ac = (6)^2 - 4 \times 1 \times 5$

$= 36 - 20$

$= 16$

$\therefore$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \dfrac{-6 \pm \sqrt{16}}{2 \times 1}$

$\therefore$ $x = \dfrac{-6 \pm 4}{2}$

$\therefore$ $x = \dfrac{-6 + 4}{2}$ or $x = \dfrac{-6 - 4}{2}$

$\therefore$ $x = \dfrac{-2}{2}$ or $x = \dfrac{-10}{2}$

$\therefore$ $x = -1$ or $x = -5$

$\therefore$ **The roots of given quadratic equation are –1 and –5.**

(ii) $x^2 - 3x - 2 = 0$ *(3 marks)*

Solution:

$x^2 - 3x - 2 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = 1, b = -3, c = -2.$

$\therefore$ $b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-2)$

$= 9 + 8$

$= 17$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \dfrac{-(-3) \pm \sqrt{17}}{2 \times 1}$

$\therefore$ $x = \dfrac{3 \pm \sqrt{17}}{2}$

$\therefore$ $x = \dfrac{3 + \sqrt{17}}{2}$ or $x = \dfrac{3 - \sqrt{17}}{2}$

$\therefore$ **The roots of given quadratic equation are $\dfrac{3 - \sqrt{17}}{2}$ and $\dfrac{3 + \sqrt{17}}{2}$.**

(iii) $3m^2 + 2m - 7 = 0$ *(3 marks)*

Solution:

$3m^2 + 2m - 7 = 0$

Comparing with $am^2 + bm + c = 0$, we get,

$a = 3, b = 2, c = -7.$

$\therefore$ $b^2 - 4ac = (2)^2 - 4 \times 3 \times (-7)$

$= 4 + 84$

$= 88$

$\therefore$ $m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ...(Formula)

$\therefore$ $m = \dfrac{-2 \pm \sqrt{88}}{2 \times 3}$

$\therefore$ $m = \dfrac{-2 \pm \sqrt{4 \times 22}}{6}$

$\therefore$ $m = \dfrac{-2 \pm 2\sqrt{22}}{6}$

$\therefore$ $m = \dfrac{2(-1 \pm \sqrt{22})}{6}$

$\therefore$ $m = \dfrac{-1 \pm \sqrt{22}}{3}$

$\therefore$ $m = \dfrac{-1 + \sqrt{22}}{3}$ or $m = \dfrac{-1 - \sqrt{22}}{3}$

$\therefore$ **The roots of given quadratic equation are $\dfrac{-1 - \sqrt{22}}{3}$ and $\dfrac{-1 + \sqrt{22}}{3}$.**

(iv) $5m^2 - 4m - 2 = 0$ *(3 marks)*

Solution:

$5m^2 - 4m - 2 = 0$

Comparing with $am^2 + bm + c = 0$, we get,

$a = 5, b = -4, c = -2.$

$\therefore$ $b^2 - 4ac = (-4)^2 - 4 \times 5 \times (-2)$

$= 16 + 40$

$= 56$

$\therefore$ $m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ...(Formula)

$\therefore$ $m = \dfrac{-(-4) \pm \sqrt{56}}{2 \times 5}$

$\therefore$ $m = \dfrac{4 \pm \sqrt{4 \times 14}}{10}$

$\therefore$ $m = \dfrac{4 \pm 2\sqrt{14}}{10}$

$\therefore$ $m = \dfrac{2(2 \pm \sqrt{14})}{10}$

$\therefore$ $m = \dfrac{2 \pm \sqrt{14}}{5}$

$\therefore \quad m = \dfrac{2+\sqrt{14}}{5}$ or $m = \dfrac{2-\sqrt{14}}{5}$

$\therefore$ **The roots of given quadratic equation are $\dfrac{2-\sqrt{14}}{5}$ and $\dfrac{2+\sqrt{14}}{5}$.**

(v) $y^2 + \dfrac{1}{3}y = 2$ *(3 marks)*

Solution:

$y^2 + \dfrac{1}{3}y = 2$

$3y^2 + y = 6$ ....(Multiplying both sides by 3)

$\therefore \quad 3y^2 + y - 6 = 0$

Comparing with $ay^2 + by + c = 0$ we get,

$a = 3, b = 1, c = -6.$

$\therefore \quad b^2 - 4ac = (1)^2 - 4 \times 3 \times (-6)$

$= 1 + 72$

$= 73$

$\therefore \quad y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ... (Formula)

$\therefore \quad y = \dfrac{-1 \pm \sqrt{73}}{2 \times 3}$

$\therefore \quad y = \dfrac{-1 \pm \sqrt{73}}{6}$

$\therefore \quad y = \dfrac{-1+\sqrt{73}}{6}$ or $y = \dfrac{-1-\sqrt{73}}{6}$

$\therefore$ **The roots of given quadratic equation are $\dfrac{-1+\sqrt{73}}{6}$ and $\dfrac{-1-\sqrt{73}}{6}$.**

(vi) $5x^2 + 13x + 8 = 0$ *(3 marks)*

Solution:

$5x^2 + 13x + 8 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = 5, b = 13, c = 8$

$\therefore \quad b^2 - 4ac = (13)^2 - 4 \times 5 \times 8$

$= 169 - 160$

$= 9$

$\therefore \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ...(Formula)

$= \dfrac{-13 \pm \sqrt{9}}{2 \times 5}$

$= \dfrac{-13 \pm 3}{10}$

$\therefore \quad x = \dfrac{-13+3}{10}$ or $x = \dfrac{-13-3}{10}$

$\therefore \quad x = \dfrac{-10}{10}$ or $x = \dfrac{-16}{10}$

$\therefore \quad x = -1$ or $x = \dfrac{-8}{5}$

$\therefore$ **The roots of given quadratic equation are -1 and $\dfrac{-8}{5}$.**

Problem Set - 2 *(Textbook Page No. 54)*

(7) Solve the following quadratic equations:

(i) $\dfrac{1}{x+5} = \dfrac{1}{x^2}$ *(3 marks)*

Solution:

$\dfrac{1}{x+5} = \dfrac{1}{x^2} \quad (x \neq 0, x + 5 \neq 0)$

$\therefore \quad x + 5 = x^2$ .....(By Invertendo)

$\therefore \quad x^2 - x - 5 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = 1, b = -1, c = -5.$

$\therefore \quad b^2 - 4ac = (-1)^2 - 4 \times 1 \times -5$

$= 1 + 20$

$= 21$

$\therefore \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ...(Formula)

$= \dfrac{-(-1) \pm \sqrt{21}}{2 \times 1}$

$= \dfrac{1 \pm \sqrt{21}}{2}$

$\therefore \quad x = \dfrac{1+\sqrt{21}}{2}$ or $x = \dfrac{1-\sqrt{21}}{2}$

$\therefore$ **The roots of given quadratic equation are $\dfrac{1-\sqrt{21}}{2}$ and $\dfrac{1+\sqrt{21}}{2}$**

(iv) $m^2 + 5m + 5 = 0$ *(3 marks)*

Solution:

$m^2 + 5m + 5 = 0$

Comparing with $am^2 + bm + c = 0$, we get,

$a = 1, b = 5, c = 5.$

$\therefore \quad b^2 - 4ac = (5)^2 - 4 \times 1 \times 5$

$= 25 - 20$

$= 5$

$\therefore \quad m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ...(Formula)

$= \dfrac{-5 \pm \sqrt{5}}{2 \times 1}$

$= \dfrac{-5 \pm \sqrt{5}}{2}$

$\therefore \quad m = \dfrac{-5+\sqrt{5}}{2}$ or $m = \dfrac{-5-\sqrt{5}}{2}$

$\therefore$ **The roots of given quadratic equation are $\dfrac{-5+\sqrt{5}}{2}$ and $\dfrac{-5-\sqrt{5}}{2}$.**

(v) $5m^2 + 2m + 1 = 0$ *(3 marks)*

Solution:

$5m^2 + 2m + 1 = 0$

Comparing with $am^2 + bm + c = 0$, we get,

$a = 5, b = 2, c = 1.$

$$\begin{aligned} \therefore \quad b^2 - 4ac &= (2)^2 - 4 \times 5 \times 1 \\ &= 4 - 20 \\ &= -16 \end{aligned}$$

$\because$ **$b^2 - 4ac < 0$, roots of given quadratic equation are not real.**

(vi) $x^2 - 4x - 3 = 0$

Solution:

$x^2 - 4x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = 1, b = -4, c = -3.$

$$\begin{aligned} \therefore \quad b^2 - 4ac &= (-4)^2 - 4 \times 1 \times -3 \\ &= 16 + 12 \\ &= 28 \end{aligned}$$

$\therefore \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (Formula)

$\therefore \quad x = \dfrac{-(-4) \pm \sqrt{28}}{2 \times 1}$

$\therefore \quad x = \dfrac{4 \pm \sqrt{4 \times 7}}{2}$

$\therefore \quad x = \dfrac{4 \pm \sqrt{4 \times 7}}{2}$

$\therefore \quad x = \dfrac{4 \pm 2\sqrt{7}}{2}$

$\therefore \quad x = \dfrac{2(2 \pm \sqrt{7})}{2}$

$\therefore \quad x = 2 \pm \sqrt{7}$

$\therefore \quad x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$

$\therefore$ **The roots of given quadratic equation are $2+\sqrt{7}$ and $2-\sqrt{7}$.**

Practice Set - 2.4 *(Textbook Page No. 44)*

(3) With the help of the flow chart given below solve the equation $x^2 + 2\sqrt{3}\,x + 3 = 0$ using formula. *(3 marks)*

Compare equations $x^2 + 2\sqrt{3}\,x + 3 = 0$ and $ax^2 + bx + c = 0$ find the values of *a, b, c* → Find the value of $b^2 - 4ac$ → Write formula to solve quadratic equation → Substitute values of *a, b, c* and find roots

Solution:

$x^2 + 2\sqrt{3}\,x + 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = 1, b = 2\sqrt{3}, c = 3.$

$$\begin{aligned} \therefore \quad b^2 - 4ac &= (2\sqrt{3})^2 - 4 \times 1 \times 3 \\ &= 12 - 12 \\ &= 0 \end{aligned}$$

$\therefore \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ...(Formula)

$\therefore \quad x = \dfrac{-2\sqrt{3} \pm \sqrt{0}}{2 \times 1}$

$= \dfrac{-2\sqrt{3} \pm 0}{2}$

$\therefore \quad x = \dfrac{-2\sqrt{3}}{2}$

$\therefore \quad x = -\sqrt{3}$

$\therefore$ **The root of given quadratic equation is $-\sqrt{3}$.**

Points to Remember:

- **Nature of roots of a quadratic equation**

We know that the roots of the quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$ are

$\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$, $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

Here the nature of the roots of the quadratic equation is determined by the value of $b^2 - 4ac$, which is called as **discriminant** of the quadratic equation and it is denoted by Δ **(Delta)**.

(1) If $b^2 - 4ac > 0$, then $\sqrt{b^2 - 4ac}$ is positive real and therefore roots of equation are real and unequal.

e.g. $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$, $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

(2) If **$b^2 - 4ac = 0$,** then $\sqrt{b^2 - 4ac} = 0$ and therefore roots of equation are **real and equal,**

So, $\alpha = \dfrac{-b}{2a}$ and $\beta = \dfrac{-b}{2a}$

i.e. both the roots are real and equal, we say that the equation has repeated roots.

(3) If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is not a real number and therefore equation **does not have real roots.**

Note : If $b^2 - 4ac > 0$ and $b^2 - 4ac$ is not a perfect square, then the roots of the quadratic equation are irrational and occur in pair. They are conjugate of each other.

Activity : Fill in the blanks. ..*(Textbook Pg. No. 44)*

	Value of discriminant		Nature of roots
(1)	50	$\longrightarrow$	**Real and unequal**
(2)	–30	$\longrightarrow$	**No real roots**
(3)	0	$\longrightarrow$	**Real and equal**

Example:

Determine the nature of roots of the quadratic equations. *(Textbook Pg. No. 45)*

(i) $\mathbf{2x^2 - 5x + 7 = 0}$

Solution:

$2x^2 - 5x + 7 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = 2,\ b = -5,\ c = 7$

$\therefore\ b^2 - 4ac = (-5)^2 - 4 \times 2 \times 7$

$= 25 - 56$

$= -31$

As $b^2 - 4ac < 0$, the roots of the equation are not real.

(ii) $\mathbf{x^2 + 2x - 9 = 0}$

Solution:

$x^2 + 2x - 9 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = \boxed{1},\ b = 2,\ c = \boxed{-9}$

$\therefore\ b^2 - 4ac = (2)^2 - 4 \times \boxed{1} \times \boxed{-9}$

$= 4 + \boxed{36}$

$= 40$

As $b^2 - 4ac > 0$, the roots of the equation are real and unequal.

(iii) $\mathbf{\sqrt{3}\,x^2 + 2\sqrt{3}\,x + \sqrt{3} = 0}$

Solution:

$\sqrt{3}\,x^2 + 2\sqrt{3}\,x + \sqrt{3} = 0$

Compare with $ax^2 + bx + c = 0$,

$a = \sqrt{3},\ b = 2\sqrt{3},\ c = \sqrt{3}$

$\therefore\ b^2 - 4ac = (2\sqrt{3})^2 - 4 \times \sqrt{3} \times \sqrt{3}$

$= 12 - 12$

$= 0$

$\because\ b^2 - 4ac = 0$, the roots of the equation are real and equal.

Practice Set - 2.5 *(Textbook Page No. 49)*

(2) Find the value of discriminant.

(i) $\mathbf{x^2 + 7x - 1 = 0}$ *(2 marks)*

Solution:

$x^2 + 7x - 1 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 1,\ b = 7,\ c = -1$

$\Delta = b^2 - 4ac$

$= (7)^2 - 4 \times 1 \times (-1)$

$= 49 + 4$

$\therefore\ \boxed{\Delta = 53}$

(ii) $\mathbf{2y^2 - 5y + 10 = 0}$ *(2 marks)*

Solution:

$2y^2 - 5y + 10 = 0$

Comparing with $ay^2 + by + c = 0$ we get,

$a = 2,\ b = -5,\ c = 10$

$\Delta = b^2 - 4ac$

$= (-5)^2 - 4 \times 2 \times 10$

$= 25 - 80$

$\therefore\ \boxed{\Delta = -55}$

(iii) $\mathbf{\sqrt{2}\,x^2 + 4x + 2\sqrt{2} = 0}$ *(2 marks)*

Solution:

$\sqrt{2}\,x^2 + 4x + 2\sqrt{2} = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = \sqrt{2},\ b = 4,\ c = 2\sqrt{2}$

$\Delta = b^2 - 4ac$

$= (4)^2 - 4 \times \sqrt{2} \times 2\sqrt{2}$

$= 16 - 16$

$\therefore\ \boxed{\Delta = 0}$

Problem Set - 2 *(Textbook Page No. 54)*

(3) Find the value of discriminant for each of the following equations.

(i) $\mathbf{2y^2 - y + 2 = 0}$ *(2 marks)*

Solution:

$2y^2 - y + 2 = 0$

Comparing with $ay^2 + by + c = 0$ we get,

$a = 2,\ b = -1,\ c = 2$

$\Delta = b^2 - 4ac$

$= (-1)^2 - 4 \times 2 \times 2$

$= 1 - 16$

$\therefore$ $\boxed{\Delta = -15}$

(ii) $5m^2 - m = 0$ *(2 marks)*

Solution:

$5m^2 - m + 0 = 0$

Comparing with $am^2 + bm + c = 0$ we get,

$a = 5, b = -1, c = 0$

$\Delta = b^2 - 4ac$

$= (-1)^2 - 4 \times 5 \times 0$

$= 1 - 0$

$\therefore$ $\boxed{\Delta = 1}$

(iii) $\sqrt{5}\,x^2 - x - \sqrt{5} = 0$ *(2 marks)*

Solution:

$\sqrt{5}\,x^2 - x - \sqrt{5} = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$\therefore$ $a = \sqrt{5}, b = -1, c = -\sqrt{5}$

$\Delta = b^2 - 4ac$

$= (-1)^2 - 4 \times \sqrt{5} \times (-\sqrt{5})$

$= 1 + 20$

$\therefore$ $\boxed{\Delta = 21}$

Practice Set - 2.5 *(Textbook Page No. 49)*

(3) Determine the nature of roots of the following quadratic equations.

(i) $x^2 - 4x + 4 = 0$ *(2 marks)*

Solution:

$x^2 - 4x + 4 = 0$

Comparing with $ax^2 + bx + c = 0$ we get,

$a = 1, b = -4, c = 4$

$\Delta = b^2 - 4ac$

$= (-4)^2 - 4 \times 1 \times 4$

$= 16 - 16$

$\therefore$ $\Delta = 0$

As $\Delta = 0$, the roots of the quadratic equation are real and equal.

(ii) $2y^2 - 7y + 2 = 0$ *(2 marks)*

Solution:

$2y^2 - 7y + 2 = 0$

Comparing with $ay^2 + by + c = 0$ we get,

$a = 2, b = -7, c = 2$

$\Delta = b^2 - 4ac$

$= (-7)^2 - 4 \times 2 \times 2$

$= 49 - 16$

$\therefore$ $\Delta = 33$

As $\Delta > 0$, the roots of the quadratic equation are real and unequal.

(iii) $m^2 + 2m + 9 = 0$ *(2 marks)*

Solution:

$m^2 + 2m + 9 = 0$

Comparing with $am^2 + bm + c = 0$ we get,

$\therefore$ $a = 1, b = 2, c = 9$

$\Delta = b^2 - 4ac$

$= (2)^2 - 4 \times 1 \times 9$

$= 4 - 36$

$\therefore$ $\Delta = -32$

As $\Delta < 0$, the roots of the quadratic equation are not real.

Problem Set - 2 *(Textbook Page No. 54)*

(6) Determine the nature of roots for each of the quadratic equations.

(i) $3x^2 - 5x + 7 = 0$ *(2 marks)*

Solution:

$3x^2 - 5x + 7 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = 3, b = -5, c = 7$

$\Delta = b^2 - 4ac$

$= (-5)^2 - 4 \times 3 \times 7$

$= 25 - 84$

$\therefore$ $\Delta = -59$

$\therefore$ **As $\Delta < 0$, the roots of the quadratic equation are not real.**

(ii) $\sqrt{3}\,x^2 + \sqrt{2}\,x - 2\sqrt{3} = 0$ *(2 marks)*

Solution:

$\sqrt{3}\,x^2 + \sqrt{2}\,x - 2\sqrt{3} = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = \sqrt{3}, b = \sqrt{2}, c = -2\sqrt{3}$

$\Delta = b^2 - 4ac$

$= \left(\sqrt{2}\right)^2 - 4 \times \sqrt{3} \times (-2\sqrt{3})$

$= 2 + 24$

$\therefore$ $\Delta = 26$

As $\Delta > 0$, the roots of the quadratic equation are real and unequal.

(iii) $m^2 - 2m + 1 = 0$ *(2 marks)*

Solution:

$m^2 - 2m + 1 = 0$

Comparing with $am^2 + bm + c = 0$ we get,

$a = 1, b = -2, c = 1$

$\Delta = b^2 - 4ac$

$= (2)^2 - 4 \times 1 \times 1$

$= 4 - 4$

$\therefore \quad \Delta = 0$

As $\Delta = 0$, the roots of the quadratic equation are real and equal.

Practice Set - 2.5 *(Textbook Page No. 50)*

(7) The roots of the each of the following quadratic equations are real and equal, find *k*.

(i) $3y^2 + ky + 12 = 0$ *(3 marks)*

Solution:

$3y^2 + ky + 12 = 0$

Comparing the above equation with $ay^2 + by + c = 0$ $[y = x]$, we get,

$a = 3, b = k, c = 12$

$\Delta = b^2 - 4ac$

$= (k)^2 - 4 \times 3 \times 12$

$\therefore \quad \Delta = k^2 - 144$

The roots are real and equal, so Δ must be zero.

$\therefore \quad k^2 - 144 = 0$

$\therefore \quad k^2 = 144$

$\therefore \quad \mathbf{k = \pm 12}$

$\boxed{\mathbf{k = 12 \text{ or } k = -12}}$

(ii) $kx(x - 2) + 6 = 0$ *(3 marks)*

Solution:

$kx(x - 2) + 6 = 0$

$\therefore \quad kx^2 - 2kx + 6 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = k, b = -2k, c = 6$

$\Delta = b^2 - 4ac$

$= (-2k)^2 - 4 \times k \times 6$

$\therefore \quad \Delta = 4k^2 - 24k$

The roots are real and equal, so Δ must be zero.

$\therefore \quad 4k^2 - 24k = 0$

$\therefore \quad 4k(k - 6) = 0$

$\therefore \quad 4k = 0 \text{ or } k - 6 = 0$

$\therefore \quad k = \dfrac{0}{4} \text{ or } k = 6$

i.e. $k = 0$ or $k = 6$.

As $a = k$ and in quadratic equation $a \neq 0$

$\therefore \quad k \neq 0$

$\therefore \quad \boxed{\mathbf{k = 6}}$

Problem Set - 2 *(Textbook Page No. 54)*

(8) Find m if $(m - 12)x^2 + 2(m - 12)x + 2 = 0$ has real and equal roots. *(4 marks)*

Solution:

$(m - 12)x^2 + 2(m - 12)x + 2 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = m - 12, b = 2(m - 12), c = 2$

$\Delta = b^2 - 4ac$

$= [2(m - 12)]^2 - 4 \times (m - 12) \times 2$

$\therefore \quad \Delta = 4(m^2 - 24m + 144) - 8(m - 12)$

$= 4m^2 - 96m + 576 - 8m + 96$

$\therefore \quad \Delta = 4m^2 - 104m + 672$

The roots are real and equal so Δ must be zero.

$\therefore \quad 4m^2 - 104m + 672 = 0$

$\therefore \quad m^2 - 26m + 168 = 0$ ..(Dividing both sides by 4)

$\therefore \quad m^2 - 14m - 12m + 168 = 0$

$\therefore \quad m(m - 14) - 12(m - 14) = 0$

$\therefore \quad (m - 14)(m - 12) = 0$

$\therefore \quad m - 14 = 0 \text{ or } m - 12 = 0$

$\therefore \quad m = 14 \text{ or } m = 12$

Since $m - 12 = a$ and in quadratic equation $a \neq 0$

$\therefore \quad m \neq 12$

$\therefore \quad \boxed{\mathbf{m = 14}}$

Points to Remember:

Relation between roots of the quadratic equation and coefficients:

If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$\alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2b}{2a}$$

$$\alpha + \beta = \frac{\mathbf{-b}}{\mathbf{a}}$$

$$\text{Also, } \alpha \times \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(-b + \sqrt{b^2 - 4ac}) \times (-b - \sqrt{b^2 - 4ac})}{4a^2}$$

$$= \frac{(-b)^2 - (b^2 - 4ac)}{4a^2}$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2}$$

$$= \frac{4ac}{4a^2}$$

$$\alpha \times \beta = \frac{c}{a}$$

Activity :

Fill in the empty boxes below properly

(Textbook pg. no. 46)

For $10x^2 + 10x + 1 = 0$, $\alpha + \beta = \boxed{-1}$ and $\alpha \times \beta = \boxed{\dfrac{1}{10}}$

Working:

$$\alpha + \beta = \frac{-b}{a}$$

$$= \frac{-10}{10} = -1$$

$$\alpha \times \beta = \frac{c}{a}$$

$$= \frac{1}{10}$$

Some Identities:

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

(iii) $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$

(iv) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

Practice Set - 2.5 *(Textbook Page No. 49)*

(1) Fill in the gaps and complete. *(1 mark each)*

(i)

Quadratic equation $ax^2 + bx + c = 0$ → $b^2 - 4ac = 5$ → **Real and unequal**

Quadratic equation $ax^2 + bx + c = 0$ → $b^2 - 4ac = -5$ → **No real roots**

Nature of roots

(ii)

Sum of the roots $= -7$ ← *Quadratic equation* $\boldsymbol{x^2 + 7x + 5 = 0}$ → Product of roots $= 5$

(iii) If α, β are the roots of quadratic equation, then

$2x^2 - 4x - 3 = 0$ → $\boldsymbol{\alpha + \beta = 2}$

$2x^2 - 4x - 3 = 0$ → $\alpha \times \beta = \boldsymbol{\dfrac{-3}{2}}$

Working: $\alpha + \beta = \dfrac{-b}{a} = \dfrac{-(-4)}{2} = 2$

$$\alpha \times \beta = \frac{c}{a} = \frac{-3}{2}$$

***(5) Sum of the roots of a quadratic equation is double their product. Find k if equation is $x^2 - 4kx + k + 3 = 0$.** *(3 marks)*

Solution:

Let α and β are the roots of the given quadratic equation

$x^2 - 4kx + k + 3 = 0$

Comparing with $ax^2 + bx + c = 0$ we get,

$a = 1$, $b = -4k$, $c = k + 3$

$$\therefore \quad \alpha + \beta = \frac{-b}{a} = \frac{-(-4k)}{1} = 4k \text{ and}$$

$$\alpha \times \beta = \frac{c}{a} = \frac{k+3}{1} = k + 3$$

Now, $\alpha + \beta = 2 \times (\alpha \times \beta)$ .... (Given)

$4k = 2 \times (k + 3)$

$4k = 2k + 6$

$\therefore \quad 4k - 2k = 6$

$\therefore \quad 2k = 6$

$\therefore \quad k = \dfrac{6}{2}$

$\therefore \quad \boxed{k = 3}$

***(6) α, β are roots of $y^2 - 2y - 7 = 0$ find,**

(i) $\alpha^2 + \beta^2$ **(ii)** $\alpha^3 + \beta^3$ *(3 marks)*

Solution:

$y^2 - 2y - 7 = 0$

Comparing with $ay^2 + by + c = 0$ we get,

$a = 1$, $b = -2$, $c = -7$

$$\therefore \quad \alpha + \beta = \frac{-b}{a} = \frac{-(-2)}{1} = 2 \text{ and}$$

$$\alpha \times \beta = \frac{c}{a} = \frac{-7}{1} = -7$$

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (2)^2 - 2 \times (-7)$

$= 4 + 14$

$\therefore \quad \boxed{\alpha^2 + \beta^2 = \mathbf{18}}$

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (2)^3 - 3 \times (-7) \times (2)$

$= 8 + 42$

$\therefore \quad \boxed{\alpha^3 + \beta^3 = \mathbf{50}}$

Points to Remember:

To obtain a quadratic equation having given roots.

If α and β are the roots of the quadratic equation, then $x = \alpha$ or $x = \beta$

$\therefore \quad x - \alpha = 0$ or $x - \beta = 0$

$\therefore \quad (x - \alpha)(x - \beta) = 0$

$\therefore \quad x^2 - \alpha x - \beta x + \alpha\beta = 0$

$\therefore \quad x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e x^2 – (Sum of the roots)x + Product of the roots = 0

Activity I :

Write the quadratic equation if addition of the roots is 10 and product of roots is 9. *(Textbook Pg. no. 48)*

$\therefore$ Quadratic equation : $x^2 - \boxed{10}\,x + \boxed{9} = 0$

Activity II :

What will be the quadratic equation if $\alpha = 2$, $\beta = 5$. *(Textbook Pg. no. 48)*

It can be written as:

$x^2 - (\boxed{2} + \boxed{5})x + \boxed{2} \times \boxed{5} = 0$

$\therefore$ $x^2 - \boxed{7}\,x + \boxed{10} = 0$

Practice Set - 2.5 *(Textbook Page No. 50)*

(4) Form the quadratic equation from the roots given below.

(i) 0 and 4 *(2 marks)*

Solution

Let α and β are the roots of the quadratic equation

Let $\alpha = 0$ and $\beta = 4$

$\therefore$ $\alpha + \beta = 0 + 4 = 4$ and

$\alpha \times \beta = 0 \times 4 = 0$

Then, the quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore$ $x^2 - 4x + 0 = 0$

$\therefore$ $\boxed{x^2 - 4x = 0}$

(ii) 3 and –10 *(2 marks)*

Solution

Let α and β are the roots of the quadratic equation

Let $\alpha = 3$ and $\beta = -10$

$\therefore$ $\alpha + \beta = 3 + (-10) = -7$ and

$\alpha \times \beta = 3 \times (-10) = -30$

Then the quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore$ $x^2 - (-7)x - 30 = 0$

$\therefore$ $\boxed{x^2 + 7x - 30 = 0}$

(iii) $\frac{1}{2}$ and $\frac{-1}{2}$ *(2 marks)*

Solution

Let α and β are the roots of the quadratic equation.

Let $\alpha = \frac{1}{2}$ and $\beta = \frac{-1}{2}$

$\therefore$ $\alpha + \beta = \frac{1}{2} + \left(\frac{-1}{2}\right) = 0$ and

$\alpha \times \beta = \frac{1}{2} \times \frac{-1}{2} = \frac{-1}{4}$

Then the quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - (0)x + \left(\frac{-1}{4}\right) = 0$

$\therefore$ $x^2 - \frac{1}{4} = 0$

$\therefore$ $\boxed{4x^2 - 1 = 0}$ .... [Multiplying both sides by 4]

(iv) $2 - \sqrt{5}$ and $2 + \sqrt{5}$ *(2 marks)*

Solution

Let α and β are the roots of the quadratic equation.

Let $\alpha = 2 - \sqrt{5}$ and $\beta = 2 + \sqrt{5}$

$\therefore$ $\alpha + \beta = 2 - \sqrt{5} + 2 + \sqrt{5} = 4$ and

$\alpha \times \beta = (2 - \sqrt{5}) \times (2 + \sqrt{5})$

$= 2^2 - (\sqrt{5})^2$

$= 4 - 5$

$= -1$

Then, required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore$ $\boxed{x^2 - 4x - 1 = 0}$

Problem Set - 2 *(Textbook Page No. 54)*

(5) Two roots of quadratic equations are given; frame the equation.

(i) 10 and –10 *(2 marks)*

Solution:

Let α and β are the roots of the quadratic equation

Let $\alpha = 10$ and $\beta = -10$

$\therefore$ $\alpha + \beta = 10 + (-10) = 0$ and

$\alpha \times \beta = 10 \times -10 = -100$

Then required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore$ $x^2 - (0)x + (-100) = 0$

$\therefore$ $\boxed{x^2 - 100 = 0}$

(ii) $1 - 3\sqrt{5}$ and $1 + 3\sqrt{5}$ *(2 marks)*

Solution:

Let α and β are the roots of the quadratic equation

Let $\alpha = 1 - 3\sqrt{5}$ and $\beta = 1 + 3\sqrt{5}$

$\therefore$ $\alpha + \beta = 1 - 3\sqrt{5} + 1 + 3\sqrt{5} = 2$ and

$\alpha \times \beta = (1 - 3\sqrt{5}) \times (1 + 3\sqrt{5})$

$= (1)^2 - (3\sqrt{5})^2$

$= 1 - 45$

$= -44$

Then required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore$ $\boxed{x^2 - 2x - 44 = 0}$

(iii) 0 and 7 *(2 marks)*

Solution:

Let α and β are the roots of the quadratic equation.

Let $\alpha = 0$ and $\beta = 7$

$\therefore$ $\alpha + \beta = 0 + 7 = 7$ and

$\alpha \times \beta = 0 \times 7 = 0$

Then, required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore$ $x^2 - 7x + 0 = 0$

$\therefore$ $\boxed{x^2 - 7x = 0}$

***(9) The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35, find the equation.** *(4 marks)*

Solution

Let α and β are the roots of the quadratic equation.

Let $\alpha + \beta = 5$ and $\alpha^3 + \beta^3 = 35$ ... (Given)

$\therefore$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$\therefore$ $35 = (5)^3 - 3\alpha\beta(5)$

$\therefore$ $35 = 125 - 15\alpha\beta$

$\therefore$ $15\alpha\beta = 125 - 35$

$\therefore$ $15\alpha\beta = 90$

$\therefore$ $\alpha\beta = \dfrac{90}{15}$

$\therefore$ $\alpha\beta = 6$

$\therefore$ The required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore$ $\boxed{x^2 - 5x + 6 = 0}$

(10) Find quadratic equation such that its roots are square of sum of the roots and square of difference of the roots of equation $2x^2 + 2(p + q)x + p^2 + q^2 = 0$ *(4 marks)*

Solution

$2x^2 + 2(p + q)x + p^2 + q^2 = 0$

Comparing with $ax^2 + bx + c = 0$, we get,

$a = 2$, $b = 2(p + q)$, $c = p^2 + q^2$

Let α and β are the roots of the quadratic equation.

$\alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

$\alpha + \beta = \dfrac{-2(p+q)}{2}$ and $\alpha\beta = \dfrac{p^2+q^2}{2}$ ....(ii)

$\alpha + \beta = -(p + q)$ .....(i)

Let α_1 and β_1 are the roots of the quadratic equation.

$\alpha_1 = (\alpha + \beta)^2$ ... (given)

$\beta_1 = (\alpha - \beta)^2$

$\therefore$ $\alpha_1 = [-(p + q)]^2$ ..... [from (i)]

$\therefore$ $\alpha_1 = (p + q)^2$ . ...(iii)

$\beta_1 = (\alpha - \beta)^2$

$\beta_1 = (\alpha + \beta)^2 - 4\alpha\beta$ $\{\because (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta\}$

$\beta_1 = [-(p + q)]^2 - 4\left(\dfrac{p^2+q^2}{2}\right)$

$\beta_1 = (p + q)^2 - 2(p^2 + q^2)$

$\beta_1 = p^2 + 2pq + q^2 - 2p^2 - 2q^2$

$\beta_1 = -p^2 + 2pq - q^2$

$\beta_1 = -(p^2 - 2pq + q^2)$

$\beta_1 = -(p - q)^2$ ....(iv)

Sum of the roots of the new equation

$= \alpha_1 + \beta_1$

$= (p + q)^2 + [-(p - q)^2]$ ....[from (iii) and (iv)]

$= p^2 + 2pq + q^2 - (p^2 - 2pq + q^2)$

$= p^2 + 2pq + q^2 - p^2 + 2pq - q^2$

$= 4pq$

Product of roots of new equation $= \alpha_1 \beta_1$

$= (p + q)^2 [-(p - q)^2]$ ... [from (iii) and (iv)]

$= -(p + q)^2 . (p - q)^2$

$= -[(p + q)(p - q)]^2$

$= -(p^2 - q^2)^2$

The required new equation

$x^2 - (\alpha_1 + \beta_1)x + \alpha_1 \beta_1 = 0$

$x^2 - 4pqx + [-(p^2 - q^2)^2] = 0$

$\therefore$ $\boxed{x^2 - 4pqx - (p^2 - q^2)^2 = 0}$

Points to Remember:

- **Application of quadratic equation**

Quadratic equations are used to solve problems arising in our day to day life.

Look out for comparative statements whichever quantity comes in the later part of the statement is assumed as 'x'.

Practice Set - 2.6 *(Textbook Page No. 52)*

(1) Product of Pragati's age 2 years ago and 3 years hence is 84. Find her present age. *(3 marks)*

Solution:

Let the present age of Pragati be x years.

Pragati's age 2 years ago $= (x - 2)$ years and 3 years hence $= (x + 3)$ years.

According to given condition,

$(x - 2)(x + 3) = 84$

$x(x + 3) - 2(x + 3) = 84$

$\therefore$ $x^2 + 3x - 2x - 6 = 84$

$\therefore$ $x^2 + x - 6 - 84 = 0$

$\therefore$ $x^2 + x - 90 = 0$

$\therefore$ $x^2 + 10x - 9x - 90 = 0$

$\therefore$ $x(x + 10) - 9(x + 10) = 0$

$\therefore$ $(x + 10)(x - 9) = 0$

$\therefore$ $x + 10 = 0$ or $x - 9 = 0$

$\therefore$ $x = -10$ or $x = 9$

Now, age cannot be negative. $\therefore x \neq -10$

$\therefore$ $x = 9$

$\therefore$ **The present age of Pragati is 9 years.**

(4) Vivek is older than Kishor by 5 years. The sum of the reciprocals of their ages is $\frac{1}{6}$. Find their present age. *(3 marks)*

Solution:

Let the present age of Kishor be x years.

∴ The present age of Vivek is $(x + 5)$ years.

According to given condition,

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

∴ $\frac{x+5+x}{x(x+5)} = \frac{1}{6}$

∴ $\frac{2x+5}{x^2+5x} = \frac{1}{6}$

∴ $x^2 + 5x = 12x + 30$

∴ $x^2 + 5x - 12x - 30 = 0$

∴ $x^2 - 7x - 30 = 0$

∴ $x^2 - 10x + 3x - 30 = 0$

∴ $x(x - 10) + 3(x - 10) = 0$

∴ $(x - 10)(x + 3) = 0$

∴ $x - 10 = 0$ or $x + 3 = 0$

∴ $x = 10$ or $x = -3$

$x \neq -3$ as age cannot be negative.

∴ $x = 10$ and $x + 5 = 10 + 5 = 15$.

∴ **The present age of Kishor is 10 years and present age of Vivek is 15 years.**

(2) The sum of squares of two consecutive even natural number is 244. Find the numbers. *(3 marks)*

Solution:

Let the two consecutive even natural numbers be x and $x + 2$ years.

According to given condition,

$x^2 + (x + 2)^2 = 244$

∴ $x^2 + x^2 + 4x + 4 = 244$ ...[Using $(a + b)^2 = a^2 + 2ab + b^2$]

∴ $2x^2 + 4x + 4 - 244 = 0$

∴ $2x^2 + 4x - 240 = 0$

∴ $x^2 + 2x - 120 = 0$ ...[Dividing both sides by 2]

∴ $x^2 + 12x - 10x - 120 = 0$

∴ $x(x + 12) - 10(x + 12) = 0$

∴ $(x + 12)(x - 10) = 0$

∴ $x + 12 = 0$ or $x - 10 = 0$

∴ $x = -12$ or $x = 10$

$x \neq -12$ as natural numbers cannot be negative.

∴ $x = 10$ and $x + 2 = 10 + 2 = 12$

∴

Problem Set- 2 *(Textbook Page No. 54)*

*** (12) The difference between squares of two numbers is 120. The square of smaller number is twice the greater number. Find the numbers.** *(4 marks)*

Solution:

Let the bigger number be x.

Now, the difference between squares of two numbers is 120.

∴ Square of bigger number – Square of smaller number = 120

∴ x^2 – square of smaller number = 120

∴ Square of smaller number = $x^2 - 120$

According to given condition,

$x^2 - 120 = 2x$

∴ $x^2 - 2x - 120 = 0$

∴ $x^2 - 12x + 10x - 120 = 0$

∴ $x(x - 12) + 10(x - 12) = 0$

∴ $(x - 12)(x + 10) = 0$

∴ $x - 12 = 0$ or $x + 10 = 0$

∴ $x = 12$ or $x = -10$

If $x = -10$ then $x^2 - 120 = (-10)^2 - 120$

$= 100 - 120$

$= -20$

∴ $x = -10$ is not acceptable because square of smaller number cannot be negative.

∴ $x = 12$

Now, square of smaller number $= x^2 - 120$

$= (12)^2 - 120$

$= 144 - 120$

$= 24.$

∴ Smaller number = $\sqrt{24}$ or $-\sqrt{24}$

... (Taking square root)

∴ **The required numbers are 12 and $\sqrt{24}$ or 12 and $-\sqrt{24}$.**

Practice Set - 2.6 *(Textbook Page No. 52)*

*** (9) If 460 is divided by a natural number, quotient is 6 more than 5 times the divisor and remainder is 1 then find quotient and divisor.** *(4 marks)*

Solution:

Let the divisor be x

∴ The quotient = $5x + 6$

Here, dividend = 460 and remainder = 1

Dividend = Divisor × Quotient + Remainder

∴ 460 $= x \times (5x + 6) + 1$

∴ 460 $= 5x^2 + 6x + 1$

$\therefore$ $5x^2 + 6x + 1 - 460 = 0$

$\therefore$ $5x^2 + 6x - 459 = 0$

$\therefore$ $5x^2 - 45x + 51x - 459 = 0$

$\therefore$ $5x(x - 9) + 51(x - 9) = 0$

$\therefore$ $(x - 9)(5x + 51) = 0$

$\therefore$ $x - 9 = 0$ or $5x + 51 = 0$

$\therefore$ $x = 9$ or $x = \frac{-51}{5}$

$x \neq \frac{-51}{5}$ as natural numbers cannot be negative.

$\therefore$ $x = 9$ and $5x + 6 = 5 \times 9 + 6 = 51.$

$\therefore$ **The divisor is 9 and the quotient is 51.**

Problem Set- 2 *(Textbook Page No. 54)*

(11) Mukund possesses `₹ 50 more than what Sagar possesses. The product of the amount they have is ₹ 15,000. Find the amounts each one has.

(3 marks)

Solution:

Let the amount with Sagar be ₹ x.

$\therefore$ The amount with Mukund is = ₹$(x + 50)$

According to given condition,

$x \times (x + 50) = 15000$

$\therefore$ $x^2 + 50x - 15000 = 0$

$\therefore$ $x^2 + 150x - 100x - 15000 = 0$

$\therefore$ $x(x + 150) - 100(x + 150) = 0$

$\therefore$ $(x + 150)(x - 100) = 0$

$\therefore$ $x + 150 = 0$ or $x - 100 = 0$

$\therefore$ $x = -150$ or $x = 100$

$x \neq -150$ amount of rupees cannot be negative.

$\therefore$ $x = 100$ and $x + 50 = 100 + 50$

$= 150$

$\therefore$ **The amount with Sagar is ₹ 100 and amount with Mukund is ₹ 150.**

Practice Set - 2.6 *(Textbook Page No. 52)*

(6) Mr. Kasam runs a small business of making earthen pots. He makes certain number of pots on daily basis. Production cost of each pot is ₹ 40 more than 10 times total number of pots, he makes in one day. If production cost of all pots per day is ₹ 600, find production cost of one pot and number of pots he makes per day.(4 marks)*

Solution:

Let the number of pots made by Mr Kasam in each day be x.

$\therefore$ The cost price of each pot = ₹$(10x + 40)$

Total cost price of all pots made by him in one day = ₹600

According to given condition,

$x \times (10x + 40) = 600$

$\therefore$ $10x^2 + 40x = 600$

$\therefore$ $10x^2 + 40x - 600 = 0$

$\therefore$ $x^2 + 4x - 60 = 0$ ...(Dividing both sides by 10)

$\therefore$ $x^2 + 10x - 6x - 60 = 0$

$\therefore$ $x(x + 10) - 6(x + 10) = 0$

$\therefore$ $(x + 10)(x - 6) = 0$

$\therefore$ $x + 10 = 0$ or $x - 6 = 0$

$\therefore$ $x = -10$ or $x = 6$

$x \neq -10$ as number of pots cannot be negative.

$\therefore$ $x = 6$ and $10x + 40 = 10 \times 6 + 40$

$= 60 + 40$

$= 100$

$\therefore$ **Mr. Kasam is making 6 pots everyday and cost of each pot is ₹ 100.**

Problem Set- 2 *(Textbook Page No. 54)*

***(13) Ranjana wants to distribute 540 oranges among some students. If 30 students were more each would get 3 oranges less. Find the number of students.** *(4 marks)*

Solution:

Let the number of students be x.

Number of oranges = 540.

$\therefore$ Number of oranges each student get = $\frac{540}{x}$

If there are $x + 30$ students, then number of oranges each student get = $\frac{540}{x + 30}$

According to given condition,

$$\frac{540}{x} - \frac{540}{x + 30} = 3$$

$$\therefore \quad 540\left(\frac{1}{x} - \frac{1}{x + 30}\right) = 3$$

$$\therefore \quad \frac{x + 30 - x}{x(x + 30)} = \frac{3}{540}$$

$$\therefore \quad \frac{30}{x^2 + 30x} = \frac{1}{180}$$

$\therefore$ $x^2 + 30x = 5400$

$\therefore$ $x^2 + 30x - 5400 = 0$

$\therefore$ $x^2 + 90x - 60x - 5400 = 0$

$\therefore$ $x(x + 90) - 60(x + 90) = 0$

$\therefore$ $(x + 90)(x - 60) = 0$

$\therefore$ $x + 90 = 0$ or $x - 60 = 0$

$\therefore$ $x = -90$ or $x = 60$

$x \neq -90$ as number of students can not be negative.

$\therefore$ $x = 60$

$\therefore$ **The required number of students is 60.**

Practice Set - 2.6 *(Textbook Page No. 52)*

(5) Suyash scored 10 marks more in second test than that in first. 5 times the score of the second test is same as square of the score in first test. Find his score in first test. *(3 marks)*

Solution:

Let the marks scored by Suyash in first unit test be x.

$\therefore$ The marks scored by him in second unit test is $(x + 10)$

According to given condition,

$5 \times (x + 10) = x^2$

$\therefore$ $5x + 50 = x^2$

$\therefore$ $x^2 - 5x - 50 = 0$

$\therefore$ $x^2 - 10x + 5x - 50 = 0$

$\therefore$ $x(x - 10) + 5(x - 10) = 0$

$\therefore$ $(x - 10)(x + 5) = 0$

$\therefore$ $x - 10 = 0$ or $x + 5 = 0$

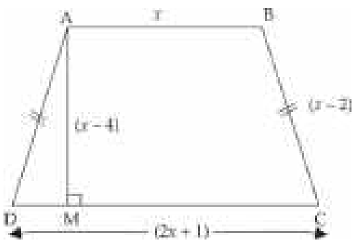
$\therefore$ $x = 10$ or $x = -5$

$x \neq -5$ as marks scored in unit test cannot be negative.

$\therefore$ $x = 10$

$\therefore$ **The marks scored by Suyash in first test is 10.**

***(10)**



In the above fig. □ABCD is a trapezium AB II CD and its area is 33 cm². From the information given in the figure, find the lengths of all sides of the □ABCD. Fill in the empty boxes to get the solution. *(3 marks)*

Solution:

□ABCD is a trapezium and AB II CD

$A(\square ABCD) = \frac{1}{2}(AB + CD) \times \boxed{AM}$

$\therefore$ $33 = \frac{1}{2}(x + 2x + 1) \times \boxed{(x - 4)}$

$\therefore$ $\boxed{66} = (3x + 1) \times \boxed{(x - 4)}$

$\therefore$ $3x^2 + \boxed{(-11x)} - \boxed{70} = 0$

$\therefore$ $3x(x - 7) + 10(x - 7) = 0$

$\therefore$ $(3x + 10)(x - 7) = 0$

$\therefore$ $3x + 10 = 0$ or $\boxed{x - 7} = 0$

$\therefore$ $x = \frac{-10}{3}$ or $x = \boxed{7}$

But, length cannot be negative

$\therefore$ $x \neq \frac{-10}{3}$ $\therefore x = \boxed{7}$

$\therefore$ **AB = 7 cm, CD = 15 cm, AD = BC = 5 cm**

Problem Set- 2 *(Textbook Page No. 54)*

***(14) Mr. Dinesh owns an agricultural farm at village Talvel. The length of the farm is 10 meter more than twice the breadth. In order to harvest rain water, he dug a square shape pond inside the farm. The side of pond is $\frac{1}{3}$ times of the breadth of the farm. The area of the farm is 20 times the area of the pond. Find the length and breadth of the farm and of the pond.** *(4 marks)*

Solution:

Let the breadth of rectangular field is x m.

$\therefore$ Its length $= (2x + 10)$

Area of the farm $= (2x + 10) \times x$ sq.m

Now, side of a square pond $= \frac{1}{3}x$ m

$\therefore$ Area of square pond $=$ side^2

$= \left(\frac{1}{3}x\right)^2$

$= \frac{1}{9}x^2$ sq.m

According to given condition,

$(2x + 10) \times x = 20 \times \frac{1}{9}x^2$

$\therefore$ $2x^2 + 10x = \frac{20x^2}{9}$

$\therefore$ $18x^2 + 90x = 20x^2$

$\therefore$ $90x = 20x^2 - 18x^2$

$\therefore$ $2x^2 - 90x = 0$

$\therefore$ $2x(x - 45) = 0$

$\therefore$ $2x = 0$ or $x - 45 = 0$

$\therefore$ $x = 0$ or $x = 45$

$x \neq 0$ as breadth of field can not be zero.

$\therefore$ $x = 45$ and $(2x + 10) = 2 \times 45 + 10$

$= 90 + 10$

$= 100$

Also, $\frac{1}{3}x = \frac{1}{3} \times 45 = 15$

$\therefore$ **The length of rectangular field is 100 m and breadth is 45 m. Also, length of each side of square pond is 15 m.**

Practice Set - 2.6 *(Textbook Page No. 52)*

***(8) Pintu takes 6 days more than those of Nishu to complete certain work. If they work together, they finish it in 4 days. How many days would it take to complete the work if they work alone?** *(4 marks)*

Solution:

Let Nishu take x days to complete the work.

$\therefore$ Pintu takes $(x + 6)$ days to complete the work.

$\therefore$ Work done by Nishu in 1 day $= \frac{1}{x}$

$\therefore$ Work done by Pintu in 1 day $= \frac{1}{x+6}$

$\therefore$ Work done by both of them in 1 day $= \frac{1}{x} + \frac{1}{x+6}$

But, together they complete the work in 4 days.

$\therefore$ Work done by both of them in 1 day $= \frac{1}{4}$

According to given condition,

$$\frac{1}{x} + \frac{1}{x+6} = \frac{1}{4}$$

$$\therefore \frac{x+6+x}{x(x+6)} = \frac{1}{4}$$

$$\therefore \frac{2x+6}{x^2+6x} = \frac{1}{4}$$

$\therefore$ $x^2 + 6x = 8x + 24$

$\therefore$ $x^2 + 6x - 8x - 24 = 0$

$\therefore$ $x^2 - 2x - 24 = 0$

$\therefore$ $x^2 - 6x + 4x - 24 = 0$

$\therefore$ $x(x - 6) + 4(x - 6) = 0$

$\therefore$ $(x - 6)(x + 4) = 0$

$\therefore$ $x - 6 = 0$ or $x + 4 = 0$

$\therefore$ $x = 6$ or $x = -4$

$x \neq -4$ as number of days cannot be negative.

$\therefore$ $x = 6$ and $x + 6 = 6 + 6 = 12$

$\therefore$ **Nishu takes 6 days and Pintu takes 12 days to complete the work.**

Problem Set- 2 *(Textbook Page No. 54)*

***(15) A tank fills completely in 2 hours if both the taps are open. If only one of the taps is open at the given time, the smaller tap takes 3 hours more than the larger one to fill the tank. How much time does each tap take to fill the tank completely.** *(4 marks)*

Solution:

Let the bigger tap requires x hours to fill the tank.

$\therefore$ The smaller tap require $(x + 3)$ hours.

According to given condition,

$$\frac{1}{x} + \frac{1}{x+3} = \frac{1}{2}$$

$$\therefore \frac{x+3+x}{x(x+3)} = \frac{1}{2}$$

$$\therefore \frac{2x+3}{x^2+3x} = \frac{1}{2}$$

$\therefore$ $x^2 + 3x = 4x + 6$

$\therefore$ $x^2 + 3x - 4x - 6 = 0$

$\therefore$ $x^2 - x - 6 = 0$

$\therefore$ $x^2 - 3x + 2x - 6 = 0$

$\therefore$ $x(x - 3) + 2(x - 3) = 0$

$\therefore$ $(x - 3)(x + 2) = 0$

$\therefore$ $x - 3 = 0$ or $x + 2 = 0$

$\therefore$ $x = 3$ or $x = -2$

$x \neq -2$ as time cannot be negative.

$\therefore$ $x = 3$ and $x + 3 = 3 + 3 = 6$

$\therefore$ **The bigger tap required 3 hours to fill the tank and the smaller tap required 6 hours.**

Practice Set - 2.6 *(Textbook Page No. 52)*

***(7) Pratik takes 8 hours to travel 36 km downstream and return to same spot. The speed of boat in still water is 12 km/hr. Find the speed of the water current.** *(4 marks)*

Solution:

Let the speed of the river current be x *km/h.*

The speed of the boat in still water is 12 km/hr.

$\therefore$ The speed of the boat in downstream is $(12 + x)$ *km/hr* and the speed of boat in upstream is $(12 - x)$ *km/hr.* $[x < 12]$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time taken by boat to travel 36 km downstream.

$$= \left(\frac{36}{12+x}\right) \text{hours}$$

Time taken by boat to travel 36 km upstream.

$$= \left(\frac{36}{12-x}\right) \text{hours}$$

According to given condition,

$$\frac{36}{12+x} + \frac{36}{12-x} = 8$$

$$\therefore 36\left(\frac{1}{12+x} + \frac{1}{12-x}\right) = 8$$

$$\therefore \frac{12-x+12+x}{(12+x)(12-x)} = \frac{8}{36}$$

$\therefore \quad \dfrac{24}{12^2 - x^2} = \dfrac{2}{9}$ ....[Using $(a + b)(a - b) = a^2 - b^2$]

$\therefore \quad \dfrac{24}{144 - x^2} = \dfrac{2}{9}$

$\therefore \quad 24 \times 9 = 2(144 - x^2)$

$\therefore \quad \dfrac{24 \times 9}{2} = 144 - x^2$

$\therefore \quad 12 \times 9 = 144 - x^2$

$\therefore \quad 108 = 144 - x^2$

$\therefore \quad x^2 = 144 - 108$

$\therefore \quad x^2 = 36$

$\therefore \quad x^2 - 36 = 0$

$\therefore \quad (x + 6)(x - 6) = 0$

$\therefore \quad x + 6 = 0$ or $x - 6 = 0$

$\therefore \quad x = -6$ or $x = 6$

$x \neq -6$ as speed of river current cannot be negative.

$\therefore \quad x = 6.$

$\therefore$ **The speed of river current is 6 km/hr.**

(3) In the orange garden of Mr. Madhusudan there are 150 orange trees. The number of trees in each rows are 5 more than that in each column. Find the number of trees in each rows and each column with the help of following flow chart. *(3 marks)*

No. of trees in a column is x → No. of trees in rows = ... → Total no. of trees = ...
→ Frame quadratic equation → Find x → No. of trees in columns → No. of trees in a row.

Solution:

Let the number of trees planted in vertical rows be x.

$\therefore$ The number of trees planted in horizontal rows is $(x + 5)$.

$\therefore$ The total number of trees planted $= x \times (x + 5)$

But, the total number of trees planted is 150.

$x \times (x + 5) = 150$

$x^2 + 5x = 150$

$\therefore \quad x^2 + 5x - 150 = 0$

$\therefore \quad x^2 + 15x - 10x - 150 = 0$

$\therefore \quad x(x + 15) - 10(x + 15) = 0$

$\therefore \quad (x + 15)(x - 10) = 0$

$\therefore \quad x + 15 = 0$ or $x - 10 = 0$

$\therefore \quad x = -15$ or $x = 10$

$x \neq -15$ as number of trees cannot be negative.

$\therefore \quad x = 10$ and $x + 5 = 10 + 5 = 15$

$\therefore$ **The number of trees planted in horizontal rows is 15 and 10 trees planted in vertical rows.**

Problem Set - 2 *(Textbook Page No. 53)*

MCQ's

(1) Choose the correct answers for the following questions. *(1 mark each)*

(1) Which one is the quadratic equation ?

(A) $\dfrac{5}{x} - 3 = x^2$ (B) $x(x + 5) = 2$

(C) $n - 1 = 2n$ (D) $\dfrac{1}{x^2}(x + 2) = x$

(2) Out of the following equations which one is not a quadratic equation ?

(A) $x^2 + 4x = 11 + x^2$ (B) $x^2 = 4x$

(C) $5x^2 = 90$ (D) $2x - x^2 = x^2 + 5$

(3) The roots of $x^2 + kx + k = 0$ are real and equal, find k.

(A) 0 (B) 4

(C) 0 or 4 (D) 2

(4) For $\sqrt{2}x^2 - 5x + \sqrt{2} = 0$, find the value of the discriminant.

(A) -5 (B) 17

(C) $\sqrt{2}$ (D) $2\sqrt{2} - 5$

(5) Which of the following quadratic equations has roots 3, 5 ?

(A) $x^2 - 15x + 8 = 0$ (B) $x^2 - 8x + 15 = 0$

(C) $x^2 + 3x + 5 = 0$ (D) $x^2 + 8x - 15 = 0$

(6) Out of the following equations, find the equation having the sum of its roots -5.

(A) $3x^2 - 15x + 3 = 0$ (B) $x^2 - 5x + 3 = 0$

(C) $x^2 + 3x - 5 = 0$ (D) $3x^2 + 15x + 3 = 0$

(7) $\sqrt{5}m^2 - \sqrt{5}m + \sqrt{5} = 0$ which of the following statement is true for this given equation ?

(A) real and unequal roots

(B) real and equal roots

(C) no real roots

(D) three roots

(8) One of the roots of equation $x^2 + mx - 5 = 0$ is 2 find m.

(A) -2 (B) $\dfrac{-1}{2}$ (C) $\dfrac{1}{2}$ (D) 2

Additional MCQ's

(9) The sum of roots $(\alpha + \beta) =$ ....

(A) $\frac{-b}{a}$ (B) $\frac{b}{a}$ (C) $\frac{-c}{a}$ (D) $\frac{c}{a}$

(10) The roots of a quadratic equation $y^2 - 16y + 63 = 0$ are ....

(A) –9 and –7 (B) –9 and 7
(C) 9 and –7 (D) 9 and 7

(11) If the roots of a quadratic equation are real and equal, then Δ must be ....

(A) zero (B) greater than zero
(C) less than zero (D) equal to one

(12) If one root of quadratic equation $kx^2 - 7x + 12 = 0$ is 3, then value of k is ....

(A) –1 (B) 1 (C) 3 (D) none of these

(13) If one root of quadratic equation is $1 - \sqrt{3}$ then the other root is ....

(A) $1 - \sqrt{3}$ (B) $-1 - \sqrt{3}$
(C) $1 + 2\sqrt{3}$ (D) $1 + \sqrt{3}$

(14) If the roots of $ax^2 + bx + c = 0$ are real and equal then ....

(A) $b^2 - 4ac < 0$ (B) $b^2 - 4ac = 0$
(C) $b^2 - 4ac > 0$ (D) cannot say

(15) The value of discriminant of the equation $x^2 + x + 1 = 0$ is ....

(A) –4 (B) –3 (C) 3 (D) 4

(16) In a quadratic equation, $\alpha + \beta = -4$ and $\alpha\beta = -1$, then required equation is ....

(A) $x^2 - 4x - 1 = 0$ (B) $x^2 + 4x - 1 = 0$
(C) $x^2 + 4x + 1 = 0$ (D) $x^2 - 4x + 1 = 0$

(17) The standard form of quadratic equation

$x - \frac{5}{x} = 3x - 7$ is ....

(A) $2x^2 - 8x + 7 = 0$ (B) $2x^2 + 7x + 5 = 0$
(C) $2x^2 - 7x + 5 = 0$ (D) $2x^2 - 5x + 7 = 0$

(18) What is the nature of roots of quadratic equation $9x^2 - 12x + 4 = 0$?

(A) real (B) equal
(C) unequal (D) both A and B

(19) Three times the square of natural number is 363 is written in the mathematical equation form as ....

(A) $x^2 + 3 = 363$ (B) $x^2 - 3 = 363$
(C) $3x^2 = 363$ (D) $\frac{x^2}{3} = 363$

(20) Which of the following is not a quadratic equation?

(A) $\frac{-5}{3}x^2 = 2x + 9$ (B) $(x + 3)(x + 4)$
(C) $\frac{5}{x} - 3 = x^2$ (D) $\frac{7}{m} = 3m + 5$

(21) The product of the roots $(\alpha \times \beta) =$ ....

(A) $\frac{-b}{a}$ (B) $\frac{-c}{a}$ (C) $\frac{b}{a}$ (D) $\frac{c}{a}$

(22) $\alpha^3 + \beta^3 =$ ....

(A) $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
(B) $(\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$
(C) $(\alpha + \beta)^3 - 3\alpha\beta(\alpha - \beta)$
(D) $(\alpha - \beta)^3 - 3\alpha\beta(\alpha - \beta)$

(23) If one root of quadratic equation is $5 + \sqrt{5}$, then the product of roots is ....

(A) 30 (B) –20 (C) 20 (D) 125

ANSWERS

(1) (B) $x(x + 5) = 2$ (2) (A) $x^2 + 4x = 11 + x^2$

(3) (C) 0 or 4

Working:

$x^2 + kx + k = 0$

Comparing with $ax^2 + bx + c = 0$, we get $a = 1$, $b = k$, $c = k$

Since the equation has equal roots

$\Delta = 0$

$\therefore$ $b^2 - 4ac = 0$

$\therefore$ $k^2 - 4(1)(k) = 0$

$\therefore$ $k^2 - 4k = 0$

$\therefore$ $k(k - 4) = 0$

$\therefore$ $k = 0$ or $k - 4 = 0$

$\therefore$ $k = 0$ or $k = 4$

(4) (B) 17

Working:

$\sqrt{2}x^2 - 5x + \sqrt{2} = 0$

Comparing with $ax^2 + bx + c = 0$ we get,

$a = \sqrt{2}$, $b = -5$, $c = \sqrt{2}$

$\Delta = b^2 - 4ac$

$= (-5)^2 - 4(\sqrt{2})(\sqrt{2})$

$= 25 - 8$

$= 17$

(5) (B) $x^2 - 8x + 15 = 0$

Working:

$\alpha = 3, \beta = 5$

$\alpha + \beta = 3 + 5 = 8$

$\alpha\beta = 3 \times 5 = 15$

The required equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$$x^2 - 8x + 15 = 0$$

(6) (D) $3x^2 + 15x + 3 = 0$

Working:

Sum of roots is –5.

