The Faraday Effect

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Abstract

This study investigates the Faraday effect using the TeachSpin FR1-A apparatus, aiming to verify Malus' law and determine the Verdet constant for SF-57 glass. By measuring the rotation of the polarization plane under various magnetic field strengths, I bridge theoretical predictions with experimental observations in magneto-optics. Additionally, I address the technical challenges encountered (detector saturation, component degradation, and thermal effects) to provide insights into systematic errors that can affect the accuracy of optical measurements.

1. Introduction

In the 1840s, Michael Faraday made a groundbreaking observation that forever altered our understanding of physics by linking the realms of electromagnetism and optics. Faraday discovered that the plane of polarization of light rotates as it passes through a medium subjected to a magnetic field. This shocking phenomenon, now known as the Faraday effect, provided one of the first experimental evidences of the interplay between light and magnetic fields. Faraday's experiments not only revealed a previously unknown physical phenomenon but also laid the conceptual groundwork for the field of magneto-optics. His work hinted at deeper connections between subatomic particles and electromagnetic interactions, sparking a wave of inquiry that would eventually influence theoretical frameworks and practical applications in physics. With the later advent of quantum mechanics, the underlying theoretical basis of the Faraday effect was refined, offering a complete picture of how light interacts with magnetic fields at the microscopic level.

Over the ensuing decades, what began as a curious laboratory observation has evolved into a robust and widely utilized tool in both research and industry. The Faraday effect now plays a pivotal role in enhancing the performance of optical isolators, devices essential for protecting sensitive laser systems from detrimental back reflections. Moreover, this magneto-optical phenomenon is instrumental in mapping the magnetic fields of the Milky Way, thereby contributing to our understanding of cosmic structures and dynamics. The ability to control and manipulate light using magnetic fields has opened up numerous avenues for innovation in optical communications, sensor technology, and beyond.

In this paper, I explore the nature of the Faraday effect through a detailed experimental investigation using the TeachSpin FR1-A apparatus. By directing polarized light through a sample of SF-57 glass in the presence of a controlled magnetic field, the experiment is designed to both reaffirm the validity of Malus' law

$I = I_O cos^2(\theta + \phi)$

and to provide an accurate estimation of the Verdet constant for the material. This constant, which quantifies the strength of the magneto-optic interaction, serves as a critical parameter in understanding and designing optical devices that rely on precise control of light polarization. Through careful calibration and rigorous data analysis, the study aims to bridge the gap between theoretical predictions and experimental observations, offering deeper insight into the sensitivity and potential applications of materials in precision magneto-optics.

2. Theory

The Faraday effect, as it is now known, is characterized by a rotation of the polarization plane that is directly proportional to the strength of the applied magnetic field and the distance that light travels through the material. This relationship is quantified by the Verdet constant, a material-specific parameter that encapsulates the extent of rotation per unit magnetic field and length. The rotation angle theta (θ) of the polarization plane is given by:

$\theta = \text{VBL}$
(Derivation in appendix 7.2.3)

Where **V** is Verdet's constant, a material-dependent parameter, **B** is the applied magnetic field strength, and **L** is the length of the material through which the light travels (SF-57 glass rod).

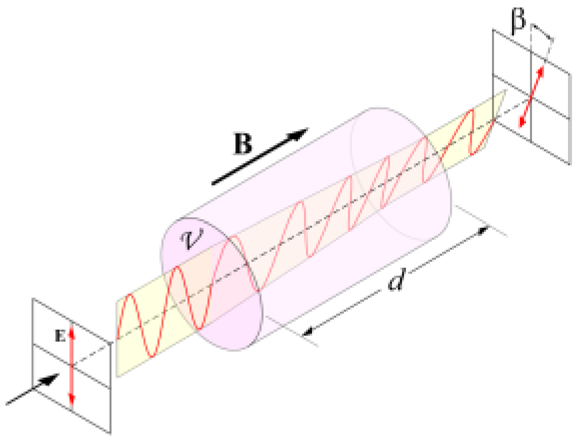

Figure 1. Faraday rotation schematic. This shows linearly polarized light (E) entering a material (V) under the influence of a magnetic field (B) across a distance (d). As the light exits there is a phase shift (β applied.

Verdet's constant for a given material can be determined experimentally using the Faraday effect. As the light travels through a material in the presence of a magnetic field it will rotate as depicted above. Verdet's constant (V) can be estimated by changing the magnetic field strength (B) and recording the change in angle (θ). Verdet's constant can be calculated directly by hand or by plotting the rotation angle against the magnetic field strength, and extracting the slope of the resulting line.

In order to understand the physics of rotation polarization, lets dive into the underlying nature of electromagnetic forces and magneto-optics. In classical terms, the Faraday effect is a consequence of the Lorentz force. Electrons moving through the medium experience a net force perpendicular to their motion. The electric and magnetic forces on an electron are described by Coulomb's law:

$\vec{F_E} = q\vec{E}$

$\vec{F_B} = q\vec{v} \times \vec{B}$

Where **q** is the charge of the electron, **v** is the velocity of the electron, **E** is the electric field strength and **B** is the magnetic field strength. F_E and F_B are the forces on the electron due to the electric and magnetic fields. The sum of these forces gives us the Lorentz force:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

This is the force that acts on electrons within a medium when exposed to an external magnetic field. When a magnetic field is applied parallel to the direction of propagation, the Lorentz force acts on the electrons, causing them to move perpendicular relative to their motion and the Lorentz force (right hand rule). As the electrons move, the direction of the Lorentz force changes, and the electrons follow helical trajectories.

