MCS 320 Project 5

Problem 1. Using Sage only, solve the following problem and show the code and output as used in the computation. Scores on an 100 points exam in a class of 500 students were normally distributed. The mean score was 78 and the standard deviation was 8.

a. what is the probability that a randomly selected student scored 80 or higher?

b. what is the probability that a randomly selected student scored less than 40?

c. what is the probability that a randomly selected student scored between 55 and 75?

d. find the exam score of those students whose exam z-score was -0.5.

e. plot a 50-bin histogram of 500 random scores, simulating scores of this class as specified in the problem description.

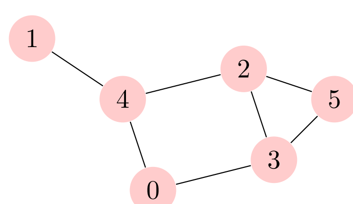
Problem 2. Consider the data contained in the file $project5-data.txt$. Read in the data (in any way you want) and place it in format (such as list, tuple, or dictionary - this is up to you), and solve the following problems. Assume that the data is generated by a function

$$f(x) = \frac{1}{s\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-m}{s})^2}$$

a. based on the data in $project5-data.txt$ file, estimate quantities m and s.

b. based on the data in $project5-data.txt$ file, find the area underneath the graph of $f(x)$, between the two inflection points of the graph.

Problem 3. Consider the following graph.



We can associate the following two matrices with this graph. Matrix D is called the degree matrix and the matrix A is called an adjacency matrix.

$$D = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

The graph vertex labels, 0 through 5, represent the columns and rows of the two matrices. They will always be distinct and sequential, without gaps.

The degree matrix D encodes the degree of each vertex. The degree of a vertex is defined as number of edges, incident to that vertex. For example, the row 0 and column 0 location in the matrix D shows a value of 2 since vertex 0 has two incident edges. Row 1 and column 1 location shows 1 since vertex 1 has one incident edge. Furthermore, the degree matrix D is always going to be a diagonal matrix.

The adjacency matrix A is always going to be a 0/1 symmetric matrix. It encodes the edge connection of a particular vertex to all other vertices, using 1 if an edge connection exists, and 0 if it does not. For example, row 0, column 3 location shows a value of 1. That is because vertex 0 is connected with an edge to vertex 3. Note also the symmetry: row 3 and column 0 locations shows a value of 1 since vertex 3 is connected with an edge to vertex 0.

We define a **Laplacian** matrix L = D - A. For our example, matrix L is looks like this:

$$L = \begin{bmatrix} 2 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 3 & -1 & -1 & -1 \\ -1 & 0 & -1 & 3 & 0 & -1 \\ -1 & -1 & -1 & 0 & 3 & 0 \\ 0 & 0 & -1 & -1 & 0 & 2 \end{bmatrix}$$

a. Write a Sage function which accepts a graph G as input and returns the Laplacian matrix L as output. Illustrate your function on G = graphs.PetersenGraph(). You must write the function to compute L yourself. **Simply calling a built-in function like kirkhoff_matrix() will not receive credit.**

b. Write a Sage function which accepts a Laplacian matrix L on input and returns the corresponding graph G. Illustrate your function on

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ -1 & -1 & 4 & -1 & -1 & 0 \\ 0 & 0 & -1 & 2 & 0 & -1 \\ 0 & -1 & -1 & 0 & 3 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

Project Guidelines, Submission Details, and Plagiarism Warning

This project is due on **Friday, November 20, 2020 at 11:59 PM**. No late submissions will be accepted and no extensions will be given!

Your solution to this project must consist of a single, computer typed (not hand-written), PDF document, called **project5.pdf**, containing the **Sage code** **and the answers** for the problems, along with answers. For each problem, write a Sage function/code that solves the problem. Clearly separate problems from one another. Upload the file **project5.pdf** through Blackboard. We must be able to copy and paste your code in order to evaluate it! Show output for all your code, below each problem.

This project must be solved **individually**. Under no circumstances are you allowed to copy or to collaborate with anyone else. All submitted files will be automatically checked for plagiarism. Regardless of who copied from whom, all caught in the act of plagiarism will be penalized.

In particular, using internet resources of any kind is **not** allowed. Internet sites are routinely checked for similarity to your submission, both for code content and code logic. Changing code order or variable names will not prevent plagiarism detection. In addition, do not post any content of this project to any internet sites or make it public in any other form. **The content of this project is not in the public domain!**

You are free, however, to use our course resources, such as lecture notes, text books, and official Sage documentation during the solving of this project.

If you have questions about this project, come to my online office hours using the usual Blackboard link.