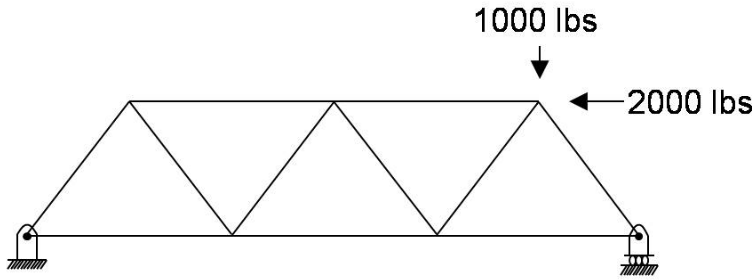
Trusses: FE Formulation

1 Introduction



These structures are typically classified as: *Trusses* or *Frames*

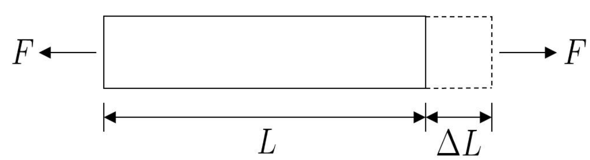
Trusses have the following characteristics

1. All two-force Members (NO bending; only tension and compression)
2. Pinned Joints (Good assumption even with bolts or welds)
3. Loads applied at joints

- Statically *determinate* Trusses are assumed to be "Rigid Bodies" and solved using:
    1. Method of Joints
    2. Method of Sections
- Statically *indeterminate* Trusses must be solved by allowing them to displace (i.e. deformable bodies) and use geometric constraints (i.e. boundary conditions). Long and tedious process!
- The Finite Element Method allows us to solve *both statically determinate and indeterminate* in a very straight forward manner.

2 Finite Element Formulation (Direct Stiffness Method)

2.1 Two-Force Member



$$\sigma = \frac{F}{A} \quad \text{and} \quad \varepsilon = \frac{\Delta L}{L}$$

From Hooke's Law, we substitute in σ and ε:

$$\sigma = E\varepsilon$$

$$\left(\frac{F}{A}\right) = E\left(\frac{\Delta L}{L}\right)$$

$$F = \left(\frac{AE}{L}\right)\Delta L$$

Note analogy to the linear spring equation, F = kx

Our axially loaded two-force members are *linear springs* with spring constant of k_{eq}

$$k_{eq} = \frac{AE}{L}$$

$$F = k_{eq}\Delta L$$

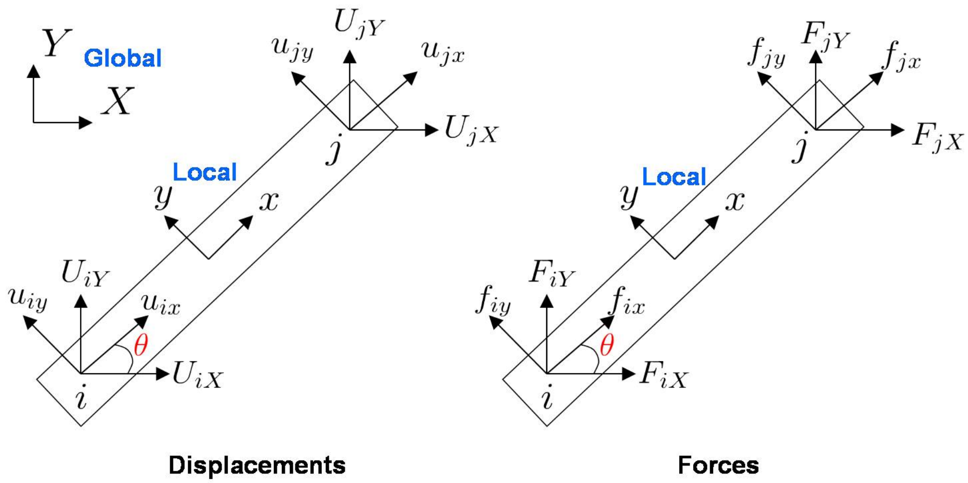
To continue formulation, we need *two frames of reference*:

1. Global Coordinate System, XY (GCS)
2. Local Coordinate System, xy (LCS)

2.2 Coordinate Systems

- The *Global* coordinate system will be used to define:
  1. Node Locations
  2. Element Angles, θ
  3. Constraints (i.e. Boundary Conditions)
  4. External Loads
  5. Solution (i.e. Displacements)

- The *Local* coordinate system will be used to define individual element behavior and values:
  1. Internal Forces
  2. Element Displacements
  3. Element Stress



Global Coordinate System Notation

Displacements	Forces
$U_{iX},\ U_{iY}$	$F_{iX},\ F_{iY}$
$U_{jX},\ U_{jY}$	$F_{jX},\ F_{jY}$

Local Coordinate System Notation

Displacements	Forces
$u_{iX},\ u_{iY}$	$f_{iX},\ f_{iY}$
$u_{jX},\ u_{jY}$	$f_{jX},\ f_{jY}$

2.3 Trig Relationship between G.C.S. and L.C.S.

$$
\begin{aligned}
U_{iX} &= u_{ix}\cos\theta - u_{iy}\sin\theta & F_{iX} &= f_{ix}\cos\theta - f_{iy}\sin\theta \\
U_{iY} &= u_{ix}\sin\theta + u_{iy}\cos\theta & F_{iY} &= f_{ix}\sin\theta + f_{iy}\cos\theta \\
U_{jX} &= u_{jx}\cos\theta - u_{jy}\sin\theta & F_{jX} &= f_{jx}\cos\theta - f_{jy}\sin\theta \\
U_{jY} &= u_{jx}\sin\theta + u_{jy}\cos\theta & F_{jY} &= f_{jx}\sin\theta + f_{jy}\cos\theta
\end{aligned}
$$

With:

$$
\{U\} = \begin{Bmatrix} U_{iX} \\ U_{iY} \\ U_{jX} \\ U_{jY} \end{Bmatrix} \quad \{u\} = \begin{Bmatrix} u_{ix} \\ u_{iy} \\ u_{jx} \\ u_{jy} \end{Bmatrix} \quad \{F\} = \begin{Bmatrix} F_{iX} \\ F_{iY} \\ F_{jX} \\ F_{jY} \end{Bmatrix} \quad \{f\} = \begin{Bmatrix} f_{ix} \\ f_{iy} \\ f_{jx} \\ f_{jy} \end{Bmatrix}
$$

$\{U\} \equiv$ Global Node Desplacements
$\{u\} \equiv$ Local Node Desplacements
$\{F\} \equiv$ Global Node Forces
$\{f\} \equiv$ Local Node Forces

And:

$$[T] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$[T] \equiv \text{Transformation Matrix}$$

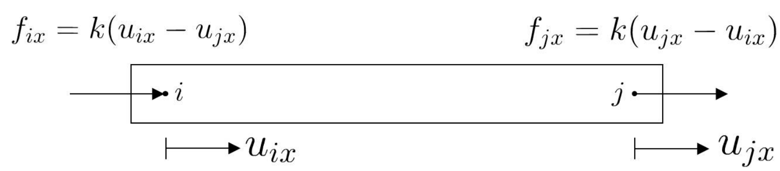
Then:

$$\{U\} = [T]\{u\}$$

$$\{F\} = [T]\{f\}$$

The *Transformation Matrix* ”transforms” Local values to Global values.

2.4 Truss Element Forces and Displacements



$$\begin{aligned} f_{ix} &= k(u_{ix} - u_{jx}) \\ f_{iy} &= 0 \\ f_{jx} &= k(u_{jx} - u_{ix}) \\ f_{jy} &= 0 \end{aligned} \quad \xrightarrow{\textbf{Matrix form}} \quad \begin{Bmatrix} f_{ix} \\ f_{iy} \\ f_{jx} \\ f_{jy} \end{Bmatrix} = \begin{bmatrix} k & 0 & -k & 0 \\ 0 & 0 & 0 & 0 \\ -k & 0 & k & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_{ix} \\ u_{iy} \\ u_{jx} \\ u_{jy} \end{Bmatrix}$$

where $k = k_{eq} = \frac{AE}{L}$
Notice the matrix equation's form:

$$\{f\} = [k]\{u\}$$

Let's convert this to Global Coordinates:
Knowing

$$\{F\} = [T]\{f\} \quad \text{and} \quad \{U\} = [T]\{u\}$$

…then

$$\{f\} = [T]^{-1}\{F\} \quad \text{and} \quad \{u\} = [T]^{-1}\{U\}$$

Substitute this into $\{f\} = [k]\{u\}$ from before, and we get

$$[T]^{-1}\{F\} = [k][T]^{-1}\{U\}$$

Multiply both sides by $[T]$,

$$[T][T]^{-1}\{F\} = [T][k][T]^{-1}\{U\}$$

Simplifying, we get

$$\{F\} = [T][k][T]^{-1}\{U\}$$

Global Coordinate form!

