

高等数学微积分公式大全

一、基本导数公式

$$\begin{aligned}(1) (c)' &= 0 & (2) x^\mu &= \mu x^{\mu-1} & (3) (\sin x)' &= \cos x \\(4) (\cos x)' &= -\sin x & (5) (\tan x)' &= \sec^2 x & (6) (\cot x)' &= -\csc^2 x \\(7) (\sec x)' &= \sec x \cdot \tan x & (8) (\csc x)' &= -\csc x \cdot \cot x \\(9) (e^x)' &= e^x & (10) (a^x)' &= a^x \ln a & (11) (\ln x)' &= \frac{1}{x} \\(12) (\log_a x)' &= \frac{1}{x \ln a} & (13) (\arcsin x)' &= \frac{1}{\sqrt{1-x^2}} & (14) (\arccos x)' &= -\frac{1}{\sqrt{1-x^2}} \\(15) (\arctan x)' &= \frac{1}{1+x^2} & (16) (\operatorname{arccot} x)' &= -\frac{1}{1+x^2} & (17) (x)' &= 1 & (18) (\sqrt{x})' &= \frac{1}{2\sqrt{x}}\end{aligned}$$

二、导数的四则运算法则

$$\begin{aligned}(u \pm v)' &= u' \pm v' & (uv)' &= u'v + uv' & \left(\frac{u}{v}\right)' &= \frac{u'v - uv'}{v^2}\end{aligned}$$

三、微分公式与微分运算法则

$$\begin{aligned}(1) d(c) &= 0 & (2) d(x^\mu) &= \mu x^{\mu-1} dx & (3) d(\sin x) &= \cos x dx \\(4) d(\cos x) &= -\sin x dx & (5) d(\tan x) &= \sec^2 x dx & (6) d(\cot x) &= -\csc^2 x dx \\(7) d(\sec x) &= \sec x \cdot \tan x dx & (8) d(\csc x) &= -\csc x \cdot \cot x dx \\(9) d(e^x) &= e^x dx & (10) d(a^x) &= a^x \ln a dx & (11) d(\ln x) &= \frac{1}{x} dx \\(12) d(\log_a x) &= \frac{1}{x \ln a} dx & (13) d(\arcsin x) &= \frac{1}{\sqrt{1-x^2}} dx & (14) d(\arccos x) &= -\frac{1}{\sqrt{1-x^2}} dx \\(15) d(\arctan x) &= \frac{1}{1+x^2} dx & (16) d(\operatorname{arccot} x) &= -\frac{1}{1+x^2} dx\end{aligned}$$

四、微分运算法则

$$\begin{aligned}(1) d(u \pm v) &= du \pm dv & (2) d(cu) &= cdu \\(3) d(uv) &= vdu + u dv & (4) d\left(\frac{u}{v}\right) &= \frac{vdu - u dv}{v^2}\end{aligned}$$

五、基本积分公式

$$\begin{aligned}(1) \int k dx &= kx + c & (2) \int x^\mu dx &= \frac{x^{\mu+1}}{\mu+1} + c & (3) \int \frac{dx}{x} &= \ln|x| + c\end{aligned}$$

$$(4) \int a^x dx = \frac{a^x}{\ln a} + c \quad (5) \int e^x dx = e^x + c \quad (6) \int \cos x dx = \sin x + c$$

$$(7) \int \sin x dx = -\cos x + c \quad (8) \int \frac{1}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + c$$

$$(9) \int \frac{1}{\sin^2 x} dx = \int \csc^2 x dx = -\cot x + c \quad (10) \int \frac{1}{1+x^2} dx = \arctan x + c$$

$$(11) \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$$

六、补充积分公式

$$\int \tan x dx = -\ln |\cos x| + c \quad \int \cot x dx = \ln |\sin x| + c$$

$$\int \sec x dx = \ln |\sec x + \tan x| + c \quad \int \csc x dx = \ln |\csc x - \cot x| + c$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c \quad \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + c \quad \int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + c$$

七、下列常用凑微分公式

积分型	换元公式
$\int f(ax+b)dx = \frac{1}{a} \int f(ax+b)d(ax+b)$	$u = ax+b$
$\int f(x^\mu) x^{\mu-1} dx = \frac{1}{\mu} \int f(x^\mu) d(x^\mu)$	$u = x^\mu$
$\int f(\ln x) \cdot \frac{1}{x} dx = \int f(\ln x) d(\ln x)$	$u = \ln x$
$\int f(e^x) \cdot e^x dx = \int f(e^x) d(e^x)$	$u = e^x$
$\int f(a^x) \cdot a^x dx = \frac{1}{\ln a} \int f(a^x) d(a^x)$	$u = a^x$
$\int f(\sin x) \cdot \cos x dx = \int f(\sin x) d(\sin x)$	$u = \sin x$
$\int f(\cos x) \cdot \sin x dx = -\int f(\cos x) d(\cos x)$	$u = \cos x$
$\int f(\tan x) \cdot \sec^2 x dx = \int f(\tan x) d(\tan x)$	$u = \tan x$
$\int f(\cot x) \cdot \csc^2 x dx = \int f(\cot x) d(\cot x)$	$u = \cot x$
$\int f(\arctan x) \cdot \frac{1}{1+x^2} dx = \int f(\arctan x) d(\arctan x)$	$u = \arctan x$

