ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

DISTANCE LEARNING PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Advanced)
TEST # 02
26-05-2024

JEE(Main + Advanced) : LEADER TEST SERIES / JOINT PACKAGE COURSE

Test Type : Unit Test # 02

ANSWER KEY

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4
	A.	A	B	D	A
SECTION-I (ii)	Q.	5	6	7	8
	A.	A,B	A,B	A,C	A,D
SECTION-I (iii)	Q.	9	10	11	12
	A.	B	D	A,B	C,D
SECTION-I (iv)	Q.	13	14	15	16
	A.	A	B	D	B
SECTION-II	Q.	1	2	3	4
	A.	4.50	1.33 to 1.34	30.00	2.00

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4
	A.	D	B	A	D
SECTION-I (ii)	Q.	5	6	7	8
	A.	A,B,C	A,B,D	A,B,C	A,B
SECTION-I (iii)	Q.	9	10	11	12
	A.	A,B,C	C	A,C,D	A,C
SECTION-I (iv)	Q.	13	14	15	16
	A.	C	D	A	C
SECTION-II	Q.	1	2	3	4
	A.	125.00	33.33	11.75	42.00

PART-3 : MATHEMATICS

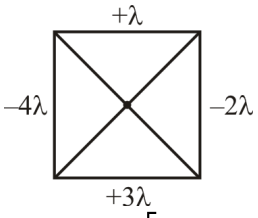
SECTION-I (i)	Q.	1	2	3	4
	A.	B	D	A	C
SECTION-I (ii)	Q.	5	6	7	8
	A.	A,B,D	B,C	B,D	A,C,D
SECTION-I (iii)	Q.	9	10	11	12
	A.	A,C	B	A,B,D	A,B
SECTION-I (iv)	Q.	13	14	15	16
	A.	A	C	A	C
SECTION-II	Q.	1	2	3	4
	A.	1.00	3.00	17.00	9.00

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

1. Ans (A)



$$E_1 = \frac{k\lambda}{\left(\frac{a}{2}\right)}\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right]\left(-\hat{j}\right) = \frac{-2\sqrt{2}k\lambda}{a}\hat{j}$$

$$E_2 = \frac{2k\lambda}{\left(\frac{a}{2}\right)}\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right]\left(\hat{i}\right) = \frac{4\sqrt{2}k\lambda}{a}\hat{i}$$

$$E_3 = \frac{3k\lambda}{\left(\frac{a}{2}\right)}\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right]\left(\hat{j}\right) = \frac{6\sqrt{2}k\lambda}{a}\hat{j}$$

$$E_4 = \frac{4k\lambda}{\left(\frac{a}{2}\right)}\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right]\left(-\hat{i}\right) = \frac{-8\sqrt{2}k\lambda}{a}\hat{i}$$

$$\vec{E}_{net} = \frac{-4\sqrt{2}}{a}k\lambda\hat{i} + \frac{4\sqrt{2}}{a}k\lambda\hat{j}$$

2. Ans (B)

Electric field inside every position of conductor is zero.

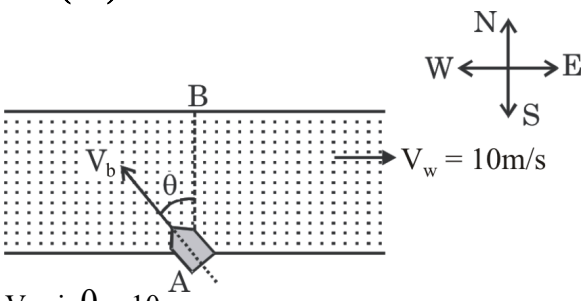
3. Ans (D)

V_{avg} = slope of secent

= tan45°

= 1 m/sec

4. Ans (A)



$V_b \sin\theta = 10$

$20\sin\theta = 10$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

PART-1 : PHYSICS

SECTION-I (ii)

5. Ans (A,B)

Since electric field due to an infinite plate is uniform, thus both the balls will reach the ground at same time and will have same velocity.

PART-1 : PHYSICS

SECTION-I (iii)

9. Ans (B)

$$V = V_q + V_{-qin} + V_{qin} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}-\frac{1}{R_1}+\frac{1}{R_2}\right]$$

10. Ans (D)

Charge distribution on the inner surface : non uniform.

