Michelson Interferometry

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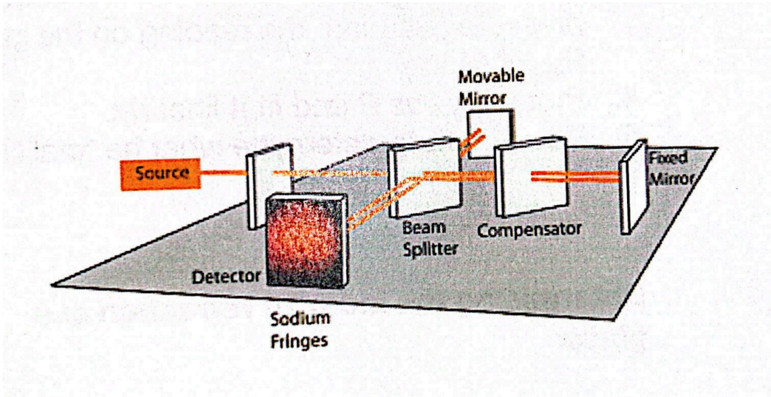


Introduction:

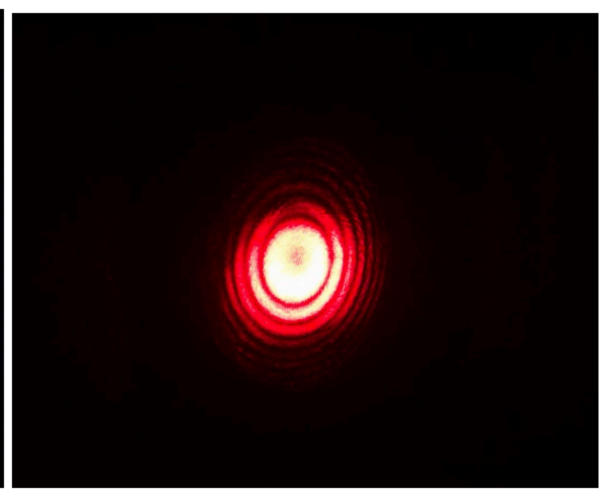
The Michelson interferometer operates on the principles of interference, a phenomenon that occurs when two waves interact. Specifically, the Michelson interferometer is used to analyze the interactions between light beams. When light beams interfere, an interference pattern can be observed. Changes in the interference pattern can be used to solve many things such as calculating the speed of light with very high precision. What we learn depends on the method of path length adjustment. Adjusting the path length difference between two laser beams causes changes in the resulting interference pattern. We can analyze this relationship and derive wonderful formulas from it. A great example is the interferometer that LIGO uses. The mirrors are 4 km away from the beam splitter in order to measure changes due to gravitational waves. In the case of this experiment, we will first analyze the effects of light passing through a cell that is decreasing in air pressure, then analyze the effects of the light's incident angle on a glass lens. In **Part 1** there is a picture of the interference pattern due to the beam splitter with and without a lens. **Part 2** includes the graphs and the index of refraction for the glass. **Part 2 Analysis** includes the calculation for the index of refraction for air. **Part 3** includes the analysis for phase shift through the glass. Data, code, and figures of experimental setup are in the appendix.

Part 1:

Two beams of light are sent orthogonal to each other and then recombined to produce an interference pattern. The two beams originate from a singular laser source which passes through a beam splitter. A fixed and a movable mirror reflect the orthogonal beams back to the beam splitter to realign them. The two beams have acquired a phase shift with respect to each other, so an interference pattern can be observed.

Procedure: Power the laser. Without the lens, position the lens holder along the path of the laser, so the laser goes through the center of the lens holder. Position the fixed mirror so the laser hits the center and is perfectly aligned. Put the beam splitter in the center of the apparatus, at 45° with respect to the laser. Place the movable mirror along the orthogonal beam path created by the beam splitter. Check that the interference pattern depends on the angle of the movable mirror. Adjust the mirror so that the multiple laser reflection overlaps as much as possible. Take a picture of the interference pattern (picture on left). Place the lens in the lens holder and take a picture of the interference pattern (picture on right).

