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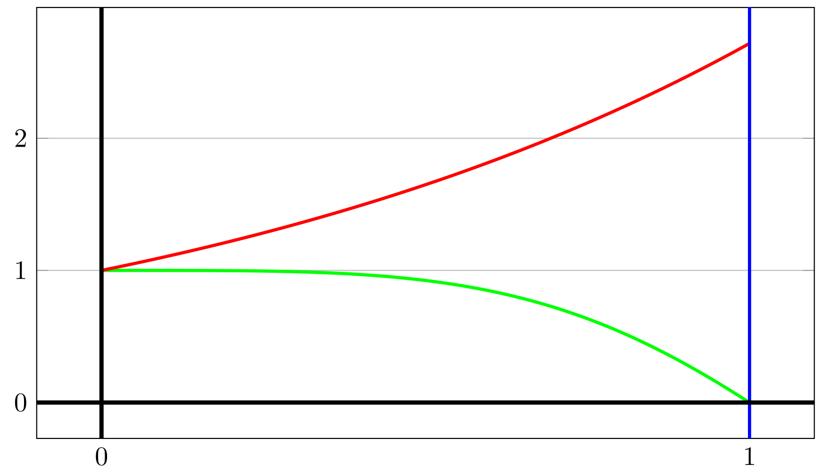
Feedback exercises

FB1 ***Find the volume of the solid generated by rotating the region bounded by the curves*** $y = e^x$***,*** $y = \cos\left(\frac{\pi x^2}{2}\right)$ ***and*** $x = 1$ ***about the*** y ***axis.***

The volume of a solid obtained by rotating about the y-axis the region under the curve $y = f(x)$ from a to b, is

$$V = \int_a^b 2\pi x f(x)\,dx$$



The shape with which has the green red and blue edges is the shape that is rotating around the y axis.
Green line is defined by the function $y = \cos\left(\frac{\pi x^2}{2}\right)$ with the domain [0:1]
Red line is defined by the function $y = e^x$ with the domain [0:1]
Blue line is defined by the function $x = 1$

If we rotate the shape around the y axis the volume of the resulting shape is equal to volume of the integral

$$\int_0^1 2\pi x \left(e^x - \cos\left(\frac{\pi x^2}{2}\right)\right)\,dx$$

$$\int_0^1 2\pi x \left(e^x - \cos\left(\frac{\pi x^2}{2}\right)\right)\,dx = \int_0^1 2\pi x e^x\,dx - \int_0^1 2\pi x \cos\left(\frac{\pi x^2}{2}\right)dx$$

I integrate by parts $\quad u = x, \quad dv = e^x dx, \quad du = dx, \quad v = e^x \qquad (\int u dv = uv - \int v du)$

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$\int_0^1 2\pi x e^x\,dx = 2\pi \int_0^1 x e^x\,dx = 2\pi(xe^x - e^x)\Big|_0^1 = 2\pi(0 - (-1)) = 2\pi$

I use substitution $\quad u = \frac{\pi x^2}{2},\ dx = \frac{1}{\pi x} du$

$\int_0^1 2\pi x \cos\left(\frac{\pi x^2}{2}\right)dx = 2\pi \int_0^1 x \cos\left(\frac{\pi x^2}{2}\right)dx = 2\pi \int_0^1 x \cos u \frac{1}{\pi x} du = 2\pi \int_0^1 \frac{1}{\pi} \cos(u) du = 2\sin u\Big|_0^1 = 2\sin\frac{\pi x^2}{2}\Big|_0^1 = 2(\sin\frac{\pi}{2} - \sin 0) = 2$

$$\int_0^1 2\pi x \left(e^x - \cos\left(\frac{\pi x^2}{2}\right)\right)\,dx = \int_0^1 2\pi x e^x\,dx - \int_0^1 2\pi x \cos\left(\frac{\pi x^2}{2}\right)dx = 2\pi - 2$$

Maths = 11/15

S	O	L	V	E
3	3	3	3	3

Total = 26/30

FB2 ***Decide whether the set A of positive integers divisible by 17 and B the set of positive integers divisible by 11 are in bijection.***

Positive integers $= \{\mathbb{Z} \mid k \geq 0\} = \mathbb{N}$

The set A which is a set of integers divisible by 17. $A\ \{a \in \mathbb{N} \mid a \equiv 0 \pmod{17}\}$

The set B which is a set of integers divisible by 11. $B\ \{b \in \mathbb{N} \mid b \equiv 0 \pmod{11}\}$

The sets A and B are in bijection because there is a bijective function between A and B. For every b in B there is a unique a in A such that $f(a) = b$

Each element of A can be uniquely represented as 17n, $n \in N$

Each element of B can be uniquely represented as 11n, $n \in N$

The bijective function maps A to B, $(A \mapsto B)$ $\quad 17n \mapsto 11n \quad n \in N$

The function itself $\quad f(a) = \frac{11a}{17} = b$

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FB3 ***Suppose that α, β are disjoint cycles in the symmetric group S_m for $m \geq 9$***

a) Let α by a cycle of length 3 and β a cycle of length 9.

What is the order of $\alpha\beta$?

By proposition 20.4 order of $\alpha\beta$ is lcm(3,9)=9

Is the permutation $\alpha\beta$ even or odd?

A permutation is even if sgn(g)=+1 and odd if sgn(g)=-1
By proposition 20.6 sgn(α)= $(-1)^{3-1} = (-1)^2 = 1$
By proposition 20.6 sgn(β)= $(-1)^{9-1} = (-1)^8 = 1$
By proposition 20.5_{ii} sgn($\alpha\beta$)=sgn(α)sgn(β)=$1 \cdot 1 = +1$; therefore $\alpha\beta$ is even

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b) Show that for every positive integer n we have

$$(\alpha\beta)^n = \alpha^n\beta^n.$$

It is given that the cycles are disjoint; therefore, they have no elements incommon and the cycles α and β commute.

$$(\alpha\beta)^n = \underbrace{\alpha\beta \cdot \alpha\beta \cdot ... \cdot \alpha\beta}_{\text{n times}} = \underbrace{\alpha\beta \cdot \beta\alpha \cdot \alpha\beta \cdot ... \cdot \alpha\beta \cdot \beta\alpha}_{\text{n times + explanation 1}} = \alpha\beta^2 \cdot \alpha^2 \cdot \beta^2 \cdot ... \cdot \alpha^2 \cdot \beta^2\alpha =$$

$$= \underbrace{\alpha\beta^2 \cdot \beta^2 \cdot \alpha^2 \cdot \alpha^2 \cdot ... \cdot \alpha^2 \cdot \alpha^2 \cdot \beta^2 \cdot \beta^2\alpha}_{\text{explanation 2}} = \ ... \ = \underbrace{\alpha^n\beta^n}_{\text{explanation 3}}$$

Explanation 1 Because cycles α and β commute, when they are multiplied their order can be changed.

Explanation 2 We know that α and β have no incommon elements; therefore, their powers too have no incommon elements and resulting cycles of $\alpha^k\beta^j$ for any j and k commute and can be reordered by explanation 1.

Explanation 3 First steps can be made over and over again, alphas can be multiplied with alphas and betas with betas, the equation can be reordered and repeat until we get $\alpha^n\beta^n$.