A Family of Sensorless Observers with Speed Estimate for Rotor Position Estimation of IM and PMSM Drives

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Abstract **– The paper discusses the problem of sensorless rotor flux angle/rotor position estimation for the permanent magnet synchronous motor (PMSM) and for the induction motor (IM) and presents a family of sensorless observer designs that use a speed estimate. In sensorless AC drive control, it is typical to measure the motor's voltages and currents and to estimate the other quantities of interest: speed, fluxes (EMFs) and rotor position. The simultaneous estimation of these quantities is possible, however, the methods available are rather complicated and the accuracy is often questionable, especially under parameter variations. The paper proposes a sequential approach which is simpler: first, estimate the drive's speed; then, use this speed estimate along with the measurements to estimate the states of the motor model and to obtain the field orientation angle. A family of observers for the IM and the PMSM is presented – these are constructed using their models in the stationary reference frame. The observers are developed assuming that the speed estimate obtained is different from the real speed; it is shown that despite this inaccuracy, with special gain designs, the correct field orientation angle is obtained. The observers are developed using Sliding Mode and/or Lyapunov methods. They can be directly applied in sensorless field-oriented drives that do not require the magnitude of the flux.**

Keywords**: sensorless control, permanent magnet synchronous motor, induction motor, rotor position estimation, rotor flux angle estimation, sliding mode observer.**

I. INTRODUCTION

The induction motor (IM) and the permanent magnet synchronous motor (PMSM) are widely used in servo and high-performance power conversion applications. The PMSM offers high efficiency and high power density since it does not have rotor windings while the squirrel cage IM is very robust and is virtually maintenance-free.

It is typical to control these drives using the field-oriented (vector control method); [1]-[3]. With this, the rotor flux angle of the IM (rotor position for the PMSM) must be known.

In a sensored control scheme of the PMSM, the rotor position angle is obtained from a shaft-mounted encoder or using Hall sensors. For the IM, the encoder signals are processed to compute the drive's speed; then, the rotor flux angle can be obtained through indirect field orientation. Generally, the encoder is undesirable, reduces the ruggedness of the drive and increases the cost, is sensitive to vibrations and may not work well at high speed. Sensorless control has been very popular lately and is a maturing technology.

In sensorless control schemes, it is typical to measure the motor voltages and currents in order to estimate the other quantities that may be needed in the control algorithm (usually: fluxes, speed and rotor position).

Currently, there is a large variety of sensorless control schemes; some may not require the speed estimate (e.g. torque controlled drives); others may not require the estimated fluxes (e.g. drives that only run in the constant torque region may simply use a constant reference current i_d^*). However, the field orientation angle is required and is the most important estimate of the control scheme – this influences the drive's region of stability.

Several estimation methods for the rotor flux angle (rotor position) are available. Magnetic saliency methods are based on the variation of the inductance between the d-q axes and are constructed using signal injection [4][5]. They are relatively complicated for real-time implementation, involve quite sensitive signal processing and are less portable from one machine to another; however, they sometimes offer the only option in the low/very low speed region.

A second class of estimation methods uses the PMSM or the IM model to construct observers for state estimation.

Several observers are available: full order observers [7],[8]; reduced-order observers [9][10]; or methods based on the Extended Kalman Filter [11][12]. State estimation can also be done using sliding mode observers ([13]-[18] for the PMSM); ([19]- [23] for the IM).

The difficulty of sensorless estimation in AC motor drives is given by the need to simultaneously estimate the states of the model and the speed of the drive – this is a complex task that often does not work very well. For example, the double manifold sliding mode observer for the induction motor presented in [13] is capable of estimating the states; however, the speed estimate is obtained by low pass filtering one of the manifolds, it has high ripple and is therefore of low quality. The estimation method for the IM shown in [14] uses two observers in cascade – an SM observer first and a linear observer after that. The speed is obtained using an adaptation

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law. While the mathematics of the method is proven under reasonable assumptions, the implementation of such a method is highly sensitive to the PWM noise in the motor currents and is quite hard to implement on a real drive setup. Note that in other methods, the mathematics may be even more complicated [24]. The observers in [25]-[28] also involve complex structures and are computational intensive.

This paper presents a family of observers that are constructed using sequential estimation instead of simultaneous estimation. The speed of the drive is estimated first. It is assumed that the speed estimate obtained is inaccurate in a reasonable range of the real speed (the paper does not give details on how exactly to estimate the speed – any method can be used). The speed estimate is fed into a state estimator that is based on the stationary reference frame of the respective motor. Then, a special feedback gain design is found such that the system produces useable estimates. The estimates obtained (EMFs or fluxes) are used to obtain the rotor flux angle (rotor position) of the drive (Fig.1).