Although the rotation is described by the Lorentz force, it does not fully describe the underlying physics of polarization. A more in-depth explanation requires quantum mechanics.

Linearly polarized light can be decomposed into two circularly polarized components; left-circularly polarized light (LCP) and right-circularly polarized light (RCP);

$$\hat{e}_L = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{y}), \quad \hat{e}_R = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{y})$$

The Zeeman effect tells us that these components experience different refractive indices, and this difference leads to a phase shift between them; a net rotation of the polarization plane. The different refractive indices arise from Zeeman splitting. Electrons orbit the atom at specific distances from the nucleus, called orbitals. Each orbital is described using discrete energy values. In the presence of a magnetic field, the energy levels of the electrons can split into several components (Zeeman splitting), affecting their interactions with the electromagnetic wave (light). The energy shift is given by:

$$\triangle E = m_l \mu_B B$$

Where m_l is the magnetic quantum number that describes the number of orbitals, μ_B is the Bohr magneton that describes the magnetic moment of an electron, and B is the magnetic field strength.

As the magnetic field strength increases, the energy shift increases and some portions of the light resonate with the appropriate energy transitions, while others do not (polarization dependence); some light interacts with electrons in the medium and some light doesn't. Due to this polarization dependence, the refractive indices for the left and right circularly polarized components change, leading to a phase shift between them. The difference in phase can be described using the following:

$$\phi_R = \frac{2\pi n_R}{\lambda} z - \omega t, \quad \phi_L = \frac{2\pi n_L}{\lambda} z - \omega t.$$

$$\Delta\phi = \phi_R - \phi_L = \frac{2\pi}{\lambda}(n_R - n_L)z.$$

When the components are combined, the result is a net rotation of the polarization plane:

$$\theta = \frac{\Delta\phi}{2} = \frac{\pi}{\lambda}(n_R - n_L)z.$$

Refer to 7.2.5 in the appendix for a derivation of polarization rotation due to circular birefringence.

Before attempting to measure Verdet's constant, the experimental setup is tested and validated. Data is taken with and without the rod and abnormalities when introducing the SF-57 sample are measured and recorded. The data is expected to follow a relationship described by Malus' law;

$$I = I_0 cos^2(\theta + \phi)$$

First understand that light is an electromagnetic wave where the electric field component (E_0) oscillates and E_0 can be split into components. If E_0 passes through a polarizer, only one component will pass through. This component can be determined using simple trigonometry:

$$E = E_o cos(\theta)$$

Intensity is proportional to E^2. If I_0 is the initial intensity, then we arrive at:

$I = I_0 cos^2(\delta\theta + \phi)$ Comparing the data allows for any deviations from the sample to be considered. Deviations such as absorption, reflections and refractions. To compare, the data can be graphed using python, and the differences can be analyzed using the curvefit() function.

The curvefit() function uses a fitting equation to obtain a line of best fit. A phase shift can be added into the fitting equation, and curvefit() will estimate the best parameter values that minimize the difference between the model (Malus' law) and the measured data. Among these parameters (I_0,, θ, ϕ, the phase shift is the one that optimally aligns the model with the data. The fitted parameters are stored in an array usually called popt, where popt[1] corresponds to the phase shift. Extract the phase shift from each data set (each array) and calculate the difference to obtain the phase shift introduced by the SF-57 sample glass rod.

After verifying Malus's law I know the experimental setup is valid. I record the phase shift and take note of differences due to the SF-57 sample and then move onto Verdet's constant.

3. Experiment

The TeachSpin Faraday Rotation Apparatus (FR1-A) is used.

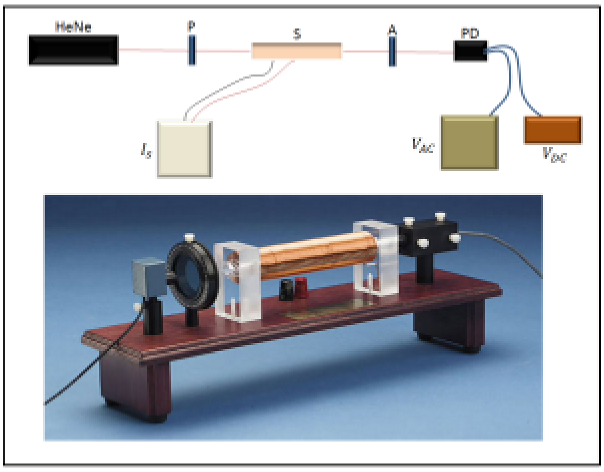

Figure 2. FR1-A.

From left to right, we have the photo detector, the polarizer, the solenoid, and the laser. The following table contains specifications on these parts.