Since $[T]^{-1} \equiv$ inverse of Transformation Matrix,

$$[T]^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

Note: $[T]^{-1} = [T]^T$ in this case

Substituting $[T], [k], [T]^{-1}$ into $\{F\} = [T][k][T]^{-1}\{U\}$ and simplifying, we get

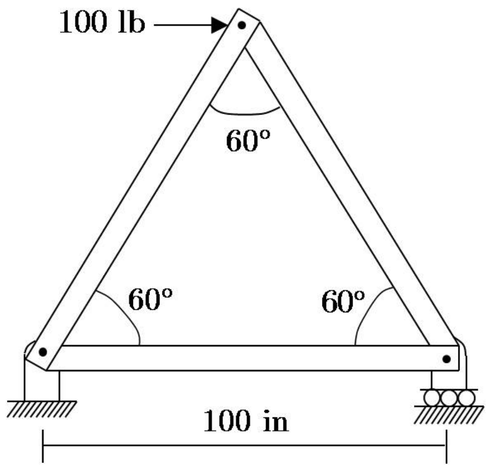
$$\begin{Bmatrix} F_{iX} \\ F_{iY} \\ F_{jX} \\ F_{jY} \end{Bmatrix} = k \underbrace{\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}} \begin{Bmatrix} U_{iX} \\ U_{iY} \\ U_{jX} \\ U_{jY} \end{Bmatrix}$$

Global Stiffness Matrix for one element

Note: the equation relates applied forces, stiffness, and displacements for *one* element, all in the Global Coordinate System.

2.5 Example

Let's do an example following the step-by-step process from Chapter 1.

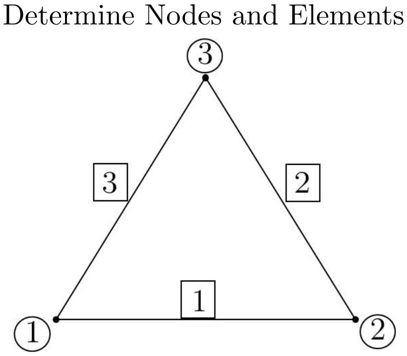


$$E = 1 \times 10^6 \text{ psi}$$
$$A = 0.1 \text{ in}^2$$
$$EA = 100,000 \text{ lb}$$
$$L = 100 \text{ in}$$

Find:

1. Displacements of each node
2. Support reactions
3. Stresses in each element

2.5.1 **Pre-processing Phase**: Determine Nodes and Elements



2.5.2 **Pre-processing Phase**: Assume a solution that approximates behavior of an Element

$$k = \frac{AE}{L}$$
$$k = \frac{100{,}000 \text{ lb}}{100 \text{ in}}$$
$$k = 1000 \ \frac{lb}{in}$$

2.5.3 **Pre-processing Phase**: Develop equations for Elements

$$[K]^{(e)} = 1000\ \frac{\text{lb}}{\text{in}} \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & -\cos^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta & -\sin\theta\cos\theta & -sin^2\theta \\ -\cos^2\theta & -\sin\theta\cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & -\sin^2\theta & \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

As discussed before to go from the local to the global coordinate system, we need to apply the Transformation matrix.

$$[K]^{(e)} = [T][k]^{(e)}[T]^{-1}$$

As a result, we'll get for Element 1, $\theta = 0°$

$$[K]^{(1)} = 1000\ \frac{\text{lb}}{\text{in}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \end{matrix}$$

