



# DISTANCE LEARNING PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Advanced)

TEST # 01

12-05-2024

## JEE(Main + Advanced) : LEADER TEST SERIES / JOINT PACKAGE COURSE

### Test Type : Unit Test # 01

#### ANSWER KEY

#### PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4		
	A.	D	C	A	B		
SECTION-I (ii)	Q.	5	6	7	8		
	A.	A,C,D	A,B,C	A,C	A,B,C		
SECTION-I (iii)	Q.	9	10	11	12		
	A.	C	B	B	D		
SECTION-III	Q.	1	2	3	4	5	6
	A.	2	1	6	2	2	2
SECTION-IV	Q.	1	2				
	A.	A->PS,B->QR,C->QS,D->PS	A->PT,B->R,C->PQ,D->QRS				

#### PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4		
	A.	B	C	C	A		
SECTION-I (ii)	Q.	5	6	7	8		
	A.	A,B,C	B,D	A,D	A,C		
SECTION-I (iii)	Q.	9	10	11	12		
	A.	A	D	D	A		
SECTION-III	Q.	1	2	3	4	5	6
	A.	2	5	5	2	5	6
SECTION-IV	Q.	1	2				
	A.	A->PRS,B->PQT,C->P,D->PQT	A->Q,B->P,C->R,D->T				

#### PART-3 : MATHEMATICS

SECTION-I (i)	Q.	1	2	3	4		
	A.	A	B	C	D		
SECTION-I (ii)	Q.	5	6	7	8		
	A.	A,B,C,D	A,D	A,C,D	A,C		
SECTION-I (iii)	Q.	9	10	11	12		
	A.	B	B	B	D		
SECTION-III	Q.	1	2	3	4	5	6
	A.	1	1	4	3	7	6
SECTION-IV	Q.	1	2				
	A.	A->Q,B->R,C->T,D->PT	A->PS,B->QR,C->PQT,D->PQT				

## HINT – SHEET

### PART-1 : PHYSICS

#### SECTION-I (i)

1. **Ans ( D )**

$$[F] = \left[ \frac{Cr}{V^2} \right]$$

$$[M^1 L^1 T^{-2}] = \left[ \frac{CL}{L^2} T^2 \right]$$

$$[C] = [M^1 L^2 T^{-4}]$$

2. **Ans ( C )**

$$\frac{d(T)}{dt} = d \frac{(T_0 + \alpha t^2 + \beta \sin t)}{dt}$$

$$= \frac{d(T_0)}{dt} + \frac{d(\alpha t^2)}{dt} + \frac{d(\beta \sin t)}{dt} = 2\alpha t + \beta \cos t$$

$$= \frac{2 \times 2}{\pi} \pi + (-4) \times \cos \pi$$

$$= 4 + (-4)(-1) = 4 + 4 = 8 \text{ K/sec}$$

3. **Ans ( A )**

Since TIR occurs at surface AC, thus

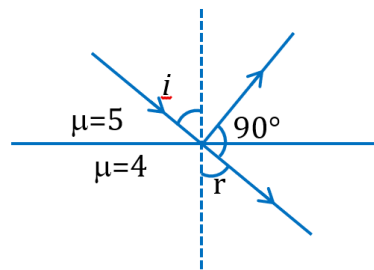
$$1.5 \sin \theta = \frac{4}{3} \sin 90^\circ \Rightarrow \sin \theta = \frac{40}{4.5} = \frac{8}{9}$$