Table 1. Apparatus Component Specifications

Component	Specifications
Red Laser	(650 nm, 3 mW)
Solenoid	(140 turns, $l = 0.15$ m)
ThorLabs PRM1 Polarizer	($360° \pm 0.01°$)
Glass Solenoid Insert	(SF-57, $l = 0.1$ m)
Photo detector resistors	(1 $k\Omega$, 3 $k\Omega$, or 10 $k\Omega$)

Other components include a Fluke multimeter, which is used to read off data values, the TeachSpin PAA1-A amplifier, which provides power to the laser from the back of the amplifier, and the TENMA DC power supply, which provides power to the solenoid. Specifications are in the following table. Uncertainty in components is measured using the multimeter.

Table 2. Other Component Specifications

Component	Specifications
Fluke multimeter	($\pm 0.001V$, $\pm 0.001A$)
TeachSpin PAA1-A amp	($4V \pm 0.001V$, $0.3A \pm 0.001A$)
TENMA DC power supply	($\pm 0.001V$, $\pm 0.001A$)

3.1. Verify Malus' law

3.1.1. Procedure

I will use a combination of the components from the tables above. 120 volts of AC current is applied to the PAA1-A amplifier from the wall outlet. The laser receives power from the back of the amplifier ($4V \pm 0.001V$, $0.3A \pm 0.001A$). The solenoid does not receive power and the glass insert (SF-57 sample) is left out of the solenoid for now. The photo detector is set to the $3k\Omega$ setting and the intensity is measured using the multimeter ($\pm 0.001V$, $\pm 0.001A$).

To begin data collection, the angle of maximum intensity is found, where the multimeter outputs the highest value. Taking note of the angle of the polarizer and the output voltage reading on the multimeter. The angle is then increased by 10 degrees. Once again, recording the angle and output voltage. I do this for a total of 36 data points.

I then plot intensity as a function of angle and examine their relationship. I expect to see intensity values that agree with Malus' law,

$$I = I_0 cos^2(\theta + \phi)$$

In order to to see this, I use the python function curvefit(), which plugs my recorded angles into the Malus' law equation and outputs the expected intensity. These intensity values are the data for my optimal line and this is plotted it on the graph. Error bars representing uncertainty in the measurement are added to the graph as well.

3.2. No sample

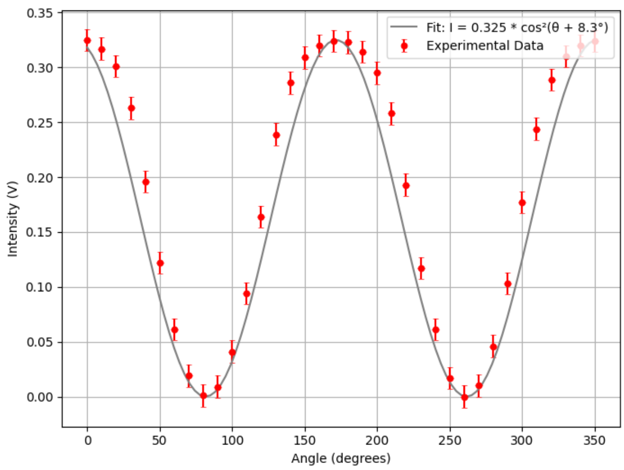

Figure 3. Intensity (V) vs angle (°). This graph represents the intensity of transmitted light as a function of the analyzer's angle. It demonstrates Malus' Law, which is described by $I = I_0 \cdot cos^2(\theta + \phi)$. The gray curve represents the standard function with a phase shift, while the red points represent the data obtained. There was a phase shift of 8.312°. Data is recorded under 7.4 in the appendix. Uncertainties in measurement devices are found using equations (5) and (6) in 7.1.5.

The peaks are truncated and the data does not fit perfectly to the expected line. The line of best fit follows this equation:

$$I = (0.325V \pm 0.001V) \cdot cos^2(\theta + 8.312°)$$

3.3. SF-57 sample

Next, I put the SF-57 glass rod sample in the solenoid and repeat the procedure.

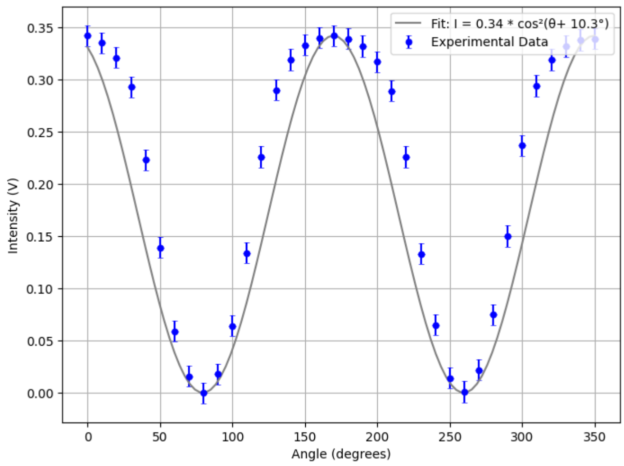

Figure 4. Intensity (V) vs angle (°). This graph represents the intensity of transmitted light as a function of the analyzer's angle. It demonstrates Malus' Law, which is described by $I = I_0 \cdot cos^2(\theta + \phi)$. The gray curve represents the standard function with a phase shift, while the blue points represent the data obtained. There was a phase shift of 10.339°. Data is recorded under 7.4 in the appendix. Uncertainties in calculated values are found using equations (5) and (6) in 7.1.5.

