4 terminal physical transmission line Tlinp4

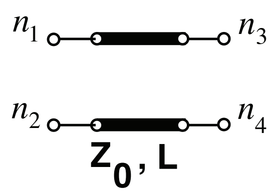


Figure 1: tlinp4 —Transmission line element.

Form: `tlinp4:⟨instance name⟩` n_1 n_2 n_3 n_4 `⟨parameter list⟩`
n_1, n_2, n_3 and n_4 are the element terminals.
Terminals n_2 and n_4 are the element reference terminals.
With `nsect` set Terminals n_2 and n_4 must be the same.

- When Terminals n_2 and n_4 are the same the device functions like a three terminal element. Transient (.TRAN), AC (.AC), DC (.DC), and convolution (.SVTR) simulations are allowed
- When terminals n_2 and n_4 are not the same, the device functions as a true 4 terminal element. Only AC (.AC), DC (.DC) and transient convolution analysis (.SVTR) permitted.

Parameters:

Parameter	Type	Default value	Required?
k: Effective dielectric constant	DOUBLE	1	no
alpha: Attenuation (dB/m)	DOUBLE	0.1	no
z0mag: Magnitude of characteristic impedance (ohms)	DOUBLE	n/a	yes
fscale: Scaling frequency for attenuation (Hz)	DOUBLE	0	no
tand: Loss tangent	DOUBLE	0	no
length: Line length (m)	DOUBLE	n/a	yes
nsect: Enable discrete approximation with n sections	INTEGER	0	no
fopt: Optimum frequency for discrete approximation	DOUBLE	0	no
fmax: Maximum frequency required for modeling	DOUBLE	0	no

Example:

```
.model c_line tlinp4 z0mag=75.00 k=7 fscale=10.e9 alpha = 59.9 nsect = 20 fopt=10e9 fmax =
9e9
tlinp4: t2 2 0 3 0 model = "c_line" length=931.69u
```

Notes:

This is the T element in the SPICE compatible netlist.

Details:
This is a linear element and is modeled differently depending on the setting of the Parameter `nsect`.

`nsect` = 0.

When `nsect` is zero (the default) the transmission line is calculated in the frequency domain using the frequency dependent characteristic impedance and propagation parameters. The model is shown in Figure 2. This model has two local reference terminals. An example netlist is:

Frequency Domain Model with `nsect` = 0.

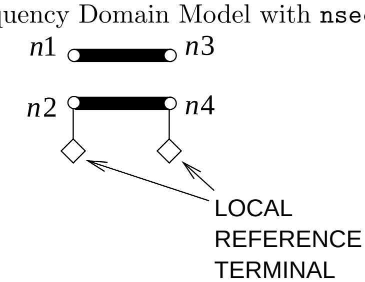


Figure 2: Multi section model of a transmission line.

```
l:1 n1 0 n3 ref1 z0=50 length=5mm
.ref "ref1"
```

Here terminals '0' and 'ref1' are the local reference terminals of the element. Terminal '0' is the global ground. However 'ref1' is a second local reference terminal of the element and either it or another terminal in the same local reference group must be specified as a reference terminal. Here 'ref1' is identified as a local reference terminal. (or a suitable terminal). Another suitable example of a circuit would be

```
vsource 2 0 vac = 1 f = 5GHz
r:1 2 n1 r=50
r:2 0 n2 r=2
tlinp4:1 n1 n2 n3 n4 z0=50 length=5mm
r:3 n2 r=5
r:4 n4 5 r=100
r:5 n4 5 r=10
.ref 5
```

tlinp4 above corresponds to Figure 2.

`nsect` > 0.

When `nsect`, the number of sections, is a positive integer the transmission line is approximated using `nsect` sections. The model is shown in Figure 3. With respect to the terminal numbers in Figure 2, terminals 'n2' and 'n4' must be the same terminal. The sectional model is used in both the time domain and frequency domain. Each series R and inductance L is modeled as a single `L` element. Each shunt G and capacitance C is modeled as a single `C` element. This is indicated in Figure 4 The RLGC parameters of the model are calculated as follows. (Note that R and L are noted as R_{DC} and L_{DC} in the skin effect section).

$$\Delta x = l/nsect \tag{1}$$

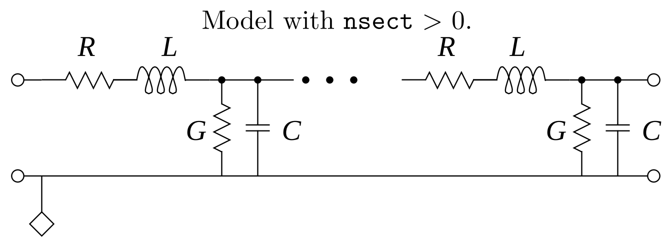


Figure 3: Multi section model of a transmission line.

