Davids Zakrevskis

FB1 ***Find a parametric equation of the line passing through the points A(1, 2, 4) and B(11, 8, 26) and find the point where this line intersects the line $L1$: $x = 1 + s$, $y = 2s$, $z = 3s$, by solving a system of linear equations.***

I'll be using the $\vec{Q} = \langle 10, -10, 22 \rangle$ notation for vectors.

Parametric equations without any values plugged in. $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$

In the parametric equation "a,b,c" are the direction values, "t" is a scalar and "x_0, y_0, z_0" are arbitrary starting points.

I calculate direction vector parallel to the line passing through A and B and call it $\vec{V}$

$\vec{V} = B - A = \langle 11 - 1, -8 - 2, 26 - 4 \rangle = \langle 10, -10, 22 \rangle$ a=10, b=-10, c=22

Now I plug in the values in the parametric equations. $x = 1 + 10t$, $y = 2 - 10t$, $z = 4 + 22t$

The "a,b,c" values can be divided by two to make the parametric equations consist of smaller numbers, with that the scalar "t" would be different for each case.

$$L_{AB} = \begin{cases} x = 1 + 10t \\ y = 2 - 10t \\ z = 4 + 22t \end{cases} \qquad L_1 = \begin{cases} x = 1 + s \\ y = 2 - s \\ z = 3s \end{cases}$$

$$x_{AB} = x_1 \qquad y_{AB} = y_1 \qquad z_{AB} = z_1$$

$$1 + 10t = 1 + s \qquad 2 - 10t = 2 - s \qquad 4 + 10t = 3s$$

$$10t - s = 0 \qquad 10t - s = 0 \qquad 22t - 3s = -4$$

$$\left[\begin{array}{cc|c} 10 & -1 & 0 \\ 10 & -1 & 0 \\ 22 & -3 & -4 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -0.1 & 0 \\ 10 & -1 & 0 \\ 22 & -3 & -4 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -0.1 & 0 \\ 22 & -3 & -4 \\ 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -0.1 & 0 \\ 0 & -0.8 & -4 \\ 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -0.1 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 0.5 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array}\right]$$

For the first matrix modification I divide the first row by 10 to get the first value 1.
For next I subtract from the 2nd row (10$\cdot$ first row).
Next I move the 2nd row with only 0's to bottom, it can be removed/ignored.
Next I subtract (22$\cdot$ first row) from the 2nd row to get the $Matrix_{21}$ element to be 0
Next I multiply the 2nd row by -1.25 to get rid of negative values and get the first element in 2nd row to be 1.
Next I add (0.1$\cdot$ 2nd row) to first row to make the matrix an identity matrix.

From the final matrix/ identity matrix scalars for the solution can be easily read. $t = 0.5 \quad s = 5$

$$x = 1 + 10t = 1 + s \qquad y = 2 - 10t = 2 - s \qquad z = 4 + 22t = 3s$$

$$x = 1 + 10 \cdot 0.5 = 1 + 5 = 6 \qquad y = 2 - 10 \cdot 0.5 = 2 - 5 = -3 \qquad z = 4 + 22 \cdot 0.5 = 3 \cdot 5 = 15$$

$$x = 6 \quad y = -3 \quad z = 15$$

The point where L_1 intersects the line that goes through points A and B is (6,-3,15)

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FB2 ***Consider an arbitrary 2 × 2 matrix with real entries, A and let B be the matrix*** $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

a) What restrictions must be placed on the entries of A in order for tr(A) = tr(AB)

let A= $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

Matrix multiplication $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$, where $C = AB$

Trace of an n x n square matrix A $tr(A) = \sum_{i=1}^{n} a_{ii}$

The trace of a matrix is only dependent on the diagonal elements in a square matrix, I calculate the necessary elements of AB.