For Element 2, $\theta = 120°$

$$[K]^{(2)} = 1000\ \frac{\text{lb}}{\text{in}} \begin{bmatrix} .25 & -.433 & -.25 & .433 \\ -.433 & .75 & .433 & -.75 \\ -.25 & .433 & .25 & -.433 \\ .433 & -.75 & -.433 & .75 \end{bmatrix} \begin{matrix} U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \end{matrix}$$

For Element 3, $\theta = 60°$

$$[K]^{(3)} = 1000\ \frac{\text{lb}}{\text{in}} \begin{bmatrix} .25 & .433 & -.25 & -.433 \\ .433 & .75 & -.433 & -.75 \\ -.25 & -.433 & .25 & .433 \\ -.433 & -.75 & .433 & .75 \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{3X} \\ U_{3Y} \end{matrix}$$

Note: I've show the corresponding Nodal Displacement Matrix beside each $[k]^{(e)}$ Matrix.

2.5.4 **Pre-processing Phase**: Assemble Elements

In order to assemble the elemental matrices, they all need to be in their global form. Therefore, we need to transpose the elemental matrices from local form to global form. Because we are going from a 4x4 (local) matrix to a 6x6 (global) matrix, our transpose matrix needs to be either a 4x6 or a 6x4. Here we'll use a 6x4, but the 4x6 version is just the transpose of the 6x4.

Looking Element 3, we note that it is composed of nodes 1 and 3. As a result, it has the following generic LOCAL form with GLOBAL coordinates.

$$\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{3X} \\ U_{3Y} \end{matrix}$$

The generic form for the transpose matrix is then the following:

$$\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \end{matrix}$$

Note that the columns (from left to right) represent i_X, i_Y, j_X, j_Y.

If $i = 1$ and $j = 3$, then we'll have...

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \end{matrix}$$

If $i = 3$ and $j = 1$, then we'll have...

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \end{matrix}$$

Be sure to convince yourself that even though θ will be different in the $[K]^{(e)}$ matrix when i and j are switched, the $[K]^{(e,G)}$ for either form will be the same.

Once we have all the global forms of the elemental matrices, we can combine them in the overall global stiffness matrix.

$$[K]^{(G)} = [K]^{(1G)} + [K]^{(2G)} + [K]^{(3G)}$$

Element $\boxed{1}$

$$[K]^{(G)} = 1000\ \tfrac{\text{lb}}{\text{in}} \begin{array}{c} \begin{matrix} U_{1X} & U_{1Y} & U_{2X} & U_{2Y} & U_{3X} & U_{3Y} \end{matrix} \\ \begin{bmatrix} 1+.25 & 0+.433 & -1 & 0 & -.25 & -.433 \\ 0+.433 & 0+.75 & 0 & 0 & -.433 & -.75 \\ -1 & 0 & 1+.25 & 0-.433 & -.25 & .433 \\ 0 & 0 & 0-.433 & 0+.75 & .433 & -.75 \\ -.25 & -.433 & -.25 & .433 & .25+.25 & -.433+.433 \\ -.433 & -.75 & .433 & -.75 & -.433+.433 & .75+.75 \end{bmatrix} \end{array} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \end{matrix}$$

Element $\boxed{3}$ Element $\boxed{2}$

$$[K]^{(G)} = 1000\ \frac{\text{lb}}{\text{in}} \begin{bmatrix} 1.25 & .433 & -1 & 0 & -.25 & -.433 \\ .433 & .75 & 0 & 0 & -.433 & -.75 \\ -1 & 0 & 1.25 & -.433 & -.25 & .433 \\ 0 & 0 & -.433 & .75 & .433 & -.75 \\ -.25 & -.433 & -.25 & .433 & .5 & 0 \\ -.433 & -.75 & .433 & -.75 & 0 & 1.5 \end{bmatrix} \begin{matrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \end{matrix}$$

2.5.5 **Pre-processing Phase**: Apply the Boundary Conditions and Loads

$$U_{1X} = 0 \quad F_{3X} = 100 \text{ lb}$$
$$U_{1Y} = 0$$
$$U_{2Y} = 0$$

$$1000\ \frac{\text{lb}}{\text{in}} \begin{bmatrix} 1.25 & .433 & -1 & 0 & -.25 & -.433 \\ .433 & .75 & 0 & 0 & -.433 & -.75 \\ -1 & 0 & 1.25 & -.433 & -.25 & .433 \\ 0 & 0 & -.433 & .75 & .433 & -.75 \\ -.25 & -.433 & -.25 & .433 & .5 & 0 \\ -.433 & -.75 & .433 & -.75 & 0 & 1.5 \end{bmatrix} \begin{Bmatrix} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 100 \text{ lb} \\ 0 \end{Bmatrix}$$