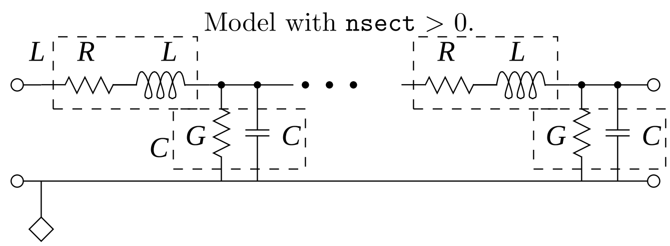


Figure 4: Multi section model of a transmission line showing pairs of primitives each modeled by a single element.

$$\alpha_f = (\alpha * 0.11512925) * \sqrt{\frac{f_{opt}}{f_{scale}}} \tag{2}$$

If f_{opt} or f_{scale} is not given or is 0 then $\alpha_f = (\alpha * 0.11512925)$

$$C = \frac{\sqrt{k}}{|Z_0| * c_0} * \Delta x \tag{3}$$

$$L = |Z_0|^2 * C \tag{4}$$

$$R = 2 * \alpha_f * |Z_0| * \Delta x \tag{5}$$

$$G = \tan d * 2\pi * f_{opt} * C \tag{6}$$

If *tand* or f_{opt} is not given or is 0 then $G=1*10^{-10}$

Use of the skin effect:
Setting of the *fmax* parameter denotes the desire to use skin effect. If *fmax* is left out, the model behaves exactly as described above. If skin effect modeling is desired, *fmax* should be set to the maximum frequency at which the user expects the model to be valid. Eg. If the user is working at frequencies no higher then 10 GHz setting *fmax* to 10e9 would be sufficient. Setting *fmax* to lower than the desired frequency will result in reduced accuracy. Note if alpha is set to 0 (ideal conductors) then the skin effect will appear not to have any effect because by definition the model will have zero series resistance. If fmax is not included, the transmission line subsections will be modeled as shown in Figure 5