11. Ans (A,B)

$mg - qE = ma$

$a = g - \frac{qE}{m}$

(A) $a' = \frac{2qE}{m} - g \Rightarrow A$

$\frac{q}{m}$ same $\Rightarrow a' = a$

(B) $g - \frac{qE}{m} > g - \frac{q'E}{m'}$

$= \frac{qE}{m} < \frac{q'E}{m'}$

12. Ans (C,D)

$$\frac{d}{4} = \frac{1}{2}\left(g - \frac{qE}{m}\right)t^2$$

$$\Rightarrow g - \frac{qE}{m} = \frac{2qE}{m} - g$$

$$\frac{3qE}{m} = 2g$$

$$\frac{qE}{m} = \frac{2g}{3}$$

$$g - \frac{q'E}{m'} = 2\left(g - \frac{qE}{m}\right) = \frac{2g}{3}$$

$$\Rightarrow \frac{q'E}{m} = \frac{q}{3} \Rightarrow \frac{q'}{m'} = \frac{q}{2m}$$

PART-1 : PHYSICS

SECTION-I (iv)

13. Ans (A)

$\vec{v} = 6\hat{i} + (20 - 4t)\hat{j}$

time when $v_y = 0$

$20 - 4t = 0$

$t = 5$

$T = 2 \times t = 10$ sec

$\tan 53 = \dfrac{20 - 4t}{6}$

$\dfrac{4}{3} = \dfrac{20 - 4t}{6}$

$t = 3$ sec

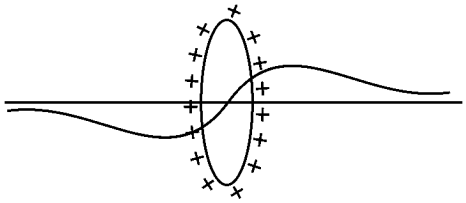
$R = v_x \times T$

$= 6 \times 10 = 60$ m

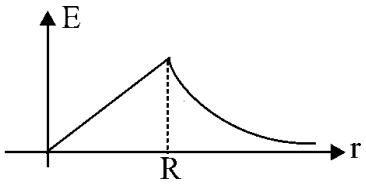
$H = \dfrac{v_y^2}{2 \times a} = \dfrac{20 \times 20}{2 \times 4} = \dfrac{400}{8} = \dfrac{200}{4} = 50$m

14. Ans (B)

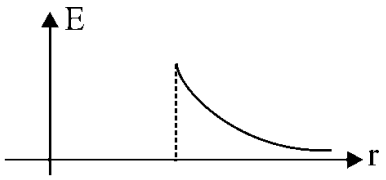
For (P) :



For (Q) : Inside point $E = \dfrac{\rho r}{3\varepsilon_0}$, outside point $E = \dfrac{K\theta}{r^2}$

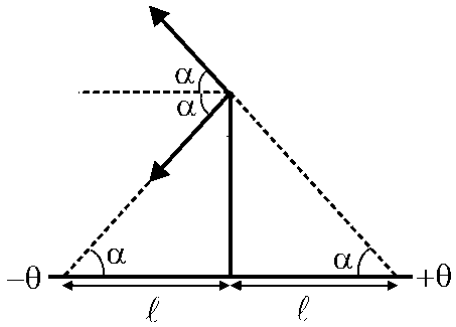


For (R) : $E_{inside} = 0$, $E_{outside} = \dfrac{K\theta}{r^2}$



For (S) : $E_{net} = 2E\cos\alpha = 2\dfrac{K\theta}{x^2 + \ell^2} \times \dfrac{\ell}{\sqrt{x^2 + \ell^2}}$;

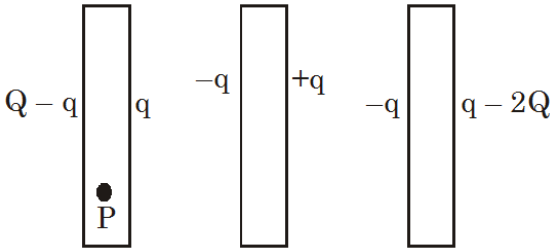
$E_{net} = \dfrac{2K\theta\ell}{(x^2 + \ell^2)^{3/2}}$



at x=0 : $E_{net} = \dfrac{2K\theta}{\ell^2}$, so only option is (P)

15. Ans (D)

Distribution of charge on different surfaces of the plates have been shown.