For the equation $3x^2 + 15x + 5 = 0$

Comparing with $ax^2 + bx + c = 0$

$a = 3, b = 15, c = 3$

Product of roots $= \frac{c}{a} = \frac{-15}{3} = -5$

(7) (C) no real roots

Working:

$\sqrt{5}\, m^2 - \sqrt{5}\, m + \sqrt{5} = 0$

Comparing with $am^2 + bm + c = 0$

$a = \sqrt{5}, b = -\sqrt{5}, c = \sqrt{5}$

$\therefore\ b^2 - 4ac = (-\sqrt{5})^2 - 4(\sqrt{5})(\sqrt{5})$

$= 5 - 20$

$= -15$

Since, $b^2 - 4ac < 0$

The equation has no real roots.

(8) (C) $\frac{1}{2}$

Working:

Since 2 is the one of the root of $x^2 + mx - 5 = 0$,

It satisfies the equation

$\therefore$ Substituting $x = 2$ in the equation, we get

$(2)^2 + m(2) - 5 = 0$

$\therefore\ 4 + 2m - 5 = 0$

$\therefore\ 2m - 1 = 0$

$\therefore\ 2m = 1$

$\therefore\ m = \frac{1}{2}$

(9) (A) $\frac{-b}{a}$ (10) (D) 9 and 7

(11) (A) zero (12) (C) 1

(13) (D) $1 + \sqrt{3}$ (14) (B) $b^2 - 4ac = 0$

(15) (B) –3 (16) (B) $x^2 + 4x - 1 = 0$

(17) (C) $2x^2 - 7x + 5 = 0$ (18) (D) both A and B

(19) (C) $3x^2 = 363$ (20) (C) $\frac{5}{x} - 3 = x^2$

(21) (D) $\frac{c}{a}$ (22) (A) $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

(23) (C) 20

PROBLEMS FOR PRACTICE

(1) If $x = 9$ is one root of the quadratic equation $x^2 - 11x + k = 0$, then find the value of k. *(2 marks)*

(2) If one root of the quadratic equation $3y^2 - ky + 8 = 0$ is $\frac{2}{3}$, then find the value of k. *(2 marks)*

(3) Determine whether the given values of x are the roots of given quadratic equation $6x^2 - x - 2 = 0$,

$x = \frac{-1}{2}, x = 5$ *(2 marks)*

(4) Which of the following are quadratic equations? *(1 mark)*

(i) $x - \frac{5}{x} = 3x - 9$ (ii) $(x + 3)(x - 4) = 0$

(iii) $\frac{5}{x} - 3 = x^2$ (iv) $n^3 - n + 4 = n^3$

(v) $x - 3 = 4x^2$

(5) Write the quadratic equations in $ax^2 + bx + c = 0$ form and find the values of a, b, c. *(2 marks)*

(i) $m(m - 7) = 0$ (ii) $\frac{x^2 - 7}{x} = 7$

(iii) $x - \frac{6}{x} = 5$ (iv) $(x + 5)(x - 11)$

(6) Solve the following quadratic equations by factorization method: *(2 marks)*

(i) $x^2 - 5x - 36 = 0$ (ii) $3y^2 - 14y + 8 = 0$

(iii) $64m^2 - 625 = 0$ (iv) $x^2 - 13x + 30 = 0$

(v) $16x^2 - 24x = 0$ (vi) $x^2 - 3\sqrt{3}\,x + 6 = 0$

(vii) $7x^2 + 4x - 20 = 0$ (viii) $m^2 - 7 = 0$

(7) Solve the following quadratic equations by completing square method: *(2 marks)*

(i) $x^2 + 8x + 15 = 0$ (ii) $m^2 - 2m - 1 = 0$

(iii) $x^2 + 3x + 1 = 0$ (iv) $3y^2 + 7y + 1 = 0$

(v) $4p^2 + 7 = 12p$ (vi) $6m^2 + m = 2$

(8) Solve the following quadratic equations by using formula method: *(2 marks)*

(i) $x^2 + 2x - 7 = 0$ (ii) $3x^2 + 8x + 3 = 0$

(iii) $5m^2 + 5m = 1$ (iv) $x^2 + 4x - 1 = 0$

(v) $4x^2 + x - 5 = 0$ (vi) $9y^2 - 5y - 4 = 0$

(9) Find the value of discriminant for each of the following quadratic equations: *(2 marks)*

(i) $x^2 + 4x + 4 = 0$ (ii) $x^2 + 4x + 1 = 0$

(iii) $3x^2 + 2x + 1 = 0$

(10) Determine the nature of roots of the following quadratic equations from their discriminants: *(2 marks)*

(i) $x^2 - 8x + 16 = 0$ (ii) $2x^2 - 3x - 4 = 0$

(iii) $4x^2 - 8x + 9 = 0$

(11) Form the quadratic equation if its roots are

(i) 0 and –4 (ii) $\frac{1}{2}$ and $\frac{-3}{4}$

(iii) $(\sqrt{2}+\sqrt{3})$ and $(\sqrt{2}-\sqrt{3})$ *(3 marks)*

(12) Find the value of k for which given quadratic equations are real and equal roots. *(3 marks)*

(i) $k^2x^2 - 2(k-1)x + 4 = 0$ (ii) $4x^2 - 3kx + 1 = 0$

(13) If α and β are the roots of equation $x^2 - 4x + 1 = 0$, find (i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$ *(3 marks)*

(14) Find *k*, if one root of the equation $5x^2 + 6x + k = 0$ is five times the other. *(3 marks)*

(15) A man riding on a bicycle covers a distance of 60 km in a direction of wind and comes back to his original position in 8 hours. If the speed of the wind is 10 km/hr, find the speed of the bicycle. *(4 marks)*

(16) One tank can be filled up by two taps in 6 hours. The smaller tap alone takes 5 hours more than the bigger tap alone. Find the time required by each tap to fill the tank separately. *(4 marks)*

(17) The sum of the squares of two consecutive natural numbers is 113. Find the numbers. *(4 marks)*

(18) The sum of the squares of two consecutive natural numbers is 100. Find the numbers. *(4 marks)*

(19) For doing some work, Ganesh takes 10 days more than John. If both work together, they will complete the work in 12 days. Find the number of days if Ganesh work alone? *(4 marks)*

(20) In garden, there are some rows and columns. The number of trees in a row is greater than that in each column by 10. Find the number of trees in each row if the total number of trees are 200. *(4 marks)*

(21) The sum of reciprocals of Reshma's ages (in years) 3 years ago and 5 years after from now is $\frac{1}{3}$. Find her present age. *(4 marks)*

ANSWERS

(1) $k = 18$ (2) $k = 14$

(3) $x = \frac{-1}{2}$ is the root and $x = 5$ is not the root of given equation.

(4) (i), (ii) and (v) are quadratic equations.

(5) (i) $m^2 - 7m + 0 = 0$ and $a = 1, b = -7, c = 0$

(ii) $x^2 - 7x - 7 = 0$ and $a = 1, b = -7, c = -7$

(iii) $x^2 - 5x - 6 = 0$ and $a = 1, b = -5, c = -6$

(iv) $x^2 - 6x - 55 = 0$ and $a = 1, b = -6, c = -55$

(6) (i) $x = 9$ or $x = -4$ (ii) $y = 4$ or $y = \frac{2}{3}$

(iii) $m = \frac{-25}{8}$ or $m = \frac{25}{8}$ (iv) $x = 10$ or $x = 3$

(v) $x = 0$ or $x = \frac{3}{2}$ (vi) $x = \sqrt{3}$ or $x = 2\sqrt{3}$

(vii) $x = -2$ or $x = \frac{10}{7}$ (viii) $x = \sqrt{7}$ or $x = -\sqrt{7}$

(7) (i) $x = -3$ or $x = -5$

(ii) $m = 1+\sqrt{2}$ or $m = 1-\sqrt{2}$

(iii) $m = \frac{-3+\sqrt{5}}{2}$ or $m = \frac{-3-\sqrt{5}}{2}$

(iv) $y = \frac{-7+\sqrt{37}}{6}$ or $m = \frac{-7-\sqrt{37}}{6}$

(v) $p = \frac{3+\sqrt{2}}{2}$ or $p = \frac{3-\sqrt{2}}{2}$

(vi) $m = \frac{1}{2}$ or $m = \frac{-2}{3}$

(8) (i) $-1 + 2\sqrt{2}$ and $-1 - 2\sqrt{2}$

(ii) $\frac{-4+\sqrt{7}}{3}$ and $\frac{-4-\sqrt{7}}{3}$

(iii) $\frac{-5+3\sqrt{5}}{10}$ and $\frac{-5-3\sqrt{5}}{10}$

(iv) $-2+\sqrt{5}$ and $-2-\sqrt{5}$

(v) 1 and $\frac{-5}{4}$

(vi) 1 and $\frac{-4}{9}$

(9) (i) 0 (ii) 12 (iii) –8

(10) (i) real and equal (ii) real and unequal

(iii) not real

(11) (i) $x^2 + 4x = 0$ (ii) $8x^2 + 2x - 3 = 0$

(iii) $x^2 - 2\sqrt{2}\,x - 1 = 0$

(12) (i) $k = -1$ or $k = \frac{1}{3}$ (ii) $k = \frac{-4}{3}$ or $k = \frac{4}{3}$

(13) (i) 14 (ii) 52

(14) $k = 1$

(15) 20 *km/hr.*

(16) 10 hours and 15 hours (17) 7 and 8

(18) 6 and 8 (19) 30 days

(20) 20 (21) 7 years

❖ ❖ ❖

ASSIGNMENT – 2

Time : 1 Hr. **Marks : 20**

Q.1. (A) Choose the proper alternatives for the question below: **(1)**

(1) If α and β are the roots of quadratic equation $2x^2 + 4x + 3 = 0$, then the value of $\alpha + \beta = \square$

(A) -2 (B) 2 (C) 34 (D) -4

(B) Perform the following activities: **(3)**

(1) Write the following quadratic equation in standard form: $(m + 4)(m - 10) = 0$

Solution: $\square\, m^2 - \square\, m - \square = 0$ is the standard form of quadratic equation.

(3) If sum of the roots of quadratic equation is 10 and the product is 9, then form the quadratic equation:

Solution: $x^2 - \square\, x + \square = \square$

(4) Find the values of a, b, c for the following quadratic equation by comparing with standard form: $x^2 - x - 3 = 0$

Solution: Quadratic Equation is $x^2 - x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = \square$, $b = \square$, $c = \square$

Q.2. Perform of the following activities: (Any 1) **(2)**

(1) Solve by factorization method : $m^2 - 25 = 0$

Solution: $m^2 - 25 = 0$

$\therefore$ $(\ldots\ldots\ldots\ldots)^2 - (\ldots\ldots\ldots\ldots)^2 = 0$

$\therefore$ $(\ldots\ldots\ldots\ldots)\ (\ldots\ldots\ldots\ldots) = 0$

$\therefore$ $m = \ldots\ldots\ldots\ldots$ or $m = \ldots\ldots\ldots\ldots$

(2) If $\alpha = -5$ and $\beta = 9$, then form the quadratic equation.

Solution: The required quadratic equation is

$x^2 + (\square + \square)x + \square \times \square = 0$

$\therefore$ $x^2 - \square\, x - \square = 0$

Q.3. Perform the following activities: (Any 2) **(6)**

(1) Vivek is 5 years elder to Kishore. The sum of the reciprocal of their age is $\frac{1}{6}$. Find their present ages by completing the given flow chart.

Let the present age of Kishore be x years. → $\therefore$ The present age of Vivek = $\square$ → $\frac{1}{x} + \square = \frac{1}{6}$

↓

$x \neq \square$ as age can not be –ve ← Find the values of x ← Form the quadratic equation

↓

Find the present age of Vivek and Kishore

(2) Solve the following quadratic equation by formula method: $x^2 + 6x + 5 = 0$

Solution: $x^2 + 6x + 5 = 0$

Here, $a = \square$, $b = \square$, $c = \square$

$$b^2 - 4ac = \square^2 - 4 \times \square \times \square$$

$$= \square$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-\square \pm \sqrt{\square}}{2 \times \square}$$

$$x = \frac{-\square \pm \square}{\square}$$

$$\therefore \quad x = \square \text{ or } x = \square$$

(3) If α and β are the roots of quadratic equation $x^2 + 5x - 1 = 0$ then, find (i) $\alpha^3 + \beta^3$ (ii) $\alpha^2 + \beta^2$

Solution: $x^2 + 5x - 1 = 0$

Here, $a = 1$, $b = 5$, $= -1$

$$\alpha + \beta = \frac{-\square}{\square} = -\square,$$

$$\alpha \times \beta = \frac{\square}{\square} = \square$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= \square^3 - 3 \times \square \times \square$$

$$\therefore \quad \alpha^3 + \beta^3 = \square$$

$$\alpha^2 + \beta^2 = \square^2 - \square$$

$$= \square^2 - 2 \times \square$$

$$\therefore \quad \alpha^2 + \beta^2 = \square$$

Q.4. Attempt of the following: (Any 2) **(8)**

(1) For doing some work, Pintu takes 6 days more than Nishu. If both work together, they complete the work in 4 days. Find the number of days if Pintu and Nishu work alone.

(2) Find m, if the roots of the quadratic equation $(m - 12)x^2 + 2(m - 12)x + 2 = 0$ are real and equal.

(3) Pratik travels by boat 36 km down a river and back in 8 hours. If the speed of his boat in still water is 12 km/hr, find the speed of the river current.

❖ ❖ ❖

3 Arithmetic Progression

... INDEX ...

Points to Remember:

- **Sequence:**

Set of natural numbers are 1, 2, 3, 4, ...

Here number 4 is in the 4^{th} position and number 13 is in the 13^{th} position. So we can predict the position of any number in this pattern.

1, 4, 9, 16, 25, 36, 49, ...

In this pattern, $25 = 5^2$ and $49 = 7^2$.

Hence in this pattern, 25 and 49 are in 5^{th} and 7^{th} position respectively.

If set of numbers are arranged in a definite order and according to some precise rule, then it is called a **sequence**.

Each number in the sequence is called a **term of a sequence**.

The first term of the sequence is denoted by t_1 or a, the second term is denoted by t_2, the third term is denoted by t_3 and so on.

In general, the n^{th} term, is denoted by t_n.

A sequence is usually denoted by $\{t_n\}$

Activity I: Some sequences are given below. Show the position of the terms by t_1, t_2, t_3, ...

[Textbook Page No. 56]

(1) 9, 15, 21, 27, ...

Here, $t_1 = 9$, $t_2 = 15$, $t_3 = 21$ .....

(2) 7 , 7 , 7 , 7, ...

Here, $t_1 = 7$, $t_2 = \boxed{7}$, $t_3 = \boxed{7}$ ...

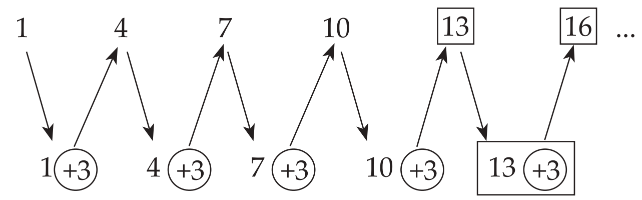
(3) –2 , –6 , –10, –16, ...

Here, $t_1 = -2$, $t_2 = \boxed{-6}$, $t_3 = \boxed{-10}$, ...

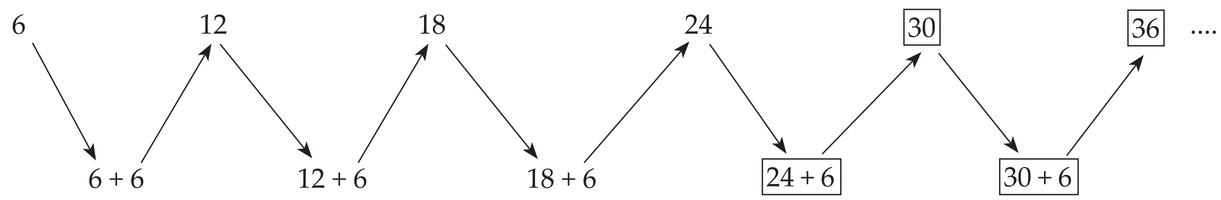
Activity II: some sequences are given below. check whether there is any rule among the terms. Find the similarity between two sequences.

(Textbook Page No. 56)

(1) 1, 4, 7, 10, 13, ...



(2) 6, 12, 18, 24, ...



(3) 3, 3, 3, 3, ...

$$3 \quad 3 \quad 3 \quad 3 \quad \boxed{3} \quad \boxed{3} \quad \ldots$$

$$3+0 \quad 3+0 \quad 3+0 \quad \boxed{3+0} \quad \boxed{3+0}$$

(4) 4, 16, 64, ...

$$4 \quad 16 \quad 64 \quad \boxed{256} \quad \boxed{1024} \quad \boxed{4096} \quad \ldots$$

$$4 \times 4 \quad 16 \times 4 \quad 64 \times 4 \quad \boxed{256 \times 4} \quad \boxed{1024 \times 4}$$

(5) –1, –1.5 , –2, –2.5, ...

$$-1 \quad -1.5 \quad -2 \quad -2.5 \quad \boxed{-3} \quad \ldots$$

$$(-1) + (-0.5) \quad -1.5 + (-0.5) \quad -2 + (0.5) \quad -2.5 + (-0.5)$$

(6) $1^3, 2^3, 3^3, 4^3, \ldots$

In sequence (1), (2), (3), (5), we add a fixed number to previous term to get the next term. Such type of sequence is called **Arithmetic Progression.** Sequence (4) is not an arithmetic progression. Here, we multiply previous term by a fixed number to get the next term. Such progression is called **Geometric Progression**.

Sequence (6) is neither arithmetic progression nor geometric progression.

- **Arithmetic Progression (A. P.):**

A sequence in which the difference between any two consecutive terms $(t_{n+1} - t_n)$ is a fixed number (constant) is called **'Arithmetic Progression' (A.P.)**.

The fixed number is called the **common difference** of the A.P. and is denoted by '*d*'.

The value of '*d*' may be positive, negative or zero. In an A.P.. if first term is *a* and common difference is *d* then,

$t_1 = a$, $t_2 = a + d$, $t_3 = (a + d) + d = a + 2d$.

Thus, an A.P. can be expressed as $a, a + d, a + 2d, a + 3d, \ldots$ where, *a* is the first term and *d* is common difference.

Activity: *(Textbook Page No. 59)*

Write one example of finite and infinite A. P. each.

Ans. Finite A. P. - The sequence of three digits natural numbers divisible by 5.

∴ 100, 105, 110, ..., 995.

Infinite A.P.- The sequence of all even numbers.

∴ 2, 4, 6, 8 ....

MASTER KEY QUESTION SET - 3

Practice Set - 3.1 *(Textbook Page No. 61)*

*(1) **Which of the following sequences are A.P.? If they are A.P. find the common difference.**

(i) **2, 4, 6, 8, ...** *(1 mark)*

Solution:

Here $t_1 = 2$, $t_2 = 4$, $t_3 = 6$, $t_4 = 8$

$t_2 - t_1 = 4 - 2 = 2$

$t_3 - t_2 = 6 - 4 = 2$

$t_4 - t_3 = 8 - 6 = 2$

This shows that the difference between any two consecutive terms is constant.

∴ **Hence, the given sequence is an A.P. and common difference (*d*) is 2.**

(ii) **$2, \frac{5}{2}, 3, \frac{7}{3}, \ldots$** *(1 mark)*

Solution:

Here, $t_1 = 2$, $t_2 = \frac{5}{2}$, $t_3 = 3$, $t_4 = \frac{7}{3}$

$$t_2 - t_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$

$$t_3 - t_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$$

$t_4 - t_3 = \frac{7}{3} - 3 = \frac{7-9}{3} = -\frac{2}{3}$

This shows that the difference between any two consecutive terms is not constant.

$\therefore$ **Hence, the given sequence is not an A.P.**

(iii) – 10, – 6, – 2, 2, ... *(1 mark)*

Solution:

Here, $t_1 = -10, t_2 = -6, t_3 = -2, t_4 = 2$

$t_2 - t_1 = -6 - (-10) = -6 + 10 = 4$

$t_3 - t_2 = -2 - (-6) = -2 + 6 = 4$

$t_4 - t_3 = 2 - (-2) = 2 + 2 = 4$

This shows that the difference between any two consecutive terms is constant.

$\therefore$ **Hence, the given sequence is an A.P. and common difference (d) is 4.**

(iv) 0.3 , 0.33 , 0.333, ... *(1 mark)*

Solution:

Here, $t_1 = 0.3, t_2 = 0.33, t_3 = 0.333$

$t_2 - t_1 = 0.33 - 0.3 = 0.03$

$t_3 - t_2 = 0.333 - 0.33 = 0.003$

$\therefore \; t_3 - t_2 \neq t_2 - t_1$

This shows that the difference between any two consecutive terms is not constant.

$\therefore$ **Hence, the given sequence is not an A.P.**

(v) 0, – 4, – 8, –12 *(1 mark)*

Solution:

Here, $t_1 = 0, t_2 = -4, t_3 = -8, t_4 = -12$

$t_2 - t_1 = -4 - 0 = -4$

$t_3 - t_2 = -8 - (-4) = -8 + 4 = -4$

$t_4 - t_3 = -12 - (-8) = -12 + 8 = -4$

This shows that difference between any two consecutive terms is constant.

$\therefore$ **Hence, the given sequence is an A.P. and common difference (d) is – 4.**

(vi) $-\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, \ldots$ *(1 mark)*

Solution:

Here $t_1 = -\frac{1}{5}, \; t_2 = -\frac{1}{5}, \; t_3 = -\frac{1}{5}$

$t_2 - t_1 = -\frac{1}{5} - \left(-\frac{1}{5}\right) = \frac{-1}{5} + \frac{1}{5} = 0$

$t_3 - t_2 = -\frac{1}{5} - \left(-\frac{1}{5}\right) = \frac{-1}{5} + \frac{1}{5} = 0$

This shows that the difference between any two consecutive terms is constant.

$\therefore$ **Hence, the given sequence is an A.P. and common difference (d) is zero.**

(vii) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$ *(2 marks)*

Solution:

Here $t_1 = 3, t_2 = 3+\sqrt{2}, t_3 = 3+2\sqrt{2},$

$t_4 = 3+3\sqrt{2}$

$t_2 - t_1 = 3+\sqrt{2} - 3 = \sqrt{2}$

$t_3 - t_2 = 3+2\sqrt{2} - (3+\sqrt{2})$

$= 3+2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

$t_4 - t_3 = 3+3\sqrt{2} - (3+2\sqrt{2})$

$= 3+3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$

This shows that the difference between any two consecutive terms is constant.

$\therefore$ **Hence, the given sequence is an A.P. and common difference (d) is $\sqrt{2}$.**

(viii) 127 , 132 , 137, ... *(1 mark)*

Solution:

Here $t_1 = 127, t_2 = 132, t_3 = 137$

$t_2 - t_1 = 132 - 127 = 5$

$t_3 - t_2 = 137 - 132 = 5$

This shows that the difference between any two consecutive terms is constant.

$\therefore$ **Hence, the given sequence is an A.P. with common difference (d) is 5.**

(2) Write an A.P. whose first term is a and common difference d in each of the following.

(i) $a = 10, d = 5$ *(1 mark)*

Solution:

$a = t_1 = 10, d = 5$

$t_2 = t_1 + d = 10 + 5 = 15$

$t_3 = t_2 + d = 15 + 5 = 20$

$t_4 = t_3 + d = 20 + 5 = 25$

$\therefore$ **Arithmetic progression is 10, 15, 20, 25, ...**

(ii) $a = -3, d = 0$ *(1 mark)*

Solution:

$a = t_1 = -3, d = 0$

$t_2 = t_1 + d = -3 + 0 = -3$

$t_3 = t_2 + d = -3 + 0 = -3$

$t_4 = t_3 + d = -3 + 0 = -3$

$\therefore$ **Arithmetic progression is –3, – 3, –3, – 3, ...**

(iii) $a = -7, d = \frac{1}{2}$ *(1 mark)*

Solution:

$a = t_1 = -7, d = \frac{1}{2}$

$t_2 = t_1 + d = -7 + \frac{1}{2} = -7 + 0.5 = -6.5$

$t_3 = t_2 + d = -6.5 + \frac{1}{2} = -6.5 + 0.5 = -6$

$t_4 = t_3 + d = -6 + \frac{1}{2} = -6 + 0.5 = -5.5$

$\therefore$ **Arithmetic progression is −7, −6.5, −6, −5.5, ...**

(iv) $a = -1.25, d = 3$ *(1 mark)*

Solution:

$a = t_1 = -1.25, d = 3$

$t_2 = t_1 + d = -1.25 + 3 = 1.75$

$t_3 = t_2 + d = 1.75 + 3 = 4.75$

$t_4 = t_3 + d = 4.75 + 3 = 7.75$

$\therefore$ **Arithmetic progression is −1.25, 1.75, 4.75, 7.75, ...**

(v) $a = 6, d = -3$ *(1 mark)*

Solution:

$a = t_1 = 6, d = -3$

$t_2 = t_1 + d = 6 + (-3) = 3$

$t_3 = t_2 + d = 3 + (-3) = 0$

$t_4 = t_3 + d = 0 + (-3) = -3$

$\therefore$ **Arithmetic progression is 6, 3, 0, −3, ...**

(vi) $a = -19, d = -4$ *(1 mark)*

Solution:

$a = t_1 = -19, d = -4$

$t_2 = t_1 + d = -19 + (-4) = -23$

$t_3 = t_2 + d = -23 + (-4) = -27$

$t_4 = t_3 + d = -27 + (-4) = -31$

$\therefore$ **Arithmetic progression is −19, −23, −27, −31, ...**

(3) Find the first term and common difference for each of the A. P.

(i) 5, 1, −3, −7 *(1 mark)*

Solution:

Here $t_1 = a = 5$,

$t_2 = 1, t_3 = -3, t_4 = -7$

For an A.P., $d = t_{n+1} - t_n$

$d = t_2 - t_1 = 1 - 5 = -4$

$d = t_3 - t_2 = -3 - 1 = -4$

$\therefore$ **First term (a) is 5 and Common difference (d) is − 4.**

(ii) 0.6, 0.9, 1.2, 1.5, ... *(1 mark)*

Solution:

Here, $t_1 = a = 0.6$,

$t_2 = 0.9, t_3 = 1.2, t_4 = 1.5$

For an A.P. $d = t_{n+1} - t_n$

$\therefore d = t_2 - t_1 = 0.9 - 0.6 = 0.3$

$d = t_3 - t_2 = 1.2 - 0.9 = 0.3$

$\therefore$ **First term (a) is 0.6 and Common difference (d) is 0.3.**

(iii) 127, 135, 143, 151, ... *(1 mark)*

Solution:

Here $t_1 = a = 127$,

$t_2 = 135, t_3 = 143, t_4 = 151$

For an A.P., $d = t_{n+1} - t_n$

$d = t_2 - t_1 = 135 - 127 = 8$

$d = t_3 - t_2 = 143 - 135 = 8$

$\therefore$ **First term (a) is 127 and Common difference (d) is 8.**

(iv) $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, ...$ *(2 mark)*

Solution:

Here $t_1 = a = \frac{1}{4}$,

$t_2 = \frac{3}{4}, \quad t_3 = \frac{5}{4}, \quad t_4 = \frac{7}{4}$

For an A. P. $d = t_{n+1} - t_n$

$$d = t_2 - t_1 = \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$d = t_3 - t_2 = \frac{5}{4} - \frac{3}{4} = \frac{5-3}{4} = \frac{2}{4} = \frac{1}{2}$$

$\therefore$ **First term (a) is $\frac{1}{4}$ and Common difference (d) is $\frac{1}{2}$.**

Points to Remember:

- **n^{th} term of an A.P.:**

  In the given A.P. $t_1, t_2, t_3, ...$ if the first term is a and the common difference is d then,

$t_1 = a$

$t_2 = t_1 + d = a + d = a + (2 - 1)\,d,$

$t_3 = t_2 + d = a + d + d = a + 2d = a + (3 - 1)\,d$

$t_4 = t_3 + d = a + 2d + d = a + 3d = a + (4 - 1)\,d$

$\therefore\ t_n = a + (n - 1)\,d$

Thus the general term i.e. the n^{th} term of an A.P. with the first term as '*a*' and the common difference '*d*' is given as

$\boxed{t_n = a + (n - 1)\,d}$

Let's think (Textbook Page No. 62)

- **Is 5, 8, 11, 14..... an A.P.? If so then what will be the 100th term? Check whether 92 is in this A.P.? Is number 61 in this A.P.?**

Solution:

5, 8, 11, 14.....

Here $t_1 = a = 5,\ t_2 = 8,\ t_3 = 11$

$t_2 - t_1 = 8 - 5 = 3,$

$t_3 - t_2 = 11 - 8 = 3$

Hence the given sequence forms an A.P. with $a = 5$ and $d = 3$.

For $n = 100$

$t_n = a + (n - 1)d$

$\therefore\ t_{100} = 5 + (100 - 1)3$

$= 5 + 99 \times 3$

$= 5 + 297$

$t_{100} = 302$

$\therefore$ **100th term = 302**

Let $t_p = 92$

$\therefore\ t_p = a + (p - 1)d$

$\therefore\ 92 = 5 + (p - 1)3$

$\therefore\ 92 - 5 = (p - 1)3$

$\therefore\ 87 = (p - 1)3$

$\therefore\ (p - 1) = \dfrac{87}{3} = 29$

$\therefore\ p = 29 + 1 = 30$

$\therefore$ **92 is the 30th term of the A.P.**

Let $t_q = 61$

$\therefore\ t_q = a + (q - 1)d$

$\therefore\ 61 = 5 + (q - 1)3$

$\therefore\ 61 - 5 = (q - 1)3$

$\therefore\ 56 = 3q - 3$

$\therefore\ 3q = 59$

$\therefore\ q = \dfrac{59}{3} = 19.66$

Since q should be positive integer

$\therefore$ **61 is not in this A.P.**

Practice Set - 3.2 *(Textbook Page No. 66)*

(1) Write the correct number in the given boxes from the following A.P.

(i) 1, 8, 15, 22, ... *(2 marks)*

Solution:

Here, $a = \boxed{1},\ t_1 = \boxed{1},\ t_2 = \boxed{8},\ t_3 = \boxed{15}, \ldots$

$t_2 - t_1 = \boxed{8} - \boxed{1} = \boxed{7}$

$t_3 - t_2 = \boxed{15} - \boxed{8} = \boxed{7}$

$\therefore\ d = \boxed{7}$

(ii) 3, 6, 9, 12, ... *(2 marks)*

Solution:

Here, $t_1 = \boxed{3},\ t_2 = \boxed{6},\ t_3 = \boxed{9},\ t_4 = \boxed{12}, \ldots..$

$t_2 - t_1 = \boxed{3},\ t_3 - t_2 = \boxed{3}$

$\therefore\ d = \boxed{3}$

(iii) –3 , –8 , –13 , –18 *(2 marks)*

Solution:

Here, $t_1 = \boxed{-3},\ t_2 = \boxed{-8},\ t_3 = \boxed{-13},\ t_4 = \boxed{-18}, \ldots$

$t_2 - t_1 = \boxed{-5},\ t_3 - t_2 = \boxed{-5}$

$\therefore\ a = \boxed{-3},\ d = \boxed{-5}$

(iv) 70, 60, 50, 40, ... *(2 marks)*

Solution:

Here, $t_1 = \boxed{70},\ t_2 = \boxed{60},\ t_3 = \boxed{50}$

$a = \boxed{70},\ d = \boxed{-10}$

working $d = t_2 - t_1$

$= 60 - 70$

$= \boxed{-10}$

(2) Decide whether following sequence is an A.P., if so find 20th term of the progression. *(3 marks)*

–12, –5, 2, 9, 16, 23, 30, ...

Solution:

Here $t_1 = -12,\ t_2 = -5,\ t_3 = 2,\ t_4 = 9, \ldots$

$t_2 - t_1 = -5 - (-12) = -5 + 12 = 7$

$t_3 - t_2 = 2 - (-5) = 2 + 5 = 7$

$t_4 - t_3 = 9 - 2 = 7$

This shows that the difference between any two consecutive terms is constant.

$\therefore$ The given sequence is an A.P.

Here $a = -12$, $d = 7$, $t_{20} = ?$

$t_n = a + (n - 1)\,d$

$t_n = -12 + (n - 1)\,d$

$\therefore\ t_{20} = -12 + (20 - 1)\,7$ $\quad [\because n = 20]$

$= -12 + 19 \times 7$

$= -12 + 133$

$\therefore\ t_{20} = 121$

$\boxed{\textbf{Thus } 20^{th} \textbf{ term of the A.P. is 121.}}$

(3) Given Arithmetic Progression 12, 16, 20, 24, ... Find the 24^{th} term of this progression. *(2 marks)*

Solution:

The A.P. 12, 16, 20, 24......

Here $a = 12$, $d = 16 - 12 = 4$, $t_{24} = ?$

$t_n = a + (n - 1)\,d$

$\therefore\ t_{24} = 12 + (24 - 1)\,4$ $\quad (\because n = 24)$

$= 12 + 23 \times 4$

$= 12 + 92$

$\therefore\ t_{24} = 104$

$\boxed{\textbf{Thus } 24^{th} \textbf{ term of the A.P. is 104.}}$

(4) Find the 19^{th} term of the following A.P. 7, 13, 19, 25, ... *(2 marks)*

Solution:

The A.P. is 7, 13, 19, 25, ...

Here $a = 7$, $d = 13 - 7 = 6$, $t_{19} = ?$

$t_n = a + (n - 1)\,d$

$t_{19} = 7 + (19 - 1)\,6$ $\quad (\because n = 19)$

$= 7 + 18 \times 6$

$= 7 + 108$

$t_{19} = 115$

$\boxed{\textbf{Thus } 19^{th} \textbf{ term of the A.P. is 115}}$

(5) Find the 27^{th} term of the following A.P.

9, 4, –1, –6, –11, ... *(2 marks)*

Solution:

The A.P. is 9, 4, –1, – 6, –11, ...

Here, $a = 9$, $d = 4 - 9 = -5$, $t_{27} = ?$

$t_n = a + (n - 1)\,d$

$\therefore\ t_{27} = 9 + (27 - 1)(-5)$ $\quad (\because n = 27)$

$= 9 + 26 \times (-5)$

$= 9 - 130$

$\therefore\ t_{27} = -121$

$\boxed{\textbf{Thus } 27^{th} \textbf{ term of the A.P. is } -121}$

Problem Set - 3 *(Textbook Page No. 79)*

(2) Find the fourth term from the end in an A.P. –11, –8, –5, ..., 49 *(3 marks)*

Solution:

Method - I

For the reverse A.P. 49, ..., –5, –8, –11

$a = 49$, $d = -8 - (-5) = -8 + 5 = -3$

$\therefore$ Fourth term from the end of given A.P. is t_4 in the reverse A.P.

We know,

$t_n = a + (n - 1)\,d$

$\therefore\ t_4 = 49 + (4 - 1)(-3)$

$\therefore\ t_4 = 49 + 3(-3)$

$\therefore\ t_4 = 49 - 9$

$\therefore\ t_4 = 40$

$\therefore$ $\boxed{\textbf{Fourth term from the end of given A.P. is 40}}$

Method - II

For an A.P., First term is a and Common difference is d. Let l be the last term of this A.P. Then n^{th} term from the end $= l - (n - 1)d$

The given A.P. is –11, –8, –5, ... 49.

Here $a = -11$, $d = -8 - (-11) = -8 + 11 = 3$ and last term $(l) = 49$

$\therefore$ 4th term from the end $= l - (n - 1)d$

$= 49 - (4 - 1)3$

$= 49 - 3 \times 3$

$= 49 - 9$

$= 40$

$\therefore$ $\boxed{\textbf{Fourth term from the end of given A.P. is 40}}$

Practice Set - 3.2 *(Textbook Page No. 66)*

(7) The 11^{th} term and the 21^{st} term of an A.P. are 16 and 29 respectively, then find the 41^{st} term of that A.P. *(3 marks)*

Solution:

Given For an A.P. : $t_{11} = 16$ and $t_{21} = 29$

To find : $t_{41} = ?$

$t_n = a + (n - 1)\,d$

$\therefore\ t_{11} = a + (11 - 1)\,d$

$\therefore\ 16 = a + 10d$

$\therefore\ a + 10d = 16$ $\quad$ ...(i)

Also $t_{21} = a + (21 - 1)\,d$

$\therefore$ 29 $= a + 20d$

$\therefore$ $a + 20d = 29$ ...(ii)

Subtracting (i) from (ii),

$$\begin{array}{rcrcr} a & + & 20d & = & 29 \\ a & + & 10d & = & 16 \\ (-) & & (-) & & (-) \\ \hline & & 10d & = & 13 \end{array}$$

$\therefore$ $d = \frac{13}{10}$

Substituting $d = \frac{13}{10}$ in equation (i)

$a + 10d = 16$

$\therefore$ $a + 10 \times \frac{13}{10} = 16$

$\therefore$ $a + 13 = 16$

$\therefore$ $a = 16 - 13$

$\therefore$ $a = 3$

Here, $a = 3$; $d = \frac{13}{10}$, $t_{41} = ?$

$t_n = a + (n - 1)\, d$

$\therefore$ $t_{41} = 3 + (41 - 1)\, \frac{13}{10}$ [$\because n = 41$]

$= 3 + 40 \times \frac{13}{10}$

$= 3 + 4 \times 13$

$= 3 + 52$

$\therefore$ $t_{41} = 55$

Thus 41st term of the A.P. is 55.

Problem Set - 3 *(Textbook Page No. 79)*

(3) In an A.P. 10th term is 46, sum of 5th and 7th term is 52. Find the A.P. *(3 marks)*

Solution:

For an A.P., $t_{10} = 46$ and $t_5 + t_7 = 52$

$t_n = a + (n - 1)\, d$

$\therefore$ $t_{10} = a + (10 - 1)\, d$

$\therefore$ $46 = a + 9d$

$\therefore$ $a + 9d = 46$ ...(i)

Now, $t_5 = a + (5 - 1)\, d$

$\therefore$ $t_5 = a + 4d$ and ...(ii)

$t_7 = a + (7 - 1)\, d$

$\therefore$ $t_7 = a + 6d$ ...(iii)

We have $t_5 + t_7 = 52$ ...(given)

$\therefore$ $a + 4d + a + 6d = 52$ ...[from (ii) and (iii)]

$\therefore$ $2a + 10d = 52$

$\therefore$ $a + 5d = 26$ .... (iv) (dividing both sides by 2)

Subtracting equation (iv) from equation (i)

$$\begin{array}{rcrcr} a & + & 9d & = & 46 \\ a & + & 5d & = & 26 \\ (-) & & (-) & & (-) \\ \hline & & 4d & = & 20 \end{array}$$

$\therefore$ $d = \frac{20}{4}$

$\therefore$ $\boldsymbol{d = 5}$

Substituting $d = 5$ in equation (i) we get,

$a + 9d = 46$

$\therefore$ $a + 9 \times 5 = 46$

$\therefore$ $a + 45 = 46$

$\therefore$ $a = 46 - 45$

$\therefore$ $\boldsymbol{a = 1}$

Now $t_1 = a = 1$

$t_2 = t_1 + d = 1 + 5 = 6$

$t_3 = t_2 + d = 6 + 5 = 11$

$t_4 = t_3 + d = 11 + 5 = 16$

Hence, the given A.P. is 1, 6, 11, 16...

(6) If sum of 3rd and 8th terms of an A.P. is 7 and sum of 7th and 14th terms is –3 then find 10th term. *(3 marks)*

Given: For an A.P. $t_3 + t_8 = 7$, $t_7 + t_{14} = -3$,

To find : t_{10}

Solution:

$t_n = a + (n - 1)\, d$

$\therefore$ $t_3 = a + (3 - 1)\, d$

$\therefore$ $t_3 = a + 2d$ ...(i)

Also $t_8 = a + (8 - 1)\, d$

$\therefore$ $t_8 = a + 7d$ ...(ii)

Now, $t_3 + t_8 = 7$ ...(given)

$\therefore$ $a + 2d + a + 7d = 7$ [...from (i) and (ii)]

$\therefore$ $2a + 9d = 7$ ...(iii)

$t_n = a + (n - 1)\, d$

$\therefore$ $t_7 = a + (7 - 1)\, d$

$\therefore$ $t_7 = a + 6d$ ... (iv)

Also $t_{14} = a + (14 - 1)\, d$

$\therefore$ $t_{14} = a + 13d$ ...(v)

Now, $t_7 + t_{14} = -3$ ...(given)

$\therefore$ $a + 6d + a + 13d = -3$ ...(from (iv) and (v))

$\therefore$ $2a + 19d = -3$ ...(vi)

Subtracting (vi) from (iii) we get,

$$\begin{array}{rcrcr} 2a & + & 9d & = & 7 \\ 2a & + & 19d & = & -3 \\ (-) & & (-) & & (+) \\ \hline & & -10d & = & 10 \end{array}$$

$\therefore \quad d = \dfrac{10}{-10}$

$\therefore \quad \mathbf{d = -1}$

Substituting $d = -1$ in (iii), we get,

$2a + 9d = 7$

$\therefore \quad 2a + 9 \times (-1) = 7$

$\therefore \quad 2a - 9 = 7$

$\therefore \quad 2a = 7 + 9$

$\therefore \quad 2a = 16$

$\therefore \quad a = \dfrac{16}{2}$

$\therefore \quad \mathbf{a = 8}$

$t_n = a + (n - 1)\, d$

$\therefore \quad t_{10} = 8 + (10 - 1)(-1)$

$\therefore \quad = 8 + (9)(-1)$

$\therefore \quad = 8 - 9$

$\therefore \quad \boxed{\mathbf{t_{10} = -1}}$

Practice set - 3.2 *(Textbook Page No. 66)*

(10) In an A.P. 17th term is 7 more than 10th term. Find the common difference? *(3 marks)*

Given: For an A.P. $t_{17} = t_{10} + 7$

To find : $d = ?$

Solution:

$t_n = a + (n - 1)\, d$

$\therefore \quad t_{17} = a + (17 - 1)\, d$

$\therefore \quad t_{17} = a + 16d \quad \text{...(I)}$

Also, $t_{10} = a + (10 - 1)\, d$

$t_{10} = a + 9d \quad \text{...(II)}$

Now, $t_{17} = t_{10} + 7 \quad \text{...(Given)}$

$\therefore \quad a + 16d = a + 9d + 7 \quad \text{...[From (i) and (ii)]}$

$\therefore \quad 7d = 7$

$\therefore \quad d = \dfrac{7}{7}$

$\therefore \quad d = 1$

$\therefore$ **The common difference is 1.**

(6) Find how many three digit natural numbers are divisible by 5. *(3 marks)*

Solution:

The three digit natural numbers divisible by 5 are 100, 105, 110, ... , 995

Here, $t_1 = 100,\ t_2 = 105,\ t_3 = 110, \ldots$

$t_2 - t_1 = 105 - 100 = 5$

$t_3 - t_2 = 110 - 105 = 5$

Since the difference between any two consecutive terms is constant.

$\therefore$ The given sequence is an A.P., where $a = 100$, $d = 5$

let $t_n = 995,\ n = ?$

$t_n = a + (n - 1)\, d$

$\therefore \quad 995 = 100 + (n - 1)\, 5$

$\therefore \quad 895 = 5n - 5$

$\therefore \quad 895 + 5 = 5n$

$\therefore \quad 5n = 900$

$\therefore \quad n = \dfrac{900}{5}$

$\therefore \quad n = 180$

There are 180 three digit natural numbers which are divisible by 5.

(9) In the natural numbers from 10 to 250, how many are divisible by 4? *(3 marks)*

Solution:

The natural numbers from 10 to 250 divisible by 4 are: 12, 16, 20, ... 248

Here $t_1 = 12,\ t_2 = 16,\ t_3 = 20, \ldots$

$t_2 - t_1 = 16 - 12 = 4,\ t_3 - t_2 = 20 - 16 = 4$

This shows that the difference between any two consecutive terms is constant.

$\therefore$ The given sequence is an A.P., where $a = 12$, $d = 4$,

let $t_n = 248,\ n = ?$

$t_n = a + (n - 1)\, d$

$\therefore \quad 248 = 12 + (n - 1)\, 4$

$\therefore \quad 248 - 12 = 4n - 4$

$\therefore \quad 236 = 4n - 4$

$\therefore \quad 236 + 4 = 4n$

$\therefore \quad 4n = 240$

$\therefore \quad n = \dfrac{240}{4}$

$\therefore \quad n = 60$

Thus, there are 60 natural numbers from 10 to 250 which are divisible by 4.

(8) 11, 8, 5, 2, ... In this A.P. which term is number –151? *(3 marks)*

Solution:

The A.P. is 11, 8, 5, 2, ...

Here, $a = 11,\ d = t_2 - t_1 = 8 - 11 = -3,\ t_n = -151$, $n = ?$

$t_n = a + (n - 1)\, d$

$\therefore \quad -151 = 11 + (n - 1)(-3)$

$\therefore \quad -151 = 11 - 3n + 3$

$\therefore \quad -151 = 14 - 3n$

$\therefore \quad 3n = 14 + 151$

$\therefore \quad 3n = 165$

$\therefore \quad n = \frac{165}{3}$

$\therefore \quad n = 55$

Hence, 55th term of the A.P. is –151.

Problem Set - 3 *(Textbook Page No. 79)*

(5) Two A.P.'s are given 9, 7, 5... and 24, 21, 18, ...If nth term of both the progressions are equal then find the value of n and n^{th} term *(3 marks)*

Solution:

The first A.P. is 9, 7, 5, ...

Here, $a = 9$, $d = 7 - 9 = -2$

$t_n = a + (n-1)\, d$

$\therefore \quad t_n = a + (n-1)\, d$

$\therefore \quad t_n = 9 + (n-1)(-2)$

$\therefore \quad t_n = 9 - 2n + 2$

$\therefore \quad t_n = 11 - 2n$ ...(i)

The second A.P. is 24, 21, 18....

Here, $a = 24$, $d = 21 - 24 = -3$

$t_n = a + (n-1)\, d$

$\therefore \quad t_n = 24 + (n-1)(-3)$

$\therefore \quad t_n = 24 - 3n + 3$

$\therefore \quad t_n = 27 - 3n$ ... (ii)

Now, n^{th} term of first A.P. = n^{th} term of second A.P.

$\therefore \quad 11 - 2n = 27 - 3n$ ...[from (i) and (ii)]

$\therefore \quad -2n + 3n = 27 - 11$

$\therefore \quad \boxed{n = 16}$

Now, $t_n = 11 - 2n$ ...[from (i)]

$\therefore \quad t_n = 11 - 2 \times 16$

$\therefore \quad t_n = 11 - 32$

$\therefore \quad \boxed{t_n = -21}$

Practice Set - 3.3 *(Textbook Page No. 73)*

***(9) It the 9th term of an A.P. is zero than show that the 29th term is twice the 19th term.** *(4 marks)*

Solution:

Given: For an A.P. $t_9 = 0$

To prove: $t_{29} = 2t_{19}$ i.e. $t_{29} - 2t_{19} = 0$

Proof:

$t_n = a + (n-1)\, d$

$\therefore \quad t_9 = a + (9-1)\, d$

$0 = a + 8d$

$a + 8d = 0$ ...(i)

$t_{29} = a + (29-1)\, d$

$= a + 28d$

$t_{19} = a + (19-1)\, d$

$= a + 18d$

$\therefore \quad t_{29} - 2t_{19} = a + 28d - 2(a + 18d)$

$= a + 28d - 2a - 36d$

$= -a - 8d$

$= -(a + 8d)$

$= 0$ ...(from (i))

$\therefore \quad t_{29} - 2t_{19} = 0$

$\therefore \quad \boxed{t_{29} = 2t_{19}}$

Problem Set - 3 *(Textbook Page No. 80)*

***(13) If m times the m^{th} term of an A.P. is equal to n times its n^{th} term then show that $(m + n)^{th}$ term of the A.P. is zero.** *(4 marks)*

Solution:

Given : $mt_m = nt_n$

Prove : $t_{m+n} = 0$

Let a be the first term and d be the common difference of the A.P.

Then m^{th} term of the A.P.

$t_m = a + (m-1)\, d$ ...(i)

and n^{th} term of the A.P.

$t_n = a + (n-1)\, d$ ...(ii)

$m(t_m) = n(t_n)$ ...(given)

$\therefore \quad m\,[a + (m-1)d] = n\,[a + (n-1)d]$...[from (i), (ii)]

$\therefore \quad m[a + md - d] = n\,[a + nd - d]$

$\therefore \quad ma + m^2d - md = na + n^2d - nd$

$\therefore \quad ma - na + m^2d - n^2d - md + nd = 0$

$\therefore \quad (m-n)\, a + (m^2 - n^2)\, d - d\,(m-n) = 0$

$\therefore \quad (m-n)\, a + (m+n)(m-n)\, d - d\,(m-n) = 0$

$\because \quad m \neq n$

$\therefore \quad m - n \neq 0$

Dividing throughout by $(m-n)$ we get,

$a + (m+n)\, d - d = 0$

$a + (m + n - 1)\, d = 0$ ...(iii)

Now $t_{m+n} = a + (m + n - 1)\, d$

$t_{m+n} = 0$ ...[from (iii)]

$\therefore \quad$ **$(m + n)^{th}$ term of the A.P. is zero.**

Points to Remember:

- **Sum of first n terms of an A.P.:**

Let us consider an A.P. $a, a + d, a + 2d, a + 3d, \ldots$ $a + (n - 1)\, d$

Here a is the first term, d is the common difference and S_n be the sum of n terms of this A.P.

$S_n = a + (a + d) + \ldots\ldots + [a + (n - 2)\, d] + [a+(n - 1)\, d]$

Re-writing it in reverse order, we get

$S_n =[a +(n - 1)\, d] +[a + (n - 2)\, d] + \ldots. + (a + d) + a$

Adding termwise in pairs, we get,

$2S_n = [a + a + (n - 1)\, d] + [a + d + a + (n - 2)\, d] + \ldots.. + [a + (n - 2)\, d + a] + [a + (n - 1)\, d + a]$

$\therefore\ 2S_n = [2a + (n -1)\, d] + [2a + (n - 1)\, d] + \ldots.$ $+ [2a + (n - 1)\, d]$ *n times*

$\therefore\ 2S_n = n\, [2a + (n - 1)\, d]$

$\therefore\ \boxed{S_n = \dfrac{n}{2}\, [2a + (n -1)\, d]}$ or

$S_n = \dfrac{n}{2}\, [a + a + (n - 1)\, d]$

$S_n = \dfrac{n}{2}\, (a + t_n)$ $\quad (\because\ t_n = a + (n - 1)\, d)$

$\therefore\ \boxed{S_n = \dfrac{n}{2}\, [t_1 + t_n]}$

- **Consecutive terms of an A.P.**

3 Consecutive terms : $a - d,\ a,\ a + d$

4 Consecutive terms : $a - d, a, a + d, a + 2d$

Note : While making assumption for any number of terms in A.P. start with first term as '$a - d$' and then increase each term by 'd'

Example : Find the sum of all odd natural numbers from 1 to 150. *(Textbook page no. 71)*

Solution:

The odd natural numbers from 1 to 150 are 1, 3, 5, ... , 149. which is an A.P.

Here $a = 1$, $d = 2$

First lets find how many odd numbers are there from 1 to 150, so find the value of n. if $t_n = 149$

$t_n = a + (n - 1)\, d$

$\therefore\ 149 = 1 + (n - 1)\, 2$

$\therefore\ 149 = 1 + 2n - 2$

$\therefore\ n = 75$

Now let's find the sum of these 75 numbers $1 + 3 + 5 + \ldots\ldots. + 149$

$a = 1$ and $d = 2$, $n = 75$

Method - I :

$S_n = \dfrac{n}{2}\, [2a + (n - 1)\, d]$

$S_n = \boxed{\dfrac{75}{2}\ [2(1) + (75 - 1)\, 2]}$

$S_n = \boxed{75} \times \boxed{75}$

$S_n = \boxed{5625}$

Method : II

$S_n = \dfrac{n}{2}\, [t_1 + t_n]$

$= \dfrac{75}{2}\, [\,1 + 149]$

$= S_n = \boxed{\dfrac{75}{2}} \times \boxed{150}$

$\therefore\ S_n = \boxed{5625}$

Practice Set - 3.3 *(Textbook Page No. 73)*

(1) First term and common difference of an A.P. are 6 and 3 respectively: Find S_{27} *(2 marks)*

Solution:

$a = 6, d = 3\,, S_{27} = ?$

$S_n = \dfrac{n}{2}\, [\,\boxed{2a} + (n - 1)\, d]$

$\therefore\ S_{27} = \dfrac{27}{2}\, [12 + (27 - 1)\,\boxed{3}\,]$

$= \dfrac{27}{2} \times \boxed{90}$

$= 27 \times 45$

$\therefore\ S_{27} = \boxed{1215}$

(2) Find the sum of first 123 even natural numbers. *(3 marks)*

Solution:

The first n even natural numbers are: 2, 4, 6, 8,.....

They form an A.P. with $a = 2$,

$d = t_2 - t_1 = 4 - 2 = 2$

We know, $S_n = \dfrac{n}{2}\, [2a + (n - 1)\, d]$

$\therefore\ S_{123} = \dfrac{123}{2}\, [2(2) + (123 - 1)2]$

$= \dfrac{123}{2}\, [4 + 122(2)]$

$= \dfrac{123}{2}\, [4 + 244]$

$= \dfrac{123}{2}\, [248]$

$= 123\, [124]$

$= 15252$

$\therefore$ **The sum of first 123 even natural numbers in 15252.**

Problem Set - 3 *(Textbook Page No. 79)*

(4) The A.P. in which 4th term is –15 and 9th term is –30. Find the sum of first 10 numbers. *(4 marks)*

Solution:

Here, $t_4 = -15$, $t_9 = -30$, $S_{10} = ?$

$t_n = a + (n-1)\,d$

$\therefore\ t_4 = a + (4-1)\,d$

$\therefore\ -15 = a + 3d$

$\therefore\ a + 3d = -15$ ...(i)

Also, $t_9 = a + (9-1)\,d$

$\therefore\ -30 = a + 8d$

$\therefore\ a + 8d = -30$ ...(ii)

Subtracting (i) from (ii) we get,

$$\begin{array}{rcrcl} a & + & 8d & = & -30 \\ a & + & 3d & = & -15 \\ (-) & & (-) & & (+) \\ \hline & & 5d & = & -15 \end{array}$$

$$d = \frac{-15}{5}$$

$\therefore$ $\boldsymbol{d = -3}$

Substituting $d = -3$ in equation (I) we get,

$a + 3d = -15$

$\therefore\ a + 3(-3) = -15$

$\therefore\ a - 9 = -15$

$\therefore\ a = -15 + 9$

$\therefore$ $\boldsymbol{a = -6}$

Now, $S_n = \frac{n}{2}\,[2a + (n-1)\,d]$

$S_{10} = \frac{10}{2}\,[2 \times (-6) + (10-1) \times (-3)]$

$= 5\,[-12 + 9 \times -3]$

$= 5\,[-12 - 27]$

$= 5 \times -39$

$\therefore\ S_{10} = \boldsymbol{-195}$

Thus, the sum of the first 10 terms of the A.P. is –195.

(8) Sum of 1 to *n* natural numbers is 36, then find the value of *n*. *(4 marks)*

Solution:

The A.P. is 1, 2, 3, ... , *n*.

They form an A.P. with $t_1 = 1$, $d = 1$, $t_n = n$ and $S_n = 36$

Now $S_n = \frac{n}{2}\,[t_1 + t_n]$

$\therefore\ 36 = \frac{n}{2}\,[1 + n]$

$\therefore\ 72 = n\,[1 + n]$

$\therefore\ 72 = n + n^2$

$\therefore\ n^2 + n - 72 = 0$

$\therefore\ n^2 + 9n - 8n - 72 = 0$

$\therefore\ n\,(n + 9) - 8\,(n + 9) = 0$

$\therefore\ n + 9 = 0$ or $n - 8 = 0$

$\therefore\ n = -9$ or $n = 8$

$\therefore\ n \neq -9$ as n is a natural number.

$\therefore\ n = 8$

$\therefore$ **Thus, the value of n is 8.**

Practice Set - 3.3 *(Textbook Page No. 72)*

***(3) Find the sum of all even numbers between 1 to 350.** *(4 marks)*

Solution:

The even natural numbers from 1 to 350 are 2, 4, 6, 8, ... , 348.

They form an A.P. where $a = 2$, $d = 4 - 2 = 2$

Let $t_n = 348$

Now, $t_n = a + (n-1)\,d$

$\therefore\ 348 = 2 + (n-1)\,2$

$\therefore\ 348 - 2 = 2n - 2$

$\therefore\ 346 = 2n - 2$

$\therefore\ 346 + 2 = 2n$

$\therefore\ 2n = 348$

$\therefore\ n = \frac{348}{2}$

$\therefore\ n = 174$

Now $S_n = [t_1 + t_n]$

$= \frac{174}{2}\,[2 + 348]$

$= \frac{174 \times 350}{2}$

$\therefore\ S_n = 30450$

Thus, the sum of even natural numbers between 1 to 350 is 30, 450.

(5) Complete the following activity to find the sum of natural numbers from 1 to 140 which are divisible by 4. *(3 marks)*

Solution:

Between 1 to 140. natural number divisible by 4

$\downarrow$

4, 8, ..., 136

$\downarrow$

How many numbers? $\therefore n = \boxed{34}$

$\downarrow$

$n = \boxed{34}, a = \boxed{4}, d = \boxed{4}$

$\downarrow$

$t_n = a + (n-1)d$

$\downarrow$

$136 = \boxed{4} + (n-1) \times \boxed{4}$

$\rightarrow n = \boxed{34} \rightarrow S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{34} = \frac{\boxed{34}}{2}[2 \times 4 + (34-1)4] = \boxed{2380}$

Working:

$a = 4, d = 4, t_n = 136$

$\therefore \quad t_n = a + (n-1)\, d$

$136 = 4 + (n-1)\, 4$

$136 = 4 + 4n - 4$

$\therefore \quad n = \frac{136}{4}$

$\mathbf{n = 34}$

$S_{34} = \frac{34}{2}[2 \times 4 + (34-1)4]$

$= 17(8 + 33 \times 4)$

$= 17(140)$

$\therefore \quad S_{34} = 2380$

$\therefore$ **The sum of numbers between 1 to 140, which are divisible by 4 = 2380**

***(6) Sum of first 55 terms in an A.P. is 3300. Find its 28th term.** *(3 marks)*

Solution:

Here $S_{55} = 3300$, $t_{28} = ?$

$S_n = \frac{n}{2}[2a + (n-1)\, \text{d}]$

$\therefore \quad S_{55} = \frac{55}{2}[2a + (55-1)d]$

$\therefore \quad 3300 = \frac{55}{2}[2a + 54d]$

$\therefore \quad 3300 = \frac{55}{2} \times 2\, [a + 27d]$

$\therefore \quad 3300 = 55\, (a + 27\text{d})$

$\therefore \quad a + 27d = \frac{3300}{55}$

$\therefore \quad a + 27d = 60$ ...(i)

Now, $t_n = a + (n-1)\, d$

$\therefore \quad t_{28} = a + (28-1)\, d$

$\therefore \quad t_{28} = a + 27d$

$\therefore \quad t_{28} = 60$ ...[from (i)]

Thus, the 28th term is 60.

(4) In an A.P. 19th term is 52 and 38th term is 128, find sum of first 56 terms. *(3 marks)*

Given : $t_{19} = 52$, and $t_{38} = 128$

To find $S_{56} = ?$

Solution:

$t_n = a + (n-1)\, d$

$\therefore \quad t_{19} = a + (19-1)\, d$

$\therefore \quad 52 = a + 18d$

$\therefore \quad a + 18d = 52$ ...(i)

Also, $t_{38} = a + (38-1)\, d$

$\therefore \quad 128 = a + 37d$

$a + 37d = 128$ ... (ii)

Adding equations (i) and (ii) we get,

$$\begin{array}{rcrcr} a & + & 18d & = & 52 \\ a & + & 37d & = & 128 \\ \hline 2a & + & 55d & = & 180 \end{array}$$ ...(iii)

$S_n = \frac{n}{2}[2a + (n-1)\, d]$

$\therefore \quad S_{56} = \frac{56}{2}[2a + (56-1)d]$

$= 28\, [2a + 55d]$

$= 28 \times 180$ ...[From (iii)]

$\therefore \quad S_{56} = 5040$

Hence, the required sum is 5040.