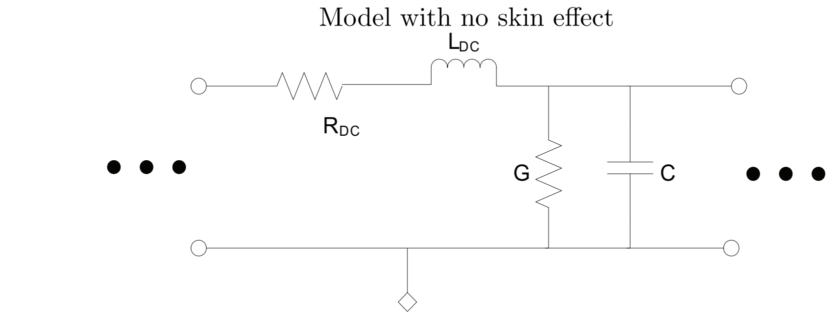


Figure 5: Transmission line subsection with skin effect not included

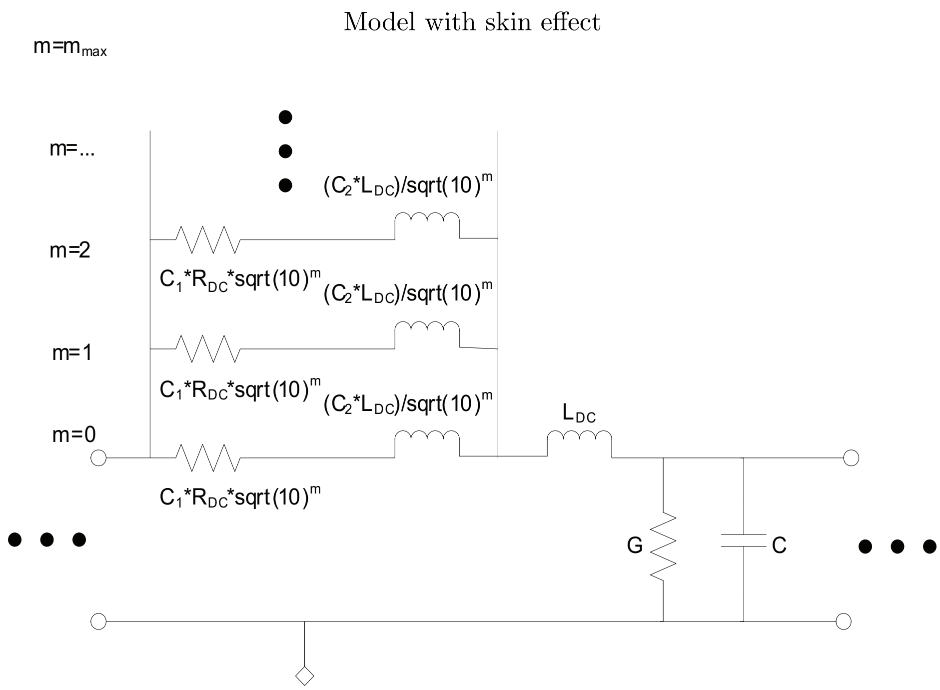


Figure 6: Transmission line subsection with skin effect included

If skin effect is included the series resistance is modeled using a R-L stack as shown in Figure 6. The series R and L values are shown below for simplicity

$$R = C_1 * R_{DC} * \left(\sqrt{10}\right)^m \tag{7}$$

$$L = C_2 * \frac{L_{DC}}{\left(\sqrt{10}\right)^m} \tag{8}$$

R_{DC} and L_{DC} are the R and L values as calculated without skin effects as shown in (5) and (4).

The constants C_1 and C_2 are constants that are chosen based on the *fmax* term, similarly the number of branches required, indexed by the m variable, is chosen based on *fmax* and will not exceed 9. The table of values for C_1 and C_2 and m is shown in Table 1.

Sqrt(10) is chosen as the factor that will increase the resistance and decrease the inductance. This value was picked experimentally such that after 1 MHz each additional branch (noted by the m variable) makes the model valid for another decade of frequency. As such the value of m_{max} is found by consideration of fmax. Eg. If fmax=10 MHz then m_{max}=2, if fmax=100 Mhz then m_{max}=3.

fmax (Hz)	C_1	C_2	m_{max}
fmax < 10	1	1	0
fmax < 10^6	1.32	1.68	1
fmax < 10^7	1.42	1.94	2
fmax < 10^8	1.45	2.03	3
fmax < 10^9	1.457	2.06	4
fmax < 10^{10}	1.461	2.07	5
fmax < 10^{11}	1.465	2.08	6
fmax < 10^{12}	1.4676	2.087	7
fmax < 10^{13}	1.4681	2.092	8
fmax < 10^{14}	1.4685	2.095	9

Table 1: *fmax* dependent model constants

Derivation of Model Constants :

The model constant C_1 was calculated such that at DC the parallel combination of all the (R) resistances are equal to R_{DC} since the inductors are shorted at DC. C_2 is somewhat more complicated but it is solved for iteratively by equating the low frequency phase of the skin effect R L with the low frequency phase of the R_{DC} and L_{DC} circuit in Figure5. These can both be solved for based on the number of branches required.

Since the skin effect on a conductor can be thought of as the conductor being made of concentric shells. At low frequencies all the shells carry currents and so the resistance seen is a DC resistance R_{DC} in the model. As frequency increases the inner shells turn off causing the DC resistance to increase and the internal inductance to decrease.

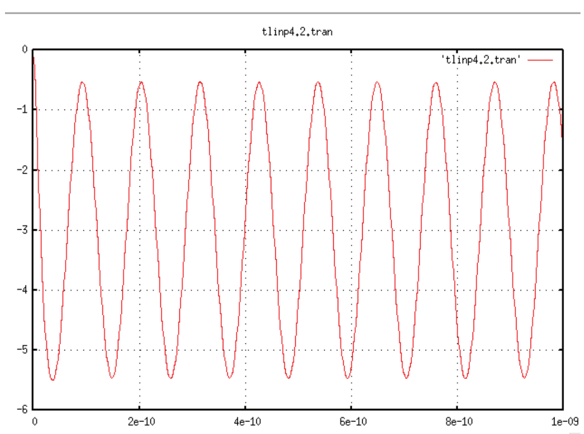
Example of Transient Analysis (.TRAN2) Fixed times steps, time-stepping nonlinear analysis.
netlist file: tlinp4.net:

```
* Transient tlinp4 test
.options f0 = 9e9
.tran2 tstop = 1e-9 tstep = .002e-9
vsource:v2 202 0 vdc= -6. vac= 5. f= f0 phase=90
resistor:rs 1 202 r=75.
tlinp4:t1  1 0 2 0
+ z0mag=75.00 nsect=1 length=978.57e-6 k=7 tand=.01 fscale=1.e10 alpha=1. fmax=1e10
resistor:rl 2 0  r=75.
.options gnuplot
.out plot term 1 vt in "tlinp4.1.tran"
.out plot term 2 vt in "tlinp4.2.tran"
.end
```