Take a point P on left most plate. The electric field at P

$E = \dfrac{Q-q}{2A\varepsilon_0} - \dfrac{q}{2A\varepsilon_0} + \dfrac{q}{2A\varepsilon_0} - \dfrac{q}{2A\varepsilon_0} + \dfrac{q}{2A\varepsilon_0} - \dfrac{(q-2Q)}{2A\varepsilon_0} = 0$

(inside a conductor, E = 0)

So, $Q - q - q + 2Q = 0$

Or $2q = 3Q \Rightarrow q = \dfrac{3Q}{2}$

The charge appearing on outer surface of right most plate

$= q - 2Q = \dfrac{3Q}{2} - 2Q = -\dfrac{Q}{2}$

The charge on outer surface of left most plate

$= Q - q = Q - \dfrac{3Q}{2} = -\dfrac{Q}{2}$

Charge appearing on middle plate is −q and +q

i.e., $-\dfrac{3Q}{2}$ and $+\dfrac{3Q}{2}$

16. Ans (B)

(P) Time of flight = $\frac{2\times 25}{10}\times\frac{4}{5}$

Range = $U_xT+\frac{1}{2}a_xT^2$

$=\left(25\times\frac{3}{5}\right)(4)+\frac{1}{2}\times(5)(4)^2$

(Q) $H_{max}=\frac{u^2\sin^2\theta}{2g}$

$=\frac{(25)^2(\sin 53)^2}{2\times 10}=20$

At $\frac{3}{4}$ th of H_{max} speed of particle is given by

$V^2=400-2\times 10\times\left(\frac{3}{4}\times 20\right)=100$

V = 10 m/s

Time difference = $\frac{2V}{g}=\frac{2\times 10^{10}}{10}=2$ sec

(R) $\vec{u}.\vec{v}=0$

t = 5 sec > Time of flight

So not possible

PART-1 : PHYSICS

SECTION-II

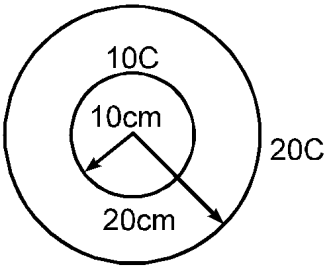
1. Ans (4.50)

potential difference due to inner 10C charge

$=K\,10\left(\frac{1}{.1}-\frac{1}{.2}\right)=9\times 10^{10}(5)$

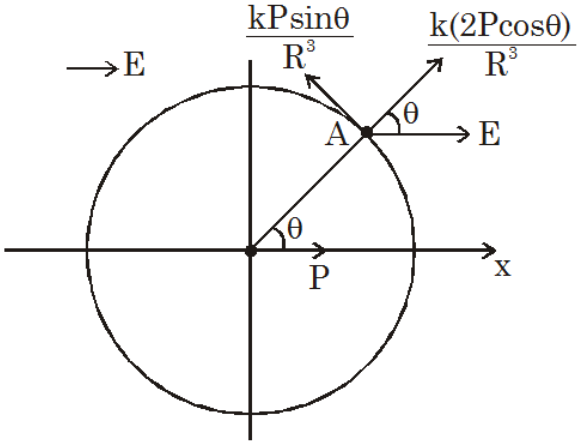
$=45\times 10^{10}=4.5\times 10^{11}$ V

potential difference due to outer charge = 0



∴ p.d. = 4.5×10^{11} V

2. Ans (1.33 to 1.34)



Consider a point on circumference at angular position θ.

For surface to be equipotential net E.F. at A should be perpendicular to A.

$\frac{kP\sin\theta}{R^3}=E\sin\theta$

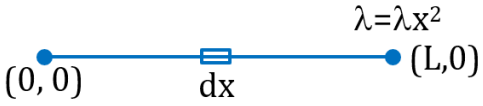
$E=\frac{kP}{R^3}$

3. Ans (30.00)

$V_A=V_C$

so C = a + b

4. Ans (2.00)



Electric field due to element dx on origin

$d\vec{E}=\frac{k\lambda dx\left(-\hat{i}\right)}{x^2}=\frac{k\lambda_0x^2}{x^2}dx\left(-\hat{i}\right)$

$\left\{k=\frac{1}{4\pi\epsilon_0}\right\}$

$dE=k\lambda_0dx$

Electric field at origin $\vec{E}=\int_0^L k\lambda_0dx\left(-\hat{i}\right)$

$E=k\lambda_0L$

so $\alpha=1,\ \beta=1\ \Rightarrow\ \alpha+\beta=2$

PART-2 : CHEMISTRY

SECTION-I (i)

1. Ans (D)

In Mn^{+2}, the number of unpaired electrons is 5.

spin only magnetic moment = $\sqrt{n(n+2)}$ B. M.

$\sqrt{5(5+2)} = \sqrt{35}$ B. M.