Problem Set - 3 *(Textbook Page No. 79)*

***(7) In an A.P., first term is – 5 and last term is 45. If sum of all the numbers in the A.P. is 120, then how many terms are there? What is the common difference.** *(3 marks)*

Solution:

Here, $t_1 = a = -5$, $t_n = 45$, $S_n = 120$, $n = ?$, $d = ?$

$\therefore$ Now $S_n = \frac{n}{2}[t_1 + t_n]$

$\therefore \quad 120 = \frac{n}{2}[-5 + 45]$

$\therefore \quad 240 = n\, [40]$

$\therefore \quad n = \dfrac{240}{40}$

$\therefore \quad \mathbf{n = 6}$

$t_n = a + (n-1)d$

$\therefore \quad 45 = -5 + (6-1)d$

$\therefore \quad 45 = -5 + 5d$

$\therefore \quad 45 + 5 = 5d$

$\therefore \quad 5d = 50$

$\therefore \quad d = \dfrac{50}{5}$

$\therefore \quad \mathbf{d = 10}$

Thus, there are 6 terms in the A.P and the common difference is 10.

Practice Set - 3.3 *(Textbook Page No. 73)*

***(7) In an A.P., sum of three consecutive terms is 27 and their product is 504, find the terms. (Assume that three consecutive terms in A.P. are *a–d, a, a + d*)** *(4 marks)*

Solution:

Let the three consecutive terms be $a-d, a, a+d$.

Their sum is 27.

$\therefore \quad a - d + a + a + d = 27$

$\therefore \quad 3a = 27$

$\therefore \quad \mathbf{a = 9}$

The product of all three terms is 504.

$\therefore \quad (a-d) \times a \times (a+d) = 504$

$\therefore \quad (a^2 - d^2) \times a = 504$

...[Using $(a+b)(a-b) = a^2 - b^2$]

$\therefore \quad (9^2 - d^2) \times 9 = 504$

$\therefore \quad 81 - d^2 = \dfrac{504}{9}$

$\therefore \quad 81 - d^2 = 56$

$\therefore \quad d^2 = 81 - 56$

$\therefore \quad d^2 = 25$

$\therefore \quad \mathbf{d = \pm 5}$ ...(Taking square root)

When $d = 5$, $a = 9$, the required terms are $a-d$, a, $a+d$, i.e. 4, 9, 14

When $d = -5$, $a = 9$ the required terms are $a-d$, a, $a+d$, i.e. 14, 9, 4

Hence, the required terms are 4, 9, 14 or 14, 9, 4

***(8) Find four consecutive terms in an A.P. whose sum is 12 and the sum of 3rd and 4th term is 14. (Assume the four consecutive terms in A.P. are *a–d, a, a + d, a + 2d*)** *(4 marks)*

Solution:

Let four consecutive terms in A.P. are $a-d$, a, $a+d$, $a+2d$

Their sum is 12

$\therefore \quad a - d + a + a + d + a + 2d = 12$

$\therefore \quad 4a + 2d = 12$

$\therefore \quad 2(2a + d) = 12$

$\therefore \quad 2a + d = 6$ ...(i)

The sum of 3rd and 4th terms is 14.

$\therefore \quad a + d + a + 2d = 14$

$\therefore \quad 2a + 3d = 14$ ...(ii)

Subtracting (ii) from (i),

$$\begin{array}{rcrcr} 2a & + & d & = & 6 \\ 2a & + & 3d & = & 14 \\ (-) & & (-) & & (-) \\ \hline & & -2d & = & -8 \end{array}$$

$d = \dfrac{-8}{-2}$

$\therefore \quad \mathbf{d = 4}$

Substituting $d = 4$ in (i) we get,

$2a + d = 6$

$\therefore \quad 2a + 4 = 6$

$\therefore \quad 2a = 6 - 4$

$\therefore \quad 2a = 2$

$\therefore \quad \mathbf{a = 1}$

$\therefore$ **Thus, the required terms are *a – d, a, a + d, a + 2d* i.e –3, 1, 5, 9.**

Problem Set - 3 *(Textbook Page No. 80)*

(9) Divide 207 in three parts, such that all parts are in A.P. and product of two smaller parts will be 4623. *(4 marks)*

Solution:

Let the three parts of 207, which are in A.P. be $a-d$, a, $a+d$

Sum of three parts is 207. ...(Given)

$\therefore \quad a - d + a + a + d = 207$

$\therefore \quad 3a = 207$

$\therefore \quad a = \dfrac{207}{3}$

$\therefore \quad \boxed{a = 69}$

Also, the product of two smaller parts is 4623 ... (Given)

$\therefore \quad (a-d) \times a = 4623$

$\therefore \quad (69 - d) \times 69 = 4623$

$\therefore \quad 69 - d = \dfrac{4623}{69}$

$\therefore \quad 69 - d = 67$

$\therefore \quad 69 - 67 = d$

$\therefore \quad \mathbf{d = 2}$

Thus, the three parts of 207 are

$a - d = 69 - 2 = 67,\ a = 69$

$a + d = 69 + 2 = 71$

$\therefore$ **Hence, three parts of 207 are 67, 69, 71**

(10) There are 37 terms in an A.P., the sum of three terms placed exactly at the middle is 225 and the sum of last three is 429. Write the A.P. *(4 marks)*

Solution:

Let the A.P. be $a, a + d, a + 2d, a + 3d, \dots, a + 36d$

Now, $n = 37$ (odd number)

$\therefore$ Exactly middle term $= \dfrac{n+1}{2} = \dfrac{37+1}{2} = 19^{th}$ term.

$\therefore$ $18^{th}, 19^{th}, 20^{th}$ terms are middle most terms.

The sum of the three middle most terms is 225.

...(Given)

$\therefore \quad t_{18} + t_{19} + t_{20} = 225$

$\therefore \quad (a + 17d) + (a + 18d) + (a + 19d) = 225$

...[$\because t_n = a + (n-1)d$]

$\therefore \quad 3a + 54d = 225$

$\therefore \quad a + 18d = 75$ ...(i) (Dividing both sides by 3)

Now, $35^{th}, 36^{th}, 37^{th}$ terms are last three terms and their sum is 429.

$\therefore \quad t_{35} + t_{36} + t_{37} = 429$

$\therefore \quad (a + 34d) + (a + 35d) + (a + 36d) = 429$

$\therefore \quad 3a + 105d = 429$

$\therefore \quad a + 35d = 143$ ...(ii) (diving both sides by 3)

Subtracting (i) from (ii) we get,

$$\begin{array}{rcrcl} & a & + & 35d & = 143 \\ & a & + & 18d & = 75 \\ (-) & (-) & & & (-) \\ \hline & & & 17d & = 68 \end{array}$$

$\therefore d = \dfrac{68}{17}$

$\therefore \quad \mathbf{d = 4}$

Substituting $d = 4$ in (i) we get,

$a + 18d = 75$

$\therefore \quad a + 18 \times 4 = 75$

$\therefore \quad a + 72 = 75$

$\therefore \quad a = 75 - 72$

$\therefore \quad \mathbf{a = 3}$

$\therefore$ **Thus, the required A.P. is 3, 7, 11, 15.... 147**

***(12) If the sum of first p terms of an A.P. is equal to the sum of first q terms, then show that the sum of its first $(p + q)$ terms is zero. $(p \neq q)$** *(4 marks)*

Solution:

Given : $S_p = S_q$

To prove $= S_{p+q} = 0$

Proof : Let a be the first term and d be the common difference of the A.P.

$\therefore \quad S_n = \dfrac{n}{2}[2a + (n-1)d]$

The sum of first p terms of the A.P. is

$S_p = \dfrac{p}{2}[2a + (p-1)d]$ ...(i)

The sum of first q terms of the A.P. is

$S_q = \dfrac{q}{2}[2a + (q-1)d]$ ...(ii)

But $S_p = S_q$ ...(Given)

$\therefore \quad \dfrac{p}{2}[2a + (p-1)d] = \dfrac{q}{2}[2a + (q-1)d]$

...[From (i) and (ii)]

$\therefore \quad p[2a + pd - d] = q[2a + qd - d]$

... (multiplying both sides by 2)

$\therefore \quad 2ap + p^2d - pd = 2aq + q^2d - qd$

$\therefore \quad 2ap - 2aq + p^2d - q^2d - pd + qd = 0$

$\therefore \quad 2a(p - q) + (p^2 - q^2)d - (p - q)d = 0$

$\therefore \quad 2a(p - q) + d(p + q)(p - q) - d(p - q) = 0$

...[Using $a^2 - b^2 = (a + b)(a - b)$]

$\therefore \quad (p - q)(2a + pd + qd - d) = 0$

$\therefore \quad 2a + (p + q - 1)d = \dfrac{0}{p-q}$

$\therefore \quad 2a + (p + q - 1)d = 0$...(iii) [$\because p \neq q \therefore p - q \neq 0$]

Now $S_{p+q} = \left(\dfrac{p+q}{2}\right)[2a + (p + q - 1)d]$

$= \left(\dfrac{p+q}{2}\right) \times 0$ ...[From (iii)]

$\therefore \quad S_{p+q} = 0$

$\therefore$ **The sum of first $(p + q)$ terms is zero.**

***(11) If first term of an A.P. is a, second term is b and last term is c, then show that sum of all the terms is $\dfrac{(a+c)(b+c-2a)}{2(b-a)}$.** *(4 marks)*

Solution:

The A.P. is $a, b, \dots, c$

Here, $t_1 = a$, $d = b - a$, $t_n = c$

We know,

$t_n = a + (n - 1)d$

$c = a + (n - 1)(b - a)$

$\therefore \quad c - a = (n - 1)(b - a)$

$\therefore \quad \dfrac{c-a}{b-a} = n - 1$

$\therefore \quad \dfrac{c-a}{b-a} + 1 = n$

$\therefore \quad \dfrac{c-a+b-a}{b-a} = n$

$\therefore \quad \dfrac{c+b-2a}{b-a} = n \qquad$ ...(i)

Method - 1

We know,

$S_n = \dfrac{n}{2}[2a + (n-1)d]$

$S_n = \dfrac{c+b-2a}{2(b-a)}\left[2a + \left(\dfrac{c+b-2a}{b-a} - 1\right)(b-a)\right]$

...[from (i) and given]

$= \dfrac{c+b-2a}{2(b-a)}\left[2a + \left(\dfrac{c+b-2a-(b-a)}{b-a}\right)(b-a)\right]$

$= \dfrac{c+b-2a}{2(b-a)}[2a+c+b-2a-b+a]$

$= \dfrac{c+b-2a}{2(b-a)}[c+a]$

$\therefore \quad \boxed{S_n = \dfrac{(a+c)(b+c-2a)}{2(b-a)}}$

Method - II

We know,

$S_n = \dfrac{n}{2}[t_1 + t_n]$

$= \dfrac{c+b-2a}{2(b-a)}[a+c]$

$\therefore \quad \boxed{S_n = \dfrac{(a+c)(b+c-2a)}{2(b-a)}}$

Practice Set - 3.4 *(Textbook Page No. 78)*

***(4)** **There is an auditorium with 27 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row and so on. Find the number of seats in 15th row and also find how many total seats are there in the auditorium?** *(4 marks)*

Solution:

The number of seats in each row are as follows: 20, 22, 24.....

The number of seats in each row form an A.P. with number of seats in first row $(a) = 20$.

Difference between number of seats in two successive rows $(d) = 2$

Total number of rows $(n) = 27$

Number of seats in 15th row $(t_{15}) = ?$

Total number of seats in 27 rows $(S_{27}) = ?$

$t_n = a + (n-1)d$

$\therefore \quad t_{15} = 20 + (15 - 1) \times 2$

$\therefore \quad t_{15} = 20 + 14 \times 2$

$\therefore \quad t_{15} = 20 + 28$

$\therefore \quad \mathbf{t_{15} = 48}$

$S_n = \dfrac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{27} = \dfrac{27}{2}[2 \times 20 + (27 - 1)\,2]$

$\therefore \quad S_{27} = \dfrac{27}{2}[40 + 52]$

$\therefore \quad S_{27} = \dfrac{27}{2} \times 92$

$\therefore \quad \mathbf{S_{27} = 1242}$

$\therefore$ **There are 48 seats in 15th row and 1242 seats in the auditorium.**

(5) **Kargil's temperature was recorded for a week i.e. Monday to Saturday. All readings were in A.P. The sum of temperatures of Monday and Saturday was 5°C more than the sum of temperatures of Tuesday and Saturday. If temperature of Wednesday was –30°C then find the temperature on the other five days.** *(4 marks)*

Solution:

Let the temperatures of Kargil from Monday to Saturday (in°C) which forms an A.P. be $a - d$, a, $a + d$, $a + 2d$, $a + 3d$, $a + 4d$ respectively.

Days	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.
Temp (in °c)	$a - d$	a	$a + d$	$a + 2d$	$a + 3d$	$a + 4d$

As per the given condition,

$(a - d) + (a + 4d) = a + (a + 4d) + 5$

$\therefore \quad a - d + a + 4d = a + a + 4d + 5$

$\therefore \quad 2a + 3d = 2a + 4d + 5$

$\therefore \quad 3d = 4d + 5$

$\therefore \quad 4d - 3d = -5$

$\therefore \quad \mathbf{d = -5}$

New temperature on Wednesday is –30°C ...(Given)

$\therefore \quad a + d = -30$

$\therefore \quad a - 5 = -30$

$\therefore \quad a = -30 + 5$

$\therefore \quad \boldsymbol{a = -25}$

Hence, $a - d = -25 - (-5) = -25 + 5 = -20$

$a = -25$

$a + d = -25 + (-5) = -30$

$a + 2d = -25 + 2(-5) = -25 + (-10) = -35$

$a + 3d = -25 + 3(-5) = -25 + (-15) = -40$

$a + 4d = -25 + 4(-5) = -25 + (-20) = -45$

$\therefore$ **The temperatures from Monday to Saturday are –20°C, –25°C, –30°C, –35°C, –40°C, –45°C respectively.**

Problem Set - 3 *(Textbook Page No. 80)*

(14) **₹ 1000 is invested at 10 percent simple interest. Check at the end of every year if the total interest amount is in A.P. If this is an A.P. then find interest amount after 20 years. For this complete the following activity.** *(3 marks)*

Solution:

Simple interest $= \dfrac{P \times R \times N}{100}$

Simple interest after 1 year $= \dfrac{1000 \times 10 \times 1}{100}$

$= \boxed{100}$

Simple interest after 2 year $= \dfrac{1000 \times 10 \times 2}{100}$

$= \boxed{200}$

Simple interest after 3 year $= \dfrac{1000 \times 10 \times 3}{100}$

$= \boxed{300}$

According to this the simple interest for 4, 5, 6 years will be 400, $\boxed{500}$, $\boxed{600}$ respectively.

From this $d = \boxed{100}$ and $a = \boxed{100}$

Amount of simple interest after 20 years.

$t_n = a + (n - 1)\, d$

$\therefore \quad t_{20} = \boxed{100} + (20 - 1) \boxed{100}$

$\therefore \quad t_{20} = \boxed{2000}$

$\therefore$ **Amount of Simple interest after 20 years is ₹ 2000**

Practice Set - 3.4 *(Textbook Page No. 78)*

(1) **On 1st Jan. 2016, Sanika decides to save ₹ 10, ₹ 11 on second day, ₹ 12 on third day. If she decides to save like this, then on 31st December 2016 what would be her total saving?** *(3 marks)*

Solution:

2016 is a leap year.

$\therefore$ Number of days from 1st January 2016 to 31st December 2016 = 366

The saving of Sanika per day, beginning with the first day are 10, 11, 12, ... for 366 days.

These every day savings form an A.P. with first days saving $(a) = 10$

Difference in saving made in two successive days $(d) = 1$

Total number of days in the year 2016 $(n) = 366$

$\therefore$ Total saving for the year 2016 $(S_{366}) = ?$

$$S_n = \frac{n}{2}\,[2a + (n - 1)\, d]$$

$$\therefore \quad S_{366} = \frac{366}{2}\,[2 \times 10 + (366 - 1)1]$$

$$= 183\,[20 + 365]$$

$$= 183 \times 385$$

$$\therefore \quad S_{366} = 70{,}455$$

$\therefore$ **Sanika saved ₹ 70,455 in the year 2016.**

(6) **On the world environment day tree plantation programme was arranged on a land which is triangular in shape. Trees are planted such that in the first row there is one tree, in the second row there are two trees, in the third row three trees and so on. Then find the total number of trees in the 25 rows.** *(3 marks)*

Solution:

The number of trees in each row upto the 25th row are as follows: 1, 2, 3, 4, ...

The no. of trees planted in each row form an A.P. with no. of trees in first row $(a) = 1$

Difference between no. of trees planted in two successive rows $(d) = 1$

No. of rows $(n) = 25$

Total no. of trees planted $(S_{25}) = ?$

$$S_n = \frac{n}{2}\,[2a + (n - 1)\, d]$$

$$\therefore \quad S_{25} = \frac{25}{2}\,[2 \times 1 + (25 - 1)1]$$

$$\therefore \quad S_{25} = \frac{25}{2}\,[2 + 24]$$

$$\therefore \quad S_{25} = \frac{25}{2} \times 26$$

$$\therefore \quad S_{25} = 25 \times 13$$

$$\therefore \quad \boldsymbol{S_{25} = 325}$$

$\therefore$ **325 trees were planted in 25 rows.**

(3) Sachin invested in a National Saving Certificate scheme. In the 1st year, he invested ₹ 5000, in 2nd year ₹ 7000, in 3rd year ₹ 9000 and so on. Find the total amount that he invested in 12 years. *(4 marks)*

Solution:

Yearly investments of Sachin are as follows:
5000, 7000, 9000, ...
The yearly investments form an A.P. with first year investment $(a) = 5000$
Difference between investments done in two successive years $(d) = 2000$
No. of years $(n) = 12$
Total investment done in 12 years $S_{12} = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore \quad S_{12} = \frac{12}{2}[2 \times 5000 + (12-1)2000]$

$\therefore \quad S_{12} = 6[10000 + 11 \times 2000]$

$\therefore \quad S_{12} = 6 \times 32{,}000$

$\therefore \quad S_{12} = 1{,}92{,}000$

$\therefore$ **Total investment done in 12 years is ₹ 1,92,000**

(2) A man borrows ₹ 8000 and agrees to repay with a total interest of ₹ 1360 in 12 monthly installments. Each installment is being less than the preceding one by ₹ 40. Find the amount of the first and last installment. *(4 marks)*

Solution:

Each installment is less than preceding investment by ₹ 40.
Therefore the installments are in A.P.
Total money repaid in 12 installments (S_{12})
$= 8000 + 1360 =$ ₹ 9360
No. of installments $(n) = 12$
Difference between two consecutive installments $(d) = -40$
First installment $(a) = ?$, Last installment $(t_{12}) = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore \quad S_{12} = \frac{12}{2}[2a + (12-1)(-40)]$

$\therefore \quad 9360 = 6[2a + 11 \times (-40)]$

$\therefore \quad \frac{9360}{6} = 2a - 440$

$\therefore \quad 1560 = 2a - 440$

$\therefore \quad 1560 + 440 = 2a$

$\therefore \quad 2a = 2000$

$\therefore \quad a = \frac{2000}{2}$

$\therefore \quad a = 1000$

$t_n = a + (n-1)d$

$\therefore \quad t_{12} = 1000 + (12-1)(-40)$

$\therefore \quad t_{12} = 1000 + 11 \times (-40)$

$\therefore \quad t_{12} = 1000 - 440$

$\therefore \quad t_{12} = 560$

$\therefore$ **First installment is ₹ 1000 and last installment is ₹ 560.**

Problem Set - 3 *(Textbook Page No. 78)*

MCQ's

Choose the correct alternative answer for each of the following sub questions. *(1 mark each)*

(1) The sequence $-10, -6, -2, 2,$ ....
(A) is an A.P. ,Reason $d = -16$
(B) is an A.P., Reason $d = 4$
(C) is an A.P., Reason $d = -4$
(D) is not an A.P.

(2) First four terms of an A.P. are ...., whose first term is -2 and common difference is -2.
(A) $-2, 0, 2, 4$ (B) $-2, 4, -8, 16$
(C) $-2, -4, -6, -8$ (D) $-2, -4, -8, -16$

(3) What is the sum of first 30 natural numbers?
(A) 464 (B) 465 (C) 462 (D) 461

(4) For an given A.P., $t_7 = 4$, $d = -4$ then $a =$ ....
(A) 6 (B) -7 (C) 20 (D) 28

(5) For an given A.P., $a = 3.5$, $d = 0$, $n = 101$ then $t_n =$ ....
(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

(6) In an A.P., first two terms are $-3, 4$ then 21st term is ....
(A) -143 (B) 143 (C) 137 (D) 17

(7) If for an any A.P., $d = 5$ theh $t_{18} - t_{13} =$ ....
(A) 5 (B) 20 (C) 25 (D) 30

(8) Sum of first five multiples of 3 is ....
(A) 45 (B) 55 (C) 15 (D) 75

(9) 15, 10, 5, ... In this A.P. the sum of first 10 terms is ....
(A) -75 (B) -125 (C) 75 (D) 125

(10) In an A.P., 1st term is 1 and the last term is 20. The sum of all terms is 399 then $n =$ ....
(A) 42 (B) 38 (C) 21 (D) 19

Additional MCQ's

(11) For an A.P. 4, 9, 14,.... $t_{11} =$ ....
(A) 49 (B) 54 (C) 59 (D) 44

(12) If $a = 6$, $d = 3$ then S_{10} = ...

(A) 192 (B) 195 (C) 198 (D) 201

(13) The sum of first 10 natural number is ....

(A) 55 (B) 155 (C) 310 (D) 210

(14) Which of the following sequence is not an A.P.?

(A) 0.5, 2, 3.5, 5 ... (B) 22, 26, 28, 31 ....
(C) 3, 5, 7, 9, ... (D) 1, 4, 7, 10, ...

(15) Find the missing term in the A.P.: –5, ... , 13, ...

(A) 1 (B) 2 (C) 3 (D) 4

(16) Find the t_2 of the following sequence for which $S_1 = 2$, $S_2 = 12$ and $S_3 = 36$.

(A) 24 (B) 2
(C) 10 (D) none of these

(17) For an A.P. if $t_4 = 12$ and $d = -10$ then find a.

(A) –18 (B) 42 (C) –5 (D) 21

(18) The next two terms of the given sequence 1, 3, 7, 15, 31, ...

(A) 42, 54 (B) 62, 124 (C) 64, 128 (D) 63, 127

(19) In an A.P., $a = 8$, $t_n = 62$, $S_n = 210$ then find n.

(A) 5 (B) 6 (C) 7 (D) 8

(20) The sum of first n terms of an A.P. S_n = ...

(A) $\frac{n}{2}[t_1 + t_n]$ (B) $\frac{n}{2}[a + (n-1)d]$
(C) $\frac{n}{2}[2 + (n-1)d]$ (D) none of these

(21) A meeting hall has 30 rows in all. There are 20 seats in the first row, 24 seats in the second row and 28 seats in the third row and so on. How many seats are there in the hall?

(A) 136 (B) 4640 (C) 2340 (D) 192

(22) State whether the given sequence is an A.P. or not: $1^3, 2^3, 3^3, 4^3, 5^3$, ...

(A) an A.P. with $d = 3$ (B) not an A.P.
(C) an A.P. with $d = 7$ (D) can't say

ANSWERS

(1) (B) is an A.P. , Reason with $d = 4$
(2) (C) –2, –4, –6, –8 **(3)** (B) 465 **(4)** (D) 28 **(5)** (B) 3.5
(6) (C) 137 **(7)** (C) 25 **(8)** (A) 45 **(9)** (A) –75
(10) (B) 38 **(11)** (B) 54 **(12)** (B) 195 **(13)** (A) 55
(14) (B) 22, 26, 28, 31,... **(15)** (D) 4 **(16)** (C) 10
(17) (B) 42 **(18)** (D) 63, 127 **(19)** (B) 6
(20) (A) $\frac{n}{2}[t_1 + t_n]$ **(21)** (C) 2340 **(22)** (B) not an A.P.

PROBLEMS FOR PRACTICE

Based on Practise Set 3.1

(1) **Which of the following sequences are Arithmetic progression? If it is an A.P. then write common difference.** *(1 mark each)*

(i) 0, 1, 0, 1, 0, 1, ... (ii) –10, –13, –16, –19, ...
(iii) $1^3, 2^3, 3^3, 4^3$, ... (iv) 31, 26, 21, 15, ...
(v) $-1, \frac{-3}{2}, -2, \frac{-5}{2}$, ...

(2) **Write an A.P. when the common difference d and the first term a are given.** *(1 mark each)*

(i) $a = 11$, $d = 1.5$ (ii) $a = 5$, $d = -5$
(iii) $a = -8$, $d = 0$ (iv) $a = -3.5$, $d = -3.5$
(v) $a = 10$, $d = -3$

Based on Practice Set 3.2

(3) **How many terms are there in the A.P. 187, 194, 201, ..., 439?** *(2 marks)*

(4) **Find n, if the n^{th} term of the following sequence is 68.**
5, 8, 11, 14, ... *(2 marks)*

(5) **If 10^{th} term and the 18^{th} term of an A.P. are 25 and 41 respectively, then find the 38^{th} term.** *(3 marks)*

(6) **How many three digit natural numbers are divisible by 4?** *(3 marks)*

(7) **Find the eighteenth term of the A.P. 1, 7, 13, 19.** *(3 marks)*

(8) **Find t_{11} from the following A.P. 4, 9, 14, ...** *(2 marks)*

(9) **Find the 10^{th} term from the end of the A.P. 4, 9, 14, ..., 254.** *(3 marks)*

(10) **For what value of n, the n^{th} term of the following two A.P.s are equal?**
23, 25, 27, 29,.... and –17, –10, –3, 4, ... *(3 marks)*

(11) **The sixth term of an A.P. is 5 times the 1^{st} term and the eleventh term exceeds twice the fifth term by 3. Find the 8^{th} term.** *(3 marks)*

Based on Practice Set 3.3

(12) **How many two digit natural numbers are divisible by 5?** *(3 marks)*

(13) **Obtain the sum of 56 terms of an A.P. whose 19^{th} and 38^{th} terms are 52 and 148 respectively.** *(3 marks)*

(14) **In an A.P., if the 5^{th} and 12th terms are 30 and 65 respectively, what is the sum of the first 20 terms.** *(3 marks)*

(15) **Split 69 in three parts such that they are in A.P. and product of two smaller parts is 483.** *(3 marks)*

(16) **The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?** *(3 marks)*

(17) **Find four consecutive terms in an A.P. such that the sum of the middle two terms is 18 and product of the two end terms is 45.** *(3 marks)*

(18) **Find three consecutive terms in an A.P. whose sum is –3 and the product of their cubes is 512.** *(3 marks)*

(19) **How many terms of the A.P.: 9, 17, 25, ... must be taken to give a sum of 636.?** *(3 marks)*

(20) **Find the sum of all natural numbers between 100 and 1000 which are multiples of 7.** *(3 marks)*

(21) **A meeting hall has 20 seats in the first row, 20 seats in the second, 28 seats in the third row and so on and has in all 30 rows. How many seats are there in the meeting hall?** *(4 marks)*

Based on Practice Set 5.4

(22) **In winter, the temperature at a hill station from Monday to Friday is in A.P., The sum of the temperatures of Monday, Tuesday and Wednesday is zero and the sum of the temperature of Thursday and Friday is 15. Find the temperature of each of the five days.** *(4 marks)*

(23) **Neeta saves in a `Mahila Bachat Gat' ₹ 2 on the first day, ₹ 4 on the second day, ₹ 6 on the third day and so on. What will be her saving in the month of February 2010?** *(4 marks)*

(24) **Mr. Shah borrows ₹ 4000 and agrees to repay with a total interest of ₹ 500 in 10 installments, each installment being less than the preceding installment by ₹ 10. What should be the first and the last installment?** *(4 marks)*

(25) **A farmer borrows ₹ 1000 and agrees to repay with a total interest of ₹ 140 in 12 installments, each installment being less than the preceding installment by ₹ 10. What should be his first installment?** *(4 marks)*

ANSWERS

(1) (i) It is not an A.P. (ii) It is an A.P., $d = -3$
(iii) It is not an A.P. (iv) It is not an A.P.
(v) It is an A.P., $d = \dfrac{-1}{2}$

(2) (i) 11, 12.5, 14, 15.5, ... (ii) 5, 0, –5, –10, ...
(iii) –8, –8, –8, –8, ... (iv) –3.5 , –7, –10.5, –14, ...
(v) 10, 7, 4, 1, ...

(3) 37 terms **(4)** $n = 22$ **(5)** $t_{38} = 81$ **(6)** 225

(7) $t_{18} = 103$ **(8)** $t_{11} = 54$ **(9)** 209 **(10)** $n = 9$ **(11)** $t_8 = 33$

(12) 18 **(13)** 5600 **(14)** 1150 **(15)** 21, 23, 25

(16) 38, 6973 **(17)** 3, 7, 11, 15 or 15, 11, 7, 3

(18) –4, –1, 2 or 2, –1, –4 **(19)** 12 **(20)** 70,336

(21) 2340 **(22)** –3 , 0, 3, 6, 9 **(23)** ₹ 812 **(24)** ₹ 495 , ₹ 405

(25) ₹ 150

ASSIGNMENT – 3

Time : 1 Hr. **Marks : 20**

Q.1. (A) Choose the proper alternative answer for the questions given below: **(2)**

(1) The sequence –10, –6, –2, 2, ...

(A) is an A.P., Reason $d = -16$ (B) is an A.P., Reason $d = 4$

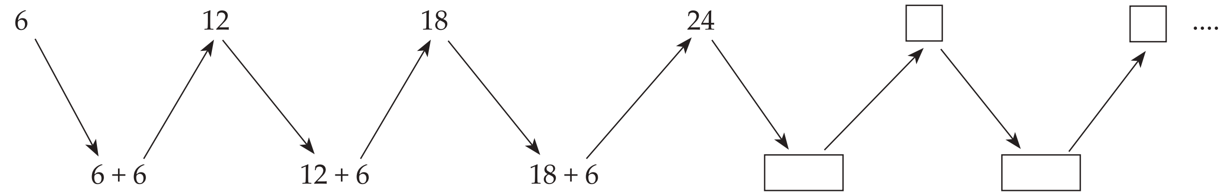
(C) is an A.P., Reason $d = -4$ (D) is not an A.P.

(2) Sum of first five multiples of 3 is ....

(A) 45 (B) 55 (C) 15 (D) 75

(B) Perform the following activities: **(3)**

(1) Write the next two terms of the following A.P.:



(2) For an A.P. 2, –2, –6, –10, ... find d

Solution:

Here, $t_1 = 2$, $t_2 = -2$, $t_3 = -6$, $t_4 = -10$

$\therefore \quad d = t_2 - t_1$

$= \square - \square$

$\therefore \quad d = \square$

(3) For an A.P., if $a = -3$ and $d = 4$ then find t_n

Solution:

$t_n = a + (n-1)d$

$\therefore \quad t_n = \square + (n-1)\square$

$\therefore \quad t_n = \square$

Q.2. Perform the following activities: (Any 2) **(4)**

(1) Determine whether following sequence is an A.P.

2, –2, –6, –10, ...

Solution: Here $t_1 = 2$, $t_2 = -2$, $t_3 = -6$, $t_4 = -10$

$t_2 - t_1 = \square - \square = \square$

$t_3 - t_2 = \square - \square = \square$

$\therefore \quad t_3 - t_2 \; \square \; t_2 - t_1$

$\therefore$ The given sequence is ....

(2) In an A.P., $a = 6$ and $d = 3$ then find S_{27}.

$$S_n = \frac{n}{2}[\square + (n-1)\,d]$$

$$\therefore \quad S_{27} = \frac{27}{2}[12 + (n-1)\square]$$

$$= \frac{27}{2} \times \square$$

$$= 27 \times 45$$

$$\therefore \quad S_{27} = \square$$

(3) Find 19th term of the following A.P.: 7, 13, 19, 25, ...

Q.3. Perform the following activities. (Any 1) **(3)**

(1) Find the sum of all numbers from 1 to 140 which are divisible by 4.

Solution:

The numbers between 1 to 140 that are divisible by 4

↓

4, 8, 12, ..., 136

↓

$a = \square$, $d = \square$, $t_n = \square$

↓

$t_n = a + (n-1)d$

↓

$136 = \square + (n-1) \times \square$

↓

$\therefore \quad n = \square$

↓

$$S_n = \frac{n}{2}[t_1 + t_n]$$

↓

$$S_{\square} = \frac{\square}{2}[\quad\square\quad]$$

$$= \square$$

(2) A village has 4000 literate people in the year 2010 and this number increases by 400 per year. How many literate people will be there till year 2020?

Solution:

Year	2010	2011	2012	....	2020
Literate people	4000	4400	4800	....	$\boxed{}$

Here $a = \boxed{}$, $d = \boxed{}$, $n = \boxed{}$

$t_n = a + (n - 1)\, d$

$\therefore \quad = \boxed{} + (\boxed{} - 1) \times \boxed{}$

$\therefore \quad = \boxed{} + \boxed{}$

$\therefore \quad = \boxed{}$

$\therefore$ There will be $\boxed{}$ literate people till year 2020.

Q.4. Attempt the following: (Any 2) **(8)**

(1) Find four consecutive terms in an A.P. whose sum is 12 and the sum of 3rd and 4th term is 14. (Let four consecutive terms be $a - d, a, a + d, a + 2d$)

(2) If the sum of p terms of an A.P. is equal to the sum of q terms then show that the sum of its p + q terms is zero.

(3) Mr. Ajay Sharma borrows ₹ 3,25,000. He paid ₹ 30,500 in the first month and then each installment being less than the preceding installment by ₹1500. How long will it take to clear his loan?

❖ ❖ ❖

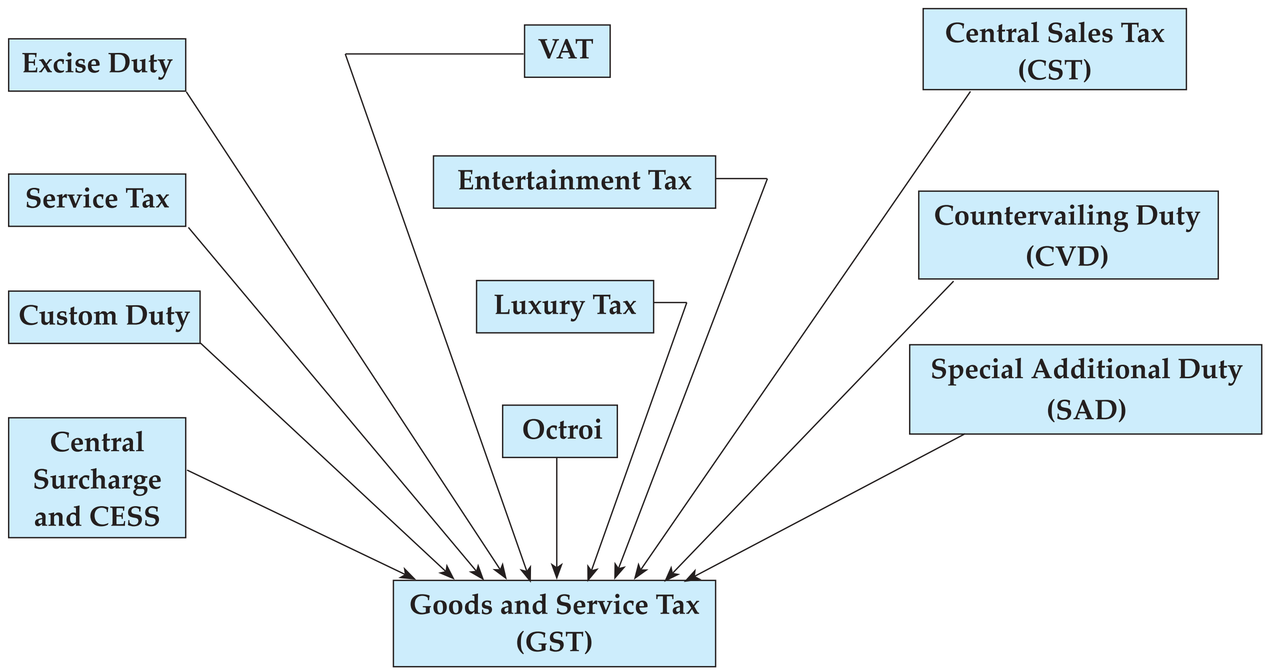
4 Financial Planning

... INDEX ...

Points to Remember:

- **GST (Goods and Service Tax):**

  GST stands for Goods and Service Tax,. Taxes that existed before were VAT, Custom duty, Excise duty, Service tax, Octroi etc. All these taxes are subsumed under GST, that is why GST is one nation, one tax, one market. GST is in effect from 1st July 2017.



- **Tax Invoice:**

Tax Invoice of goods purchase (Sample)

Supplier : A to Z Sweet Mart GSTIN : 27 ABCDE1234H1Zs
143, Shivaji Rasta, Mumbai : 400001 Maharashtra
Mob. No. 9263692111 email: atoz@gmail.com

Invoice No. GST / 110 Invoice Date : 31 July 2017

S. No	HSN Code	Name of Product	Rate	Quantity	Taxable Amount	CGST		SGST		Total
						Rate	Tax	Rate	Tax	₹
(1)	210690	Pedhe	₹ 400 per.kg.	500 gm	200.00	2.5%	5.00	2.5%	5.00	210.00
(2)	210691	Chocolate	₹ 80	1 Bar	80.00	14%	11.20	14%	11.20	102.40
(3)	2105	Ice cream	₹ 200	1 Pack (500 gm)	200.00	9%	18.00	9%	18.00	236.00
(4)	1905	Bread	₹ 35	1 Pack	35.00	0%	0.00	0%	0.00	35.00
(5)	210690	Butter	₹ 500 per kg.	250 gm	125.00	6%	7.50	6%	7.50	140.00
						Total ₹	**41.70**		**41.70**	**723.40**

Let us understand the meaning of some new words given in this bill.

CGST - Central Goods and Service Tax

SGST - State Goods and Service Tax

GSTIN - Goods and Service Tax Identification Number

HSN - Harmonized System of Nomenclature

SAC - Service Accounting Code

Now, CGST, i.e. Central Goods and Service Tax is to be paid to the Central Government whereas SGST, i.e. State Goods and Service Tax is to be paid to State Government. It is compulsory to get GSTIN, if the turn over of a dealer in previous financial year exceeds ₹ 20 lacs.

GSTIN includes 15 alpha numerals.

For ex.

27 A B C D E 1 2 3 4 H 1 Z 5

10 digit PAN of the firm

1 represents registration

Uniform for all(By Default)

Check Sum digit

2 digit for (Confirms validity of GSTIN State Code (Confirms validity of GSTIN when entered on GST portal.

'27' is Maharashtra State Code.

Form 27, one can understand that a person or a firm is registered in Maharashtra.

Complete the following tax invoice. *(Textbook Page No. 83)*

Tax Invoice of Service provided (Sample)

Food Junction, Shivpur, Pune Invoice No. : 58

Mob. No. 7588580000 E-mail: ahar.khed@yahoo.com

GSTIN : 27AAAAA 5555B1ZA Invoice Date : 25 Dec., 2017

SAC	Food Items	Qty.	Rate (in ₹)	Taxable amount	CGST		SGST	
9961	Coffee	1	20	20.00	2.5%	₹ 0.50	2.5%	₹ 0.50
9963	Masala Tea	1	10	10.00	2.5%	₹ 0.25	2.5%	₹ 0.50
9962	Masala Dosa	1	60	120	2.5%	₹ 3	2.5 %	₹ 3
			Total	150		₹ 3.75		₹ 3.75
							Grand Total =	**₹ 157.50**

The following table gives Goods and Service Tax rates:

Sr. No.	Type	Tax rate	Goods and Servive Types
(I)	Zero rated	0%	**Goods** - Essential Commodities like food grains, fruits, vegetables, milk, salt, earthen pots etc. **Services** - Charitable trust activities, transport of water, use of roads and bridges, public library, agriculture related services, Education and Health care services etc.
(II)	Low rated	5%	**Goods** - Commonly used items- LPG cylinder, Tea, coffee, oil, Honey, Frozen vegetables, spices, sweets etc. **Services** - Railway transport services, bus transport services, taxi services, Air transport (economy class), Hotels providing food and beverages etc.
(III)	Standard rated (I slab)	12%	**Goods** - Consumer goods : Ghee, dry fruits, Jam, Jelly, Sauces, Pickles, Mobile phone etc. **Services** - Printing job work (Non AC) Guest house, services related to construction business,
(IV)	Standard rated (II slab)	18% (Most of the Goods and services are included)	**Goods** - Marble, granite, perfumes, Metal items, computer, printer, monitor, CCTV etc. **Services** - Courier services, outdoor catering, Circus, Drama, Cinema, Exhibition, Currency exchange, Broker Services in share trading etc.
(V)	Highly rated	28%	**Goods** - Luxury items, Motor cycles and spare parts, luxury cars, Pan-Masala, vacuum cleaner, dish washer, AC, washing machine, fridge, tobacco products, aerated water etc. **Services** - Five star hotel accomodation,Amusement parks, water parks, theme parks, casino, Race course, IPL games, gambling,Air transport (business class) etc.

Reference : www.cbec.gov.in (Visit Central Board of Excise and customs website for more information)

Note: Electricity, Petrol, diesel etc. are not under purview of GST. The rate of GST are taken at the time of writting this chapter.

Activity 1 : Make list of 10 items you need daily and find the rate of GST applicable on them from news paper, internet, books on GST and tax invoice. *(Textbook Page No. 85)*

	Items	GST rate		Items	GST rate
(1)	Note book	12%	(6)	Pencil	12%
(2)	Compass box	12%	(7)	Pen	12%
(3)	School bag	18%	(8)	Shoes	5%
(4)	Milk	0%	(9)	School uniform	5%
(5)	Water bottle	18%	(10)	Computer	18%

Activity 2 : The following table shows 10 different services (e.g. Railway, S.T., Bus Bookings, service etc.) Find the rate and GST applicable to them.

(Textbook Page No. 85)

	Service	GST rate		Service	GST rate
(1)	Railway bookings	5%	(6)	Water Park	28%
(2)	Courier Service	18%	(7)	Five Star Hotel	28%
(3)	Cinema	18%	(8)	Cinema	18%
(4)	Exhibition	18%	(9)	Airline Economy Class	5%
(5)	IPL	28%	(10)	Airline Business Class	28%

Note : Value of Goods on which GST is levied is called taxable value. Total value or Invoice value is the value with GST. If not mentioned take the selling price as taxable price

For More Information

Composition Scheme

The Person whose annual turn over in the previous financial year is less than 1.5 crore can opt for composition scheme under GST rules. GST rates applicable to composition dealers as follows.

GST rates for composition Scheme

Sr. No.	Supplier	GST rate	(CGST + SGST)
1.	Restaurants	5%	2.5% + 2.5%
2.	Manufacturers	1%	0.5% + 0.5 %

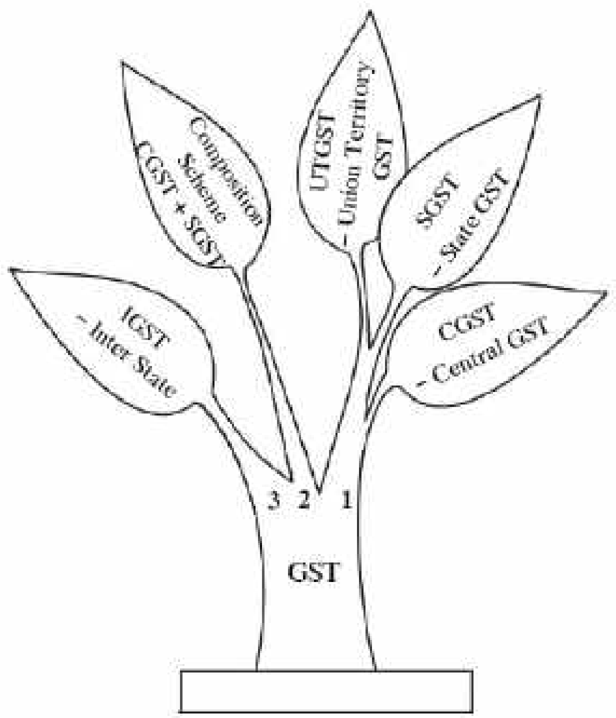
Some rules for composition dealers

- Composition dealers cannot collect tax from the customers, hence they can not issue tax invoice. They have to give '**bill of Supply**'.
- Composition dealers should file the return quarterly (i.e. Every 3 months)
- Composition dealers cannot sell goods outside the state (inter-state sale is not allowed) But they can purchase goods from other states.
- Composition dealers cannot avail the benefits of ITC
- On the signboard of the shop, he should mention 'Composition taxable person'.
- On the Bill of supply it is mandatory to print `Composition taxable person not eligible to collect tax on supplies' in bold letters.

Features of GST

- Many Indirect taxes are subsuymed under GST
- No dispute between Goods and Service
- Statewise Registration for traders
- GISTIN holder needs ti keep all the records and should pay GST in time
- Transparency in transactions
- This tax system is simple and easy to understand
- Removal of cascading effect of taxes hence the prices are controlled
- Increase in Quality of Goods and Services as they are globally competitive.
- Boost to 'Make in India' project.
- Technology driven tax system leads to speedy decisions.
- Goods and Serviced tax system is a Dual model, as equal amount of tax is levied by Central and State governments.

Types of taxes under GST



1. CGST-SGST (UTGST): Tax levied for trading within state (Intra state).

2. Composition Scheme : For those GSTIN holders whose annual turn-over is between 20 lacs to 1.5 crore. They pay CGST and SGST with special rates.

3. IGST : Tax levied by central govern-ment for Inter state trading.

For More Information

IGST- Integrated GST (for Inter state trade)

When trading of goods and services takes place between two or more states, the GST is levied only by the Central Government, and it is termed as IGST, hence the total amount is paid to the Central Government.

Suppose if a trader buys goods from another state and sells them in his state, then let us see how he can avail of the ITC, which he has paid as IGST at the time of purchase.

For example : Trader 'M' (of Maharashtra) purchased scooter parts for Rs. 20,000 from trader 'P' (of Punjab) and paid tax of Rs. 5600 as IGST (GST rate 28%) to the trader 'P'.

Trader 'M' sold these parts to local consumers for Rs. 25,000 and collected Rs. 7000 GST at the rate of 28%, bifurcated as CGST `3500 + SGST `3500

At the time of paying taxes to the Government, see, how to take ITC of ₹ 5600.

Note : For taking credit of IGST first preference to be given to pay the liability of IGST then CGST and remaining amount can be utilised to pay SGST. Here there is no IGST during the sale for trader 'M', so first the credit is used for CGST and then for SGST.

CGST payable = 3500 – 3500 = 0 Rs.

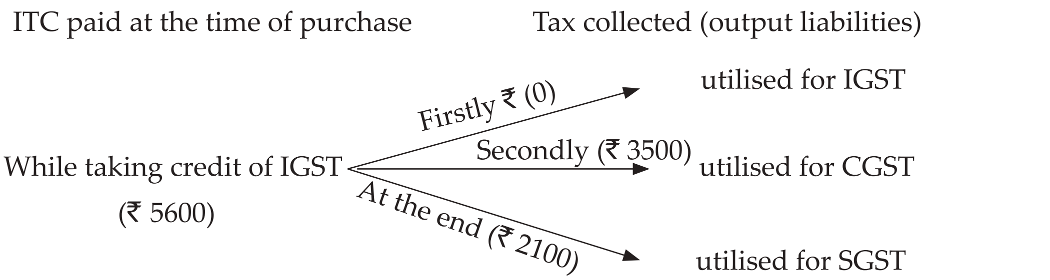
So out of Rs. 5600 credit of Rs. 3500 is utilised for CGST and the remaining amount 5600 - 3500 = 2100 is the credit available for SGST

∴ **SGST payable = 3500 – 2100 = 1400 Rs.**

Trader 'M' has to pay Rs. 1400 as SGST.

Note that, trader 'M' got full credit of Rs. 5600. (so that ITC is completely utilised)

Rule for availing ITC



Hence payable SGST is ₹1400

MASTER KEY QUESTION SET - 4

Practice Set - 4.1 *(Textbook Page No. 87)*

(1) 'Pawan Medical' supplies medicines. On some medicines the rate of GST is 12% , then what is the rate of CGST and SGST? *(1 mark)*

Solution:

Rate of GST on medicines = 12%

∴ $\boxed{\text{Rate of CGST is 6\% and Rate of SGST is 6\%}}$

(2) On certain article if rate of CGST is 9% then what is the rate of SGST? and what is the rate of GST? *(1 mark)*

Solution:

Rate of CGST = 9%

∴ $\boxed{\text{Rate of CGST = Rate of SGST = 9\%}}$

∴ Rate of GST = Rate of CGST + Rate of SGST

= 9% + 9%

∴ $\boxed{\textbf{Rate of GST = 18\%}}$

(3) 'M/s Real Paint' sold 2 tins of lustre paint and taxable value of each tin is ₹ 2800. If the rate of GST is 28%, then find the amount of CGST and SGST charged in the tax invoice. *(2 mark)*

Solution:

Taxable value of 2 boxes of paints = 2 × 2800

= ₹ 5600

Rate of GST = 28%

∴ Rate of CGST is 14% and rate of SGST is 14%.

∴ CGST = 14% of ₹ 5600

$= \frac{14}{100} \times 5600$

= ₹ 784

∴ CGST = SGST = ₹ 784

∴ **The amount of CGST is ₹ 784 and SGST is ₹ 784 charged in tax invoice.**

Problem Set - 4A *(Textbook Page No. 110)*

(4) A trader from Surat, Gujarat sold cotton clothes to a trader in Rajkot, Gujarat. The taxable value of cotton clothes is ₹ 2.5 lacs. What is the amount of GST at 5% paid by the trader in Rajkot? *(1 mark)*

Solution:

Taxable value of cotton clothes = ₹ 2.5 lacs

Rate of GST applicable = 5%

∴ GST = 5% of ₹ 2.5 lacs

$= \frac{5}{100} \times 250000$

= ₹ 12,500

∴ **The GST to be paid by businessman from Rajkot is ₹ 12,500.**

Practice Set - 4.1 *(Textbook Page No. 87)*

(4) The taxable value of Wrist watch belt is ₹ 586. Rate of GST is 18%, then what is price of the belt for the customer? *(2 marks)*

Solution:

Taxable value = ₹ 586

Rate of GST = 18%

∴ Rate of CGST = 9% and Rate of SGST = 9%

∴ CGST = 9% of ₹ 586

$= \frac{9}{100} \times 586$

= ₹ 52.74

∴ CGST = SGST = ₹ 52.74

∴ Amount to be paid by customer = Taxable value + CGST + SGST

= 586 + 52.74 + 52.74

= ₹ 691.48

∴ **The price of belt for customer is ₹ 691.48.**

Problem Set - 4A *(Textbook Page No. 110)*

(3) A ready-made garment shopkeeper gives 5% discount on the dress of ₹ 1000 and charges 5% GST on the remaining amount, then what is the purchase price of the dress for the customer? *(3 marks)*

Solution:

Price of a dress = ₹ 1000

Rate of discount = 5%

∴ Discount = 5% of ₹ 1000

$= \frac{5}{100} \times 1000$

= ₹ 50

∴ Taxable value of a dress = 1000 – 50 = ₹ 950

Rate of GST = 5%

∴ Rate of CGST is 2.5% and rate of SGST is 2.5%

∴ CGST = 2.5% of ₹ 950

$= \frac{2.5}{100} \times 950$

∴ CGST = ₹ 23.75

∴ CGST = SGST = ₹ 23.75

∴ Total cost of the dress = Taxable value + CGST + SGST

= 950 +23.75 + 23.75

= ₹ 997.50

∴ **The customer has to pay ₹ 997.50 for the dress.**

Practice Set - 4.1 *(Textbook Page No. 87)*

(5) The total value (with GST) of a remote-controlled toy car is ₹ 1770. Rate of GST is 18% on toys. Find the taxable value, CGST and SGST for this toy-car. *(4 marks)*

Solution:

Method - 1

Rate of GST = 18%

∴ If taxable value is ₹ 100 then total value of toy car including GST will be (100 + 18) = ₹ 118.

Similary, if taxable value is ₹ x, than total value of toy car including GST is ₹ 1770.

Taxable price (in ₹)	Total cost including GST (in ₹)
100	118
x	1770

$\therefore \quad \dfrac{100}{x} = \dfrac{118}{1770}$

$\therefore \quad x = \dfrac{100 \times 1770}{118}$

$\therefore \quad x = 1500$

$\therefore$ **The taxable value of a car is ₹ 1500**

Rate of GST = 18%

Rate of CGST is 9% and rate of SGST is 9%

$\therefore$ CGST = 9% of ₹ 1500

$= \dfrac{9}{100} \times 1500$

= ₹ 135

$\therefore$ **CGST = SGST = ₹ 135**

Method - 2

Let the taxable value of a car be ₹ x.

Rate of GST = 18%

$\therefore$ GST = 18% of ₹ x

$= \dfrac{18}{100} \times x$

$=$ ₹ $\dfrac{18x}{100}$ ...(i)

$\therefore$ Total value of car including GST =

Taxable value + GST

$= x + \dfrac{18x}{100}$

$= \dfrac{100x + 18x}{100}$

$=$ ₹ $\dfrac{118x}{100}$

But, total value of car including GST is ₹ 1770. ...(given)

$\therefore \quad \dfrac{118x}{100} = 1770$

$\therefore \quad x = \dfrac{1770 \times 100}{118}$

$\therefore \quad x = 1500$

$\therefore$ **The taxable value of a car is ₹ 1500**

GST = ₹ $\dfrac{18x}{100}$ ...[from (i)]

$= \dfrac{18 \times 1500}{100}$

$\therefore$ GST = ₹ 270

$\therefore$ CGST = SGST = $\dfrac{270}{2}$

$\therefore$ **CGST = SGST = ₹ 135**

Problem Set - 4A *(Textbook Page No. 110)*

(7) A dealer supplied Walky-Talky set of ₹ 84,000 (with GST) to police control room. Rate of GST is 12%. Find the amount of state and central GST charged by the dealer. Also find the taxable value of the set. *(3 marks)*

Solution:

Method - 1

Rate of GST = 12%

$\therefore$ If taxable value is ₹ 100, then total value including GST will be (100 + 12) = ₹ 112.

Similarly, if taxable value is ₹ x then total value including GST is ₹ 84,000.

Taxable value (in ₹)	Total value including GST (in ₹)
100	112
x	84,000

$\therefore \quad \dfrac{100}{x} = \dfrac{112}{84000}$

$\therefore \quad x = \dfrac{84000 \times 100}{112}$

$\therefore \quad x = 75000$

$\therefore$ **The taxable value of Walky Talky sets is ₹ 75,000**

Rate of GST = 12%

$\therefore$ Rate of CGST is 6% and rate of SGST is 6%

$\therefore$ CGST = 6% of ₹ 75000

$= \dfrac{6}{100} \times 75000$

$\therefore$ CGST = ₹ 4500

$\therefore$ **CGST = SGST = ₹ 4500**

Method - 2

Let the taxable value of Walky-Talky set be ₹ x.

Rate of GST = 12%

$\therefore$ GST = 12% of ₹ x

$= \dfrac{12}{100} \times x$

$\therefore$ GST = ₹ $\dfrac{12x}{100}$ ... (i)

$\therefore$ Total value of Walky-Talky including GST

= Taxable value + GST

$= x + \dfrac{12x}{100}$

$= \dfrac{100x + 12x}{100}$

$= \frac{112x}{100}$

But total value of Walky-Talky including GST is ₹ 84000

$\therefore \frac{112x}{100} = 84000$

$\therefore x = \frac{84000 \times 100}{112}$

$\therefore x = 75000$

$\therefore$ **The taxable value of Walky-Talky set is ₹ 75,000**

$\therefore$ GST = ₹ $\frac{12x}{100}$ ... from (i)

$= \frac{12 \times 75000}{100}$

$\therefore$ GST = ₹ 9000

$\therefore$ CGST = SGST = $\frac{9000}{2} = 4500$

$\therefore$ **The amount of CGST = SGST = ₹ 4500**

Practice Set - 4.1 *(Textbook Page No. 87)*

(6) 'Tiptop Electronics' supplied an AC of 1.5 ton to a company. Cost of the AC supplied is Rs. 51,200 (with GST). Rate of CGST on AC is 14%. Then find the following amounts as shown in the tax invoice of Tiptop Electronics.

(1) Rate of SGST (2) Rate of GST on AC

(3) Taxable value of AC (4) Total amount of GST

(5) Amount of CGST (6) Amount of SGST

(4 marks)

Solution:

(i) Rate of CGST on A.C. = 14% ...(given)

$\therefore$ **Rate of SGST on A.C. = 14%**

(ii) Rate of GST on A.C. = Rate of CGST + Rate of SGST

= 14% + 14%

$\therefore$ **Rate of GST on A.C. = 28%**

(iii) If taxable value of A.C. is ₹ 100, then total value including GST will be (100 + 28) = ₹ 128.

Similary, if taxable value of an A.C. is ₹ x then total value including GST is ₹ 51,200.

Taxable value (in ₹)	Total value including GST (in ₹)
100	128
x	51200

$\therefore \frac{100}{x} = \frac{128}{51200}$

$\therefore x = \frac{51200 \times 100}{128}$

$\therefore x = 40000$

$\therefore$ **The taxable value of an A.C. is ₹ 40,000**

(iv) Total GST = Total cost including GST – Taxable price

= 51,200 – 40,000

$\therefore$ **Total GST = ₹ 11,200**

(v) Amount of CGST = $\frac{\text{Total GST}}{2}$

$= \frac{11200}{2}$

$\therefore$ **Amount of CGST = ₹ 5600**

(vi) Amount of CGST = Amount of SGST = ₹ 5600

$\therefore$ **Amount of SGST = ₹ 5600**

(7) Prasad purchased a washing-machine from 'Maharashtra Electronic Goods'. The discount of 5% was given on the printed price of ₹ 40,000. Rate of GST charged was 28%. Find the purchase price of washing machine. Also find the amount of CGST and SGST shown in the tax invoice.

(4 marks)

Solution:

The printed price of a washing machine = ₹ 40,000

Rate of Discount = 5%

$\therefore$ Discount = 5% of ₹ 40,000

$= \frac{5}{100} \times 40000$

= ₹ 2000

$\therefore$ Taxable value of washing machine

= Printed price – Discount

= 40,000 – 2000

= ₹ 38,000

Rate of GST on washing machine = 28%

$\therefore$ Rate of CGST is 14% and rate of SGST is 14%

$\therefore$ CGST = 14% of ₹ 38000

$= \frac{14}{100} \times 38000$

$\therefore$ CGST = ₹ 5320

$\therefore$ **CGST = SGST = ₹ 5320**

Purchase price of washing machine

= Taxable value + CGST + SGST

= 38000 + 5320 + 5320

= ₹ 48,640

∴ **Prasad paid ₹ 48,640 for washing machine.**

Problem Set - 4-A *(Textbook Page No. 110)*

(2) **A dealer has given 10% discount on a showpiece of ₹ 25,000. GST of 28% was charged on the discounted price. Find the total amount shown in the tax invoice. What is the amount of CGST and SGST ?** *(4 marks)*

Solution:

The price of an article = ₹ 25,000

Rate of discount = 10%

∴ Discount = 10% of ₹ 25000

$$= \frac{10}{100} \times 25000$$

= ₹ 2500

∴ Taxable value of an article = 25000 – 2500

= ₹ 22500

Rate of GST = 28%

∴ Rate of CGST = Rate of SGST = 14%

∴ CGST = 14% of ₹ 22,500

$$= \frac{14}{100} \times 22500$$

∴ CGST = ₹ 3,150

∴ **CGST = SGST = ₹ 3,150**

∴ Total cost of an article = Taxable price + CGST + SGST

= 22500 + 3150 +3150

= ₹ 28,800

∴ **Total bill amount is ₹ 28800, CGST = ₹ 3150 and SGST = ₹ 3150**

Points to Remember:

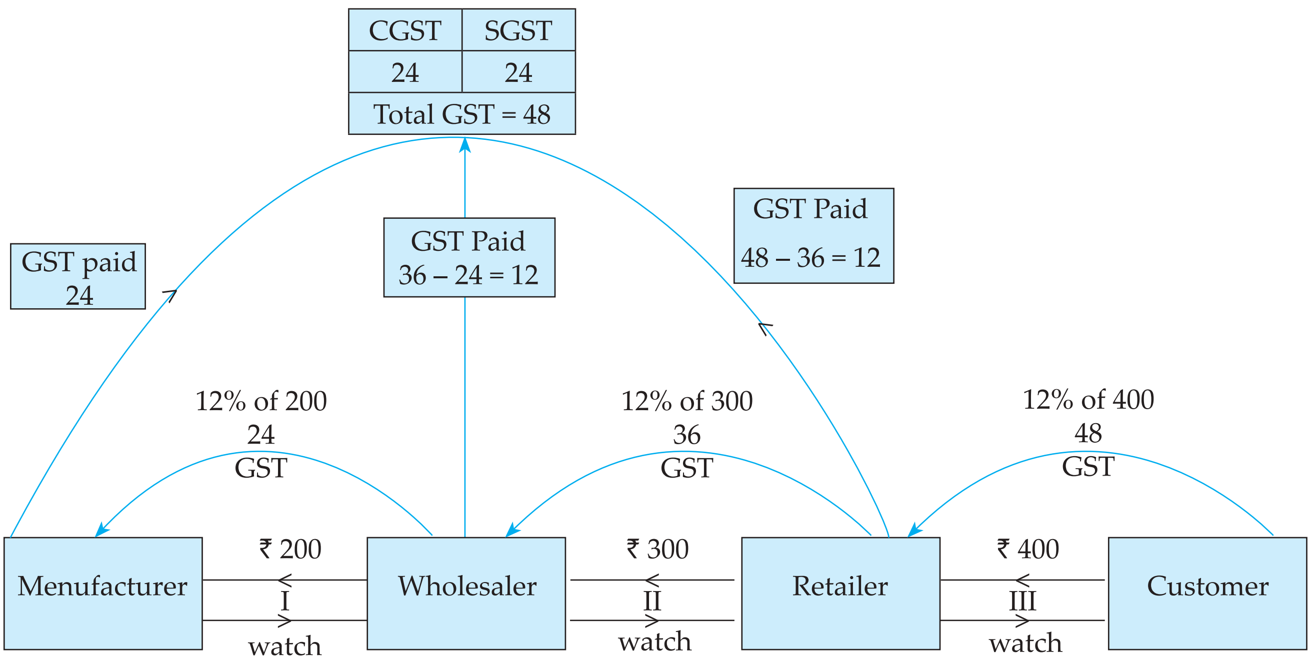
GST in trading chain

Manufacturer → Distributor or wholesaler → Retailer → Customer

(Trading chain)

In trading chain, how is GST charged and paid to Government at every stage is given below:

Example: The manufacturer of a watch sold a watch to wholesaler at ₹ 200 (including profit). The wholesaler sold it to retailer for ₹ 300 and retailer sold it to customer for ₹ 400. The rate of GST is 12%. Then how manufacturer, wholesaler and retailer will pay tax is given in the following flow chart.