The peaks are truncated and the data does not fit perfectly to the expected line. The line of best fit follows this equation:

$$I = (0.342V \pm 0.001V) \cdot cos^2(\theta + 10.339°)$$

Refer to 4.2 for the data analysis and discussion, as well as the comparison between data sets when introducing the rod.

3.4. Verdet's Constant

The setup consists of the same equipment. 120 volts of AC current is applied to the PAA1-A amplifier from the wall outlet. The laser receives power from the back of the amplifier. The solenoid receives power from the TENMA DC power supply and the glass insert (SF-57 sample) is put into the solenoid. The photo detector is set to the $3k\Omega$ setting and the intensity is measured using the multimeter.

I use a reference intensity (V) above 0. The goal is to have higher sensitivity to small changes. Thus, data is taken where the slope is the greatest, 45° from maximum extinction.

3.4.1. Procedure

To begin data collection, the power is disconnected from the solenoid. Then, the point of maximum extinction is found and the polarizer is rotated 45° from this point. Taking note of the angle and intensity. Reconnecting the solenoid and applying a current results in a larger voltage reading on the multimeter. The polarizer is rotated until the original voltage reading is achieved again. The amount of rotation is recorded. I do this for several different solenoid currents and plot the data using:

$$V = \frac{\theta}{Bl}$$

When graphing using python and curvefit(), the uncertainty is stored in a covariance matrix I have called pcov; this represents how much the connected parameters (Intensity (I) and theta (θ)) will fluctuate together. This value is stored as σ^2 in the first row and column of the matrix. Taking the square root and dividing by the length of the SF-57 sample ($L = 0.1m$) yields the uncertainty in the slope.

$$\sigma = \frac{np.sqrt(pcov[0,0])}{L}$$

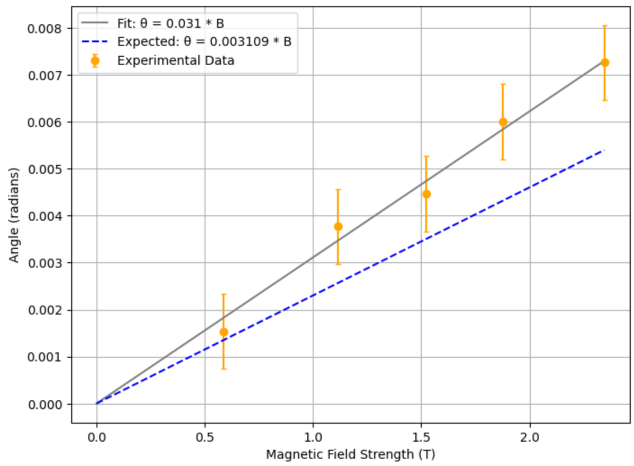

Figure 5. Change in angle (rad) with respect to magnetic field strength (B). Orange represents data with error bars. The gray line represents the curve fit. The blue line represents the expected slope. Data is recorded 7.4 of the appendix. Uncertainties in calculated values are found using equations (5) and (6) in 7.1.5.

Verdet's constant values:

$$V_{calc} = 0.026\frac{rad}{mT\cdot m} \pm 0.004\frac{rad}{mT\cdot m}$$

Error: 14%

$$V_{slope} = 0.031\frac{rad}{mT\cdot m} \pm 0.001\frac{rad}{mT\cdot m}$$

Error: 35%

$$V_{expected} = 0.023\frac{rad}{mT\cdot m}$$

The slope does not match the expected line (blue). The uncertainty is enough to account for the first data value ($V = 0.026\frac{rad}{mT\cdot m} \pm 0.004\frac{rad}{mT\cdot m}$, but no other points. Refer to 4.3 for the data analysis and discussion.

4. Data Analysis and Discussion

4.1. Malus' Law photodetector failure

The data taken to verify Malus' law have truncated peaks. This is due to one of or both of two things; Saturation of the detector and/or misuse leading to improper measurements and component failure. When attempting to re-collect data, the photo detector gave no response. An internal connection point had melted. I obtained one data set for each objective and after the component failure a replacement could not be found. The only thing to do was analyze the dirty data.

It is possible that the photo detector had previously been misused and I dealt the final blow. If the part had partially melted and re-solidified, the metal would have oxidized, leading to an increase in impurities/resistance. We can define the resistance of a wire as follows:

R = ρL / A.

As the wire melts, it slowly loses material, decreasing the cross-sectional area (**A**), **A** decreases so **R** increases. Ohm's law tells us that:

$$V = IR$$

The wire and the $3k\Omega$ resistor can be represented in this equation:

$$V = I_{wire}R_{3k} + I_{wire}R_{wire}$$

When the resistance in the wire increases, the current decreases, leading to a lower output voltage. A lower output voltage causes the data near peaks to be lower than expected, appearing as truncated peaks.

Below is a photo of the melted wire.

