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Some contributions to estimation for model-free control

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Abstract: This paper describes some contributions to Model Free Control (MFC). For example, its properties about disturbance rejection are analyzed. It is also shown that it is possible to propose new estimation methods for an extended version for MFC using time-varying parameters of an ultra-local model. In particular it is emphasized that these parameters can be estimated using alternative and standard adaptation methods. An application to a thermal process model illustrates one specific point of the new adaptive approach.

Keywords: Parameter estimation, model-free control, adaptive ultra-local model control.

1. INTRODUCTION

During the last decade a new control strategy has been proposed in Fliess and Join (2008) and Fliess et al. (2011). This control strategy is based on two specific features. On the one hand the use of an ultra-local model given by:

$$[y(t)]^{(n)} = F(t) + \alpha(t)u(t), \tag{1}$$

where :

- $y(t)$ and $u(t)$ are the scalar output and input of the system;
- n is the derivation order of the proposed ultra-local model. Usually, and for practical applications, $n = 1$ or 2 is sufficient to describe during a short time period the system behavior;
- $F(t)$ is a time-varying function which transforms as a disturbance term the higher order terms and the complexity of the model;
- $\alpha(t)$ is a time-varying gain. The advice given in papers on model-free control is to consider constant this gain and choose its value such that $y^{(n)}(t)$ and that $\alpha u(t)$ are "of the same magnitude". Let us mention here that this standpoint leads to fruitful practical applications. See for instance the concrete applications listed in Fliess and Join (2013) and the references therein. Nevertheless, one of the objective of our paper is to consider a time dependency of the gain $\alpha(t)$.

On the other hand the model-free control is designed following flatness-based principles (Sira-Ramirez and Agrawal (2004); Fliess et al. (1995); Lévine (2009)). Namely, for a desired reference trajectory on the system output, denoted $r(t)$ and defined on a given time domain $[0, t_f]$, the control is given by:

$$u(t) = \frac{1}{\hat{\alpha}(t)} \left[(r(t))^{(n)} - \hat{F}(t) + R(p)e(t) \right], \tag{2}$$

where:

- $e(t)$ is the tracking error $r(t) - y(t)$;
- $R(p)$ is a rational function in the differential operator p (Rotella and Zambetakis (2013)). Let us mention that the coefficients of $R(p)$ may be time-varying functions;
- $\hat{\alpha}(t)$ and $\hat{F}(t)$ are the instantaneous estimated values for the coefficients $\alpha(t)$ and $F(t)$ of the ultra-local model (1).

When the controller (2) is applied to the system, taking into account the ultra-local model, we obtain:

$$[y(t)]^{(n)} = F(t) + \frac{\alpha(t)}{\hat{\alpha}(t)} \left[(r(t))^{(n)} - \hat{F}(t) + R(p)e(t) \right].$$

Let us introduce the following notations:

- $k(t) = \dfrac{\alpha(t)}{\hat{\alpha}(t)}$ and $\lambda(t) = 1 - k(t)$;
- $g(t) = F(t) - k(t)\hat{F}(t)$;
- $S(p) = p^n + k(t)R(p)$,

then the previous controller (2) yields the following closed loop system:

$$S(p)e(t) = \lambda(t)\,(r(t))^{(n)} - g(t). \tag{3}$$

This relationship leads to the conclusion that estimators and the regulator $R(p)$ must be designed such that $\int_0^{t_f} e(t) < E$ where E is the admissible error in the tracking objective.