Here, all the three financial transactions took place in one state. The tax invoices given below will help you to understand the application of GST in these transactions.

GST in Tax Invoice I
Price of watch = ₹ 200
CGST 6% = ₹ 12
SGST 6% = ₹ 12
Total price = ₹ 224

Tax invoice of manufacturer (B2B)

GST in Tax Invoice II
Price of watch = ₹ 300
CGST 6% = ₹ 18
SGST 6% = ₹ 18
Total price = ₹ 336

Tax invoice of wholesaler (B2B)

GST in Tax Invoice III
Price of watch = ₹ 400
CGST 6% = ₹ 24
SGST 6% = ₹ 24
Total price = ₹ 448

Tax invoice of retailer (B2C)

Note : B2B means trading between GSTIN holders is known as Business to Business. B2C means trading between GSTIN holders and consumer.

In this transation, GST paid by each businessman is gives below.

		CGST		SGST		Total GST paid
(i)	Manufacturere	₹ 12	+	₹ 12	=	₹ 24
(ii)	Wholesaler	₹ 6	+	₹ 6	=	₹ 12
(iii)	Retailer	₹ 6	+	₹ 6	=	₹ 12
	Total	₹ 24	+	₹ 24	=	₹ 48

- **ITC - Input Tax Credit System:**

GST is levied and collected at every stage of trading from manufacturer to consumer. When trader pays GST at the time of purchase, it is called '**Input tax**' and he collects GST at the time of sale which is called '**Output tax**'. At the time of paying GST to the government a trader deducts the input tax from the output tax and pays the remaining tax. This deduction of input tax is called **Input Tax Credit.**

GST payable = Output tax - ITC

Example: The cycle manufacturer sold a cycle to wholesaler for taxable price of ₹ 4000. The wholesaler sold it to retailer for ₹ 4800, (taxable value). Retailer sold it to customer at ₹ 5200 (taxable price). The rate of GST is 12%, then complete the following activity to find CGST and SGST at each stage of trading. *(Textbook Page No. 92)*

Solution: Trading Chain

Manufacturer $\xrightarrow[12\%]{₹\ 4000}$ Wholesaler $\xrightarrow[12\%]{₹\ 4800}$ Retailer $\xrightarrow[12\%]{₹\ 5200}$ Customer

Output tax of manufacturer

= 12% of 4000

= $\boxed{4000 \times \frac{12}{100}}$

= $\boxed{₹\ 480}$

∴ GST payable by manufacturer = ₹ 480

Output tax of wholesaler

= 12% of 4800

= $\boxed{₹\ 576}$

∴ GST payable by wholesaler = Output tax – Input tax

= 576 – 480

= ₹ 96

Output tax of retailer

= 12% of 5200

= $\boxed{₹\ 624}$

∴ GST payable by Retailer = Output tax of Retailer – ITC of retailer

= $\boxed{624}$ – $\boxed{576}$

= $\boxed{₹\ 48}$

In this business chain, GST is paid as follows:

Individual	GST	CGST	SGST
Manufacturer	₹ 480	₹ 240	₹ 240
Wholesaler	₹ 96	₹ 48	₹ 48
Retailer	₹ 48	₹ 24	₹ 24
Total	₹ 624	₹ 312	₹ 312

Practice Set - 4.2 *(Textbook Page No. 93)*

(1) 'Chetana Store' paid total GST of ₹1,00,500 at the time of purchase and collected GST ₹1,22,500 at the time of sale during 1st of July 2017 to 31st July 2017. Find the GST payable by Chetana Stores. *(2 marks)*

Solution:

Input tax (ITC) = ₹ 1,00,500

Output tax = ₹ 1,22,500

∴ GST payable = Output tax – ITC

=1,22,500 – 1,00,500

∴ GST = ₹ 22,000

∴ **The GST payable by Chetana Stores is ₹ 22,000**

(2) Nazama is a proprietor of a firm, registered under GST. She has paid GST of ₹ 12,500 on purchase and collected ₹ 14,750 on sale. What is the amount of ITC to be claimed ? What is the amount of GST payable ? *(2 marks)*

Solution:

Input tax (ITC)= ₹ 12,500

Output tax = ₹ 14,750

∴ GST Payable = Output tax – ITC

= 14,750 – 12,500

∴ GST = ₹ 2250

∴ **Input tax credit for Nazma is ₹ 12500 and GST to be paid is ₹ 2250**

Problem Set - 4A *(Textbook Page No. 110)*

(5) Smt. Malhotra purchased solar panels for the taxable value of ₹ 85,000. She sold them for ₹ 90,000. The rate of GST is 5%. Find the ITC of Smt. Malhotra. What is the amount of GST payable by her ? *(3 marks)*

Solution:

Input tax (ITC) = 5% of ₹ 85,000

$= \frac{5}{100} \times 85000$

∴ ITC = ₹ 4250

Output tax = 5% of ₹ 90,000

$= \frac{5}{100} \times 90000$

∴ Output tax = ₹ 4500

∴ GST Payable = Output Tax – ITC

= 4500 – 4250

∴ GST = ₹ 250

∴ **ITC = ₹ 4250 and GST payable = ₹ 250**

(6) A company provided Z-security services for the taxable value of ₹ 64,500. Rate of GST is 18%. Company had paid GST of ₹ 1550 for laundry services and uniforms etc. What is the amount of ITC (input Tax Credit)? Find the amount of CGST and SGST payable by the company. *(4 marks)*

Solution:

Taxable value for security provided = ₹ 64,500

Rate of GST = 18%

Output tax=18% of ₹ 64,500

$= \frac{18}{100} \times 64500$

= ₹ 11,610

GST paid by company for laundry services and uniform etc. = ₹ 1550

∴ ITC = ₹ 1550

∴ GST payable by the company = Output tax – ITC

=11610 – 1550

=₹ 10,060

Now CGST = SGST = $\frac{10060}{2}$

= ₹ 5030

∴ **ITC for the company is ₹ 1550 and CGST = SGST = ₹ 5030**

Practice Set - 4.2 *(Textbook Page No. 93)*

(3) Amir Enterprise purchased chocolate sauce bottles and paid GST of ₹ 3800. He sold those bottles to Akbari Bros. and collected GST of ₹ 4100. Mayank Food Corner purchased these bottles from Akbari Bros and paid GST of ₹ 4500. Find the amount of GST payable at every stage of trading and hence find payable CGST and SGST. *(4 marks)*

Solution:

(i) For Amir Enterprises:

ITC = ₹ 3800

Output tax = ₹ 4100

∴ GST payable = Output tax – ITC

= 4100 – 3800

∴ GST = ₹ 300

∴ **GST payable by Amir Enterprises is ₹ 300 and CGST = SGST = $\frac{300}{2}$ = ₹ 150**

(ii) For Akbari Brothers

ITC = ₹ 4100

Output tax= ₹ 4500

∴ GST payable = Output tax – ITC

=4500 – 4100

∴ GST = ₹ 400

∴ **GST to be payable by Akbari Brothers is ₹ 400 and CGST = SGST = $\frac{400}{2}$ = ₹ 200**

Problem Set - 4A *(Textbook Page No. 110)*

*(8) **A wholesaler purchased electric goods for the taxable amount of ₹ 1,50,000. He sold it to the retailer for the taxable amount of ₹ 1,80,000. Retailer sold it to the customer for the taxable amount of ₹ 2,20,000. Rate of GST is 18%. Show the computation of GST in tax invoices of sales. Also find the payable CGST and payable SGST for wholesaler and retailer.** *(4 marks)*

Solution:

(i) For wholesaler:

Taxable amount of Electric goods = ₹ 1,50,000

Rate of GST = 18%

∴ Tax paid at the time of purchase (ITC)

= 18% of ₹ 1,50,000

= $\frac{18}{100} \times 150000$

= ₹ 27,000

Tax collected at time of sale (Output tax)

= 18% of ₹ 1,80,000

= $\frac{18}{100} \times 180000$

= ₹ 32,400

∴ **CGST and SGST shown in the tax invoice of wholesaler = $\frac{32400}{2}$ = ₹ 16,200**

∴ GST payable by wholesaler = Output tax – ITC

= ₹ 32,400 – ₹ 27,000

GST payable by wholesaler = ₹ 5400

∴ CGST to be paid by wholeseller = $\frac{5400}{2}$

= ₹ 2700

∴ **CGST = SGST = ₹ 2700 is payable by wholeseller**

(ii) For retailer:

Tax paid at the time of purchase (ITC) = ₹ 32,400

Tax collected at the time of sale (Output tax)

= 18% of ₹ 2,20,000

= $\frac{18}{100} \times 220000$

= ₹ 39,600

∴ CGST and SGST shown in the tax invoice of retailer = $\frac{39600}{2}$

= ₹ 19,800

GST payable by retailer = Output tax – ITC

= ₹ 39,600 – ₹ 32,400

GST payable by retailer = ₹ 7200

∴ CGST payable by wholeseller = $\frac{7200}{2}$

= ₹ 3600

∴ **CGST = SGST = ₹ 3600 payable by retailer**

Practice Set - 4.2 *(Textbook Page No. 93)*

(4) **Malik Gas Agency (Chandigarh Union Territory) purchased some gas cylinders for industrial use for ₹ 24,500, and sold them to the local customers for ₹ 26,500. Find the GST to be paid at the rate of 5% and hence the CGST and UTGST to be paid for this transaction. (for Union Territories there is UTGST instead of SGST.)** *(3 marks)*

Solution:

Tax paid at the time of purchase (Input tax)

= 5% of ₹ 24,500

= $\frac{5}{100} \times 24500$

= ₹ 1225

∴ ITC = ₹ 1225

Tax collected at the time of sale (output tax)

= 5% of ₹ 26,500

= $\frac{5}{100} \times 26500$

= ₹ 1325

∴ GST = Output tax – ITC

= 1325 – 1225

∴ GST = ₹ 100

∴ CGST = (UTGST) = $\frac{100}{2}$ = ₹ 50

∴ **GST to be paid by Malik gas agency is ₹ 100, CGST = ₹ 50 and UTGST = ₹ 50**

Problem Set - 4A *(Textbook Page No. 110)*

***(9)** **Anna Patil (Thane, Maharashtra) supplied vacuum cleaner to a shopkeeper in Vasai (Mumbai) for the taxable value of ₹ 14,000, and GST rate of 28%. Shopkeeper sold it to the customer at the same GST rate for ₹ 16,800 (taxable value) Find the following -**

(1) Amount of CGST and SGST shown in the tax invoice issued by Anna Patil.

(2) Amount of CGST and SGST charged by the shopkeeper in Vasai.

(3) What is the CGST and SGST payable by shopkeeper in Vasai at the time of filing the return. *(4 marks)*

Solution:

All the transactions are done in Maharashtra.

For Anna Patil in Thane:

Taxable value of Vaccum Cleaner = ₹ 14,000

Rate of GST = 28%

∴ GST = 28% of 14,000

$= \frac{28}{100} \times 14000$

∴ GST = ₹ 3920

∴ CGST (shown in the tax invoice of Anna Patil) $= \frac{3920}{2}$

=₹ 1960 ∴

CGST = SGST = ₹ 1960 shown in the tax invoice of Anna Patil

For Shopkeeper in Vasai:

Input tax credit (ITC) for shopkeeper = ₹ 3920

Taxable value at which Vaccum cleaner sold to customer = ₹ 16,800

Rate of GST = 28%

∴ GST = 28% of ₹ 16800

$= \frac{28}{100} \times 16800$

∴ GST = ₹ 4704

∴ CGST charged by shopkeeper $= \frac{4704}{2}$

= ₹ 2352

∴ **CGST = SGST = ₹ 2352 charged by the shopkeeper.**

Now, GST to be paid by shopkeepr at Vasai

= Output tax – ITC

= 4704 – 3920

= ₹ 784

∴ CGST to be paid by shopkeeper at Vasai $= \frac{784}{2}$ = ₹ 392

∴ **CGST = SGST = ₹ 392 to be paid by shopkeeper in Vasai**

Practice Set - 4.2 *(Textbook Page No.93)*

(5) **M/s Beauty Products paid 18% GST on cosmetics worth ₹ 6000 and sold to a customer for ₹ 10,000. What are the amounts of CGST and SGST shown in the tax invoice issued ?** *(3 marks)*

Solution:

Tax paid at the time of purchase (ITC)

= 18% of ₹ 6000

$= \frac{18}{100} \times 6000$

= ₹ 1080

Tax collected at the time of sale (Output tax)

= 18% of ₹ 10,000

$= \frac{18}{100} \times 10000$

= ₹ 1800

∴ GST = Output tax – ITC

= 1800 – 1080

∴ GST = ₹ 720

Tax invoice for GST:

Beauty Products	GST	CGST	SGST
Purchase	₹ 1080	₹ 540	₹ 540
Sale	₹ 720	₹ 360	₹ 360
Total	₹ 1800	₹ 900	₹ 900

∴ **CGST = SGST = ₹ 900**

(6) **Prepare Business to Consumer (B2C) tax invoice using given information. Write the name of the supplier, address, state, Date, invoice number, GSTIN etc. as per your choice. Supplier : M/s .... Address .... State .... Date ....**

Invoice No. .... GSTIN ....

Particulars - Rate of Mobile Battery - ₹ 200 Rate of GST 12% HSN 8507, 1 pc.

Rate of Headphone - ₹ 750 Rate of GST 18% HSN 8518, 1 pc. *(3 marks)*

Solution:

Tax Invoice

Supplier : M/s Shruti Electicals Invoice No. : 64
401/B, Jijamata Road, Andheri (East)
Mumbai - 400 093, Maharashtra
Tel.: 022-6232685
GSTIN : 27PQRST1234H126 Invoice Date: 31.01.2018

Sr. No.	HSN Code	Name of Product	Rate (₹)	Quan-tity	Taxable amount	CGST		SGST		Total (₹)
						Rate	Tax	Rate	Tax	
(1)	8507	Mobile Battery	200	1	₹ 200	6%	12	6%	12	224
(2)	8518	Head phone	750	1	₹ 750	9%	67.50	9%	67.50	885
					Grand Total **₹ 950**		**79.50**		**79.50**	**1109**

(7) Prepare Business to Business (B2B) Tax Invoice as per the details given below.
name of the supplier, address, Date etc. as per your choice.
Supplier - Name, Address, State, GSTIN, Invoice No., Date
Recipient - Name, Address, State, GSTIN,
Items : (1) Pencil boxes 100, HSN - 3924, Rate ₹ 20, GST 12%
(2) Jigsaw Puzzles 50, HSN 9503, Rate ₹ 100 GST 12%. *(3 marks)*

Solution:

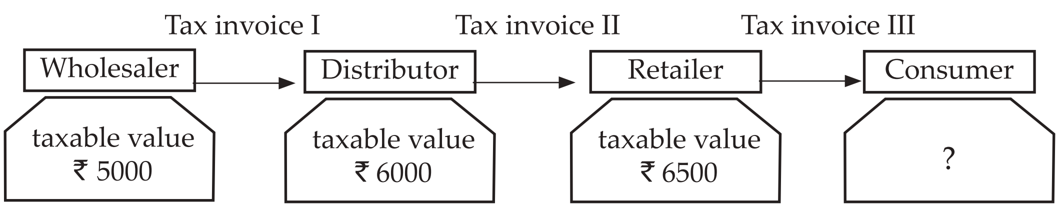
Tax Invoice

Supplier : M/s Sahil Enterprises GSTIN: 27LMOAB1397H1Z1
B/24, Paranjape Scheme, Malad (W)
Mumbai - 400 069, Maharashtra
Receiver : M/s Sandeep Book Agency GSTIN: 27ABCDE7999H1Z7
2, Ground Floor,N. M. Joshi Marg,
Lower Parel, Mumbai - 400 004, Maharashtra
Invoice No. : GST/19 Invoice Date: 12.01.2018

Sr. No.	HSN Code	Name of Product	Rate (₹)	Quantity	Taxable amount	CGST		SGST		Total (₹)
						Rate	Tax	Rate	Tax	
(1)	3924	Pencil box	20	100	₹ 2000	6%	120	6%	120	2240
(2)	9503	Jigsaw puzzles	100	50	₹ 5000	6%	300	9%	300	5600
				Grand Total :	**₹ 7000**		**420**		**420**	**7840**

Problem Set - 4A *(Textbook Page No. 111)*

***(10) For the given trading chain prepare the tax invoice I, II, III. GST at the rate of 12% was charged for the article supplied.**



(1) **Prepare the statement of GST payable under each head by the wholesaler, distributor and retailer at the time of filing the return to the government.**

(2) **At the end what amount is paid by the consumer ?**

(3) **Write which of the invoices issued are B2B and B2C ?** *(4 marks)*

Solution:

(i) GST collected by wholesaler at the time of sale (Output tax)

= 12% of ₹ 5000

$= \frac{12}{100} \times 5000$

= 600

∴ GST payable by distributor = ₹ 600

GST collected by distributor at the time of sale (Output tax)

= 12% of ₹ 6000

$= \frac{12}{100} \times 6000$

= ₹ 720

∴ GST payable by distributor = Output tax – ITC

= 720 – 600

= ₹ 120

GST collected by retailer at the time of sale (Output tax)

= 12% of ₹ 6500

$= \frac{12}{100} \times 6500$

= ₹ 780

∴ GST payable by retailer = Output tax – ITC

= 780 – 720

= ₹ 60

Persons	Payable GST	Payable CGST	Payable SGST
Wholesaler	₹ 600	₹ 300	₹ 300
Distributor	₹ 120	₹ 60	₹ 60
Retailer	₹ 60	₹ 30	₹ 30
Total Tax	₹ 780	₹ 390	₹ 390

(ii) Price at which article sold to customer

= Taxable value + GST

= 6500 + 780

= ₹ 7280

(iii) The tax invoice between wholesaler and distributor is B2B type.

Also the tax invoice between distributor and retailer is B2B type.

The tax invoice between retailer and customer is B2C type.

Points to Remember:

For a small business one person can invest sufficient capital. Capital required for a medium business can be raised in partnership. But when a large business or industry is to be established, a partnership of a few persons is not sufficient to provide the required capital. In such a case, some persons come together and form a company. These persons are known as **promoters** of the company. The company is established by these promoters under the Companies Act of 1956. Such a company is Public Limited Company. They divide the required capital into small parts called **shares**. These shares are generally worth face value of ₹ 1, ₹ 2, ₹ 5, ₹ 10 or ₹ 100 etc.

- **Share:**
  A share is the smallest unit of the capital of the company. The value of share is printed on the company's certificates with other details and it is called Share Certificate.
- **Share Holder:**
  A person who holds one or more shares is called a share holder and ultimately the part owner of the company in the proportion of number of shares he/she holds.
- **Stock Exchange:**
  The place where the transactions of buying and selling of shares takes place is known as the **Stock Exchange** or **share market** or **stock market** or **capital market** or **Equity market.** The public limited company must be a listed company in share market.
- **Face Value (FV):**
  The value of the share printed on the share certificate is called the Face value of the share. It is also called Nominal value or Printed value or par value.
- **Market Value (MV):**
  The price at which the shares are sold or purchased in the stock market is called Market value (MV) of the share. This market value (MV) may change frequently in the market.
- **Dividend:**
  Dividend is a part of a profit distributed to the share holders. Share holders gets dividend on number of shares having with him. No income

tax is payable on dividend received by share holder.

Note: The dividend is always paid on the face value of the share.

- **Comparison of Face Value (FV) and Market Value (MV):**

(i) If Market Value (MV) > Face Value (FV), then that share is called as **share is at premium.**

(ii) If market value (MV) = Face Value (FV) then that share is called as **share is at par.**

(iii) If market value (MV) < Face Value (FV) then the share is called as **share is at discount.**

- **Sum Invested:**

Sum invested is the total amount paid while purchasing shares.

∴ Sum Invested = Number of shares × Market Value (MV) of the share.

- **Rate of Return (ROR):**

It is very important to know the rate of return you get on amount invested in shares. Let us understand it by following examples.

Example (1) Shriyansh purchased a share with face value ₹ 100 at ₹ 120 market value. The company declared a dividend of 15%. Then, what is the rate of return on investment?

Solution:

Face Value = ₹ 100 , Market Value = ₹120, Dividend = 15%

Let the rate of return be x%.

Here, on investment of ₹120, Shriyansh got ₹15

If, 120 : 15

then, 100 : x

$$\therefore \frac{15}{120} = \frac{x}{100}$$

$$\therefore x = \frac{15 \times 100}{120}$$

$$= \frac{25}{2}$$

$$\therefore x = 12.5\%$$

∴ **Rate of return is 12.5%**

Example (2) Complete the following table by writing correct number or words?

Sr. No.	Face Value	Share is at	Market Value
(i)	₹ 10	Premium ₹ 7	
(ii)	₹ 25		₹ 16
(iii)		at par	₹ 5

Solution:

Sr. No.	Face Value	Share is at	Market Value
(i)	₹ 10	Premium ₹ 7	₹ 17
(ii)	₹ 25	Discount ₹ 9	₹ 16
(iii)	₹ 5	at par	₹ 5

Example (3) Smita has invested ₹ 12,000 and purchased shares of FV `10 at a premium at ` 2. Finad the number of shares she purchased. Complete the given activity to get the answer.

(Textbook Page No. 101)

Solution:

Face value = ₹ 10, Premium = ₹ 2

∴ Market value = Face value + $\boxed{\text{Premium}}$

$= \boxed{10} + \boxed{2}$

$= \boxed{₹\ 12}$

$$\therefore \text{No. of Shares} = \frac{\text{Total Investment}}{\text{Market value}}$$

$$= \frac{12000}{\boxed{12}}$$

$= \boxed{1000}$

∴ **Smita has purchased $\boxed{1000}$ shares.**

Practice Set - 4.3 *(Textbook Page No. 102)*

(1) Complete the following table by writing suitable numbers and words. *(1 mark)*

Sr. No.	Face Value	Share is at	Market Value
(i)	₹ 100	par	
(ii)		Premium ₹ 500	₹ 575
(iii)	₹ 10		₹ 5

Solution:

Sr. No.	Face Value	Share is at	Market Value
(i)	₹ 100	par	₹ 100
(ii)	₹ 75	Premium ₹ 500	₹ 575
(iii)	₹ 10	Discount ₹ 5	₹ 5

(3) Joseph purchased following shares, Find his total investment. *(3 marks)*

Company A : 200 shares, FV = ₹ 2 Premium = ₹ 18.

Company B : 45 shares, MV = ₹ 500

Company C : 1 share, MV = ₹ 10,540.

Solution:

Company A : Premium = ₹ 18 and FV = ₹ 2

∴ MV of 1 share = FV + Premium

= 2 + 18

= ₹ 20

Total investment in Company A = No. of shares × MV

= 200 × 20

= ₹ 4000

Company B : No. of shares = 45 ,

MV = ₹ 500

∴ Total investment in Company B = No. of shares × MV

= 45 × 500

= ₹ 22,500

Company C : No. of shares = 1,

MV = ₹ 10,540

∴ Total investment in Company C = No. of shares × MV

= 1 × 10,540

= ₹ 10,540

∴ **Total investment = 4000 + 22,500 + 10,540 = ₹ 37,040**

Problem Set - 4B *(Textbook Page No. 112)*

(4) Find the amount received when 300 shares of FV ₹ 100, were sold at a discount of ₹ 30. *(2 marks)*

Solution:

FV of share = ₹ 100

Discount = ₹ 30

∴ MV = FV – discount

= 100 – 30

= ₹ 70

∴ Market value of 1 share = ₹ 70

∴ Total amount obtained by selling 300 shares

= 300 × 70 = ₹ 21000

∴ **The amount obtained by selling 300 shares is ₹ 21,000.**

Practice Set - 4.3 *(Textbook Page No. 102)*

(4) Smt. Deshpande purchased shares of FV ₹ 5 at a premium of ₹ 20. How many shares will she get for ₹ 20,000 ? *(2 marks)*

Solution:

FV of share = ₹ 5, Premium = ₹ 20

∴ MV = FV + Premium

= 5 + 20 = ₹ 25

Total investment = ₹ 20,000

∴ Number of shares = $\dfrac{\text{Total Investment}}{\text{MV}}$

$= \dfrac{20000}{25}$

= 800

∴ **Mrs. Deshpande will get 800 shares.**

Problem Set - 4B *(Textbook Page No. 112)*

(5) Find the number of shares received when ₹ 60,000 was invested in the shares of FV ₹ 100 and MV ₹ 120. *(2 marks)*

Solution:

MV of share = ₹ 120

Total investment = ₹ 60,000

∴ Number of shares = $\dfrac{\text{Total Investment}}{\text{MV}}$

$= \dfrac{60000}{120}$

= 500

∴ **The number of shares purchased is 500**

Practice Set - 4.3 *(Textbook Page No. 102)*

(2) Mr. Amol purchased 50 shares of Face Value ₹ 100 when the Market value of the share was ₹ 80. Company had given 20% dividend. Find the rate of return on investment. *(3 marks)*

Solution:

FV of share = ₹ 100

Dividend per share = 20% of ₹ 100

$= \dfrac{20}{100} \times 100$

∴ Dividend per share = ₹ 20

∴ Dividend on 50 shares = 50 × 20

= ₹ 1000

∴ MV of share = ₹ 80

∴ Total investment = 50 × 80

= ₹ 4000

∴ Rate of return =

$\dfrac{\text{Total dividend received}}{\text{Total investment}} \times 100$

$= \dfrac{1000}{4000} \times 100$

∴ **Rate of return = 25%**

(5) Shri Shantilal has purchased 150 shares of FV ₹ 100, for MV of ₹ 120. Company has paid dividend at 7%. Find the rate of return on his investment. *(3 marks)*

Solution:

FV of share = ₹ 100

Dividend on one share = 7% of ₹ 100

$= \frac{7}{100} \times 100$

= ₹ 7

∴ Dividend on 150 shares = 150×7

= ₹ 1050

MV of share = ₹ 120

∴ Total investment = 150×120

= ₹ 18,000

∴ Rate of return =

$\frac{\text{Total dividend received}}{\text{Total investment}} \times 100$

$= \frac{1050}{18000} \times 100$

∴ **Rate of return = 5.83%**

Problem Set - 4B *(Textbook Page No. 112)*

(3) Prashant bought 50 shares of FV ₹ 100, having MV ₹. 180. Company gave 40% dividend on the shares. Find the rate of return on investment. *(3 marks)*

Solution:

FV of share = ₹ 100

Dividend on one share = 40% of ₹ 100

$= \frac{40}{100} \times 100$

= ₹ 40

Dividend on 50 shares = 50×40

= ₹ 2000

∴ MV of share = ₹ 180

∴ Total investment = 50×80

= ₹ 9000

∴ Rate of return =

$\frac{\text{Total dividend received}}{\text{Total investment}} \times 100$

$= \frac{2000}{9000} \times 100$

∴ **Rate of return = 22.22%**

Practice Set - 4.3 *(Textbook Page No. 103)*

(6) If the face value of both the shares is same, then which investment out of the following is more profitable ?
Company A : dividend 16%, MV = ₹ 80,
Company B : dividend 20%, MV = ₹ 120. *(4 marks)*

Solution:

Suppose the amount invested in both the companies is ₹ 9600, (i.e ₹ 80 × ₹ 120)

	Company A	Company B
Amount Invested	₹ 9600	₹ 9600
MV	₹ 80	₹ 120
No. of shares purchased = $\frac{\text{Amount Invested}}{\text{MV}}$	$\frac{9600}{80} = 120$	$\frac{9600}{120} = 80$
Dividend	16%	20%
FV	₹ 10	₹ 10
Dividend per share	16% of ₹ 10 $= \frac{16}{100} \times 10$ = ₹ 1.6	20% of ₹ 10 $= \frac{20}{100} \times 10$ = ₹ 2
Dividend on shares = Dividend per share × No. of share	120×1.6 = ₹ 192	80×2 = ₹ 160

∴ Company A gives more amount of dividend than company B,

∴ **Investment in Company A is more profitable**

Problem Set - 4B *(Textbook Page No. 112)*

(7) Market value of shares and dividend declared by the two companies is given below. Face Value is same and it is ₹ 100 for both the shares. Investment in which company is more profitable ?
(1) Company A - ₹ 132 , 12%
(2) Company B - ₹ 144, 16% *(4 marks)*

Solution:

Suppose the amount invested in both the companies is ₹ 19,008 (i.e. ₹ 132 × ₹ 144)

	Company A	Company B
Amount Invested	₹ 19,008	₹ 19,008
MV	₹ 132	₹ 144
No. of shares purchased = $\frac{\text{Amount Invested}}{\text{MV}}$	$\frac{19008}{132} = 144$	$\frac{19008}{144} = 132$
Dividend declared	12%	16%
Face value per share	₹ 100	₹ 100

Dividend per share	12% of ₹ 100 $= \frac{12}{100} \times 100$ = ₹ 12	16% of ₹ 100 $= \frac{16}{100} \times 100$ = ₹ 16
Dividend on shares =	144×12 = ₹ 1728	132×16 = ₹ 2112

$\therefore$ Company B gives more amount of dividend than company A

$\therefore$ **Investment in company B is more profitable**

Points to Remember:

- **Brokerage and taxes on share trading:**

  Brokerage: The buying and selling of shares cannot take place privately. It has to be done through a person or a body recognised by the stock exchange. Such a person is called **Share-Broker**. For transaction of buying and selling of shares, charges of the broker of particular rate paid both by the purchaser and the seller. These charges are called **brokerage**. Brokerage is paid on the MV of the share.

- **GST on Brokerage Services:**

  Share brokers provide services for purchase and sale of shares for their clients. These services are charged under GST. Rate of GST is 18% on brokerage.

Example: Nalinitai invested ₹ 6024 in the shares of FV ₹ 10 when the Market Value was ₹ 60. She sold all the shares at MV of ₹ 50 after taking 60% dividend. She paid 0.4% brokerage at each stage of transactions. What was the total gain or loss in this transaction? (4 marks)

Solution: Rate of GST is not given in the example, so it is not considered.

Shares Purchased : FV = ₹ 10, MV = ₹ 60

Brokerage per share = $\frac{0.4}{100} \times 60 =$ ₹ $\boxed{0.24}$

$\therefore$ Cost of one share = 60 + 0.24 = ₹ $\boxed{60.24}$

$\therefore$ Number of shares = $\frac{6024}{60.24} = 100$

Shares sold : FV ₹ 10, MV = ₹ 50

$\therefore$ Brokerage per share = $\frac{0.4}{100} \times 50$ = ₹ 0.20

$\therefore$ Selling price per share = 50 − 0.20 = ₹ 49.80

$\therefore$ Selling price of 100 shares = 100 × 49.80 = ₹ 4980

Dividend received 60%

$\therefore$ Dividend per share = $\frac{60}{100} \times 10$ = ₹ 6

$\therefore$ Dividend on 100 shares = 6 × 100 = ₹ 600

$\therefore$ Nalinitai's income = 4980 + 600 = ₹ 5580

Sum invested = ₹ 6024

$\therefore$ Loss = 6024 − 5580 = ₹ 444

Ans. Nalinitai's loss is Rs. 444

- **Mutual Fund (M.F.) :**

  Mutual Fund is a professionally managed investment scheme, usually run by an AMC i.e. Asset Management Company. They invest the money given by the investors in different schemes e.g. equity fund (in shares), debt fund (in debentures, bonds etc.) or balanced funds as per the investor's choice.

  As we get 'shares' for the investment in sharemarket, in Mutual Fund investment we get 'units' when we invest in mutual fund.

  The market value of 'a unit' is called **'NAV'** (Net Asset Value)

  NAV of one unit × Number of units = Total fund value

  As the market value of share changes frequently in the same way NAV of a unit also changes. One can redeem the units when needed.

- **Systematic Investment Plan (SIP):**

  Suppose, one does not want to invest a big amount at once, then invest small amounts at regular time intervals e.g. Rs. 500 per month could be invested in mutual fund. Investment could be done monthly or quarterly. This way of investment is called SIP.

- **Benefits of Mutual Funds:**

  (1) Professional fund managers

  (2) Diversification of funds

  (3) Transparency and sufficiently safe investment

  (4) Liquidity- redemption of units can be done

  (5) limited risks

  (6) Advantage of long term and short term gain

  (7) Investment in funds like ELSS are admissible for deduction under section 80C of Income tax .

Practice Set - 4.4 *(Textbook Page No. 109)*

(1) Market value of a share is ₹ 200. If the brokerage rate is 0.3% then find the purchase value of the share. *(2 marks)*

Solution:

MV = ₹ 200

Brokerage = 0.3%

Brokerage per share = 0.3% of ₹ 200

$= \frac{0.3}{100} \times 200$

$= ₹ 0.60$

Purchase value per share = Market value + Brokerage

$= 200 + 0.60$

$= ₹ 200.60$

∴ **Purchase value of the share is ₹ 200.60**

Problem Set - 4B *(Textbook Page No. 111)*

(2) Find the purchase price of a share of FV ₹ 100 if it is at premium of ₹ 30. The brokerage rate is 0.3%. *(2 marks)*

Solution:

FV = ₹ 100 and Premium = ₹ 30

∴ MV = FV + Premium

= 100 + 30

= ₹ 130

Brokerage per share = 0.3% of ₹ 130

$= \frac{0.3}{100} \times 130$

= ₹ 0.39

∴ Purchase price per share = 130 + 0.39

= ₹ 130.39

∴ **The purchase price of one share is ₹ 130.39**

Practice Set - 4.4 *(Textbook Page No. 109)*

(2) A share is sold for the market value of ₹ 1000. Brokerage is paid at the rate of 0.1%. What is the amount received after the sale ? *(3 marks)*

Solution:

MV = ₹ 1000

Brokerage = 0.1%

Brokerage per share = 0.1% of ₹ 1000

$= \frac{0.1}{100} \times 1000$

= ₹ 1

∴ Selling price per share = MV – Brokerage

= 1000 – 1

= ₹ 999

∴ **The amount obtained on selling the share is ₹ 999.**

(3) Fill in the blanks given in the contract note of sale-purchase of shares. (B - buy S - sell) *(3 marks)*

No. of shares	MV of shares	Total value	Brokerage 0.2%	9% CGST on brokerage	9% SGST on brokerage	Total Value of shares
100 B	₹ 45					
75 S	₹ 200					

Solution:

No. of shares	MV of shares	Total value	Brokerage 0.2%	9% CGST on brokerage	9% SGST on brokerage	Total Value of shares
100 B	₹ 45	₹ 4500	₹ 9	₹ 0.81	₹ 0.81	₹ 4510.62
75 S	₹ 200	₹ 15000	₹ 30	₹ 2.70	₹ 2.70	₹ 14964.60

(4) Smt. Desai sold shares of face value ₹ 100 when the market value was ₹ 50 and received ₹ 4988.20. She paid brokerage 0.2% and GST on brokerage 18%, then how many shares did she sell ? *(4 marks)*

Solution:

Market value of share = ₹ 50

Brokerage = 0.2%

∴ Brokerage per share = 0.2% of ₹ 50

$= \frac{0.2}{100} \times 50$

= ₹ 0.10

∴ GST on brokerage per share = 18 % of ₹ 0.10

$= \frac{18}{100} \times 0.10$

= ₹ 0.018

Total selling price per share = MV – Brokerage – GST

= 50 – 0.10 – 0.018

= ₹ 49.882

∴ Number of shares sold = $\frac{\text{Total amount received}}{\text{Selling price of one share}}$

$= \frac{4988.20}{49.882}$

= 100

∴ **Smt. Desai sold 100 shares.**

Problem Set - 4B *(Textbook Page No. 112)*

*(8) **Shri. Aditya Sanghavi invested ₹ 50,118 in shares of FV ₹ 100, when the market value is ₹ 50. Rate of brokerage is 0.2% and Rate of GST on brokerage is 18%, then How many shares were purchased for ₹ 50,118 ?** *(4 marks)*

Solution:

Market value of share = ₹ 50

Brokerage = 0.2%

Brokerage per share = 0.2% of ₹ 50

$= \frac{0.2}{100} \times 50$

= ₹ 0.10

GST on brokerage per share = 18% of ₹ 0.10

$= \frac{18}{100} \times 0.10$

= ₹ 0.018

Total purchase price per share =

Market value + Brokerage + GST

= 50 + 0.10 + 0.018

= ₹ 50.118

Number of shaers purchased =

$\frac{\text{Total investment}}{\text{Purchase price per share}}$

$= \frac{50118}{50.118}$

= 1000

∴ **Mr. Sanghvi purchased 1000 shares**

Practice Set - 4.4 *(Textbook Page No. 109)*

(5) **Mr. D'souza purchased 200 shares of FV ₹ 50 at a premium of ₹ 100. He received 50% dividend on the shares. After receiving the dividend he sold 100 shares at a discount of ₹ 10 and remaining shares were sold at a premium of ₹ 75. For each trade he paid the brokerage of ₹ 20. Find whether Mr. D'souza gained or incurred a loss ? by how much ?** *(4 marks)*

Solution:

Share at premium = ₹ 100,

FV = ₹ 50 and dividend = 50%

Dividend per share = 50% of ₹ 50

$= \frac{50}{100} \times 50$

= ₹ 25

Dividend on 200 share = 200 × 25

= ₹ 5000

Purchase price per share = FV + Premium

= 50 + 100

= ₹ 150

∴ Purchase price of 200 shares

= 200 × 150 + Brokerage

= 30,000 + 20

= ₹ 30,020

Now, 100 shares were sold at ₹ 10 discount

∴ Selling price per share = FV – discount

= 50 – 10

= ₹ 40

∴ Selling price of first 100 shares

= 100 × 40 – Brokerage

= 4000 – 20 = ₹ 3980

Now, remaining 100 shares were sold at ₹ 75 premium.

∴ Selling price per share = FV + Premium

= 50 + 75

= ₹ 125

∴ Selling price of 100 shares

= 100 × 125 – Brokerage

= 12,500 – 20 = ₹ 12,480

∴ Amount received on selling 200 shares

= Dividend + Selling Price

= 3,980 + 12,480

= ₹ 16,460

Net amount received = 5000 + 16,450

= ₹ 21,460

Here, net amount invested = ₹ 30,020 and net amount received = ₹ 21,460

∴ Net amount received < Net amount invested

There is a loss in the transaction

Loss incurred =

Net amount invested – Net amount received

= 30,020 – 21,460

= ₹ 8560

∴ **Mrs. D'Souza made a loss of ₹ 8560 in the whole transaction.**

Problem Set- 4B *(Textbook Page No. 112)*

(6) **Smt. Mita Agrawal invested ₹ 10,200 when MV of the share is ₹ 100. She sold 60 shares when the MV was ₹ 125 and sold remaining shares when the MV was ₹ 90. She paid 0.1% brokerage for each trading. Find whether she made profit or loss ? and how much ?** *(4 marks)*

Solution:

Amount invested by Mrs. Agarwal = ₹ 10,200

MV = ₹ 100

Brokerage per share = 0.1% of ₹ 100

$= \frac{0.1}{100} \times 100$

= ₹ 0.10

Purchase price per share = MV + Brokerage

= 100 + 0.10

= ₹ 100.10

Number of shares purchased =

$\frac{\text{Amount invested}}{\text{Purchase price per share}}$

$= \frac{10200}{100.10}$

= 102 (approx.)

Number of shares sold at ₹ 125 per share = 60

∴ MV per share while selling 60 shares = ₹ 125

Brokerage per share = 0.1% of ₹ 125

$= \frac{0.1}{100} \times 125$

= ₹ 0.125

Total selling price per share = MV – Brokerage

= 125 – 0.125

= ₹ 124.875

Amount received on selling 60 shares

= 60 × 124.875

= ₹ 7492.50

Remaining shares = 102 – 60

= 42

MV per share while selling 42 shares = ₹ 90

Brokerage per share = 0.1% of ₹ 90

$= \frac{0.1}{100} \times 90$

= ₹ 0.09

Total selling price per share = MV – brokerage

= 90 – 0.09

= ₹ 89.91

Amount received on selling 42 shares

= 42 × 89.91

= ₹ 3776.22

Net amount received = Amount received on selling 60 shares + Amount received on selling 42 shares

= 7492.50 + 3776.22

= ₹ 11268.72

Here,

Net amount received > Amount invested

∴ There is a gain in the transaction.

Gain incurred =

Net amount received – Amount invested

= ₹ 11268.72 – ₹ 10200

= ₹ 1068.72

∴ **Mrs. Agarwal made a profit of ₹ 1068.72.**

***(11)** **Smt. Anagha Doshi purchased 22 shares of FV ₹ 100 for Market Value of ₹ 660. Find the sum invested. After taking 20% dividend, she sold all the shares when market value was ₹ 650. She paid 0.1% brokerage for each trading done. Find the percent of profit or loss in the share trading. (Write your answer to the nearest integer.)**

(4 marks)

Solution:

Number of shares purchased = 22

MV = ₹ 660

∴ Brokerage per share = 0.1% of ₹ 660

$= \frac{0.1}{100} \times 660$

= ₹ 0.66

Purchase price per share = MV + Brokerage

= 660 + 0.66

= ₹ 660.66

Purchase price of 22 shares = 22 × 660.66

= ₹ 14534.52

FV = ₹ 100

Dividend per share = 20% of ₹ 100

$= \frac{20}{100} \times 100$

= ₹ 20

∴ Dividend on 22 shares = 22 × 20

= ₹ 440

MV per share while selling = ₹ 650

Brokerage per share = 0.1% of ₹ 650

$= \frac{0.1}{100} \times 650$

= ₹ 0.65

Total selling price per share = MV – Brokerage

= 650 – 0.65

= ₹ 649.35

Total selling price of 22 shares

= 22 × 649.35

= ₹ 14,285.70

Amount invested = ₹ 14,534.52 and

Total amount received = Dividend + Selling Price

440 + 14285.70 = ₹ 14,725.70

∴ Amount received > Amount invested

∴ There is a profit in the transaction

∴ Profit = Amount received – Amount invested

= 14725.70 – 14534.52

= ₹ 191.18

$$\text{Profit percent} = \frac{\text{Profit}}{\text{Amount invested}} \times 100$$

$$= \frac{191.18}{14534.52} \times 100$$

$$= 1.32 \text{ (approx)}$$

Profit percent = 1

∴ **Mrs. Anagha Doshi got profit of 1%**

***(9) Shri. Batliwala sold shares of ₹ 30,350 and purchased shares of ₹ 69,650 in a day. He paid brokerage at the rate of 0.1% on sale and purchase. 18% GST was charged on brokerage. Find his total expenditure on brokerage and tax.** *(4 marks)*

Solution:

Shri Batliwala sold shares of ₹ 30,350

Brokerage paid while selling shares = 0.1% of ₹ 30,350

= ₹ 30.35

GST paid while selling shares = 18% of ₹ 30.35

$$= \frac{18}{100} \times 30.35$$

= ₹ 5.463

Also, he purchased shares of ₹ 69,650

Brokerage paid while purchasing shares

= 0.1% of ₹ 69,650

$$= \frac{0.1}{100} \times 69650$$

= ₹ 69.65

GST paid while purchasing shares

= 18% of ₹ 69.65

$$= \frac{18}{100} \times 69.65$$

= ₹ 12.537

Total brokerage paid in this transaction

= 30.35 + 69.65

= ₹ 100

Total GST paid in this transaction = 5.463 + 12.537

= ₹ 18

∴ **Total expenditure on brokerage and tax = 100 + 18 = ₹ 118**

(10) Smt. Aruna Thakkar purchased 100 shares of FV 100 when the MV is ₹ 1200. She paid brokerage at the rate of 0.3% and 18% GST on brokerage. Find the following - *(4 marks)*

(1) Net amount paid for 100 shares.

(2) Brokerage paid on sum invested.

(3) GST paid on brokerage.

(4) Total amount paid for 100 shares.

Solution:

MV = ₹ 1200 and number of shares purchased = 100

∴ Net amount paid for 100 shares = 1200 × 100

= ₹ 1,20,000

Brokerage = 0.3%

∴ Brokerage per share = 0.3% of ₹ 1200

$$= \frac{0.3}{100} \times 1200$$

= ₹ 3.6

∴ Brokerage on 100 shares = 100 × 3.6

= ₹ 360

GST per share = 18% of ₹ 3.6

$$= \frac{18}{100} \times 3.6$$

= ₹ 0.648

GST on 100 shares = 100 × 0.648

= ₹ 64.80

Total purchase price per share

= MV + brokerage + GST

= 1200 + 3.6 + 0.648

= ₹ 1204.248

Total purchase price of 100 shares

= 100 × 1204.248

= ₹ 1,20,424.80

Problem Set - 4A *(Textbook Page No. 109)*

MCQ's

Write the correct alternative for each of the following. *(1 mark each)*

(1) Rate of GST on essential commodities is ....

(A) 5% (B) 12% (C) 0% (D) 18%

(2) The tax levied by the central government for trading within state is ....

(A) IGST (B) CGST (C) SGST (D) UTGST

(3) GST system was introduced in our country from ....

(A) 31st March 2017 (B) 1st April 2017
(C) 1st January 2017 (D) 1st July 2017

(4) The rate of GST on stainless steel utensils is 18%, then the rate of State GST is ....
(A) 18% (B) 9% (C) 36% (D) 0.9%

(5) In the format of GSTIN there are .... alpha-numerals.
(A) 15 (B) 10 (C) 16 (D) 9

(6) The business between two GSTIN businessman is known as ....
(A) BB (B) B2B (C) BC (D) B2C

Problem Set - 4B *(Textbook Page No. 111)*

(7) If the Face Value of a share is ₹ 100 and Market value is ₹ 75, then which of the following statements is correct ?
(A) The share is at premium of ₹ 175
(B) The share is at discount of ₹ 25
(C) The share is at premium of ₹ 25
(D) The share is at discount of ₹ 75

(8) What is the amount of dividend received per share of face value ₹ 10 and dividend declared is 50%.
(A) ₹ 50 (B) ₹ 5 (C) ₹ 500 (D) ₹ 100

(9) The NAV of a unit in mutual fund scheme is ₹ 10.65 then find the amount required to buy 500 such units.
(A) 5325 (B) 5235 (C) 532500 (D) 53250

(10) Rate of GST on brokerage is ....
(A) 5% (B) 12% (C) 18% (D) 28%

(11) To find the cost of one share at the time of buying the amount of Brokerage and GST is to be .... the MV of share.
(A) added to (B) substracted from
(C) Multiplied with (D) divided by

Additional MCQ's

(12) Rate of GST on Health care services is ....
(A) 5% (B) 0% (C) 12% (D) 18%

(13) Trading between GSTIN holder and consumer is termed as
(A) BB (B) B2B (C) BC (D) B2C

(14) If the face value of the share is ₹ 100 and market value is ₹ 150. Rate if brokerage is 0.5%. The buying price of 1 share is ....
(A) ₹ 149.25 (B) ₹ 99.5
(C) ₹ 150.75 (D) ₹ 100.5

(15) If 75 shares of FV ₹ 100 each are purchased for MV ₹ 130 then the sum invested is ....
(A) ₹ 9750 (B) ₹ 7500 (C) ₹ 13000 (D) ₹ 6000

(16) If NAV of one unit is ₹ 35 then the number of units allotted for investment of ₹ 4270 is ....
(A) 12 (B) 200 (C) 45 (D) 122

(17) If the face value of share is ₹ 100 when market value was ₹ 80. Company declared 30% dividend. The dividend per share is ....
(A) ₹ 24 (B) ₹ 30 (C) ₹ 300 (D) ₹ 150

(18) The rate of GST on mobile phone is 12% then the rate of central GST is ....
(A) 12% (B) 1.2% (C) 36% (D) 6%

(19) In GST, all goods are classified by given numerical code called .... code.
(A) HSN (B) GSTIN (C) SAC (D) NAV

(20) If FV < MV the share is at ....
(A) discount (B) par
(C) Premium (D) None of these

(21) When trader collects GST at the time of sale, it is called ....
(A) CGST (B) Output tax (C) Input tax (D) SGST

ANSWERS

(1) (C) 0% **(2)** (B) CGST **(3)** (D) 1st July 2017
(4) (B) 9% **(5)** (A) 15 **(6)** (B) B2B
(7) (B) The share is at discount of ₹ 25 **(8)** (B) ₹ 5
(9) (A) 5325 **(10)** (C) 18% **(11)** (A) added to
(12) (B) 0% **(13)** (D) B2C **(14)** (C) ₹ 150.75
(15) (A) ₹ 9750 **(16)** (D) 122 **(17)** (B) ₹ 30 **(18)** (D) 6%
(19) (A) HSN **(20)** (C) Premium **(21)** (B) Output tax

PROBLEMS FOR PRACTICE

(1) **'Sony Electronic' sold a computer set to a customer. The rate of GST on computer is 18%, then find the rate of CGST and SGST.** *(1 mark)*

(2) **The taxable price of a Nokia mobile is ₹ 11,000. The rate of CGST is 6%. Find the total GST printed in the tax invoice.** *(1 mark)*

(3) **On an article CGST is 2.5%, then what is the rate of SGST? Also, find the rate of GST.** *(1 mark)*

(4) **The total cost of a perfume bottle including GST is ₹ 590. The rate of GST is 18%. Find the taxable price of the perfume bottle.** *(2 marks)*

(5) **'Sadhana Electronics' sold a dish washer to a customer. The total cost price including GST is ₹ 8960. The rate of GST on dish water is 28%.**

Find (i) Amount of GST (ii) Amount of SGST (iii) Taxable price *(3 marks)*

(6) **Disha purchased an A.C. Unit from a dealer. The printed price of an A.C. unit is ₹ 45,000. Dealer offered a discount of 15% on it. The rate of GST on it is 14%. Then at what price A.C. unit was sold to Disha by dealer? Find CGST and SGST amount printed on tax invoice.** *(4 marks)*

(7) **A courier company delivered a parcel from Mumbai to Pune. The customer paid ₹ 531 to the courier company. Now, tax invoice shows ₹ 450 as taxable price, CGST is ₹ 40.50 and SGST is ₹ 40.50, then find the rate of GST applicable in this transaction.** *(4 marks)*

(8) **Disha purchased some beauty products and paid GST of ₹ 1500. She sold all the beauty products to one customer and collected GST of ₹ 1700. Find the CGST and SGST to be paid.** *(3 marks)*

(9) **M/S Shridhar Chemicals purchased washing powder for ₹ 10,000 taxable amount. They sold it to a shopkeeper for ₹ 12,000 taxable amount. The rate of GST is 18%, then find the CGST and SGST to be paid by M/S Shridhar Chemicals.** *(4 marks)*

(10) **A T.V. manufacturer sold a T.V. to wholesaler for taxable price of ₹ 10,500. The wholeseller sold it to retailer at ₹ 12,000 taxable price and retailer sold it to customer at ₹ 14,500 taxable price. The rate of GST is 18%, then find the CGST and SGST applicable at every transaction.** *(4 marks)*

(11) **From the given information, prepare the tax invoice for Business to Business (B2B). Write any name, address, date, etc.** *(3 marks)*

Supplier : Name, Address, State, GSTIN, Invoice number, date

Receiver : Name, address, State, GSTIN

Name of products:

(i) Compass box : 100, HSN 3497, ₹ 60, GST 12%

(ii) Writing Pads : 50, HSN 9607, ₹ 35 , GST 12%

(12) **From the information, prepare the tax invoice for business to customer. (B2C)** *(3 marks)*

Write any name, address, date, etc.

Supplier: M/s .... Address.............Date ....

Invoice No. .... GSTIN ....

Name of products:

(i) Jam bottle ₹ 75, 1 piece, Rate of GST 12%, HSN1207

(ii) Honey bottle ₹ 60, 1 piece, Rate of GST 5% , HSN3607

(iii) Perfume bottle ₹ 225, 1 piece, Rate of GST 18%, HSN9319

(13) **Mr. Gokhale invested ₹ 22,500 in shares of face value ₹ 100 at market value ₹ 125. If the company declared 60% dividend at the end of the year, what was the income from dividend?** *(3 marks)*

(14) **A share of the value ₹ 100 was purchased for ₹ 175. The company declared a dividend of 30%. What is the rate of return on investment?** *(3 marks)*

(15) **Mrs. Parekh invested an equal amount in two companies by purchasing equity shares with market price ₹ 145 and ₹ 160 each. At the end of the year, both the companies declared the dividend of 20% and 30% each. In which company was her investment profitable?** *(4 marks)*

(16) **If 500 shares of face value ₹ 100 were sold at ₹ 50 premium, then how much amount is obtained?** *(3 marks)*

(17) **A sum of ₹ 75,000 invested in shares of face value ₹ 100 at ₹ 125 market value, then how many shares were purchased?** *(2 marks)*

(18) **Complete the following table:** *(2 marks)*

Sr. No.	Face Value	Type	Market Value
(i)	₹ 100	Premium ₹ 25	
(ii)		At par	₹ 175
(iii)	₹ 100	Discount ₹ 40	

(19) **Mr. Deshmukh's investment in shares is given below. Find his total investment in shares.** *(3 marks)*

Company A: 450 shares, face value = ₹ 100
Premium = ₹ 25

Company B: 500 shares, face value = ₹ 100
Market Value = ₹ 205

Company C: 80 shares, face value = ₹ 100
Discount = ₹ 15

(20) Fill in the blanks in the following tax invoice of buying - selling of share. (B, Buying, S - Selling)

S.N.	No. of shares	Market Value	Value of shares	Brokerage 0.2%	CGST 9%	SGST 9%	Total value of shares
(i)	50B	80					
(ii)	100B	120					
(iii)	50S	70					
(iv)	200S	140					

(21) A share was sold at ₹ 950 market value and brokerage of 0.2% was paid then how much amount is obtained on selling it? *(2 marks)*

(22) Mr. Kumar invested ₹ 25,000 in a mutual fund scheme. The NAV of one unit is ₹ 125, then how many units were obtained? *(2 marks)*

(23) Mrs. Sita invested ₹ 92,124 in shares of face value ₹ 10 each at ₹ 90 market value. She paid 2% brokerage and 18% GST on it. Company declared a dividend of 60% on then. Find her dividend. *(3 marks)*

(24) Mr. Chavan purchased 100 shares of ₹ 100 face value at ₹ 150 market value. He paid 0.2% brokerage and GST 18% on brokerage. Then, find his total investment on 100 shares. *(3 marks)*

(25) Mr. Ramesh Sawant invested ₹ 2,50,295 in shares of face value ₹ 100 each at ₹ 250 market value. He gave brokerage of 0.1% and GST of 18% on brokerage then how many shares are purchased by him? *(4 marks)*

ANSWERS

(1) Rate of CGST = Rate of SGST = 9%

(2) ₹ 1320

(3) Rate of SGST = 2.5%, rate of GST = 5%

(4) ₹ 500

(5) Taxable price = ₹ 7000, Amount of GST = ₹ 1960, Amount of SGST = ₹ 980

(6) ₹ 43,605, CGST = SGST = ₹ 2677.50 (7) 18%

(8) CGST = SGST = ₹ 100 (9) CGST = SGST = ₹ 180

(10)

Individuals	GST	CGST	SGST
Manufacturer	₹ 1890	₹ 945	₹ 945
Wholeseller	₹ 270	₹ 135	₹ 135
Retailer	₹ 450	₹ 225	₹ 225
Total	₹ 2610	₹ 1305	₹ 1305

(11) Ans Missing (12) Ans Missing

(13) ₹ 10,800 (14) 17.14%

(15) Second Company (16) ₹ 75,000 (17) 600

(18) (i) Market Value - ₹ 125 (ii) Face Value - ₹ 175
(iii) Market Value - ₹ 60

(19) Company A : ₹ 56,250 Company B : ₹ 1,02,500
Company C : ₹ 6800
Total investment = ₹ 1,65,550

(20) (i) 50B - ₹ 4000, ₹ 8, ₹ 0.72, ₹ 0.72, ₹ 4009.44
(ii) 100B - ₹ 12,000, ₹ 24, ₹ 2.16, ₹ 2.16, ₹ 12028.32
(iii) 50S - ₹ 3500, ₹ 7, ₹ 0.63, ₹ 0.63, ₹ 3491.74
(iv) 200S - ₹ 28000, ₹ 56, ₹ 5.04, ₹ 5.04, ₹ 27933.92

(21) ₹ 948.10 (22) 200 (23) ₹ 6000

(24) ₹ 15035.4 (25) 1000

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ASSIGNMENT – 4

Time : 1 Hr. **Marks : 20**

Q.1. (A) Choose the proper alternative answer for the question given below: **(1)**

(1) Rate of GST on essential commodities is ....

(A) 5% (B) 12%(C) 0%(D) 18%

(B) Perform the following activities: **(3)**

(1) If NAV of one unit is ₹ 25, then how many units will be alloted for the investment of ₹ 10,000?

(2) ' Pawan Medical' supplies medicines on some medicines, the rate of GST 12% then what is the rate of CGST and SGST?

Solution:

Rate of GST = $\boxed{}$ %

Rate of CGST = $\dfrac{\boxed{}}{\boxed{}}$ %

Rate of SGST = $\boxed{}$ %

(3) A person paid ₹ 75 brokerage for buying 100 shares. The rate of GST on brokerage is 18%. Find the amount of GST to be paid to the broker.

Q.2. Perform the following activities: (Any 1) **(2)**

(1) Find the purchase price of a share of FV ₹ 100 if it is at premium of ₹ 30. The brokerage rate is 0.3%

(2) Smita has invested ₹ 12,000 and purchased shares FV ₹ 10 at a premium of ₹ 2. Find the number of shares she purchased. Complete the given activity.

Solution:

FV = ₹ 10, Premium = ₹ 2

$\therefore$ MV = FV + $\boxed{}$ = $\boxed{}$ + $\boxed{}$ = $\boxed{}$

Number of shares = $\dfrac{\textit{Total investment}}{MV}$

= $\dfrac{12000}{\boxed{}}$

= $\boxed{}$ shares,

Q.3. Perform the following activities: (Any 2) **(6)**

(1) If 50 shares of FV ₹ 10 were purchased for MV of ₹ 25. Company declared 30% dividend on the shares then find

(i) Sum invested (ii) Dividend received(3) Rate of return

(2) Prepare Business to consumer (B2C) tax invoice using given information. Write the name of the supplier, address, state, date of invoice number, GSTIN etc. as per your choice.

Supplier : M/s. .... Address : ....

State : .... Date : .... Invoice No. : .... GSTIN : ....

Particulars - Rate of mobile battery - ₹ 200 , Rate of GST 12%, HSN 8507, 1 pc.

Rate of Headphone - ₹ 750 , Rate of GST 18%, HSN 8518, 1 pc.