$$\text{Therefore } \sin \theta \geq \frac{8}{9}$$

### PART-1 : PHYSICS

#### SECTION-I (ii)

5. **Ans ( A,C,D )**



$$i + r = 90^\circ$$

$$r = 90 - i$$

By snell's law

$$\mu_i \sin i = \mu_2 \sin r$$

$$5 \sin i = 4 \sin(90 - i)$$

$$\tan i = \frac{4}{5}; \sin i = \frac{4}{\sqrt{41}}$$

6. **Ans ( A,B,C )**

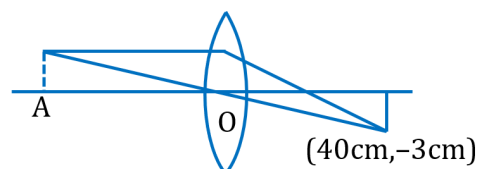


Image is real & inverted

Lens is converging

$$v + u = 100$$

$$m = \frac{v}{u} = \frac{3}{2}$$

$$v = \frac{3}{2}u$$

$$\frac{3}{2}u + u = 100 \quad 2.5u = 100$$

$$v = 60$$

$$OA = -60$$

$$r_A - r_0 = OA = -60$$

$$r_0 = r_A - 60$$

$$r_0 = 40 - 60 = -20$$

7. Ans (A,C)

$$\frac{\mu}{v} - \frac{1}{\infty} = \frac{(\mu - 1)}{10}$$

$$v = \frac{10\mu}{\mu - 1} = \frac{10}{1 - \frac{1}{\mu}}$$

$$v_v < v_r$$

$$v_v = \frac{10}{0.615} \times 1.615$$

$$v_r = 10 \times \frac{1.6}{0.6}$$

$$\begin{aligned}
 v_v - v_r &= \frac{16.15}{0.615} - \frac{16}{0.6} \\
 &= \frac{16.15 \times 0.6 - 16 \times 0.615}{0.6 \times 0.615} = 0.40 \text{ cm}
 \end{aligned}$$

8. Ans (A,B,C)

For  $\lambda_{\text{yellow}}$ , the R.I of liquid and glass are same,  
so not deviation for yellow light.

The RI of blue light is higher in liquid i.e. light enters from denser to rarer medium, so bends away from normal

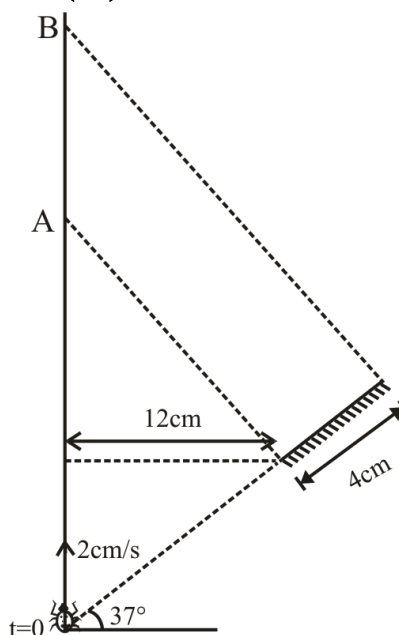
For Red light

$$\mu_g > \mu_l$$

## PART-1 : PHYSICS

### SECTION-I (iii)

10. Ans (B)



By geometry insect will observe its image at A,  
distance OA = 25cm

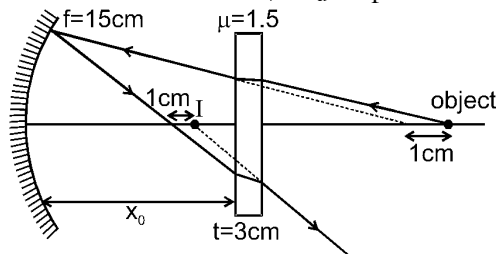
So time = 12.5 sec

It will observe its image for distance  $AB = \frac{20}{3}$  cm,  $t = \frac{10}{3}$

12. Ans (D)

The shift due to slab is  $t \left( 1 - \frac{1}{\mu} \right) = 3 \left( 1 - \frac{1}{1.5} \right) = 1$  cm towards left. Hence the object appears to mirror at a distance  $61 - 1 = 60$  cm.

From mirror formula  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$  we get  $v = 20$  cm.



Hence the mirror forms the image at  $v = 20$  cms towards right. The slab again causes a shift of 1cm towards right. hence the final image is formed at a distance of 21 cm from pole.

Shifting of slab towards left does no cause any change to position of final image .

