DDPM (Forward Path)

Denoising Diffusion Probabilistic Model.

Can be described in two processes

→ a fixed forward diffusion process that gradually mixes the input with noise.

→ a learned reverse process that generates data by a gradual denoising.

Forward diffusion

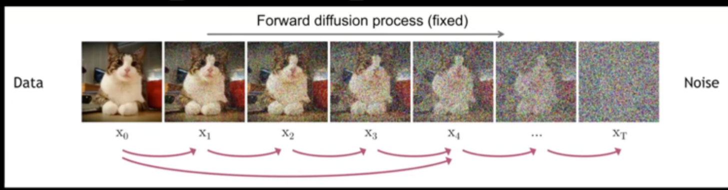
The process starts with x_0 and generates sequentially $x_1, x_2, x_3 - - - - x_T$. $\simeq$ 1000 steps

Each step performs a simple mixture of previous state and weighted white gaussian iid noise.

$$(0 < \beta \ll 1) \quad \boxed{x_t = \sqrt{1-\beta_t}\, x_{t-1} + \sqrt{\beta_t}\, v_t} \quad \text{where } v_t \sim N(0, I)$$

Diffusion Kernel

- We could easily tie every instance $\underline{x}_t$ to the initial $\underline{x}_0$ as an accumulation of noise vectors, using a wider diffusion kernel:



We should choose β_t such that $\alpha_T \approx 0$ so that $\underline{x}_T \sim \mathbb{N}(\underline{0}, \mathbf{I})$

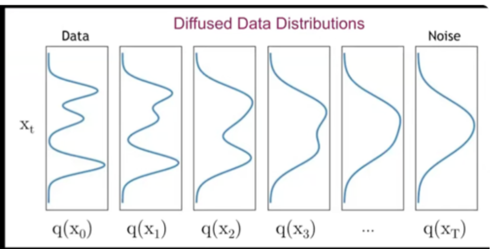
$$\underline{x}_t = \sqrt{1-\beta_t}\,\underline{x}_{t-1} + \sqrt{\beta_t}\,\underline{v}_t$$

$$= \cdots = \sqrt{\alpha_t}\,\underline{x}_0 + \sqrt{1-\alpha_t}\,\underline{\epsilon}_t \quad \text{for } \alpha_t = \prod_{s=1}^{t}(1-\beta_s) \text{ and } \underline{\epsilon}_t \sim \mathbb{N}(\underline{0}, \mathbf{I})$$

$$\Rightarrow q(\underline{x}_t|\underline{x}_0) = \mathbb{N}(\underline{x}_t; \sqrt{\alpha_t}\,\underline{x}_0, (1-\alpha_t)\mathbf{I})$$

Flow of Distribution

- In the two extremes of this flow, we get:
  - $\underline{x}_0 \sim P(\underline{x})$ (the data PDF)
  - $\underline{x}_T \sim \mathbb{N}(\underline{0}, \mathbf{I})$
- In between the distribution varies smoothly as a convolution between a dilated version of the data PDF and an isotropic Gaussian



- Another way to look at it: the intermediate PDF are a convolution of $P(\underline{x})$ with shifted Gaussians of growing width

P(x) - the data PDF

$$q(\underline{x}_t) = \int q(\underline{x}_t, \underline{x}_0)\,d\underline{x}_0 = \int q(\underline{x}_t|\underline{x}_0)\,q(\underline{x}_0)\,d\underline{x}_0$$

A shifted Gaussian: $\mathbb{N}(\underline{x}_t; \sqrt{\alpha_t}\,\underline{x}_0, (1-\alpha_t)\mathbf{I})$

Writing $q(x_t|x_{t-1})$ is easy! what about the reversed direction, $\underbrace{q(x_{t-1}|x_t)\ ??}_{\text{Why?}}$

Because with these conditional probabilities we could offer a generative process:

→ Draw: $x_t \sim N(0, I)$

→ update iteratively by drawing x_{t-1} randomly from $q(x_{t-1}|x_t)$

→ we get x_0.

Terrific right???

Bayes doesn't work ' ' ' '