$\int f(\arcsin x) \cdot \frac{1}{\sqrt{1-x^2}} dx = \int f(u) du$	$u = \arcsin x$
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八、分部积分公式

(1) 形如 $\int x^n e^{ax} dx$, 令 $u = x^n$, $dv = e^{ax} dx$

形如 $\int x^n \sin x dx$ 令 $u = x^n$, $dv = \sin x dx$

形如 $\int x^n \cos x dx$ 令 $u = x^n$, $dv = \cos x dx$

(2) 形如 $\int x^n \arctan x dx$, 令 $u = \arctan x$, $dv = x^n dx$

形如 $\int x^n \ln x dx$, 令 $u = \ln x$, $dv = x^n dx$

(3) 形如 $\int e^{ax} \sin x dx$, $\int e^{ax} \cos x dx$ 令 $u = e^{ax}$, $\sin x, \cos x$ 均可。

九、第二换元积分法中的三角换元公式

(1) $\sqrt{a^2 - x^2}$ 令 $x = a \sin t$ (2) $\sqrt{a^2 + x^2}$ 令 $x = a \tan t$ (3) $\sqrt{x^2 - a^2}$ 令 $x = a \sec t$

十、重要公式

$$(1) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad (2) \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e \quad (3) \lim_{n \rightarrow \infty} \sqrt[n]{a} (a > 0) = 1$$

$$(4) \lim_{n \rightarrow \infty} \sqrt[n]{n} = 1 \quad (5) \lim_{x \rightarrow \infty} \arctan x = \frac{\pi}{2} \quad (6) \lim_{x \rightarrow -\infty} \arctan x = -\frac{\pi}{2}$$

$$(7) \lim_{x \rightarrow \infty} \operatorname{arccot} x = 0 \quad (8) \lim_{x \rightarrow -\infty} \operatorname{arccot} x = \pi \quad (9) \lim_{x \rightarrow -\infty} e^x = 0$$

$$(10) \lim_{x \rightarrow +\infty} e^x = \infty \quad (11) \lim_{x \rightarrow 0^+} x^x = 1$$

$$(12) \lim_{x \rightarrow \infty} \frac{a_0 x^n + a_1 x^{n-1} + \cdots + a_n}{b_0 x^m + b_1 x^{m-1} + \cdots + b_m} = \begin{cases} \frac{a_0}{b_0} & n = m \\ 0 & n < m \\ \infty & n > m \end{cases} \quad (\text{系数不为 } 0 \text{ 的情况})$$

十一、下列常用等价无穷小关系 ($x \rightarrow 0$)

$$\sin x \sim x \quad \tan x \sim x \quad \arcsin x \sim x \quad \arctan x \sim x \quad 1 - \cos x \sim \frac{1}{2} x^2$$

$$\ln(1+x) \sim x \quad e^x - 1 \sim x \quad a^x - 1 \sim x \ln a \quad (1+x)^\alpha - 1 \sim \alpha x$$

十二、三角函数公式

1. 两角和公式

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \quad \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \quad \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\begin{aligned} \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} & \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\ \cot(A+B) &= \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A} & \cot(A-B) &= \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A} \end{aligned}$$

2. 二倍角公式

$$\sin 2A = 2 \sin A \cos A \quad \cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

3. 半角公式

$$\sin \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}} \quad \cos \frac{A}{2} = \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A} \quad \cot \frac{A}{2} = \sqrt{\frac{1 + \cos A}{1 - \cos A}} = \frac{\sin A}{1 - \cos A}$$

4. 和差化积公式

$$\sin a + \sin b = 2 \sin \frac{a+b}{2} \cdot \cos \frac{a-b}{2} \quad \sin a - \sin b = 2 \cos \frac{a+b}{2} \cdot \sin \frac{a-b}{2}$$

$$\cos a + \cos b = 2 \cos \frac{a+b}{2} \cdot \cos \frac{a-b}{2} \quad \cos a - \cos b = -2 \sin \frac{a+b}{2} \cdot \sin \frac{a-b}{2}$$

$$\tan a + \tan b = \frac{\sin(a+b)}{\cos a \cdot \cos b}$$

5. 积化和差公式

$$\sin a \sin b = -\frac{1}{2} [\cos(a+b) - \cos(a-b)] \quad \cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$\sin a \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)] \quad \cos a \sin b = \frac{1}{2} [\sin(a+b) - \sin(a-b)]$$

6. 万能公式

$$\begin{aligned} \sin a &= \frac{2 \tan \frac{a}{2}}{1 + \tan^2 \frac{a}{2}} & \cos a &= \frac{1 - \tan^2 \frac{a}{2}}{1 + \tan^2 \frac{a}{2}} & \tan a &= \frac{2 \tan \frac{a}{2}}{1 - \tan^2 \frac{a}{2}} \end{aligned}$$

7. 平方关系

$$\sin^2 x + \cos^2 x = 1 \quad \sec^2 x - \tan^2 x = 1 \quad \csc^2 x - \cot^2 x = 1$$

8. 倒数关系

$$\tan x \cdot \cot x = 1 \quad \sec x \cdot \cos x = 1 \quad \csc x \cdot \sin x = 1$$