| S | O | L | V | E |
|---|---|---|---|---|
| 3 | 3 | 3 | 3 | 3 |

MATHS $\frac{15}{15}$

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Feedback exercises

$\frac{30}{30}$

**FB1** ***Find the remainder r (between 0 and 8) that we get when we divide $7^{96}$ by 9. Be sure to show your work.***

I'll be using the method of successive squares

$$7^{96} \equiv r \pmod 9$$

$$\begin{aligned}
7 &\equiv 7 \pmod 9 && \text{By preposition 13.2}\\
7^2 &\equiv 7^2 \pmod 9 = 4 \pmod 9 && \text{By preposition 13.4}\\
7^4 &\equiv 4^2 \pmod 9 = 7 \pmod 9 && \text{By preposition 13.4}\\
7^8 &\equiv 7^2 \pmod 9 = 4 \pmod 9 && \text{By preposition 13.4}\\
7^{16} &\equiv 4^2 \pmod 9 = 7 \pmod 9 && \text{By preposition 13.4}\\
7^{32} &\equiv 7^2 \pmod 9 = 4 \pmod 9 && \text{By preposition 13.4}\\
7^{64} &\equiv 4^2 \pmod 9 = 7 \pmod 9 && \text{By preposition 13.4}
\end{aligned}$$

$$7^{96} = 7^{64}7^{32}$$

$$7^{64} \equiv 7 \pmod 9$$

$$7^{32} \equiv 4 \pmod 9$$

$$7^{64} \cdot 7^{32} \equiv 4 \cdot 7 \pmod 9 \qquad \text{By preposition 13.3}$$

$$7^{96} \equiv 28 \pmod 9 = 1 \pmod 9$$

$$7^{96} \equiv 1 \pmod 9$$

Hence the remainder of $\frac{7^{96}}{9}$ is 1

5/5

**FB2** ***Does the congruence equation $6x \equiv 7 \pmod{25}$ have a solution for x (notice that congruence only makes sense if $x \in \mathbb{Z}$ so you are looking for integer solutions)? If it does, find the solution. If it does not, prove that it does not.***

Euclidean algorithm to find hcf(6,25)

$$\begin{aligned}
25 + 6(-4) &= 1 \\
6 + 1(-6) &= 0
\end{aligned}$$

Final non-zero remainder in the Euclidean algorithm is the highest common factor, hence hcf(6,25)=1

1 | 7 therefore there is a solution

By preposition 10.3 there are such integers s,t that satisfy the inequality $1 = 6s + 25t$.
Therefore $6s = 1 - 25t$ which also means $6s \equiv 1 \pmod{25}$.
I multiply the congruence by 7 and get $6 \cdot 7s \equiv 7 \pmod{25}$.
$6x \equiv 7 \pmod{25}$ and $6 \cdot 7s \equiv 7 \pmod{25}$ hence $x = 7s$.
The integers s and t can be easily read from the first line of the Euclidean algorithm, s=-4 and t=1
Therefore $x = 7 \cdot -4 = -28$

5/5

**FB3** ***Find the acute angle between the lines*** $3x + y = 5$ ***and*** $x - 2y = 4$***.***

The angle between lines is only dependent on arguments of x and y, constants added only shift the lines vertically.

I will write vectors as $\vec{a}$ and use the $a_0 i + a_1 j + a_2 k$ notation.
From $3x + y = 5$ I get direction vector $\vec{a} = 3i + j$
From $x - 2y = 4$ I get direction vector $\vec{b} = i - 2j$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|a||b|}$$

$$\vec{a} \cdot \vec{b} = (3 \cdot 1) + (1 \cdot -2) = 1 \qquad |\vec{a}| = \sqrt{3^2 + 1^2} = \sqrt{10} \qquad |\vec{b}| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$

$$\cos\theta = \frac{1}{\sqrt{10} \cdot \sqrt{5}} = \frac{1}{5\sqrt{2}}$$

$$\theta = \arccos\frac{1}{5\sqrt{2}} = \cos^{-1}\frac{1}{5\sqrt{2}}$$

If we plug in the expressions into a calculator we get $\theta \approx 81.87°$

5/5