The output log file is

```
**********  fREEDA 1.3 running on Wed Apr 29 11:13:35 2009  **********


   *** Parsing input netlist ...
```

```
*** Expanding subcircuits ... done.

*** Initializing and Expanding Elements ... done.

*** Checking reference terminals ... done.

*** Starting analysis ...

------------------------------------------------------------------------------
   Matrix size = 13
   Matrix nnz = 47
   Using line search method.
   Nonlinear analysis tolerance (ftol) = 6.12865e-06
   Maximum number of nonlinear iterations per time-point (maxit) = 250
   Using Lee and Lee's quasi-Newton updates.
   --- Starting transient simulation ...

   Number of nonlinear state variables: 0
------------------------------------------------------------------------------
   |  Step  |     Time (s)   |   Residual    |  Recent Max    |      Max       |
------------------------------------------------------------------------------
   |      0 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00   | 0.000000e+00 |
   |    200 | 4.000000e-10 | 0.000000e+00 | 0.000000e+00   | 0.000000e+00 |
   |    400 | 8.000000e-10 | 0.000000e+00 | 0.000000e+00   | 0.000000e+00 |
   --- Maximum Residual: 0
------------------------------------------------------------------------------

 Plotting output file: tlinp4.1.tran.
 Plotting output file: tlinp4.2.tran.

**************  fREEDA 1.3 stopping on Wed Apr 29 11:13:35 2009  ***********
```



Example of Transient Analysis using Convolution (.SVTR).
In this example the transmission line is calculated in the frequency domain and convolution analysis is used to convert the model into an impulse response used directly in transient analysis.
netlist file: tlinp4_conv.net:

```
* Transient tlinp4 test. Convolution is used
.options f0 = 9e9
.svtr tstop = 1e-9 tstep = .0002e-9
vsource:v2 202 0 vdc= -6. vac= 5. f= f0 phase=90
resistor:rs 1 202 r=75.
tlinp4:t1  1 0 2 ref2
+ z0mag=75.00 length=978.57e-6 k=7 tand=.01 fscale=1.e10 alpha=1. fmax=1e10
resistor:r3 2 ref2  r=75.
open:1 2 ref2
.ref "ref2"
.options gnuplot
.out plot element "open:1" 0 ut in "tlinp4.2.svtr"
.end
```

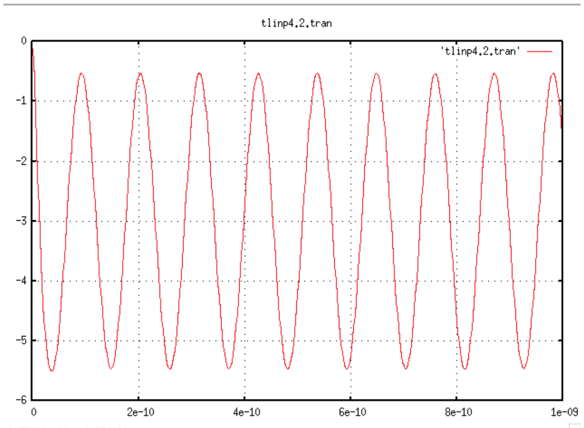
Note that in convolution analysis augmentation elements are used to provide the proper interface between the linear and nonlinear partitions of the circuit analyzed in the time-domain and frequency-domain. As well all linear elements internal to the linear circuit partition care not available. Consequently the voltages at terminals of linear elements are not available and the voltages at the terminals of nonlinear elements are modified by the augmentation circuit. Voltages and currents must be obtained from nonlinear elements (using `ut` and `it`). Additional nonlinear elements can be introduced using the `open` element which is treated as tough it is a nonlinear element. So to get the voltage at a particular terminal either an existing nonlinear element can be exploited or an `open` element introduced. The voltage determine the voltage (ut) of that element is used. In the above `element "open:1" 0 ut` indicates that the first (the zero'th) `ut` voltage of the element `open:1` is pushed on the stack.