## PART-1 : PHYSICS

### SECTION-III

1. **Ans ( 2 )**

$$\left[ \begin{array}{l} p \propto m_0^a \\ p \propto v^b \\ p \propto t_0^c \end{array} \right]$$

$$p = k[M]^a [LT^{-1}]^b [T]^c$$

$$ML^2T^{-3} = M^a L^b T^{-b+c}$$

$$a = 1$$

$$b = 2$$

$$-b + c = -3$$

$$-2 + c = -3$$

$$c = -1$$

$$p \propto v^2 \Rightarrow p = kv^2$$

2. **Ans ( 1 )**

$$V = \frac{1}{3}\pi r^2 h$$

$$\text{Also } \frac{h}{r} = \frac{10}{5} \Rightarrow r = \frac{h}{2}$$

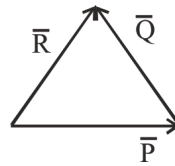
$$\Rightarrow V = \frac{1}{3}\pi \frac{h^3}{4} = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{\pi h^2}{4} \frac{dh}{dt}$$

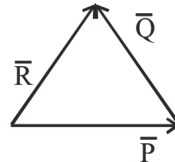
$$9 = \frac{\pi(6)^2}{4} \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{\pi}$$

3. **Ans ( 6 )**



$$\theta_1 = \frac{\pi}{3}$$



$$\theta_2 = \frac{2\pi}{3}$$

$$\therefore \theta_1 = \frac{\theta_2}{2}$$

$$\Rightarrow x = 2$$

4. **Ans ( 2 )**

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{\mu}{2R} - \frac{1}{\infty} = \frac{\mu - 1}{R}$$

$$\frac{\mu}{2R} = \frac{\mu - 1}{R}$$

$$\mu = 2$$

5. **Ans ( 2 )**

At point O

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\frac{3}{2} \times \sin \theta_c = \frac{4}{3} \sin r$$

$$\sin r = \frac{3}{4}$$

At point E

$$\frac{4}{3} \times \frac{3}{4} = 1 \times \sin \theta$$

$$\sin \theta = \frac{\pi}{2}$$

6. Ans ( 2 )

$$\frac{1}{10} = \left( \frac{1.5}{1} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{-20} = \left( \frac{1.5}{\mu} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$-2 = \frac{1.5 - 1}{\left( \frac{1.5}{\mu} - 1 \right)}$$

$$-2 \left( \frac{1.5}{\mu} - 1 \right) = 1.5 - 1$$

$$\frac{-3}{\mu} + 2 = 0.5$$

$$\frac{-3}{\mu} = -1.5$$

$$\mu = 2 = 1 + \frac{1}{10}t$$

$$\frac{1}{10}t = 1$$

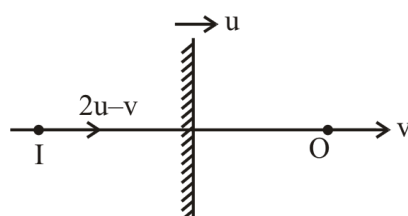
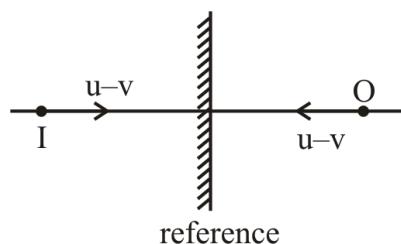
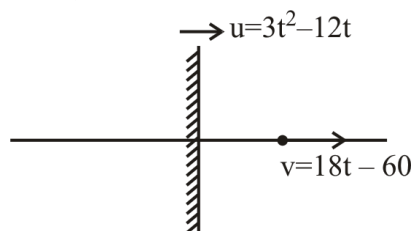
$$t = 10 = 5n$$

$$n = 2 \text{ sec}$$

## PART-1 : PHYSICS

### SECTION-IV

2. Ans ( A->PT,B->R,C->PQ,D->QRS )



$$\begin{aligned} \therefore v_{\text{img}} &= (6t^2 - 24t) - (18t - 60) \\ &= 6t^2 - 42t + 60 \end{aligned}$$