(3) M/s. Jay Chemicals purchased a liquid soap having taxable value ₹ 8,000 and sold it to the consumer for the taxable value ₹ 10,000. Rate of GST is 18%. Find the CGST and SGST payable by M/s. Jay Chemicals.

Q.4. Attempt the following: (Any 2) **(8)**

(1) Shri Aditya Sanghavi invested ₹ 50,118 in shares of FV of ₹ 100, when the market value is ₹ 50. Rate of brokerage is 0.2% and Rate of GST on brokerage is 18%, then how many shares were purchased for ₹ 50,118?

(2) Smt. Mita Agrawal invested ₹ 10,200 when MV of the shares is ₹ 100. She sold 60 shares when the MV was ₹ 125 and sold remaining shares when the MV was ₹ 90. She paid 0.1% brokerage for each trading. Find whether she made profit or loss? and how much?

(3) Mr. D'Souza purchased 200 shares of FV of ₹ 50 at a premium of ₹ 100. He received 50% dividend on the shares. After receiving the dividend he sold 100 shares at a discount of ₹ 10 and remaining shares were sold at premium of ₹ 75. For each trade he paid the brokerage of ₹ 20. Find whether Mr. D'Souza gained or incurred a loss? by how much?

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5 Probability

... INDEX ...

Points to Remember:

- **Probability:**

  The words probably, fair, likely, possibly and chance are commonly used in our day-to -day conversations.

- **Terms of probability:**

  Random Experiment: An experiment in which all the possible results are known in advance but none of them can be predicted with certainty and there is equal possibility for each result is known as a 'Random experiment'.

  For example:

  (i) Tossing a coin.

  (ii) Picking a card from a set of cards.

  Outcome : The result of a random experiment is known as an 'Outcome'.

  For example:

  (i) If a coin is tossed, the two possible outcomes are Head (H) or Tail (T),

  (ii) If a die is thrown, six possible outcomes are 1 or 2 or 3 or 4 or 5 or 6.

  Equally Likely Outcomes : A given number of outcomes are said to be equally likely if none of them occurs in preferance to others.

  If an unbiased die is rolled, each number, i.e. from 1, 2, 3, 4, 5, 6 may appear on the upper face. That means the outcomes are equally likely to happen.

  If a die is so formed that a particular face occurs most often, then the die is biased and the outcomes are not likely to occur equally.

MASTER KEY QUESTION SET - 5

Practice Set - 5.1 *(Textbook Page No. 116)*

(1) How many possibilities are there in each of the following?

(i) Vanita knows the following sites in Maharashtra. She is planning to visit one of them in her summer vacation.

Ajintha, Mahabaleshwar, Lonar Sarovar, Tadoba wild life sanctuary, Amboli, Raigad, Matheran, Anandavan. *(1 mark)*

Solution:

The place to be covered during vacation: Ajanta, Mahabaleshwar, Lonar Sarovar, Tadoba Sanctuary, Amboli, Raigad, Matheran, Anandvan.

$\because$ There are 8 places can be chosen randomly,

Total number of ways any one place can be chosen = 8.

(ii) Any day of a week is to be selected randomly. *(1 mark)*

Solution:

A week has 7 days : Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday.

$\therefore$ **Total number of ways a day can be chosen randomly in a week = 7**

(iii) Select one card from the pack of 52 cards. *(1 mark)*

Solution:

Total no. of cards = 52.

$\therefore$ **Total number of ways a card can be chosen among the 52 cards randomly = 52.**

(iv) One number from 10 to 20 is written on each card. Select one card randomly. *(1 mark)*

Solution:

Number on cards are 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

∴ Total number of cards = 11.

∴ **Total no. of ways a card can be chosen randomly = 11.**

Points to Remember:

- **Sample Space:**

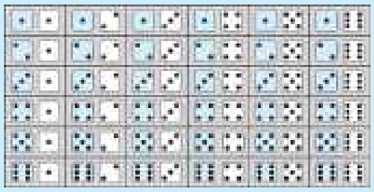
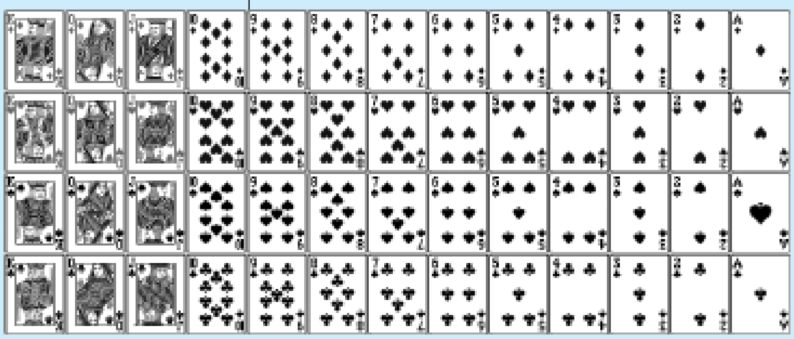
  The set of all the possible outcomes of a random experiment is called the sample space. It is denoted by 'S' or 'Ω' (omega).

  Each element of sample space is called a 'sample point'.

  Number of elements of the set S is denoted by n(s).

- **Finite Sample Space:**

  If n(S) is finite, then the sample space is called **finite sample space**.

No.	Random Experiment	Sample Space (S)	No. of sample points
(1)	One coin is tossed	S = {H, T}	n (S) = 2
(2)	Two coins are tossed	S = {HH, HT, TH, TT}	n (S) = 4
(3)	Three coins are tossed	S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}	n (S) = 8
(4)	A die is thrown	S = {1, 2, 3, 4, 5, 6}	n (S) = 6
(5)	Two dice are thrown	S = {  } S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}	n (S) = 36
(6)	Two digit numbers are formed using the digits 0, 1, 2, 3, 4 without repeating the digits	S = {10, 12, 13, 14, 20, 21, 23, 24, 30, 31, 32, 34, 40, 41, 42, 43}	n (S) = 16
(7)	Drawing a card at random from a pack of 25 cards numbered 1 to 25.	S = {1, 2, 3, 4... 25}	n (S) = 25
(8)	A committee of 2 is to be formed from a group of 3 boys and 2 girls	S = {B1B2, B1B3, B2B3 ,B1G1, B1G2, B2G1, B2G2, B3G1, B3G2, G1G2}	n (S) = 10
(9)	A coin is tossed and a die is thrown simultaneously	S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}	n (S) = 12
(10)	A card is drawn from a pack of 52 playing cards	There are 52 cards in a pack. 	n (S) = 52

- The sample space for a coin tossed twice is the same as that of two coins tossed simultaneously. The same is true for three coins.
- The sample space for a die rolled twice is the same as two dice rolled simultaneously.

Practice Set - 5.2 *(Textbook Page No. 117)*

(1) For each of the following experiments write sample space 'S' and number of sample points n(S).:

(i) One coin and one die are thrown simultaneously. *(1 mark)*

Solution:

When a coin and a die are thrown simultaneously

S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

$\therefore$ $\boxed{n(S) = 12}$

(ii) Two digit numbers are formed using digits 2, 3 and 5 without repeating a digits. *(1 mark)*

Solution:

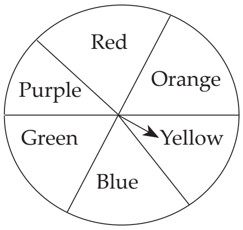
The two digit number formed using digits 2, 3 and 5.

S = {23, 25, 32, 35, 52, 53}

$\therefore$ $\boxed{n(S) = 6}$

(2) The arrow is rotated and it stops randomly on the disc. Find out on which colour it may stop. *(1 mark)*

Solution:



An arrow stops on a circular disc made of six colours.

S = {Red, Purple, Green, Blue, Yellow, Orange}

$\therefore$ $\boxed{n(S) = 6}$

(3) In the month of March 2019, find the days on which the date is a multiple of 5. (see the given page of the calender) *(1 mark)*

MARCH 2019

M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Solution:

The days of March 2019 which has date as multiple of 5 are:

5th March 2019 Tuesday

10th March 2019 Sunday

15th March 2019 Friday

20th March 2019 Wednesday

25th March 2019 Monday

30th March 2019 Saturday

S = {Tuesday, Sunday, Friday, Wednesday, Monday, Saturday}

$\therefore$ $\boxed{n(S) = 6}$

(4) Form a 'Road safety committee' of two, from 2 boys (B_1, B_2) and 2 girls(G_1, G_2). Complete the following activity to write the sample space. *(1 mark)*

Solution:

(1) Committee with 2 boys = $\boxed{B_1, B_2}$

(2) Committee with 2 girls = $\boxed{G_1, G_2}$

(3) Committee of one boy and one girl = $\boxed{B_1, G_1}$, $\boxed{B_1, G_2}$, $\boxed{B_2, G_1}$, $\boxed{B_2, G_2}$

$\therefore$ Sample space = $\{B_1B_2, B_1G_1, B_1G_2, B_2G_1, B_2G_2, G_1G_2\}$

Points to Remember:

- **Event:**

A set of favourable outcomes of a given space is an '**event**'.

Event is always denoted by capital letter.

Ex. : Getting at least one tail, when two coins are used is an event.

A = {TT, TH, HT} = n(A) = 3

- **Types of Events:**

(a) Certain Event/Sure event: An event is known as the Certain Event when it contains all the sample points of sample space.

e.g. **Random experiment** : A die is thrown. Let A be the event of getting a score on the upper face which is less than 7.

Here, S = { 1, 2, 3, 4, 5, 6 } and A = { 1, 2, 3, 4, 5, 6 }

Here, event A has all the elements of S, therefore event A is a certain event.

(b) Impossible Event : An event is called an Impossible Event if it does not contain any sample point of sample space.

e.g. **Random experiment :** A die is thrown. Let A be the event of getting a score on the upper face which is divisible by 7.

Here, S = { 1, 2, 3, 4, 5, 6 } and A = { }

Event A is an Impossible event if A = ϕ or A = { }

(c) Simple/Elementary Event : An event consisting of only one sample point of a sample space is called an elementary event.

e.g. **Random experiment :** A die is thrown. Let A be the event of getting a score on the upper face which is divisible by 5.

Here S = { 1, 2, 3, 4, 5, 6 } and A = { 5 }

$\therefore$ $n(A) = 1$

$\therefore$ Event A is an elementary event.

(d) Mutually exclusive Events : Two events are called Mutually exclusive Events if they do not have any sample points in common.

Example : If A = {1, 2, 3} and B = {4, 5} are two events then,

A and B are Mutually exclusive events, since they do not have any sample point in common.

Mathematically : $A \cap B = \phi$ (Empty set)

(e) Exhaustive Event : Two events are said to be exhaustive events if their union is sample space.

e.g. Let S be the sample space

S = { 1, 2, 3, 4, 5, 6 }

A = { 1, 2 , 3 }

B = { 3, 4, 5, 6 }

$A \cup B$ = { 1, 2, 3, 4, 5, 6 } which is sample space.

$\therefore$ $A \cup B = S$

$\therefore$ Events A and B are called Exhaustive events.

(f) Complementary Events : Two events are said to be Complementary Events if they satisfy the following two conditions :

(i) The events should not have any sample points in common, i.e. they should be Mutually Exclusive Events, i.e. $A \cap B = \phi$

(ii) The Union of both the events should give us the Sample Space, i.e. $A \cup B = S$

Example : Lets S be the Sample space. Two events are complementary events if they are mutually exclusive as well as exhaustive.

S = { 1, 2, 3, 4, 5, 6 }

If A = { 1, 3, 5 }

B = { 2, 4, 6 }

$A \cap B = \phi$ and $A \cup B = S$

In the above example, events A and B are complementary events since they do not have any sample points in common, the Union of both the events gives us the Sample Space, i.e. S.

Practice Set - 5.3 *(Textbook Page No. 121)*

(1) Write sample space 'S' and number of sample point n(S) for each of the following experiments. Also write events A, B, C in the set form and write n(A), n(B), n(C).

(i) One die is rolled,

Event A : Even number on the upper face.

Event B : Odd number on the upper face.

Event C : Prime number on the upper face.

(3 marks)

Solution:

When a die is thrown

S = {1, 2, 3, 4, 5, 6}

$\therefore$ $\boxed{n\,(S) = 6}$

Event A: Even number on the upper face.

A = {2, 4, 6}

$\therefore$ $\boxed{n\,(A) = 3}$

Event B : Odd number on the upper face.

B = {1, 3, 5}

$\therefore$ $\boxed{n\,(B) = 3}$

Event C : Prime number on the upper face.

C = {2, 3, 5}

$\therefore$ $\boxed{n\,(C) = 3}$

(ii) Two dice are rolled simultaneously :

Event A : The sum of the digits on upper faces is a multiple of 6.

Event B : The sum of the digits on the upper faces is minimum 10.

Event C : The same digit on both the upper faces.

(4 marks)

Solution :

When two dice are thrown

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore$ $\boxed{n\,(S) = 36}$

Event A : The sum of the digits on upper faces is a multiple of 6.

A = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 6)}

$\therefore$ $\boxed{n\,(A) = 6}$

Event B : The sum of the digits on the upper faces is minimum 10.

B = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}

∴ $\boxed{n\text{ (B)} = 6}$

Event C : The same digit on both the upper faces.

C = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

∴ $\boxed{n\text{ (C)} = 6}$

(iii) Three coins are tossed simultaneously :
Event A : To get at least two heads.
Event B : To get no head.
Event C : To get head on the second coin.

(3 marks)

Solution :

When three coins are tossed simultaneously.

S = {HHH, HTH, THH, TTH,
HHT, HTT, THT, TTT}

∴ $\boxed{n\text{ (S)} = 8}$

Condition for event A : To get at least two heads.

A = {HHH, HTH, THH, HHT}

∴ $\boxed{n\text{ (A)} = 4}$

Condition for event B : To get no head.

B = {TTT}

∴ $\boxed{n\text{ (B)} = 1}$

Condition for event C : To get head on the second coin.

C = {HHH, THH, HHT, THT}

∴ $\boxed{n\text{ (C)} = 4}$

(iv) Two digit numbers are formed using digits 0, 1, 2, 3, 4, 5 without repetition of the digits.

Condition for event A : The number formed is even

Condition for event B : The number formed is divisible by 3.

Condition for event C : The number formed is greater than 50.

(3 marks)

Solution :

Two digit numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 without repeating digits are as follows :

S = {10, 12, 13, 14, 15, 20, 21, 23, 24, 25, 30, 31, 32, 34, 35, 40, 41, 42, 43, 45, 50, 51, 52, 53, 54}

∴ $\boxed{n\text{ (S)} = 25}$

Condition for event A : The number formed is even

A = {10, 12, 14, 20, 24, 30, 32, 34, 40, 42, 50, 52, 54}

∴ $\boxed{n\text{ (A)} = 13}$

Condition for event B : The number formed is divisible by 3.

B = {12, 15, 21, 24, 30, 42, 45, 51, 54}

∴ $\boxed{n\text{ (B)} = 9}$

Condition for event C : The number formed is greater than 50.

C = {51, 52, 53, 54}

∴ $\boxed{n\text{ (C)} = 4}$

(v) From three men and two women, environment committee of two persons is to be formed.

Condition for event A : There must be at least one woman member.

Condition for event B : One man, one woman committee to be formed.

Condition for event C : There should not be a woman member.

(3 marks)

Solution :

Let three men be denoted by M_1, M_2 and M_3 and two women be denoted as W_1 and W_2.

A committee of two is formed in the following ways.

S = $\{M_1M_2, M_1M_3, M_1W_1, M_1W_2, M_2M_3, M_2W_1, M_2W_2, M_3W_1, M_3W_2, W_1W_2\}$

$\boxed{n\text{ (S)} = 10}$

Condition for event A : There must be at least one woman member.

A = $\{M_1W_1, M_1W_2, M_2W_1, M_2W_2, M_3W_1, M_3W_2, W_1W_2\}$

∴ $\boxed{n\text{ (A)} = 7}$

Condition for event B : One man, one woman committee to be formed.

B = $\{M_1W_1, M_1W_2, M_2W_1, M_2W_2, M_3W_1, M_3W_2\}$

∴ $\boxed{n\text{ (B)} = 6}$

Condition for event C : There should not be a woman member.

C = $\{M_1M_2, M_1M_3, M_2M_3\}$

∴ $\boxed{n\text{ (C)} = 3}$

(vi) One coin and one die are thrown simultaneously.

Condition for event A : To get head and an odd number.

Condition for event B : To get a head or tail and an even number.

Condition for event C : Number on the upper face is greater than 7 and tail on the coin.

(3 marks)

Solution :

A coin is tossed and a die is thrown

S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

$\therefore$ $\boxed{n\,(S) = 12}$

Condition for event A : To get head and an odd number.

A = {H1, H3, H5}

$\therefore$ $\boxed{n\,(A) = 3}$

Condition for event B : To get a head or tail and an even number.

B = {H2, H4, H6, T2, T4, T6}

$\therefore$ $\boxed{n\,(B) = 6}$

Condition for event C : Number on the upper face is greater than 7 and tail on the coin.

C = { }

$\therefore$ $\boxed{n\,(C) = 0}$

Points to Remember:

- **Probability of an event**

In mathematical language, when possibility of an expected event is expressed in number, it is called 'probability'. It is expressed as a fraction or percentage using the following formula.

The probability of an event 'A' in finite sample space S is written as P(A) and is defined as,

$$P\,(A) = \frac{\text{Number of elements in event A}}{\text{Number of elements in sample space S}}$$

$$= \frac{n\,(A)}{n\,(S)}$$

- **Properties of P (A)**

(1) If S is the finite sample space and A is an event of S, then $0 \le P(A) \le 1$ or $0\% \le P(A) \le 100\%$

(2) Probability of an Impossible event is zero.

(3) Probability of a Certain event is one.

(4) If S is the sample space and A is an event of S then, $P(A') = 1 - P\,(A)$, where A' is the complement of an event A

Problem Set - 5 *(Textbook Page No. 126)*

(2) Basketball players John, Vasim, Akash were practising the ball drop in the basket. The probabilities of success for John, Vasim and Akash are $\frac{4}{5}$, 0.83 and 58% respectively. Who had the greatest probability of success? *(2 Marks)*

Solution:

Let the probability of John throwing the ball in the basket be P(A), Vasim be P(B) and Akash be P(C)

$\therefore$ $P(A) = \frac{4}{5} = 0.80$, $P(B) = 0.83$

and $P(C) = 58\% = \frac{58}{100} = 0.58$

Since P(B) > P(A) > P(C)

Vasim has the greatest probability of success.

Practice Set - 5.4 *(Textbook Page No. 125)*

(1) If two coins are tossed, find the probability of the following events :

(i) Getting atleast one head

(ii) Getting no head *(3 marks)*

Solution :

When two coins are tossed

S = {HH, HT, TH, TT}

$\therefore$ $\boxed{n(S) = 4}$

(i) Let A be the event that atleast one head.

A = {HT, TH, HH}

$\therefore$ $n(A) = 3$

$P(A) = \frac{n\,(A)}{n\,(S)} = \frac{3}{4}$

$\therefore$ $\boxed{P(A) = \frac{3}{4}}$

(ii) Let B be event of getting no head.

B = {TT}

$n(B) = 1$

$P(B) = \frac{n\,(B)}{n\,(S)} = \frac{1}{4}$

$\therefore$ $\boxed{P(B) = \frac{1}{4}}$

(2) If two dice are rolled simultaneously, find the probability of the following events.

(i) The sum of the digits on the upper faces is at least 10.

(ii) The sum of the digits on the upper faces is 33.

(iii) The digit on the first die is greater than the digit on second die. *(4 marks)*

Solution:

When two dice are thrown

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore$ $\boxed{n(S) = 36}$

(i) Let A be the event that the sum of the digits on the upper faces is atleast 10

A = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}

∴ $n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

∴ $\boxed{P(A) = \frac{1}{6}}$

(ii) Let B is the event that the sum of the digits on the upper faces is 33.

B = { }

∴ $n(B) = 0$

$$P(B) = \frac{n(B)}{n(S)} = \frac{0}{36} = 0$$

∴ $\boxed{P(B) = 0}$

(iii) Let C is the event that the digits on the first die is greater than the digits on second die.

C = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)}

∴ $n(C) = 15$

$$P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

∴ $\boxed{P(C) = \frac{5}{12}}$

Problem Set - 5 *(Textbook Page No. 127)*

(7) Six faces of a die are as shown below.

A B C D E A

If the die is rolled once, find the probability of -

(i) 'A' appears on upper face.

(ii) 'D' appears on upper face. *(3 marks)*

Solution:

The sample space for the experiment is

S = {A, B, C, D, E, A}

∴ $\boxed{n(S) = 6}$

(i) Let A be the event that the letter appears on the uppermost face is A.

∴ $n(A) = 2$ (As there are two faces with letter A)

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

∴ $\boxed{P(A) = \frac{1}{3}}$

(ii) Let B be the event that the letter appearing on the uppermost face is D.

∴ $n(B) = 1$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{6}$$

∴ $\boxed{P(B) = \frac{1}{6}}$

***(16) The faces of a die bear numbers 0, 1, 2, 3, 4, 5. If the die is rolled twice, then find the probability that the product of digits on the upper face is zero.** *(2 marks)*

Solution:

The Sample space for the experiment is

S = {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5),
(1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (1, 5)
(2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5)
(3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5)
(4, 0), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5)
(5, 0), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5)}

∴ $\boxed{n(S) = 36}$

Let A be the event that the product of the digits on the upper face is zero.

A = {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (1, 0), (2, 0), (3, 0), (4, 0), (5, 0)}

∴ $n(A) = 11$

$$P(A) = \frac{n(A)}{n(S)} = \frac{11}{36}$$

∴ $\boxed{P(A) = \frac{11}{36}}$

Practice Set - 5.4 *(Textbook Page No. 125)*

(5) A card is drawn at random from a pack of well shuffled 52 playing cards. Find the probability that the card drawn is -

(i) an ace (ii) a spade *(3 marks)*

Solution:

There are 52 cards in a pack.

$n(S) = 52$

(i) Let A be the event that the card drawn is an ace.

∴ $n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

∴ $\boxed{P(A) = \frac{1}{13}}$

(ii) Let B be the event that the card drawn is a spade.

∴ $n(B) = 13$

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

$\therefore$ $\boxed{\mathbf{P(B) = \frac{1}{4}}}$

(3) There are 15 tickets in a box, each bearing one of the numbers from 1 to 15. One ticket is drawn at random from the box. Find the probability of event that the ticket drawn-

(i) shows an even number.

(ii) shows a number which is a multiple of 5.

(3 marks)

Solution:

The sample space for box containing 15 tickets numbered from 1 to 15 is

$n(S)$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

$\therefore$ $\boxed{\mathbf{n(S) = 15}}$

(i) Let A be the event that the ticket drawn shows an even number.

A = {2, 4, 6, 8, 10, 12, 14}

$\therefore$ $n(A) = 7$

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{15}$$

$\therefore$ $\boxed{\mathbf{P(A) = \frac{7}{15}}}$

(ii) Let B be the event that the ticket drawn shows a number which is a multiple of 5.

B = {5, 10, 15}

$\therefore$ $n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{15} = \frac{1}{5}$$

$\therefore$ $\boxed{\mathbf{P(B) = \frac{1}{5}}}$

Problem Set - 5 *(Textbook Page No. 126)*

(4) Joseph kept 26 cards in a cap, bearing one English alphabet on each card. One card is drawn at random. What is the probability that the card drawn is a vowel card ? *(2 marks)*

Solution:

The alphabets of English are

$n(S)$ = {a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}

$\therefore$ $\boxed{\mathbf{n(S) = 26}}$

Let A be the event that the card is a vowel card.

A = {a, e, i, o, u}

$\therefore$ $n(A) = 5$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{26}$$

$\therefore$ $\boxed{\mathbf{P(A) = \frac{5}{26}}}$

(8) A box contains 30 tickets, bearing only one number from 1 to 30 on each. If one ticket is drawn at random, find the probability of an event that the ticket drawn bears

(i) an odd number

(ii) a complete square number. *(4 marks)*

Solution:

The sample space for box containing tickets bearing only one number from 1 to 30 on each.

$n(S)$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10
11, 12, 13, 14, 15, 16, 17, 18, 19, 20
21, 22, 23, 24, 25, 26, 27, 28, 29, 30}

$\therefore$ $\boxed{\mathbf{n(S) = 30}}$

(i) Let A be the event that the ticket drawn bears an odd number.

A = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29}

$\therefore$ $n(A) = 15$

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{30} = \frac{1}{2}$$

$\therefore$ $\boxed{\mathbf{P(A) = \frac{1}{2}}}$

(ii) Let B be the event that the ticket drawn bears a complete square number.

B = {1, 4, 9, 16, 25}

$\therefore$ $n(B) = 5$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{30} = \frac{1}{6}$$

$\therefore$ $\boxed{\mathbf{P(B) = \frac{1}{6}}}$

(11) There are six cards in a box, each bearing a number from 0 to 5. Find the probability of each of the following events, that a card drawn shows,

(i) a natural number.

(ii) a number less than 1.

(iii) a whole number.

(iv) a number is greater than 5. *(4 marks)*

Solution:

The sample space for the experiment is

$n(S)$ = {0, 1, 2, 3, 4, 5}

$\therefore$ $\boxed{\mathbf{n(S) = 6}}$

(i) Let A be the event that the card drawn shows a natural number.

A $=$ {1, 2, 3, 4, 5}

$\therefore$ $n(A) = 5$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{6}$$

$\therefore$ $\boxed{\mathbf{P(A) = \frac{5}{6}}}$

(ii) Let B be the event that the card drawn is shows a number less than 1

B $=$ {0}

$\therefore$ $n(B) = 1$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{6}$$

$\therefore$ $\boxed{\mathbf{P(B) = \frac{1}{6}}}$

(iii) Let C be the event that the card drawn shows a whole number.

C $=$ {0, 1, 2, 3, 4, 5}

$\therefore$ $n(C) = 6$

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{6} = 1$$

$\therefore$ $\boxed{\mathbf{P(C) = 1}}$

(iv) Let D be the event that the card drawn shows a number greater than 5.

D $=$ { }

$\therefore$ $n(D) = 0$

$$P(D) = \frac{n(D)}{n(S)} = \frac{0}{6} = 0$$

$\therefore$ $\boxed{\mathbf{P(D) = 0}}$

(13) Each card bears one letter from the word 'mathematics' The cards are placed on a table upside down. Find the probability that a card drawn bears the letter 'm'. *(2 marks)*

Solution:

There are 11 alphabets in the word 'mathematics'.

$n(S)$ $=$ {m, a, t, h, e, m, a, t, i, c, s}

$\therefore$ $\boxed{\mathbf{n(S) = 11}}$

Let A be the event that the letter is m

$\therefore$ $n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{11}$$

$\therefore$ $\boxed{\mathbf{P(A) = \frac{2}{11}}}$

Practice Set - 5.4 *(Textbook Page No. 125)*

(4) A two digit number is formed with digits 2, 3, 5, 7, 9 without repetition. What is the probability that the number formed is

(i) an odd number ? **(ii) a multiple of 5 ?** *(4 marks)*

Solution:

The two digit numbers formed using the digits 2, 3, 5, 7, 9 without repetition.

S ={23, 25, 27, 29, 32, 35, 37, 39, 52, 53, 57, 59, 72, 73, 75, 79, 92, 93, 95, 97}

$\therefore$ $\boxed{\mathbf{n(S) = 20}}$

(i) Let A be the event that the number formed is an odd number.

A = {23, 25, 27, 29, 35, 37, 39, 53, 57, 59, 73, 75, 79, 93, 95, 97}

$\therefore$ $n(A) = 16$

$$P(A) = \frac{n(A)}{n(S)} = \frac{16}{20}$$

$$= \frac{4}{5}$$

$\therefore$ $\boxed{\mathbf{P(A) = \frac{4}{5}}}$

(ii) Let B be the event that the number formed is a multiple of 5.

B $=$ {25, 35, 75, 95}

$\therefore$ $n(B) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{20}$$

$$= \frac{1}{5}$$

$\therefore$ $\boxed{\mathbf{P(B) = \frac{1}{5}}}$

Problem Set - 5 *(Textbook Page No. 127)*

(10) In a game of chance, a spinning arrow comes to rest at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8.

All these are equally likely outcomes.

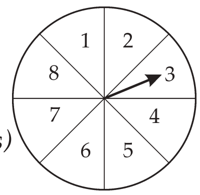
Find the probability that it will rest at

(i) 8.

(ii) an odd number.

(iii) a number greater than 2.

(iv) a number less than 9. *(4 marks)*



Solution:

The Sample space for the game of chance of spinning an arrow is

S $=$ {1, 2, 3, 4, 5, 6, 7, 8}

$\therefore$ $\boxed{\mathbf{n(S) = 8}}$

(i) Let A be the event that the arrow will rest at 8.

$\therefore$ $n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

$\therefore$ $\boxed{P(A) = \frac{1}{8}}$

(ii) Let B be the event that the arrow will rest at an odd number.

B = {1, 3, 5, 7}

$\therefore$ $n(B) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

$\therefore$ $\boxed{P(B) = \frac{1}{2}}$

(iii) Let C be the event that the arrow will rest at a number greater than 2.

C = {3, 4, 5, 6, 7, 8}

$\therefore$ $n(C) = 6$

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

$\therefore$ $\boxed{P(C) = \frac{3}{4}}$

(iv) Let D be the event that the arrow will rest at a number less than 9.

D = {1, 2, 3, 4, 5, 6, 7, 8}

$\therefore$ $n(D) = 8$

$$P(D) = \frac{n(D)}{n(S)} = \frac{8}{8} = 1$$

$\therefore$ $\boxed{P(D) = 1}$

***(15) A two digit number is to be formed from the digits 0, 1, 2, 3, 4. Repetition of the digits is allowed. Find the probability that the number so formed is a -**

(i) prime number (ii) multiple of 4

(iii) multiple of 11. *(4 marks)*

Solution:

The two digit numbers that can be formed using the digits 0, 1, 2, 3, 4 with repeating the digits is

S = {10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44}

$\therefore$ $\boxed{n(S) = 20}$

(i) Let A be the event that the number so formed is a prime number.

A = {11, 13, 23, 31, 41, 43}

$\therefore$ $n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{20} = \frac{3}{10}$$

$\therefore$ $\boxed{P(A) = \frac{3}{10}}$

(ii) Let B be the event that the number so formed is a multiple of 4.

B = {12, 20, 24, 32, 40, 44}

$\therefore$ $n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)} = = \frac{6}{20} = \frac{3}{10}$$

$\therefore$ $\boxed{P(B) = \frac{3}{10}}$

(iii) Let C be the event that number so formed is a multiple of 11.

C = {11, 22, 33, 44}

$\therefore$ $n(C) = 4$

$$P(C) = \frac{n(C)}{n(S)} = \frac{4}{20} = \frac{1}{5}$$

$\therefore$ $\boxed{P(C) = \frac{1}{5}}$

(3) In a hockey team there are 6 defenders, 4 offenders and 1 goalee. Out of these, one player is to be selected randomly as a captain. Find the probability of the selection that -

(i) The goalee will be selected.

(ii) A defender will be selected. *(4 marks)*

Solution:

In a hockey team,

No. of Defenders = 6

No. of Offenders = 4

No. of Goalee = 1

Total no. of players = 11.

$\therefore$ $\boxed{n(S) = 11}$

(i) Let A be the event that goalee will be selected as a captain.

$\therefore$ $n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{11}$$

$\therefore$ $\boxed{P(A) = \frac{1}{11}}$

(ii) Let B be the event that a defender will be selected.

$\therefore$ $n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{11}$$

$\therefore$ $\boxed{P(B) = \frac{6}{11}}$

(5) A balloon vendor has 2 red, 3 blue and 4 green balloons. He wants to choose one of them at random to give it to Pranali. What is the probability of the event that Pranali gets,

(i) a red balloon (ii) a blue balloon

(iii) a green balloon. *(4 marks)*

Solution:

Let R_1, R_2 be the red balloons, B_1, B_2, B_3 be the blue balloons and G_1, G_2, G_3, G_4 be the green balloons.

S = $\{R_1, R_2, B_1, B_2, B_3, G_1, G_2, G_3, G_4\}$

$\therefore$ $\boxed{n(S) = 9}$

(i) Let A be the event that a Pranali gets a red balloon.

A = $\{R_1, R_2\}$

$\therefore$ $n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{9}$$

$\therefore$ $\boxed{P(A) = \frac{2}{9}}$

(ii) Let B be the event that Pranali gets a blue balloon.

A = $\{B_1, B_2, B_3\}$

$\therefore$ $n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{9} = \frac{1}{3}$$

$\therefore$ $\boxed{P(B) = \frac{1}{3}}$

(iii) Let C be the event that Pranali gets a green balloon.

C = $\{G_1, G_2, G_3, G_4\}$

$\therefore$ $n(C) = 4$

$$P(C) = \frac{n(C)}{n(S)} = \frac{4}{9}$$

$\therefore$ $\boxed{P(C) = \frac{4}{9}}$

(6) A box contains 5 red, 8 blue and 3 green pens. Rutuja wants to pick a pen at random. What is the probability that the pen is blue? *(3 marks)*

Solution:

Number of Red pens = 5,

Number of Blue pens = 8,

Number of green pens = 3

$\therefore$ Total no. of pens = 16

S = $\{R_1, R_2, R_3, R_4, R_5, B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, G_1, G_2, G_3\}$

$\therefore$ $\boxed{n(S) = 16}$

Let A be the event that the pen picked up is blue.

A = $\{B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8\}$

$\therefore$ $n(A) = 8$

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{16} = \frac{1}{2}$$

$\therefore$ $\boxed{P(A) = \frac{1}{2}}$

(12) A bag contains 3 red, 3 white and 3 green balls. One ball is taken out of the bag at random. What is the probability that the ball drawn is -

(i) red (ii) not red (iii) either red or white.

(4 marks)

Solution:

Let three red balls be denoted as R_1, R_2, R_3, three white balls be W_1, W_2, W_3 and three green balls be G_1, G_2, G_3

S = $\{R_1, R_2, R_3, W_1, W_2, W_3, G_1, G_2, G_3\}$

$\therefore$ $\boxed{n(S) = 9}$

(i) Let A be the event that the ball drawn is red.

A = $\{R_1, R_2, R_3\}$

$\therefore$ $n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{9} = \frac{1}{3}$$

$\therefore$ $\boxed{P(A) = \frac{1}{3}}$

(ii) Let B be the event that the ball drawn is not red.

B = $\{W_1, W_2, W_3, G_1, G_2, G_3\}$

$\therefore$ $n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{9} = \frac{2}{3}$$

$\therefore$ $\boxed{P(B) = \frac{2}{3}}$

(iii) Let C be the event that the ball drawn is either red or white.

C = $\{R_1, R_2, R_3, W_1, W_2, W_3\}$

$\therefore$ $n(C) = 6$

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{9} = \frac{2}{3}$$

$\therefore$ $\boxed{P(C) = \frac{2}{3}}$

(14) Out of 200 students from a school, 135 like Kabbaddi and the remaining students do not like the game. If one student is selected at random from all the students, find the probability that the student selected dosen't like Kabbaddi.

(3 marks)

Solution:

Let S = Total no. of students.

$\therefore$ $\boxed{n(S) = 200}$

No. of students who like to play Kabbaddi = 135

$\therefore$ No. of students who do not like the game = 200 – 135 = 65

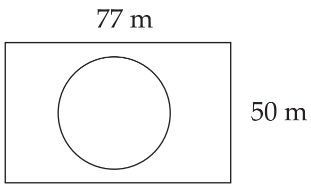
(i) Let A be the event that the student selected doesn't like to play kabaddi.

$\therefore$ $n(A) = 65$

$$P(A) = \frac{n(A)}{n(S)} = \frac{65}{200} = \frac{13}{40}$$

$\therefore$ $\boxed{\mathbf{P(A) = \frac{13}{40}}}$

(9) Length and breadth of a rectangular garden are 77 m and 50 m. There is a circular lake in the garden having diameter 14 m. Due to wind, a towel from a terrace on a nearby building fell into the garden. Then find the probability of the event that it fell in the lake. *(4 marks)*



Solution:

S : Area of the garden.

Length of the garden = 77 m,

Breadth of the garden = 50 m

$\therefore$ Area of the garden = 77 × 50 = 3850 sq m.

Let A be the event that the towel fell in a lake.

Diameter of the lake = 14 m

$\therefore$ Radius of the lake = 7 m

$\therefore$ Area of the lake $= \pi r^2$

$$= \frac{22}{7} \times 7 \times 7$$

$$= 154 \text{ sq m.}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{154}{3850} = \frac{1}{25} = 0.04$$

$\therefore$ **P(Towel fell in the lake) = 0.04**

Problem Set - 5 *(Textbook Page No. 126)*

MCQ's

Choose the correct alternative answer for each of the following questions. *(1 mark each)*

(1) Which number cannot represent a probability?

(A) $\frac{2}{3}$ (B) 1.5 (C) 15% (D) 0.7

(2) A die is rolled. What is the probability that the number appearing on upperface is less than 3?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 0

(3) What is the probability of the event that a number chosen from 1 to 100 is a prime number?

(A) $\frac{1}{5}$ (B) $\frac{6}{25}$ (C) $\frac{1}{4}$ (D) $\frac{13}{50}$

(4) There are 40 cards in a bag. Each bears a number from 1 to 40. One card is drawn at random. What is the probability that the card bears a number which is a multiple of 5?

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{1}{3}$

(5) If $n(A) = 2$, $P(A) = \frac{1}{5}$, then $n(S) = ?$

(A) 10 (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ (D) $\frac{1}{3}$

Additional MCQ's

(6) Which of the following cannot be the probability of an event?

(A) $\frac{3}{5}$ (B) $\frac{7}{2}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$

(7) The value of probability always lie between

(A) 0 to 1 (B) unlimited

(C) 1 to ∞ (D) – ∞ to 1

(8) The probability of throwing a number smaller than 2 in a fair die is

(A) $\frac{2}{3}$ (B) $\frac{1}{6}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$

(9) A card is drawn from a well shuffled pack of 52 cards. The probability that the card drawn is a black face card is

(A) $\frac{2}{13}$ (B) $\frac{3}{13}$ (C) $\frac{3}{26}$ (D) $\frac{1}{13}$

(10) Three coins are tossed. Find the probability that tail does not appear.

(A) $\frac{3}{8}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{7}{8}$

ANSWERS

(1) (B) 1.5 **(2)** (B) $\frac{1}{3}$ **(3)** (C) $\frac{1}{4}$ **(4)** (A) $\frac{1}{5}$

(5) (A) 10 **(6)** (B) $\frac{7}{2}$ **(7)** (A) 0 to 1 **(8)** (B) $\frac{1}{6}$

(9) (C) $\frac{3}{26}$ **(10)** (B) $\frac{1}{8}$

PROBLEMS FOR PRACTICE

Based on Practice Set 5.2

(1) In each of the following experiments, find the sample space S and the sample points $n(S)$ *(1 mark each)*

(i) A two digit number is to be formed from the digits 0, 2, 4, 6 without repetitions of digits.

(ii) A ball is drawn from a bag containing 3 red, 3 green and 4 white balls.

(iii) A day is chosen randomly for the meeting of Gram Sevaks in the month of February 2016.

(iv) A committee of two is to be formed from 2 men and 3 women for Gram Swachhatta Abhiyan.

(v) A card is drawn from a box containing cards numbered from 1 to 25.

Based on Practice Set 5.3

(2) A box contains cards numbered from 1 to 30. Write the sample space S and no. of sample points n(S) and if a card is drawn at random, write A and n(A) if the card is divisible by 5. *(3 marks)*

(3) An urn contains 10 red and 8 white balls. Write sample space S and n(S). Write the events A and B using Set and mention n(A) and n(B) if A is the event that ball is white, B is the event that ball is neither white nor red. *(3 marks)*

(4) A card is picked up randomly from well shuffled pack of cards. Write the n(S), n(A), n(B) and n(C)

Event A: A red face card.

Event B: An ace of spade

Event C: Not a black king *(3 marks)*

(5) A box contains 3 apples, 4 oranges and 5 bananas. One fruit is drawn at random from the box. Write S, n(S) and sample points of each of the following events.

Event A: Fruit is orange or banana

Event B: Fruit is not an apple.

Event C: Fruit is neither apple nor banana

Event D : Fruit is banana *(3 marks)*

Based on Practice Set 5.4

(6) Tickets numbered 1 to 30 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket drawn will be a multiple of 7? *(3 marks)*

(7) Find the probability that a leap year selected at random will have 53 sundays. *(3 marks)*

(8) A box contains 300 electrical bulbs out of which 18 are defective. What is the probability that bulb chosen will not be defective? *(3 marks)*

(9) Three coins are tossed together. Find the probability of getting exactly two heads. *(3 marks)*

(10) A card is drawn at random from a well shuffled pack of 52 cards. Find the probability that the card drawn is

(i) neither ace nor king.

(ii) black and a king

(iii) 10 of spades. *(3 marks)*

ANSWERS

(1)

(i) S = {20, 24, 26, 40, 42, 46, 60, 62, 64} n(S) = 9

(ii) S = {R_1, R_2, R_3, G_1, G_2, G_3, W_1, W_2, W_3, W_4} n(S) = 10

(iii) S = {1, 2, 3, ...29} n(S) = 29

(iv) S = { M_1M_2; M_1W_1; M_1W_2; M_1W_3; M_2W_1; M_2W_2; M_2W_3; W_1, W_2; W_1, W_3; W_2, W_3} n(S) = 10

(v) S = { 1, 2, 3, ..., 24, 25} n(S) = 25

(2) S = {1, 2, 3, 4, 5, ... 30} n(S) = 30

A = {5, 10, 15, 20, 25, 30} n(A) = 6

(3) S = {R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8} n(S) = 18

A = {W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8} = n(A) = 8

B = { } = n(B) = 0

(4) n(S) = 52, n(A) = 6, n(B) = 1, n(C) = 50

(5) n(S) = 12, n(A) = 9, n(B) = 9, n(C) = 4, n(D) = 5

(6) $\frac{2}{15}$, **(7)** $\frac{2}{7}$, **(8)** $\frac{141}{150}$, **(9)** $\frac{3}{8}$

(10) (i) $\frac{11}{13}$ (ii) $\frac{1}{26}$ (iii) $\frac{1}{52}$

ASSIGNMENT – 5

Time : 1 Hr. **Marks : 20**

Q.1. Attempt the following: **(2)**

(1) If $n(A) = 2$, $P(A) = \frac{1}{5}$, then $n(S)$ -= ?

(A) 10 (B) $\frac{5}{2}$ (C) $\frac{4}{5}$ (D) $\frac{1}{3}$

(2) One coin and one die are towel simultaneously. Write the sample space S and $n(S)$.

Q.2. Attempt the following: (Any 2) **(4)**

(1) Six faces of a die are as shown below.

A	B	C	D	E	A

If the die is rolled once, find the probability of -

(i) 'A' appears on upper face. **(ii)** 'D' appears on upper face.

Solution:

The sample space for the experiment is

S = $\boxed{\{ \qquad \qquad \}}$

$n(S) = \square$

(i) Let A be the event that the letter appears on the uppermost face is A.

$\therefore$ $n(A) = \square$

$P(A) = \frac{\square}{\square} = \frac{\square}{\square}$

(ii) Let B be the event that the letter appearing on the uppermost face is D.

$\therefore$ $n(B) = \square$

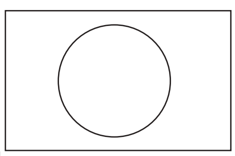
$P(B) = \frac{\square}{\square} = \frac{\square}{\square}$

(2) Basketball players John, Vasim, Akash were practising the ball drop in the basket.

The probabilities of success for John, Vasim and Akash are $\frac{4}{5}$, 0.83 and 58% respectively. Who had the greatest probability of success?

(3) Length and breadth of a rectangular garden are 77 m and 50 m. There is a circular lake in the garden having diameter 14 m. Due to wind, a towel from a terrace on a nearby building fell into the garden. Then find the probability of the event that it fell in the lake.

77 m



50 m

Solution:

Area of the garden = $\square \times \square = \square$ sq m.

Let A be the event that the towel fell in a lake.

Area of the lake = $\square = \square \times \square \times \square = \square$ sq.m.

$P(A) = \frac{\square}{\square} = \frac{\square}{\square}$

Q.3. Attempt the following: (Any 2) **(6)**

(1) A card is drawn at random from a pack of well shuffled 52 playing cards. Find the probability that the card drawn is - (i) an ace (ii) a spade

Solutions:

There are $\boxed{}$ cards in a pack.

$n(S) = \boxed{}$

(i) Let A be the event that the card drawn is an ace.

$n(A) = \boxed{}$

$P(A) = \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$

(ii) Let B be the event that the card drawn is a spade.

$n(B) = \boxed{}$

$P(B) = \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$

(2) If two coins are tossed, find the probability of the following events :

(i) Getting atleast one head (ii) Getting no head

(3) There are 15 tickets in a box, each bearing one of the numbers from 1 to 15. One ticket is drawn at random from the box. Find the probability of event that the ticket drawn-

(i) shows an even number. (ii) shows a number which is a multiple of 5.

Solution:

The sample space for box containing 15 tickets numbered from 1 to 15 is

$\therefore \; n(S) = \boxed{}$

(i) Let A be the event that the ticket drawn shows an even number.

$A = \{\boxed{}\}$

$\therefore \; n(A) = \boxed{}$

$P(A) = \frac{n(A)}{n(S)} = \frac{\boxed{}}{\boxed{}}$

(ii) Let B be the event that the ticket drawn shows a number which is a multiple of 5.

$B = \{\boxed{}\}$

$P(B) = \frac{n(B)}{n(S)} = \frac{3}{15} = \frac{\boxed{}}{\boxed{}}$

Q.4. Attempt the following: (Any 2) **(8)**

(1) In a hockey team there are 6 defenders, 4 offenders and 1 goalee. Out of these, one player is to be selected randomly as a captain. Find the probability of the selection that -

(i) The goalee will be selected. **(ii)** A defender will be selected.

(2) A bag contains 3 red, 3 white and 3 green balls. One ball is taken out of the bag at random. What is the probability that the ball drawn is - (i) red (ii) not red (iii) either red or white.

(3) If two dice are rolled simultaneously, find the probability of the following events.

(i) The sum of the digits on the upper faces is at least 10

(ii) The sum of the digits on the upper faces is 33.

(iii) The digit on the first die is greater than the digit on second die.

❖ ❖ ❖

6 Statistics

... INDEX ...

Points to Remember:

- **Introduction:**

Statistics is something that surrounds us every day - we are constantly bombarded with statistics, in the form of polls, tests, ratings etc. It is a branch of Mathematics which deals with the collection, presentation and analysis of numerical data and drawing the conclusions on the basis of the same.

Statistics is used to study the problems in Biology, Psychology, Economics, Education, Sociology, Trade etc. It is widely used in industries to control the quality of products.

In our Xth standard syllabus, we have four things to learn :

(A) Measures of Central Tendency of Grouped Data.

(B) Pie-Diagram.

(C) Histogram.

(D) Frequency Polygon or Frequency Curve

- **Measures of Central Tendency of Grouped data:**

The most basic concept in statistics is the idea of an **average (arithmetic mean)**. An **average** is a single number which represents the idea of a typical value. There are three different numbers which can represent the idea of an average value. The three values are the mean, the median and the mode.

Mean : Given a set of values, the mean is sum of all the values, divided by the total number of values. Mean summarizes the properties of entire data.

There are three different methods to find Mean for the given data.

Mean by Direct Method:

- Classes need not be continuous.
- Classmark (x_i) has to be found for every class of the given data.

- **Method 1 : Direct Method**

In this method, table will have four columns.

Step 1 : In 1st column, write the given Classes.

Step 2 : In 2nd column, write the Classmark/Mid-values (x_i) for each of the class.

Find the class mark of the first class, the remaining class marks could be found by adding class width (h) to the previous class mark.

Step 3 : In 3rd column, write the Frequency (f_i) of all the classes. Find the sum of all frequencies i.e. Σf_i.

Step 4: In 4th column, find the product of Classmark/Mid values (x_i) and frequency (f_i), i.e. $f_i x_i$, for each of the classes. Find the sum of all ($f_i x_i$) values i.e. $\Sigma f_i x_i$

Step 5 : Calculate, Mean ($\bar{x}$) = $\dfrac{\sum f_i x_i}{\sum f_i}$

MASTER KEY QUESTION SET - 6

Practice Set - 6.1 *(Textbook Page No. 138)*

(1) **The following table shows the number of students and the time they utilized daily for their studies. Find the mean time spent by students for their studies by direct method.** *(3 marks)*

Time *(hrs.)*	0 - 2	2 - 4	4 - 6	6 - 8	8 - 10
No. of students	7	18	12	10	3

Solution:

Time *(in hrs.)*	Class Mark (x_i)	No. of students (f_i)	f_ix_i
0 - 2	1	7	7
2 - 4	3	18	54
4 - 6	5	12	60
6 - 8	7	10	70
8 - 10	9	3	27
Total		$N = \Sigma f_i = 50$	$\Sigma f_ix_i = 218$

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{218}{50}$$

$$= 4.36$$

$\therefore$ **Mean of the time spent by students for studies is 4.36 hrs.**

(3) **A milk centre sold milk to 50 customers. The table below gives the number of customers and the milk they purchased. Find the mean of the milk sold by direct method.** *(3 marks)*

Milk sold (Litre)	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6
No. of customers	17	13	10	7	3

Solution:

Milk sold *(in litres)*	Class Mark (x_i)	No. of families (f_i)	f_ix_i
1 - 2	1.5	17	25.5
2 - 3	2.5	13	32.5
3 - 4	3.5	10	35
4 - 5	4.5	7	31.5
5 - 6	5.5	3	16.5
Total		$N = \Sigma f_i = 50$	$\Sigma f_ix_i = 141$

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{141}{50}$$

$$= 2.82$$

$\therefore$ **Mean quantity of milk sold is 2.82 litres**

Problem Set - 6 *(Textbook Page No. 165)*

(2) **The following table shows the income of farmers in a grape season. Find the mean of their income.** *(3 marks)*

Income (Thousand ₹)	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Farmers	10	11	15	16	18	14

Solution:

Production *(in thousand rupees)*	Class Mark (x_i)	No. of families (f_i)	f_ix_i
20 - 30	25	10	250
30 - 40	35	11	385
40 - 50	45	15	675
50 - 60	55	16	880
60 - 70	65	18	1170
70 - 80	75	14	1050
Total		$N = \Sigma f_i = 84$	$\Sigma f_ix_i = 4410$

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{4410}{84}$$

$$= 52.5$$

Mean of production = 52.5 × 1000

= 52,500

$\therefore$ **Mean of production in rupees is ₹ 52,500.**

Points to Remember:

- **Method II:**

  Assumed Mean Method or shift of origin method:

  In this method, table will have five columns.

  Step 1 : In 1st column, write the given classes.

  Step 2 : In 2nd column, write the Classmark/Mid values (x_i) for each of the class.

Step 3 : Select any value in the Classmark (x_i) column as Assumed mean. It is denoted by A. Generally, middle value of the column is selected.

Step 4 : In 3rd column, find deviations using formula $d_i = x_i - A$ for first class and the remaining d_i could be found by adding class width to the previous d_i.

Step 5 : In 4th column, write the Frequency (f_i) of all the classes. Also, find the sum of all frequencies, i.e. Σf_i

Step 6 : In 5th column, find the product of deviations (d_i) and frequency (f_i), i.e. $f_i d_i$, for each of the classes. Find the sum of all ($f_i d_i$) values i.e. $\Sigma f_i d_i$

Step 7 : Calculate $\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$

Step 8 : Calculate **Mean** $\bar{x} = A + \bar{d}$

Practice Set - 6.1 *(Textbook Page No. 138)*

(2) In the following table, the toll paid by drivers and the number of vehicles is shown. Find the mean of the toll by 'assumed mean' method.

Toll (₹)	300 - 400	400 - 500	500 - 600	600 - 700	700 - 800
No. of Vehicles	80	110	120	70	40

(4 marks)

Solution:

Assumed Mean (A) = 550

Toll collected *(in rupees)*	Class Mark (x_i)	$d_i = x_i - A$ $= x_i - 550$	No. of vehicles (f_i)	$f_i d_i$
300 - 400	350	–200	80	–16000
400 - 500	450	–100	110	–11000
500 - 600	550→A	0	120	0
600 - 700	650	100	70	7000
700 - 800	750	200	40	8000
Total			$N = \Sigma f_i = 420$	$\Sigma f_i d_i = -12000$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$$

$$= \frac{-12000}{420}$$

$$= -28.57$$

Mean ($\bar{x}$) = **A +** $\bar{d}$

$= [550 + (-28.57)]$

$= 521.43$

$\therefore$ **Mean of the money collected is ₹ 521.43.**

(4) A frequency distribution table for the production of oranges of some farm owners is given below. Find the mean production of oranges by 'assumed mean' method. *(4 marks)*

Production (Thousand ₹)	25 - 30	30 - 35	35 - 40	40 - 45	45 - 50
No. of farm owners	20	25	15	10	10

Solution:

Assumed Mean (A) = 37.5

Production *(in thousand rupees)*	Class Mark (x_i)	$d_i = x_i - A$ $= x_i - 37.5$	frequency (f_i)	$f_i d_i$
25 - 30	27.5	–10	20	–200
30 - 35	32.5	–5	25	–125
35 - 40	37.5→A	0	15	0
40 - 45	42.5	5	10	50
45 - 50	47.5	10	10	100
Total			$\Sigma f_i = 80$	$\Sigma f_i d_i = -175$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$$

$$= \frac{-175}{80}$$

$$= -2.19$$

Mean ($\bar{x}$) = **A +** $\bar{d}$

$= [37.5 + (-2.19)]$

$= 35.31$

$\therefore$ Mean of production Amount = 35.31 × 1000

= ₹ 35,310

$\therefore$ **Mean of production is ₹ 35,310.**

Problem Set - 6 *(Textbook Page No. 165)*

(3) The loans sanctioned by a bank for construction of farm ponds are shown in the following table. Find the mean of the loans. *(4 marks)*

Loan (Thousand ₹)	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90
No. of farm ponds	13	20	24	36	7

Solution:

Assumed Mean (A) = 65

Loan (*in thousand rupees*)	Class Mark (x_i)	$d_i = x_i - A$ $= x_i - 65$	Number of farm pond (f_i)	$f_i d_i$
40 - 50	45	–20	13	–260
50 - 60	55	–10	20	–200
60 - 70	65 → A	0	24	0
70 - 80	75	10	36	360
80 - 90	85	20	7	140
Total			$\Sigma f_i = 100$	$\Sigma f_i d_i = 40$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$$

$$= \frac{40}{100}$$

$$= 0.4$$

Mean ($\bar{x}$) = A + $\bar{d}$

$= 65 + 0.4$

$= 65.4$

Total amount $= 65.4 \times 1000$

$= 65{,}400$

∴ **Mean of loan given by bank is ₹ 65,400.**

(5) The following frequency distribution table shows the amount of aid given to 50 flood affected families. Find the mean of the amount of aid. *(4 marks)*

Amount of aid (Thousand ₹)	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
No. of families	7	13	20	6	4

Solution:

Assumed Mean (A) = 75

Money (*in thousand rupees*)	Class Mark (x_i)	$d_i = x_i - A$ $= x_i - 75$	frequency (f_i)	$f_i d_i$
50 - 60	55	–20	7	–140
60 - 70	65	–10	13	–130
70 - 80	75 → A	0	20	0
80 - 90	85	10	6	60
90 - 100	95	20	4	80
Total			$\Sigma f_i = 50$	$\Sigma f_i d_i = -130$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$$

$$= \frac{-130}{50}$$

$$= 2.6$$

Mean ($\bar{x}$) = A + $\bar{d}$

$= 75 + (-2.6) = 72.4$

Total amount $= 72.4 \times 1000 =$ ₹ 72,400

∴ **Mean of money given is ₹ 72,400.**

Points to Remember:

Method III : Step Deviation Method or shift of origin and scale method.

Width of class (g) = Difference between any two consecutive upper limits or lower limits. In this method, table will have 6 columns.

Step 1 : In 1st column, write the given classes.

Step 2 : In 2nd column, write the Classmark/ Mid-values (x_i) for each of the class.

Step 3 : In 3rd column, write the values of the d_i, where $d_i = x_i - A$

Step 4 : In 4th column, write the value of the u_i where $u_i = \frac{x_i - A}{g} = \frac{d_i}{g}$

Step 5 : In 5th column, write the frequencies f_i and find its sum.

Step 6 : In 6th column, find the product of u_i and frequency, f_i i.e. $f_i u_i$ for each of the classes. Find the sum of all ($f_i u_i$) values. i.e. $\Sigma f_i u_i$

Step 7 : Find mean of u_i using the formula

$$\bar{u_i} = \frac{\sum f_i u_i}{\sum f_i}$$

Step 8 : Calculate, Mean $(\bar{x}) = A + h\bar{u}$

Practice set - 6.1 *(Textbook Page No. 138)*

(5) **A frequency distribution of funds collected by 120 workers in a company for the drought affected people are given in the following table. Find the mean of the funds by 'step deviation' method.** *(4 marks)*

Fund (₹)	0 - 500	500 - 1000	1000 - 1500	1500 - 2000	2000 - 2500
No. of workers	35	28	32	15	10

Solution:

$h = 500$, Assumed Mean (A) = 1250

Fund (in rupees)	Class Mark (x_i)	$d_i = x_i - A = x_i - 1250$	$u_i = \frac{d_i}{h}$	No. of workers (f_i)	$f_i u_i$
0 - 500	250	–1000	–2	35	–70
500 - 1000	750	–500	–1	28	–28
1000 - 1500	1250→A	0	0	32	0
1500 - 2000	1750	500	1	15	15
2000 - 2500	2250	1000	2	10	20
Total				$\Sigma f_i = 120$	$\Sigma f_i u_i = -63$

$$\bar{u} = \frac{\sum f_i u_i}{\sum f_i}$$

$$= \frac{-63}{120}$$

$$= -0.525$$

Mean $(\bar{x}) = A + \bar{u}.h$

$= 1250 + 500(-0.525)$

$= 1250 - 262.5$

$= 987.50$

$\therefore$ **Mean of the Fund collected is ₹ 987.50.**

(6) **The following table gives the information of frequency distribution of weekly wages of 150 workers of a company. Find the mean of the weekly wages by 'step deviation' method.** *(4 marks)*

Weekly wages (₹)	1000 - 2000	2000 - 3000	3000 - 4000	4000 - 5000
No. of workers	25	45	50	30

Solution:

$h = 1000$, Assumed Mean (A) = 2500

Weekly salary (in rupees)	Class Mark (x_i)	$d_i = x_i - A = x_i - 2500$	$u_i = \frac{d_i}{h}$	No. of workers (f_i)	$f_i u_i$
1000 - 2000	1500	–1000	–1	25	–25
2000 - 3000	2500→A	0	0	45	0
3000 - 4000	3500	1000	1	50	50
4000 - 5000	4500	2000	2	30	60
Total				$N = \Sigma f_i = 150$	$\Sigma f_i u_i = 85$

$$\bar{u} = \frac{\sum f_i u_i}{\sum f_i}$$

$$= \frac{85}{150}$$

$$= 0.567$$

Mean $(\bar{x}) = A + \bar{u}.h$

$= 2500 + 0.567 \times 1000$

$= 2500 + 567$

$= 3067$

$\therefore$ **Mean of salary is ₹ 3067.**

Problem Set - 6 *(Textbook Page No. 166)*

(4) **The weekly wages of 120 workers in a factory are shown in the following frequency distribution table. Find the mean of the weekly wages.** *(4 marks)*

Weekly wages (₹)	0 - 2000	2000 - 4000	4000 - 6000	6000 - 8000
No. of workers	15	35	50	20

Solution:

$h = 2000$, Assumed Mean (A) = 3000

Weekly salary (in rupees)	Class Mark (x_i)	$d_i = x_i - A = x_i - 2500$	$u_i = \frac{d_i}{h}$	No. of employee (f_i)	$f_i u_i$
0 - 2000	1000	–2000	–1	15	–15
2000 - 4000	3000→A	0	0	35	0
4000 - 6000	5000	2000	1	50	50
6000 - 8000	7000	4000	2	20	40
Total				$N = \Sigma f_i = 120$	$\Sigma f_i u_i = 75$

$$\bar{u} = \frac{\sum f_i u_i}{\sum f_i}$$

$$= \frac{75}{120}$$

$$= 0.625$$

Mean $(\bar{x}) = A + \bar{u} \cdot g$

$= 3000 + 2000 \times 0.625$

$= 3000 + 1250$

$= 4250$

$\therefore$ **Mean of weekly salary is ₹ 4250.**

Points to Remember:

- **Median:**

To find Median for the given data.

Median is a measure of the 'Middle' value of the given data. When it is arranged. Median is often a better representative of a typical member of a group. If we take all values in a list and arrange them in increasing order, the number at the centre will be the median. Median gives you the central number of the group. Median is a number which divides data into two equal parts.

- **Steps to calculate Median**

Step 1 : Make classes continuous if they are not.

Step 2 : In next column, write the frequencies.