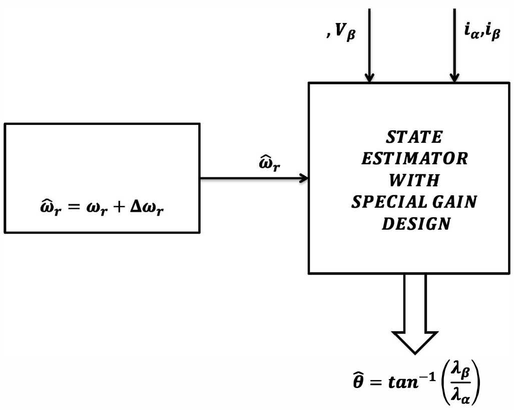

Fig.1. Sensorless State Observer with speed estimate

II. OBSERVER FOR THE PMSM DRIVE WITH SPEED ESTIMATE

The fluxes and the EMFs of the PMSM in the stationary reference frame are defined as:

$$\begin{cases}\lambda_{PM\alpha} = \lambda_{PM} cos\theta \\ \lambda_{PM\beta} = \lambda_{PM} sin\theta\end{cases} \qquad \begin{cases} e_\alpha = p\lambda_{PM\alpha} = -K_E\omega_e sin\theta \\ e_\beta = p\lambda_{PM\beta} = K_E\omega_e cos\theta\end{cases} \quad (1)$$

where $\theta = \omega_e t$ is the rotor position angle; K_E is the EMF constant. The EMFs are differentiated considering that the speed varies much slower than the electrical quantities ($\dot{\omega}_e \approx 0$) – this is a very reasonable assumption for motors with low inductance for which there is a wide separation between the electrical and mechanical time constants. After appending the equations of the currents, the model of the PMSM in the stationary reference frame is:

$$\begin{cases} pe_\alpha = -\omega_e e_\beta \\ pe_\beta = \omega_e e_\alpha \\ pi_\alpha = -\frac{R}{L} i_\alpha - \frac{1}{L} e_\alpha + \frac{1}{L} V_\alpha \\ pi_\beta = -\frac{R}{L} i_\beta - \frac{1}{L} e_\beta + \frac{1}{L} V_\beta \end{cases} \quad (2)$$

Based on this model, several types of state observers could be designed: full-order, reduced-order, sliding mode. For sensorless control, since the speed ω_e appears in (2), this should be estimated simultaneously.

A simple method to obtain the EMFs that is applicable in a sensorless algorithm is to construct a sliding mode observer based only on the current equations in (2) – in this way, ω_e is not involved. This observer uses discontinuous feedback (switching terms) that are functions of the current mismatches. The EMFs are obtained by low pass filtering the switching terms. However, at low speed, the EMF estimates do not quite match the real signals (due to filtering). When Direct Field Orientation (DFO) is used, the phase errors in the EMFs propagate in the rotor position.

The estimation method proposed here is developed using system (2) and, as explained, is rather unusual for a couple of reasons: it does not attempt to estimate ω_e and the states of the model simultaneously (instead, estimation is sequential). It uses the full model of the PMSM (therefore, it involves ω_e and requires implementation of all four state equations). Also, it will be shown that the process of estimating the states can be made insensitive to the errors in the speed signal.

With this method, the speed of the PMSM is estimated first - this is a standalone algorithm that can be tuned and tested separately. Then, the speed estimate is fed in a state observer built along the lines of system (2) in order to obtain the EMFs. Low pass filtering is not needed.

Consider that the speed estimator yields a signal that is of the form:

$$\hat{\omega}_e = \omega_e + \Delta\omega_e \quad (3)$$

where ω_e is the real speed and the speed mismatch $\Delta\omega_e$ is relatively small and generally unknown.

The state observer for the PMSM is a Sliding Mode observer that uses the speed estimate (3) and is designed based on system (2). The equations are:

$$\begin{cases} p\hat{e}_\alpha = -\hat{\omega}_e \hat{e}_\beta + l_{11} u_\alpha \\ p\hat{e}_\beta = \hat{\omega}_e \hat{e}_\alpha + l_{22} u_\beta \\ p\hat{\imath}_\alpha = -\frac{R}{L}\hat{\imath}_\alpha - \frac{1}{L}\hat{e}_\alpha + \frac{1}{L}V_\alpha - \frac{1}{L}u_\alpha \\ p\hat{\imath}_\beta = -\frac{R}{L}\hat{\imath}_\beta - \frac{1}{L}\hat{e}_\beta + \frac{1}{L}V_\beta - \frac{1}{L}u_\beta \end{cases} \quad (4)$$

The switching controls u_α, u_β are:

$$\begin{cases} u_\alpha = M \cdot sign(s_\alpha) \\ u_\beta = M \cdot sign(s_\beta) \end{cases} ; \quad \begin{cases} s_\alpha = \hat{\imath}_\alpha - i_\alpha \\ s_\beta = \hat{\imath}_\beta - i_\beta \end{cases} \quad (5)$$

For the observer implementation, the voltages V_α, V_β and currents i_α, i_β are measured quantities.

In (5), M is a design gain, $M > 0$. The feedback gains l_{11}, l_{22} are design parameters. The PMSM equations (2) are subtracted from (4) and the result is:

$$\begin{cases} p\bar{e}_\alpha = -\omega_e \bar{e}_\beta - \Delta\omega_e \hat{e}_\beta + l_{11} u_\alpha \\ p\bar{e}_\beta = \omega_e \bar{e}_\alpha + \Delta\omega_e \hat{e}_\alpha + l_{22} u_\beta \\ \dot{s}_\alpha = -\frac{R}{L} s_\alpha + \frac{1}{L}\bar{e}_\alpha - \frac{1}{L} u_\alpha \\ \dot{s}_\beta = -\frac{R}{L} s_\beta + \frac{1}{L}\bar{e}_\beta - \frac{1}{L} u_\beta \end{cases} \quad (6)$$