Since $U_{1X} = U_{1Y} = U_{2Y} = 0$, we can eliminate the first, second, and fourth rows and columns. To do this via matrix math, we need to apply another transpose matrix. Noting where the boundary conditions indicate that the displacement is 0, we create a transpose matrix that will only keep non-zero displacements.

$$[BC] = \begin{array}{cc} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} U_{1X} \\ U_{1Y} \\ U_{2X} \\ U_{2Y} \\ U_{3X} \\ U_{3Y} \end{array} \end{array}$$

where the columns are (from left to right) U_{2X}, U_{3X}, and U_{3Y}. Applying it in the following manner reduces the global stiffness matrix to a 3x3 matrix.

$$[K]^{(G,reduced)} = [BC]^T [K]^{(G)} [BC]$$

To reduce the load matrix, we use the following:

$$[F]^{(G,reduced)} = [BC]^T [F]^{(G)}$$

Thus, the equation reduces to:

$$[K]^{(G,reduced)} [U]^{(reduced)} = [F]^{(G,reduced)}$$

$$1000 \ \frac{\text{lb}}{\text{in}} \begin{bmatrix} 1.25 & -.25 & .433 \\ -.25 & .5 & 0 \\ .433 & 0 & 1.5 \end{bmatrix} \begin{Bmatrix} U_{2X} \\ U_{3X} \\ U_{3Y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 100 \text{ lb} \\ 0 \end{Bmatrix}$$

2.5.6 **Solution Phase**: Solve Algebraic Equations

$$\begin{Bmatrix} U_{2X} \\ U_{3X} \\ U_{3Y} \end{Bmatrix} = \begin{Bmatrix} 0.0500 \\ 0.225 \\ -0.0144 \end{Bmatrix} \text{ in}$$

2.5.7 **Post-processing Phase**: Obtain other information

Reaction forces:

$$\{R\} = [K]^{(G)} \{U\} - \{F\}$$

$$\{R\} = \begin{Bmatrix} R_{1X} \\ R_{1Y} \\ R_{2X} \\ R_{2Y} \\ R_{3X} \\ R_{3Y} \end{Bmatrix}$$

Perform matrix operations

$$\{R\} = \begin{Bmatrix} -100 \\ -86.6 \\ 0 \\ 86.6 \\ 0 \\ 0 \end{Bmatrix} \text{ lb}$$

Displacements, Internal Forces, and Normal Stresses for each element:

$$\{U\} = [T]\{u\}$$
$$\therefore \{u\} = [T]^{-1}\{U\}$$

Expanding

$$\begin{Bmatrix} u_{ix} \\ u_{iy} \\ u_{jx} \\ u_{jy} \end{Bmatrix} = k \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} U_{iX} \\ U_{iY} \\ U_{jX} \\ U_{jY} \end{Bmatrix}$$

Local displacements can be found for each element since θ's and $\{U\}$ are known

$$\{f\} = [k]\{u\} \ \ \textbf{for each element}$$

Element stresses can be found using

$$\sigma = \frac{f}{A}$$

OR

$$\sigma = k\frac{-u_{ix} + u_{jx}}{A} = \frac{\frac{AE}{L}(-u_{ix} + u_{jx})}{A}$$

$$\sigma = E\frac{-u_{ix} + u_{jx}}{L}$$

Note: The Δu is altered from the book. This done so that + tension and - compression.

