FINC2012
CORPORATE FINANCE 2
Semester 2 2024
Lecture

OPTIONS – II: Valuing Options

BMAE Ch.22

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OPTION VALUATION MODELS

- Standard discounted cash flow (DCF) method will not work for option valuation. More rigorous option valuation techniques needed.
- The first method is to use an option equivalents approach (also called the *replicating portfolio*), which requires setting up an investment strategy with the stock and then borrowing or lending so that the payoffs are the same as that of the option.
- The second is the *risk-neutral method*.
- Both of these are simplified versions of the **binomial method** that uses discrete-time approach

SIMPLE OPTION-VALUATION MODEL: Replicating Portfolio

- Valuation approach → Create *Option Equivalent*
- Combining ordinary share investment and borrowing = option equivalent or "replicating portfolio".
- Net cost of buying the option equivalent ("replicating portfolio") = value of the option.

Example of Option Equivalent: Replicating Portfolio

- Suppose in Jan 2020 you purchase a six-month **call** options on Amazon shares with an exercise price of $1,830. The price of Amazon shares is also $1830.
- The short-term, risk-free interest rate was r = 2% for six months, or just over 4% p.a.
- Amazon stock can do only two things over the option's six-month life. The price could rise by 20% to 1.20 × $1,830 = $2,196. Alternatively, it could fall by the same proportion to $1,830 ÷ 1.20 = $1,525. The upward move is sometimes written as $u = 1.2$, and the downward move as $d = 1/u = 1/1.2 = 0.833$.

Possible payoffs of the **call** option:

	Stock Price = $1,525	**Stock Price = $2,196**
1 call option	$0	$366
	Max[0, (1,525 – 1,830)] = 0	Max[0, (2,196 – 1,830)] = 366

Example of Option Equivalent: Replicating Portfolio

- Now compare these payoffs with what you would get if you bought 0.54545 Amazon share and borrowed the present value of $831.82 from the bank.

	Stock Price = $1,525	Stock Price = $2,196
0.54545 share	$831.82	$1,197.82
Repayment of loan + interest	−831.82	−831.82
Total payoff	$ 0	$366.00

Notice that the payoffs from this levered investment in the stock are identical to the payoffs from the call option. Therefore, the law of one price tells us that both investments must have the same value today.

Value of call = value of 0.54545 shares − value of bank loan

$$= 0.54545 \times \$1{,}830 - \$831.82 / 1.02 = \$182.67$$

Presto! You've valued a call option. How did we arrive at this?

Example of Call Option Equivalent: Replicating Portfolio

- When you buy a call option, you're effectively taking a long position in the stock but putting up less of your own money than if you had bought the stock directly $\rightarrow$ a *levered investment* in the stock.
- ***Value of option = Value of replicating portfolio (option equivalent)***
- ***Replicating portfolio***: borrowing money & buying stock in such a way that we exactly replicate the payoffs from a call option.
- Can create an *option equivalent* by replicating a strategy of:
  (i) buying or selling "***delta***" shares and
  (ii) borrowing or lending the balance.

Example of Option Equivalent: Replicating Portfolio

- The number of shares needed to replicate one call is called the ***hedge ratio*** or ***option delta*** (δ).
- **Option Delta** (δ): the spread of possible option prices divided by the spread of possible share prices.

$$\textbf{Option delta}(\delta) = \frac{\textbf{spread of possible option prices}}{\textbf{spread of possible share prices}}$$

[Option Delta x spread of possible share prices] = spread of possible option prices

Example of Call Option Equivalent: Replicating Portfolio

- You can create a *call option equivalent* by :
  - 1. buying ***delta*** (δ) shares and
  - 2. *borrowing* to help finance that purchase
- Implementation
  - Pick delta (hedge ratio) and the amount borrowed to replicate the possible option payoffs.
  - You want to make your delta share position give you a spread of payoffs equal to the spread of option payoffs.
  - Delta x Share Price Spread = Option Payoff Spread
- *Note: When you buy a call, you are taking a position in the stock but putting up less of your own money than if you had bought the stock directly. Since this is a levered position in a share, the option is riskier than a share - it has a higher beta.*

Call Option Equivalent: Replicating Portfolio

- If the stock price S can rise by a factor of u to S_u or fall by a factor $d = 1/u$ to S_d, then the value of a call option is equal to the value of a portfolio composed of an investment of δS in the stock, and debt of B, where the option delta(δ) is given by:

$$\textbf{Option delta } (\delta) = \frac{\textbf{spread of possible option prices}}{\textbf{spread of possible share prices}}$$

$$= \frac{C_{\mathrm{u}} - C_{\mathrm{d}}}{S_{\mathrm{u}} - S_{\mathrm{d}}}$$

where C_u and C_d are the values of the call option in the “up” and “down” states