2. Ans (B)

Electronic configuration of

Mn^{2+} : $1s^2 2s^2 2p^6 3s^2 3p^6 3d^5$

max. number of electrons having $(m_l = -1)$ is 5

$(2p^6 \longrightarrow 2e^-$, $3p^6 \longrightarrow 2e^-$, $3d^5 \longrightarrow 1e^-)$

3. Ans (A)

$$CH_3CHO(g) \longrightarrow CH_4(g) + CO(g)$$

t = 0	80	0	0
t = 20min	80 − x	x	x

$(80 - x) + x + x = 120 \Rightarrow x = 40$

$t_{1/2} =$ 20min

$$K = \frac{0.693}{20} = 3.465 \times 10^{-2} \text{ min}^{-1}$$

4. Ans (D)

$Li^{+2} \longrightarrow$ single electron : energy : 3s = 3p = 3d

PART-2 : CHEMISTRY

SECTION-I (ii)

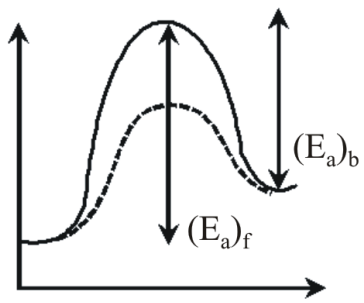
5. Ans (A,B,C)

(A) Enthalpy of reaction always depends on physical state of reactants.

(B) It is not necessory that it always increases with increase in temperature.

(C) ΔH is different between $[E_a]_f - [E_a]_b$ not ratio.

(D) Catalyst decreases activation energy but do not change ΔH.



$\Delta H = [E_a]_f - [E_a]_b$

6. Ans (A,B,D)

$$= -\frac{1}{3}\frac{d[H_2]}{dt} = \frac{1}{2}\frac{d[NH_3]}{dt}$$

$$\therefore \frac{-d[H_2]}{dt} = \frac{3}{2} \times \frac{0.001}{17} \frac{\text{Kmole}}{\text{hr}}$$

$$= \frac{3}{2} \times \frac{0.001}{17} \times 2 \text{ Kg/hr}$$

$$= 1.76 \times 10^{-4} \text{ Kg/hr.}$$

7. Ans (A,B,C)

$V4+ \Rightarrow 4s^\circ\ 3d^1$

$\ell = 2$

orbital angular momentum

$$= \sqrt{\ell(\ell+1)}\frac{h}{2\pi}$$

$$= \sqrt{2(n)}\frac{h}{2\pi}$$

$$= \sqrt{6}\frac{h}{2\pi}$$

8. Ans (A,B)

Fact

PART-2 : CHEMISTRY

SECTION-I (iii)

11. Ans (A,C,D)

[A] Incorrect

$$\underset{\substack{\Downarrow \\ \text{fully–filled orbitals}}}{Be} \xrightarrow{EA} Be^- \text{ endothermic}$$

fully-filled orbitals

[B] Correct

Since $\underset{\substack{\Downarrow \\ \text{fully–filled orbitals}}}{He} \longrightarrow He^-$ is endothermic.

that's why $He^- \longrightarrow$ He is exothermic

[C] Incorrect

$$\underset{1s^2 2s^1}{Li} \rightarrow \underset{1s^2 2s^2}{Li^-} \text{ exothermic } \underset{\text{(fully–filled)}}{}$$

[D] Incorrect

$P \longrightarrow P^-$ is slightly exothermic

PART-2 : CHEMISTRY

SECTION-I (iv)

13. Ans (C)

(P) Ca

Belongs to 2^{nd} group

Has lower size than Cs (across the period size decreases)

Have higher IE than Cs (as zeff increases across the period)

(P) ⟶ 1, 3, 4

(Q) Ba

Belong to 2^{nd} group

Has lower size than Cs (across the period size decreases) have higher IE than Cs(as zeff increases accross the period)

(Q) ⟶ 1, 3, 4

(R) K

Belong to 1^{st} group.

Has lower size than cs (Top to bottom size increases)

Have higher IE than cs (top to bottom IE decreases.

(R) ⟶ 2, 3, 4

(S) Na

Belong to 1^{st} group

Has lower size than Cs (Top to bottom size increases)

Have higher IE than cs (Top to bottom IE decreases

(S) ⟶ 2, 3, 4

14. Ans (D)

[P] Ionisation energy ⟶ [3] HF > Zr [I.E.]

[Q] Radii ⟶ [1] Ne > Na^+

[R] Electronegativity ⟶

[4] $[He]2s^2 2p^4 > [He]2s^2\ 2p^3$

⇓ ⇓

O [E.N.] > N[E.N.]