Step 3 : In next column, calculate the 'Cumulative frequency less than type'.

Step 4 : Determine the median class, frequency of median class (f) and cumulative frequency of the class preceeding the median class ($c.f.$)

Step 5 : Calculate $\boxed{\textbf{Median} = L + \left(\frac{\frac{N}{2} - c.f.}{f}\right) h}$

L = Lower limit of median class.

N = Total frequency.

$c.f.$ = Cumulative frequency of class preceeding the median class.

f = Frequency of median class.

h = class interval of the median class

- Steps to determine Median class

Step 1 : Find the value of $\frac{N}{2}$, where N = Total frequency.

Step 2 : Find the cumulative frequency less than type which is just greater than or equal to $\frac{N}{2}$. The corresponding class is a median class.

For Median,

- Classes have to be continuous.
- Cumulative frequency less than type has to be found for every sum.

Practice set - 6.2 *(Textbook Page No. 145)*

(1) **The following table shows classification of number of workers and the number of hours they work in a software company. Find the median of the number of hours they work.** *(4 marks)*

Daily No. of hours	8 - 10	10 - 12	12 - 14	14 - 16
No. of workers	150	500	300	50

Solution:

Class width (h) = 2

Class intervals *(Daily working hours)*	Frequency *(No. of employees)*	Cumulative frequency *(less than type)*
8 - 10	150	$150 \rightarrow c.f.$
10 - 12	$500 \rightarrow f$	150 + 500 = 650
12 - 14	300	650 + 300 = 950
14 - 16	50	950 + 50 = 1000
Total	**1000 → N**	

Here, total frequency (N) = 1000.

$\therefore \quad \frac{N}{2} = \frac{1000}{2} = 500$

Cumulative frequency (less than type) which is just greater than 500 is 650.

$\therefore$ Corresponding class 10 - 12 is the median class.

$\therefore$ $f = 500$, $c.f. = 150$, L = 10 and $h = 2$

$$\textbf{Median} = L + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$$

$$= 10 + \left[\frac{500 - 150}{500}\right] \times 2$$

$$= 10 + 1.4$$

$$= 11.4$$

$\therefore$ **Median of no. of hours worked is 11.4**

(2) The frequency distribution table shows the number of mango trees in a grove and their yield of mangoes. Find the median of data. *(4 marks)*

No. of mangoes	50 - 100	100 - 150	150 - 200	200 - 250	250 - 300
No. of trees	33	30	90	80	17

Solution:

Class intervals (*No. of mangoes*)	Frequency (*No. of trees*)	Cumulative frequency (*less than type*)
50-100	33	33
100-150	30	33 + 30 = 63 → *c.f.*
150 - 200	90 → *f*	63 + 90 = 153
200 - 250	80	153 + 80 = 233
250 - 300	17	233 + 17 = 250
Total	**250 → N**	

Here, total frequency (N) = 250.

∴ $\frac{N}{2} = \frac{250}{2} = 125$

Cumulative frequency (less than type) which is just greater than 125 is 153.

∴ Corresponding class 150 - 200 is the median class.

∴ $f = 90$, $c.\,f. = 63$, L = 150 and $h = 50$

$$\textbf{Median} = L + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$$

$$= 150 + \left[\frac{125 - 63}{90}\right] \times 50$$

$$= 150 + \frac{62}{90} \times 50$$

$$= 150 + 34.44$$

$$= 184.44 \approx 184$$

∴ **Median of no. of mangoes is 184 (approx.)**

(4) The production of electric bulbs in different factories is shown in the following table. Find the median of the productions. *(4 marks)*

No. of bulbs produced (Thousands)	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
No. of factories	12	35	20	15	8	7	8

Solution:

Class intervals (*No. of bulbs*) (*in Thousands*)	Frequency (No. of factories)	Cumulative frequency (*less than type*)
30 - 40	12	12
40 - 50	35	35 + 12 = 47 → *c.f.*
50 - 60	20 → *f*	47 + 20 = 67
60 - 70	15	67 + 15 = 82
70 - 80	8	82 + 8 = 90
80 - 90	7	90 + 7 = 97
90 -100	8	97 + 8 = 105
Total	**105 → N**	

Here, total frequency (N) = 105.

∴ $\frac{N}{2} = \frac{105}{2} = 52.5$ and $h = 10$

Cumulative frequency (less than type) which is just greater than 52.5 is 67.

∴ Corresponding class 50 - 60 is the median class.

∴ $f = 20$, $c.\,f. = 47$, L = 50

$$\textbf{Median} = L + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$$

$$= 50 + \left[\frac{52.5 - 47}{20}\right] \times 10$$

$$= 50 + \frac{5.5}{20} \times 10$$

$$= 50 + 2.75$$

$$= 52.75$$

Median production = 52.75 × 1000 = 52750

∴ **Median of production of bulbs is 52750**

Problem Set - 6 *(Textbook Page No. 166)*

(6) The distances covered by 250 public transport buses in a day is shown in the following frequency distribution table. Find the median of the distances. *(4 marks)*

Distance (km)	200 - 210	210 - 220	220 - 230	230 - 240	240 - 250
No. of buses	40	60	80	50	20

Solution:

Class (Distance km)	Frequency (No. of buses)	Cumulative frequency (less than type)
200 - 210	40	40
210 - 220	60	40 + 60 = 100 → *c.f.*
220 - 230	80 → *f*	100 + 80 = 180
230 - 240	50	180 + 50 = 230
240 - 250	20	230 + 20 = 250
Total	**250 → N**	

Here, total frequency = Σf_i = N = 250 and h = 10

$\therefore$ $\dfrac{N}{2} = \dfrac{250}{2} = 125$

Cumulative frequency (less than type) which is just greater than 125 is 180.

$\therefore$ Corresponding class 220 - 230 is the median class.

$\therefore$ $f = 80$, $c.f. = 100$, L = 220

$$\textbf{Median} = L + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$$

$$= 220 + \left(\frac{125 - 100}{80}\right) \times 10$$

$$= 220 + \left(\frac{25}{80}\right) \times 10$$

$$= 220 + 3.125$$

$$= 223.13$$

$\therefore$ **Median distance covered is 223.13 km.**

(7) **The prices of different articles and demand for them is shown in the following frequency distribution table. Find the median of the prices.**

Price (₹)	less than 20	20 - 40	40 - 60	60 - 80	80 - 100
No. of articles	140	100	80	60	20

(4 marks)

Solution:

Class intervals (Price in Rs.)	Frequency (No. of articles)	Cumulative frequency (less than type)
less than 20	140	140 → *c.f.*
20 - 40	100 → *f*	140 + 100 = 240
40 - 60	80	240 + 80 = 320
60 - 80	60	320 + 60 = 380
80 - 100	20	380 + 20 = 400
Total	**400 → N**	

Here, total frequency = Σf_i = N = 400.

$\therefore$ $\dfrac{N}{2} = 200$ and $h = 20$

Cumulative frequency (less than type) which is just greater than 200 is 240.

$\therefore$ Corresponding class 20 - 40 is the median class.

$\therefore$ $f = 100$, $c.f. = 140$, L = 20

$$\textbf{Median} = L + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$$

$$= 20 + \left(\frac{200 - 140}{100}\right) 20$$

$$= 20 + \frac{60}{100} \times 20$$

$$= 20 + 12 = 32$$

$\therefore$ **Median of Amount is ₹ 32.**

Practice set - 6.2 *(Textbook Page No. 145)*

(3) **The following table shows the classification of number of vehicles and their speeds on Mumbai-Pune express way. Find the median of the data.** *(4 marks)*

Average Speed of Vehicles (Km/hr)	60-64	65-69	70-74	75 - 79	80 - 84	85 - 89
No. of Vehicles	10	34	55	85	10	6

Solution: First make continuous classes

Class intervals (vehicle speed)	Continuous class intervals	Frequency (No. of vehicles)	Cumulative frequency (less than type)
60 - 64	59.5 - 64.5	10	10
65 - 69	64.5 - 69.5	34	44
70 - 74	69.5 - 74.5	55	99 → *c.f.*
75 - 79	**74.5 - 79.5**	85 → *f*	184
80 - 84	79.5 - 84.5	10	194
85 - 89	84.5 - 89.5	6	200
Total		**200 → N**	

Here, total frequency (N) = 200.

$\therefore$ $\dfrac{N}{2} = \dfrac{200}{2} = 100$ and $h = 5$

Cumulative frequency (less than type) which is just greater than 100 is 184.

$\therefore$ Corresponding class 74.5 - 79.5 is the median class.

$\therefore$ $f = 85$, $c.f. = 99$, $L = 74.5$

$$\textbf{Median} = L + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$$

$$= 74.5 + \left[\frac{100 - 99}{85}\right] \times 5$$

$$= 74.5 + \frac{1}{85} \times 5$$

$$= 74.5 + 0.058$$

$$= 74.559 \approx 75 \text{ (approx)}$$

$\therefore$ **Median of speed of the vehicles is 75 km/h (approx)**

Points to Remember:

Mode :

Mode is the observation which occurs most frequently. The observation having maximum frequency is called **mode**. A data set has no mode when all the observations appearing in the data have the frequency 1. A data set has multiple modes when two or more values appear with the same highest frequency.

To find mode for the given data,

- Classes have to be continuous.
- If maximum frequency occurs twice in the data, then data will have two Modal classes.

Hence, Mode has to be found for both modal class.

- **Steps to calculate Mode :**

Step 1 : Make the classes continuous if they are not.

Step2 : Determine Modal class, Maximum frequency (f_1), frequency of class preceding modal class (f_0), frequency of class succeeding the modal class (f_2)

Step 3 : Calculate,

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

L = Lower limit of modal class.

N = Total frequency.

f_1 = Maximum frequency.

f_0 = Frequency of class preceding modal class.

f_2 = Frequency of class succeeding the modal class.

h = class interval of the modal class.

- **Steps to determine Modal class :**

Step 1 : The class having maximum frequency is termed as Modal class.

Practice Set - 6.3 *(Textbook Page No. 149)*

(1) The following table shows the information regarding the milk collected from farmers on a milk collection centre and the content of fat in the milk, measured by a lactometer. Find the mode of fat content. *(4 marks)*

Content of fat (%)	2 - 3	3 -4	4 -5	5 -6	6 -7
Milk collected (Litre)	30	70	80	60	20

Solution:

Class interval *(fat content in milk)*	Frequency *(milk in litres)*
2 - 3	30
3 - 4	$70 \rightarrow f_0$
4 - 5	$80 \rightarrow f_1$
5 - 6	$60 \rightarrow f_2$
6 - 7	20

f_1 = Maximum frequency = 80.

The corresponding class 4 - 5 is modal class.

$f_0 = 70$, $f_2 = 60$, $L = 4$ and $h = 1$

$$\textbf{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 4 + \left[\frac{80 - 70}{2 \times 80 - 70 - 60}\right] \times 1$$

$$= 4 + \left[\frac{10}{160 - 130}\right]$$

$$= 4 + \frac{10}{30}$$

$$= 4 + 0.33$$

$$= 4.33$$

$\therefore$ **Mode of weight of fat in milk is 4.33 %**

(2) Electricity used by some families is shown in the following table. Find the mode for use of electricity. *(4 marks)*

Use of electricity (Unit)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
No. of families	13	50	70	100	80	17

Solution:

Class interval *(Units of electricity)*	Frequency *(No. of families)*
0 - 20	13
20 - 40	50
40 - 60	$70 \to f_0$
60 - 80	$100 \to f_1$
80 - 100	$80 \to f_2$
100 - 120	17

f_1 = Maximum frequency = 100.

The corresponding class 60 - 80 is modal class.

$f_0 = 70, f_2 = 80$, L = 60 and $h = 20$

$$\textbf{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 60 + \left[\frac{100 - 70}{2 \times 100 - 70 - 80}\right] \times 20$$

$$= 60 + \frac{30}{50} \times 20$$

$$= 60 + 12$$

$$= 72$$

$\therefore$ **Mode of no. of units consumed is 72**

(3) Grouped frequency distribution of supply of milk to hotels and the number of hotels is given in the following table. Find the mode of the supply of milk. *(4 marks)*

Milk (litre)	1-3	3-5	5-7	7-9	9-11	11-13
No. of hotels	7	5	15	20	35	18

Solution:

Class interval *(Milk in litres)*	Frequency *(No. of Hotels)*
1 - 3	7
3 - 5	5
5 - 7	15
7 - 9	$20 \to f_0$
9 - 11	$35 \to f_1$
11 - 13	$18 \to f_2$

f_1 = Maximum frequency = 35.

The corresponding class 9 - 11 is the modal class.

$f_0 = 20, f_2 = 18$, L = 9 and $h = 2$

$$\textbf{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 9 + \left[\frac{35 - 20}{2 \times 35 - 20 - 18}\right] \times 2$$

$$= 9 + \frac{15}{32} \times 2$$

$$= 9 + 0.94 = 9.94$$

$\therefore$ **Modal of quantity of milk consumed by hotels is 9.94 litres.**

(4) The following frequency distribution table gives the ages of 200 patients treated in a hospital in a week. Find the mode of ages of the patients. *(4 marks)*

Age (years)	Less than 5	5 - 9	10 - 14	15 - 19	20 - 24	25 - 29
No. of patients	38	32	50	36	24	20

Solution:

Age	Contribution class Interval	No. of patients
less than 5	0.5 - 4.5	38
5 - 9	4.5 - 9.5	$32 \to f_0$
10 - 14	**9.5 - 14.5**	$50 \to f_1$
15 - 19	14.5 - 19.5	$36 \to f_2$
20 - 24	19.5 - 24.5	24
25 - 29	24.5 - 29.5	20

f_1 = Maximum frequency = 50.

The corresponding class 9.5 - 14.5 is the modal class.

$f_0 = 32, f_2 = 36$, L = 9.5 and $h = 5$

$$\textbf{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 9.5 + \left[\frac{50 - 32}{2 \times 50 - 32 - 36}\right] \times 5$$

$$= 9.5 + \frac{18}{32} \times 5$$

$$= 9.5 + 2.81$$

$$= 12.31 \text{ years.}$$

$\therefore$ **Mode of age of patients is is 12.31 years**

Problem Set - 6 *(Textbook Page No. 166)*

(8) The following frequency table shows the demand for a sweet and the number of customers. Find the mode of demand of sweet.

Weight of sweet (gram)	0 - 250	250 - 500	500 - 750	750 - 1000	1000 - 1250
No. of customers	10	60	25	20	15

(4 marks)

Solution:

Class interval (*Weight of sweets in gms*)	Frequency (*No. of customers*)
0 - 250	$10 \rightarrow f_0$
250 - 500	$60 \rightarrow f_1$
500 - 750	$25 \rightarrow f_2$
750 - 1000	20
1000 - 1250	15

Here, the maximum frequency $f_1 = 60$.

∴ The corresponding class 250 - 500 is the Modal class.

$f_1 = 60$, $f_0 = 10$, $f_2 = 25$, L = 250 and $h = 250$

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 250 + \left[\frac{60 - 10}{2 \times 60 - 10 - 25}\right] \times 250$$

$$= 250 + \left(\frac{50}{85}\right) \times 250$$

$$= 250 + 147.06$$

$$= 397.06$$

∴ **Mode of weight of sweets is 397.06 gm.**

Points to Remember:

- **Pictorial Representation of Statistical Data:**

It is difficult to arrive at a conclusion with a raw data. So, it needs to be processed. Processing involves arranging data in order, classification, tabulation. Such a compact or condensed form of data are easy to understand. Further if it is graphically or diagrammatically represented, it becomes simple to understand and interpret.

The specific graphs associated with frequency distribution are as follows :

(i) Histogram (ii) Frequency polygon or Frequency curve (iii) Pie diagram

(i) Histogram :

Steps to draw histogram :

Step 1 : Make the classes continuous if they are not continuous.

Step 2 : Take continuous classes on X-axis with suitable scale. For writing the classes, we can either use 1 cm or 2 cm for each class.

Step 3 : Take frequency on Y-axis with suitable scale.

Step 4 : Draw rectangles on the X-axis for each class. The height of the rectangles is equal to the frequency of the respective class.

Note: Thus, the histogram is series of joint rectangles:

If there is a difference in the gap between the origin and lower limit of the first class and class width, then a ‘ ’ krink mark is drawn near the origin on the X-axis. The mark can be used on Y-axis to draw a graph of optimum site

Practice Set - 6.4 *(Textbook Page No. 153)*

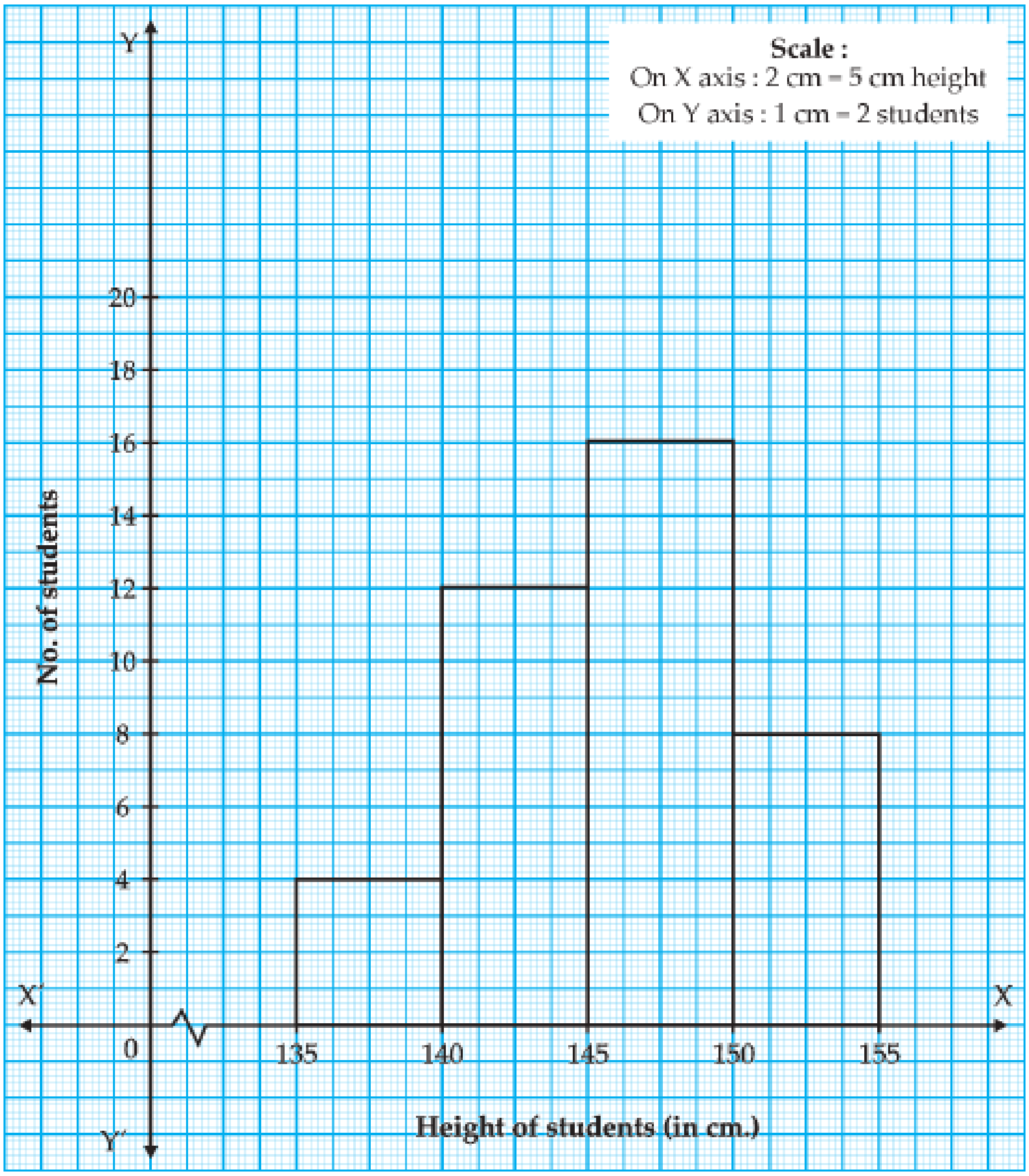
(1) Draw a histogram of the following data.

Height of student (cm)	135 - 140	140 - 145	145 - 150	150 - 155
No. of students	4	12	16	8

(3 marks)

Solution:

Height (in cms.)	No. of Students
135 - 140	4
140 - 145	12
145 - 150	16
150 - 155	8



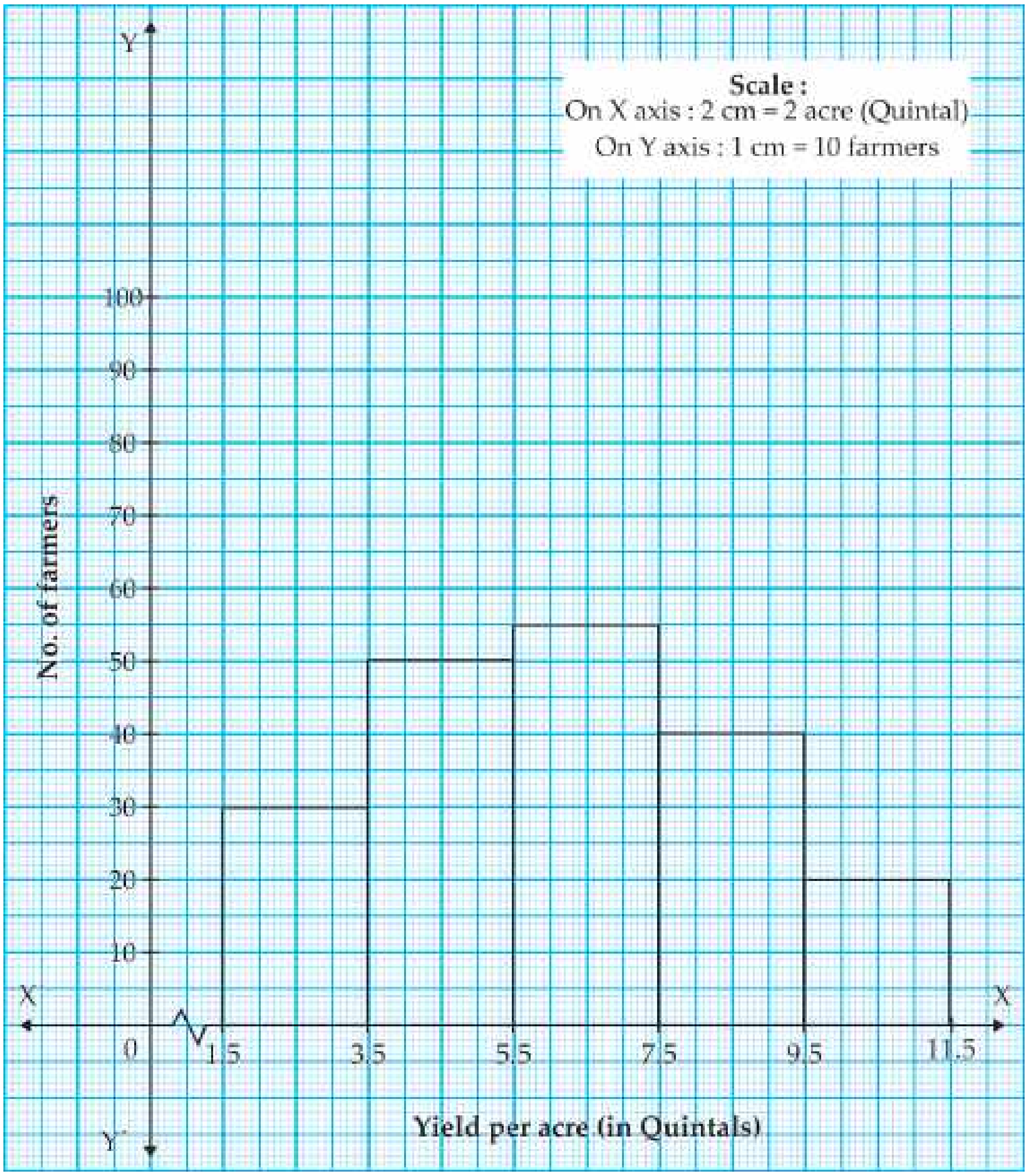
(2) **The table below shows the yield of jowar per acre. Show the data by histogram.**

Yield per acre (quintal)	2 - 3	4 - 5	6 - 7	8 - 9	10 - 11
No. of farmers	30	50	55	40	20

(4 marks)

Solution:

Class	Continuous class	Frequency
2 - 3	1.5 - 3.5	30
4 - 5	3.5 - 5.5	50
6 - 7	5.5 - 7.5	55
8 - 9	7.5 - 9.5	40
10 -11	9.5 - 11.5	20



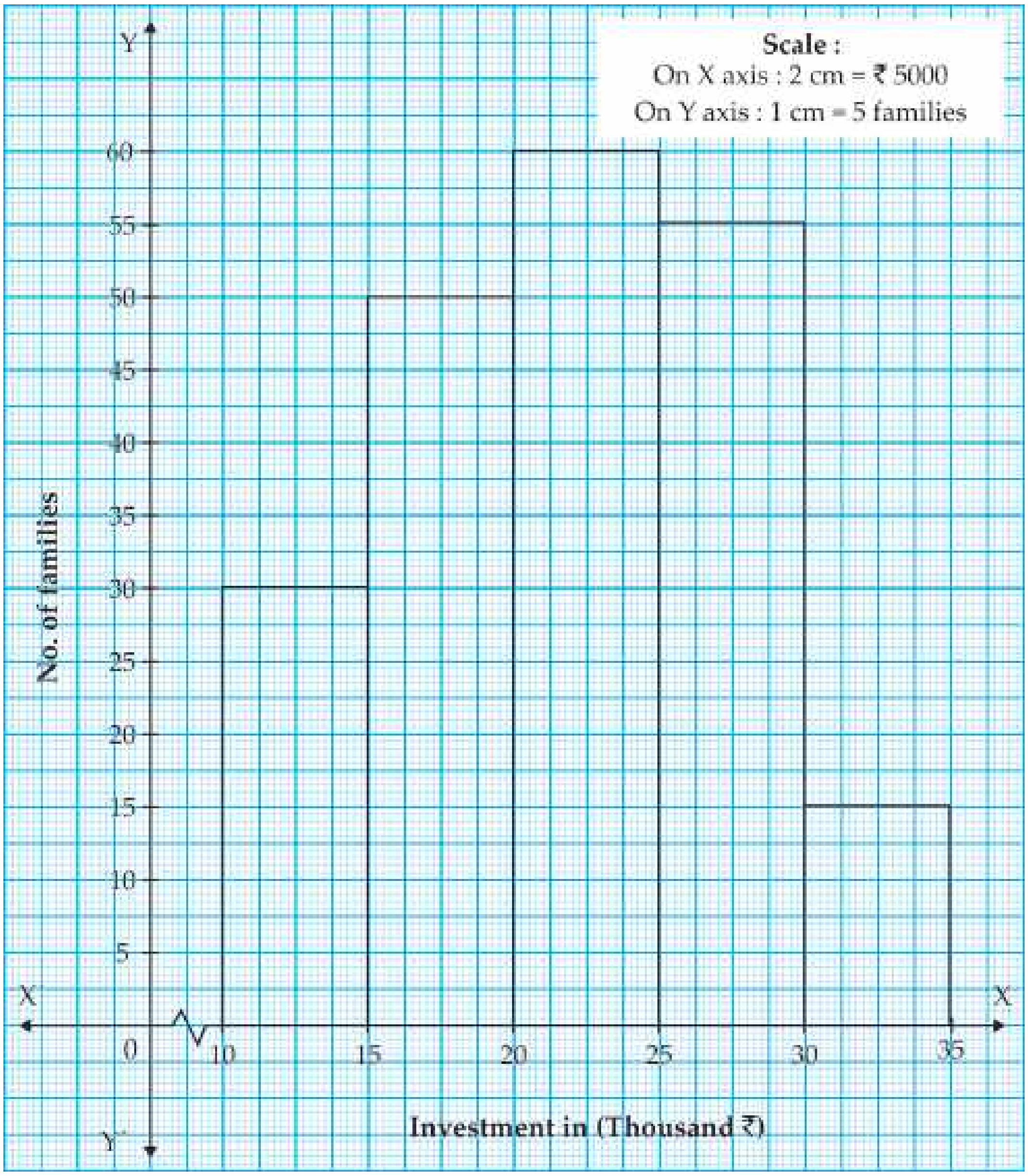
(3) **In the following table, the investment made by 210 families is shown. Present it in the form of a histogram.**

Investment (Thousand ₹)	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35
No. of families	30	50	60	55	15

(3 marks)

Solution:

Investment (Thousand ₹)	No. of families
10 - 15	30
15 - 20	50
20 - 25	60
25 - 30	55
30 - 35	15



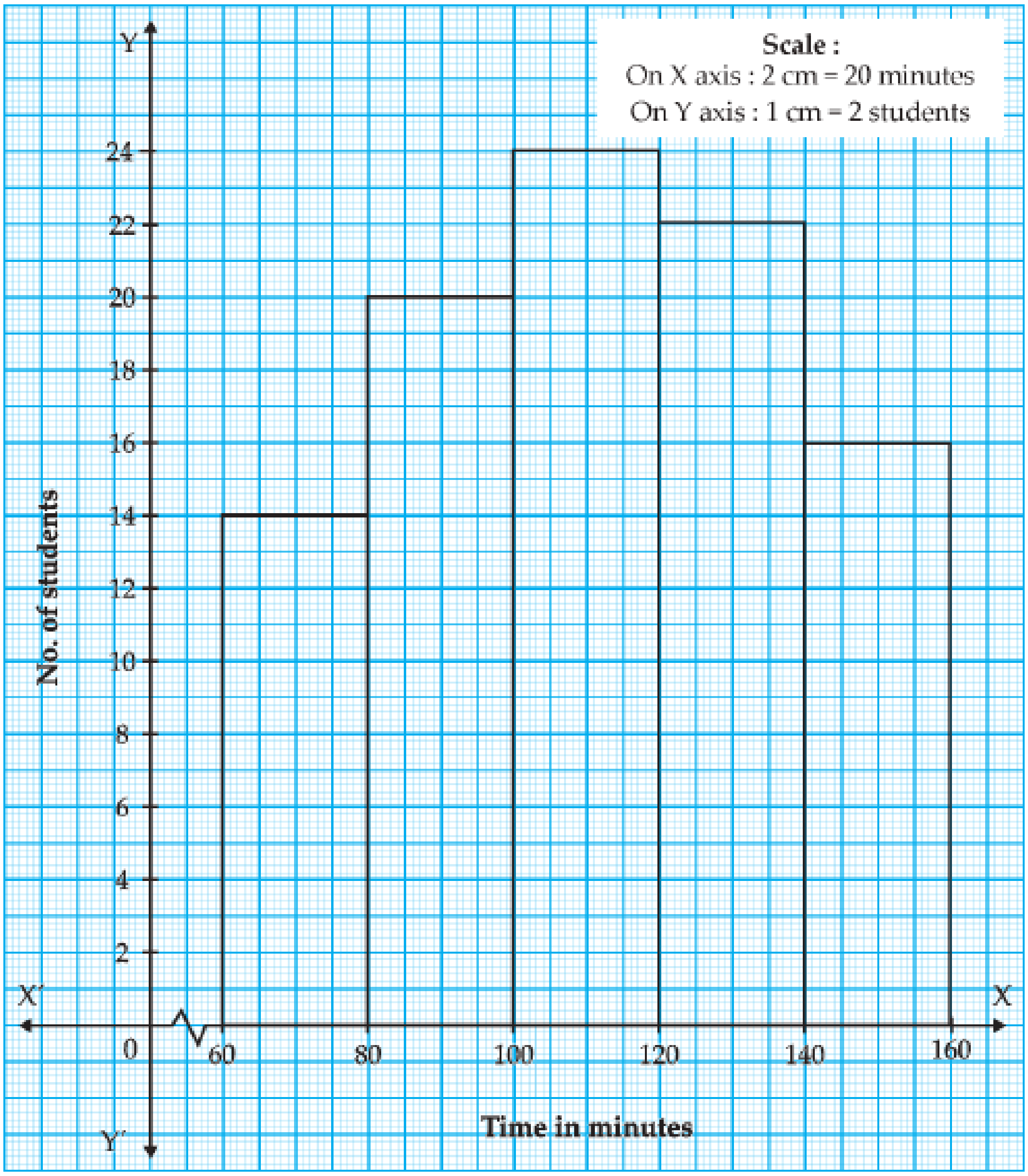
(4) **Time alloted for the preparation of an examination by some students is shown in the table. Draw a histogram to show the information.**

Time (minutes)	60 - 80	80 - 100	100 - 120	120 - 140	140 - 160
No. of students	14	20	24	22	16

(3 marks)

Solution:

Time (in minutes)	No. of students
60 - 80	14
80 - 100	20
100 - 120	24
120 - 140	22
140 - 160	16



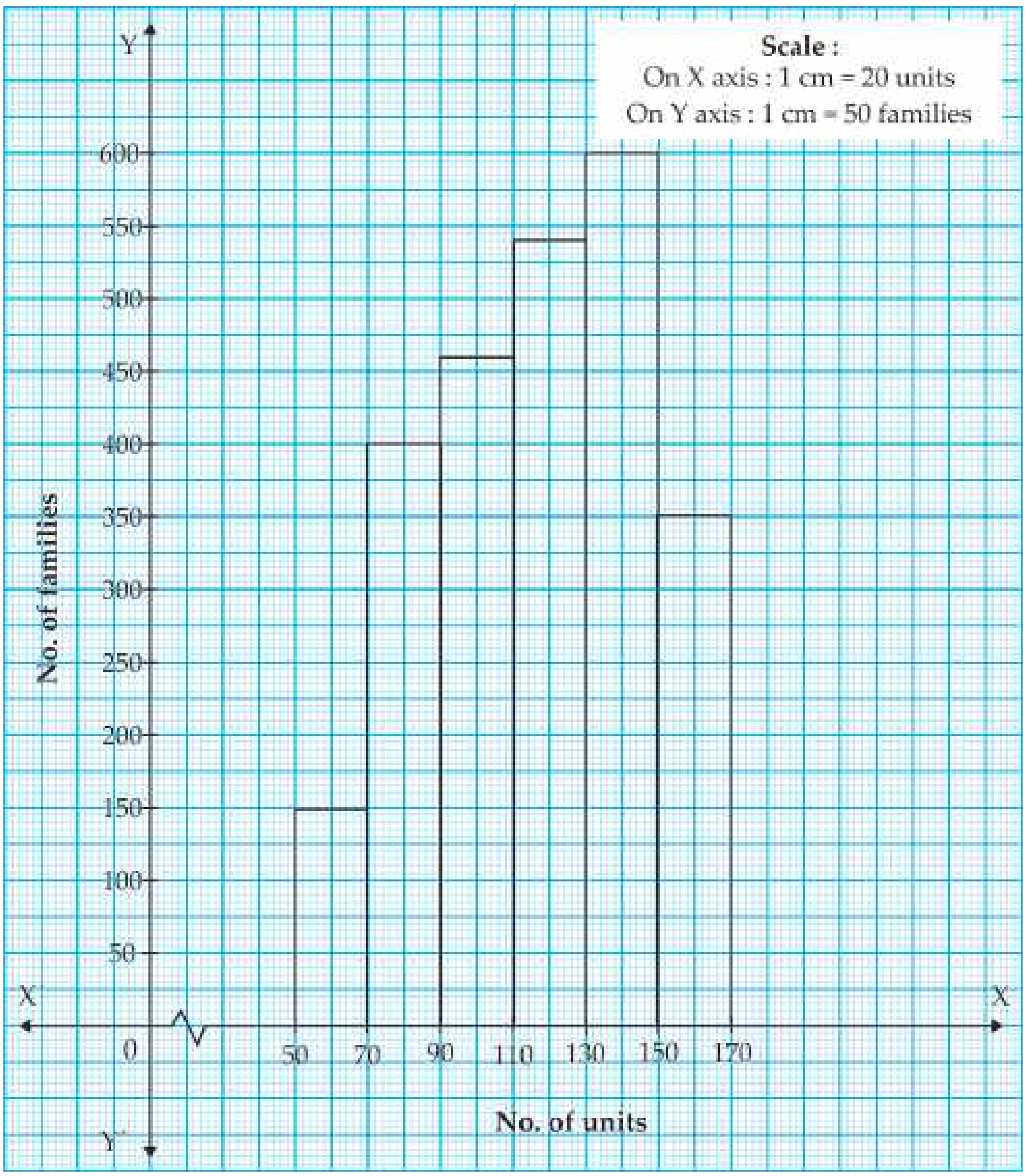
Problem Set - 6 *(Textbook Page No. 166)*

(9) **Draw a histogram for the following frequency distribution.** *(3 marks)*

Use of electricity (Unit)	50 - 70	70 - 90	90 - 110	110 - 130	130 - 150	150 - 170
No. of families	150	400	460	540	600	350

Solution:

Electricity electricity (Units)	No. of families
50 - 70	150
70 - 90	400
90 - 110	460
110 - 130	540
130 - 150	600
150 - 170	350



Points to Remember:

(ii) Frequency Polygon:

There are two methods to draw frequency polygon or curve:

(i) Using histogram

(ii) Plotting the points.

In the first method, we have to draw histogram and then mark all the mid-points on the upper side of the rectangles and then we join all the points with scale. To close the polygon or curve, we have join the midpoint of the class before the first class and the mid point of the class after the last class.

In the second method, we plot the points on the graph paper with midpoint of class as x co-ordinate and frequency as y co-ordinate (midpoint, frequency) and then join all the points in order with scale and then close the polygon or curve as mentioned in the previous method.

- **Steps to draw frequency polygon or frequency curve :**

Step 1 : Prepare frequency distribution table with class marks.

Step 2 : Draw X-axis, Y-axis, choose proper scale.

Step 3 : Take class marks on X-axis.

Step 4 : Take frequencies (f_i) on Y-axis

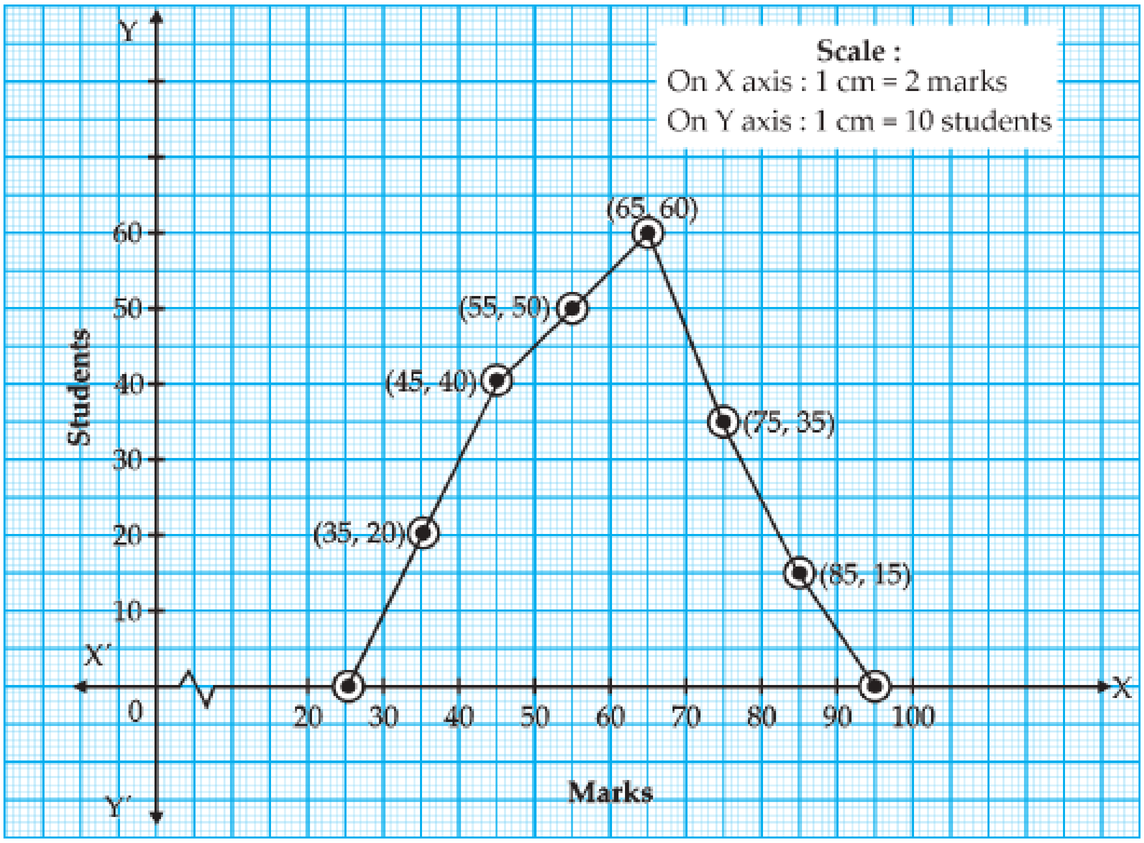
Step 5 : Plot the points (x_i, f_i).

Step 6 : Plot two additional points with two additional classes one preceding the first class with frequency zero (0) and the other succeeding the last class with frequency zero (0).

Step 7 : Join all the successive points including the two additional extreme points by straight lines. Eventually, the closed figure so obtained is frequency polygon.

Practice Set - 6.5 *(Textbook Page No. 157)*

(1) Observe the following frequency polygon and write the answers of the questions below it. *(3 marks)*



(1) Which class has the maximum number of students?

Ans. Maximum number of students are present in the group 60 - 70.

(2) Write the classes having zero frequency.

Ans. Zero students are present in 20 - 30 and 90 - 100 groups.

(3) What is the class-mark of the class, having frequency of 50 students?

Ans. 50 students are present in the group 50 - 60 whose class mark is 55.

(4) Write the lower and upper class limits of the class whose class mark is 85.

Ans. 85 is the class mark of the group 80 - 90.

(5) How many students are in the class 80-90?

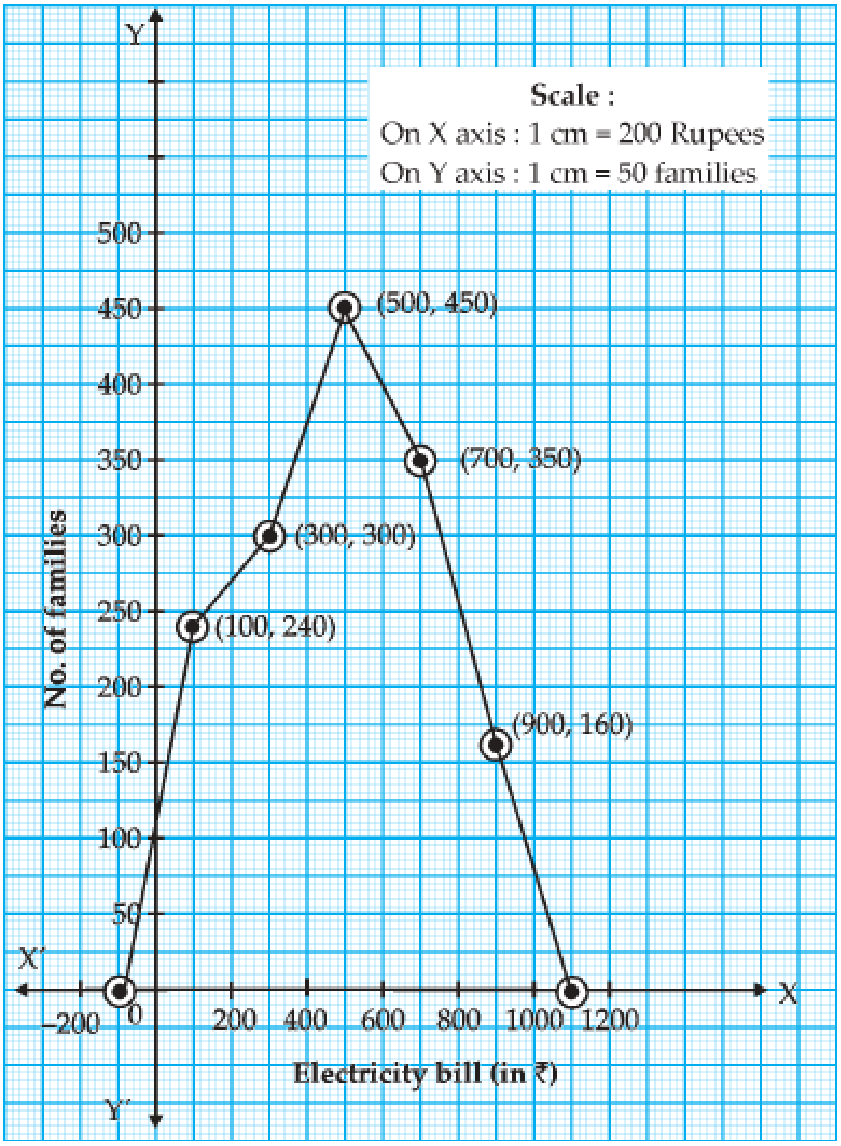
Ans. There are 15 students in the group 80 - 90.

(2) Show the following data by a frequency polygon. *(4 marks)*

Electricity bill (₹)	0 - 200	200 - 400	400 - 600	600 - 800	800 - 1000
No. of families	240	300	450	350	160

Solution:

Class	Class mark	Frequency	Coordinate of points
–200 - 0	–100	0	(–100, 0)
0 - 200	100	240	(100, 240)
200 - 400	300	300	(300, 300)
400 - 600	500	450	(500, 450)
600 - 800	700	350	(700, 350)
800 - 1000	900	160	(900, 160)
1000 - 1200	1100	0	(1100, 0)



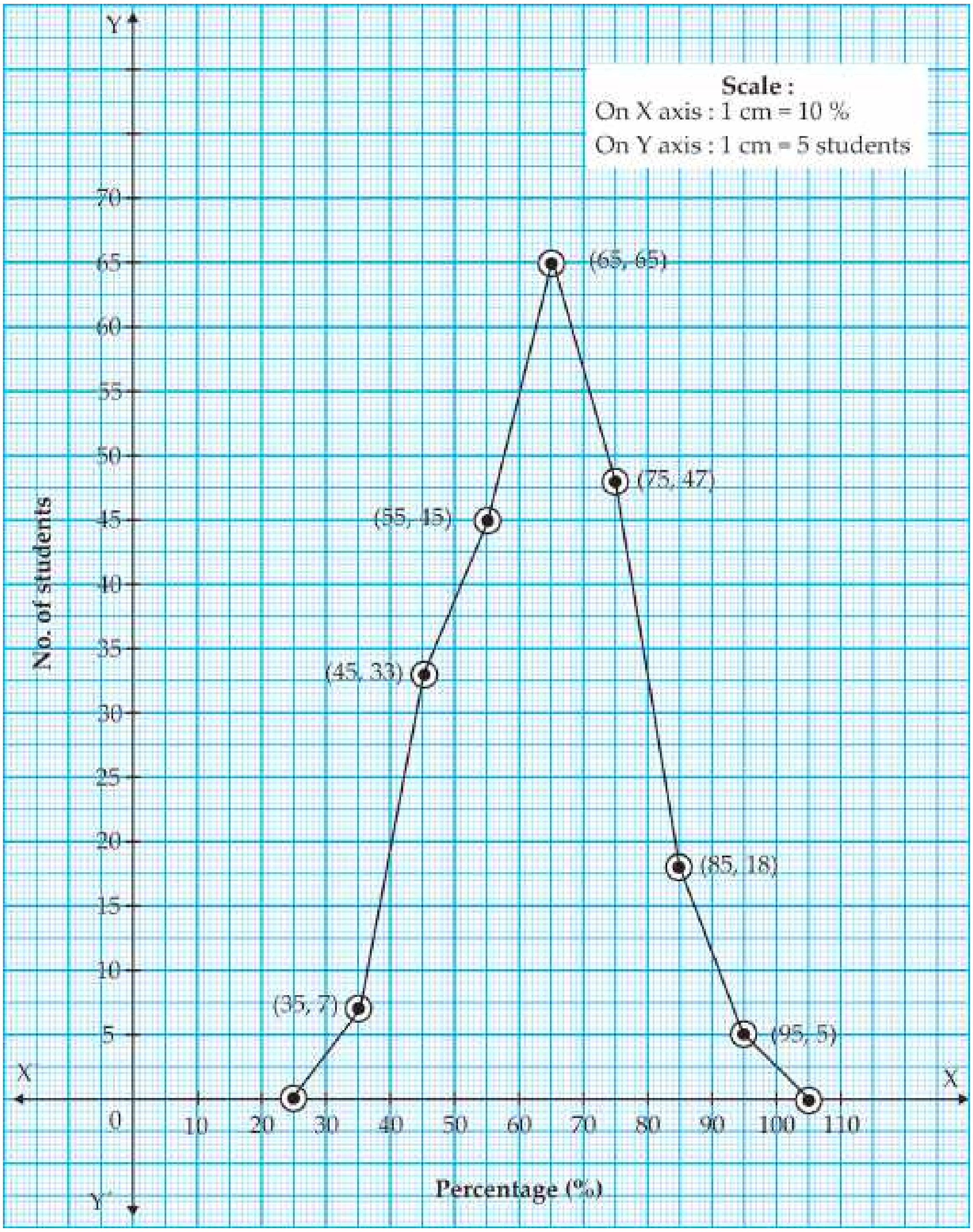
(3) **The following table shows the classification of percentages of marks of students and the number of students. Draw a frequency polygon from the table.**

(4 marks)

Result (Percentage)	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
No. of students	7	33	45	65	47	18	5

Solution:

Class	Class mark	Frequency	Coordinate of points
20 - 30	25	0	(25, 0)
30 - 40	35	7	(35, 7)
40 - 50	45	33	(45, 33)
50 - 60	55	45	(55, 45)
60 - 70	65	65	(65, 65)
70 - 80	75	47	(75, 47)
80 - 90	85	18	(85, 18)
90 - 100	95	5	(95, 5)
100 - 110	105	0	(105, 0)



Problem Set - 6 *(Textbook Page No. 167)*

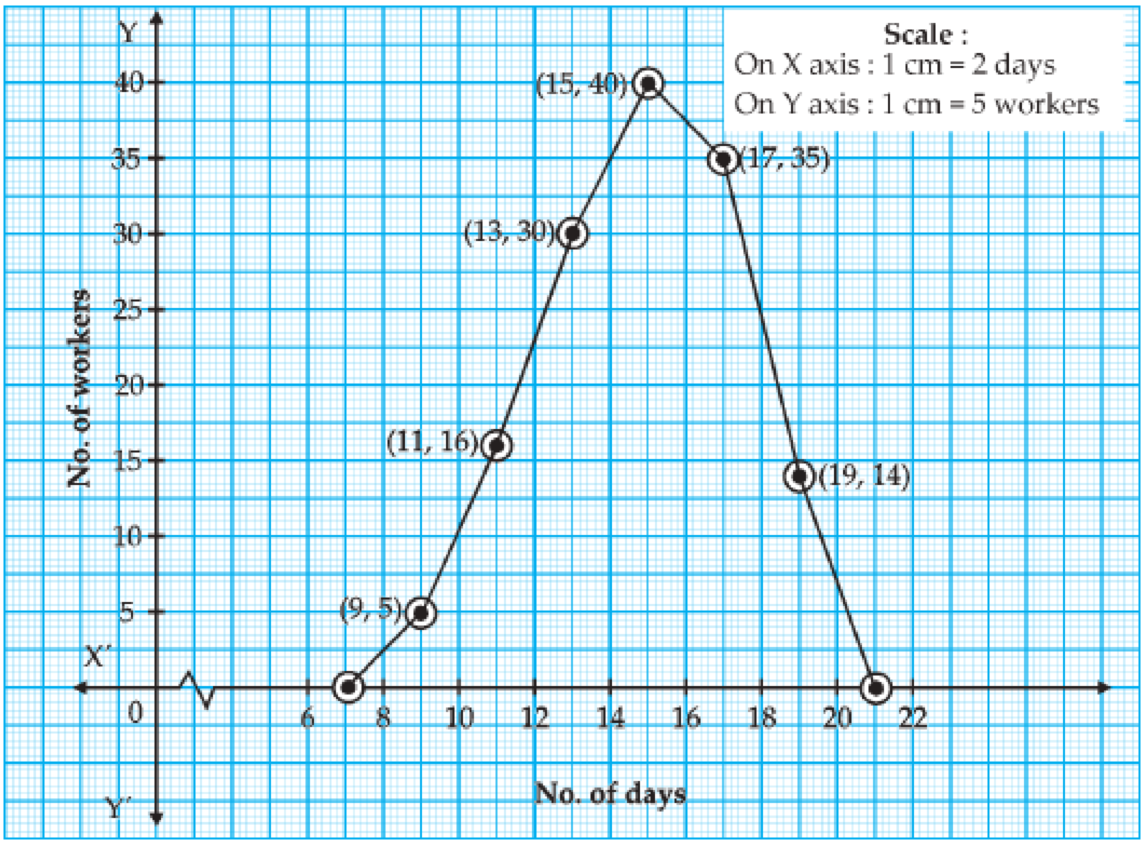
(10) **In a handloom factory different workers take different periods of time to weave a saree. The number of workers and their required periods are given below. Present the information by a frequency polygon.**

(4 marks)

No. of days	8 - 10	10 - 12	12 - 14	14 - 16	16 - 18	18 - 20
No. of workers	5	16	30	40	35	14

Solution:

Class	Class mark	Frequency	Coordinate of points
6 - 8	7	0	(7, 0)
8 - 10	9	5	(9, 5)
10 - 12	11	16	(11, 16)
12 - 14	13	30	(13, 30)
14 - 16	15	40	(15, 40)
16 - 18	17	35	(17, 35)
18 - 20	19	14	(19, 14)
20 - 22	21	0	(21, 0)

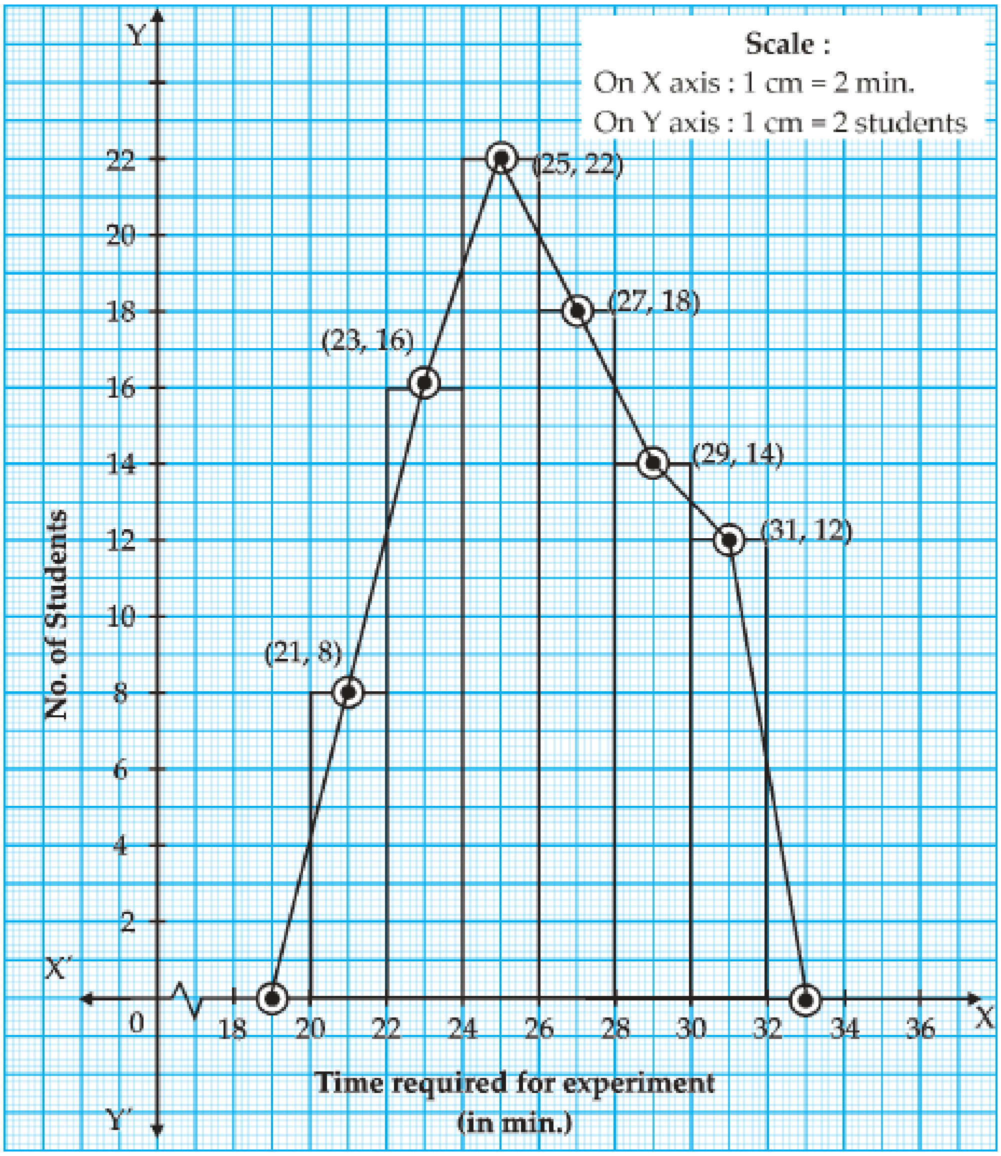


(11) **The time required for students to do a science experiment and the number of students is shown in the following grouped frequency distribution table. Show the information by a histogram and also by a frequency polygon.** *(4 marks)*

Time required for experiment (minutes)	20 - 22	22 - 24	24 - 26	26 - 28	28 - 30	30 - 32
No. of students	8	16	22	18	14	12

Solution:

Class	Class mark	Frequency	Coordinate of points
18 - 20	19	0	(19, 0)
20 - 22	21	8	(21, 8)
22 - 24	23	16	(23, 16)
24 - 26	25	22	(25, 22)
26 - 28	27	18	(27, 18)
28 - 30	29	14	(29, 14)
30 - 32	31	12	(31, 12)
32 - 34	33	0	(33, 0)



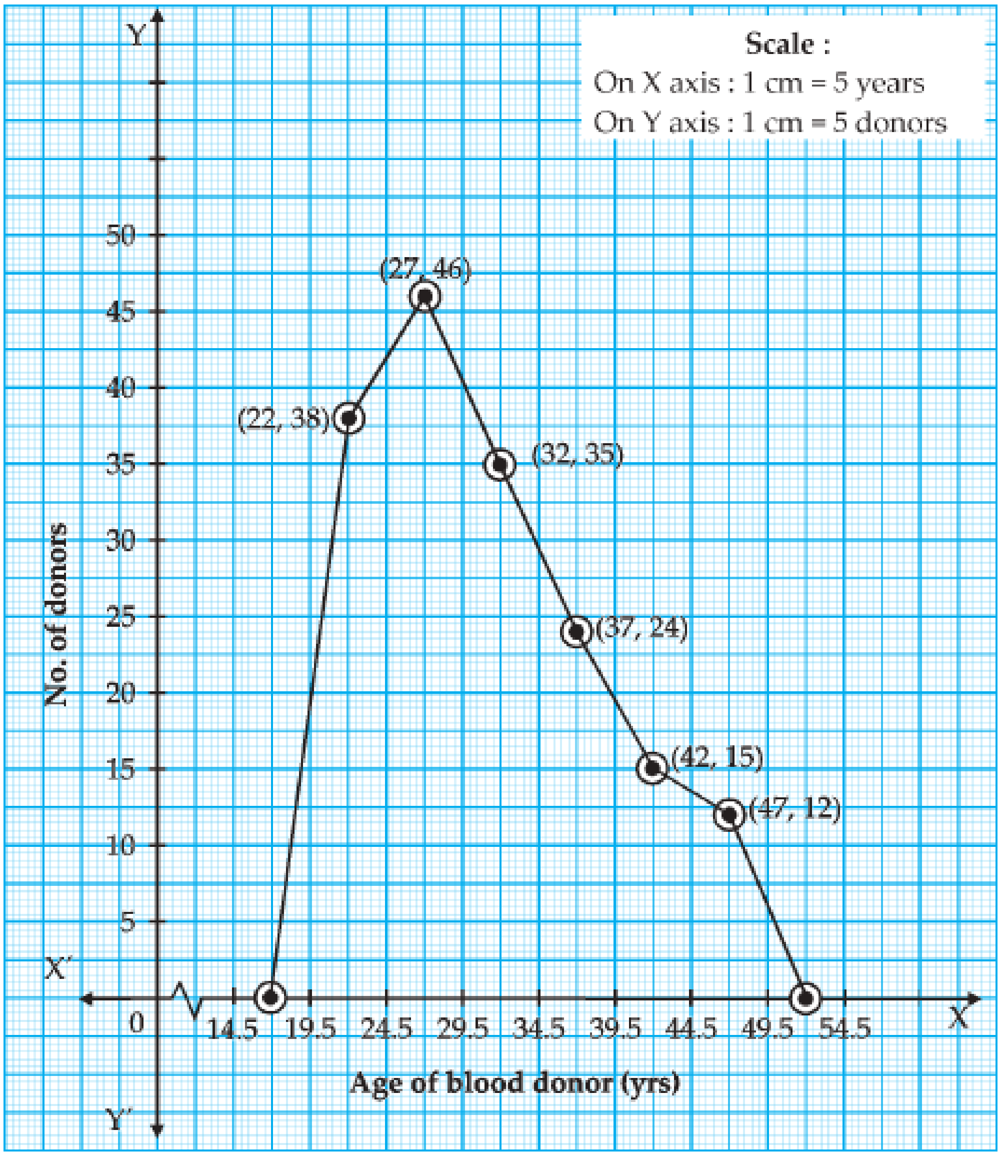
(12) **Draw a frequency polygon for the following grouped frequency distribution table.**

(4 marks)

Age of the donor (Yrs.)	20 - 24	25 - 29	30 - 34	35 - 39	40 - 44	45 - 49
No. of blood donors	38	46	35	24	15	12

Solution:

Class	Continuous class	Class mark	Frequency	Coordinate of points
15 - 19	14.5 - 19.5	17	0	(17, 0)
20 - 24	19.5 - 24.5	22	38	(22, 38)
25 - 29	24.5 - 29.5	27	46	(27, 46)
30 - 34	29.5 - 34.5	32	35	(32, 35)
35 - 39	34.5 - 39.5	37	24	(37, 24)
40 - 44	39.5 - 44.5	42	15	(42, 15)
45 - 49	44.5 - 49.5	47	12	(47, 12)
50 - 54	49.5 - 54.5	52	0	(52, 0)

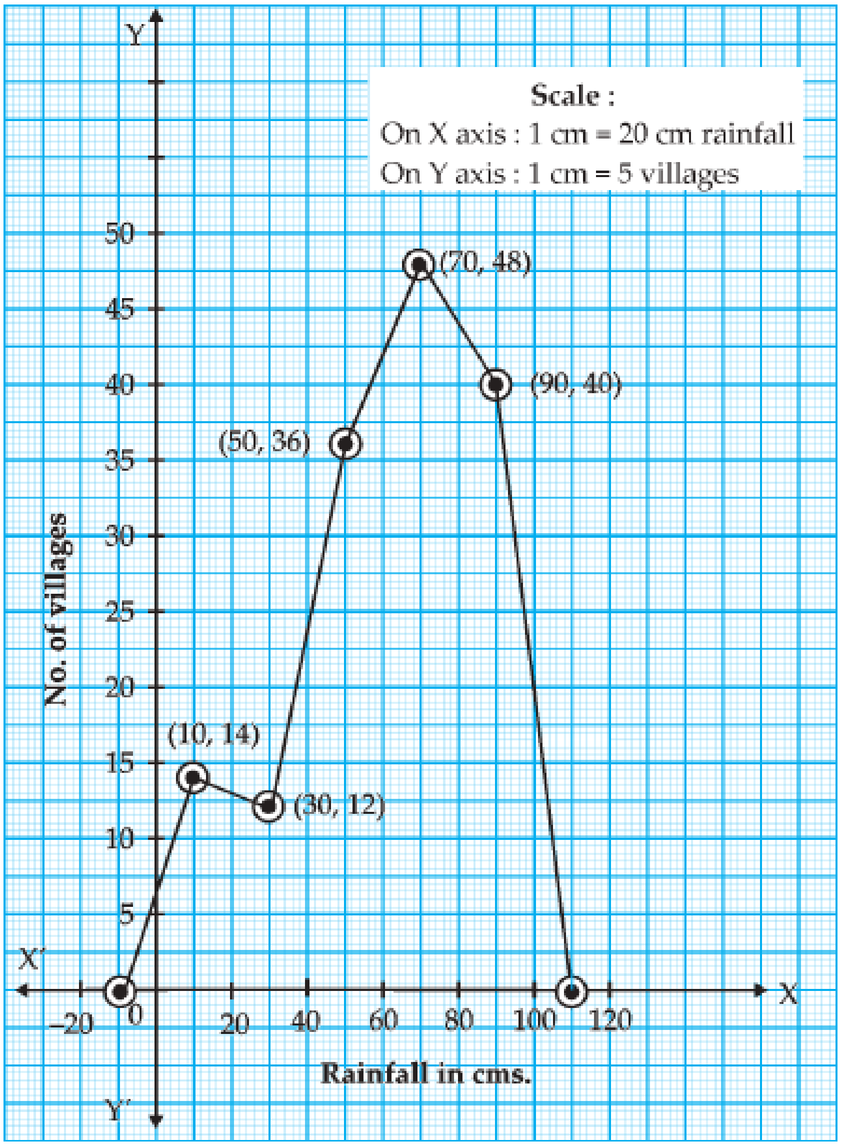


(13) **The following table shows the average rainfall in 150 towns. Show the information by a frequency polygon.** *(4 marks)*

Average rainfall (cm)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
No. of towns	14	12	36	48	40

Solution:

Class	Class mark	Frequency	Coordinate of points
–20 - 0	–10	0	(–10, 0)
0 - 20	10	14	(10, 14)
20 - 40	30	12	(30, 12)
40 - 60	50	36	(50, 36)
60 - 80	70	48	(70, 48)
80 - 100	90	40	(90, 40)
100 - 120	110	0	(110, 0)



Points to Remember:

(iii) Pie diagram:

The relative values of items are represented by a sector of circle. Since the sectors resembles the slices of pie, therefore it is called a **pie diagram**.

