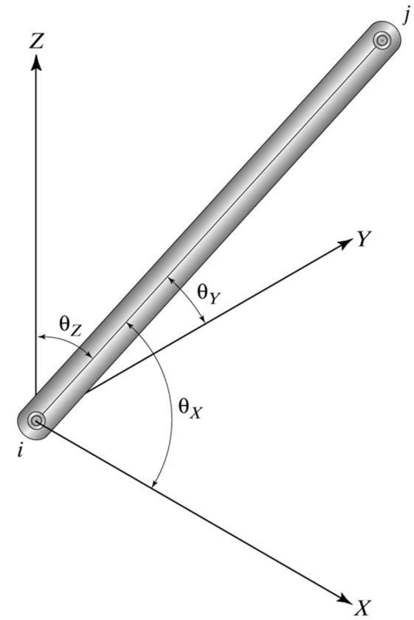
Space Trusses and Verification of Results

1 Space Trusses (3-D Truss)



$$\cos\theta_X = \frac{X_j - X_i}{L}$$
$$\cos\theta_Y = \frac{Y_j - Y_i}{L}$$
$$\cos\theta_Z = \frac{Z_j - Z_i}{L}$$

Through a similar formulation to what we did with 2-D trusses, we can find $[K]^{(e)}$

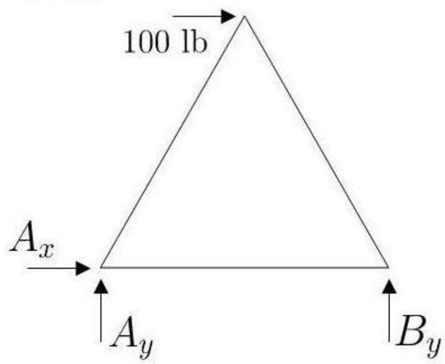
$$[\mathbf{K}]^{(e)} = [\mathbf{T}][\mathbf{K}][\mathbf{T}]^{-1}$$

$$[\mathbf{K}]^{(e)} = k\begin{bmatrix} \cos^2\theta_X & \cos\theta_X\cos\theta_Y & \cos\theta_X\cos\theta_Z & -\cos^2\theta_X & -\cos\theta_X\cos\theta_Y & -\cos\theta_X\cos\theta_Z \\ \cos\theta_X\cos\theta_Y & \cos^2\theta_Y & \cos\theta_Y\cos\theta_Z & -\cos\theta_X\cos\theta_Y & -\cos^2\theta_Y & -\cos\theta_Y\cos\theta_Z \\ \cos\theta_X\cos\theta_Z & \cos\theta_Y\cos\theta_Z & \cos^2\theta_Z & -\cos\theta_X\cos\theta_Z & -\cos\theta_Y\cos\theta_Z & -\cos^2\theta_Z \\ -\cos^2\theta_X & -\cos\theta_X\cos\theta_Y & -\cos\theta_X\cos\theta_Z & \cos^2\theta_X & \cos\theta_X\cos\theta_Y & \cos\theta_X\cos\theta_Z \\ -\cos\theta_X\cos\theta_Y & cos^2\theta_Y & -\cos\theta_Y\cos\theta_Z & \cos\theta_X\cos\theta_Y & \cos^2\theta_Y & \cos\theta_Y\cos\theta_Z \\ -\cos\theta_X\cos\theta_Z & -\cos\theta_Y\cos\theta_Z & -\cos^2\theta_Z & \cos\theta_X\cos\theta_Z & \cos\theta_Y\cos\theta_Z & \cos^2\theta_Z \end{bmatrix}$$