where $\bar{e}_\alpha = \hat{e}_\alpha - e_\alpha$, $\bar{e}_\beta = \hat{e}_\beta - e_\beta$. In the equations of the manifolds $\dot{s}_\alpha$, $\dot{s}_\beta$, with high enough M, the terms $-M \cdot sign(s_\alpha)$, $-M \cdot sign(s_\beta)$ overcome the other terms on the right side of (6). Then, s_α, s_β and their derivatives have opposite signs. Therefore, the manifolds tend to zero and sliding mode occurs: $s_\alpha \to 0$, $s_\beta \to 0$. This means that the estimates of the currents converge: $\hat{\imath}_\alpha \to i_\alpha$, $\hat{\imath}_\beta \to i_\beta$. Once sliding mode starts, s_α, s_β and their derivatives are equal to zero. Using the equivalent control method in [13], the equivalent controls are:

$$\begin{cases} u_{\alpha,eq} = \bar{e}_\alpha \\ u_{\beta,eq} = \bar{e}_\beta \end{cases} \tag{7}$$

To examine the behavior of the mismatches $\bar{e}_\alpha$, $\bar{e}_\beta$ after SM occurs; replace the switching terms u_α, u_β in the upper two equations of (6) with the equivalent controls. This gives:

$$\begin{cases} p\bar{e}_\alpha = -\omega_e \bar{e}_\beta - \Delta\omega_e \hat{e}_\beta + l_{11}\bar{e}_\alpha \\ p\bar{e}_\beta = \omega_e \bar{e}_\alpha + \Delta\omega_e \hat{e}_\alpha + l_{22}\bar{e}_\beta \end{cases} \tag{8}$$

To study the convergence of the EMF estimates, select the positive definite Lyapunov function:

$$V = \frac{1}{2}(\bar{e}_\alpha^2 + \bar{e}_\beta^2) \tag{9}$$

Function V is differentiated and the derivatives from (8) are replaced. The expression of $\dot{V}$ is:

$$\dot{V} = l_{11}\bar{e}_\alpha^2 + l_{22}\bar{e}_\beta^2 - \Delta\omega_e \hat{e}_\beta \bar{e}_\alpha + \Delta\omega_e \hat{e}_\alpha \bar{e}_\beta \tag{10}$$

With the design gains $l_{11} = l_{22} = -k$ $(k > 0)$, the derivative of V is:

$$\dot{V} = -k(\bar{e}_\alpha^2 + \bar{e}_\beta^2) + \Delta\omega_e(\hat{e}_\beta \bar{e}_\alpha - \hat{e}_\alpha \bar{e}_\beta) \tag{11}$$

Equation (11) is an important result: note that $\dot{V} < 0$ if $\Delta\omega_e = 0$ and, in this case, the observer is asymptotically stable. However, when $\Delta\omega_e \neq 0$, $\dot{V}$ is negative at the beginning of the motion (and V decays); the decays stops when $\dot{V} = 0$ - this is equivalent to:

$$k(\bar{e}_\alpha^2 + \bar{e}_\beta^2) = \Delta\omega_e(\hat{e}_\beta \bar{e}_\alpha - \hat{e}_\alpha \bar{e}_\beta) \tag{12}$$

It can be assumed that the mismatches $\bar{e}_\alpha$, $\bar{e}_\beta$ are of the same order of magnitude; then, using the notation $m\bar{e}_\alpha = \bar{e}_\beta$ (m is unknown), equation (12) is manipulated to obtain the values of $\bar{e}_\alpha$, $\bar{e}_\beta$ at the equilibrium point (where V stops decaying):

$$\bar{e}_\alpha = \Delta\omega_e \frac{m\hat{e}_\beta - \hat{e}_\alpha}{k(1+m^2)} \qquad \bar{e}_\beta = \Delta\omega_e \frac{m(m\hat{e}_\beta - \hat{e}_\alpha)}{k(1+m^2)} \tag{13}$$

The expressions in (13) are a remarkable result because the design parameter k appears in the denominator. Therefore, despite $\Delta\omega_e \neq 0$, if k is increased, the EMF mismatches can be made as small as desired (theoretically zero). Therefore, the observer allows accurate estimation of the EMFs. A more complete treatment can be found in [29].

The method has a few desirable properties: the sequential estimation process is simple and it allows use of any reasonably accurate speed estimator. Estimation does not involve low pass filtering. The only special design requirement is to use a relatively high value for k. In practice, a moderate value of k will work fine.

After the EMFs are found, the rotor position of the PMSM is obtained through direct field orientation.

The SM observer was simulated using Simulink. The rated PMSM parameters are given in Table I. The drive is operated in speed control mode with synchronous d-q current control; $i_d^* = 0$. The motor is started towards 1000 rpm with a load torque of 0.4 Nm. In the simulation, the speed fed in the observer is 25% lower than the real speed from 0 to 0.2 s; is equal to the real speed from 0.2 to 0.4 s; is 25% higher than the real speed from 0.4 s to 0.6s. The simulation is run with discrete-time blocks with a sampling time of 50 μs; it does not use PWM switching.

Fig. 2 shows the real and estimated EMFs with k= 50; with this relatively low gain and with incorrect speed, the estimates do not match the real EMFs.