The main terms related to pie diagrams are sector and central angle.

Steps to draw a pie diagram:

Step 1: **Find** the total of all values of items or that of components corresponds to 360°. Hence, find the measure of central angle (θ) for each given information.

Step 2: Draw a circle of convinient radius (usually within 4 - 5 cm range preferred.)

Step 3: Divide the circle into number of sectors using the value of central angle (θ)

to represent the various items or components.

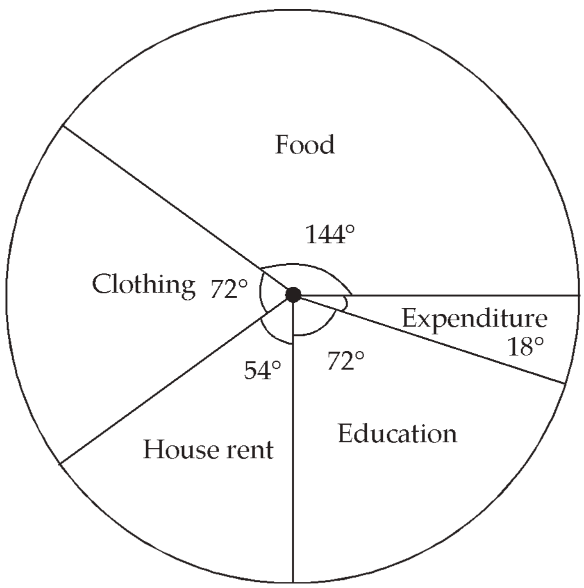
Central angle $(\theta) = \dfrac{\textit{value of the component}}{\textit{Total value}} \times 360°$

Activity :

The monthly expenditure of a family of different items is shown in the following table. Calculate the related central angles and draw a pei chart.

Different Items	% of Expenditure	Measure of Central angle
Food	40	$\frac{40}{100} \times 360 = \boxed{144°}$
Clothing	20	$\frac{\boxed{20}}{\boxed{100}} \times \boxed{360} = \boxed{72°}$
House rent	15	$\frac{\boxed{15}}{\boxed{100}} \times \boxed{360} = \boxed{54°}$
Education	20	$\frac{\boxed{20}}{\boxed{100}} \times \boxed{360} = \boxed{72°}$
Expenditure	05	$\frac{\boxed{05}}{\boxed{100}} \times \boxed{360} = \boxed{72°}$
Total	**100**	**360°**

Pie Diagram:



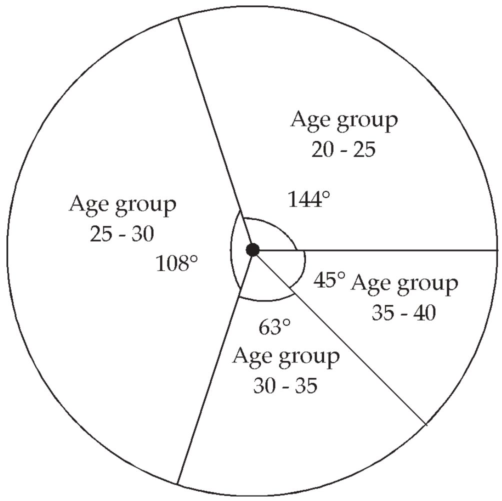
Practice Set - 6.6 *(Textbook Page No. 163)*

(1) **The age group and number of persons, who donated blood in a blood donation camp is given below. Draw a pie diagram from it.** *(4 marks)*

Age group (Yrs.)	20 - 25	25 - 30	30 - 35	35 - 40
No. of persons	80	60	35	25

Solution:

Age group (Years)	No. of persons	Measure of Central angle
20 - 25	80	$\frac{80}{200} \times 360 = 144°$
25 - 30	60	$\frac{60}{200} \times 360 = 108°$
30 - 35	35	$\frac{35}{200} \times 360 = 63°$
35 - 40	25	$\frac{25}{200} \times 360 = 45°$
Total	**200**	**360°**

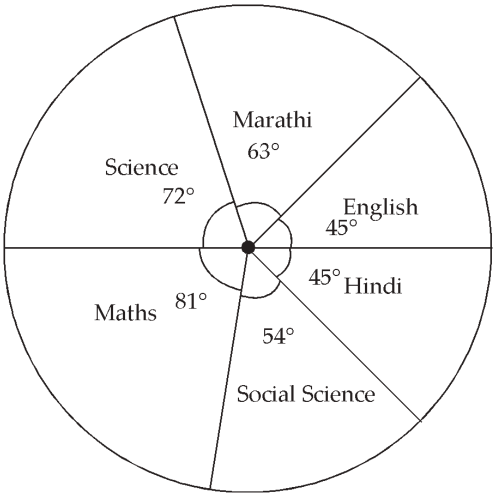


(2) **The marks obtained by a student in different subjects are shown. Draw a pie diagram showing the information.** *(4 marks)*

Subject	English	Marathi	Science	Mathematics	Social science	Hindi
Marks	50	70	80	90	60	50

Solutions:

Subjects	Marks	Measure of Central angle
English	50	$\frac{50}{400} \times 360 = 45°$
Marathi	70	$\frac{70}{400} \times 360 = 63°$
Science	80	$\frac{80}{400} \times 360 = 72°$
Maths	90	$\frac{90}{400} \times 360 = 81°$
Social Science	60	$\frac{60}{400} \times 360 = 54°$
Hindi	50	$\frac{50}{400} \times 360 = 45°$
Total	**400**	**360°**

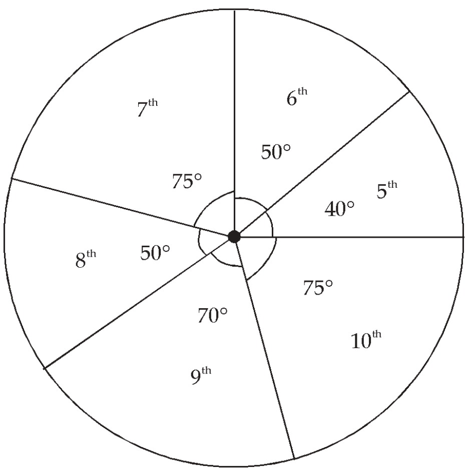


(3) **In a tree plantation programme, the number of trees planted by students of different classes is given in the following table. Draw a pie diagram showing the information.** *(4 marks)*

Standard	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}
No. of trees	40	50	75	50	70	75

Solutions:

Standard Class	**No. of trees**	**Measure of Central angle**
5^{th}	40	$\frac{40}{360} \times 360 = 40°$
6^{th}	50	$\frac{50}{360} \times 360 = 50°$
7^{th}	75	$\frac{75}{360} \times 360 = 75°$
8^{th}	50	$\frac{50}{360} \times 360 = 50°$
9^{th}	70	$\frac{70}{360} \times 360 = 70°$
10^{th}	75	$\frac{75}{360} \times 360 = 75°$
Total	**360**	**360°**

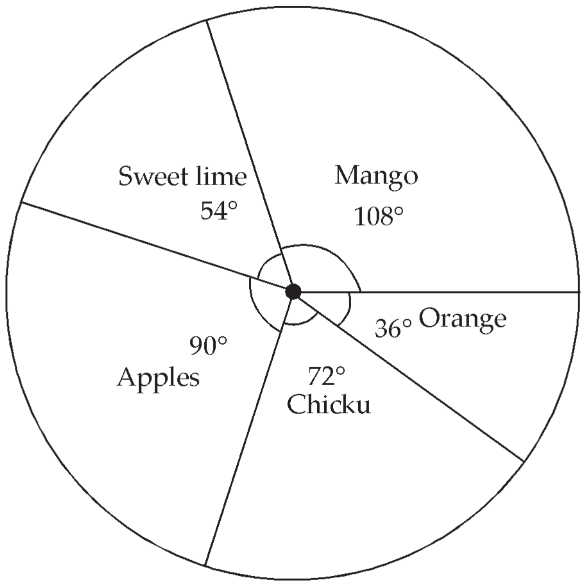


(4) **The following table shows the percentages of demands for different fruits registered with a fruit vendor. Show the information by a pie diagram.** *(4 marks)*

Fruits	Mango	Sweet lime	Apples	Cheeku	Oranges
% of demand	30	15	25	20	10

Solution:

Fruits	**Demand (%)**	**Measure of Central angle**
Mango	30	$\frac{30}{100} \times 360 = 108°$
Sweet lime	15	$\frac{15}{100} \times 360 = 54°$
Apples	25	$\frac{25}{100} \times 360 = 90°$
Chicku	20	$\frac{20}{100} \times 360 = 72°$
Oranges	10	$\frac{10}{100} \times 360 = 36°$
Total	**100**	**360°**



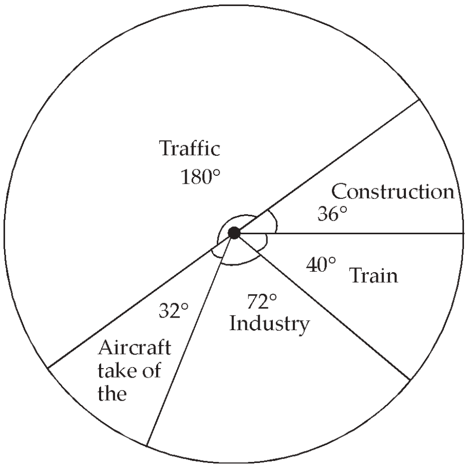
Problem Set - 6 *(Textbook Page No. 167)*

(15) **The following table shows causes of noise pollution. Show it by a pie diagram.** *(4 marks)*

Construction	Traffic	Aircraft take offs	Industry	Trains
10%	50%	9%	20%	11%

Solution:

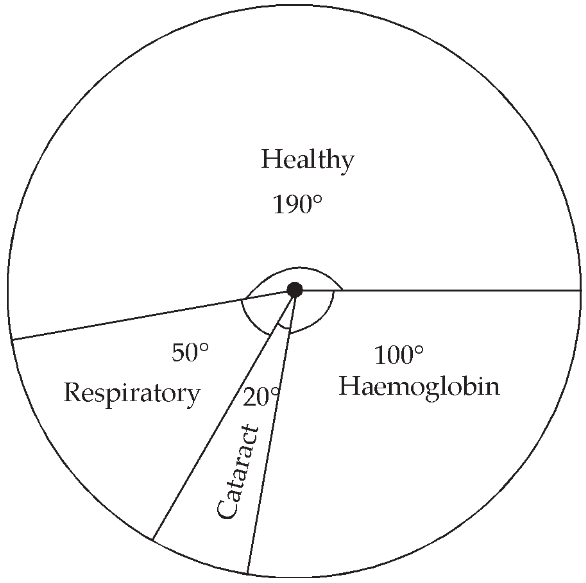
Components	**%**	**Measure of Central angle**
Construction	10	$\frac{10}{100} \times 360 = 36°$
Traffic	50	$\frac{50}{100} \times 360 = 180°$
Aircraft take offs	9	$\frac{9}{100} \times 360 = 32.4° \approx 32°$
Industry	20	$\frac{20}{100} \times 360 = 72°$
Train	11	$\frac{11}{100} \times 360 = 39.6° \approx 40°$



(17) **Medical check up of 180 women was conducted in a health centre in a village. 50 of them were short of haemoglobin, 10 suffered from cataract and 25 had respiratory disorders. The remaining women were healthy. Show the information by a pie diagram.** *(4 marks)*

Solution:

Disease	**No. of women**	**Measure of Central angle**
Haemoglobin	50	$\frac{50}{180} \times 360 = 100°$
Cataract	10	$\frac{10}{180} \times 360 = 20°$
Respiratory	25	$\frac{25}{180} \times 360 = 50°$
Healthy	180 – 95 = 95	$\frac{95}{180} \times 360 = 190°$
Total	**180**	**360°**

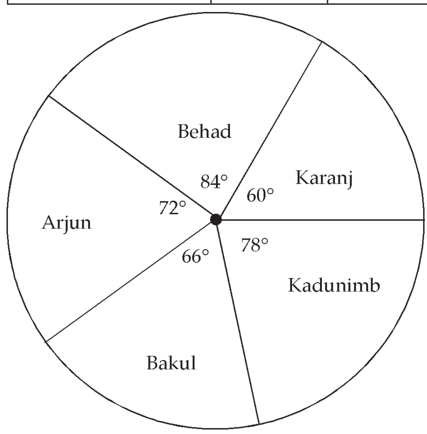


(18) **On an environment day, students in a school planted 120 trees under plantation project. The information regarding the project is shown in the following table. Show it by a pie diagram.** *(4 marks)*

Tree Name	Karanj	Behada	Arjun	Bakul	Kadunimb
No. of trees	20	28	24	22	26

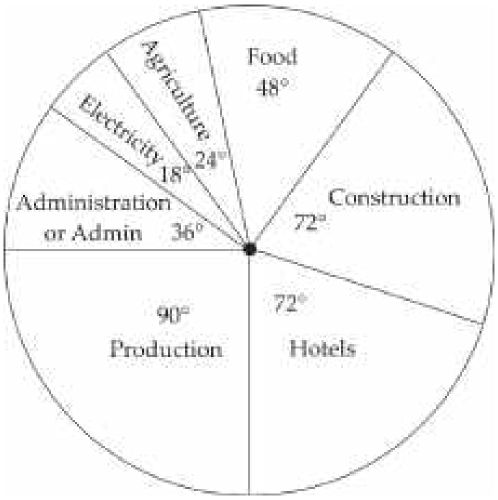
Solution:

Name of trees	**No. of trees**	**Measure of Central angle**
Karanj	20	$\frac{20}{120} \times 360 = 60°$
Behad	28	$\frac{28}{120} \times 360 = 84°$
Arjun	24	$\frac{24}{120} \times 360 = 72°$
Bakul	22	$\frac{22}{120} \times 360 = 66°$
Kadunimb	26	$\frac{26}{120} \times 360 = 78°$
Total	**120**	**360°**



Practice Set - 6.6 *(Textbook Page No. 164)*

(5) The pie diagram in figure 6.13 shows the proportions of different workers in a town. Answer the following questions with its help. *(3 marks)*



(i) If the total workers is 10,000; how many of them are in the field of construction?

(ii) How many workers are working in the administration?

(iii) What is the percentage of workers in production?

Solution:

(i) Total no. of workers = 10,000

Total 10,000 workers corresponds to central angle = 360°

The measure of central angle for construction = 72°

∴ No. of professionals in construction industry

$$= \frac{\textit{Central angle for construction}}{360} \times \text{Total}$$

$$= \frac{72}{360} \times 10{,}000 = 2{,}000$$

∴ **No. of professionals in construction = 2,000**

(ii) The measure of central angle for administration = 36°

∴ No. of people in administration

$$= \frac{\textit{Central angle for administration}}{360} \times \text{Total}$$

$$= \frac{36}{360} \times 10{,}000 = 1{,}000$$

∴ **No. of professionals in administration = 1,000**

(iii) Central angle for production = 90°

No. of professionals in production industry

$$= \frac{\textit{Central angle for production}}{360} \times \text{Total}$$

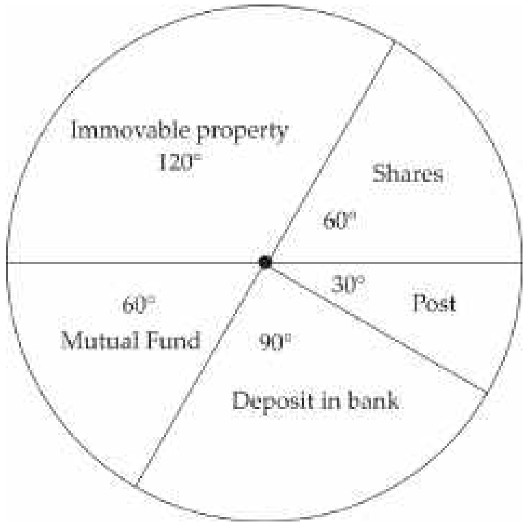
$$= \frac{90}{360} \times 10{,}000 = 2500$$

∴ % of professionals in production industry

$$= \frac{2500}{10000} \times 100 = 25\%$$

∴ **25% of professionals are in production industry.**

(6) The annual investments of a family are shown in the adjacent pie diagram. Answer the following questions based on it. *(4 marks)*



(i) If the investment in shares is Rs. 2000/, find the total investment.

(ii) How much amount is deposited in bank?

(iii) How much more money is invested in immovable property than in mutual fund?

(iv) How much amount is invested in post?

Solution:

(i) Let x be the total amount invested.

Total amount 'x' corresponds to central angle = '360'

Central angle for shares

$$= \frac{\textit{Amount invested in shares}}{\textit{Total amount invested}}$$

$$60 = \frac{2000}{\textit{Total amount invested}}$$

∴ Total amount invested $= \dfrac{2000 \times 360}{60}$

$= 12{,}000$

Total amount invested = ₹ 12,000

(ii) Central angle for investment in Bank = 90°

Amountdeposited in Bank

$$= \frac{\textit{Central angle for bank}}{360} \times \text{Total}$$

$$= \frac{90}{360} \times 12,000 = 3,000$$

Amount deposited in Bank = ₹ 3000

(iii) Central angle for Mutual fund = 60°

Amount invested in Mutual fund

$$= \frac{\textit{Central angle for mutual fund}}{360} \times \text{Total}$$

$$= \frac{60}{360} \times 12,000 = 2,000$$

Amount invested in immovable property

$$= \frac{\textit{Central angle for property}}{360}$$

$$= \frac{120}{360} \times 12,000 = 4,000$$

∴ **Amount invested in property is ₹ 2,000 more than in mutual fund.**

(iv) Central angle for Post Office = 30°

Amount invested in Post office

$$= \frac{\textit{Central angle for post office}}{360} \times \text{Total}$$

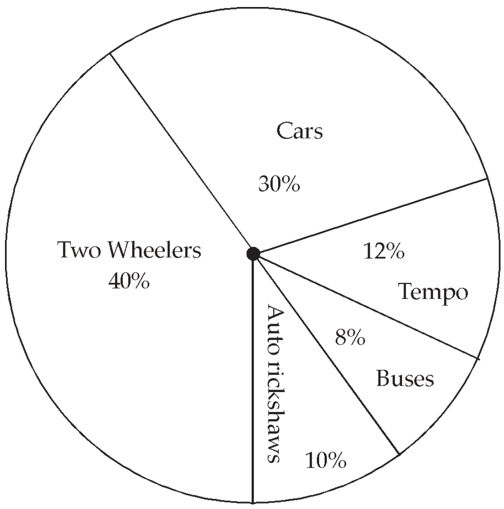
$$= \frac{30}{360} \times 12,000 = 1,000$$

∴ **Amount invested in post is ₹ 1,000.**

Problem Set - 6 *(Textbook Page No. 167)*

(14) Observe the below pie diagram. It shows the percentages of number of vehicles passing a signal in a town between 8 am and 10 am

(4 marks)



(i) Find the central angle for each type of vehicle.

(ii) If the number of two-wheelers is 1200, find the number of all vehicles.

Solution:

(i)

Mode of transport	%	Measure of Central angle
Two wheelers	40%	$\frac{40}{100} \times 360 = 144°$
Rickshaws	10%	$\frac{10}{100} \times 360 = 36°$
Buses	8%	$\frac{8}{100} \times 360 = 28.8 \approx 29°$
Tempos	12%	$\frac{12}{100} \times 360 = 43.2 \approx 43°$
Cars	30%	$\frac{30}{100} \times 360 = 108°$
Total	**100%**	**360°**

(ii) Total no. of two wheelers = 1200

Central angle of two wheeler

$$= \frac{\textit{No. of two wheelers}}{\textit{Total no. of vehicles}} \times 360$$

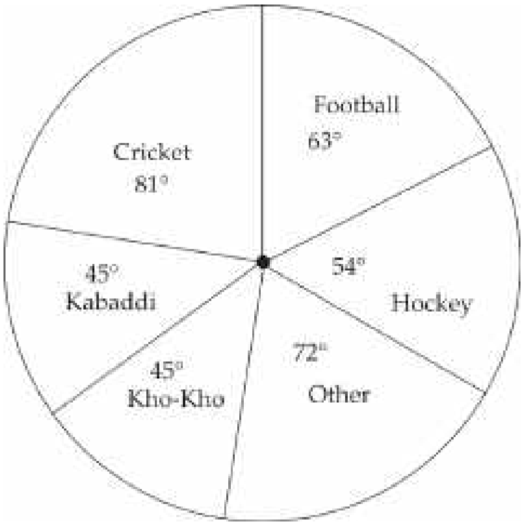
$$144 = \frac{1200}{\textit{Total no. of vehicles}} \times 360$$

$$\text{Total no. of vehicles} = \frac{1200 \times 360}{144}$$

$$= 3000$$

∴ **Total number of vehicles is 3000.**

(16) A survey of students was made to know which game they like. The data obtained in the survey is presented in the below pie diagram. If the total number of students are 1000, *(3 marks)*



(i) How many students like cricket?

(ii) How many students like football?

(iii) How many students prefer other games?

Solution:

(i) Total no. of students = 1000

Central angle for cricket = 81°

∴ No. of students interested in cricket

$$= \frac{\text{Central angle}}{360} \times \text{Total no. of students}$$

$$= \frac{81}{360} \times 1000 = 225$$

∴ **No. of students interested in cricket = 225**

(ii) Central angle for Football = 63°

∴ No. of students interested in football

$$= \frac{\text{Central angle}}{360} \times \text{Total no. of students}$$

$$= \frac{63}{360} \times 1000 = 175$$

∴ **No. of students interested in football = 175**

(iii) Central angle for other sports = 72°

∴ No. of students interested in other sports

$$= \frac{\text{Central angle}}{360} \times \text{Total no. of students}$$

$$= \frac{72}{360} \times 1000 = 200$$

∴ **No. of students interested in other sports = 200.**

Problem Set - 6 *(Textbook Page No. 164)*

MCQ's

Find the correct answer from the alternatives given. *(1 mark each)*

(1) The persons of O- blood group are 40%. The classification of persons based on blood groups is to be shown by a pie diagram. What should be the measures of angle for the persons of O- blood group?

(A) 114° (B) 140° (C) 104° (D) 144°

(2) Different expenditures incurred on the construction of a building were shown by a pie diagram. The expenditure Rs. 45,000 on cement was shown by a sector of central angle of 75°. What was the total expenditure of the construction ?

(A) 2,16,000 (B) 3,60,000

(C) 4,50,000 (D) 7,50,000

(3) Cumulative frequencies in a grouped frequency table are useful to find . . .

(A) Mean (B) Median

(C) Mode (D) All of these

(4) The formula to find mean from a grouped frequency table is $\bar{x} = A + g\frac{\sum f_i u_i}{\sum f_i}$ *h In the formula* $u_i = \ldots\ldots$

(A) $\frac{x_i + A}{g}$ (B) $(x_i - A)$

(C) $\frac{x_i - A}{g}$ (D) $\frac{A - x_i}{g}$

(5)

Distance covered per litre (km)	12-14	14-16	16-18	18-20
No. of cars	11	12	20	7

The median of the distances covered per litre shown in the above data is in the group . . . . . .

(A) 12-14 (B) 14-16 (C) 16-18 (D) 18-20

(6)

No. of trees planted by each student	1-3	4-6	7-9	10-12
No. of students	7	8	6	4

The above data is to be shown by a frequency polygon. The coordinates of the points to show number of students in the class 4-6 are . . . .

(A) (4, 8) (B) (3, 5) (C) (5, 8) (D) (8, 4)

Additional MCQ's

(7) The formula to find mean by direct method is

(A) $A + \bar{d}$ (B) $\frac{\sum f_i d_i}{\sum f_i}$ (C) $\frac{\sum f_i d_i}{\sum d_i}$ (D) $\frac{\sum f_i x_i}{\sum f_i}$

(8) Class width of any class interval is ....

(A) Lower limit + upper limit

(B) Lower limit – upper limit

(C) Upper limit – lower limit

(D) None of the above.

(9) The formula for median is

(A) $\frac{L}{2} + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$ (B) $L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$

(C) $L + \left(\frac{N}{2} - c.f.\right)\frac{f}{h}$ (D) $L + \left(\frac{N}{2} - f\right)\frac{c.f}{h}$

(10) The formula to find the mean deviation ($\bar{d}$) in Assumed mean method is

(A) $\bar{d}\ \frac{\sum f_i d_i}{\sum d_i}$ (B) $\sum f_i d_i$

(C) $\frac{\sum f_i d_i}{\sum f_i}$ (D) None of the above.

(11) In step deviation method, u_i is given by

(A) $A + \bar{d}$ (B) $x_i - A$ (C) $\frac{x_i + A}{g}$ (D) $\frac{x_i - A}{g}$

(12) The median class for the following frequency distribution is

Class interval	0-10	10-20	20-30	30-40
Frequency	5	7	13	18

(A) 0 - 10 (B) 10 - 20 (C) 20-30 (D) 30-40

(13) The value of lower limit (L) in the above example to calculate median is

(A) 0 (B) 10 (C) 20 (D) 30

(14) If $\Sigma f_i d_i = 1885$ and $\Sigma f_i = 100$, then the value of $\bar{d}$ is

(A) 1.885 (B) 18.85 (C) 188.5 (D) 1885

(15) The formula of mode is

(A) $L+\left[\dfrac{f_1-f_0}{2f_1-f_0-f_2}\right]\times h$

(B) $L-\left[\dfrac{f_1-f_0}{2f_1-f_0-f_2}\right]\times h$

(C) $L+\left[\dfrac{f_1-f_0}{2f_1-f_0+f_2}\right]\times h$

(D) $L-\left[\dfrac{f_1-f_0}{2f_1-f_0-f_2}\right]\times \dfrac{1}{h}$

(16) The formula to find mean by step deviation method is

(A) $\dfrac{\sum f_i u_i}{\sum f_i}$ (B) $A+\bar{d}$

(C) $A+g\bar{u}$ (D) None of these

ANSWERS

(1) (D) 144° (2) (A) 2,16,000 (3) (B) Median

(4) (C) $\dfrac{x_i-A}{h}$ (5) (C) 16-18 (6) (C) (5, 8)

(7) (D) $\dfrac{\sum f_i x_i}{\sum f_i}$ (8) (C) Upper limit – lower limit

(9) (B) $L+\left(\dfrac{N}{2}-c.f.\right)\dfrac{h}{f}$ (10) (C) $\dfrac{\sum f_i d_i}{\sum f_i}$ (11) (D) $\dfrac{x_i-A}{g}$

(12) (C) 20-30 (13) (C) 20 (14) (B) 18.85

(15) (A) $L+\left[\dfrac{f_1-f_0}{2f_1-f_0-f_2}\right]\times h$ (16) (C) $A+g\bar{u}$

PROBLEMS FOR PRACTICE

Based on Practice Set 6.1

(1) A study related to time (in months) taken to settle a dispute in a lower court resulted the following data.

Time (in months)	0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 -12
No. of disputes	15	90	120	75	70	50

Find the mean time taken to settle a dispute in a lower court. *(3 marks)*

(2) Find the mean marks of the student from the following cumulative frequency distribution. *(3 marks)*

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
No. of students	5	9	17	29	45

Marks	Below 60	Below 70	Below 80	Below 90
No. of students	60	70	78	83

(3) The following table gives the distribution of total household expenditure (in rupee) of workers in a city.

Expenditure (in rupees)	100-150	150-200	200-250	250-300
No. of workers	24	40	33	28

Expenditure (in rupees)	300-350	350-400	400-450
No. of workers	30	22	16

Find the mean expenses per household by step deviation method. *(3 marks)*

(4) Rainfall (in mm) recorded in 50 cities in a partial region on a particular day is given below.

Rainfall in mm	36-39	40-43	44-47	48-51	52-55	56-59	60-63
No. of cities	6	7	10	7	7	9	4

Find the mean rainfall on that day in a city by Assumed Mean method. *(3 marks)*

(5) The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Complete the missing frequencies f_1 and f_2. *(3 marks)*

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

Based on Practice Set 6.2

(6) The following table shows the ages of 300 patients getting medical treatment in a hospital on a particular day. *(3 marks)*

Age in years	10-20	20-30	30-40	40-50	50-60	60-70
No. of patients	60	42	55	70	53	20

Find the median age of patients. *(4 marks)*

(7) Below is given the frequency distribution of no. of packages received at a post office per day.

No. of packages	10-20	20-30	30-40	40-50	50-60	60-70
No. of days	2	8	16	24	30	20

Find the median no. of packages received by the post-office per day. *(4 marks)*

(8) Following table shows frequency distribution of no. of rooms occupied in a hotel per day.

No. of rooms occupied	0-10	10-20	20-30	30-40	40-50	50-60
No. of days	5	27	17	11	9	1

Find the median no. of rooms occupied in a hotel per day. *(4 marks)*

(9) Time spent in queue by 100 passengers is classified and given below:

Time in mins	20-29	30-39	40-49	50-59	60-69	Total
No. of passengers	8	20	32	28	12	100

Find the median no. of passengers. *(4 marks)*

(10) Following is the cumulative frequency distribution of employees in a certain office according to age.

Age	less than 20	less than 25	less than 30	less than 35	less than 40
Cumulative frequency	0	2	8	22	51

Age	less than 45	less than 50	less than 55	less than 60
Cumula-tive fre-quency	88	105	108	110

Find the median age of employee.

Based on Practice Set 6.3

(11) Automatic filling machine was listed for its performance. A sample 100 filled packets generalised the data, which is classified as follows:

Weight in gms.	485-490	490-495	495-500	500-505	505-510	510-515
No. of packets	12	18	20	22	24	4

Find the modal weight of the packets. *(4 marks)*

(12) The following table gives profit earned by companies in lacs. *(4 marks)*

Profit	5-10	10-15	15-20	20-25	25-30
No. of companies	4	28	49	17	2

Find the modal profit earned by the companies.

(13) The following table shows the heights of students.

Height of students (cm)	140-144	145-149	150-154	155-159
No. of students	2	12	10	4

Find the modal height of the students. *(4 marks)*

(14) Below is given frequency distribution of 'Dai;y wages (in ₹) of 130 workers.

Daily wages (₹)	80-84	85-89	90-94	95-99	100-104	105-109
No. of workers	10	20	25	40	30	5

Find the modal wage of a worker. *(4 marks)*

(15) The following is the frequency distribution of blood pressure measured for 100 patients.

Blood pressure in suitable units.	110-115	115-120	120-125	125-130	130-135
No. of patients	2	35	52	8	3

Find the modal Blood pressure of the patient. *(4 marks)*

Based on Practice Set 6.4 and 6.5

(16) Draw the frequency polygon for the following data. *(4 marks)*

Body Mass Index.	18-19	19-20	20-21	21-22	22-23	23-24
No. of patients	5	12	18	25	16	4

(17) Below is given frequency distribution of marks (out of 100) obtained by students.

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
No. of students	3	5	7	10	12	15	12	6	2	8

Draw the histogram and frequency polygon on the same graph papers. *(4 marks)*

(18) Draw the frequency polygon using mid-ponts for the following data on time required to do a certain job. *(4 marks)*

Time required (in hours)	**50-70**	**70-90**	**90-110**	**110-130**	**130-150**	**150-170**	**170-190**
No. of workers	2	8	30	25	14	19	2

Based on Practice Set 6.6

(19) Draw a pie diagram to represent the world population given in the following table after determining the value of a. *(4 marks)*

Country	**India**	**China**	**Rus-sia**	**USA**	**Oth-ers**	**To-tal**
Percentage poulation	15	20	a	a	25	100

(20) The following table shows the expenditure incurred by a publisher in publishing a book. *(4 marks)*

Item	**Paper**	**Print-ing**	**Bind-ing**	**Adver-tising**	**Miscel-laneous**
Expendi-ture in %	35%	20%	10%	5%	30%

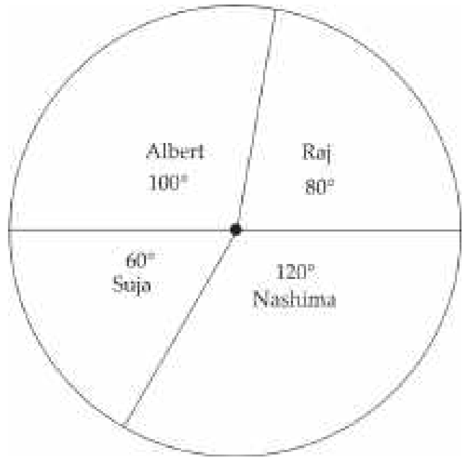
Draw a pie chart representing the above data.

(21) Following is the breakup of expenditure of a family on different times of consumption. *(4 marks)*

Item	**Food**	**Clothing**	**Rent**	**Education**	**Fuel**	**Medicine**	**Miscellaneous**
Expenditure in ₹	1600	200	600	150	100	80	270

Draw a pie diagram of the above data.

(22) The following pie diagram represents the number of valid votes obtained by four students who contested for school captain. The total of valid votes polled was 720. Answer the following questions. *(4 marks)*

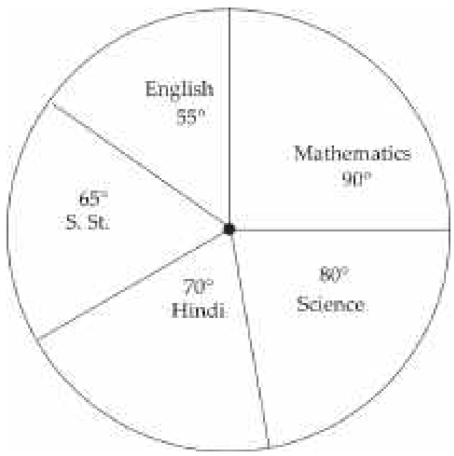


(i) Who has won the election?

(ii) What is the minimum no. of votes? Who got it?

(iii) By how many votes did the winner defeat the nearest contestant?

(23) The following pie chart gives the marks scored in an examination by a student in various subjects. If the total marks obtained by the student were 360, answer the following questions. *(4 marks)*



(i) Find the marks obtained in each subject.

(ii) How many more marks he got in Mathematics than in Science?

(iii) Which subject he get the least marks?

ANSWERS

(1) 6.07 months **(2)** 48.41 **(3)** 258.7 **(4)** 49.1 mm

(5) $f_1 = 8, f_2 = 12$ **(6)** 38.73 years **(7)** 50 packets

(8) 21.76 **(9)** 47 **(10)** 40.5 years **(11)** 505.45 gm.

(12) 16.98 lacs **(13)** 148.67 cm **(14)** ₹97.5 **(15)** 121.39

(22) **(i)** Nashima **(ii)** 120, Suja **(iii)** 40

(23) **(i)** Eng. - 55, S. St. - 65, Hindi - 70, Science - 80 Maths - 90

(ii) He got 10 more marks in Mathematics than Science.

(iii) He got least marks in English.

ASSIGNMENT – 6

Time : 1 Hr. **Marks : 20**

Q.1. (A) Choose the correct alternative among the following: **(3)**

(1) The class mark of the class interval 120-130 is

(A) 120 (B) 130 (C) 122.5 (D) 125

(2) The height of the class interval 133-139 is

(A) 3 (B) 5 (C) 6 (D) 4

(3) The persons of O- blood group are 40%. The classification of persons based on blood groups is to be shown by a pie diagram. What should be the measures of angle for the persons of O- blood group?

(A) 114° (B) 140° (C) 104° (D) 144°

(B) Answer the following : **(2)**

Class Interval	Frequency	*c.f.*
0 - 20	4	4
20 - 40	20	$\boxed{}$
40 - 60	$\boxed{}$	54
60 - 80	40	94
80 - 100	6	100
	N = $\boxed{}$	

Q.2. Answer the following: (Any 2) **(4)**

(1) For a certain frequency distribution median class is 129.5 to 134.5, f is 30 and c.f. is 24 and N = 100, find the median

(2) Find the measure of central angle of Afternoon and night from the following informaton.

Part of a day	Morning	Afternoon	Evening	Night
Percentage of electricity used	20	25	15	10

(3) Find the mean of the following data

12, 18, 15, 17, 11, 10

Q.3. Answer the following: (Any 1) **(3)**

(1) Draw a pie chart using the following information

Subject	English	Marathi	Science	Maths	Social Studies	Hndi
Marks	50	70	80	90	60	50

(2) Following is the frequency distribution of students who participated in a sports event. Find the mode of students who participated in different sports.

Age group (in years)	6 - 8	8 - 10	10 - 12	12 - 14	14 - 16
No. of students	43	58	70	42	27

Q.4. Answer the following: (any 2) **(8)**

(1) The following frequency distribution shows the no. of persons who visited the museum on a particular day. Find the median age of the visiters of that day.

Age (in years)	No of person
less than 10	3
less than 20	10
less than 30	22
less than 40	40
less than 50	54
less than 60	71

(2) Draw a histogram and frequency polygon of the following data.

Area in hectare	12 - 20	21 - 30	31 - 40	41 - 52	51 - 60	61 - 70	71 - 80
No of farmers	58	103	208	392	112	34	12

(3) The following frequency distribution table gives the ages of 200 patients treated in a hospital in a week. Find the mode of ages of the patients.

Age (years)	Less than 5	5 - 9	10 - 14	15 - 19	20 - 24	25 - 29
No. of patients	38	32	50	36	24	20

❖ ❖ ❖

Challenging Questions

1. Linear Equations in Two Variables

(1) The weight of a bucket is 15 kg, when it is filled with water upto $\frac{3}{5}$ of its capacity. And the weight is 19 kg, if it is filled with water upto $\frac{4}{5}$ of its capacity. Find the weight of bucket, if it is completely filled with water. *(4 marks)*

Solution:

Let the weight of empty bucket be x kg and the weight of water the bucket can hold to its full capacity be y kg.

According to the first condition,

$x + \frac{3}{5}y = 15$

$\therefore \quad 5x + 3y = 75 \quad$ ...(i)

According to the second condition,

$x + \frac{4}{5}y = 19$

$\therefore \quad 5x + 4y = 95 \quad$ ...(ii)

Subtracting (ii) from (i),

$$\begin{array}{rcrcr} 5x & + & 3y & = & 75 \\ 5x & + & 4y & = & 95 \\ (-) & & (-) & & (-) \\ \hline & & -y & = & -20 \end{array}$$

$\therefore \quad$ **$y = 20$**

Substituting $y = 20$ in (i), we get,

$5x + 3(20) = 75$

$\therefore \quad 5x + 60 = 75$

$\therefore \quad 5x = 75 - 60$

$\therefore \quad 5x = 15$

$\therefore \quad$ **$x = 3$**

$\therefore \quad$ **The weight of bucket when it is completely filled with water $= x + y$**

$= 3 + 20$

$= 23$ kg.

(2) Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi, he takes 6 minutes longer. Find the speed of the train and that of the taxi. *(4 marks)*

Solution:

Let the speed of the train be x km/hr and speed of the taxi be y km/hr.

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

Time taken to cover 300 km by train $= \frac{300}{x}$ hrs.

Time taken to cover 200 km by taxi $= \frac{200}{y}$ hrs.

According to the first condition,

$\frac{300}{x} + \frac{200}{y} = 5\frac{30}{60} \quad$ [1 hour = 60 min.]

$\therefore \quad \frac{300}{x} + \frac{200}{y} = \frac{11}{2} \quad$ ...(i)

Similarly,

Time taken to cover 260 km by train $= \frac{260}{x}$ hrs.

Time taken to cover 240 km by taxi $= \frac{240}{y}$ hrs.

According to the second condition,

$\frac{260}{x} + \frac{240}{y} = 5\frac{36}{60}$

$\therefore \quad \frac{260}{x} + \frac{240}{y} = \frac{28}{5} \quad$ ...(ii)

Substituting $\frac{1}{x} = m$ and $\frac{1}{y} = n$ in equations (i) and (ii),

$300m + 200n = \frac{11}{2}$

i.e. $600m + 400n = 11 \quad$ ...(iii)

Similarly, $260m + 240n = \frac{28}{5}$

i.e. $1300m + 1200n = 28 \quad$ ...(iv)

Multiplying eq. (iii) by 3,

$1800m + 1200n = 33 \quad$ ...(v)

Subtracting (iv) from (v),

$$\begin{array}{rcrcr} 1800m & + & 1200n & = & 33 \\ 1300m & + & 1200n & = & 28 \\ (-) & & (-) & & (-) \\ \hline 500m & & & = & 5 \end{array}$$

$\therefore \quad m = \frac{5}{500}$

$\therefore \quad m = \frac{1}{100}$

Substituting value of m in equation (iii) we get,

$$600 \times \frac{1}{100} + 400n = 11$$

$\therefore$ $6 + 400n = 11$

$\therefore$ $400n = 11 - 6$

$\therefore$ $400n = 5$

$\therefore$ $n = \frac{5}{400}$

$\therefore$ $n = \frac{1}{80}$

Resubstituting the values of m and n,

$$\frac{1}{x} = \frac{1}{100} \text{ and } \frac{1}{y} = \frac{1}{80}$$

$\therefore$ $x = 100$ and $y = 80$

The speed of the train is 100 km/hr and the speed of the taxi is 80 km/hr.

(3) **When the son will be as old as his father today, the sum of their ages then will be 126; when the father was as old as his son is today, the sum of their ages then was 38. Find their present ages.** *(4 marks)*

Solution:

Let the present age of father be x years and son be y years.

Son's age when he will be as old as his father today = x years.

$\therefore$ Father's age at that time = $[x + (x - y)]$ years

According to the first condition,

$x + x + (x - y) = 126$

$\therefore$ $3x - y = 126$ ...(i)

Father's age when he was as old as son today

= y years

$\therefore$ Son's age at that time = $[y - (x - y)]$ years

According to the second condition,

$y + y - (x - y) = 38$

$\therefore$ $y + y - x + y = 38$

$\therefore$ $-x + 3y = 38$ ...(ii)

Multiplying (ii) by 3,

$-3x + 9y = 114$ ...(iii)

Adding (i) and (iii), we get,

$$\begin{array}{r} 3x - y = 126 \\ -3x + 9y = 114 \\ \hline 8y = 240 \end{array}$$

$\therefore$ $y = \frac{240}{8}$

$\boldsymbol{y = 30}$

Substituting the value of y in equation (i) we get,

$3x - 30 = 126$

$\therefore$ $3x = 126 + 30$

$\therefore$ $3x = 156$

$\therefore$ $x = \frac{156}{3}$

$\therefore$ $\boldsymbol{x = 52}$

$\therefore$ **The present age of father and son are 52 years and 30 years respectively.**

(4) **The forewheel of a carriage makes 6 revolutions more than the rearwheel in going 120 m. If the diameter of the forewheel be increased by $\frac{1}{4}$ its present diameter and the diameter of the rearwheel be increased by $\frac{1}{5}$ of its present diameter, then the forewheel makes 4 revolutions more than the rearwheel in going the same distance. Find the circumference of each wheel of the carriage.** *(4 marks)*

Solution:

Let the circumference of the forewheel be x m and circumference of rearwheel be y m.

$$\text{Number of revolutions} = \frac{\text{Distance covered}}{\text{circumference}}$$

$\therefore$ Distance covered by each wheel = 120 m

$\therefore$ Number of revolutions made by forewheel = $\frac{120}{x}$

$\therefore$ Number of revolutions made by rearwheel = $\frac{120}{y}$

According to the first condition,

$$\frac{120}{x} = \frac{120}{y} + 6$$

$\therefore$ $\frac{120}{x} - \frac{120}{y} = 6$

$\therefore$ $\frac{20}{x} - \frac{20}{y} = 1$ ...(i)

Now, $c = \pi d$

$\therefore$ As diameter increases, circumference also increases.

Increase in circumference of forewheel = $\frac{x}{4}$

$$\text{New circumference of forewheel} = x + \frac{x}{4} = \frac{4x + x}{4} = \left(\frac{5x}{4}\right) \text{m}$$

$$\text{New circumference of rearwheel} = y + \frac{y}{5} = \frac{5y + y}{5} = \left(\frac{6y}{5}\right) \text{m}$$

According to the second condition,

$$\frac{120}{\frac{5x}{4}} = \frac{120}{\frac{6y}{5}} + 4$$

$$\therefore \quad \frac{120 \times 4}{5x} = \frac{120 \times 5}{6y} + 4$$

$$\therefore \quad \frac{96}{x} = \frac{100}{y} + 4$$

$$\therefore \quad \frac{96}{x} - \frac{100}{y} = 4 \qquad \text{...(ii)}$$

Substituting $\frac{1}{x} = a$ and $\frac{1}{y} = b$, we get,

$20a - 20b = 1$ ...(iii)

$96a - 100b = 4$ ...(iv)

Multiplying equation (iii) by 5, we get,

$100a - 100b = 5$ ...(v)

Subtracting (v) from (iv),

$$\begin{array}{rcrl} 96a & - & 100b & = 4 \\ 100a & - & 100b & = 5 \\ (-) & & (+) & (-) \\ \hline -4a & & & = -1 \end{array}$$

$\therefore \quad a = \frac{1}{4}$

Substituting value of $a = \frac{1}{4}$ in (iii), we get,

$20 \times \frac{1}{4} - 20b = 1$

$\therefore \quad 5 - 20b = 1$

$\therefore \quad 5 - 1 = 20b$

$\therefore \quad 4 = 20b$

$\therefore \quad b = \frac{4}{20}$

$\therefore \quad b = \frac{1}{5}$

Resubstituting the values of a and b,

$a = \frac{1}{x} = \frac{1}{4}$ and $b = \frac{1}{y} = \frac{1}{5}$

$\therefore \quad x = 4$ and $y = 5$

$\therefore$ **Circumference of forewheel is 4 m and circumference of rearwheel is 5 m.**

(5) Find the area of the triangle formed by the following lines and X axis.

$4x - 3y + 4 = 0$ and $4x + 3y - 20 = 0$ *(4 marks)*

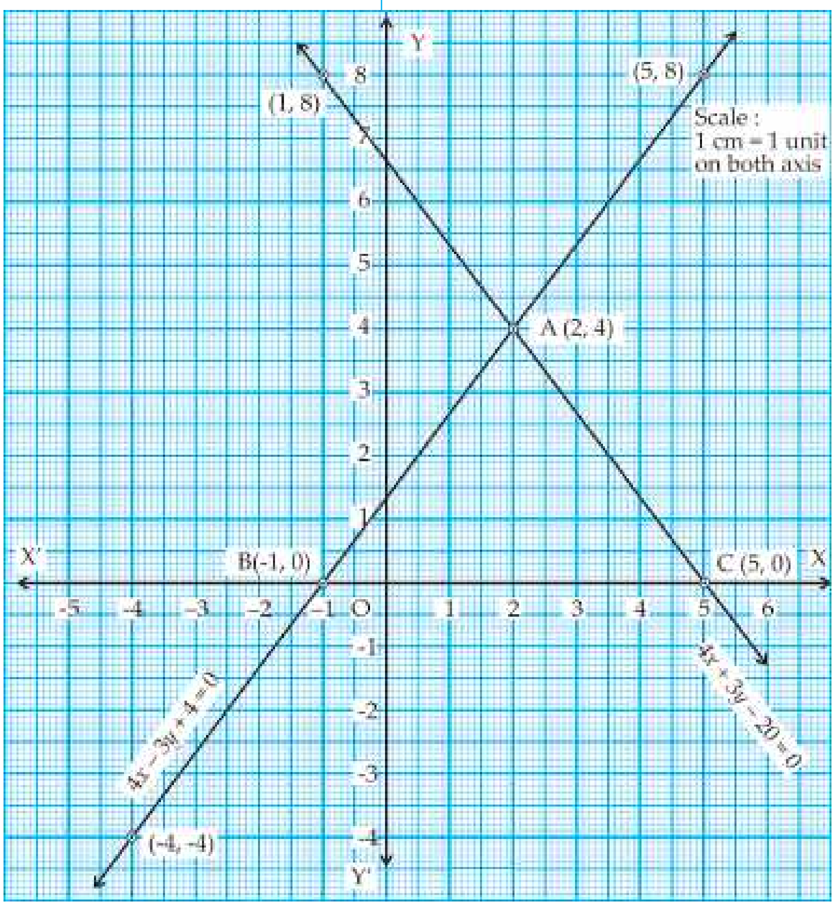
Solution:

$4x - 3y + 4 = 0 \quad \therefore \quad y = \frac{4x + 4}{3}$

x	2	5	–4
y	4	8	–4
(x, y)	(2, 4)	(5, 8)	(–4, –4)

$4x + 3y - 20 = 0 \quad \therefore \quad y = \frac{20 - 4x}{3}$

x	2	–1	5
y	4	8	0
(x, y)	(2, 4)	(–1, 8)	(5, 0)



For $\triangle ABC$, A(2, 4), B(–1, 0) and C(5, 0)

$\therefore \quad A(\triangle ABC) = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6 \times 4$

$\therefore$ $\boxed{\mathbf{A(\triangle\ ABC) = 12\ sq.\ units}}$

(6) Solve: $2^x + 3^y = 17$, $2^{x+2} - 3^{y+1} = 5$ *(4 marks)*

Solution:

$2^x + 3^y = 17$ ...(i)

$2^{x+2} - 3^{y+1} = 5$

$\therefore \quad 2^x.2^2 - 3^y.3^1 = 5$

$\therefore \quad 4.\ 2^x - 3.\ 3^y = 5$ ...(ii)

Substituting $2^x = m$ and $3^y = n$, we get,

$m + n = 17$ ...(iii)

$4m - 3n = 5$ ...(iv)

Multiplying eq. (iii) by 3,

$3m + 3n = 51$ ...(v)

Adding (iv) and (v) we get,

$$\begin{array}{rcl} 4m - 3n &=& 5 \\ 3m + 3n &=& 51 \\ \hline 7m &=& 56 \end{array}$$

$\therefore \quad m = \frac{56}{7}$

$\therefore \quad m = 8$

Substituting $m = 8$ in (iii),

$8 + n = 17$

$\therefore \quad n = 17 - 8$

$\therefore \quad n = 9$

Resubstituting the value of m and n, we get,

$m = 2^x$	$n = 3^y$
$\therefore$ $8 = 2^x$	$9 = 3^y$
$\therefore$ $2^3 = 2^x$	$3^2 = 3^y$
$\therefore$ $x = 3$	$y = 2$

$\therefore$ $\boxed{\mathbf{x = 3 \text{ and } y = 2 \text{ is the solution.}}}$

(7) A person deposits ₹ x in savings bank account at the rate of 5% per annum and ₹ y in fixed deposit at 10% per annum. At the end of one year, he gets ₹ 400 as total interest. If he deposits ₹ y in savings bank account and ₹ x in fixed deposit, he would get ₹ 350 as total interest. Find the total amount he deposited. *(4 marks)*

Solution:

Amount deposited in savings account = ₹ x

Amount deposited in fixed deposit account = ₹ y

$\therefore$ Total amount deposited = ₹ $(x + y)$

Interest received on Savings account ₹ $\frac{5x}{100}$

Interest received on fixed deposit account ₹ $\frac{10y}{100}$

According to the first condition,

$\frac{5x}{100} + \frac{10y}{100} = 400$

$\therefore \quad 5x + 10y = 40{,}000$ ...(i)

According to the second condition,

$\frac{10x}{100} + \frac{5y}{100} = 350$

$\therefore \quad 5y + 10x = 35{,}000$

$\therefore \quad 10x + 5y = 35{,}000$ ...(ii)

Adding equation (i) and (ii), we get,

$$\begin{array}{rcl} 5x + 10y &=& 40{,}000 \\ 10x + 5y &=& 35{,}000 \\ \hline 15x + 15y &=& 75{,}000 \end{array}$$

$\therefore \quad x + y = 5{,}000$ [Dividing both sides by 15]

$\therefore$ $\boxed{\textbf{Total amount deposited is ₹ 5000.}}$

(8) The sum of the digits of a number consisting of three digits is 12. The middle digit is equal to half of the sum of the other two. If the order of the digit be reversed, the number diminished by 198. Find the number. *(4 marks)*

Solution:

Digits	H	T	U
Original number	x	$\frac{1}{2}(x + y)$	y
Reversed number	y	$\frac{1}{2}(y + x)$	x

Let the digit in the hundredth place be x and the digit in the units place be y.

$\therefore$ The middle digit = $\frac{1}{2}(x+y)$

The sum of the digits is 12. (Given)

$\therefore \quad x + \frac{1}{2}(x+y) + y = 12$

$\therefore \quad 2x + x + y + 2y = 24$

$\therefore \quad 3x + 3y = 24$

$\therefore \quad x + y = 8$ ... (i)

The original number = $100x + 10 \times \frac{1}{2}(x+y) + y$

$= 100x + 5(x+y) + y$

$= 100x + 5x + 5y + y$

$= 105x + 6y$

The reversed number = $100y + 10 \times \frac{1}{2}(y+x) + x$

$= 100y + 5(y + x) + x$

$= 100y + 5y + 5x + x$

$= 6x + 105y$

Now,

Reversed number = Original number – 198 (Given)

$\therefore \quad 6x + 105y = 105x + 6y - 198$

$\therefore \quad 6x - 105x + 105y - 6y = -198$

$\therefore \quad -99x + 99y = -198$

$\therefore \quad x - y = 2$ ....(ii)

Adding (i) and (ii), we get,

$$\begin{array}{rcrcl} x & + & y & = & 8 \\ x & - & y & = & 2 \\ \hline 2x & & & = & 10 \end{array}$$

$\therefore \quad \boxed{x = 5}$

Substituting $x = 5$ in (i),

$5 + y = 8$

$\therefore \quad y = 8 - 5$

$\therefore \quad y = 3$

$\therefore$ The required number $= 105x + 6y$

$= 105 \times 5 + 6 \times 3$

$= 525 + 18$

$= 543$

$\therefore$ **The required number is 543**

(9) A train covered a certain distance at a uniform speed. If the train would have been 6 km/hr faster, it would have taken 4 hours less than the schedule time. And, if train were slower by 6 km/hr, it would have taken 6 hours more than the schedule time. Find the length of the journey. *(4 marks)*

Solution:

Let the original speed of train be x km/hr and the actual time taken be y hours.

Then, distance = speed × time

= (xy) km

According to the first condition,

$(x + 6)(y - 4) = xy$

$\therefore \quad xy - 4x + 6y - 24 = xy$

$\therefore \quad -4x + 6y = 24$

$\therefore \quad -2x + 3y = 12$ ...(i)

According to the second condition,

$(x - 6)(y + 6) = xy$

$\therefore \quad xy + 6x - 6y - 36 = xy$

$\therefore \quad 6x - 6y = 36$

$\therefore \quad x - y = 6$ ...(ii)

Multiplying (ii) by 2, we get,

$2x - 2y = 12$ ...(iii)

Adding equations (i) and (iii) we get,

$$\begin{array}{rcrcl} -2x & + & 3y & = & 12 \\ 2x & - & 2y & = & 12 \\ \hline & & y & = & 24 \end{array}$$

Substituting the value of y in equation (i) we get,

$x - 24 = 6$

$\therefore \quad x = 6 + 24$

$\therefore \quad x = 30$

$\therefore$ The original speed = 30 km/hr and time = 24 hours.

Hence, length of the journey = speed × time

$= 30 \times 24$

$= 720$ km.