- The debt, B, is given by: $$B = \frac{1}{(1+r)} \frac{uC_d - dC_{\mathrm{u}}}{u-d}$$

Example of Call Option Equivalent: Replicating Portfolio

$u = 1.2$ $d = 0.833$
$C_u = \$366$ $C_d = 0$
$S_u = \$2,196$ $S_d = \$1,525$
$r = 0.02$

$$Option\ delta(\delta) = \frac{spread\ of\ possible\ option\ prices}{spread\ of\ possible\ share\ prices}$$

$$\delta = \frac{C_u - C_d}{S_u - S_d} = \frac{\$366 - \$0}{\$2,196 - \$1,525} = 0.54545$$

Therefore, you need to buy 0.54545 (option delta) of one share in Amazon and borrow \$815.51:

$$B = \frac{1}{(1+0.02)} \frac{1.2 \text{ x } 0 - 0.833 \text{ x } 366}{1.2 - 0.833} = \frac{-831.82}{(1+0.02)} = -815.51$$

Notice that *B* is negative. Borrowing is equivalent to a negative investment in the risk-free asset.

Value of call = $\delta S + B$ = 0.54545 × \$1,830 – \$815.51 = **\$182.67**

Example of Call Option Equivalent: Replicating Portfolio

Amount to borrow

= Present value of the difference between the option payoffs and the payoffs from holding 0. 54545 of a share of Amazon.

$\Rightarrow$ Borrow the present value of $831.82 = $815.51.

(0. 54545) x ($2,196)
= $1,197.82

(0. 54545) x ($1,525)
= $831.82

	Stock Price = $1,525	Stock Price = $2,196
0.54545 share	$831.82	$1,197.82
Option payoff	$0	$366
Repayment of loan + interest	$ 831.82	$ 831.82

Example of Call Option Equivalent: Replicating Portfolio

The Amazon call option is equal to leveraged position in 0.54545 Amazon shares.

Call option payoffs would be replicated if you bought 0. 54545 Amazon shares and borrowed the present value of $831.82 from the bank.

	Stock Price = $1,525	Stock Price = $2,196
0.54545 share	$831.82	$1,197.82
Repayment of loan + interest	−831.82	−831.82
Total payoff	$ 0	$366.00

- The replicating portfolio consisted of an investment of just under $1,000 (0.54545 x $1,830 = $998.17) in the share, largely financed by a loan of about $815.
- Call option on a share equivalent to financing purchase of share with borrowed money. That is why call option is always riskier than the share and why we say that call options create leverage.

Example of Call Option Equivalent: Risk-Neutral Valuation

- Why should the Amazon call option sell for $182.67?
- If call option price > $182.67, you could make a certain profit by buying 0.54545 shares of stock, selling a call option, and borrowing the present value of **$831.82**.
- Similarly, if the call option price < $182.67, you could make an equally certain profit by selling 0.54545 shares, buying a call, and lending the balance.
- In either case there would be a risk-less arbitrage opportunity!
- Such a proposition that the option price had to be $182.67 or there would be an arbitrage opportunity, does not depend on the need to know anything about investor attitudes to risk.
- This suggests an alternative way to value the option.

Example of Call Option Equivalent: Risk-Neutral Valuation

- Risk-Neutral Valuation → alternative way to value the option.
- *Pretend* that all investors are *indifferent* about risk, work out the expected future value of the option in such a "*risk-neutral*" world, and discount it back at the risk-free interest rate to determine option's current value.
- If investors are indifferent to risk, the *expected return* on the stock must be equal to the *risk-free rate of interest*:

$\Rightarrow$ Expected returns on Amazon stock = 2.0% per six months

- Amazon stock can either rise by 20% to $2,196 or fall by 16.667% to $1,525.

$$\text{Expected Return} = [\text{prob of rise} \times 20\%] + [(1 - \text{prob of rise})] \times (-16.667\%)]$$
$$\text{Expected Return} = 2.0\% = \mathbf{6{-}month}\ \text{risk}{-}\text{free interest rate}$$
$$\text{Probability of rise (Pu)} = 0.50909 \text{ or } 50.909\%$$

[p* (+20%)] + [(1 - p*)(-16.667%)] = 2.0% ⇒ p* = 0.50909 or 50.909%

Example of Call Option Equivalent: Risk-Neutral Valuation

- This is not the true probability that Amazon stock will rise. Since investors dislike risk, they will almost surely require a higher expected return than the risk-free interest rate from Amazon stock. Therefore, the true probability is > 0.50909 or 50.909%