As a particular case the adaptive control (2) is defined through the PID structure

$$R(p) = k_P + \frac{k_I}{p} + \frac{k_D p}{1 + \tau_F p}, \tag{4}$$

where k_P, k_I and k_D are the gains of the proportional, integral and derivative actions, respectively, and, τ_F is the time-constant for the filtered derivative. Let us remark

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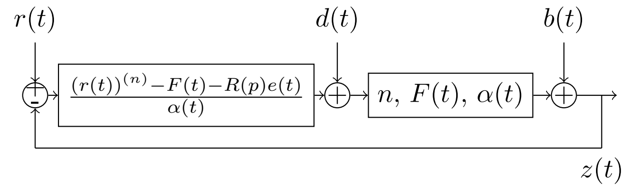

Fig. 1. Closed-loop control.

that the control with (4) is denoted by Fliess and Join (2008); Fliess et al. (2011) as an intelligent-PID controller. We prefer here the adjective adaptive due to the fact that the coefficients of the controller (2) are adapted through the estimation of $\alpha(t)$ and $F(t)$ with a differential equation (see for instance Becedas et al. (2009) or Wu (1999)). Let us mention also that in the case where $\alpha(t)$ is fixed at the beginning to a given value (e.g. $\alpha(t)\equiv$ 0.9 or 1) then the closed-loop controlled becomes:

$$(p^n + R(p))\, e(t) = F(t) - \hat{F}(t), \tag{5}$$

and the adaptation mechanism has a consequence on the right-side term of the error dynamics only. With respect to the proposed model, let us mention that some similar, but different, control strategy has been given in Ilchmann and Ryan (2009). The main difference is that in our approach are estimated the drift term and the input gain. Considering the estimation for $\alpha(t)$ we are led beyond the i-control and we are going towards adaptive control (a-control).

In this paper we restrict the scope to single-input single-output systems. Nevertheless, the case of multivariable systems could be considered with the same guidelines as in Fliess et al. (2006); Fliess and Join (2008). The paper is organized as follows. Section 2 describes some remarks about disturbance rejection properties and the use of prior knowledge in the extended MFC. Section 3 proposes some alternative and standard parameter estimation methods for MFC. In section 4 an application to a thermal process is presented. Concluding remarks are given in section 5.

2. SOME REMARKS ON MODEL-FREE CONTROL

As it can readily be seen we have considered here a little bit more complex model than those used in the usual literature or applications on this topic by introducing the time-varying parameter $\alpha(t)$. Indeed we consider in the following this parameter as a parameter which must be estimated. This point of view leads to a light difference with the implementation of intelligent controllers, namely i-P and i-PI controllers which are using derivatives estimators. Nevertheless, we are in position to perform some remarks which reenforce the advantages for the free-model control. These comments are to be added to the proposed ones in D'Andrea-Novel et al. (2010); Fliess et al. (2011).

2.1 Implicit integral action

Let us suppose that the closed-loop control (2) is performed with exact estimations for $F(t)$ and $\alpha(t)$ (Fig. 1) where b and d are disturbances. With $e(t) = r(t) - z(t)$ and $z(t) = y(t) + b(t)$, we obtain

$$\begin{aligned}[y(t)]^{(n)} &= F(t) + \alpha(t) \times \left\{ \frac{1}{\alpha(t)} \left[(r(t))^{(n)} \right.\right. \\ &\qquad \left.\left. - F(t) + R(p)e(t) \right] + d(t) \right\}, \\ &= (r(t))^{(n)} + R(p)e(t) + \alpha(t)d(t),\end{aligned}$$

thus we get

$$(p^n + R(p))\, e(t) = -p^n b(t) - \alpha(t)d(t). \tag{6}$$

When the closed-loop system is stable, the first consequence of this result is the reject of output perturbations $b(t) = t^i$ for $i = 0$ to $n-1$. In the case where integral action are included in $R(p)$, namely $R(p) = \dfrac{R^*(p)}{p^m}$ with $m > 0$, we obtain

$$\left(p^{n+m} + R^*(p)\right) e(t) = -p^{n+m} b(t) - p^m \left(\alpha(t)d(t)\right). \tag{7}$$

Obviously, when $\alpha(t)d(t) = t^j$, for $j = 0$ to $m-1$, the disturbance rejection is ensured.