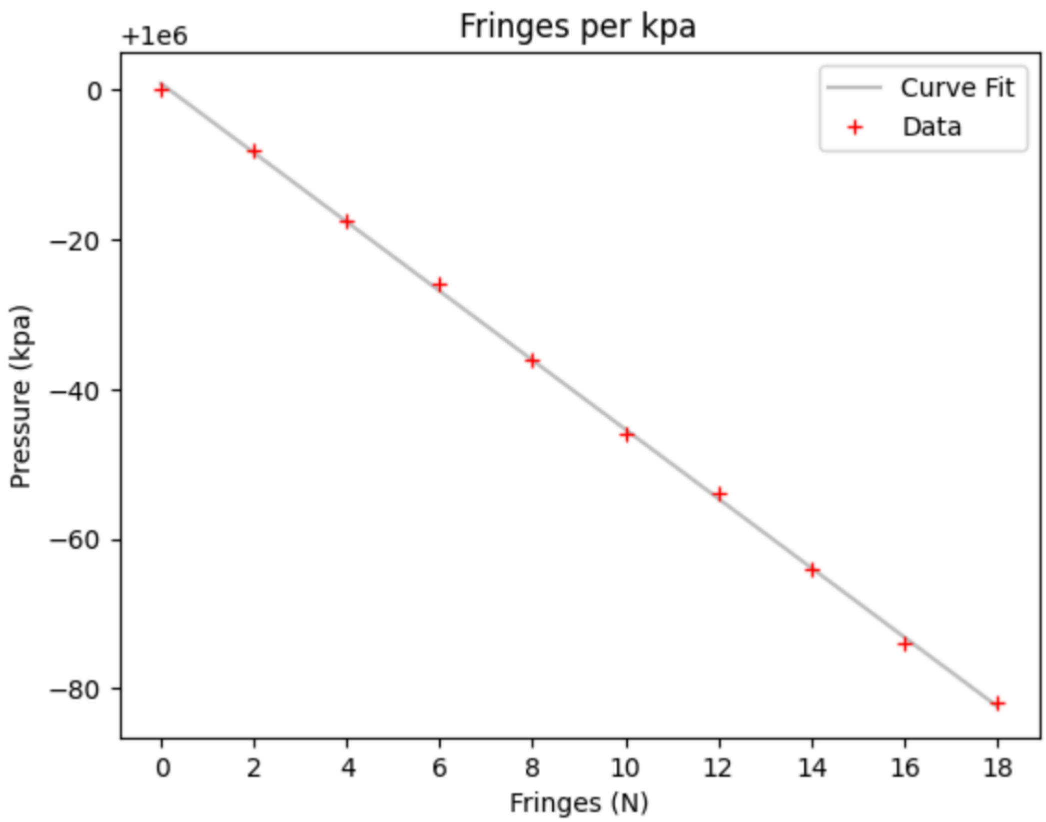
Pictures:



Part 2: Air

Procedure: With the same setup as in **Part 1**, position a component holder orthogonal to the laser source. Place a vacuum cell at atmospheric pressure in the component holder. Check that the pattern still depends on the angle of the mirror. Decrease the pressure in the vacuum cell until two fringes have passed. Record the pressure in kilo-pascals. Repeat this for a total of 18 fringes. Plot the number of fringes vs pressure and provide a linear curve fit. Using the curve fit, determine the number of fringes when the pressure equals zero (0 kpa).

Plot:



Fringes when P = 0 kpa:

$$y = mx + b, \ m = -46,230.30, \ b = 1,000,000.85$$

When y = 0, $x = \frac{y-b}{m}$, $x = \frac{0-1e6}{-46230.3} = 21.63$

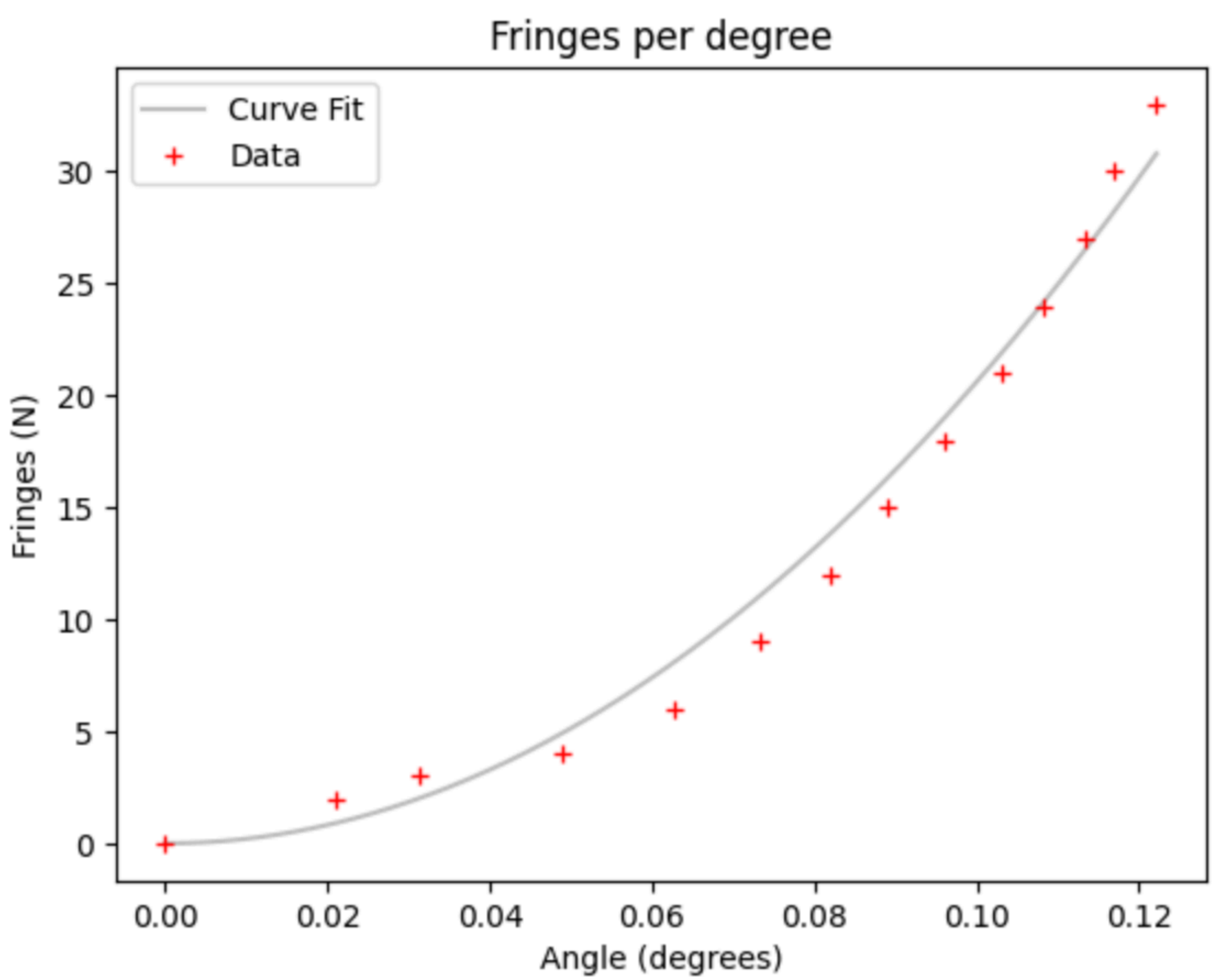
$$\boxed{N_{fringes} = 21.63, \ n_{air} = 1.000228}$$

Part 2: Glass

Procedure: With the same setup as in **Part 1**, add the rotational pointer with the test glass slide. (Reference image in appendix) Increase the angle of inclination of the rotational pointer until three fringes pass. Record the angle of inclination. Repeat this for a total of 33 fringes. Plot the number of fringes vs the angle of inclination and provide a curve fit using the equation derived in **Part 3**. Have python output an approximation for n.

Plot:

n = 1.2283



Part 2 Analysis:

d = 30 mm, λ = 633 nm

1) Determine the phase shift accumulated during one round trip.

Phase shift going through the cell is: $\varphi = \frac{2\pi dn}{\lambda}$

Due to round trip the phase shift happens twice: $\varphi = \frac{4\pi dn}{\lambda}$

$$\boxed{\varphi = \frac{4\pi dn}{\lambda}}$$

2) Determine the phase shift when it is fully pumped out.