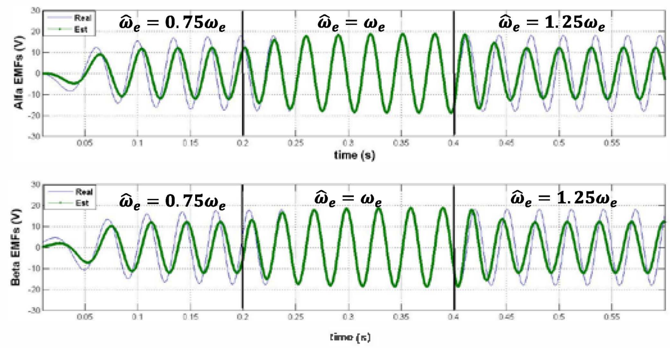

Fig.2. Real versus estimated EMFs, k=50

Fig. 3 shows the same situation for $k = 400$. With this higher gain, the estimated EMFs tend to the real ones - this confirms equations (13). In practice, this value of this gain can be easily implemented and the method works well with currents that have PWM ripple. Once the EMFs have been estimated, the rotor position angle is computed using the tan^{-1} function.

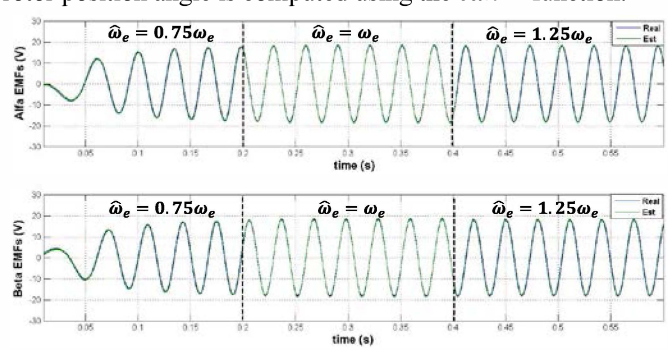

Fig.3. Real versus estimated EMFs, k=400

III. SLIDING MODE OBSERVER FOR THE IM DRIVE WITH SPEED ESTIMATE

The stationary frame model of the induction motor is:

$$\begin{cases} \frac{d\lambda_\alpha}{dt} = -\eta\lambda_\alpha - \omega_r\lambda_\beta + \eta L_m i_\alpha \\ \frac{d\lambda_\beta}{dt} = \omega_r\lambda_\alpha - \eta\lambda_\beta + \eta L_m i_\beta \\ \frac{di_\alpha}{dt} = \eta\beta\lambda_\alpha + \omega_r\beta\lambda_\beta - \gamma i_\alpha + \frac{1}{\sigma L_s}v_\alpha \\ \frac{di_\beta}{dt} = -\omega_r\beta\lambda_\alpha + \eta\beta\lambda_\beta - \gamma i_\beta + \frac{1}{\sigma L_s}v_\beta \end{cases} \tag{14}$$

The parameters σ,β,γ are:

$$\sigma = 1 - \frac{L_m^2}{L_s L_r};\ \beta = \frac{L_m}{\sigma L_s L_r};\ \ \gamma = \frac{1}{\sigma L_s}\left(\frac{L_m^2}{L_s L_r}R_r + R_s\right) \quad (15)$$

For the IM, consider an estimate of the rotor electrical speed which is of the form:

$$\hat{\omega}_r = \omega_r + \Delta\omega_r \quad (16)$$

($\Delta\omega_r$ is unknown). The objective is to design a state observer for the system (14) that uses the speed estimate $\hat{\omega}_r$.This is a sliding mode observer that employs the current mismatches for feedback. The equations are:

$$\begin{cases} \begin{bmatrix} \frac{d\hat{\lambda}_\alpha}{dt} \\ \frac{d\hat{\lambda}_\beta}{dt} \end{bmatrix} = \begin{bmatrix} -\eta & -\hat{\omega}_r \\ \hat{\omega}_r & -\eta \end{bmatrix} \begin{bmatrix} \hat{\lambda}_\alpha \\ \hat{\lambda}_\beta \end{bmatrix} + \eta L_m \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} - K \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \\ \begin{bmatrix} \frac{d\hat{i}_\alpha}{dt} \\ \frac{d\hat{i}_\beta}{dt} \end{bmatrix} = \begin{bmatrix} \eta\beta & \hat{\omega}_r\beta \\ -\hat{\omega}_r\beta & \eta\beta \end{bmatrix} \begin{bmatrix} \hat{\lambda}_\alpha \\ \hat{\lambda}_\beta \end{bmatrix} - \gamma \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{1}{\sigma L_s} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} - \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \end{cases} \quad (17)$$

The switching terms u_α, u_β are:

$$\begin{cases} u_\alpha = M \cdot sign(s_\alpha) \\ u_\beta = M \cdot sign(s_\beta) \end{cases} \qquad \begin{cases} s_\alpha = \hat{i}_\alpha - i_\alpha \\ s_\beta = \hat{i}_\beta - i_\beta \end{cases} \quad (18)$$