2 Verification of Results

2.1 Check the Reaction Forces

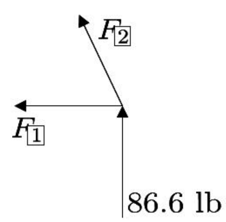
FBD



$$
\begin{array}{lrl}
+\rightarrow \sum F_x = 0 & 100 \text{ lb} + A_x = 0 \;\rightrightarrows & \underline{A_x = -100 \text{ lb}} \\
+\curvearrowleft \sum M_{nodeA} = 0 & (B_y)(100 \text{ in}) - (100)[100 \text{ in}(\sin 60)] = 0 \;\rightrightarrows & \underline{B_y = 86.6 \text{ lb}} \\
+\uparrow \sum F_y = 0 & A_y + B_y = 0 \;\rightrightarrows & \underline{A_y = -86.6 \text{ lb}}
\end{array}
$$

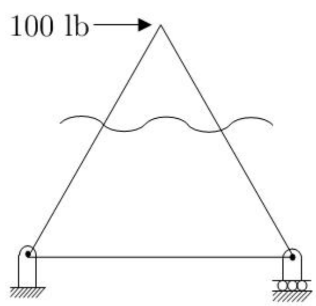
2.2 The sum of forces at each node should be zero

Node 2

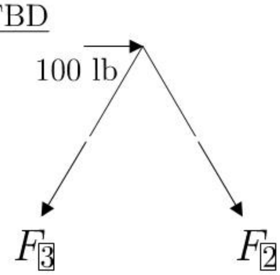


$$
\begin{array}{ll}
+\rightarrow \sum F_x = 0 & -F_1 - F_2 \cos 60 = 0 \\
+\uparrow \sum F_y = 0 & 86.6 \text{ lb} + F_2 \sin 60 = 0
\end{array}
$$

2.3 Pass an arbitrary section through the truss



FBD

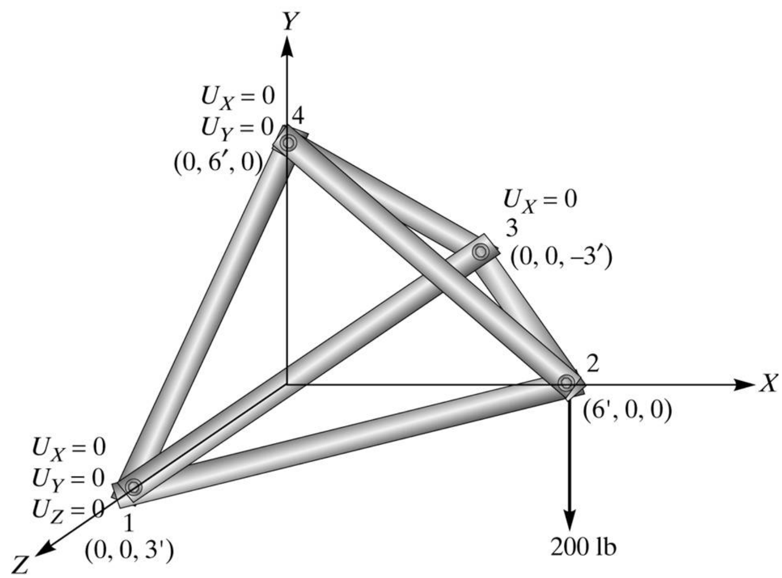


$$
\begin{array}{ll}
+\rightarrow \sum F_x = 0 & 100 \text{ lb} + F_2 \sin 30 - F_3 \sin 30 = 0 \\
+\uparrow \sum F_y = 0 & -F_2 \cos 30 - F_3 \cos 30 = 0 \\
+\curvearrowleft \sum M_{node} = 0 \text{ (if needed)} &
\end{array}
$$

3 Examples

Example #1

All members are made from aluminum with a modulus of elasticity of $E = 10.6 \times 10^6$ lb/in^2 and a cross-sectional area of 1.56 in^2.



Example 3-2

Example #2

The 3-D truss is made of steel ($E = 29 \times 10^6$ psi) and is to support the load shown in the figure. The Cartesian coordinates of the joints with respect to the system shown are given in feet. The cross-sectional area of each member is 3.093 in^2. Using ANSYS, determine the deflection of each joint, the stresses in each member, and the reaction forces. Verify your results.