2.2 A priori knowledge

The main advantage of the model-free control is to generate a control design without an a priori model for the system. In this framework, the reference trajectory to track is fixed at the beginning. Nevertheless, we are going to see that taking into account a dynamic model for the system can improve the convergence properties for the estimators or to see that the parameter α cannot be considered as a constant. Let us suppose that the system is described by the nonlinear input-output model

$$[y(t)]^{(\nu)} = \Phi(y(t), u(t)), \tag{8}$$

where $\Phi(y(t), u(t))$ is a function which depends on input-output variables and their derivatives up to the $(\nu-1)$-th order. Let us mention here that, in practice, the order ν of this model is greater or equal to n. Firstly, from the flatness standpoint, the first task is to generate the reference to track, $r(t)$ such that $r^{(n)}(t) = \Phi(r(t), u_r(t))$ where $u_r(t)$ stands for the control which ensures that $y(t) = r(t)$. Secondly, from the model (8) we can build

$$\begin{aligned}&[y(t)]^{(n)} = \Phi(y(t), u(t)) + \\ &\quad [y(t)]^{(n)} - [y(t)]^{(\nu)} + \alpha(t)u(t) - \alpha(t)u(t),\end{aligned}$$

where n is an arbitrary integer and $\alpha(t)$ an arbitrary possibly time-varying gain. Obviously, $\alpha(t)$ can be fixed at the beginning to 1, but the identification of $F(t)$ in the ultra-local model (1) leads to

$$F(t) = \Phi(y(t), u(t)) + [y(t)]^{(n)} - [y(t)]^{(\nu)} - \alpha(t)u(t).$$

We deduce that

$$\alpha(t) = \frac{\Phi(y(t), u(t)) + [y(t)]^{(n)} - [y(t)]^{(\nu)} - F(t)}{u(t)}. \tag{9}$$

As a consequence, an estimator for $F(t)$ may lead to a possibly time-varying gain $\alpha(t)$. Let us mention here that the construction of $F(t)$ and $\alpha(t)$ is not unique for a given model. This fact will be proved in the parameter estimation section 3.

In order to be more precise consider the model of an active magnetic bearing used in De Miras et al. (2013). This model can be written as

$$\ddot{y}(t) = D(t) + \frac{\lambda \text{sign}(i(t)) i^2(t)}{2m\left(g - \text{sign}(i(t))y(t)\right)}, \tag{10}$$

where the output is the displacement $y(t)$, the control is the current $i(t)$ in the coil, $D(t)$ is a load disturbance, g the nominal gap between the coil and the shaft, m is the shaft mass and λ an electromagnetic constant parameter. With $n = 2$, two models such as (1) can be deduced. On the one hand

$$\ddot{y}(t) = D(t) + \frac{\lambda \mathrm{sign}(i(t)) i^2(t)}{2m\,(g - \mathrm{sign}(i(t)) y(t))} - i(t) + i(t),$$

which leads to the identification

$$F(t) = D(t) + \frac{\lambda \mathrm{sign}(i(t)) i^2(t)}{2m\,(g - \mathrm{sign}(i(t)) y(t))} - i(t),$$
$$\alpha(t) = 1.$$

It is the model used in De Miras et al. (2013) which implies an estimation of $F(t)$ only. On the other hand, another point of view, among others, can be used here. Namely, we can write (10) as

$$\ddot{y}(t) = D(t) + \left[\frac{\lambda \mathrm{sign}(i(t)) i(t)}{2m\,(g - \mathrm{sign}(i(t)) y(t))}\right] i(t),$$

which leads to the identification

$$F(t) = D(t),$$
$$\alpha(t) = \frac{\lambda \mathrm{sign}(i(t)) i(t)}{2m\,(g - \mathrm{sign}(i(t)) y(t))},$$

where the gain $\alpha(t)$ is a time-varying gain. To our point of view, the previous remark leads us to consider an estimator for $\alpha(t)$ too.