After subtracting the original equations, the result is:

$$\begin{cases} \begin{bmatrix} \frac{d\bar{\lambda}_\alpha}{dt} \\ \frac{d\bar{\lambda}_\beta}{dt} \end{bmatrix} = \begin{bmatrix} -\eta & -\omega_r \\ \omega_r & -\eta \end{bmatrix} \begin{bmatrix} \bar{\lambda}_\alpha \\ \bar{\lambda}_\beta \end{bmatrix} + \Delta\omega_r \begin{bmatrix} -\hat{\lambda}_\beta \\ \hat{\lambda}_\alpha \end{bmatrix} - K \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \\ \begin{bmatrix} \frac{d\bar{i}_\alpha}{dt} \\ \frac{d\bar{i}_\beta}{dt} \end{bmatrix} = \begin{bmatrix} \eta\beta & \omega_r\beta \\ -\omega_r\beta & \eta\beta \end{bmatrix} \begin{bmatrix} \bar{\lambda}_\alpha \\ \bar{\lambda}_\beta \end{bmatrix} + \Delta\omega_r\beta \begin{bmatrix} \hat{\lambda}_\beta \\ -\hat{\lambda}_\alpha \end{bmatrix} - \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \end{cases} \quad (19)$$

In the lower equations of (19), if gain M is high enough, sliding mode occurs and the current mismatches tend to zero. The equivalent controls are:

$$\begin{cases} u_{\alpha,eq} = \eta\beta\bar{\lambda}_\alpha + \omega_r\beta\bar{\lambda}_\beta + \Delta\omega_r\beta\hat{\lambda}_\beta \\ u_{\beta,eq} = -\omega_r\beta\bar{\lambda}_\alpha + \eta\beta\bar{\lambda}_\beta - \Delta\omega_r\beta\hat{\lambda}_\alpha \end{cases} \quad (20)$$

The dynamics of the flux mismatches during the sliding mode motion are obtained using the equivalent controls:

$$\begin{bmatrix} \frac{d\bar{\lambda}_\alpha}{dt} \\ \frac{d\bar{\lambda}_\beta}{dt} \end{bmatrix} = \begin{bmatrix} -\eta & -\omega_r \\ \omega_r & -\eta \end{bmatrix} \begin{bmatrix} \bar{\lambda}_\alpha \\ \bar{\lambda}_\beta \end{bmatrix} + \Delta\omega_r \begin{bmatrix} -\hat{\lambda}_\beta \\ \hat{\lambda}_\alpha \end{bmatrix} - \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} u_{\alpha,eq} \\ u_{\beta,eq} \end{bmatrix} \quad (21)$$

To investigate convergence, select the Lyapunov function $V = \frac{1}{2}(\bar{\lambda}_\alpha^2 + \bar{\lambda}_\beta^2)$. After differentiation, the derivatives of $\bar{\lambda}_\alpha$, $\bar{\lambda}_\beta$ are replaced from (21). Then, $\dot{V}$ is:

$$\dot{V} = -(\eta + k_{11}\eta\beta - k_{12}\omega_r\beta)(\bar{\lambda}_\alpha^2 + \bar{\lambda}_\beta^2) + [k_{12}\beta\hat{\lambda}_\alpha - (1 + k_{11}\beta)\hat{\lambda}_\beta]\Delta\omega_r\bar{\lambda}_\alpha + [-k_{21}\beta\hat{\lambda}_\beta + (1 + k_{22}\beta)\hat{\lambda}_\alpha]\Delta\omega_r\bar{\lambda}_\beta \quad (22)$$

In (22), since $\Delta\omega_r \neq 0$, there are two undesirable terms and $\dot{V}$ cannot be made negative. Instead, $\dot{V}$ is negative for a while and eventually settles into an equilibrium point given by the equality:

$$(\eta + k_{11}\eta\beta - k_{12}\omega_r\beta)(\bar{\lambda}_\alpha^2 + \bar{\lambda}_\beta^2) = [k_{12}\beta\hat{\lambda}_\alpha - (1 + k_{11}\beta)\hat{\lambda}_\beta]\Delta\omega_r\bar{\lambda}_\alpha + [-k_{21}\beta\hat{\lambda}_\beta + (1 + k_{22}\beta)\hat{\lambda}_\alpha]\Delta\omega_r\bar{\lambda}_\beta \quad (23)$$

The problem is to find a set of gains such that the estimates corresponding to this equilibrium point are useable.

Consider the following set of gains:

$$\begin{cases} k_{11} = k_{22} = -\frac{1}{\beta} \\ k_{12} = -k_{21} = k \cdot sign(\omega_r) \end{cases} \quad (24)$$

The value of k_{11} was chosen to cancel the first two terms in the parenthesis that multiplies $(\bar{\lambda}_\alpha^2 + \bar{\lambda}_\beta^2)$ in (23). With this, the equation of the equilibrium point becomes:

$$-k\omega_r sign(\omega_r)\beta(\bar{\lambda}_\alpha^2 + \bar{\lambda}_\beta^2) = k \cdot \Delta\omega_r\beta(\hat{\lambda}_\alpha\bar{\lambda}_\alpha + \hat{\lambda}_\beta\bar{\lambda}_\beta)\, sign(\omega_r) \quad (25)$$

After simplifying k,β and $sign(\omega_r)$, the result is:

$$-\omega_r(\bar{\lambda}_\alpha^2 + \bar{\lambda}_\beta^2) = \Delta\omega_r(\hat{\lambda}_\alpha\bar{\lambda}_\alpha + \hat{\lambda}_\beta\bar{\lambda}_\beta) \quad (26)$$

The observer was simulated with an incorrect speed $\hat{\omega}_r$ and with the gains (24) – the result is very interesting. While the observer does not converge, *the estimated fluxes are in phase with the real fluxes*. The magnitude of the fluxes obtained is inaccurate and is inverse proportional with the value of $\hat{\omega}_r$ used. The observer converges in a few ms.

