Davids Zakrevskis
Feedback exercises

SPCKUE Maths
3 3 2 3 2 $\frac{14}{15}$

FB1 ***Find the second derivative of*** y ***using implicit differentiation of*** $x^2 - y^2 = 5$.

$$x^2 - y^2 = 5$$

Taking the 1st derivative of the function, using the difference rule and implicit differentiation.

$$\frac{d}{dx}x^2 - \frac{d}{dx}y^2 = \frac{d}{dx}5$$
$$2x - 2y\frac{d}{dx}y = 0$$
$$\frac{d}{dx}y = \frac{2x}{2y} = \frac{x}{y}$$
$$y' = \frac{x}{y}$$

Taking the 2nd derivative using the quotient rule and implicit differentiation. (From the first part we know that $y' = \frac{x}{y}$)

$$y'' = \frac{x'y - xy'}{y^2} = \frac{y - x \times \frac{x}{y}}{y^2} = \frac{1}{y} - \frac{x^2}{y^3}$$
$$y'' = \frac{d^2y}{dx^2} = \frac{1}{y} - \frac{x^2}{y^3}$$

$\frac{5}{5}$

FB2 ***Prove by induction that*** $n! \leq n^n$***, for all*** $n \in \mathbb{N}$

$$n! \leqslant n^n, n \in \mathbb{N}, k \in \mathbb{N}$$

We check if it is true at 1 and if it's true we assume $k! \leqslant k^k$ is correct for some k. Then we check if $(k+1)! \leqslant (k+1)^{k+1}$ is true.

$$n_1 = 1 \qquad n! \leqslant n^n \quad => \quad 1! \leqslant 1^1 \quad => \quad 1 \leqslant 1$$
$$n_2 = 2 \qquad n! \leqslant n^n \quad => \quad 2! \leqslant 2^2 \quad => \quad 2 \leqslant 4$$

(0) Unnecessary

I assume that k!$\leqslant k^k$ *is true for some* k

(C)

$$n = k+1 \qquad (k+1)! \leqslant (k+1)^{k+1}$$
$$\Leftarrow \quad (k+1)k! \leqslant (k+1)(k+1)^n \qquad \textit{I divide both sides by } (k+1)$$
$$\Leftarrow \quad k! \leqslant (k+1)^k$$

backwards

- We made the assumption that $k! \leqslant k^k$ was correct, which it is at $k = 1$ and $k = 2$.
- $k^k \leqslant (k+1)^k \ \forall\ k,\ k \in \mathbb{N}$. That is because we can take the k$_{th}$ *root of both sides and we will get* k $\leqslant k+1$ which is true. We can take the k$_{th}$ *root because* $k \in \mathbb{N}$.
- $k! \leqslant (k+1)^k$ is true because of the assumption $k! \leqslant k^k$ and $k^k \leqslant (k+1)^k$. $\qquad (k! \leqslant k^k \leqslant (k+1)^k)$
- Therefore, $(k+1)! \leqslant (k+1)^{k+1}$ is true because $k! \leqslant (k+1)^k$ was derived from it and is equal.

$n! \leq n^n$ is true when n=k+1 if it is true at $n = k$, $k \in \mathbb{N}$ and we know that it is true for n=1 and n=2.

$\frac{4}{5}$

I have proved that $n! \leq n^n$, $\forall\ n \in \mathbb{N}$ using the principles of mathematical induction.

FB3 ***Suppose the roots of the equation*** $2x^2 - 5x - 6 = 0$ ***are*** α ***and*** β***. Find the quadratic equation with roots*** $\frac{1}{\alpha}$ ***and*** $\frac{1}{\beta}$

$$a_1x^2 + b_1x + c_1 = 2x^2 - 5x - 6 = 0$$

Vieta's formulas applied to quadratic polynomial: $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$, the same formulas are given and proved in the book "A concise introduction to pure mathematics" by M. W. Liebeck on page 56.

$$\alpha + \beta = \frac{-b_1}{a_1} = \frac{5}{2} = 2.5 \quad \textit{and} \quad \alpha\beta = \frac{c_1}{a_1} = \frac{-6}{2} = -3$$

The new equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, $x^2 + b_2x + c_2 = 0$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} \quad \textit{and} \quad \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta}$$

$$-b_2 = \frac{\alpha + \beta}{\alpha\beta} = \frac{2.5}{-3} = -\frac{5}{6} \quad \textit{and} \quad c_2 = \frac{1}{\alpha\beta} = \frac{1}{-3} = -\frac{1}{3}$$

$$b_2 = \frac{5}{6} \quad \textit{and} \quad c_2 = -\frac{1}{3}$$

$\frac{5}{5}$

Substitute b_2 and c_2 in the equation: $x^2 + b_2x + c_2 = 0$

$$x^2 + \frac{5}{6}x - \frac{1}{3} = 0$$

To get rid of fractions as arguments we can multiply the equation by 6

$$6x^2 + 5x - 2 = 0$$

The equation $x^2 + \frac{5}{6}x - \frac{1}{3} = 0$ which is equal to $6x^2 + 5x - 2 = 0$ has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(E)