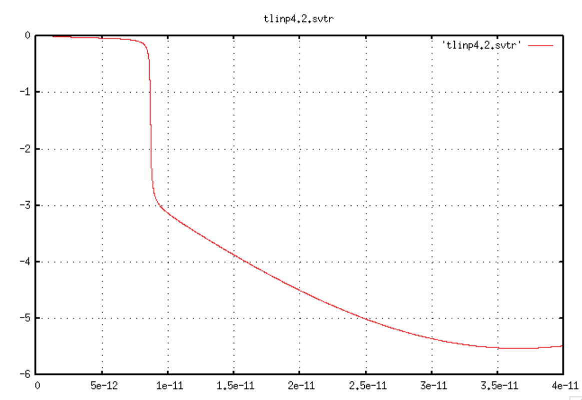
The output log file follows:

```
**********  fREEDA 1.3 running on Wed Apr 29 11:31:42 2009  **********


   *** Parsing input netlist ...

   *** Expanding subcircuits ... done.

   *** Initializing and Expanding Elements ... done.

   *** Checking reference terminals ... done.

   *** Starting analysis ...

-----------------------------------------------------------------------------
   *** State Variable-Based Transient Analysis ***
-----------------------------------------------------------------------------

   n_samples = 1792
   Frequency step = 2.43902e+09 Hz
   Maximum frequency = 2.49756e+12 Hz
   Number of time steps = 5001

   --- Building Msv(f) ...
   Number of state variables = 2
   --- Converting Msv to the time domain ...
```

```
Warning: Last 35 samples impulse contribution: 1.72 %

Warning: Last 35 samples impulse contribution: 25.01 %

Warning: Last 35 samples impulse contribution: 25.01 %

Warning: Last 35 samples impulse contribution: 5.57 %
--- Starting transient simulation ...

Using line search method.
Nonlinear analysis tolerance (ftol) = 6.12865e-06
Maximum number of nonlinear iterations per time-point (maxit) = 250
Using Lee and Lee's quasi-Newton updates.
-----------------------------------------------------------------------------
   |   Step   |    Time (s)    |   Residual (V)   |
-----------------------------------------------------------------------------
   |      100 |  2.000000e-11  |  1.333760e-14    |
   |      200 |  4.000000e-11  |  1.959041e-14    |
   |      300 |  6.000000e-11  |  2.203514e-14    |
   |      400 |  8.000000e-11  |  2.237927e-14    |
   |      500 |  1.000000e-10  |  2.250559e-14    |
   |      600 |  1.200000e-10  |  2.388845e-14    |
   |      700 |  1.400000e-10  |  2.675828e-14    |
   |      800 |  1.600000e-10  |  3.062149e-14    |
   |      900 |  1.800000e-10  |  3.157902e-14    |
   |     1000 |  2.000000e-10  |  3.168229e-14    |
   |     1100 |  2.200000e-10  |  3.193301e-14    |
   |     1200 |  2.400000e-10  |  3.328842e-14    |
   |     1300 |  2.600000e-10  |  3.689638e-14    |
   |     1400 |  2.800000e-10  |  3.872710e-14    |
   |     1500 |  3.000000e-10  |  3.905102e-14    |
   |     1600 |  3.200000e-10  |  3.912686e-14    |
   |     1700 |  3.400000e-10  |  3.974990e-14    |
   |     1800 |  3.600000e-10  |  4.197954e-14    |
   |     1900 |  3.800000e-10  |  4.506976e-14    |
   |     2000 |  4.000000e-10  |  4.588288e-14    |
   |     2100 |  4.200000e-10  |  4.596716e-14    |
   |     2200 |  4.400000e-10  |  4.612777e-14    |
   |     2300 |  4.600000e-10  |  4.749799e-14    |
   |     2400 |  4.800000e-10  |  4.974311e-14    |
   |     2500 |  5.000000e-10  |  5.141199e-14    |
   |     2600 |  5.200000e-10  |  5.173273e-14    |
   |     2700 |  5.400000e-10  |  5.176453e-14    |
   |     2800 |  5.600000e-10  |  5.215110e-14    |
   |     2900 |  5.800000e-10  |  5.358540e-14    |
   |     3000 |  6.000000e-10  |  5.559405e-14    |
   |     3100 |  6.200000e-10  |  5.638657e-14    |
   |     3200 |  6.400000e-10  |  5.647929e-14    |
   |     3300 |  6.600000e-10  |  5.658068e-14    |
   |     3400 |  6.800000e-10  |  5.732690e-14    |
```

```
|     3500  |  7.000000e-10  |  5.913543e-14     |
|     3600  |  7.200000e-10  |  6.068913e-14     |
|     3700  |  7.400000e-10  |  6.108066e-14     |
|     3800  |  7.600000e-10  |  6.111576e-14     |
|     3900  |  7.800000e-10  |  6.129309e-14     |
|     4000  |  8.000000e-10  |  6.279185e-14     |
|     4100  |  8.200000e-10  |  6.440435e-14     |
|     4200  |  8.400000e-10  |  6.524856e-14     |
|     4300  |  8.600000e-10  |  6.533710e-14     |
|     4400  |  8.800000e-10  |  6.541732e-14     |
|     4500  |  9.000000e-10  |  6.602423e-14     |
|     4600  |  9.200000e-10  |  6.745446e-14     |
|     4700  |  9.400000e-10  |  6.890652e-14     |
|     4800  |  9.600000e-10  |  6.934587e-14     |
|     4900  |  9.800000e-10  |  6.937813e-14     |
|     5000  |  1.000000e-09  |  6.954280e-14     |
-----------------------------------------------------------------------------

   --- Residual: 6.95428e-14
   --- Writing output vectors ...
 Plotting output file: tlinp4.2.svtr.

**************  fREEDA 1.3 stopping on Wed Apr 29 11:31:50 2009  ***********
```