We assume there is a perfect vacuum when the cell is fully pumped out. In other words,

n = 1.

$$\varphi = \frac{4\pi dn}{\lambda} = \frac{(4)(\pi)(0.03m)(1)}{(633e-9m)} = 595562.5884\, rad$$

$$\boxed{d\varphi = 595,563\ radians}$$

3) Determine n-1 as a function of $N_{fringes}$.

$$\varphi = \frac{4\pi d(n-1)}{\lambda},\ so\ (n - 1) = \frac{d\varphi\lambda}{4\pi d}$$

2π corresponds with a phase shift for one fringe. In terms of $N_{fringes}$, $\varphi = 2\pi N$

$$\varphi = 2\pi N,\ so\ (n - 1) = \frac{N\lambda}{2d}$$

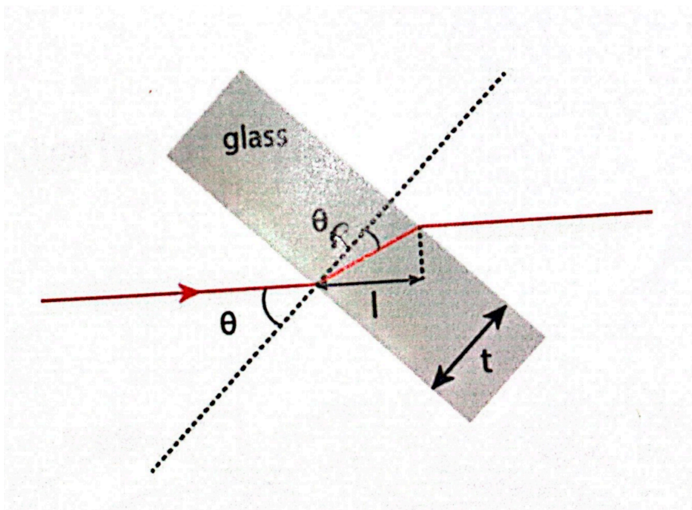
$$\boxed{\begin{gathered}(n - 1) = \frac{N\lambda}{2d} \\ n_{air} = \frac{(21.6)(633e-9)}{2(0.03)} + 1 = 1.000228\end{gathered}}$$

Part 3 Analysis:

d is the distance traveled through glass.

l is the distance the light would've traveled if in air.

t is the thickness of the slab.



1) **Show that the total phase shift is:** $\varphi = \frac{4\pi nt}{\lambda cos(\theta_R)}$.

The refractive index (n) of the glass affects the distance traveled and the wavelength of the light beam. The distance traveled (d) can be calculated using trigonometry as follows, $cos(\theta_R) = \frac{t}{d}$, $so\ d = \frac{t}{cos(\theta_R)}$. The wavelength (λ) is affected by n such that $\Delta\lambda = \frac{\lambda}{n}$. The difference in phase shift is calculated using the following formula, $\varphi = kd - \omega t$. The time traveled in the glass is minimal and thus negligible. Plugging in, $k = \frac{2\pi}{\lambda}$, $d = \frac{t}{cos(\theta_R)}$, $\Delta\lambda = \frac{\lambda}{n}$, $so\ \varphi = \frac{2\pi t}{\frac{\lambda}{n}cos(\theta_R)} = \frac{2\pi nt}{\lambda cos(\theta_R)}$. The light beam travels through the glass and hits a mirror then travels back through the glass, so total distance traveled is doubled. $k(2 * d) = 2 * \varphi = 2(\frac{2\pi nt}{\lambda cos(\theta_R)}) \ldots \varphi = \frac{4\pi nt}{\lambda cos(\theta_R)}$.

2) Determine I as a function of d.

On the diagram there is a right triangle draw with **I** and **d**. By analyzing the diagram we can see that $\theta = \theta - \theta_R$. Using trigonometry, $cos(\theta - \theta_R) = \frac{I}{d},\ so\ I = dcos(\theta - \theta_R)$.