Back to the example:

Finding *Local Displacements* for **Element** $\boxed{\mathbf{1}}$

$$\{u\} = [T]^{-1}\{U\}$$

$$\begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.050 \text{ in} \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0.050 \text{ in} \\ 0 \end{Bmatrix}$$

Finding *Local Forces* for **Element** $\boxed{\mathbf{1}}$

$$\{f\} = [k]\{u\}$$

$$\begin{Bmatrix} f_{ix} \\ f_{iy} \\ f_{jx} \\ f_{jy} \end{Bmatrix} = 1000 \ \frac{\text{lb}}{\text{in}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.050 \text{ in} \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} f_{ix} \\ f_{iy} \\ f_{jx} \\ f_{jy} \end{Bmatrix} = \begin{Bmatrix} -50.0 \text{ lb} \\ 0 \\ 50.0 \text{ lb} \\ 0 \end{Bmatrix}$$

Finding *Internal Stress* for **Element** $\boxed{\mathbf{1}}$

(For $A = 0.1$ in^2 and $E = 1 \times 10^6$ psi)

$$\sigma^{(1)} = E(\frac{-u_{1x} + u_{2x}}{L}) = (1 \times 10^6 \text{ psi})(\frac{-0 + 0.050 \text{ in}}{100 \text{ in}})$$

$$\sigma^{(1)} = 500 \text{ psi T}$$

Likewise, for Elements $\boxed{2}$ and $\boxed{3}$

Local Displacements

$$\{u\} = [T]^{-1}\{U\}$$

Element $\boxed{2}$

$$\begin{Bmatrix} u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \end{Bmatrix} = \begin{bmatrix} \cos 120° & \sin 120° & 0 & 0 \\ -\sin 120° & \cos 120° & 0 & 0 \\ 0 & 0 & \cos 120° & \sin 120° \\ 0 & 0 & -\sin 120° & \cos^1 20° \end{bmatrix} \begin{Bmatrix} 0.0500 \text{ in} \\ 0 \\ 0.235 \text{ in} \\ -0.0144 \text{ in} \end{Bmatrix} \quad \begin{Bmatrix} u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \end{Bmatrix} = \begin{Bmatrix} -0.025 \text{ in} \\ -0.0433 \text{ in} \\ -0.125 \text{ in} \\ -0.1877 \text{ in} \end{Bmatrix}$$

Element $\boxed{3}$

$$\begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{3x} \\ u_{3y} \end{Bmatrix} = \begin{bmatrix} \cos 60° & \sin 60° & 0 & 0 \\ -\sin 60° & \cos 60° & 0 & 0 \\ 0 & 0 & \cos 60° & \sin 60° \\ 0 & 0 & -\sin 60° & \cos^6 0° \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.225 \text{ in} \\ -0.0144 \text{ in} \end{Bmatrix} \quad \begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{3x} \\ u_{3y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0.1000 \text{ in} \\ -0.2021 \text{ in} \end{Bmatrix}$$

Local Forces

$$\{f\} = [k]\{u\}$$

Element $\boxed{2}$

$$\begin{Bmatrix} f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \end{Bmatrix} = 1000 \ \frac{\text{lb}}{\text{in}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} -0.025 \text{ in} \\ -0.0433 \text{ in} \\ -0.125 \text{ in} \\ -0.1877 \text{ in} \end{Bmatrix} \quad \begin{Bmatrix} f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \end{Bmatrix} = \begin{Bmatrix} 100 \text{ lb} \\ 0 \text{ lb} \\ -100 \text{ lb} \\ 0 \text{ lb} \end{Bmatrix}$$

Element $\boxed{3}$

$$\begin{Bmatrix} f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \end{Bmatrix} = 1000 \ \frac{\text{lb}}{\text{in}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.1000 \\ -0.202 \text{ in} \end{Bmatrix} \quad \begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{3x} \\ f_{3y} \end{Bmatrix} = \begin{Bmatrix} -100 \text{ lb} \\ 0 \\ 100 \text{ lb} \\ 0 \end{Bmatrix}$$

Internal Stress

$$\sigma^{(2)} = E(\frac{-u_{2x} + u_{3x}}{L}) = (1 \times 10^6 \text{ psi})(\frac{0.025 \text{ in} - 0.125 \text{ in}}{100 \text{ in}})$$

$$\sigma^{(2)} = -1000 \text{ psi C}$$

$$\sigma^{(3)} = E(\frac{-u_{2x} + u_{3x}}{L}) = (1 \times 10^6 \text{ psi})(\frac{0.025 \text{ in} - 0.125 \text{ in}}{100 \text{ in}})$$

$$\sigma^{(3)} = 1000 \text{ psi T}$$