$$(A) 0 = 6(t^2 - 7t + 10) = 6(t - 2)(t - 5)$$

$$\therefore t = 2 \text{ \& } 5$$

$$(B) a_{\text{img}} = 12t - 42$$

$$(C) a_{\text{img}} < 0$$

$$= 12t - 42 < 0$$

$$= t < 3.5$$

## PART-2 : CHEMISTRY

### SECTION-I (i)

1. Ans ( B )

A is present on corner and Face centre, B is present in Alternate T.Vs

Along one Body diagonal two atom of A and one atom of B is present will be removed

$$A \rightarrow 6 \times \frac{1}{8} + 6 \times \frac{1}{2} = \frac{15}{4}$$

$$B \rightarrow 4 - 1 = 3$$

So the formula will be  $A_5B_4$

3. **Ans (C)**

$$\frac{32}{2x + 3y} = 0.2$$

$$\frac{92.8}{3x + 4y} = 0.4$$

Hence  $x = 56$  &  $y = 16$ .

4. **Ans (A)**

$$P = P_B^\circ X_B + P_T^\circ X_T$$

$$120 = 150(X_B) + 50(1 - X_B)$$

$$100X_B = 70$$

$$X_B = 0.7$$

$$Y_B = \frac{X_B P_B^0}{P} = \frac{0.7 \times 150}{120} = 0.875$$

$$\frac{Y_B}{Y_T} = \frac{7}{1}$$

$$Y_T = 1 - 0.875 = 0.125$$

## PART-2 : CHEMISTRY

### SECTION-I (ii)

5. **Ans (A,B,C)**

(A) For solution  $Z_2$  at  $P_1$  pressure

$$X_A = 0.25 \quad X_B = 0.75$$

$$Y_A = 0.5 \quad Y_B = 0.5$$

(B) For solution  $Z_2$  at  $P_3$  pressure  $\rightarrow$  solution

will not vapourise so

$$X_A = 0.4 ; X_B = 0.6$$

(C) For solution  $Z_1$  at  $P_2$  pressure  $\rightarrow$  solution

will not vapourise so

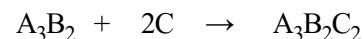
$$X_A = 0.2 ; X_B = 0.8$$

6. **Ans (B,D)**



$$\text{initial mole} \quad 3 \quad 3 \quad 0$$

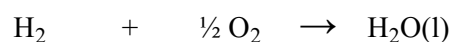
$$\text{final mole} \quad 0 \quad 3-2 \quad 1$$



$$\text{initial mole} \quad 1 \quad 1 \quad 0$$

$$\text{final mole} \quad 1 - \frac{1}{2} \quad 0 \quad \frac{1}{2}$$

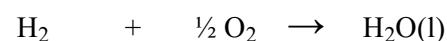
8. **Ans (A,C)**



$$\Rightarrow \quad 30 \quad 25 \quad -$$

$$0 \quad 10 \quad -$$

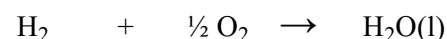
$$\text{Volume contraction} = 55 - 10 = 45$$



$$\Rightarrow \quad 10 \quad 45 \quad 0$$

$$0 \quad 40 \quad -$$

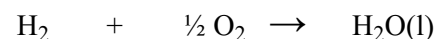
$$\therefore \text{Volume contraction} = 55 - 40 = 15 \text{ mL}$$



$$\Rightarrow \quad 40 \quad 15 \quad 0$$

$$10 \quad 0 \quad -$$

$$\therefore \text{Volume contraction} = 55 - 10 = 45$$



$$\Rightarrow \quad 35 \quad 20 \quad 0$$

$$0 \quad 2.5 \quad -$$

$$\text{Volume contraction} = 55 - 2.5 = 52.5 \text{ mL}$$

## PART-2 : CHEMISTRY

### SECTION-III

#### 1. Ans ( 2 )

$$\text{Formula of density} = \frac{Z \times M}{N_A \times a^3}$$

For FCC unit cell  $Z = 4$

Edge length  $a = 4 \times 10^{-8} \text{ cm}$

$$M = \frac{d \times N_A \times a^3}{Z}$$

$$= \frac{8 \times 6 \times 10^{23} \times 64 \times 10^{-24}}{4} \text{ gm/mol}$$

