Ans-1. Convolutional Networks

Input image is X with size $=(u,v)$

Set of weights are $= W^K$

$\{$ where $K = 1, \ldots K \}$

Convolution $\longrightarrow$

$$Y^K = X * W^K$$

So

$$Y^K_{(p,q)} = \sum_{i=0}^{S-1} \sum_{j=0}^{T-1} X(p+i, q+j) W^K(i,j)$$

where W^K = filter

S, T = size of the filter

p is range from $0, 1 \ldots U-S$

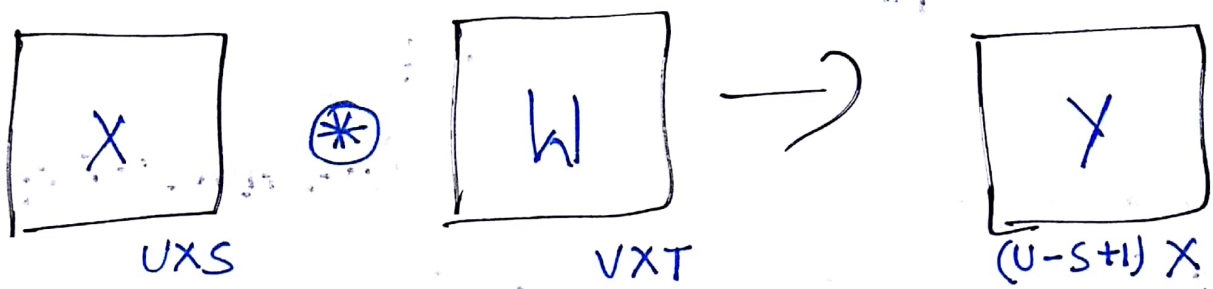
q is range from $0, 1 \ldots V-T$

Y^K size $= (U-S+1,\ V-T+1)$

J = Cost function

Filter output $= \dfrac{\partial J}{\partial Y^K}$

Convolution Block



here bias will be $\sigma(\gamma+b)$

for each sampling output will be

$$Y_{11} = W_{11}X_{11} + W_{12}X_{12} + \cdots W_{1T}X_{1T} + W_{21}X_{21} + \cdots W_{2T}X_{2T} \vdots \vdots + W_{V1}X_{V1} + \cdots W_{ST}X_{ST} \quad (1)$$

$$Y_{12} = W_{11}X_{12} + W_{12}X_{13} + \cdots W_{1T} X_{1(T+1)} \quad (2)$$

Assume that $\left[\frac{\partial J}{\partial Y}\right]_{(U-S+1,\ V-T+1)} = \left[\frac{\partial J}{\partial Y_{11}}, \frac{\partial J}{\partial Y_{12}} \cdots\right]$

by using eq ① & ②

$$\frac{\partial J}{\partial w_{11}} = \frac{\partial J}{\partial Y_{11}} \cdot \frac{\partial Y_{11}}{\partial w_{11}} + \frac{\partial J}{\partial Y_{12}} \cdot \frac{\partial Y_{12}}{\partial w_{12}} + \cdots$$

$$\frac{\partial E}{\partial Y_{(U-S+1)\times(V-T+1)}}$$

using eq ①

$$\frac{\partial J}{\partial w_{11}} = \frac{\partial J}{\partial Y_{11}} \cdot X_{11} + \frac{\partial J}{\partial Y_{12}} \cdot X_{12} + \cdots$$

$$\frac{\partial J}{\partial Y_{(U-S+1)\times(V-T+1)}} \cdot X_{(U-S+1)\times(V-T+1)}$$

Similarly $\frac{\partial J}{\partial w_{12}} = \frac{\partial J}{\partial Y_{11}} \times X_{12} + \frac{\partial J}{\partial Y_{12}} \times X_{13} + \cdots$

So we can conclude that

$$\frac{\partial J}{\partial w_K} = X \frac{\partial J}{\partial Y^K}$$

hence proved.