3. PARAMETER ESTIMATION

The first three subsections of this part are independent. The first one points out the non uniqueness of the estimated model. The second one deals with some obtained tracking property. The third subsection proposes a new adaptation mechanism for the drift term and the input gain of the ultra-local model. In the fourth subsection an improvement is indicated for the case of a fast time-varying input gain.

In the following we take $n = 1$ in the ultra-local model (1).

3.1 Generalized inverse based estimator

A possibility for the estimation is to consider the ultra-local model (1) as the regressor

$$\dot{y}(t) = U(t)\theta(t), \tag{11}$$

where $U(t) = [\,1\ u(t)\,]$ and $\theta(t)^T = [\,F(t)\ \alpha(t)\,]$. Usually, the term $\alpha(t)$ is fixed at the outset or, in some cases as in (Gédouin et al. (2011)) is a nonlinear known function. The basic principle is to estimate the drift term $F(t)$. The purpose of this section is to point out that, on the one hand, $\alpha(t)$ can be estimated too, and, on the other hand, this estimation introduces an arbitrary parameter which allows a significance of the parameter $\alpha(t)$. Indeed, the estimation $\hat{\theta}(t)$ of $\theta(t)$ can be thought as the solution of the undetermined linear equation (11). As, for every t, $\mathrm{rank}\,(U(t)) = 1$, this system is consistent (or compatible) which leads to

$$\hat{\theta}(t) = U(t)^{\{1\}}\dot{y}(t) + \left(I_2 - U(t)^{\{1\}}U(t)\right) z(t), \tag{12}$$

where :

- $U(t)^{\{1\}}$ is an arbitrary generalized inverse of $U(t)$, namely it fulfills the relationship (Ben-Israel and Greville (2003))
$$U(t)U(t)^{\{1\}}U(t) = U(t);$$
- $z(t)$ is an arbitrary 2-dimensional vector. Because $\mathrm{rank}\,(U(t)) = 1$, we get
$$\dim\left(\ker\{U(t)\}\right) = \mathrm{rank}\left(I_2 - U(t)^{\{1\}}U(t)\right) = 1.$$

Consequently, if we denote $v(t)$ a vector such that $\mathrm{span}\,\{v(t)\} = \ker\{U(t)\}$, the arbitrary part of (12) can be reduced to

$$\left(I_2 - U(t)^{\{1\}}U(t)\right) z(t) = \lambda(t)v(t),$$

where $\lambda(t)$ is the arbitrary scalar parameter. This parameter allows to span the solution set of (11) which can be written as

$$\hat{\theta}(t) = U(t)^{\{1\}}\dot{y}(t) + \lambda(t)v(t),$$

The estimations of $F(t)$ and $\alpha(t)$ are depending of this parameter which can be tuned with respect to some supplementary constraints.

Let us consider now a particular, but important, case. Let us chose the following generalized inverse for $U(t)$

$$U(t)^{\{1\}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

which leads to

$$I_2 - U(t)^{\{1\}}U(t) = \begin{bmatrix} 0 & -u(t) \\ 0 & 1 \end{bmatrix}.$$

This particular case allows to consider $\lambda(t)$ as the second component of $z(t)$ and, the solution set can be read as :

$$\hat{F}(t) = \dot{y}(t) - \lambda(t)u(t),$$
$$\hat{\alpha}(t) = \lambda(t),$$

where the meaning of the parameter appears clearly. As a first remark the case of constant $\lambda(t)$ has been considered in Fliess and Join (2013), and secondly, we get

$$\hat{F}(t) = \dot{y}(t) - \hat{\alpha}(t)u(t). \tag{13}$$

As another example let us take $U(t)^{\{1\}} = U(t)^+$ the pseudo-inverse of $U(t)$ (Ben-Israel and Greville (2003)). Namely,

$$U(t)^+ = \frac{1}{1 + u(t)^2}\begin{bmatrix} 1 \\ u(t) \end{bmatrix},$$

which leads to

$$I_2 - U(t)^+U(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{1}{1 + u(t)^2}\begin{bmatrix} 1 & u(t) \\ u(t) & u(t)^2 \end{bmatrix} = \frac{1}{1 + u(t)^2}\begin{bmatrix} u(t) \\ -1 \end{bmatrix}[\,u(t)\ -1\,].$$