$$\text{No. of atoms} = \frac{\text{wt (gm)}}{\text{molar mass}} \times N_A$$

$$= \frac{256 \times 10 \times 6 \times 10^{23}}{8 \times 6 \times 16} = 2 \times 10^{24} \text{ (Value of } N = 2)$$

#### 3. Ans ( 5 )

$$\Delta T_f = K_f \times m \times i$$

$$7 = 14 \times \frac{75.2}{94} \times \left(1 - \alpha + \frac{\alpha}{2}\right)$$

$$\alpha = 0.75 = 75\%$$

#### 4. Ans ( 2 )

12 gm  $\text{CH}_3\text{COOH}$  is present in 100 ml of solution

120 gm  $\text{CH}_3\text{COOH}$  is present in 1000 ml of solution

$$M_2 = \frac{120}{60} = 2, \quad \text{Now we are mixing}$$

500 ml, 2M  $\text{CH}_3\text{COOH}$  + 2M, 600 ml  $\text{CH}_3\text{COOH}$

$$M_1V_1 + M_2V_2 = M_3V_3$$

$$500 \times 2 + 600 \times 2 = M_3 \times 1100,$$

$$M_3 = \frac{2200}{1100} = 2$$

#### 5. Ans ( 5 )

$$\text{Ions in } \text{Al}_2(\text{SO}_4)_3 = \frac{1368}{342} \times N_A \times 5 = 20 \times N_A$$

$$\text{Ions in } \text{Na}_3\text{PO}_4 = (n \text{ moles}) \times N_A \times 4$$

$$\Rightarrow n \times N_A \times 4 = 20 \times N_A \Rightarrow n = 5 \text{ moles.}$$

#### 6. Ans ( 6 )

$$P_T = x = P_A^\circ x_A + P_B^\circ x_B \dots\dots\dots (1)$$

$$x_A = \frac{n_A}{n_A + n_B} = \left( \frac{\frac{w}{80}}{\frac{w}{80} + \frac{w}{120}} \right)$$

$$x_A = \frac{3}{5}, \quad x_B = \frac{2}{5}$$

From equation (1)

$$x = 40 \times \frac{3}{5} + 30 \times \frac{2}{5}$$

$$x = 36 \text{ torr}$$

$$\frac{x}{6} = \frac{36}{6} = 6$$

## PART-3 : MATHEMATICS

### SECTION-I (i)

#### 1. Ans ( A )

$$\text{Put } x = \cos \theta$$

$$y = \frac{|\sin \theta|}{1 + |\cos \theta|}$$

2. **Ans (B)**

$$\text{Put } x \rightarrow \frac{1}{x}$$

$$\therefore 2f\left(\frac{1}{x^2}\right) + 3f(x^2) = \frac{1}{x^2} - 1$$

$$2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$$

$$\therefore f(x^2) = \frac{(1-x^2)(3+2x^2)}{5x^2}$$

3. **Ans (C)**

$$x^{100} = (x^2 - 3x + 2)Q(x) + ax - b$$

$$\text{Put } x=1$$

$$1 = a - b$$

4. **Ans (D)**

$$\alpha + \beta \Rightarrow \frac{-b}{a} < 0 \Rightarrow \frac{b}{a} > 0$$

$$\alpha\beta < 0 \Rightarrow \frac{c}{a} < 0$$

### PART-3 : MATHEMATICS

#### SECTION-I (ii)

5. **Ans (A,B,C,D)**

$$\text{clearly, } \operatorname{cosec}^{-1}x = \pm \frac{\pi}{2}$$

$$\text{Also, } \operatorname{cosec}^{-1}y = \pm \frac{\pi}{2} \text{ \& } \operatorname{cosec}^{-1}z = \pm \frac{\pi}{2}$$

