S O L V E
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FB1 ***Using the Euclidean algorithm, find*** $hcf(86, 100)$***, and use this to find integers*** s, t ***such that*** $hcf(86, 100) = 86s + 100t$***.***

$$hcf(86, 100)$$
$$100 = 86 \times 1 + 14$$
$$86 = 14 \times 6 + 2$$
$$14 = 2 \times 7 + 0$$

Final non-zero remainder in the Euclidean algorithm is the highest common factor, hence hcf(86,100)=2

hcf(86,100) = 86s+100t   hcf(86,100)=2 ✓   2=86s+100t

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$$100 = 86 \times 1 + 14 \qquad => \qquad 14 = 100 + 86 \times (-1)$$

$I\ substitute\ 100 + 86 \times (-1)\ for\ 14\ in\ the\ equation\ 2 = 86 + 14 \times (-6)$

$$2 = 86 + (100 + 86(-1))(-6)$$
$$2 = 86 + 100(-6) + 86(6)$$
$$2 = 86 \times 7 + 100 \times (-6)$$

$s = 7$ and $t = -6$ ✓

FB2 ***Which positive integers have exactly three positive divisors?***

If a number has 3 positive divisors one of them is a 1, another is itself and the 3rd one is some other integer.

Lets say that x resembles the integers we are looking for, y represents the 3rd divisor apart from 1 and x itself and b represents the outcome of $x \div y$. $x, y \in \mathbb{N}$
Also $b \in \mathbb{N}$ because if it doesn't then $x \nmid y$.

$x \div y = b$ y must be equal to b because if it wasn't then a and b would be 2 separate divisors of x and that would mean that there would be at least 4 divisors for x.   (1, a, b, x)

$x \div y = y$ y must be a prime because if it wasn't it could be factored into smaller integers and they too would be divisors for x.

We know that $x \div y = y$ and y is a prime, we can solve the equation for x and we get $x = y^2$.

The integers which have exactly 3 divisors are prime squares, $x = y^2$ $\{y \in \mathbb{N} |\ y\ is\ prime\}$ ✓

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See Solutions

FB3 ***Let*** $f(x) = 2 - |4x - 2|$***. Show that there is no value of*** c ***such that*** $f(3) - f(0) = f'(c)(3 - 0)$***. Why does this not contradict the Mean Value Theorem?***

$$f(3) = 2 - |4 \times 3 - 2| = 2 - 10 = -8$$

$$f(0) = 2 - |4 \times 0 - 2| = 2 - 2 = 0$$

$$f(3) - f(0) = -8 - 0 = -8$$

$$\frac{d}{dx}|u| = \frac{u}{|u|} \cdot \frac{d}{dx}u$$

$$f'(x) = \frac{d}{dx}2 - \frac{d}{dx}|4x - 2| = 0 - \frac{4x - 2}{|4x - 2|} \cdot 4 = -\frac{4(4x - 2)}{|4x - 2|}$$

$$f'(x) domain \{x \in \mathbb{R} | x \neq 0.5\}$$

$$f(3) - f(0) = f'(c)(3 - 0)$$

$$f'(c) = \frac{f(3) - f(0)}{3 - 0}$$

I substitute $f(3),\ f(0)$ and $f'(c)$ in $f'(c) = \frac{f(3)-f(0)}{3-0}$.

$$-\frac{4(4c - 2)}{|4c - 2|} = \frac{-8}{3} \qquad c \neq 0.5$$

$$\frac{4c - 2}{|4c - 2|} = \frac{2}{3}$$

$$3(4c - 2) = 2|4c - 2|$$

$f(3) - f(0) = f'(c)(3 - 0)$ is false $\forall\ c$ because of the 3 bullet points bellow.

- $f(3) - f(0) = f'(c)(3 - 0)$ is not defined for c=0.5 because $f'(0.5)\nexists$
- For $c > 0.5$ the equation $f(3) - f(0) = f'(c)(3 - 0)$ is false.

$$if\ c > 0.5$$

$$3(4c - 2) = 2|4c - 2|$$

$$3(4c - 2) = 2(4c - 2)$$

$$3 = 2$$

- For $c < 0.5$ the equation $f(3) - f(0) = f'(c)(3 - 0)$ is false.

$$if\ c < 0.5$$

$$3(4c - 2) = 2|4c - 2|$$

$$3(4c - 2) = 2(2 - 4c)$$

$$-3(2 - 4c) = 2(2 - 4c)$$

$$-3 = 2$$

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It does not contradict the mean value theorem because it can not be applied, and that is because it is not differentiable at $c = 0.5$. Cannot be applied, therefore it has to say about the mean value theorem and it does not contradict it.