- If investors are indifferent to risk, the expected return on the stock must be equal to the risk-free rate of interest. The general formula for calculating the risk-neutral probability of a rise in value is:

$$p* = \frac{1 + \textit{interest rate} - \textit{downside change}}{\textit{upside change} - \textit{downside change}} = \frac{(1+r)-d}{u-d}$$

- In the case of Amazon:

$$p* = \frac{(1+r)-d}{u-d} = \frac{(1.02)-(0.8333)}{(1.20)-(0.8333)} = 0.50909$$

Example of Call Option Equivalent: Risk-Neutral Valuation

- Value of Amazon 6-mth Call Option, X =$1,830

- If stock price rises, call option will be worth $366; if it falls, the call will be worth nothing. Therefore, if investors are risk-neutral, the *expected future value* of the call option is:

  = [Probability of rise] x 366] + [(1 - Probability of rise) x 0]
  = p* x 366 + (1 – p*) x 0
  = (0.50909) x 366 + (0 .49091) x 0 = $186.33

- *Current* value of the call is given by:

$$\text{Present value of the call option} = \frac{\text{Expected future value}}{1 + \text{interest rate}} = \frac{\$186.33}{1.02} = \$182.67$$

Exactly the same answer that we got earlier using option equivalent approach!

Valuation of Amazon Put Option

- Valuation of 6-mth Amazon Put Option; X = $1,830

uS = $2,196 and dS = $1,525

Case 1	Case 2
Stock price falls to $1,525	**Stock price rises to $2,196**
Option value = $305	**Option value = $0**

$P_d = Max[(X – dS), 0]$
$P_d = Max[(1,830 – 1,525), 0]$
$P_d = Max[(305), 0] = 305$

$P_u = Max[(X – uS), 0]$
$P_u = Max[(1,830 – 2,196), 0]$
$P_u = Max[(– 366), 0] = 0$

Valuation of Amazon Put Option

- Option delta for this Amazon put option is equal to:

$$\delta = \frac{\text{spread of possible option prices}}{\text{spread of possible share prices}} = \frac{P_u - P_d}{S_u - S_d}$$

$$= \frac{0 - 305}{2{,}196 - 1{,}525} = \frac{-305}{671}$$

$$= -0.45455$$

Notice that the *delta of a put option* is always negative; i.e. you need to ***sell*** delta shares of stock to replicate the put.

The delta of a put option is always equal to the delta of a call option with the same exercise price minus 1;
in this example, delta of put = 0.54545 − 1 = −0.45455

Valuing the Amazon Put Option

- Replicate option payoffs: Sell (go short) 0.45455 Amazon shares and lend the present value of \$998.18 @ 2.0% for 6 months.

	Stock Price = \$1,525	Stock Price = \$2,196
Sale of 0.45455 share	−\$693.18	−\$998.18
Repayment of loan + interest	+ 998.18	+ 998.18
Total payoff	\$305.00	\$ 0

$$\text{Value of put} = -(0.45455)\text{ shares} + \text{value of bank loan}$$

$$= -(0.45455) \times \$1{,}830 + \$998.18 / 1.02 = \$146.79$$

Risk-Neutral Put Valuation

- We already know that the probability of a rise in the stock price is 0.50909
  $\Rightarrow$ expected value of the put option in a risk-neutral world is:

$$\text{Expected value of the put option} = \left(p^{*} \times 0\right) + \left(\left[1 - p^{*}\right] \times \$305\right)$$

$$= \left(0.50909 \times \$0\right) + \left(0.49091 \times \$305\right)$$

$$= \$149.73$$

$$\text{Present value of the put option} = \frac{\$149.73}{1.02} = \$146.79$$

Relationship between Call and Put Prices

- For European options there is a simple relationship between the values of the call and the put options.

Value of put = value of call + PV(X) present value of exercise price – share price

- Since value of the Amazon call option was already calculated, we could also have used this relationship to find the value of the put:

Value of put = 182.67 + (1,830/1. 02) − 1,830 = **$146.79**

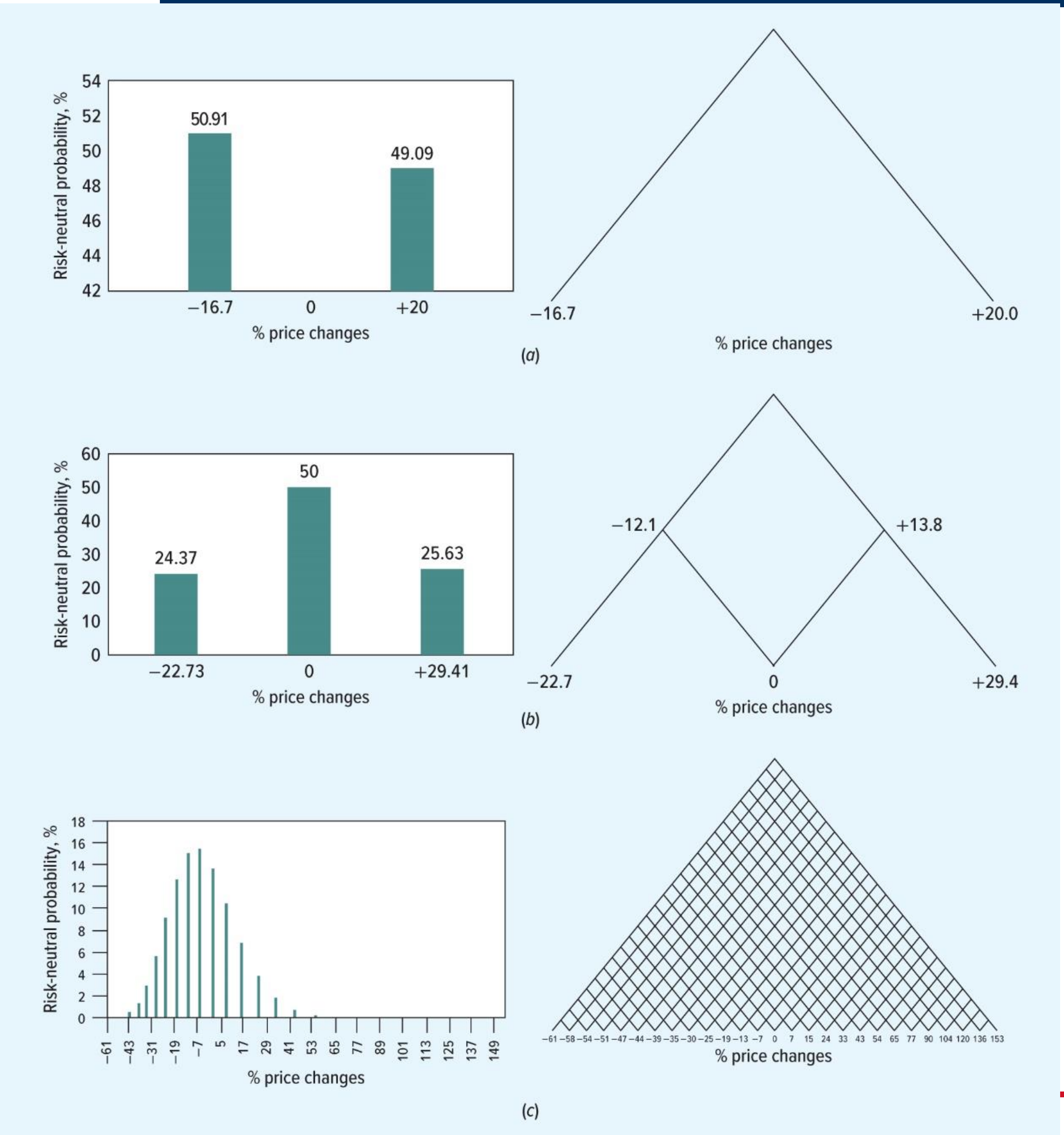
- Everything checks out!

The Binomial Method for Valuing Options

Binomial method:

- The method starts by reducing the possible changes in the next period's stock price to two, an "up" move and a "down" move.
- Assumption that there are just 2 possible prices for Amazon stock at the end of six months is clearly fanciful.
- Could make it more realistic by assuming that there are 2 possible price changes in each three-month period $\Rightarrow$ a wider variety of six-month prices. Could go on to take shorter & shorter intervals, with each interval showing two possible changes in Amazon's stock price and giving an even wider selection of prices by month 6.
- Figure 22.1 illustrate this process.

The Binomial Method for Valuing Options



This figure shows the possible six-month price changes for Amazon stock assuming that the stock makes a single up or down move each six months (panel a); two moves, one every three months (panel b); or 26 moves, one every week (panel c). Beside each tree is a histogram of the possible six-month price changes, assuming investors are risk-neutral.

One could continue with shorter and shorter intervals, until stock price is changing continuously and there is continuum of possible future stock prices.