[S] Electron gain enthalpy ⟶ [2] S > O

↓

[More space]

PART-2 : CHEMISTRY

SECTION-II

1. Ans (125.00)

$$\Delta H = \Delta H_1 \times \frac{3}{4} + \Delta H_2 \times \frac{1}{4}$$

$$= 100 \times \frac{3}{4} + 200 \times \frac{1}{4} = 125 \text{ cal}$$

2. Ans (33.33)

$$t = \frac{t_{1/2}}{\log 2} . \log \frac{V_\infty}{V_\infty - V_t}$$

$$20 = \frac{10}{\log 2} . \log \frac{V_\infty}{V_\infty - 25}$$

$\therefore V_\infty = 33.33$ ml

3. Ans (11.75)

$E_{ab} - E_{af} = 4RT$; $A_f = 2 \times A_b$

$K_f = A_f e^{\frac{-E_{af}}{RT}}$ .....(1)

$K_b = A_b e^{\frac{-E_{ab}}{RT}}$ .....(2)

$$K_{eq} = \frac{K_f}{K_b} = \frac{A_f}{A_b} e$$

$K_{eq} = 2e^4$

$\Delta^oG = -RT\ln K_{eq} = -2500 \times [4.7]$

$= 11.75$ kJ/mol

4. Ans (42.00)

$Z = 29 \Rightarrow$ copper

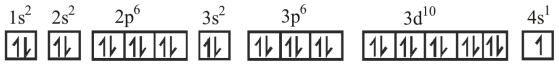
$Cu_{29} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^1$

non directional orbitals = $1s^2, 2s^2, 3s^2, 4s^1$

Total $7e^-$

So X = 7

$|m_\ell| = 1, m_\ell = \pm 1$



Y = 6

$X.Y = 7 \times 6 = 42$

PART-3 : MATHEMATICS

SECTION-I (i)

1. Ans (B)

$$\lim_{x\to 0}\left(\frac{(4^x-1)^3}{x^3}\right)\left(\frac{\left(\frac{x}{p}\right)}{\sin\frac{x}{p}}\right)\left(\frac{\frac{x^2}{3}}{\log\left(1+\frac{x^2}{3}\right)}\right)\times 3p$$

$= 3p\,(\log_e 4)^3$

2. Ans (D)

$$T_n = \frac{1+2+3+\ldots+n}{1^3+2^3+3^3+\ldots+n^3}$$

$$= \frac{\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2} = \frac{2}{n(n+1)} = 2\left[\frac{1}{n}-\frac{1}{n+1}\right]$$

$$\Rightarrow S_n = 2\left[\frac{1}{1}-\frac{1}{n+1}\right] \quad \therefore S_\infty = 2$$

3. Ans (A)

$10^n + 3.4^{n+2} + 5$

Put $n = 1 = 10^1 + 3.4^{1+2} + 5$

$= 10 + 3.64 + 5 = 207$

Which is divisible by 9

4. Ans (C)

$5^{\log_5 x.\log_5 x} + x^{\log_5 x} = 1250$

$$\Rightarrow \left(5^{\log_5 x}\right)^{\log_5 x} + x^{\log_5 x} = 1250$$

$$\Rightarrow x^{\log_5 x} + x^{\log_5 x} = 1250 \Rightarrow 2.x^{\log_5 x} = 1250$$

$$\Rightarrow x^{\log_5 x} = 625 \Rightarrow \log_5 x^{(\log_5 x)} = \log_5 625$$

$$\Rightarrow (\log_5 x)(\log_5 x) = 4 \Rightarrow (\log_5 x)^2 = 4$$

$$\Rightarrow \log_5 x = 2 \text{ or } \log_5 x = -2 \Rightarrow x = 25 \text{ or } x = \frac{1}{25}$$

$$\text{So, } \log_{25}\left(\frac{1}{25}\right) = -1$$

PART-3 : MATHEMATICS

SECTION-I (ii)

5. Ans (A,B,D)

$$\ell_1 = \lim_{x\to 0}\frac{e^{2x}-1-2x}{x^2} = \lim_{x\to 0}\frac{2e^{2x}-2}{2x} = 2$$

$$\ell_2 = \lim_{x\to 0}\frac{\sin 3x - 3x}{x^3}$$

$$= \lim_{x\to 0}\frac{3(\cos 3x - 1)}{3x^2} = \frac{-3\sin 3x}{2x} = -\frac{9}{2}$$

$$\ell_3 = \lim_{x\to 0}\frac{\tan x(\cos x - 1)}{x^3} = -\frac{1}{2}$$

6. Ans (B,C)

$$A = \lim_{x\to 0}\frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$$

$$\lim_{x\to 0^-}\frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)} = \lim_{x\to 0^-}\frac{x}{-x} = -1$$

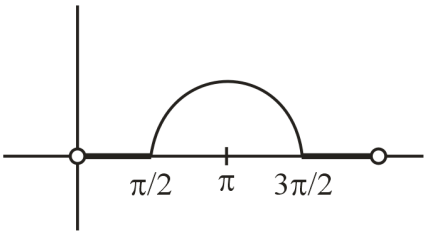
$$\lim_{x\to 0^+}\frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)} = \lim_{x\to 0^+}\frac{x}{x} = 1$$