Example of AC Analysis using single subsection calculations.
netlist file: tlinp4AC.net:

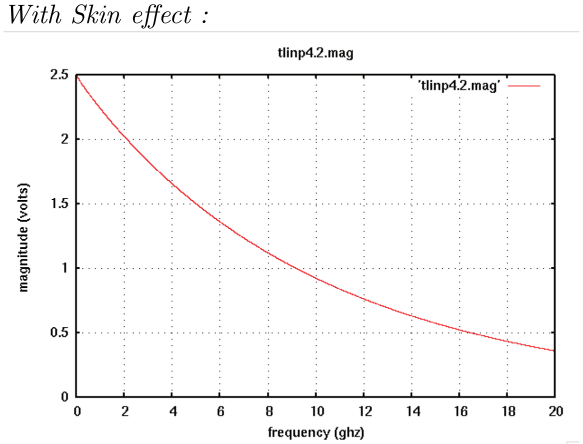
```
* AC tlinp4 test
.ac start = 1 stop = 20GHz n_freqs=10000
vsource:v2 202 0  vac= 5
resistor:rs 1 202 r=75.
tlinp4:t1  1 0 2 ref2
+ z0mag=75.00 length=0.03 k=7 tand=.1 fscale=1.e10 alpha=1 fmax=20e9
resistor:rl 2 ref2  r=75.
.ref "ref2"
.options gnuplot
* Get the magnitude of the voltage at terminal 2. This is with respect to ref2
.options preamble1="set term x11 font 'helvetica,13';
```

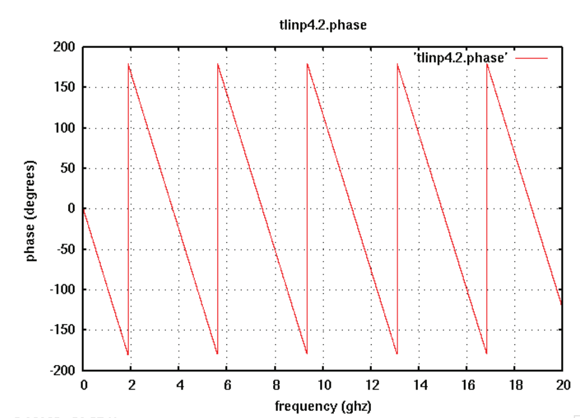
```
set xlabel 'FREQUENCY (GHz)'; set ylabel 'MAGNITUDE (VOLTS)"
.out plot term 2 vf mag 1e-9 scalex preamble1 in "tlinp4.2.mag"
* Get the phase of the voltage at terminal 2. This is with respect to ref2
* prinphase gets the principal phase as opposed to the continuous phase.
.options preamble2="set term x11 font 'helvetica,13';
set xlabel 'FREQUENCY (GHz)'; set ylabel 'phase (DEGREES)"
.out plot term 2 vf prinphase 1e-9 scalex rad2deg preamble2 in "tlinp4.2.phase"
.end
```