With $\mu(t) = [\,u(t)\ -1\,]\,z(t)$, the estimator can be read as

$$\begin{aligned}\hat{\theta}(t) &= U(t)^+\dot{y}(t) + \mu(t)\frac{1}{1 + u(t)^2}\begin{bmatrix} u(t) \\ -1 \end{bmatrix}, \\ &= \frac{1}{1 + u(t)^2}\left\{\dot{y}(t)\begin{bmatrix} 1 \\ u(t) \end{bmatrix} + \mu(t)\begin{bmatrix} u(t) \\ -1 \end{bmatrix}\right\}, \\ &= \frac{1}{1 + u(t)^2}\begin{bmatrix} \dot{y}(t) + \mu(t)u(t) \\ u(t)\dot{y}(t) - \mu(t) \end{bmatrix},\end{aligned}$$

which leads to

$$\hat{F}(t) = \frac{\dot{y}(t) + \mu(t)u(t)}{1 + u(t)^2},$$
$$\hat{\alpha}(t) = \frac{u(t)\dot{y}(t) - \mu(t)}{1 + u(t)^2}.$$

Let us remark that the elimination of $\mu(t)$ leads to the same relationship, namely $\hat{F}(t) = \dot{y}(t) - \hat{\alpha}(t)u(t)$. Nevertheless, the existence of the degree of freedom in the estimation induces that no a priori exact values for $F(t)$ and $\alpha(t)$ exist independently of outer constraints.

For instance some outer constraints on the control (e.g. saturations,...) need $\hat{\alpha}(t) > \alpha_{\min}$ to ensure the implementation. With the parameters $\lambda(t)$ or $\mu(t)$ this point can be obviously satisfied.

The formula (13) indicates clearly that two estimation strategies can be used. On the one hand, from the algebraic method presented in Fliess and Join (2008); Fliess et al. (2011); Mboup et al. (2009) we can obtain the estimation $\widehat{\dot{y}(t)}$ of the first derivative of $y(t)$. In this framework $F(t)$ and $\alpha(t)$ has to be performed in two successive steps. An estimate $\hat{\alpha}(t)$ of $\alpha(t)$ leads to the estimation $\hat{F}(t) = \hat{\dot{y}}(t) - \hat{\alpha}(t)u(t)$. The details can be found in the cited works. Let us mention here that the algebraic methods can be used or can be extended to the estimation of a gain α which can be considered as constant during a short period of time. For shortness sake the developments are not given in the present paper but will be the subject of future works. Indeed, we are interested in an adaptive estimation strategy to get $F(t)$ and $\alpha(t)$.

3.2 Tracking for derivative

With the control
$$u(t) = \frac{\dot{r}(t) - \hat{F}(t) + k\,(r(t) - y(t))}{\hat{\alpha}(t)},$$
and the estimator $\hat{F}(t) = \hat{\dot{y}}(t) - \hat{\alpha}(t)u(t)$, where $\hat{\dot{y}}(t)$ is an estimator for the derivative of the output. We are led to write
$$u(t) = \frac{\dot{r}(t) - \hat{\dot{y}}(t) + \hat{\alpha}(t)u(t) + k\,(r(t) - y(t))}{\hat{\alpha}(t)}.$$
The elimination of $u(t)$ leads to
$$\dot{r}(t) - \hat{\dot{y}}(t) + k\,(r(t) - y(t)) = 0.$$
With the error term $e(t) = r(t) - y(t)$ the previous equation can be read as
$$(k + p)e(t) = \hat{\dot{y}}(t) - \dot{y}(t),$$
where p stands for the derivative operator.