$\Rightarrow$ A does not exist

$$B = \lim_{x\to 0}\frac{[|x|]}{x} = 0 \text{ as } [|x|] = 0 \text{ when } x \longrightarrow 0.$$

7. **Ans (B,D)**

$$\cos x\,[\cos x] = \begin{cases} 0 & 0 < x \leqslant \frac{\pi}{2} \\ -\cos x & \frac{\pi}{2} < x \leqslant \pi \\ -\cos x & \pi < x < \frac{3\pi}{2} \\ 0 & \frac{3\pi}{2} \leqslant x < 2\pi \end{cases}$$



8. **Ans (A,C,D)**

$T_{40} = 119 \quad t_{50} = 101$

common term series = 5, 11, 17, ..., T'_n

$T'_n \leq 101 \Rightarrow 5 + (n-1)\,6 \leq 101 \Rightarrow n \leq 17$

$\Rightarrow n = 17$

$\Rightarrow T'_{17} = 101$

sum of all common terms $= \frac{17}{2}(5 + 101) = 901$

PART-3 : MATHEMATICS

SECTION-I (iii)

9. **Ans (A,C)**

$ar = 4 \quad \frac{a}{1-r} = 25$

$r = \frac{1}{5}, \frac{4}{5}$ i.a = 20, a = 5.

10. **Ans (B)**

$$S_n = \frac{a(1-r^n)}{1-r}$$

for $r = \frac{4}{5}$ & a = 5;

$$\frac{5\left(1-\left(\frac{4}{5}\right)^n\right)}{1-\frac{4}{5}} > 24$$

$$\left(\frac{4}{5}\right)^n < \frac{1}{25}$$

$$n > \frac{2\log 5}{\log 5 - \log 4}$$

$$\Rightarrow \left(\frac{1}{5}\right)^n < \frac{1}{25} \Rightarrow 5^n > 25$$

$$n > \frac{2(1-0.3)}{1-0.3-2\times 0.3} \quad \Rightarrow n > 2 \Rightarrow n \geq 3$$

$n > 14$

smallest value n = 3

for $r = \frac{1}{5}$ & a = 20

$$\frac{20\left(1-\left(\frac{1}{5}\right)^n\right)}{1-\frac{1}{5}} > 24$$

$$\Rightarrow 1-\left(\frac{1}{5}\right)^n > \frac{24}{25}$$

11. **Ans (A,B,D)**

$A = \log_2[\log_4(16\log_{10}2 + 16\log_{10}5)] - 1$

$= \log_2[\log_4 16] - 1 \Rightarrow \log_2 2 - 1 = 0$

$$B = \log\left(5\sqrt{5\sqrt{5\sqrt{5\sqrt{5\sqrt{5}}}}}\right)$$

$\sqrt{5\sqrt{5}} = 5^{\frac{3}{4}}, \sqrt{5\sqrt{5\sqrt{5}}} = 5^{\frac{7}{8}}$

$\sqrt{5\sqrt{5\sqrt{5\sqrt{5}}}} = 5^{\frac{15}{16}}, \sqrt{5\sqrt{5\sqrt{5\sqrt{5\sqrt{5}}}}} = 5^{\frac{31}{32}}$

$\sqrt{5\sqrt{5...6\text{times}}} = 5^{\frac{63}{64}},$

$1\sqrt{5\sqrt{5...7\text{times}}} = 5.5^{\frac{63}{64}} = 5^{\frac{127}{64}}$

(C) $\log_4 10 + 1 = x + 1 = \log_4 40$

$\log_2 20 + 1 = y + 1 = \log_2 40$

$\log_5 8 + 1 = z + 1 = \log_5 40$

$$\Rightarrow \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$$

$= \log_{40}4 + \log_{40}2 + \log_{40}5 = 1$

(D) $(\log_2 10)(\log_2 80) - (\log_2 5)(\log_2 160)$

$(1 + \log_2 5)(4 + \log_2 5) - (\log_2 5)(5 + \log_2 5)$

$= 4$

12. Ans (A,B)

$A = \log_2\left[\log_4\left(\log 16_{10}^4 + \log 25_{10}^8\right)\right] - 1$

$A = \log_2\left[\log_4\left(\log 4_{10}^8 + \log 25_{10}^8\right)\right] - 1$

$A = \log_2[\log_4(8 \times 2)] - 1$

$A = \log_2\left[2\log_4^4\right] - 1$

$\boxed{A = 0}$

$B = (2q - P)$

$$\log\left(\sqrt{5\sqrt{5\sqrt{5\sqrt{5\sqrt{5\sqrt{5}}}}}}\right) = \frac{P}{q}\log 5$$