The output log file is:

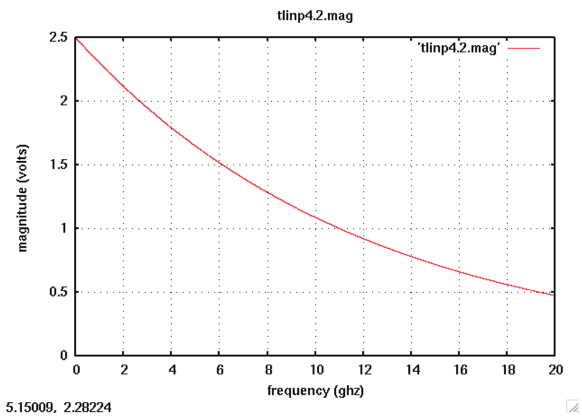
```
**********  fREEDA 1.3 running on Wed Apr 29 11:46:15 2009  **********

   *** Parsing input netlist ...

   *** Expanding subcircuits ... done.

   *** Initializing and Expanding Elements ... done.

   *** Checking reference terminals ... done.

   *** Starting analysis ...

------------------------------------------------------------------------------
   *** AC Analysis ***
------------------------------------------------------------------------------

   Frequency step = 2.0002e+06 Hz
   --- Writing output vectors ...
 Plotting output file: tlinp4.2.mag.
 Plotting output file: tlinp4.2.phase.
**************  fREEDA 1.3 stopping on Wed Apr 29 11:46:15 2009  ***********
```

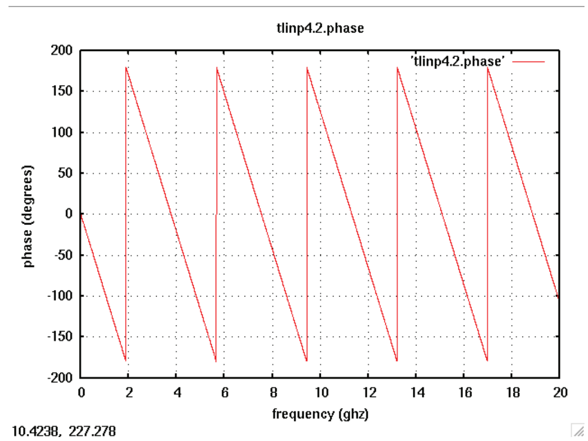
And the results are:
With Skin effect :





Without Skin effect :





Notice how the plots appear to be the exact same with or without skin effect, this is to be expected because this is only a one subsection model of the transmission line and as such is a very poor approximation. The series inductor, L_{DC}, dominates at the higher frequencies and begins to look like an open circuit, as such the output voltage shown in the plot moves exponentially towards zero. See the next section for the example of a multiple subsection simulation.

Example of AC Analysis using a multi subsection approximation
netlist file: tlinp4AC2.net:

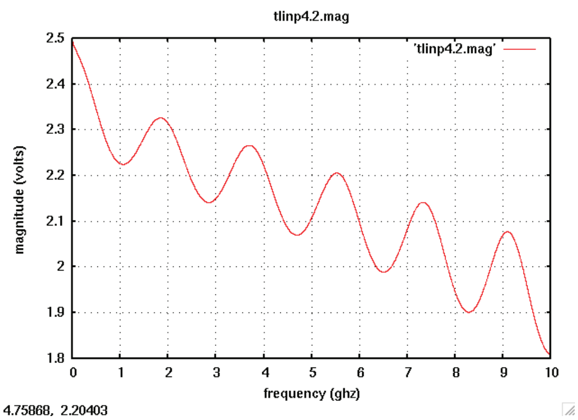
```
* AC tlinp4 test
.ac start = 1 stop = 10GHz n_freqs=10000
vsource:v2 202 ref2  vac= 5
resistor:rs 1 202 r=75.
tlinp4:t1  1 ref2 2 ref2
+ z0mag=50.00 length=.03 k=7 tand=.1 nsect=20 alpha=1 fmax=2e10
resistor:rl 2 ref2  r=75.
.ref "ref2"
.options gnuplot
* Get the magnitude of the voltage at terminal 2. This is with respect to ref2
.options preamble1="set term x11 font 'helvetica,13';
set xlabel 'FREQUENCY (GHz)'; set ylabel 'MAGNITUDE (VOLTS)"
.out plot term 2 vf mag 1e-9 scalex preamble1 in "tlinp4.2.mag"
* Get the phase of the voltage at terminal 2. This is with respect to ref2
* prinphase gets the principal phase as opposed to the continuous phase.
.options preamble2="set term x11 font 'helvetica,13';
set xlabel 'FREQUENCY (GHz)'; set ylabel 'phase (DEGREES)"
.out plot term 2 vf prinphase 1e-9 scalex rad2deg preamble2 in "tlinp4.2.phase"
.end
```

