MCS 320 Project 1 : Introduction to Symbolic Computing with Sage

We will use Sage to explore the basics of symbolic computing and the basic functionality of a computer algebra system.

Problem One.

Write a Sage function which takes on input as its argument an integer n and returns a random n degree polynomial with randomly generated integer coefficients. The randomly generated integer coefficients range from -100 to +100, inclusive. In addition to returning the randomly generated polynomial of degree n, the Sage function must return the number of **real roots** of the randomly generated polynomial. In particular, this Sage function returns the randomly generated polynomial and the number of its real roots in form of a Python tuple or a list.

Input example : program1(6), where $n = 6$.
Output example:

```
(89*x^6 - 19*x^5 - 80*x^4 - 76*x^3 + 54*x^2 - 39*x + 45, 2)
```

As your solution to this problem, provide the Sage function and the output for $n = 4$ and $n = 10$.

Problem Two. Write a Sage function which takes on input as its arguments a polynomial f with *integer* coefficients and a list L. The polynomial f is a symbolic expression in the variable x. The list L consists of an unspecified number of sub-lists, each of which contains two *integers*. These two integers represent the x and y coordinates. As an example of L, consider $L = [[x_1, y_1], [x_2, y_2], [x_3, y_3], ...[x_k, y_k]]$. This Sage functions returns all sub-lists in L, for which $f(x_i) = y_i$, for all x_i and y_i in sub-lists of L.
Input example :

```
problem2( 2*x^2-2*x+3, [[0, 3], [1, 4], [2, 7], [3, 17], [4, 33], [5, 43], [6, 63],
[7, 87], [8, 116], [9, 147]])
```

Output example:

```
[[0, 3], [2, 7], [5, 43], [6, 63], [7, 87], [9, 147]]
```

As your solution to this problem, provide the Sage function and the output for $f = x^3 - x^2 - 1$ and $L = [[0, -1], [1, -2], [2, 4], [3, 17], [4, 48], [5, 99], [6, 189], [7, 332]]$.

Problem Three. Write a Sage function which takes on input as its argument a polynomial f. The polynomial f is a symbolic expression with *integer* coefficients. This function returns the count of how often the sign of the coefficients has changed.
Input example :

```
problem3(13*x^6 - 88*x^5 + 53*x^4 - 58*x^3 - 57*x^2 + 93*x + 22)
```

Output example: 4

As your solution to this problem, provide the Sage function and the output for $f = -27 * x^7 + 3 * x^6 - 28 * x^5 + 44 * x^4 - 11 * x^3 - 71 * x^2 + 6 * x - 14$

Problem Four. Two cars leave an intersection at the same time. One car heads north at the constant speed of s_1 (ft/sec). The second car heads east at the constant speed of s_2 (ft/sec). Write a Sage function which takes on input as its argument the constant speed s_1 of the first car, the constant speed s_2 of the second car, h hours and m minutes, the time the two cars left the intersection. This Sage function returns the distance (in miles) between the two cars after h hours and m minutes.

Input example :

```
problem4(65, 65, 1, 12)
```

Output example: 75.2104485443873

As your solution to this problem, provide the Sage function and the output for problem4(25, 45, 1, 30), where $s_1 = 25$, $s_2 = 45$, $h = 1$, and $m = 30$.

Project Guidelines and Submission Details

This project is due on **Friday, September 25, 2020 at 9 AM**. No late submissions will be accepted!

Your solution to this project must consist of a single PDF document, called **project1.pdf**, containing the **Sage code and the answers** for the four problems. For each problem, write a Sage function, titled *def problem1(), def problem2(), etc*, with the appropriate input arguments. Do not change the function names. Upload the file **project1.pdf** through Blackboard. **No other format will be accepted**.

This project must be solved **individually** but you may brainstorm ideas with each other. Under no circumstances are you allowed to copy or to collaborate with anyone else beyond big-picture discussion. All submitted files will be automatically checked for plagiarism. Regardless of who copied from whom, all caught in the act of plagiarism will be penalized. Using the internet resources is also off limits. However, you are free to use our course resources, such as lecture notes, text books, and official Sage documentation during the solving of this project.

If you have questions about this project, come to my online office hours, using the usual Zoom meeting.