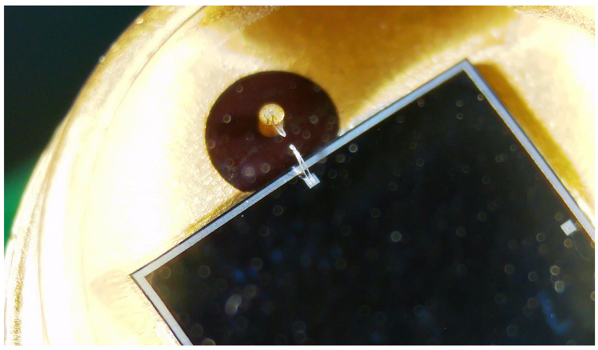

Figure 6. FR1-A photo detector failure. The gap in the thin silver colored wire connecting the gold circle (inductor) with the square (diode) is the failure point. Failure is due to misuse.

4.2. Malus' Law phase shift

When the glass rod was introduced into the system without applying a magnetic field, a noticeable shift in intensity values was observed across all angles. This change in intensity is likely due to multiple optical effects introduced by the rod:

Reflections; A portion of the light is reflected, reducing the overall transmitted intensity.

Internal absorption; Slight absorption within the material can attenuate light passing through.

Birefringence; Minor refractions of light could occur, leading to small variations in intensity.

However, since no magnetic field was applied, the observed differences cannot be attributed to the Faraday effect. Instead, they arise from the optical properties of the glass itself, which influence how light propagates through the system.

I calculate the phase shift introduced by the rod using python. The magic lies in the curvefit() function, which fits our data to the Malus' Law equation. I add a phase shift (ϕ) into the Malus law equation:

$I = I_0 cos^2(\theta + \phi)$

Curvefit() estimates the best parameter values that minimize the difference between the model and the measured data. Among these parameters, the phase shift is the one that optimally aligns the model with the data. The fitted parameters are stored in an array called popt, where popt[1] corresponds to the phase shift. This value is extracted for both data sets and the phase shift introduced by the glass rod is determined by computing their difference.

$\phi_1 = 8.312°, \phi_2 = 10.339°$

$\triangle\phi = \phi_1 - \phi_2 = 8.312° - 10.339°$

$\triangle\phi = -2.027° = -0.035rad$

The phase shift in graph 2 (SF-57 sample included) is subtracted from the phase shift in graph 1 (No SF-57 sample). The negative symbol in the phase shift means that graph 2 is shifted to the left, when compared to graph 1.

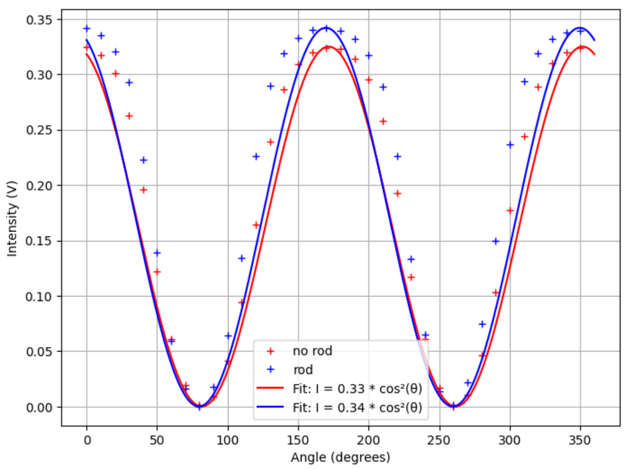

Figure 7. Intensity (V) vs angle (°). With and without glass rod (SF-57 sample). Blue represents the data with the glass rod, red without. Data is recorded in 7.4 of the appendix, link to the code is in 7.7 of the appendix. The phase shift is found to be $2.027° = 0.035rad$.

$\triangle\phi = -2.027° = -0.035rad$

This effect is subtle and cannot be seen on the decreasing slope; an error due to the different maximum intensity values. Below is another graph with just the best-fit lines. I have changed the fitting parameters to give them the same maximum intensity and now the phase shift is clearly seen.

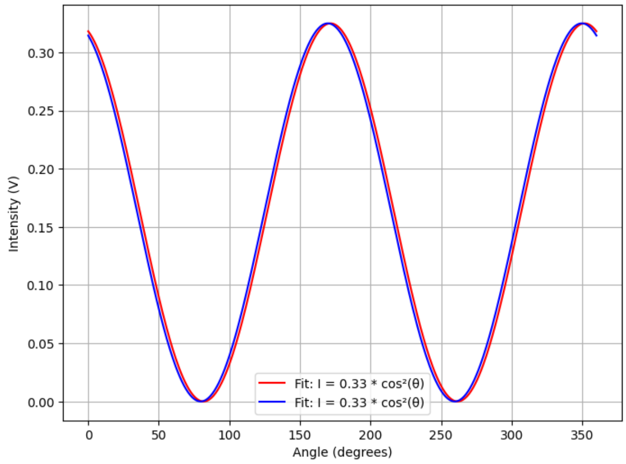

Figure 8. Intensity (V) vs angle (°). With and without glass rod (SF-57 sample). Blue represents the data with the glass rod, red without. Data is recorded in 7.4 of the appendix, link to the code is in 7.7 of the appendix. The phase shift is found to be $2.027° = 0.035rad$.

