Aircraft Performance - Physics and Simulation

Simulation assignment – Climb and descent

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Introduction

The general objective of this assignment is to perform realistic calculations of the climb and descent phase by means of a numerical simulation model, for a typical large commercial transport aircraft with four wing-mounted turbofan engines. The general equations of motion for symmetric climbing flight are the following:

$$\frac{W}{g}\frac{dV}{dt} = T\cos\alpha_T - D - W\sin\gamma$$

$$\frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma + T\sin\alpha_T$$

Data required for the simulation model

The following data are available for a generic large transport aircraft with four turbofan engines:

Mass and dimensions

Maximum Take-off Weight: $MTOW = 3,600,000$ N

Useable fuel mass: $W_{fuel} = 1,600,000$ N

Wing surface area: $S = 500$ m^2

Aerodynamic characteristics

The parabolic lift drag polar of this aircraft is dependent on the Mach number according to the following table:

Mach number	$C_D = C_{D_0} + kC_L^2$	
	C_{D_0}	k
0.3	0.0132	0.056
0.5	0.0131	0.057
0.6	0.0131	0.058
0.7	0.0130	0.061
0.8	0.0130	0.067
0.85	0.0128	0.074

Table 1: High speed lift drag polars (clean configuration)

The lift coefficient has a linear relation with the angle of attack (α).

$$C_L = 0.03 + 4.4\alpha \ (\alpha \text{ in rad})$$

Operational conditions

Atmosphere: International standard atmosphere (ISA)

Gravitational acceleration: $g = 9.80665$ m/s^2

Propulsion system characteristics

Maximum sea level static thrust for a single engine (take-off rating – 100%): $T_0 = 270$ [kN]

Bypass ratio: $BPR = 5$

To determine the variation of maximum engine thrust (T) with altitude and Mach number (M), the following set of equations can be used. This set of equations is based on the work of Bartel and Young[1].

$$\frac{T}{T_0} = A - \frac{0.377(1+BPR)}{\sqrt{(1+0.82BPR)G_0}} ZM + (0.23 + 0.19\sqrt{BPR})XM^2$$

The empirical parameters A, X and Z are altitude dependent according to the following relations

$$A = -0.4327\left(\frac{p}{p_0}\right)^2 + 1.3855\left(\frac{p}{p_0}\right) + 0.0472$$

$$Z = 0.9106\left(\frac{p}{p_0}\right)^3 - 1.7736\left(\frac{p}{p_0}\right)^2 + 1.8697\left(\frac{p}{p_0}\right)$$

$$X = 0.1377\left(\frac{p}{p_0}\right)^3 - 0.4374\left(\frac{p}{p_0}\right)^2 + 1.3003\left(\frac{p}{p_0}\right)$$

Finally, the parameter G_0 is a function of the bypass ratio (BPR)

$$G_0 = 0.6375 + 0.0604BPR$$

The engines are aligned with the nose of the aircraft. Hence, $\alpha_T = \alpha$.

The thrust specific fuel consumption (c_T) is a function of Mach number and altitude.

$$F = c_T T$$

$$\frac{c_T}{\sqrt{\theta}} = 11(1+M)$$

The thrust specific fuel consumption has the units [mg/s / N]. The parameter θ is defined as the temperature relative to sea level conditions.

$$\theta = \frac{T}{T_{sealevel}}$$

This essentially makes the thrust specific fuel consumption dependent on the altitude.

Pilot behaviour

After take-off, various dynamic flight segments are flown in which the landing gear is retracted, the flaps are retracted and the aircraft is accelerated up to the airspeed for the en-route climb. For the en-route climb, a high speed climb procedure is performed. It starts at 2000 m altitude and is completed at

[1] Bartel, M., Young, T. M., "Simplified thrust and fuel consumption models for modern two-shaft turbofan engines," *Journal of aircraft*, Vol. 45, No. 4, 2008, pp. 1450-1456.

10.000 m altitude. The weight at the start of the en-route climb is 3,000,000 N (which is lower than the maximum take-off weight).

During the en-route climb phase, the pilot will apply maximum climb thrust, which is 95% of the maximum take-off thrust at that altitude. Furthermore, the indicated airspeed is kept constant at 170 m/s during the climb up to the altitude (crossover altitude) where the aircraft reaches the cruise Mach number ($M = 0.85$). Above the crossover altitude, the Mach number is kept constant at 0.85. It can be assumed that the indicated airspeed is the same as the calibrated airspeed (CAS). **Note that this results in an accelerated flight, hence the true airspeed (TAS) is not constant** (see video about “en-route climb”). The indicated airspeed is controlled by applying pitch changes. In case the aircraft flies too fast, the pilot will increase the pitch attitude and in case it flies too slow, it will decrease the pitch attitude. Pitch attitude changes are applied proportional to the speed error.

$$\theta = \left(V - V_{ref}\right) K_P + \theta_{trim}$$

The proportional gain K_P can be found through trial and error using the simulation model. It is recommended to start with small values and slowly increase these until a good response is found. The trim pitch attitude is the attitude required at the start of the climb simulation (a positive pitch attitude and thus a positive angle of attack is required in case the speed error is zero)

Assignments

Task 1
At 2000 meter altitude, the aircraft is assumed to be in a steady and straight flight condition (constant true airspeed and constant flight path angle). The indicated airspeed equals 170 m/s. Create a simulation model that calculates the required values for the aircraft state and control variables at initial condition (true airspeed, flight path angle, angle of attack, thrust setting). The aircraft weight at the start of the en-route climb is assumed to be equal 3,000,000 N.

Task 2
Simulate the en-route climb up to an altitude of 10.000 meter. Note that a suitable value for the gain K_P representing the pilot should be found manually.

Results

First create a figure with the workflow of your simulation program in a PDF and upload it. The results of the simulations (task 1 – 2) must be combined in a single PDF file and uploaded. At least the following figures should be created.

- Horizontal distance travelled relative to the ground (y-axis) – time (x-axis)
- Altitude (y-axis) – time (x-axis)
- Altitude (y-axis) - Horizontal distance travelled relative to the ground (x-axis)
- True air speed (x-axis) – Altitude (y-axis)
- Mach number (y-axis)– time (x-axis)
- True air speed (y-axis) – time (x-axis)
- Fuel burn (y-axis) – time (x-axis)
- Flight path angle (y-axis) – time (x-axis)

Make sure these figures are clearly annotated. If needed, additional figures with other parameters can be included. It is allowed to support your figures with text but this is not mandatory. If you provide supporting text and / or additional figures, make sure the report is concise.

Optional assignments

The assignments in this section are optional and **not mandatory** to complete.

Task 1

Extend the program to allow for descent simulations. For this, a low throttle setting must be used. The minimum thrust setting (flight idle), is 5% of the maximum take-off thrust.

Task 2

Use the simulation program to investigate climb performance for various aircraft weights and airspeeds. Note that the airspeed used in the first task is the maximum value.