$$\therefore x, y, z \in \{-1, 1\}$$

6. **Ans (A,D)**

$$a = \sin^{-1}(\sin 4) = \pi - 4$$

$$b = \cos^{-1}(\cos 4) = 2\pi - 4$$

$$c = \tan^{-1}(\tan 4) = 4 - \pi$$

$$d = \cot^{-1}(\cot 4) = 4 - \pi$$

7. **Ans (A,C,D)**

$$\sin^{-1}(x^2 + x + 1) + \cos^{-1}(ax + 1) = \frac{\pi}{2} \quad \dots (1)$$

$$\Rightarrow x^2 + x + 1 = ax + 1 \quad \dots (2)$$

$$\Rightarrow x^2 + (1-a)x = 0$$

It has exactly two solutions.

$$\Rightarrow \text{Exactly two real roots (different)}$$

$$\Rightarrow \text{Disc.} > 0 \Rightarrow (1-a)^2 - 4(1)(0) > 0$$

$$\Rightarrow (1-a)^2 > 0 \Rightarrow a \neq 1$$

$\therefore 1$  is the only integer that 'a' can't attain to satisfy equation 2.

$$\text{Now } \frac{3}{4} \leq x^2 + x + 1 \leq 1 \text{ and } ax + 1 \in \left[\frac{3}{4}, 1\right]$$

$$\Rightarrow \frac{-1}{4} \leq x^2 + x \leq 0$$

$$x^2 + x \leq 0 \text{ and } 4x^2 + 4x + 1 \geq 0$$

$$\Rightarrow x(x+1) \leq 0 \text{ and } (2x+1)^2 \geq 0$$

$$\Rightarrow x \in (-1, 0]$$

$$\text{For } a = -1$$

$$x^2 + x + 1 = -x + 1$$

$$\Rightarrow x^2 + 2x = 0 \Rightarrow x(x+2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = -2$$

$$\Rightarrow ax + 1 = 0 \text{ or } 3 \text{ but } (ax + 1) \in \left[\frac{3}{4}, 1\right]$$

$$\text{and also } -2 \notin [1, 0]$$

$$\therefore a \neq 1; \text{ (only one solution, not two solution)}$$

$$\text{For } a = 0; x^2 + x + 1 = 1$$

$$\Rightarrow x(x+1) = 0 \Rightarrow x = 0 \text{ or } x = -1$$

$$\therefore \sin^{-1}(x^2 + x + 1) + \cos^{-1}(ax + 1) = \frac{\pi}{2} \text{ is}$$

$$\text{satisfied by } x = 0, x = -1 \text{ and } a = 0$$

$$\text{For } a = 2$$

$$x^2 + x + 1 = 2x + 1 \Rightarrow x^2 - x = 0$$

$$\Rightarrow x = 0 \text{ or } 1, \text{ but } 1 \notin [-1, 0]$$

$$\text{Thus } a \neq -1, 1 \text{ and } 2.$$



8. **Ans (A,C)**

$$\sin^{-1} \left( \frac{\sqrt{x}}{2} \right) + \sin^{-1} \left( \sqrt{\frac{1-x}{4}} \right) + \tan^{-1} y = \frac{2\pi}{3}$$

Here  $x \in [0,4]$

Now we have

$$\sin^{-1} \left( \frac{\sqrt{x}}{2} \right) + \cos^{-1} \frac{\sqrt{x}}{2} + \tan^{-1} y = \frac{2\pi}{3}$$

$$\Rightarrow y = \frac{1}{\sqrt{3}}$$

$$\text{maximum value of } x^2 + y^2 = 16 + \frac{1}{3} = \frac{49}{3}$$

$$\& \text{ minimum value of } x^2 + y^2 = 0^2 + \frac{1}{3} = \frac{1}{3}$$

### PART-3 : MATHEMATICS

#### SECTION-I (iii)

9. **Ans (B)**

Since it's an invertible function so it must be

linear so  $x = k$  must be a factor of numerator.