3) Show that the phase shift for air with no glass would've been: $\varphi = \frac{4\pi tcos(\theta-\theta_R)}{\lambda cos(\theta_R)}$

From Q1, $d = \frac{t}{cos(\theta_R)}$. From Q2, $I = dcos(\theta - \theta_R)$. So, $d = \frac{t}{cos(\theta_R)} = \frac{I}{cos(\theta-\theta_R)}$. In the case of no glass, the distance traveled **d** is now **I**. $I = \frac{tcos(\theta-\theta_R)}{cos(\theta_R)}$. Now, $\varphi = k(2 * I)$, and $n = 1$ for air, so $\varphi = \frac{4\pi tcos(\theta-\theta_R)}{\lambda cos(\theta_R)}$.

4) Show that the phase shift accumulated because of the glass can be simplified to:

$$d\varphi = \frac{4\pi nt}{\lambda cos(\theta_R)} - \frac{4\pi tcos(\theta-\theta_R)}{\lambda cos(\theta_R)} = \frac{4\pi t}{\lambda}\left(\frac{n}{cos(\theta_R)} - cos(\theta) - sin(\theta\ )tan(\theta_R)\right)$$

The phase shift between the unobstructed wave and the wave passing through the glass can be determined by taking the difference between the two using

$d\varphi = \frac{4\pi nt}{\lambda cos(\theta_R)} - \frac{4\pi tcos(\theta-\theta_R)}{\lambda cos(\theta_R)}$. First, pull common terms out,

$$d\varphi = \frac{4\pi t}{\lambda}\left(\frac{n}{cos(\theta_R)} - \frac{cos(\theta-\theta_R)}{cos(\theta_R)}\right).$$

Using the trigonometric identity $cos(a - b) = cos(a)cos(b) + sin(a)sin(b)$, $cos(\theta - \theta_R) = cos(\theta)cos(\theta_R) + sin(\theta)sin(\theta_R)$. Plugging in,

$d\varphi = \frac{4\pi t}{\lambda}\left(\frac{n}{cos(\theta_R)} - \left(\frac{cos(\theta)cos(\theta_R)+sin(\theta)sin(\theta_R)}{cos(\theta_R)}\right)\right)$. Simplifying,

$d\varphi = \frac{4\pi t}{\lambda}\left(\frac{n}{cos(\theta_R)} - \frac{cos(\theta)cos(\theta_R)}{cos(\theta_R)} - \frac{sin(\theta)sin(\theta_R)}{cos(\theta_R)}\right)$. Simplify further to obtain the final expression, $d\varphi = \frac{4\pi t}{\lambda}\left(\frac{n}{cos(\theta_R)} - cos(\theta) - sin(\theta\ )tan(\theta_R)\right)$.

5) **Determine $cos(\theta_R)$ as a function of n and theta, using Snell's law.**

Snell's Law: $n_{air} sin(\theta) = n_{glass} sin(\theta_R)$, $n_{air} = 1$. Using the trigonometric identity

$sin^2(\theta) = 1 - cos^2(\theta)$, $1 - cos^2(\theta) = n_{glass}^{\ 2}(1 - cos^2(\theta_R))$. Rearrange to obtain,

$$cos(\theta_R) = \sqrt{1 - \frac{1-cos^2(\theta)}{n^2}}.$$

6) **Explain why the number of fringes that are observed is equal to:**

$$(\delta\varphi(\theta) - \delta\varphi(0))/2\pi$$

Fringes are 2π periodic; $\varphi = 2\pi N$, so $N = \frac{\varphi}{2\pi}$. $(\delta\varphi(\theta) - \delta\varphi(0))/2\pi =$