$\therefore$ **Length of the journey is 720 km.**

(10) Solve : $(a - b)x + (a + b)y = a^2 - 2ab - b^2$ and $(a + b)(x + y) = a^2 + b^2$ *(4 marks)*

Solution:

$(a - b)x + (a + b)y = a^2 - 2ab - b^2$ ...(i)

$(a + b)(x + y) = a^2 + b^2$

$\therefore \quad (a + b)x + (a + b)y = a^2 + b^2$ ...(ii)

Subtracting (ii) from (i),

$$\begin{array}{lcccl} (a-b)x & + & (a+b)y & = & a^2 - 2ab - b^2 \\ (a+b)x & + & (a+b)y & = & a^2 \qquad + b^2 \\ (-) & & (-) & & (-) \qquad (-) \\ \hline [a - b - (a + b)]x & & & = & -2ab - 2b^2 \end{array}$$

$\therefore \quad (a - b - a - b)\,x = -2b(a + b)$

$\therefore \quad -2bx = -2b(a + b)$

$\therefore \quad x = \frac{-2b(a+b)}{-2b}$

$\therefore \quad x = (a + b)$

Substituting value of x in (i), we get,

$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$

$\therefore \quad a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$

$\therefore \quad (a + b)y = -2ab$

$\therefore \quad y = \frac{-2ab}{a+b}$

$\therefore$ **$x = (a + b)$ and $y = \frac{-2ab}{a+b}$ is the solution of the given equations.**

2. Quadratic Equations

(1) **Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels 5 km/hr faster than second train. If after two hours, they are 50 km apart, find the speed of each train.** *(4 marks)*

Solution:

Let the speed of second train be x km/hr.

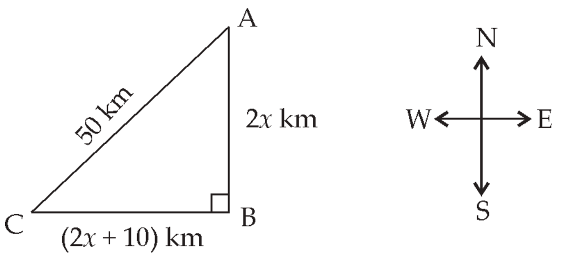
$\therefore$ Speed of first train = $(x + 5)$ km/hr

Distance = speed × time

Distance covered by second train = $x \times 2 = 2x$ km

Distance covered by first train $= (x + 5) \times 2$

$= (2x + 10)$ km



In ΔABC, $m\angle ABC = 90°$

$AB^2 + BC^2 = AC^2$ ... [Pythagoras theorem]

$\therefore$ $(2x)^2 + (2x + 10)^2 = (50)^2$

$\therefore$ $4x^2 + 4x^2 + 40x + 100 = 2500$

$\therefore$ $8x^2 + 40x + 100 - 2500 = 0$

$\therefore$ $8x^2 + 40x - 2400 = 0$

$\therefore$ $x^2 + 5x - 300 = 0$ ... (Dividing both sides by 8)

$\therefore$ $x^2 + 20x - 15x - 300 = 0$

$\therefore$ $x(x + 20) - 15(x + 20) = 0$

$\therefore$ $(x + 20)(x - 15) = 0$

$\therefore$ $x + 20 = 0$ or $x - 15 = 0$

$\therefore$ $x = -20$ or $x = 15$

$x \neq -20$ as speed cannot be negative.

$\therefore$ $x = 15$ and $x + 5 = 15 + 5 = 20$

$\therefore$ **The speeds of the trains are 20 km/hr and 15 km/hr respectively.**

(2) **If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then prove that $2a^2c = c^2b + b^2a$.** *(4 marks)*

Solution:

Let α and β be the roots of the given quadratic equation.

$\therefore$ $\alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

As per given condition,

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\therefore \quad \alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$\therefore \quad \alpha + \beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\ \beta)^2}$$

$$\therefore \quad \frac{-b}{a} = \frac{\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$\therefore \quad \frac{-b}{a} = \left[\frac{b^2}{a^2} - \frac{2c}{a}\right] \div \frac{c^2}{a^2}$$

$$\therefore \quad \frac{-b}{a} = \left[\frac{b^2 - 2ac}{a^2}\right] \div \frac{c^2}{a^2}$$

$$\therefore \quad \frac{-b}{a} = \frac{b^2 - 2ac}{a^2} \times \frac{a^2}{c^2}$$

$$\therefore \quad \frac{-b}{a} = \frac{b^2 \square 2ac}{c^2}$$

$\therefore$ $bc^2 = a(b^2 - 2ac)$

$\therefore$ $bc^2 = ab^2 - 2a^2c$

$\therefore$ $\boxed{2a^2c = c^2b + b^2a}$ **... Hence proved.**

(3) **If the roots of the quadratic equation $ax^2 + cx + c = 0$ are in the ratio p : q, then show that**

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{c}{a}} = 0$$ *(4 marks)*

Solution:

Let α and β be the roots of the given quadratic equation.

$\therefore$ $\alpha + \beta = \dfrac{-c}{a}$ and $\alpha\beta = \dfrac{c}{a}$

Also, $\dfrac{\alpha}{\beta} = \dfrac{p}{q}$ ... (Given)

$$\therefore \quad \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{c}{a}} = \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\ \beta}$$

$$= \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \sqrt{\alpha\beta}$$

$$= \frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \sqrt{\alpha\beta}$$

$$= \frac{\alpha+\beta+\alpha\beta}{\sqrt{\alpha\beta}}$$

$$= \frac{\frac{-c}{a}+\frac{c}{a}}{\sqrt{\alpha\beta}}$$

$$= \frac{0}{\sqrt{\alpha\beta}}$$

$$= 0$$

$\therefore$ $\boxed{\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{c}{a}}=0}$

(4) A businessman bought some items for ₹ 600. Keeping 10 items for himself he sold the remaining items at a profit of ₹ 5 per item. From the amount received in this deal he could buy 15 more items. Find the original price of each item. *(4 marks)*

Solution:

Total cost price of some items = ₹ 600 (given)

Let the original cost price of 1 item = ₹ x

$\therefore$ Number of items purchased $= \frac{600}{x}$

Number of items sold after keeping 10 items

$$= \frac{600}{x} - 10$$

Selling price of each item = ₹ $(x + 5)$

Total selling price of all items = ₹ $(x+5)\left(\frac{600}{x}-10\right)$

Profit = S.P. – C.P.

$\therefore$ Net profit made in the deal

$$= ₹\ (x+5)\left(\frac{600}{x}-10\right) - 600$$

But, profit is equal to cost price of 15 items = ₹ $15x$

As per given condition,

$$(x+5)\left(\frac{600}{x}-10\right) - 600 = 15x$$

$\therefore$ $x\left(\frac{600}{x}-10\right) + 5\left(\frac{600}{x}-10\right) - 600 = 15x$

$\therefore$ $600 - 10x + \frac{3000}{x} - 50 - 600 - 15x = 0$

$\therefore$ $-10x + \frac{3000}{x} - 50 - 15x = 0$

$\therefore$ $-25x + \frac{3000}{x} - 50 = 0$

$\therefore$ $-25x^2 + 3000 - 50x = 0$

...(Multiplying both sides by x)

$\therefore$ $x^2 - 120 + 2x = 0$ ...(Dividing both sides by –25)

$\therefore$ $x^2 + 2x - 120 = 0$

$\therefore$ $x^2 + 12x - 10x - 120 = 0$

$\therefore$ $x(x + 12) - 10(x + 12) = 0$

$\therefore$ $(x + 12)(x - 10) = 0$

$\therefore$ $x + 12 = 0$ or $x - 10 = 0$

$\therefore$ $x = -12$ or $x = 10$

$\therefore$ $x \neq -12$ as cost price cannot be negative.

$\therefore$ $x = 10$

$\therefore$ **Original price of each item is ₹ 10.**

(5) A man travels by boat 36 km down a river and back in 8 hours. If the speed of his boat in still water is 12 km/hr, find the speed of the river current. *(4 marks)*

Solution:

Let the speed of river current = x km/hr

Speed of boat in still water = 12 km/hr

$\therefore$ Speed of boat up the river = $(12 - x)$ km/hr

Speed of boat down the river = $(12 + x)$ km/hr

$$\text{Time} = \frac{\text{Distance}}{\text{speed}}$$

$\therefore$ Time taken by boat to travel 36 km down the river

$$= \left(\frac{36}{12+x}\right) \text{hrs.}$$

$\therefore$ Time taken by boat to travel 36 km up the river

$$= \left(\frac{36}{12-x}\right) \text{hrs.}$$

As per given condition,

$$\frac{36}{12+x} + \frac{36}{12-x} = 8$$

$$36\left(\frac{1}{12+x}+\frac{1}{12-x}\right) = 8$$

$\therefore$ $\frac{12-x+12+x}{(12+x)(12-x)} = \frac{8}{36}$

$\therefore$ $\frac{24}{144-x^2} = \frac{2}{9}$

$\therefore$ $108 = 144 - x^2$

$\therefore$ $x^2 = 144 - 108$

$\therefore$ $x^2 = 36$

$\therefore$ $x = \pm 6$

$\therefore$ $x \neq -6$ as speed cannot be negative.

$\therefore$ $x = 6.$

$\therefore$ **The speed of river current = 6 km/hr.**

(6) A number consists of two digits whose product is 56. When 9 is subtracted from the number, the digits interchange their places. Find the number. *(4 marks)*

Solution:

Let the ten's digit = x and unit's digit = y

So, the number $= 10x + y$

Product of the digits $= 56$

$\therefore$ $xy = 56$ ...(i)

On subtracting 9 from the number, the digits are interchanged.

$\therefore$ $(10x + y) - 9 = 10y + x$

$\therefore$ $10x - x + y - 10y = 9$

$\therefore$ $9x - 9y = 9$

$\therefore$ $x - y = 1$

$\therefore$ $x - \frac{56}{x} = 1$ ... [From (i)]

$\therefore$ $x^2 - 56 = x$

$\therefore$ $x^2 - x - 56 = 0$

$\therefore$ $x^2 - 8x + 7x - 56 = 0$

$\therefore$ $x(x - 8) + 7(x - 8) = 0$

$\therefore$ $(x - 8)(x + 7) = 0$

$\therefore$ $x - 8 = 0$ or $x + 7 = 0$

$\therefore$ $x = 8$ or $x = -7$

$\therefore$ $x \neq -7$ as it is a digit and cannot be negative.

$\therefore$ $x = 8.$

Now, $8 \times y = 56$ ...[From (i)]

$\therefore$ $y = \frac{56}{8}$

$\therefore$ $y = 7$

$\therefore$ **Hence, the number is 10 × 8 + 7 = 87.**

(7) Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe would fill the cistern. *(4 marks)*

Solution:

Let the time taken by other pipe to fill the cistern be x minutes.

$\therefore$ Time taken by first pipe to fill the cistern $= (x + 3)$ minutes

Portion of cistern filled by other pipe in one minute $= \frac{1}{x}$ and portion of cistern filled by first pipe in one minute $= \frac{1}{x+3}$

Position of cistern filled by both the pipes in one

$$\text{minute} = \frac{1}{3\frac{1}{13}} = \frac{1}{\frac{40}{13}} = \frac{13}{40}$$

As per given condition,

$$\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$$

$\therefore$ $\frac{x+3+x}{x(x+3)} = \frac{13}{40}$

$\therefore$ $\frac{2x+3}{x^2+3x} = \frac{13}{40}$

$\therefore$ $40(2x + 3) = 13(x^2 + 3x)$

$\therefore$ $80x + 120 = 13x^2 + 39x$

$\therefore$ $13x^2 - 41x - 120 = 0$

$\therefore$ $13x^2 - 65x + 24x - 120 = 0$

$\therefore$ $13x(x - 5) + 24(x - 5) = 0$

$\therefore$ $(x - 5)(13x + 24) = 0$

$\therefore$ $x - 5 = 0$ or $13x + 24 = 0$

$\therefore$ $x = 5$ or $x = \frac{-24}{13}$

$\therefore$ $x \neq \frac{-24}{13}$ as time cannot be negative.

$\therefore$ $x = 5$ and $x + 3 = 5 + 3 = 8$

$\therefore$ **Time taken by two pipes to fill the cistern separately are 5 minutes and 8 minutes.**

(8) If the sum of roots of the quadratic equation is $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ is zero, show that the product of the roots is $-\left(\frac{p^2+q^2}{2}\right)$. *(4 marks)*

Solution

$$\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$$

$\therefore \quad \dfrac{x+q+x+p}{(x+p)(x+q)} = \dfrac{1}{r}$

$\therefore \quad \dfrac{2x+q+p}{x^2+qx+px+pq} = \dfrac{1}{r}$

$\therefore \quad r(2x+p+q) = x^2+qx+px+pq$

$\therefore \quad 2rx+pr+qr = x^2+qx+px+pq$

$\therefore \quad 0 = x^2+qx+px+pq-2rx-pr-qr$

$\therefore \quad x^2+qx+px-2rx+pq-pr-qr = 0$

$\therefore \quad x^2+x(q+p-2r)+pq-pr-qr = 0$

$\therefore \quad x^2+(p+q-2r)x+(pq-pr-qr) = 0$ ...(i)

Comparing with $ax^2+bx+c=0$, we get,

$a=1$, $b=p+q-2r$, $c=pq-pr-qr$

Let α and β be the roots of equation (i).

Let $\alpha+\beta = \dfrac{-b}{a}$

$= \dfrac{-(p+q-2r)}{1}$

$\alpha+\beta = -p-q+2r$

But $\alpha+\beta=0$ .....[Given]

$\therefore \quad -p+q-2r = 0$

$\therefore \quad 2r = p+q$

$\therefore \quad r = \dfrac{p+q}{2}$

Also, $\alpha\beta = \dfrac{c}{a}$

$= \dfrac{pq-pr-qr}{1}$

$= pq-pr-qr$

$= pq-r(p+q)$

$= pq-\left(\dfrac{p+q}{2}\right)(p+q)$ .... $[r = \dfrac{p+q}{2}]$

$= pq-\dfrac{(p+q)^2}{2}$

$= \dfrac{2pq-(p+q)^2}{2}$

$= \dfrac{2pq-(p^2+2pq+q^2)}{2}$

$= \dfrac{2pq-p^2-2pq-q^2}{2}$

$= \dfrac{-p^2-q^2}{2}$

$\therefore \quad \alpha\beta = -\left(\dfrac{p^2+q^2}{2}\right)$

(9) Solve : $\sqrt{x^2-16}-\sqrt{x^2-8x+16}=\sqrt{x^2-5x+4}$ (4 marks)

Solution

$\sqrt{x^2-16}-\sqrt{x^2-8x+16}=\sqrt{x^2-5x+4}$

$\therefore \quad \sqrt{(x+4)(x-4)}-\sqrt{(x-4)^2}=\sqrt{(x-4)(x-1)}$

$\therefore \quad \sqrt{(x+4)(x-4)}-\sqrt{(x-4)^2}-\sqrt{(x-4)(x-1)}=0$

$\therefore \quad \sqrt{x-4}\left[\sqrt{x+4}-\sqrt{(x-4)}-\sqrt{x-1}\right]=0$

$\therefore \quad \sqrt{x-4}=0$ or $\sqrt{x+4}-\sqrt{x-4}-\sqrt{x-1}=0$

$\therefore \quad x-4=0$ or $\sqrt{x+4}-\sqrt{x-4}=\sqrt{x-1}$

$\therefore \quad x=4$ or $\sqrt{x+4}-\sqrt{x-4}=\sqrt{x-1}$

Now, $\sqrt{x+4}-\sqrt{x-4}=\sqrt{x-1}$ ...(i)

$\therefore \quad \left(\sqrt{x+4}-\sqrt{x-4}\right)^2 = x-1$ ...(Squaring both sides)

$\therefore \quad x+4-2\sqrt{(x+4)(x-4)}+x-4=x-1$

$\therefore \quad 2x-2\sqrt{x^2-16} = x-1$

$\therefore \quad 2x-x+1 = 2\sqrt{x^2-16}$

$\therefore \quad x+1 = 2\sqrt{x^2-16}$

$\therefore \quad (x+1)^2 = 4(x^2-16)$...(Squaring both sides)

$\therefore \quad x^2+2x+1 = 4x^2-64$

$\therefore \quad 4x^2-x^2-2x-64-1 = 0$

$\therefore \quad 3x^2-2x-65=0$

$\therefore \quad 3x^2-15x+13x-65=0$

$\therefore \quad 3x(x-5)+13(x-5)=0$

$\therefore \quad (3x+13)(x-5)=0$

$\therefore \quad 3x+13=0$ or $x-5=0$

$\therefore \quad x=\dfrac{-13}{3}$ or $x=5$

Now, $x \neq \dfrac{-13}{3}$ as it does not satisfy the (i)

$\therefore$ **Hence, the roots of the given equation are 4 and 5.**

❖ ❖ ❖

3. Arithmetic Progression

(1) 200 logs of wood are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows 200 logs are placed and how many logs are there in the top row? *(4 marks)*

Solution:

The arrangement of logs

20, 19, 18, ... forms an A.P. with $a = 20$, $d = -1$

Let 200 logs be arranged in n rows.

$S_n = 200$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore\ 200 = \frac{n}{2}[2 \times 20 + (n-1)(-1)]$

$\therefore\ 400 = n[40 - n + 1]$

$\therefore\ 400 = n[41 - n]$

$\therefore\ 400 = 41n - n^2$

$\therefore\ n^2 - 41n + 400 = 0$

$\therefore\ n^2 - 25n - 16n + 400 = 0$

$\therefore\ n(n - 25) - 16(n - 25) = 0$

$\therefore\ (n - 25)(n - 16) = 0$

$\therefore\ n - 25 = 0$ or $n - 16 = 0$

$\therefore\ n = 25$ or $n = 16$

If $n = 25$

$t_n = a + (n-1)d$

$\therefore\ t_{25} = 20 + (25 - 1)(-1)$

$= 20 + 24(-1)$

$= 20 - 24$

$t_{25} = -4$

No. of logs in the 25th row cannot be negative

$\therefore\ n \neq 25 \quad \therefore\ n = 16$

$t_n = a + (n-1)d$

$\therefore\ t_{16} = 20 + (16 - 1)(-1)$

$= 20 + 15(-1)$

$= 20 - 15$

$t_{16} = 5$

$\therefore$ **200 logs are placed in 16 rows and there are 5 logs in the top row.**

(2) If sum of m terms is n and sum of n terms is m, then show that the sum of $(m + n)$ terms is $-(m + n)$. *(4 marks)*

Solution:

Given: $S_m = n$, $S_n = m$

To prove : $S_{m+n} = -(m+n)$

Proof:

$S_m = \frac{m}{2}[2a + (m-1)d]$

$\therefore\ n = \frac{m}{2}[2a + (m-1)d]$ ...(i)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$m = \frac{n}{2}[2a + (n-1)d]$ ...(ii)

Subtracting (ii) from (i)

$\therefore\ n - m = \frac{m}{2}[2a + (m-1)d] - \frac{n}{2}[2a + (n-1)d]$

$\therefore\ 2(n - m) = m[2a + (m-1)d] - n[2a + (n-1)d]$

$\therefore\ 2(n - m) = m[2a + md - d] - n[2a + nd - d]$

$\therefore\ 2(n - m) = 2am + m^2d - md - 2an - n^2d + nd$

$\therefore\ 2(n - m) = 2am - 2an + m^2d - n^2d - md + nd$

$\therefore\ 2(n - m) = 2a(m - n) + d(m^2 - n^2) - d(m - n)$

$\therefore\ -2(m - n) = 2a(m-n) + d(m+n)(m-n) - d(m - n)$

Dividing throughout by $m - n$,

$\therefore\ -2 = 2a + d(m + n) - d$

$\therefore\ -2 = 2a + d(m + n - 1)$ ...(iii)

Now, $S_{m+n} = \frac{m+n}{2}[2a + d(m + n - 1)]$

$\therefore\ S_{m+n} = \frac{m+n}{2}[-2]$ ....[From (iii)]

$\therefore\ S_{m+n} = m + n(-1)$

$\therefore\ \boxed{S_{m+n} = -(m+n)}$

(3) How many terms of the A.P. 16, 14, 12, ... are needed to given the sum 60? Explain why we get two answers. *(4 marks)*

Solution:

For the A.P. 16, 14, 12, ...

$a = 16$, $d = 14 - 16 = -2$

Let $S_n = 60$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore\ 60 = \frac{n}{2}[2 \times 16 + (n-1)(-2)]$

$\therefore\ 120 = n[32 - 2n + 2]$

$\therefore\ 120 = n[34 - 2n]$

$\therefore\ 120 = 34n - 2n^2$

$\therefore\ 2n^2 - 34n + 120 = 0$

$\therefore\ n^2 - 17n + 60 = 0$ ... [Dividing both sides by 2]

$\therefore \quad n^2 - 12n - 5n + 60 = 0$

$\therefore \quad n(n-12) - 5(n-12) = 0$

$\therefore \quad (n-12)(n-5) = 0$

$\therefore \quad n - 12 = 0$ or $n - 5 = 0$

$\therefore \quad n = 12$ or $n = 5$

$\therefore$ The number of terms required to give a sum of 60 are 12 or 5.

The reason for getting two answers is the common difference is –2 and the progression is towards the negative side.

The A. P. is as follows:

16, 14, 12, 10, 8, 6, 4, 2, 0, –2, –4, –6.....

$S_5 = 16 + 14 + 12 + 10 + 8 = 60$

$S_{12} = 16 + 14 + 12 + 10 + 8 + 6 + 4 + 2 + 0 - 2 - 4 - 6 = 60$

(4) If p^{th}, q^{th} and r^{th} terms of an A. P. are l, m, n respectively, show that
$(q-r)l + (r-p)m + (p-q)n = 0$ *(4 marks)*

Solution:

Given : $t_p = l$, $t_q = m$, $t_r = n$

Let the 1st term of given A.P. be a and d be the common difference.

$t_n = a + (n-1)\,d$

$\therefore \quad t_p = a + (p-1)\,d$

$\therefore \quad l = a + (p-1)\,d \quad$ ...(i)

$\therefore \quad t_q = a + (q-1)\,d$

$\therefore \quad m = a + (q-1)\,d \quad$ ...(ii)

$\therefore \quad t_r = a + (r-1)\,d$

$\therefore \quad n = a + (r-1)\,d \quad$ ...(iii)

Subtracting (ii) from (i), we get

$$\begin{array}{rcl} l & = & a + (p-1)\,d \\ -\ m & = & a + (q-1)\,d \\ (-) & & (-) \qquad (-) \\ \hline l - m & = & (p-1)\,d - (q-1)\,d \end{array}$$

$\therefore \quad l - m = (p - 1 - q + 1)d$

$\therefore \quad l - m = (p-q)d$

$\therefore \quad \dfrac{l-m}{d} = (p-q) \quad$ ...(iv)

Subtracting (iii) from (ii), we get

$$\begin{array}{rcl} m & = & a + (q-1)\,d \\ n & = & a + (r-1)\,d \\ \hline (-) & (-) & \qquad (-) \end{array}$$

$\therefore \quad m - n = (q-1)\,d - (r-1)\,d$

$\therefore \quad m - n = (q - 1 - r + 1)d$

$\therefore \quad m - n = (q-r)d$

$\therefore \quad \dfrac{m-n}{d} = (q-r) \quad$ ...(v)

Subtracting (i) from (iii), we get

$$\begin{array}{rcl} n & = & a + (r-1)\,d \\ l & = & a + (p-1)\,d \\ \hline (-) & (-) & \qquad (-) \end{array}$$

$\therefore \quad n - l = (r-1)\,d - (p-1)\,d$

$\therefore \quad n - l = (r - 1 - p + 1)d$

$\therefore \quad n - l = (r-p)d$

$\therefore \quad \dfrac{n-l}{d} = (r-p) \quad$ ...(vi)

Now, $(q-r)l + (r-p)m + (p-q)n$

$= \left(\dfrac{m-n}{d}\right)l + \left(\dfrac{n-l}{d}\right)m + \left(\dfrac{l-m}{d}\right)n$

...[From (iv), (v), (vi)]

$= \dfrac{1}{d}[lm - ln + mn - lm + ln - mn]$

$= 0$

Hence proved.

(5) Jinal saves ₹ 1600 during the first year, ₹ 2100 in the second year, ₹ 2600 in the third year, If she continues her saving in this pattern, in how many years will she save ₹ 38,500? *(4 marks)*

Solution:

Pattern of her saving is ₹ 1600, ₹ 2100, ₹ 2600, ...

It is an A.P. with $a = 1600$, $d = 2100 - 1600 = 500$

Also, $S_n = 38500$

$S_n = \dfrac{n}{2}[2a + (n-1)\,d]$

$\therefore \quad 38500 = \dfrac{n}{2}[2 \times 1600 + (n-1)500]$

$\therefore \quad 77000 = n\,[3200 + 500n - 500]$

$\therefore \quad 77000 = n\,[2700 + 500n]$

$\therefore \quad 77000 = 2700n + 500n^2$

$\therefore \quad 500n^2 + 2700n - 77000 = 0$

$\therefore \quad 5n^2 + 27n - 770 = 0$

...[Dividing both sides by 100]

$\therefore \quad 5n^2 + 77n - 50n - 770 = 0$

$\therefore \quad n(5n+77) - 10(5n+77) = 0$

$\therefore \quad (n-10)(5n+77) = 0$

$\therefore \quad n - 10 = 0$ or $5n + 77 = 0$

$\therefore \quad n = 10$ or $n = \dfrac{-77}{5}$

Now $n = \dfrac{-77}{5}$ (Not possible as n is natural number)

$\therefore \quad n = 10$

$\therefore$ **Jinal will save ₹ 38500 in 10 years.**

(6) Find the middle term of sequence formed by all three digit numbers which leave a remainder 3 when divided by 4. Also find sum of all numbers on both sides of the middle term. *(4 marks)*

Solution:

The sequence formed by the given numbers is 103, 107, 111, 115, ... , 999.

This is an A.P. with $a = 103$, $d = 107 - 103 = 4$ and $t_n = 999$

$t_n = a + (n-1)d$

$\therefore\ 999 = 103 + (n-1)4$

$\therefore\ 896 = 4n - 4$

$\therefore\ 4n = 900$

$\therefore\ n = \dfrac{900}{4}$

$\therefore\ n = 225$

$\therefore$ Middle term $= \left(\dfrac{n+1}{2}\right)^{th}$ term

$= \left(\dfrac{225+1}{2}\right)^{th}$ term

$= \dfrac{226}{2}$

$= 113^{th}$ term.

$t_n = a + (n-1)d$

$\therefore\ t_{113} = 103 + (113-1)4$

$\therefore\ = 103 + 112 \times 4$

$= 103 + 448$

$t_{113} = 551$

Now, $t_{112} = 551 - 4 = 547$

So, we have to find S_{112} and $(S_{225} - S_{113})$

$S_n = \dfrac{n}{2}[t_1 + t_n]$

$\therefore\ S_{112} = \dfrac{112}{2}[103 + 547]$

$\therefore\ = \dfrac{112}{2} \times 650$

$\therefore\ S_{112} = 36400$

Similarly ,

$S_{225} - S_{113} = \dfrac{225}{2}(103 + 999) - \dfrac{113}{2}(103 + 551)$

$= (225 \times 551) - (113 \times 327)$

$= 123975 - 36951$

$= 87024$

$\therefore$ **Sum of all members on LHS of middle term is 36400 and sum of all numbers on RHS of the middle term is 87024.**

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4. Financial Planning

(1) Mr. Modi invested ₹ 30,120 in equity shares of FV ₹10, when the market value was ₹ 60. After receiving dividend on them at 90%, he sold them at MV of ₹ 55. In each transaction he paid 0.4%. What was the total gain or loss in this transaction? *(4 marks)*

Solution:

Rate of GST is not given, so it is not considered.

Shares Purchased : FV = ₹ 10, MV = ₹ 60

Brokerage per share $= \dfrac{0.4}{100} \times 60$

= ₹ 0.24

$\therefore$ Cost of one share = 60 + 0.24

= ₹ 60.24

Number of shares purchased $= \dfrac{30120}{60.24}$

= 500

Shares sold : FV = ₹ 10, MV = ₹ 55

$\therefore$ Brokerage per share $= \dfrac{0.4}{100} \times 55$

= ₹ 0.22

$\therefore$ Selling price per share = 55 – 0.22

= ₹ 54.78

$\therefore$ Selling price of 500 shares = 500 × 54.78

= ₹ 27,390

Dividend received 90%

$\therefore$ Dividend per share $= \dfrac{90}{100} \times 10$

= ₹ 9

$\therefore$ Dividend of 500 shares = 9 × 500

= ₹ 4,500

Mr. Modi's income = ₹ 27,390 + ₹ 4,500

= ₹ 31,890

Sum invested = ₹ 30,120

$\therefore$ Profit = 31,890 – 30,120 = ₹ 1,770

$\therefore$ **Mr. Modi made a profit of ₹ 1770 in the whole transaction.**

(2) Usha Joshi invested an equal amount in two companies by purchasing equity shares with MV ₹ 145 and ₹ 160 each. The FV is same and it is ₹ 100 for both the shares. At the end of the year, both companies declared the dividends at 20% and 30% each; In which company was her investment profitable? *(4 marks)*

Solution:

Rate of GST and brokerage is not given, so it is not considered.

Suppose the amount invested by Usha Joshi in both the companies is ₹ 23,200 (i.e. ₹ 145 × ₹ 160)

	Company 1	Company 2
Amount Invested	₹ 23,200	₹ 23,200
MV per share	₹ 145	₹ 160
Brokerage per share [∵ Brokerage is not mentioned] ∴ cost of one share	₹ 0 ₹ 145	₹ 0 ₹ 160
No. of shares purchased	$\frac{23200}{145} = 160$	$\frac{23200}{160} = 145$
Dividend declared	20%	30%
FV per share	₹ 100	₹ 100
Dividend per share	$= \frac{20}{100} \times 100$ = ₹ 20	$= \frac{30}{100} \times 100$ = ₹ 30
Dividend on shares =	160 × 20 = ₹ 3200	145 × 30 = ₹ 4350

∴ Company 2 gives more amount of dividend than Company 1.

∴ **Investment in company 2 is more profitable.**

(3) **Mr. Deepak Pal invested ₹ 1,00,354 in shares of FV ₹ 100, when the market value is ₹ 50. Rate of brokerage is 0.3% and rate of GST on brokerage is 18%, then how many shares were purchased?** *(4 marks)*

Solution:

FV = ₹ 100, MV = ₹ 50

∴ Brokerage per share = 0.3% of ₹ 50

$= \frac{0.3}{100} \times 50$

= ₹ 0.15

∴ GST per share = 18 % of ₹ 0.15

$= \frac{18}{100} \times 0.15$

= ₹ 0.027

Cost of 1 share = MV + Brokerage + GST

= 50 + 0.15 + 0.027

= ₹ 50.177

∴ Number of shares = $\frac{\text{Total investment}}{\text{Cost of 1 share}}$

$= \frac{1,00,354}{50.177}$

= 2000

∴ **Mr. Deepak Pal purchased 2000 shares.**

(4) **Star Pharma purchased some chemicals for ₹ 8,000 (with GST) and sold it to the M/s. Pooja Chemicals for ₹ 10,000 (with GST). Rate of GST is 18%. Find the amount of CGST and SGST to be paid by Star Pharma.** *(4 marks)*

Solution:

Total value (including GST) = Taxable value + GST

∴ The ratio of $\frac{\text{Total Value}}{\text{Taxable Value}}$ is constant as the rate of GST is same.

(i) For total value of ₹ 118, the taxable value is ₹ 100 and for total value of ₹ 8000, let taxable value be ₹ x.

∴ $\frac{x}{8000} = \frac{100}{118}$

∴ $x = \frac{8000 \times 100}{118}$

∴ x = ₹ 6779.66

∴ GST paid at the time of the purchase

= 8000 – 6779.66

= ₹ 1220.34

∴ Input tax = ₹ 1220.34

∴ ITC = ₹ 1220.34 ...(i)

(ii) For total value of ₹ 10,000, let taxable value be ₹ y.

∴ $\frac{y}{10000} = \frac{100}{118}$

∴ $y = \frac{10000 \times 100}{118}$

∴ y = ₹ 8474.58

∴ Output Tax = 10000 – 8474.58

= ₹ 1525.42 ...(ii)

∴ GST payable = Output tax – ITC

= ₹ 1525.42 – ₹ 1220.34

[From i, ii]

= ₹ 305.08

∴ Payable CGST = Payable SGST

$$= \frac{305.08}{2}$$

$= ₹\ 152.54$

∴ **Star Pharma has to pay ₹ 152.54 CGST and ₹ 152.54 SGST.**

(5) A manufacturer sold electric goods for a taxable value of ₹ 40,000 to the wholeseller. Wholeseller sold it to the retailer for ₹ 48,000 (taxable value). Retailer sold it to a customer for ₹ 52,000 (taxable value). Rate of GST is 18%. Find the CGST and SGST payable at each stage of trading. Also show statement of GST payable at each stage of trading. *(4 marks)*

Solution:

Output tax of manufacturer = 12% of 40,000

$$= \frac{12}{100} \times 40{,}000$$

$= ₹\ 4800$

∴ GST payable by manufacturer = ₹ 4800

Output tax of wholeseller = 12% of 48,000

$$= \frac{12}{100} \times 48{,}000$$

$= ₹\ 5760$

∴ GST payable by wholeseller = Output tax – ITC

= 5760 – 4800

= ₹ 960

Output tax of Retailer = 12% of 52,000

$$= \frac{12}{100} \times 52{,}000$$

$= ₹\ 6240$

∴ GST payable by Retailer = Output tax – ITC

= 6240 – 5760

= ₹ 480

Statement of GST payable at each stage of trading.

Persons	Payable GST	Payable CGST	Payable SGST
Manufacturer	₹ 4800	₹ 2400	₹ 2400
Wholeseller	₹ 960	₹ 480	₹ 480
Retailer	₹ 480	₹ 240	₹ 240
Total Tax	**₹ 6240**	**₹ 3120**	**₹ 3120**

(6) Mr. Joshi purchased 250 shares of FV ₹ 100 for MV of ₹ 500. Find the sum invested. After taking 40% dividend, he sold all the shares when market value was ₹ 400. He paid 0.1% brokerage for each trading done. Find the percentage of profit or loss in the share trading. *(4 marks)*

Solution:

Shares Purchased : FV = ₹ 100, MV = ₹ 500

No. of shares = 250

$$\text{Brokerage per share} = \frac{0.1}{100} \times 500$$

= ₹ 0.5

∴ Cost of one share = MV + Brokerage

= 500 + 0.5

= ₹ 500.5

∴ Cost of 250 shares = 250 × 500.5

= ₹ 1,25,125

∴ **Sum invested by Mr. Joshi is ₹ 1,25,125.**

Shares sold : FV = ₹ 100, MV = ₹ 400

$$\text{Brokerage per share} = \frac{0.1}{100} \times 400$$

= ₹ 0.4

∴ Selling price of per share = MV – Brokerage

= 400 – 0.4

= ₹ 399.6

∴ Selling price of 250 shares = 250 × 399.6

= ₹ 99,900

Dividend received 40%.

$$\therefore \text{Dividend per share} = \frac{40}{100} \times 100$$

= ₹ 40

∴ Dividend on 250 shares = 40 × 250

= ₹ 10,000

∴ Mr. Joshi's income = ₹ 99,900 + ₹ 10,000

= ₹ 1,09,900

But, sum invested by Mr. Joshi = ₹ 1,25,125

∴ Loss = 1,25,125 – 1,09,900

= ₹ 15,225

$$\text{Loss Percent} = \frac{\text{Loss}}{\text{Amount invested}} \times 100$$

$$= \frac{15{,}225}{1{,}25{,}125} \times 100$$

= 12.17

∴ **Mr. Joshi incurred a loss of 12.17% in this trading.**

5. Probability

(1) Two customers Sumit and Amit are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on:
(i) the same day (ii) different days (iii) Consecutive days. *(4 marks)*

Solution:

Total possible ways of visiting shop by them

$= 5 \times 5 = 25$

$\therefore \; n(S) = 25$

(i) Let A be the event of visiting the shop by them on the same day.

$\therefore \; n(A) = 5$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{5}{25} = \frac{1}{5}$$

(ii) Let B be the event of visiting the shop by them on the consecutive days.

Sumit :	Tuesday	Wednesday	Thursday	Friday
Amit :	Wednesday	Thursday	Friday	Saturday

Amit :	Tuesday	Wednesday	Thursday	Friday
Sumit :	Wednesday	Thursday	Friday	Saturday

$\therefore \; n(B) = 8$

$$P(B) = \frac{n(B)}{n(S)} = \boxed{\frac{8}{25}}$$

(iii) Let C be the event of visiting the shop by them on different days.

$\therefore \; n(C) = 25 - 5 = 20$

$$P(C) = \frac{n(C)}{n(S)}$$

$$P(V) = \frac{20}{25} = \boxed{\frac{4}{5}}$$

(2) What is the probability than an ordinary year has 53 Sundays? *(4 marks)*

Solution:

An ordinary year has 365 days.

i.e. 52 weeks and 1 extra day.

52 weeks will have 52 Sundays.

The sample space for one extra day is

S = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

$\therefore \; n(S) = 7$

Let A be the event that the extra day is a Sunday.

A = {Sunday}

$\therefore \; n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{7}$$

$\therefore$ **The probability that an ordinary year has 53 Sundays is $\frac{1}{7}$.**

(3) What is the probability that a leap year has 53 Sundays ? *(4 marks)*

Solution:

A leap year has 366 days, which is equivalent to 52 weeks and 2 days.

52 weeks will have 52 Sundays.

The sample space for remaining two days can be as follows:

$\therefore$ S= {Sunday-Monday, Monday-Tuesday, Tuesday -Wednesday, Wednesday Thursday, Thursday-Friday, Friday-Saturday}

$\therefore \; n(S) = 7$

Let A be the event of getting 53rd Sunday in remaining 2 days.

A = {Saturday-Sunday, Sunday-Monday}

$\therefore \; n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

$\therefore$ **The probability that a leap year has 53 Sundays is $\frac{2}{7}$.**

(4) A missing helicopter is reported to have crashed somewhere in the rectangular region as shown in figure. What is the probability that it crashed inside the lake shown in the figure? *(4 marks)*

Solution:

For rectangular region:

Length = 9 km, Breadth = 4.5 km

Area of rectangular region = Length × Breadth

$= 9 \times 4.5$

$= 40.5$ sq. km

For rectangular Lake:

length = 9 – 6 = 3 km

breadth = 4.5 – 2 = 2.5 km

Area of rectangular lake = length × breadth

= 3 × 2.5

= 7.5 sq. km

Let A be the event that the helicopter crashed inside the lake.

$$P(A) = \frac{\text{Area of lack}}{\text{Area of rectangular region}}$$

$$= \frac{7.5}{40.5}$$

$$= \frac{70}{405}$$

$$P(A) = \boxed{\frac{5}{27}}$$

(5) Each coefficient in equation $ax^2 + bx + c = 0$ is obtained by throwing an ordinary die. Find the probability that the equation has real roots. *(4 marks)*

Solution:

$ax^2 + bx + c = 0$

a, b and c can be selected from 1, 2, 3, 4, 5, 6 in 6^3 ways.

$\therefore$ $n(S) = 216$

For the equation to have real roots,

$b^2 - 4ac > 0$

$b^2 \geq 4ac$

The values of a, b and c satisfying the above condition can be tabulated:

b	a and c	No. of favourable cases
2	(1, 1)	1
3	(1, 1), (1, 2), (2, 1)	3
4	(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (3, 1), (4, 1)	8
5	(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1), (5, 1) (6, 1)	14
6	(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1) (6, 1)	17

There are 43 favourable caseds.

Let A be the event that the value of a, b, c selected in such a way that the roots of equation are real.

$\therefore$ $n(A) = 43$

$$P(A) = \frac{n(A)}{n(S)} = \boxed{\frac{43}{216}}$$

$$\therefore \quad P(A) = \boxed{\frac{43}{216}}$$

(6) A box contains 12 balls, out of which x are black,
(i) If one ball is drawn at random, what is the probability that it will be a black ball?
(ii) If 6 more black balls are put in the bag, the probability of drawing a black ball will double than that in (i), find x. *(4 marks)*

Solution:

(i) Total number of balls = 12

Number of black balls = x

$\therefore$ Probability of getting a black ball = $\frac{x}{12}$

(ii) If 6 more black balls added in the box, then total number of black balls = $x + 6$

Total no. of balls in the box = 12 + 6 = 18

$\therefore$ Probability of getting a black balls = $\frac{x+6}{18}$

According to given condition,

$$\frac{x+6}{18} = 2 \times \left(\frac{x}{12}\right)$$

$$\therefore \quad \frac{x+6}{18} = \frac{x}{6}$$

$\therefore$ $6x + 36 = 18x$

$\therefore$ $36 = 18x - 6x$

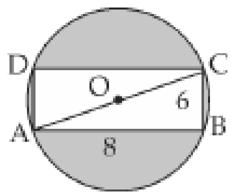
$\therefore$ $36 = 12x$

$\therefore$ $x = \frac{36}{12}$

$\therefore$ $x = 3$

$\therefore$ **Number of black balls are 3.**

(7) In the adjoining figure, a dart thrown lands in the interior of the circle. What is the probability that the dart will land in the shaded region. *(4 marks)*



Solution:

☐ABCD is a rectangle.

$\therefore$ m $\angle ABC = 90°$ [Angle of a rectangle]

In Δ ABC,

$AC^2 = AB^2 + BC^2$ [Pythagoras theorem]

$= 8^2 + 6^2$

$= 64 + 36$

$\therefore$ $AC^2 = 100$

$\therefore$ AC = 10 units

$\therefore$ radius of circle $= \frac{10}{2} = 5$ units.

Area of circle $= \pi r^2$

$= 3.14 \times 5^2$

$= 3.14 \times 25$

$= 78.5$ sq. units

Area of rectangle ABCD $= l \times b$

$= 8 \times 6$

$= 48$ sq. units

$\therefore$ Area of shaded region = Area of circle – Area of rectangle

$= 78.5 - 48$

$= 30.5$ sq. units

Let A be the event that dart lands in the shaded region.

$$P(A) = \frac{\text{Area of Shaded region}}{\text{Area of circle}}$$

$$= \frac{30.5}{78.5}$$

$$= \frac{305}{785}$$

$$\therefore P(A) = \boxed{\frac{61}{157}}$$

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6. Statistics

(1) The mean of the following frequency distribution is 50. Find the value of f: *(4 marks)*

Class interval	10 - 20	20 - 40	40 - 60	60- 80	80 - 100
Frequency	14	f	32	24	19

Solution:

Class intervals	Class marks (x_i)	frequency (f)	$f_i x_i$
0 - 20	10	17	170
20 - 40	30	f	$30f$
40 - 60	50	32	1600
60 - 80	70	24	1680
80 - 100	90	19	1710
Total		Σf_i	$\Sigma f_i x_i =$ **5160 + 30 f**

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

$$\therefore \quad 50 = \frac{5160 + 30f}{92 + f}$$

$\therefore$ $50(92 + f) = 5160 + 30f$

$\therefore$ $4600 + 50f = 5160 + 30f$

$\therefore$ $50f - 30f = 5160 - 4600$

$\therefore$ $20f = 560$

$\therefore$ $f = \frac{560}{20} = 28$

$\therefore$ $\boxed{f = 28}$

(2) An incomplete frequency distribution is given as follows:

Class interval	Frequency
10 - 20	12
20 - 30	30
30 - 40	?
40 - 50	65
50 - 60	?
60 - 70	25
70 - 80	18
Total	**229**

Given that median value is 46, determine the missing frequencies using the medians formula. *(4 marks)*

Solution:

Median is 46 it lies in the class 40 - 50 and the corresponding frequency is 65.

Classes	Frequency	Cumulative frequency (c.f.)
10 - 20	12	12
20 - 30	30	42
30 - 40	f_1	$42 + f_1$
40 - 50	$65 \rightarrow f$	$42 + f_1 + 65 = 107 + f_1$
50 - 60	f_2	$107 + f_1 + f_2$
60 - 70	25	$107 + f_1 + f_2 + 25 = 132 + + f_1 + f_2$
70 - 80	18	$132 + f_1 + f_2 + 18 = 150 + + f_1 + f_2 = 229$
Total	**229**	

From the last $c.f.$

$150 + f_1 + f_2 = 229$

$\therefore$ $f_1 + f_2 = 229 - 150$

$\therefore$ $f_1 + f_2 = 79$

$\therefore$ $f_2 = 79 - f_1$ ....(i)

$$\text{Median} = L + \left(\frac{N}{2} - c.f.\right)\frac{h}{f}$$

Where L = 40, N = 229, $c.f. = 42 + f_1$, $h = 10$, $f = 65$

$$\text{Median} = 40 + \left(\frac{229}{2} - (42 + f_1)\right)\frac{10}{65}$$

$$\therefore \quad 46 = 40 + \left(\frac{229}{2} - (42 + f_1)\right)\frac{2}{13}$$

$$\therefore \quad 46 - 40 = \left(\frac{229}{2} - 42 - f_1\right)\frac{2}{13}$$

$$\therefore \quad 6 \times \frac{13}{2} = \frac{229}{2} - 42 - f_1$$

$$\therefore \quad 39 = \frac{229}{2} - 42 - f_1$$

$$\therefore \quad 78 = 229 - 84 - 2f_1$$

$$\therefore \quad 2f_1 = 229 - 84 - 78$$

$$\therefore \quad 2f_1 = 67$$

$$\therefore \quad f_1 = \frac{67}{2}$$

$$\therefore \quad f_1 = 33.5 = 34$$

$$\therefore \quad f_2 = 79 - f_1 \qquad \text{[From (i)]}$$

$$\therefore \quad f_2 = 79 - 34$$

$$\therefore \quad f_2 = 45$$

Hence, $f_1 = 34$ and $f_2 = 45$

(3) The following data gives the information on the observed life time (in hour) of 225 electrical components:

Lifetime (in hours)	Frequency
0 - 20	10
20 - 40	35
40 - 60	52
60 - 80	61
80 - 100	38
100 - 120	29

Determine the modal life time of the components. *(3 marks)*

Solution:

The class having maximum frequency is 60 - 80 is the modal class.

Lifetime (in hour)	Frequency
0 - 20	10
20 - 40	35
40 - 60	$52 \rightarrow f_1$
60 - 80	$61 \rightarrow f_0$
80 - 100	$38 \rightarrow f_2$
100 - 120	29

The modal class is 60 - 80 and L = 60, $f_1 = 52$, $f_0 = 61$, $f_2 = 38$, $h = 20$

$$\text{Mode} = L + \left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2}\right)h$$

$$= 60 + \left(\frac{61 - 52}{2(61) - 52 - 38}\right)20$$

$$= 60 + \left(\frac{9}{122 - 90}\right)20$$

$$= 60 + \left(\frac{9}{32}\right)20$$

$$\therefore \quad \text{Mode} = 60 + 5.625 = 65.625$$

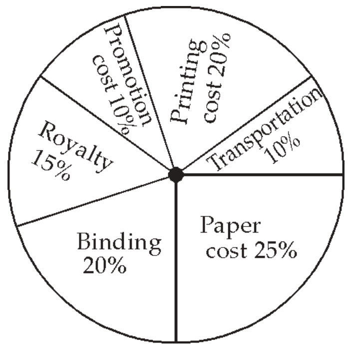
$\therefore$ **Mode = 65.625 hours.**

(4) The adjoining pie-diagram shows the percentage distribution of the expenditure incurred in publishing a book. Study the diagram and answer the questions:

(a) If for certain quantity of books, the publisher has to pay ₹ 30,600 as printing, then what will be the amount of royalty to be paid for these books?

(b) What is the central angle of the sector corresponding to the expenditure incurred on Royalty? *(3 marks)*

Solution:



(a) Let the total cost for certain quantity of books be ₹ 'x'

Printing cost = 20% of x

$$30600 = \frac{20}{100} \times x$$

$$\frac{30600 \times 100}{20} = x$$

$$x = ₹\ 1,53,000$$

Royalty paid for these books = 15% of total cost

$$= \frac{15}{100} \times 153000$$

$$= 15 \times 1530$$

$$= ₹\ 22,950$$

(b) Measure of central angle for Royalty

$$= \frac{15}{100} \times 360 = 54°$$

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Model Activity Sheet – 1

Time : 2 Hrs. **Marks : 40**

Q.1. (A) Attempt any four of the following questions: **(4)**

(1) What is the ratio of 1 mm to 1 cm?

(2) Yamini and Fatima, two students of class X of a school, together contributed ₹ 1500 towards the Prime minister's Relief fund to help the flood victims. Write a linear equation using two variables which satisfies the data.

(3) If T = { 1, 2, 3, 4, 5} and M = { 4, 5, 6, 7, 8}, then what is $T \cap M$?

(4) What is the class mark of 45 – 55.

(5) Write the following sum in simplest form.

$\frac{-4}{5}\sqrt{75}$

(6) Find the value of $\frac{a+9b}{a-9b}$ if $\frac{a}{b} = \frac{3}{7}$

Q.1. (B) Solve the following questions: (Any 2) **(4)**

(1) Determine whether $(x + 1)$ is a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$ or not.

(2) Mrs. Hinduja's age is 50 years. Last year her taxable income was ₹ 16,30,000. How much income tax has she to pay?

(3) Consider a small unit of a factory where there are 5 employees, a supervisor and four labourers. The labourers draw a salary of ₹ 9,000 per month each while the supervisor get ₹ 23,000 per month. Calculate median salary of this unit of factory.

Q.2. (A) Choose the correct alternative: **(4)**

(1) If one card is drawn from well shuffled deck of 52 cards, then probability of getting red face card is.

(a) $\frac{3}{13}$ (b) $\frac{3}{4}$ (c) $\frac{3}{26}$ (d) $\frac{1}{13}$

(2) $\sqrt{5}\,m^2 - \sqrt{5}\,m + \sqrt{5} = 0$ which of the following statement is true for this given equation?

(a) Real and unequal roots (b) Real and equal roots (c) Roots are not real (d) Three roots

(3) The equations 3x – 4y = 7 and 9x – 12y = 12 will have

(a) Unique solution (b) Infinitely many solutions

(c) No solution (d) Cannot be determined

(4) 28% GST was charged on a scooter having cost ₹ 50,000 then find the amount of CGST charged.

(a) ₹ 8000 (b) ₹ 7500 (c) ₹ 14000 (d) ₹ 7000

Q.2. (B) Solve the following: (Any 2) **(4)**

(1) Find the 25^{th} term of A.P 12, 16, 20, 24, ...

(2) Obtain the quadratic equation if roots are –3, –7

(3) The record of a weather station shows that out of the past 250 consecutive days, if weather forecasts were correct 175 times. What is the probability that on a given day it was correct?

Q.3. (A) Complete the following activities: (Any 2) **(4)**

(1) If $x = 5$ is the root of a equation $Kx^2 - 14x - 5 = 0$, then find the value of K by completing the following activity?

One of the roots of equation $Kx^2 - 14x - 5 = 0$ $\boxed{}$

$\therefore$ New let $x =$ $\boxed{}$ in the equation.

K $\boxed{}^2 - 14 \boxed{} - 5 = 0$ the equation.

$25K - 70 - 5 = 0$

$25K - \boxed{} = 0$

$K = \dfrac{\boxed{}}{\boxed{}} = 3$

(2) Form a Road safety committee of two from 2 boys (B1, B2) and 2 girls (G1, G2). Complete the following activity to write the sample space.

(a) Committee of 2 boys = $\boxed{}$

(b) Committee of 2 girls = $\boxed{}$

(c) Committee of one boy and one girl

$\boxed{B, G}$ $\boxed{}$ $\boxed{}$ $\boxed{}$

$\therefore$ Sample space = $\{\boxed{}, \boxed{}, \boxed{}, \boxed{}, \boxed{}, \boxed{}\}$

(3) Complete the following table:

	Face value	The share is at	Market value
(i)	₹ 100	discount of ₹ 15	....
(ii)	₹ 25	....	₹ 360

Q.3. (B) Solve the following questions: (Any 2) **(4)**

(1) The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35. Find the equation.

(2) Prashant bought 50 shares of FV ₹ 100 having MV ₹ 180. Company gave 40% dividend on the shares. Find the rate of return on investment.

(3) Medical check-up of 180 women was conducted in a health centre in a village. 50 of them were short of haemoglobin, 10 suffered from cataract and 25 had respiratory disorders. The remaining women were healthy. Show the information in a pie chart.

Q.4. Solve the following questions: (Any 3) **(9)**

(1) Fifty seeds were selected at random from each of 5 bags of seeds, and were kept under standardised conditions favourable to germination. After 20 days, the number of seeds which had geminated in each collection were conducted and recorded as follows

Bag	1	2	3	4	5
No. of seeds germinated	40	48	42	39	41

What is the probability of germination of

(i) More than 40 seeds in a bag.

(ii) 49 seeds in a bag

(iii) More than 35 seeds in a bag

(2) Solve $5x - 6y + 30 = 0$ and $5x + 4y - 20 = 0$ graphically.

(3) A trader from Surat, Gujarat sold cotton clothes to a trader in Rajkot, Gujarat. The taxable value of cotton clothes is ₹ 2.5 lacs. What is the amount of GST at 5% paid by the trader in Rajkot.

(4) Find the sum of all odd numbers from 1 to 150.

Q.5. Solve the following: (Any 1) **(4)**

(1) If the sum of first p terms of an A.P is equal to the sum of the first q terms, then show that the sum of its first $(p + q)$ terms is zero. $(p \neq q)$

(2) M/s. Jay Chemicals purchased a liquid soap for ₹ 8000 (with GST) and sold it to the consumers for ₹ 10,000 (with GST). Rate of GST is 18%. Find the amount of CGST and SGST to be paid by Jay chemicals.

Q.6. Solve the following questions: (Any 1) **(3)**

(1) The sum of father's age and twice the age of his son is 70. If we double the age of the father and add it to the age of his son the sum is 95. Find their present ages.

(2) Grouped frequency distribution of supply of milk to hotels and the number of hotels is given in the following table. Find the mode of the supply of milk.

Milk (litre)	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
No. of hotels	7	5	15	20	35	18

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Model Activity Sheet – 2

Time : 2 Hrs. **Marks : 40**

Q.1. (A) Attempt any four of the following questions: **(4)**

(1) Find the median of the observations 10, 7, 5, 3, 9, 6, 9.

(2) Mr. Ahmed's payable amount of income tax is ₹ 8,000. He needs to pay education cess at 2% on income tax. Hence how much total income tax will he have to pay?

(3) Write any two solutions for the equation $x - 2y = 5$.

(4) a, b, c are in continued proportion. If $a = 3$ and $c = 27$ then find b.

(5) Add the following polynomials. $5x^3 + 2x + 9$, $x^2 - 7x - 9$

(6) Write the following surd in simplest form.

$$\frac{5}{9}\sqrt{45}$$

Q.1. (B) Solve the following questions: (Any 2) **(4)**

(1) If A = {1, 2, 3, 4, 5}, B = {3, 4, 7, 8} then find $A \cup B$ and $A \cap B$.

(2) Write the mathematical form of the statement: 'When 5 is subtracted from length and breadth of the rectangle, the perimeter becomes 26'.

(3) If $\frac{a}{b} = \frac{2}{3}$ then find the value of $\frac{4a + 3b}{3b}$.

Q.2. (A) Choose the correct alternative: **(4)**

(1) One of the roots of equation $x^2 + mx - 5 = 0$ is 2; find m.

(a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2

(2) 15, 10, 5, ... In this A.P. sum of first 10 terms is ....

(a) -75 (b) -125 (c) 75 (d) 125

(3) In the format of GSTIN there are .... alpha-numerals.

(a) 15 (b) 10 (c) 16 (d) 9

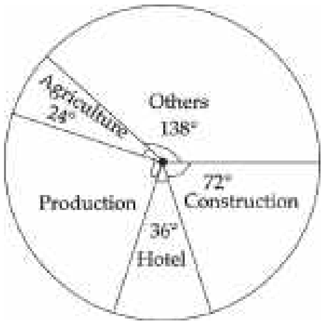
(4) The probability of getting a prime number in single throw of a dice is ....

(a) zero (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Q.2. (B) Solve the following: (Any 2) **(4)**

(1) If one coin and one die are thrown simultaneously then find the probability of getting head and prime number.

(2) As deduced from a survey, the classification of skilled workers is shown in the pie diagram. If the number of workers in the production sector is 4500, then what is the total number of skilled workers in all fields?



(3) The age group and number of persons, who donated blood donation camp is given below. Draw a pie diagram from it.

Age group (years)	20 - 25	25 - 30	30 - 35	35 - 40
No. of persons	80	60	35	25

Q.3. (A) Complete the following activities: (Any 2) **(4)**

(1) Find the sum of these 75 numbers. 1 + 3 + 5 + ... + 149.

$S_n = \frac{n}{2}[t_1 + t_n]$

$\therefore S_n = \frac{75}{2}[\boxed{} + \boxed{}]$

$\therefore S_n = \boxed{} \times \boxed{}$

$\therefore S_n = \boxed{}$

(2) Smita has invested ₹ 12,000 and purchased shares of FV ₹ 10 at a premium of ₹ 2. Find the number of shares she purchased. Complete the given activity to get the answer.

Solution:

FV = ₹ 10, Premium = ₹ 2

$\therefore$ MV = FV + $\boxed{}$

= $\boxed{} + \boxed{}$

$\therefore$ MV = ₹ $\boxed{}$

Number of shares = $\frac{\text{Total investment}}{\text{MV}}$

= $\frac{12000}{\boxed{}}$

= $\boxed{}$ shares.

(3) Complete the following activity.

There are 52 cards in a pack.

In each pack, total black face cards are $\boxed{}$.

There are $\boxed{}$ kings of red colour.

There are $\boxed{}$ cards which are not of diamond.

In each pack, total club cards are $\boxed{}$.

Q.3. (B) Solve the following questions: (Any 2) **(4)**

(1) Solve the following quadratic equation -

$5x^2 - 4x - 7 = 0$

(2) How many two digit numbers are divisible by 4?

(3) The weekly wages of 120 workers in a factory are shown in the following frequency distribution table. Find the mean of the weekly wages.

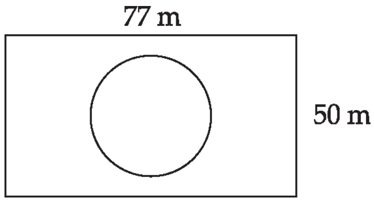
Weekly wages (₹)	0 - 2000	2000 - 4000	4000 - 6000	6000 - 8000
No. of workers	15	35	50	20

Q.4. Solve the following questions: (Any 3) **(9)**

(1) A certain amount is equally distributed among certain number of students. Each would get ₹ 2 less if 10 students were more and each would get ₹ 6 more if 15 students were less. Find the number of students and the amount distributed.

(2) There are 37 terms in an A.P., the sum of three terms placed exactly at the middle is 225. and the sum of last three terms is 429. Write the A.P.

(3) Shruti invested ₹ 6024 in the share of FV ₹ 10 when the market value was ₹ 60. She sold all the shares at MV of ₹ 50 after taking 60% dividend. She paid 0.4% brokerage at each stage of transactions. What was the total gain or loss in this transaction?

(4) 

Length and breadth of a rectangular garden are 77 m and 50 m respectively. There is a circular lake in the garden having diameter 14 m. Due to wind, a towel from a terrace on a nearby building fell into the garden. Then find the probability of the event that it fell in the lake.

Q.5. Solve the following: (Any 1) **(4)**

(1) Two tractors, one moving towards north and the other towards east, leave the same place at the same time. The speed of one of them is greater than that of the other by 5 km/hr. At the end of two hours they are at a distance of 50 km from each other. Find the speed of each tractor.

(2) Draw the graphs representing the equations $2x = y + 2$ and $4x + 3y = 24$ on the same graph paper. Find the area of the triangle formed by these lines and the X axis.

Q.6. Solve the following questions: (Any 1) **(3)**

(1) **Tax Invoice:**

Tax Invoice of goods purchase (Sample)

Supplier : A to Z Sweet Mart GSTIN : 27 ABCDE1234H1Z5
143, Shivaji Rasta, Mumbai : 400001 Maharashtra
Mob. No. 9263692111 email: atoz@gmail.com

Invoice No. GST / 110 Invoice Date : 31 July 2017

S. No	HSN Code	Name of Product	Rate	Quantity	Taxable Amount	CGST		SGST		Total
						Rate	Tax	Rate	Tax	₹
(1)	210690	Pedhe	₹ 400 per kg.	500 gm	200.00	2.5%	5.00	2.5%	5.00	210.00
(2)	210691	Chocolate	₹ 80	1 Bar	80.00	14%	11.20	14%	11.20	102.40
(3)	2105	Ice cream	₹ 200 per pack	1 Pack (500 gm)	200.00	9%	18.00	9%	18.00	236.00
(4)	1905	Bread	₹ 35	1 Pack	35.00	0%	0.00	0%	0.00	35.00
(5)	210690	Butter	₹ 500 per kg.	250 gm	125.00	6%	7.50	6%	7.50	140.00
						Total ₹	**41.70**		**41.70**	**723.40**

Observe the given bill and fill in the boxes with the appropriate number.

- CGST at the rate of 2.5% is ₹ $\boxed{}$
- Rate of GST on the ice-cream is $\boxed{}$ %, hence the total cost of ice-cream is ₹ $\boxed{}$.
- On butter CGST rate is $\boxed{}$ %, and SGST rate is also $\boxed{}$ % So GST rate on butter is $\boxed{}$ %.

(2) A two digit number is to be formed from the digits 0, 1, 2, 3, 4. Repetition of the digits is allowed. Find the probability that the number so formed is a ....

(a) multiple of 4 (b) prime number.

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