$$f(x) = \frac{x^2 - 2x - 8}{x^2 - 4} = \frac{(x-4)(x+2)}{(x-2)(x+2)}$$

So,  $x \neq 2, -2$

$$f(x) = \frac{x-4}{x-2} = y, y \neq \frac{3}{2}$$

$$\Rightarrow x - 4 = xy - 2y$$

$$\Rightarrow 2y - 4 = x(y - 1)$$

$$\Rightarrow x = \frac{2y-4}{y-1}$$

$$\therefore f^{-1}(x) = \frac{2x-4}{x-1}, x \neq 1$$

10. **Ans (B)**

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$$\Rightarrow 2y - 4 = x(y - 1)$$

$$\Rightarrow x = \frac{2y-4}{y-1}$$

$$\therefore f^{-1}(x) = \frac{2x-4}{x-1}, x \neq 1$$

11. **Ans (B)**

$$k \in \left( -2, \frac{1}{4} \right]$$

2 integer

12. **Ans (D)**

$$k \in \phi$$

### PART-3 : MATHEMATICS

#### SECTION-III

1. **Ans (1)**

Multiply all the three equations

$$\left( x + \frac{1}{y} \right) \left( y + \frac{1}{z} \right) \left( z + \frac{1}{x} \right) = \frac{28}{3}$$

$$xyz + \frac{1}{xyz} + \left( x + \frac{1}{y} \right) + \left( y + \frac{1}{z} \right) + \left( z + \frac{1}{x} \right) = \frac{28}{3}$$

$$xyz + \frac{1}{xyz} = 2$$

$$\Rightarrow xyz = 1$$

2. **Ans ( 1 )**

$$x^3 - 12x + 1 = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$$

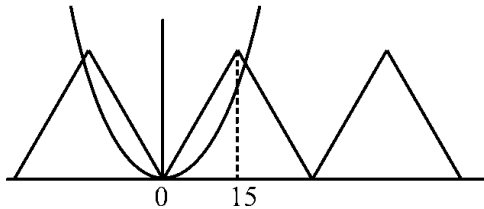
use  $\alpha + \beta + \gamma = 0$  &  $\alpha\beta\gamma = -1$

3. **Ans ( 4 )**

$$\alpha + \beta = 4, \quad \alpha\beta = 1$$

$$\begin{aligned} \text{Let } \frac{a_{n+1} + a_{n-1}}{a_n} &= \frac{\alpha^{n+1} + \beta^{n+1} + \alpha^{n-1} + \beta^{n-1}}{\alpha^n + \beta^n} \\ &= \frac{\alpha^{n+1} + \beta^{n+1} + \alpha\beta\alpha^{n-1} + \alpha\beta\beta^{n-1}}{\alpha^n + \beta^n} \\ &= \frac{(\alpha^n + \beta^n)(\alpha + \beta)}{(\alpha^n + \beta^n)} \\ &= \alpha + \beta = 4 \end{aligned}$$

4. **Ans ( 3 )**



Number of solutions are three

5. **Ans ( 7 )**

$$f(x) + f(-x) = 12$$

put  $x = \frac{1}{2}$

6. **Ans ( 6 )**

$$y = x + \frac{1}{x}; \quad x = \frac{y \pm \sqrt{y^2 - 4}}{2}$$

## PART-3 : MATHEMATICS

### SECTION-IV

1. **Ans ( A->Q, B->R, C->T, D->PT )**

$$\begin{cases} x^2 - 8x + a = 0 < \frac{\alpha}{3\beta} & \& \quad x^2 - bx + 16 = 0 < \frac{\alpha}{4\beta} \\ \alpha + 3\beta = 8 & \text{--- ①} & \quad \alpha + 4\beta = b & \text{--- ③} \\ \alpha(3\beta) = a & \text{--- ②} & \quad \alpha(4\beta) = 16 & \text{--- ④} \end{cases}$$

By (2) & (4)  $\Rightarrow a = 12$

$\therefore$  Equation  $x^2 - 8x + a = 0$  is  $x^2 - 8x + 12 = 0$

New roots are  $x = 2$  or  $6$ .