Fig. 4 shows the flux estimates of the observer. The motor runs at 1000 rpm with a load of 0.3Nm. The speed used is 25% smaller from 0-0.2 s, equal to the real speed from 0.2-0.4s and 25% bigger from 0.4-0.6 s. It can be seen that the estimates are in phase with the real fluxes.

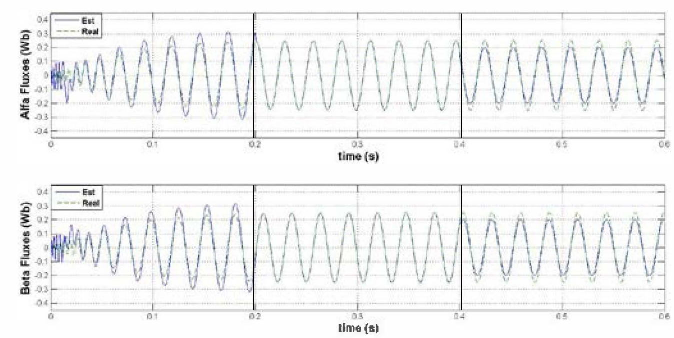

Fig.4. Real versus estimated fluxes of the SM Observer

According to the simulations, the flux estimates $\hat{\lambda}_\alpha$, $\hat{\lambda}_\beta$ and the flux mismatches $\bar{\lambda}_\alpha$, $\bar{\lambda}_\beta$ are of the form:

$$\begin{cases} \hat{\lambda}_\alpha = \hat{\lambda} \cdot \cos(\theta) \\ \hat{\lambda}_\beta = \hat{\lambda} \cdot \sin(\theta) \end{cases} \qquad \begin{cases} \bar{\lambda}_\alpha = \Delta\lambda \cdot \cos(\theta) \\ \bar{\lambda}_\beta = \Delta\lambda \cdot \sin(\theta) \end{cases} \quad (27)$$

where θ is the rotor flux angle and $\Delta\lambda = \hat{\lambda} - \lambda$ is the magnitude mismatch. It will be shown that the fluxes and mismatches from (27) are a solution to the equation of the equilibrium point. After replacing (27) into (26), this is:

$$-\omega_r \cdot \Delta\lambda^2[sin^2(\theta) + cos^2(\theta)] = \Delta\omega_r \cdot \hat{\lambda} \cdot \Delta\lambda\,[sin^2(\theta) + cos^2(\theta)] \quad (28)$$

After some algebra, the magnitude of the flux estimate is:

$$\hat{\lambda} = \frac{\lambda}{1 + \frac{\Delta\omega_r}{\omega_r}} = \lambda\frac{\omega_r}{\hat{\omega}_r} \quad (29)$$

Qualitatively, relationship (29) corresponds to the simulations: when the speed fed in the observer is higher than real ($\Delta\omega_r > 0$), then, $1 + \frac{\Delta\omega_r}{\omega_r} > 1$ and the flux estimates are smaller than real [30]. The opposite happens when $\Delta\omega_r < 0$. Relationship (29) can be verified through simulations.

In conclusion, the estimates of the SM observer with the proposed gain design are of the form:

$$\begin{cases} \hat{\lambda}_\alpha = \lambda \frac{\omega_r}{\widehat{\omega}_r} \cdot \cos(\theta) \\ \hat{\lambda}_\beta = \lambda \frac{\omega_r}{\widehat{\omega}_r} \cdot \sin(\theta) \end{cases} \tag{30}$$

Based on the above expressions, using the tan^{-1} function and DFO, an accurate value for θ is obtained. The method can be used to provide field orientation in a sensorless IM algorithm that does not require the magnitude of the rotor flux.

IV. TIME-VARYING LINEAR OBSERVER FOR THE IM DRIVE WITH SPEED ESTIMATE

Using the same model of the IM, a linear, Luenberger-type observer can be constructed to estimate the states of the model (14). This is an observer with continuous feedback – it uses correction terms that depend on the current mismatches. The observer is of the form:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L(\hat{y} - y) \\ \hat{y} = C\hat{x} \end{cases}; \qquad L = \begin{bmatrix} l_{11} l_{12} \\ l_{21} l_{22} \\ l_{31} l_{32} \\ l_{41} l_{42} \end{bmatrix} \tag{31}$$

where $y = [i_\alpha \; i_\beta]$; L is a 4x2 matrix of gains that needs to be designed. For the estimation method discussed, the voltages V_α, V_β and currents i_α, i_β are measured. In the initial development, the speed $\widehat{\omega}_r$ is considered known. Later, the speed estimate (16) will be used to obtain a sensorless observer. The equations of the proposed observer are:

$$\begin{cases} \frac{d\hat{\lambda}_\alpha}{dt} = -\eta\hat{\lambda}_\alpha - \omega_r\hat{\lambda}_\beta + \eta L_m \hat{\imath}_\alpha + l_{11}\bar{\imath}_\alpha + l_{12}\bar{\imath}_\beta \\ \frac{d\hat{\lambda}_\beta}{dt} = \omega_r\hat{\lambda}_\alpha - \eta\hat{\lambda}_\beta + \eta L_m \hat{\imath}_\beta + l_{21}\bar{\imath}_\alpha + l_{22}\bar{\imath}_\beta \\ \frac{d\hat{\imath}_\alpha}{dt} = \eta\beta\hat{\lambda}_\alpha + \omega_r\beta\hat{\lambda}_\beta - \gamma\hat{\imath}_\alpha + \frac{1}{\sigma L_s}v_\alpha + l_{31}\bar{\imath}_\alpha + l_{32}\bar{\imath}_\beta \\ \frac{d\hat{\imath}_\beta}{dt} = -\omega_r\beta\hat{\lambda}_\alpha + \eta\beta\hat{\lambda}_\beta - \gamma\hat{\imath}_\beta + \frac{1}{\sigma L_s}v_\beta + l_{41}\bar{\imath}_\alpha + l_{42}\bar{\imath}_\beta \end{cases} \tag{32}$$

where $\bar{\imath}_\alpha = \hat{\imath}_\alpha - i_\alpha$, $\bar{\imath}_\beta = \hat{\imath}_\beta - i_\beta$. After subtraction, the mismatches are:

$$\begin{cases} \frac{d\bar{\lambda}_\alpha}{dt} = -\eta\bar{\lambda}_\alpha - \omega_r\bar{\lambda}_\beta + \eta L_m \bar{\imath}_\alpha + l_{11}\bar{\imath}_\alpha + l_{12}\bar{\imath}_\beta \\ \frac{d\bar{\lambda}_\beta}{dt} = \omega_r\bar{\lambda}_\alpha - \eta\bar{\lambda}_\beta + \eta L_m \bar{\imath}_\beta + l_{21}\bar{\imath}_\alpha + l_{22}\bar{\imath}_\beta \\ \frac{d\bar{\imath}_\alpha}{dt} = \eta\beta\bar{\lambda}_\alpha + \omega_r\beta\bar{\lambda}_\beta - \gamma\bar{\imath}_\alpha + l_{31}\bar{\imath}_\alpha + l_{32}\bar{\imath}_\beta \\ \frac{d\bar{\imath}_\beta}{dt} = -\omega_r\beta\bar{\lambda}_\alpha + \eta\beta\bar{\lambda}_\beta - \gamma\bar{\imath}_\beta + l_{41}\bar{\imath}_\alpha + l_{42}\bar{\imath}_\beta \end{cases} \tag{33}$$

Using system (9), the design gains corresponding to matrix L must be designed.

For that, consider the Lyapunov function:

$$V = \tfrac{1}{2}(\bar{\lambda}_\alpha^2 + \bar{\lambda}_\beta^2 + \bar{\imath}_\alpha^2 + \bar{\imath}_\beta^2) \tag{34}$$

Function V is differentiated; the derivatives of the mismatches are replaced with their expressions from (33) and the terms alike are combined. The resulting expression of $\dot{V}$ is:

$$\begin{aligned} \dot{V} = &-\eta\bar{\lambda}_\alpha^2 + \eta L_m\bar{\lambda}_\alpha\bar{\imath}_\alpha + l_{11}\bar{\lambda}_\alpha\bar{\imath}_\alpha + l_{12}\bar{\lambda}_\alpha\bar{\imath}_\beta \\ &-\eta\bar{\lambda}_\beta^2 + \eta L_m\bar{\lambda}_\beta\bar{\imath}_\beta + l_{21}\bar{\lambda}_\beta\bar{\imath}_\alpha + l_{22}\bar{\lambda}_\beta\bar{\imath}_\beta \\ &+\eta\beta\bar{\lambda}_\alpha\bar{\imath}_\alpha + \omega_r\bar{\lambda}_\beta\bar{\imath}_\alpha - \gamma\bar{\imath}_\alpha^2 + l_{31}\bar{\imath}_\alpha^2 + l_{32}\bar{\imath}_\alpha\bar{\imath}_\beta \\ &-\omega_r\beta\bar{\lambda}_\alpha\bar{\imath}_\beta + \eta\beta\bar{\lambda}_\beta\bar{\imath}_\beta - \gamma\bar{\imath}_\beta^2 + l_{41}\bar{\imath}_\alpha\bar{\imath}_\beta + l_{42}\bar{\imath}_\beta^2 \end{aligned} \tag{35}$$

Based on (35), the gains of the observer should be designed to have $\dot{V} < 0$; then, V decays, the mismatches tend to zero and the proposed observer is stable. To make $\dot{V}$ negative, the gains should be selected as:

$$\begin{cases} l_{32} = -l_{41} = 0 \\ l_{11} = l_{22} = -\eta(L_m + \beta) \\ l_{12} = \omega_r\beta \\ l_{21} = -\omega_r\beta \\ l_{31} = l_{42} = -k \end{cases} \tag{36}$$

In (36), k is a design parameter. With these gains, the expression of $\dot{V}$ is:

$$\dot{V} = -\eta(\bar{\lambda}_\alpha^2 + \bar{\lambda}_\beta^2) - (\gamma + k)(\bar{\imath}_\alpha^2 + \bar{\imath}_\beta^2) \tag{37}$$

Since η and γ are positive; it follows that for $k > -\gamma$, $\dot{V} < 0$. Therefore, V decays; in the end, V settles in the equilibrium point $\bar{\lambda}_\alpha = 0$, $\bar{\lambda}_b = 0$, $\bar{\imath}_\alpha = 0$, $\bar{\imath}_\beta = 0$ [31]. Note that some of the gains are time varying since l_{12} and l_{21} depend on speed. Also, knowledge of ω_r is needed for implementation (this needs to be a sensored observer).