Let us suppose that $k > 0$ and $\lim_{t\to\infty} e(t) = 0$, namely, the regulator ensures perfect tracking, then we have proved that the derivative estimator converges to the real derivative of the output signal. In other words, the estimation error and the tracking error are related through a filtering as
$$\hat{\dot{y}}(t) - \dot{y}(t) = \frac{1}{k + p} e(t).$$

3.3 Adaptation mechanism

Let us suppose here that α and F are unknown constants such that:
$$\dot{y}(t) = F + \alpha u(t) = \theta^T \phi, \tag{14}$$
where $\theta^T = [F\ \alpha]$ and $\phi^T = [1\ u]$.

Due to the fact that the system output derivative is not measurable, we proceed by filtering as in the parametric estimation of continuous-time systems (Baysse et al. (2011)). Namely, considering the filtered signals $y_f(t) = G(p)y(t)$ and $u_f(t) = G(p)u(t)$, where $G(p)$ is the transfer of the filter, yields the filtered version of the ultra-local model. Then, from (14) it can be written that

$$\dot{y}_f(t) = [\,F\ \ \alpha\,] \begin{bmatrix} 1_f(t) \\ u_f(t) \end{bmatrix} = \theta^T \phi_f(t),$$

where $1_f(t)$ stands for the filtered unit step $G(p)H(t)$ where $H(t)$ is the Heaviside step function and $\phi_f^T(t) = [\,1_f(t)\ u_f(t)\,]$. When the filter is supposed to be strictly proper $\dot{y}_f(t)$ is obtained with the output filtering
$$\dot{y}_f(t) = pG(p)y(t).$$

Then the filtered estimated ultra-local model is proposed as

$$\dot{y}_f(t) = \hat{\theta}^T \phi_f(t) = [\,\hat{F}\ \ \hat{\alpha}\,] \begin{bmatrix} 1_f(t) \\ u_f(t) \end{bmatrix}.$$

Because this estimator is to be used in an adaptive controller, the parameters can be obtained from the minimization of the following normalized quadratic criterion

$$J = \min_{\hat{\theta}} \left[\frac{1}{2} \frac{\left(\dot{y}_f - \hat{\theta}^T \phi_f\right)^2}{m^2} \right],$$

where m^2 is chosen to satisfy that $\frac{\phi_f}{m} \in \mathcal{L}_\infty$ as proposed in Ioannou and Sun (1996). For example we can use:

$$m^2 = 1 + \gamma \phi_f^T \phi_f,$$

where $\gamma > 0$. The minimization can be then obtained using the gradient algorithm

$$\epsilon = \frac{(\dot{y}_f - \hat{\theta}^T \phi_f)}{1 + \gamma \phi_f^T \phi_f} \text{ and } \dot{\hat{\theta}} = -\Gamma \nabla J(\hat{\theta}) = \Gamma \phi_f \epsilon.$$

Where the adaptive gain Γ is a symmetric definite positive matrix. This algorithm have good tracking alertness properties. In practice, first- or second-order filters $G(p)$ with unit static gain provide good results. For example it can be used:

$$G(p) = \frac{c}{p + c}; \qquad G(p) = \frac{c^2}{(p + c)^2};$$

Of course, some knowledge about the system bandwidth frequency is useful.

The convergence and stability properties of this new adaptive controller are under study and will be the subject of a future paper. In particular it could be necessary to use adaptive gradient with projection laws as discussed in Ioannou and Sun (1996) to obtain a more robust design.

