Chapter 6 作业.

1. 求 $u(x)$ 和 $v(x)$, s.t. $(f(x), g(x)) = u(x)f(x) + v(x)g(x)$.

(1) $f(x) = x^4+2x^3-x^2-4x-2$, $g(x) = x^4+x^3-x^2-2x-2$

解: $x^4+2x^3-x^2-4x-2 = x^4+x^3-x^2-2x-2+(x^3-2x)$

$x^4+x^3-x^2-2x-2 = (x+1)(x^3-2x)+x^2-2)$

$x^3-2x = x(x^2-2) \quad \Rightarrow (f(x), g(x)) = x^2-2$

$\therefore \; x^2-2 = x^4+x^3-x^2-2x-2-(x+1)(x^3-2x)$

$= g(x)-(x+1)f(x)+(x+1)g(x) = (x+2)g(x)-(x+1)f(x)$

$\therefore u(x) = -(x+1)$, $V(x) = (x+2)$.

(2) $f(x) = 4x^4-2x^3-16x^2+5x+9$, $g(x) = 2x^3-x^2-5x+4$

解: $4x^4-2x^3-16x^2+5x+9 = 2x\cdot(2x^3-x^2-5x+4)+(-6x^2-3x+9)$

$2x^3-x^2-5x+4 = -\frac{1}{3}(x-1)(-6x^2-3x+9)+1$

$-6x^2-3x+9 = (-6x^2-3x+9)\cdot 1 \quad \therefore (f(x), g(x)) = 1$

$\therefore 1 = 2x^3-x^2-5x+4+\frac{1}{3}(x-1)(-6x^2-3x+9)$

$= g(x)+\frac{1}{3}(x-1)\cdot[f(x)-2xg(x)]$

$= g(x)+\frac{1}{3}(x-1)f(x)-\frac{2}{3}x(x-1)g(x) = \frac{1}{3}(x-1)f(x)+(1-\frac{2}{3}x^2+\frac{2}{3}x)g(x)$

$\therefore u(x) = \frac{1}{3}(x-1)$, $V(x) = (1-\frac{2}{3}x^2+\frac{2}{3}x)$

(3) $f(x) = x^4-x^3-4x^2+4x+1$, $g(x) = x^2-x-1$

解: $x^4-x^3-4x^2+4x+1 = (x^2-3)(x^2-x-1)+(x-2)$

$x^2-x-1 = x(x-2)+x-1$

$x-2 = 1\cdot(x-1)-1 \qquad \Rightarrow 1 = (x^3+x^2-3x-2)g(x)$

$x-1 = -(x-1)\cdot(-1). \qquad -(x+1)f(x)$

$\Rightarrow (f(x), g(x)) = 1$

$\therefore u(x) = -(x+1)$, ~~$\mathscr{g}$~~ $V(x) = x^3+x^2-3x-2$

二、证明：如果 $f(x)$ 与 $g(x)$ 不全为0，且 $(f(x),g(x))=u(x)f(x)+v(x)g(x)$，那么 $(u(x),v(x))=1$

证明：设 $f(x)=f_1(x)\cdot(f(x),g(x))$

$g(x)=g_1(x)\cdot(f(x),g(x))$

代入：$u(x)f_1(x)\cdot(f(x),g(x))+v(x)g_1(x)(f(x),g(x))=(f(x),g(x))$

消去：$u(x)f_1(x)+v(x)g_1(x)=1$

$\Rightarrow (u(x),v(x))=1$ ∴得证.