To obtain a sensorless observer, the speed ω_r is replaced with the speed estimate given by (16). Therefore, $\widehat{\omega}_r$ is used to implement equations (32) and to compute the gains in (36). The updated expression of $\dot{V}$ is:

$$\begin{aligned} \dot{V} = &-\eta(\bar{\lambda}_\alpha^2 + \bar{\lambda}_\beta^2) - (\gamma + k)(\bar{\imath}_\alpha^2 + \bar{\imath}_\beta^2) + \\ &-\Delta\omega_r(\bar{\lambda}_\alpha\hat{\lambda}_\beta - \bar{\lambda}_\beta\hat{\lambda}_\alpha) - \beta \cdot \Delta\omega_r(\hat{\lambda}_\alpha\bar{\imath}_\beta - \hat{\lambda}_\beta\bar{\imath}_\alpha) \end{aligned} \tag{38}$$

In this case, $\dot{V}$ cannot be made negative anymore. At the beginning of the motion (when the mismatches are large), the negative terms in (38) make $\dot{V}$ negative for a while and V decays. The decay ends when $\dot{V} = 0$, this is equivalent to:

$$\begin{aligned} \eta(\bar{\lambda}_\alpha^2 + \bar{\lambda}_\beta^2) + (\gamma + k)(\bar{\imath}_\alpha^2 + \bar{\imath}_\beta^2) = &-\Delta\omega_r(\bar{\lambda}_\alpha\hat{\lambda}_\beta - \bar{\lambda}_\beta\hat{\lambda}_\alpha) \\ &-\beta \cdot \Delta\omega_r(\hat{\lambda}_\alpha\bar{\imath}_\beta - \hat{\lambda}_\beta\bar{\imath}_\alpha) \end{aligned} \tag{39}$$

The observer with this gain design exhibits the same property as the SM observer shown in Section III: *despite the improper speed, the estimated fluxes are in phase with the real fluxes.* The estimates of the currents also do not converge (the errors are very small but not quite zero).

Fig. 5 shows the estimated fluxes under improper speed. The drive runs in speed control mode at 1000 rpm with 0.3 Nm of load. The speed fed in the observer is 25% smaller than real from 0-0.2 s, equal to the real speed from 0.2-0.4s and 25% bigger from 0.4-0.6 s. The parameters of the induction motor used in simulation are given in Table III.

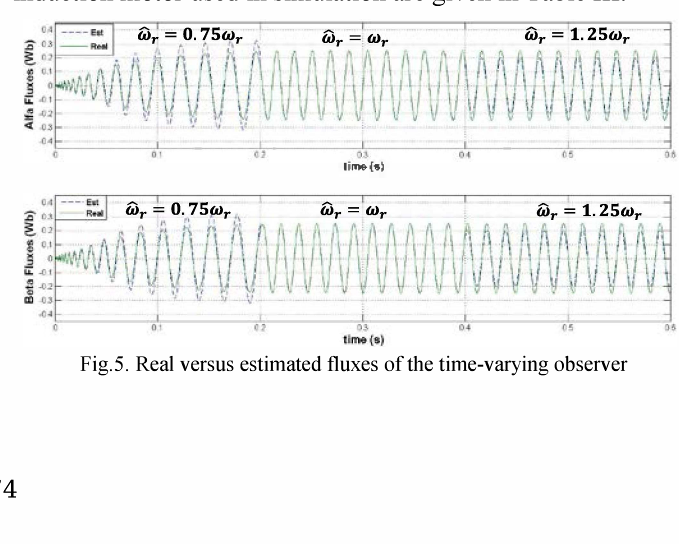

Fig.5. Real versus estimated fluxes of the time-varying observer

V. CONCLUSIONS

The paper presents a family of sensorless observers for the PMSM and for the IM that are constructed using a speed estimate. One observer for the PMSM and two observers for the IM are shown. Estimation is done using the motors' models in the stationary reference frame - the states of the models are sinusoidal quantities. It is assumed that the speed estimate is different from the real speed in a $\pm 25\%$ interval. For the PMSM, it is shown that, with high feedback gain design, the EMFs of the proposed SM observer tend to the real EMFs. For the IM, with special sets of gains, the SM observer and the linear observer do not converge but they produce flux estimates that are in phase with the real fluxes. This property allows accurate estimation of the rotor flux angle based on the tan^{-1} function and direct field orientation.

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TABLE I – PMSM PARAMETERS

Stator resistance	R	2.5 Ω
Synchronous inductance	L	1.8 mH
Rated voltage	V_n	18.2 V
Rated continuous torque	T_C	50 oz-in
Rated speed	n	6000 rpm
Number of poles	P	4

TABLE II. INDUCTION MOTOR SPECIFICATIONS AND PARAMETERS

Rating	*¼ hp*	Pole #	4
Speed	*1732 rpm*	Voltage	220 V
R_s		10.9 Ω	
L_{ls}, L_{lr}		0.015 H	
L_m		0.30 H	
Rr		5.57 Ω	