3.4 Estimation for a time-varying $\alpha(t)$

In the case of time-varying functions $F(t)$ and $\alpha(t)$ the previous procedure can be also used. In order to give a simplified description we consider here a first-order filter $G(p) = \frac{c}{c+p}$. The objective here is to determine a filtered ultra-local model which takes time into account in the functions to estimate. From the filtered ultra-local model we have

$$\dot{y}_f(t) = FG(p)H(t) + G(p)\left(\alpha(t)u(t)\right).$$

As we can formally write

$$G(p) = \frac{1}{1+\tau p} = 1 - \tau p + (\tau p)^2 - \cdots$$

with $\tau = \frac{1}{c}$, we obtain

$$\begin{aligned}\dot{y}_f(t) &= FG(p)H(t) + \alpha(t)u(t) - \tau\left(\dot{\alpha}(t)u(t) + \alpha(t)\dot{u}(t)\right) + \\ &\quad \tau^2\left(\ddot{\alpha}(t)u(t) + 2\dot{\alpha}(t)\dot{u}(t) + \alpha(t)\ddot{u}(t)\right) - \cdots \\ &= FG(p)H(t) + \alpha(t)\left(1 - \tau p + (\tau p)^2 - \cdots\right)u(t) + \\ &\quad \dot{\alpha}(t)\left(-\tau + 2\tau^2 p - 3\tau^3 p^2 + \cdots\right)u(t) + \cdots\end{aligned}$$

For simplicity sake, let us consider we look for estimations for $\alpha(t)$ and $\dot{\alpha}(t)$. Thus taking into account the first terms in the previous series leads to the ultra-local (truncated) filtered model

$$\dot{y}_f(t) = FG(p)H(t) + \alpha(t)u_f(t) + \dot{\alpha}(t)u_{ff}(t),$$

where $u_f(t) = G(p)u(t)$ and

$$\begin{aligned}u_{ff}(t) &= \left(-\tau + 2\tau^2 p - 3\tau^3 p^2 + \cdots\right)u(t), \\ &= \left[\frac{\mathrm{d}}{\mathrm{d}p}\left\{\frac{1}{1+\tau p}\right\}\right]u(t) = -\frac{\tau}{(1+\tau p)^2}u(t).\end{aligned}$$

In this case the regressor can be written (using again that $1_f(t)$ stands for the filtered unit step):

$$\dot{y}_f(t) = \hat{\Theta}^T\Phi_f(t) = \left[\begin{array}{ccc}\hat{F} & \hat{\alpha} & \hat{\dot{\alpha}}\end{array}\right]\left[\begin{array}{c}1_f(t) \\ u_f(t) \\ u_{ff}(t)\end{array}\right],$$

and, the previous adaptation strategy can be used.

4. EXAMPLE

The simplified model of the thermal process trainer Feedback PT320 is given by

$$y(t) = \frac{Ke^{-T_d p}u(t)}{(1+\tau_1 p)(1+\tau_2 p)},$$

where the parameters K, T_d,τ_1 and τ_2 can vary because they depend on the operating point.

For a nominal point with medium air flow and temperature set-point of 40°C (in the figure 2 the corresponding sensor voltage value is around 5 volts) the following values can be obtained (Baysse et al. (2011), Baysse (2010))

$$y(t) = \frac{0.8e^{-0.22p}u(t)}{(1+0.45p)(1+0.15p)}.$$

The simulations results are obtained with an a-P controller (with $k_P = 3$) using Matlab-Simulink. A second-order estimation filter $G(p)$ with $c = 10$ is used; $\gamma = 1$ and $\Gamma = 500I$, where I is the identity matrix. The initial value for the $\hat{\theta}$ vector is $\hat{\theta}(0) = \begin{bmatrix}1 \\ 1\end{bmatrix}$.