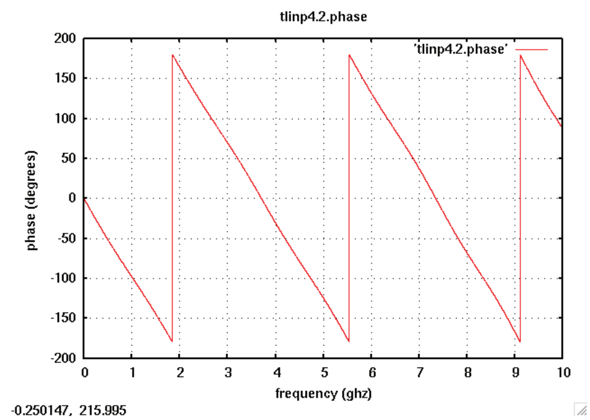
The output log file is:

```
**********  fREEDA 1.3 running on Wed Apr 29 13:37:27 2009  **********


   *** Parsing input netlist ...

   *** Expanding subcircuits ... done.
```

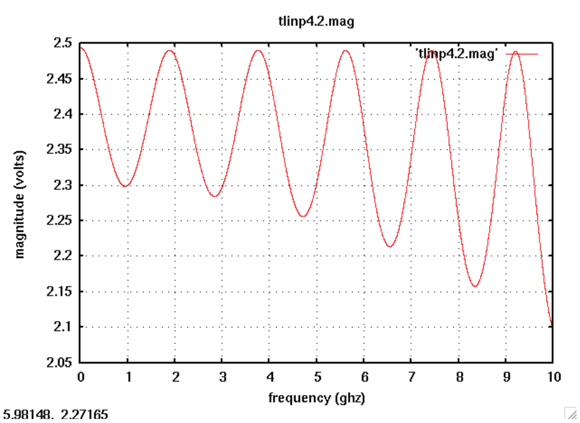
```
   *** Initializing and Expanding Elements ... done.

   *** Checking reference terminals ... done.

   *** Starting analysis ...

----------------------------------------------------------------------------
   *** AC Analysis ***
----------------------------------------------------------------------------

   Frequency step = 1.0001e+06 Hz
   --- Writing output vectors ...
 Plotting output file: tlinp4.2.mag.
 Plotting output file: tlinp4.2.phase.

**************  fREEDA 1.3 stopping on Wed Apr 29 13:37:27 2009  ***********
```

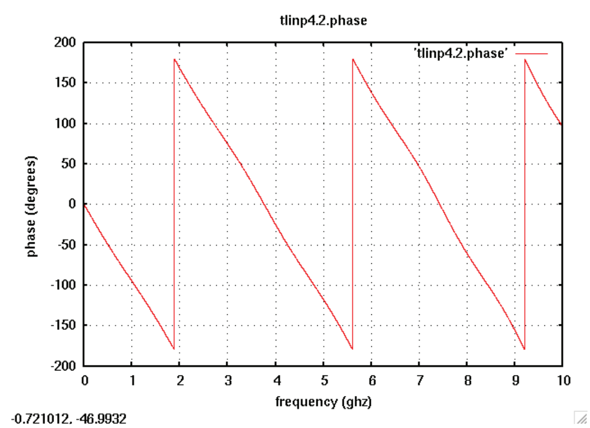
And the results are:
With Skin effect :





Without Skin effect :





In the above graphs notice how the plots without skin effect lack any significant attenuation at higher frequencies while the skin effect model attenuates more at the higher frequencies. These plots perhaps best illustrate the use of the skin effect model.

This time we are using a 20 subsection approximation, it is a much better approximation than the single subsection approximation above. In the graphs below, standing waves can be seen with peaks corresponding to the phase wrapping in the phase graph. This is because the model is using LC pairs and as such it is a resonance structure.

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