$\triangle\phi = -2.027° = -0.035rad$

4.3. Verdet's's Constant

The accepted value for the SF-57 sample is:

$0.023\frac{rad}{mT \cdot m}$.

When plotting the data, the result was as follows:

$0.031\frac{rad}{mT \cdot m} \pm 0.001\frac{rad}{mT \cdot m}$.

This results in a percent error of 35%. When graphing using python and curvefit(), the uncertainty is stored in a covariance matrix; this represents how much the connected parameters will fluctuate together. This value is stored as σ^2. Taking the square root yields the uncertainty in the slope. The uncertainty is only enough to account for the first data point. This data point yielded a value of:

$0.026\frac{rad}{mT \cdot m} \pm 0.004\frac{rad}{mT \cdot m}$.

The percent error in this data point is 13%. The data shows that at higher magnetic field strengths the percent error increases. Verdet's constants are highly dependent on material, wavelength, and temperature. During the experiment, the solenoid grew quite warm to the touch. It is possible this residual heat caused a few problems, inflating Verdet's constant for the SF-57 sample.

The SF-57 sample has a very delicate crystal lattice structure. While heating up the sample would expand and contract, stressing the structure of the sample and altering the refractive index. If there was localized heating and cooling, the sample would have different refractive indices, leading to birefringent behavior. Consider the following equation:

$V = \frac{\theta}{Bl}$

An induced birefringence would lead to an increase in the polarization angle. As θ increases with constant magnetic field strength (B), Verdet's constant increases.

Alternatively, an increased temperature would also increase the number of electron interactions. According to the equipartition theorem:

$$E_e = \frac{1}{2}k_B T$$

the energy in an electron (E_e) is proportional to the temperature (T). At higher magnetic field strengths the temperature increased, increasing the energy in the electrons and the amount of electron interactions. Instead of holding split energy levels, electrons would be excited and move around, destroying their delicate refractive indices through a decrease in Zeeman splitting; the electrons' split energy levels that cause the polarization rotation of the light. This would result in a lower rotation angle and thus a lower Verdet constant.

Without knowing the condition of the SF-57 sample; wether it induced a thermal gradient or physically changed, I cannot conclude that the increase in Verdet's constant came from the residual heat.

In the case of a slowly melting wire in the photodetector; the output voltage would decrease due to the increase in resistance. A lower voltage reading would result in a lower rotation angle, lowering Verdet's constant.

However, When taking data multiple times, the value of Verdet's constant consistently hovered 35% higher than the actual value. This leads me to believe that the system is biased towards higher values of Verdet's constant. The only error that would result in a larger than expected value of Verdet's constant would be due to the heat implied stress on the sample. In conclusion, the errors associated with the measured Verdet constant are attributed to systematic error.

4.4. Uncertainty

Intensity is measured using a Fluke multimeter, and uncertainties in measured quantities are obtained from the least count and fluctuations. The calculated uncertainties are found using the uncertainty propagation equations (5) and (6) in 7.1.5. Uncertainty for the polarizer is measured using the micrometer attachment.

Table 3. Uncertainty

Component	Uncertainty
Fluke multimeter	$(\pm 0.001V, \pm 0.001A)$
TeachSpin PAA1-A amp	$(4V \pm 0.001V, 0.3A \pm 0.001A)$
TENMA DC power supply	$(\pm 0.001V, \pm 0.001A)$
ThorLabs PRM1 Polarizer	$(360° \pm 0.01°)$

Before calculating the uncertainty in the measured Verdet constant, the uncertainy in the magnetic field must be found. Using the following equation:

$$B = u_0 n I$$

The uncertainty in B depends only on the current, it is a linear relationship, thus:

$$\delta B = B \cdot \delta I$$

Using the following equation the uncertainty in the measured Verdet constant can be calculated:

$$\delta V = V \cdot \sqrt{(\frac{\delta B}{B})^2 + (\frac{\delta L}{L})^2 + (\frac{\delta\theta}{\theta})^2}$$

The uncertainty in the length of the SF-57 sample (L) is negligible:

$$\delta V \approx V \cdot \sqrt{(\frac{\delta B}{B})^2 + (\frac{\delta\theta}{\theta})^2}$$

or

$$\delta V \approx V \cdot \sqrt{(\frac{B\delta I}{B})^2 + (\frac{\delta\theta}{\theta})^2}$$

$$\delta V \approx V \cdot \sqrt{(\delta I)^2 + (\frac{\delta\theta}{\theta})^2}$$

Calculated Uncertainty:

$$\delta V \approx 0.026 \cdot \sqrt{(0.001)^2 + (\frac{0.0002}{0.0015})^2}$$

$$\delta V \approx \pm 0.004 \frac{rad}{mT \cdot m}$$

Slope Uncertainty (Extracted from covariance matrix):

$$\delta V \approx \pm 0.001 \frac{rad}{mT \cdot m}$$

4.5. Errors

In the Malus law section, the graphs have truncated peaks. A connection point in the photo detector melted. The phase difference is also difficult to see since the maximum intensities are different.

There are major errors in the fitted estimates of Verdet's constant when extracting it from the slope. The percentage error is high; 35%. The uncertainty is not enough to account for the error.