Lets say AB = C, take in mind that all elements of B are 1, and the b_{kj} part of matrix multiplication formula is redundant; therefore I exclude it.

n=2 no need for summation notation

$$c_{11} = \sum_{k=1}^{n} a_{1k} = a_{11} + a_{12} \qquad c_{22} = \sum_{k=1}^{n} a_{2k} = a_{21} + a_{22}$$

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Substitute in the matrix pars in the trace formula for A and AB

$$tr(A) = a_{11} + a_{12} \qquad \text{and} \qquad tr(AB) = tr(C) = a_{11} + a_{12} + a_{21} + a_{22}$$

$$tr(A) = tr(AB)$$

$$a_{11} + a_{12} = a_{11} + a_{12} + a_{21} + a_{22}$$

$$a_{21} + a_{22} = 0$$

The condition required for tr(A) = tr(AB) to be true is that matrices A elements a_{12} and a_{21} have to satisfy the expression $a_{21} + a_{22} = 0$, both must be equal to zero or one must be the other multiplied my -1. Note: This is true specifically for the given matrix B.

this includes the case both zero!

b) Show that if det(A) = det(AB), then A is not invertible.

I continue from where I calculated a_{11} and a_{22} in part "a" of the task and calculate the 2 other values in matrix C.

$$c_{21} = \sum_{k=1}^{n} a_{2k} = a_{21} + a_{22} \qquad c_{12} = \sum_{k=1}^{n} a_{1k} = a_{11} + a_{12}$$

$$\text{C=AB=} \begin{bmatrix} a_{11} + a_{12} & a_{21} + a_{22} \\ a_{11} + a_{12} & a_{21} + a_{22} \end{bmatrix}$$

$$det(A) = det(AB)$$

$$a_{11}a_{22} - a_{21}a_{12} = (a_{11} + a_{12})(a_{21} + a_{22}) - (a_{21} + a_{22})(a_{11} + a_{12})$$

$$a_{11}a_{22} - a_{21}a_{12} = 0$$

$$det(A) = 0$$

By theorem 3.8 matrix A isn't invertible if det(A)=det(AB), because if it's true then $det(A) = a_{11}a_{22} - a_{21}a_{12} = 0$

det A = 0 4

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FB3

a) Write down and simplify the expansion of $(a + \frac{b}{a})^6$, for a,b $\in \mathbb{R}$.

Binomial theorem $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

I'll expand it for the special case when n=6

$(a+b)^6 = \sum_{k=0}^{6} \binom{6}{k} a^{6-k} b^k = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

I substitute $\frac{b}{a}$ in $(a+b)^6$ for b.

$(a + \frac{b}{a})^6 = a^6 + 6a^5\frac{b}{a} + 15a^4(\frac{b}{a})^2 + 20a^3(\frac{b}{a})^3 + 15a^2(\frac{b}{a})^4 + 6a(\frac{b}{a})^5 + (\frac{b}{a})^6$

I multiply a^i and $(\frac{b}{a})^i$ together where i is the i_{th} element in the sum.

$a^6+6a^5\frac{b}{a}+15a^4(\frac{b}{a})^2+20a^3(\frac{b}{a})^3+15a^2(\frac{b}{a})^4+6a(\frac{b}{a})^5+(\frac{b}{a})^6 = a^6+6a^4b+15a^2b^2+20b^3+15a^{-2}b^4+6a^{-4}b^5+b^6$

Simplified version $(a + \frac{b}{a})^6 = a^6 + 6a^4b + 15a^2b^2 + 20b^3 + 15a^{-2}b^4 + 6a^{-4}b^5 + b^6$

b) What is the coefficient of the b^3 term?

The coefficient of the b^3 term can be easily read from the simplified version of $(a + \frac{b}{a})^6$ which I've found in part "a" of the task and it is 20.

c) Let b = 1. What is the simplified form of the expression now?

I simply replace all b's in the simplified version of $(a + \frac{b}{a})^6$ with ones. Note: $1^x = 1 \; \forall \; x, \; x \in \mathbb{R}$ and as a multiplier a 1 does nothing and can be excluded.

$(a + \frac{b}{a})^6 = a^6 + 6a^4b + 15a^2b^2 + 20b^3 + 15a^{-2}b^4 + 6a^{-4}b^5 + b^6 = a^6 + 6a^4 + 15a^2 + 20 + 15a^{-2} + 6a^{-4} + 1$

Simplified version $(a + \frac{1}{a})^6 = a^6 + 6a^4 + 15a^2 + 20 + 15a^{-2} + 6a^{-4} + 1$

? ✓ 4

Maths: 15

SOLVE
3 1 1 2 3

Can be written much more concisely