$= \log 5^{\left(\frac{127}{64}\right)} = \frac{p}{q}\log 5$

$\Rightarrow \frac{p}{q} = \frac{127}{64}$

$\Rightarrow p = 127, q = 64$

$B = 2q - p$

$= 2 \times 64 - 127$

$\boxed{B = 1}$

$\rightarrow C = \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$;

$x = \log_4^{10}$, $y = \log_2^{20}$ $\log^8{}_5 = z$

$C = \frac{1}{\log_4^{10}+1} + \frac{1}{\log_2^{20}+1} + \frac{1}{\log_5^8+1}$

$C = \frac{1}{\log_4^{10}+\log_4^4} + \frac{1}{\log_2^{20}+\log_2^2} + \frac{1}{\log_5^8+\log_5^5}$

$C = \frac{1}{\log_4^{40}} + \frac{1}{\log_2^{40}} + \frac{1}{\log_5^{40}}$

$C = \frac{\log^4 + \log^2 + \log^5}{\log^{40}}$

$\boxed{c = 1}$

$\Rightarrow D = \left(\log_2^{10}\right)\left(\log_2^{80}\right) - \left(\log_2^5\right)\left(\log_2^{160}\right)$

$D = \left(1+\log_2^5\right)\left(4+\log_2^5\right) - \log_2^5\left(5+\log_2^5\right)$

$D = (1+t)(4+t) - t(5+t)$ $\left\{\text{Let } t = \log_2^5\right.$

$D = t^2 + 5t + 4 - 5t - t^2$

$\boxed{D = 4}$

option - (A) $BC^2D = 1 \times 1^2 \times 4 = 4$

option - (B) $2B + D = 2 \times 1 + 4 = 6$

option - (C) $B + C + 4d = 1 + 1 + 4 \times 4 = 18$

option - (D) $A^2 + D + B = (O)^2 + 4 + 1 = 5$

option (A,B)

PART-3 : MATHEMATICS

SECTION-I (iv)

13. Ans (A)

(P) $a + b + c = 0$ and $a + c = 2b \Rightarrow b = 0$

In 17 terms of A.P., b is 9th (middle) term

$\Rightarrow$ 8 (+)ve terms & 8(–)ve terms of equal magnitude

$\Rightarrow$ sum = 0

(Q) $\frac{1}{a^2} = \frac{2\left(\frac{1}{a+2}\right)\left(\frac{1}{4a+5}\right)}{\frac{1}{a+2}+\frac{1}{4a+5}}$

$\Rightarrow 2a^2 - 5a - 7 = 0$ $\begin{cases} a = \frac{7}{2} \\ a = -1 \end{cases}$

(R) $S_1 = \alpha + \beta + \gamma + 2 = \alpha \Rightarrow \gamma + 2 = -\beta$

A.P. $\Rightarrow 2\beta = \alpha + \gamma + 2 \Rightarrow 3\beta = \alpha$

$S_2 = \alpha\beta + \beta(\gamma + 2) + \alpha(\gamma + 2) = \beta$

$\Rightarrow 3\beta^2 - \beta^2 + 3\beta(-\beta) = \beta \Rightarrow \beta = -1,$ 0(rejected)

$\Rightarrow \alpha = -3 \Rightarrow \gamma = -1$

$\Rightarrow$ roots are –3,–1,1 (in order)

(S) $\sum_{r=0}^{\infty}(\alpha^r + \beta^r) = \frac{1}{1-\alpha} + \frac{1}{1-\beta}$

$= \frac{2-(\alpha+\beta)}{1+\alpha\beta-(\alpha+\beta)} = 7$

14. Ans (C)

(P) $d = \dfrac{2-102}{19+1} = -5$

$A_{17} = 102 + 17(-5) = 17$

$A_{10} = 102 + 10(-5) = 52$

(Q) $P_n = \dfrac{1}{1\cdot3\cdot5} + \dfrac{1}{3\cdot5\cdot7} + \ldots + \dfrac{1}{(2n-1)(2n+1)(2n+3)}$