If common root  $\alpha = 2$ , then  $\beta = 2$  for which we

get  $3\beta$  &  $4\beta$  both are integers.

But, if common root is  $\alpha = 6$ , then  $\beta = \frac{2}{3}$  for

which  $(4\beta)$  is not integer.

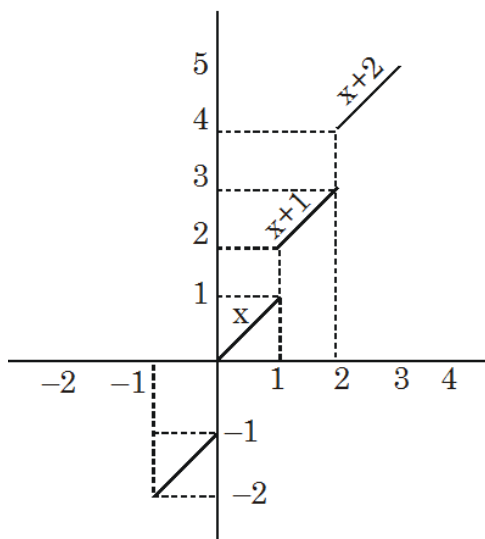
Hence, common root is 2.

$\therefore a = 12 \quad \& \quad b = \alpha + 4\beta = 2 + (4(2)) = 10$

$\therefore (a + b) = 12 + 10 = 22$

2. Ans ( A->PS,B->QR,C->PQT,D->PQT )

$$\begin{aligned}
 & \vdots \\
 (A) \quad f(x) = x + [x] = & \begin{cases} x-2 & x \in (-2, -1) \\ x=1 & x \in [-1, 0) \\ x & x \in [0, 1) \\ x+1 & x \in [1, 2) \\ x+2 & x \in [2, 3) \end{cases} \\
 & \vdots
 \end{aligned}$$

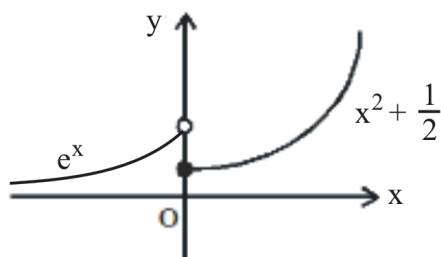


$f(x)$  is injective but not surjective

(B)  $f : \mathbb{R} \rightarrow (0, \infty)$

$$f(x) = \begin{cases} e^x & x < 0 \\ x^2 + \frac{1}{2} & x \geq 0 \end{cases}$$

Non injective surjective



(C)  $f : (0, \infty) \rightarrow \mathbb{R}$

$$f(x) = x^3 - 2x^2 + 2x + \log x$$

$$f'(x) = \underbrace{3x^2 - 4x + 2}_{a>0} + \frac{1}{x} \Rightarrow f'(x) > 0$$

$D < 0$

$\therefore$  Always +ve

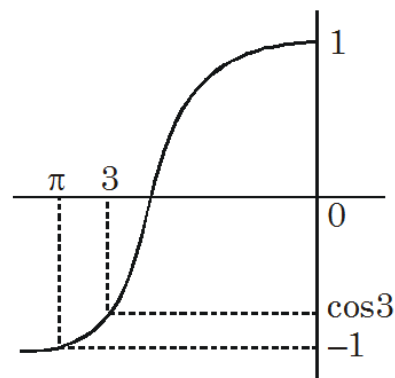
$\therefore f(x)$  is increasing  $\Rightarrow$  one one

Also Range of  $f(x)$  is  $\mathbb{R}$

$\therefore f(x)$  is one one onto

(D)  $f : [-3, 0] \rightarrow [\cos 3, 1]$

$$f(x) = \cos x$$



one one onto