4.6. Improvements

The accuracy in measurements can always be improved with more accurate measuring equipment or taking more measurements. Accuracy can also be improved by isolating the experiment; decreasing noise via lights, **temperature** or sound.

When setting up the experiment, first make sure you can achieve maximum extinction, then proceed. If you cannot achieve maximum extinction, either your photo detector is detecting ambient light or your equipment is faulty.

Proceed with the experiment in a dark or dim room. You only want data for the light coming from the laser through the polarizer.

There was also a lack of a specification sheet or part number. It was impossible to determine what photo detector it was for replacement. Have a part number or replacement device for your equipment.

Attempt to get the same maximum intensity value for each data set when verifying Malus' law so that the phase shift is more obvious on the graph.

Obtain more data points for Verdet's constant and use lower magnetic field strengths.

4.7. Conclusion

In this work, I explored the Faraday effect by experimentally verifying Malus' law and estimating the Verdet constant for SF-57 glass. The results confirm that the polarization rotation is proportional to the applied magnetic field and the sample length, in agreement with theoretical predictions. However, the measured Verdet constant deviated from accepted values, largely due to systematic errors arising from detector saturation, thermal fluctuations, and component failures. These challenges highlight the importance of rigorous experimental control and equipment calibration in magneto-optical studies. Overall, despite the encountered difficulties, my findings reinforce the significance of Faraday's discovery, validating his findings while also offering practical insights for future improvements in experimental design.

5. Appendix

5.1. Equations

5.1.1. Malus' Law

$$I = I_0 \cdot cos^2(\delta\theta + \phi) \tag{1}$$

5.1.2. Faraday Rotation

$$\theta = V \cdot B \cdot L \tag{2}$$

5.1.3. Magnetic Field of a Solenoid

$$B = \mu_0 \cdot n \cdot I \tag{3}$$

5.1.4. Turns per meter

$$n = \frac{N}{l} \tag{4}$$

5.1.5. Uncertainty

$$\delta B = B \cdot \delta I \tag{5}$$

$$\delta V = V \cdot \sqrt{(\frac{\delta B}{B})^2 + (\frac{\delta L}{L})^2 + (\frac{\delta\theta}{\theta})^2} \tag{6}$$

$$\delta V \approx V \cdot \sqrt{(\frac{\delta B}{B})^2 + (\frac{\delta\theta}{\theta})^2} \tag{7}$$

5.1.6. Plotting

Use equation (4) and curvefit() to verify Malus' law.

Using equation (1), solve for V. Plot theta vs B and extract the slope.

$$V = \frac{\theta}{B \cdot L} = \frac{slope}{L} \tag{8}$$

5.2. Derivations

5.2.1. Malus' Law

First understand that light is an electromagnetic wave where the electric field component (E_0) oscillates and E_0 can be split into components. If E_0 passes through a polarizer, only one component will pass through. This component can be determined using simple trigonometry:

$$E = E_o cos(\theta)$$

Intensity is proportional to E^2. If I_0 is the initial intensity, then we arrive at:

$$I = I_0 cos^2(\delta\theta + \phi)$$

5.2.2. Magnetic Field of a Solenoid

Starting with Ampere's law:

$$\int B \cdot dl = \mu_0 I_{total}$$

Where B is the magnetic field strength, l is the length of the solenoid, μ_0 is the permeability of free space, and I_{total} is the total current which is equal to the number of turns times the input current.

$$B \cdot l = \mu_0 I_{total}$$
$$I_{total} = NI, n = \frac{N}{l}$$

N is the number of turns and l is the length of the solenoid.

$$B \cdot l = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{l}$$

$$B = \mu_0 nI$$

5.2.3. Faraday Rotation

In a uniform medium, the wave vector (k) is related to the wavelength (λ) by:

$$k = \frac{2\pi}{\lambda}$$

The two circularly polarized components experience a difference in their magnitudes and directions as they propagate through a material of a length (L). This difference can be described using:

$$\theta = k \cdot \triangle n \cdot L$$

$$\theta = \frac{2\pi}{\lambda} \cdot \triangle n \cdot L$$

where n is the difference in the refractive indices and it is equal to some constant (K) times the magnetic field strength (B):

$$\triangle n = KB$$

$$\theta = \frac{2\pi}{\lambda} \cdot KBL$$

Now, Verdet constant is identified as in the equation as:

$$V = \frac{2\pi}{\lambda} \cdot K$$

$$\theta = VBL$$

5.2.4. Lorentz Force

The electric and magnetic forces on an electron are described by Coulomb's law:

$$F_E = qE$$

$$F_B = qv \times B$$

Combining these forces gives us the Lorentz force:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

5.2.5. Polarization Rotation Due to Circular Birefringence

Consider a linearly polarized electromagnetic wave propagating in the z-direction with an electric field:

$$\mathbf{E}(z,t) = E_0 e^{i(kz-\omega t)}\,\hat{p},$$

where E_0 is the amplitude, $k = \frac{2\pi n}{\lambda}$ is the wavenumber in a medium with refractive index n, and $\hat{p}$ is the polarization unit vector.