$P_n = \dfrac{1}{4}\left(\dfrac{1}{1\cdot3} - \dfrac{1}{(2n+1)(2n+3)}\right)$

$P_\infty = \dfrac{1}{12}$

(R) $\sum_{i=1}^{n}\left(\sum_{j=1}^{i}1\right)^2 = \sum_{i=1}^{n} i^2 = 385$

$\dfrac{n(n+1)(2n+1)}{6} = 385$

$\Rightarrow n = 10$

(S) $(G_1G_2G_3G_4G_5G_6)^{1/6} = \sqrt{2\times5}$

$\prod_{i=1}^{6} G_i = 10^3$

15. Ans (A)

(P) $x = \log_\pi e + \dfrac{1}{\log_\pi e} > 2$

(Q) $x = 3\log_{10}a + \dfrac{3}{\log_{10}a} < -6$

$y = 4\log_{10}b + \dfrac{4}{\log_{10}b} > 8 \Rightarrow x < y$

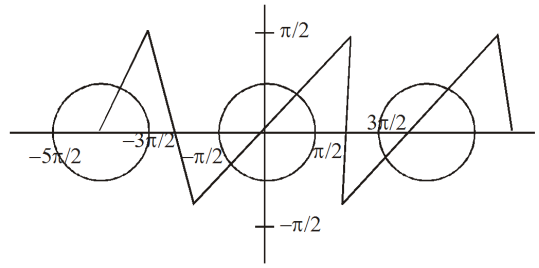
(R) $x = y$

(S) $x = 2 - \sqrt{3} + \sqrt{3} - \sqrt{2} + \sqrt{2} - 1 = 1$

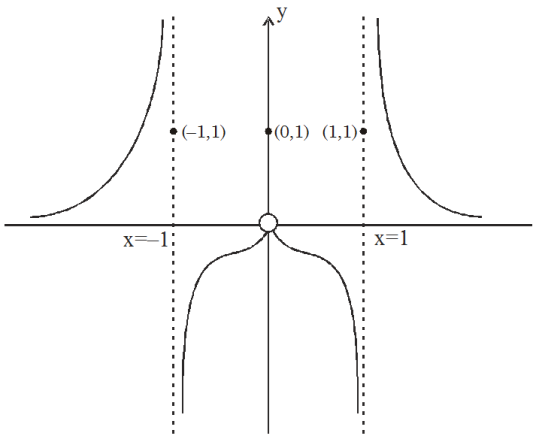
$y = 3$

16. Ans (C)

(P)



(Q)



(R) $\lim_{x\to0}\left(1 + x\ln\left(1+b^2\right)\right)^{1/x} = 2b\sin^2\theta,\ b > 0$

$\Rightarrow 1 + b^2 = 2b\sin^2\theta \Rightarrow b^2 - 2b\sin\theta + 1 = 0$

$D \geqslant 0 \Rightarrow \sin^2\theta = 1 \Rightarrow \sin\theta = \pm1$

$\Rightarrow \theta = \pm\dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}$

$\therefore$ Sum of all possible value of $\theta = 4\pi$

$\therefore k = 4$

(S) for a limit of the function to exist $\alpha + \beta = 1$

since function gain form of 1^∞ we get $\alpha = -3$,

$\beta = 4$

$\therefore (4\beta + 3\alpha) = 7 \Rightarrow Q$

PART-3 : MATHEMATICS

SECTION-II

1. Ans (1.00)

$$\lim_{x\to 0}\frac{(ax+b)-\sqrt{4+\sin x}}{\tan x}=\frac{27}{4}$$

As $x \to 0$, $Dr \to 0$

So, $Nr \to 0$ for existence of limit. So

$b-2=0 \Rightarrow b=2$

$$\text{Now, } \lim_{x\to 0}\frac{(ax+2)-\sqrt{4+\sin x}}{x}=\frac{27}{4}\left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\frac{(ax+2)^2-(4+\sin x)}{x\cdot\left(ax+2+\sqrt{4+\sin x}\right)}=\frac{27}{4}$$

$$\lim_{x\to 0}\left(\frac{a^2x^2+4ax-\sin x}{x}\right)=27$$

$$\Rightarrow 4a-1=27 \Rightarrow a=\frac{28}{4}=7$$

Hence, $(a-3b)=7-3(2)=1$ Ans.

2. Ans (3.00)

For continuity at $x=1$

$8=b^2-3b+10 \Rightarrow b^2-3b+2=0$

Sum of all b's = 3

3. Ans (17.00)

tan (px) is discontinuous at points $x=\left(\frac{n+1}{2}\right)$,

ie. $\frac{-7}{2},\frac{-5}{2},\frac{-3}{2},\frac{-1}{2},\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2}$, total 8 points

signum function in discontinuous at points where

$\tan\pi x=0$

$\Rightarrow -4,-3,-2,-1,0,.....4$

total 9 points

4. Ans (9.00)

Let $\log_2 x=t$

$3\sqrt{t}-t-3+1=0$

$3\sqrt{t}=t+2$

$9t=t^2+4+4t$

$t^2-5t+4=0$

$t=1,4$

$\log_2 x=1,4$

$x=2,16$