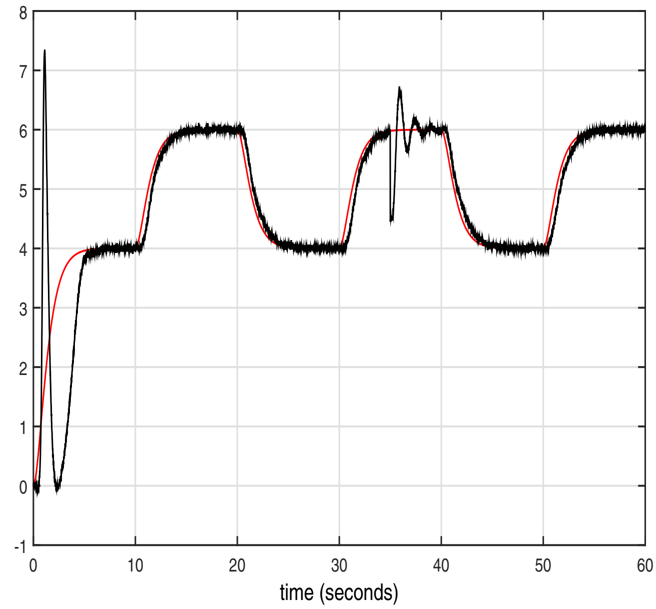

Fig. 2. Tracking with parameters change (black: controlled output, red: reference, 4 stands for 35°C and 6 for 45°C).

In order to evaluate the adaptation properties of the proposed estimation strategy the parameters of the process to be controlled are changed at time $t = 35$ seconds to:

$$y(t) = \frac{0.6e^{-0.18p}}{(1+0.35p)(1+0.15p)}u(t),$$

which means a change of the operating point (e.g. the air flow has been increased). Simulation results are shown in the figure 2 for the behavior of the process (where it can be clearly seen the adaptation transients starting at $t = 0$ and $t = 35$ seconds and showing the good tracking alertness properties of the normalized gradient algorithm) and, in the figure 3 the time evolution of the parameters of the ultra-local model. Notice that the estimated parameters are not constant and the adaptation mechanism follows the process variations. In our simulation example, a control with a constant value for α could be used, nevertheless, our adaptation algorithm gives an admissible set of estimated varying parameters. Other simulations (not shown here due to space limitations) permit to see that the obtained regulation performances with the a-P controller have equivalent settling times to those obtained with a fixed PID tuned with the Ziegler-Nichols frequency response model as described in Aström and Häglund (2006).

5. CONCLUSION

This paper proposes some contributions for the analysis and for the implementation of adaptive ultra-local model control. In particular it was proposed a new extended version of MFC using time-varying parameters. Indeed the simplest version considers that the α parameter is constant. Also alternative adaptive versions giving estimates

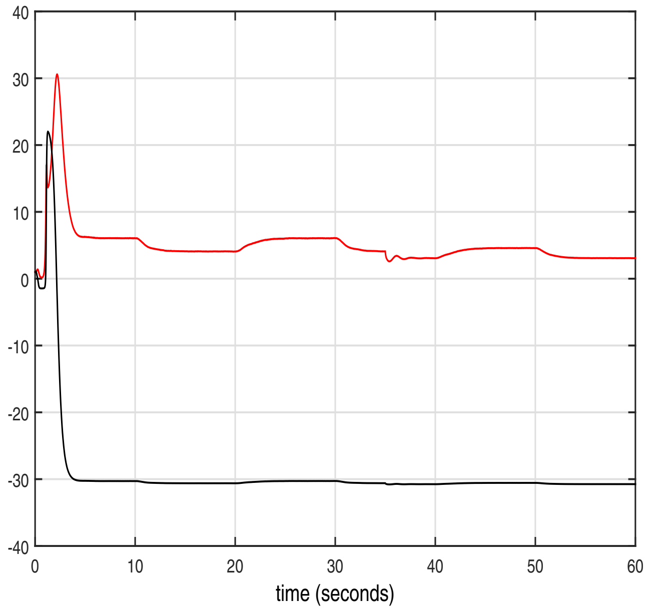

Fig. 3. Estimation with parameters change (black: $\hat{F}$, red: $\hat{\alpha}$).

for the parameters of the ultra-local model were proposed. Simulations were provided which point out the efficiency of the reference tracking and the good convergence properties. In the paper some other research lines have been proposed and will be the subject of next papers.

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