A linearly polarized wave can be decomposed into its right- and left-circularly polarized components using the basis vectors:

$$\hat{e}_R = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{y}), \quad \hat{e}_L = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{y}).$$

Thus, the electric field can be written as:

$$\mathbf{E}(z,t) = \frac{E_0}{\sqrt{2}}\Big[e^{i\phi_R}\hat{e}_R + e^{i\phi_L}\hat{e}_L\Big],$$

where the phases are given by:

$$\phi_R = \frac{2\pi n_R}{\lambda}z - \omega t, \quad \phi_L = \frac{2\pi n_L}{\lambda}z - \omega t.$$

Define an average phase,

$$\phi_0 = \frac{2\pi n}{\lambda}z - \omega t,$$

and the phase difference:

$$\Delta\phi = \phi_R - \phi_L = \frac{2\pi}{\lambda}(n_R - n_L)z.$$

Then we can express:

$$e^{i\phi_R} = e^{i\phi_0}e^{i\Delta\phi/2}, \quad e^{i\phi_L} = e^{i\phi_0}e^{-i\Delta\phi/2}.$$

Substitute these back into the expression for $\mathbf{E}(z,t)$:

$$\mathbf{E}(z,t) = \frac{E_0\,e^{i\phi_0}}{\sqrt{2}}\Big[e^{i\Delta\phi/2}\,\hat{e}_R + e^{-i\Delta\phi/2}\,\hat{e}_L\Big].$$

Replacing $\hat{e}_R$ and $\hat{e}_L$ with their definitions:

$$\mathbf{E}(z,t) = \frac{E_0\,e^{i\phi_0}}{\sqrt{2}}\left[e^{i\Delta\phi/2}\frac{1}{\sqrt{2}}(\hat{x} - i\hat{y}) + e^{-i\Delta\phi/2}\frac{1}{\sqrt{2}}(\hat{x} + i\hat{y})\right].$$

Simplify by combining the prefactors:

$$\mathbf{E}(z,t) = \frac{E_0\,e^{i\phi_0}}{2}\left[\left(e^{i\Delta\phi/2} + e^{-i\Delta\phi/2}\right)\hat{x} + \left(-i\,e^{i\Delta\phi/2} + i\,e^{-i\Delta\phi/2}\right)\hat{y}\right].$$

Using the Euler identities:

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta, \quad e^{i\theta} - e^{-i\theta} = 2i\sin\theta,$$

with $\theta = \Delta\phi/2$, the expression becomes:

$$\mathbf{E}(z,t) = E_0\,e^{i\phi_0}\left[\cos\!\left(\frac{\Delta\phi}{2}\right)\hat{x} - \sin\!\left(\frac{\Delta\phi}{2}\right)\hat{y}\right].$$

This shows that the recombined wave is still linearly polarized, but its polarization plane is rotated by an angle:

$$\theta = \frac{\Delta\phi}{2} = \frac{\pi}{\lambda}(n_R - n_L)z.$$

Thus, the phase difference between the right- and left-circularly polarized components, resulting from their different refractive indices, produces a rotation of the polarization plane by θ.

5.3. Data

Table 4. Malus Law no rod

$V \pm 1mV$	$\theta \pm 0.005°$
0.342	0
0.335	10
0.321	20
0.293	30
0.223	40
0.139	50
0.059	60
0.016	70
0.000	80
0.018	90
0.064	100
0.134	110
0.226	120
0.290	130
0.319	140
0.333	150
0.340	160
0.342	170
0.339	180
0.332	190
0.317	200
0.289	210
0.226	220
0.133	230
0.065	240
0.014	250
0.001	260
0.022	270
0.075	280
0.150	290
0.237	300
0.294	310
0.319	320
0.332	330
0.338	340
0.339	350

Note: Intensity with respect to angle of the polarizer.

Table 5. Malus' Law rod

$V \pm 1mV$	$\theta \pm 0.005°$
0.325	0
0.317	10
0.301	20
0.263	30
0.196	40
0.122	50
0.061	60
0.019	70
0.001	80
0.009	90
0.041	100
0.094	110
0.164	120
0.239	130
0.286	140
0.309	150
0.320	160
0.324	170
0.323	180
0.314	190
0.295	200
0.258	210
0.193	220
0.117	230
0.061	240
0.017	250
0.000	260
0.010	270
0.046	280
0.103	290
0.177	300
0.244	310
0.289	320
0.310	330
0.320	340
0.324	350

Note: Intensity with respect to angle of the polarizer.

Table 6. Verdet's Constant DC

$B \pm 0.01mT$	$radians \pm 0.0002rad$
0.59	0.0015
1.11	0.0038
1.52	0.0045
1.88	0.0060
2.35	0.0073
2.82	0.0101

Note: Change in angle with respect to the magnetic field strength.

5.4. Helpful people

Lab partner: *Eva Rissanen*
Lab instructors: *Dr. Roshani Silwal, Dr. Zach Russell*

5.5. Helpful resources

Google Colab for graphing the data. It has many built in libraries and a built in AI to help understand the code better.
ChatGPT for acquiring better vocabulary and formatting my sources to APA style.

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5.6. Links

Code
Lab Notebook

The Faraday Effect ☻

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