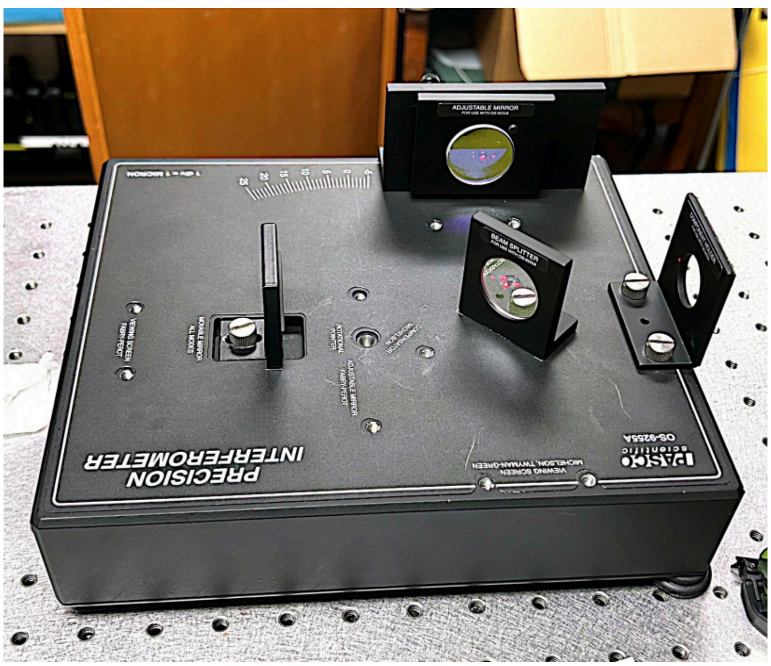
$$\frac{\frac{4\pi t}{\lambda}\left(\sqrt{n^2 - sin^2(\theta)} - cos(\theta) - n + 1\right)}{2\pi}.$$

Appendix:

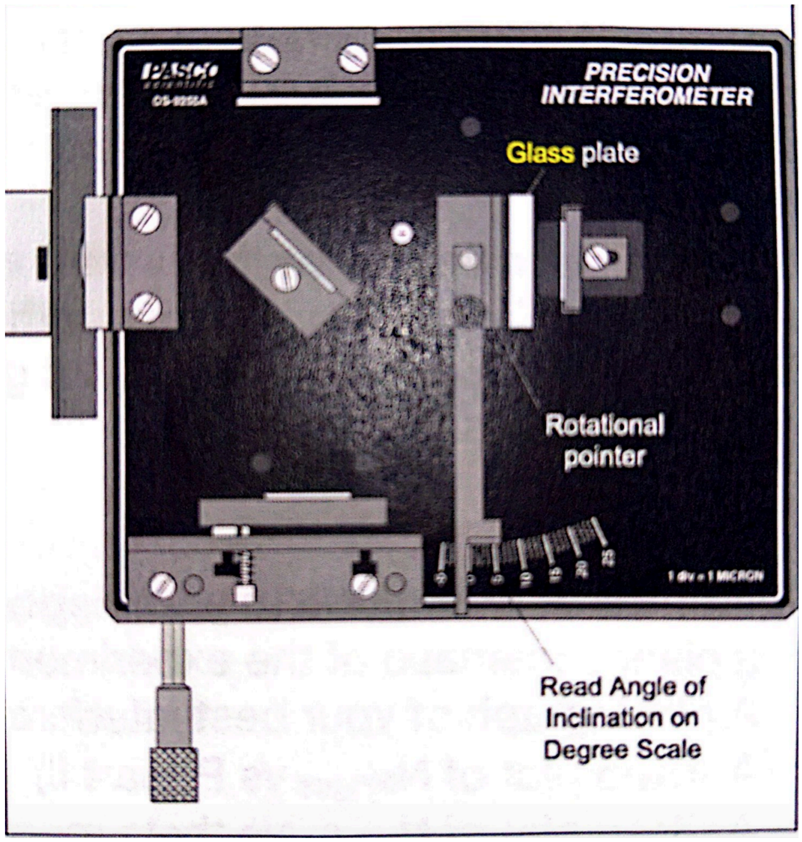
Code: https://colab.research.google.com/drive/1oGb1m3EQcAb8hNrUMQl8opsXcl3bOGGd?usp=sharing

Data: https://docs.google.com/spreadsheets/d/1GvfsLYyVGwp-HeScWdvVsHmjPcJ0DhQlWVrzkDyLCsY/edit?usp=sharing

Experimental Setup:



Part 2 Glass Slider:


PASCO
PRECISION INTERFEROMETER
Glass plate
Rotational pointer
1 div = 1 MICRON
Read Angle of Inclination on Degree Scale