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Example: The Two-Step Binomial Method

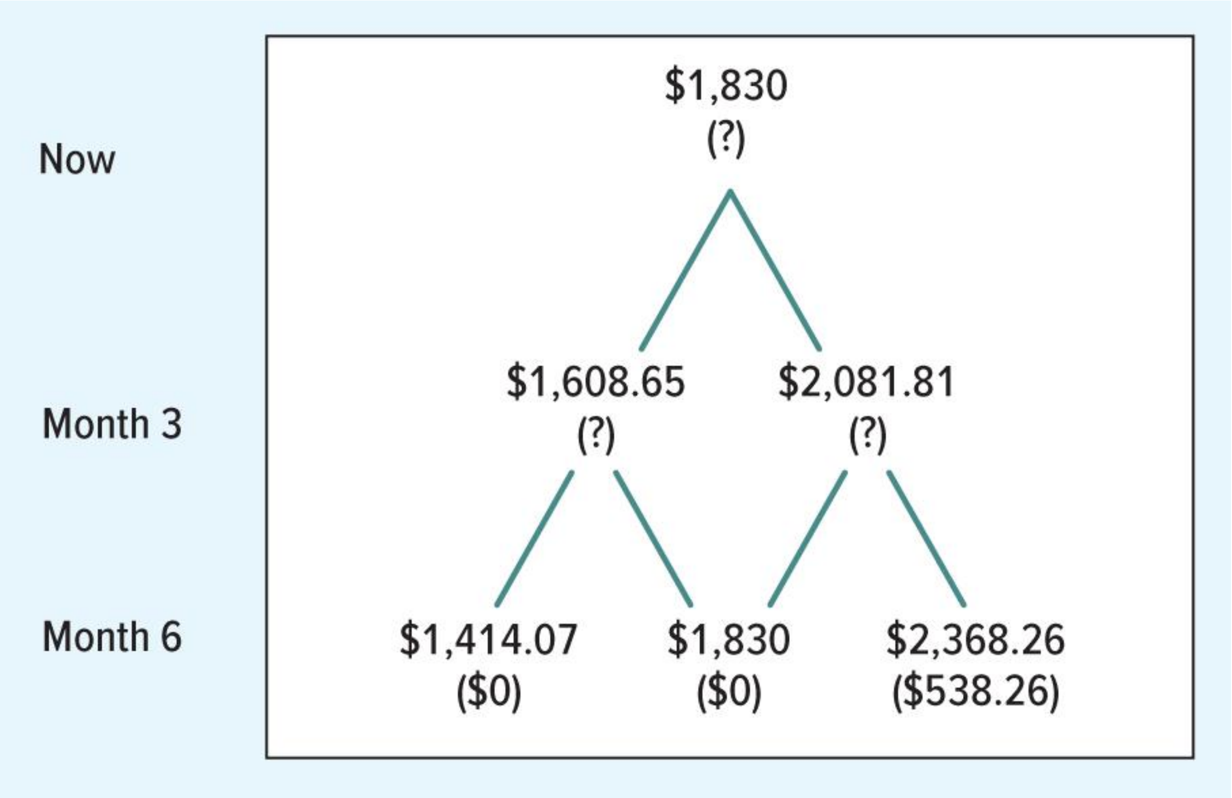
- Start out and demonstrate Binomial Method with simple two-step case in Figure 22.1b.
- Dividing the period into shorter intervals doesn’t alter the basic approach for valuing a call option.
- Can use either the *replicating portfolio* approach or the *risk-neutral* pricing method.
- In this case simpler to use the *risk-neutral* method.

Example: The Two-Step Binomial Method

- Start out and demonstrate Binomial Method with simple two-step case in Figure 22.1b.
- Dividing the period into shorter intervals doesn't alter the basic approach for valuing a call option.
- Can use either the *replicating portfolio* approach or the *risk-neutral* pricing method.
- In this case simpler to use the *risk-neutral* method.

Example: The Two-Step Binomial Method

Present and possible future prices of Amazon stock assuming that in each three-month period the price will either rise by u = 1.1376 or fall by d = 0.8790.
Figures in parentheses show the corresponding values of a six-month call option with an exercise price of $1,830. The interest rate is just under 1% a quarter.



For example, if Amazon's stock price turns out to be $1,414.07 in month 6, the call option will be worthless; at the other extreme, if the stock value is $2,368.26, the call will be worth $2,368.26 – $1,830 = $538.26. Haven't worked out yet what the option will be worth before maturity, hence '?' for now.

Example: The Two-Step Binomial Method

We continue to assume an interest rate of 2.0% for six months $\Rightarrow$ about 1.00% a quarter. If investors demand a return of 1.0% a quarter, what is the probability (p^*) at each stage that the stock price will rise?

$$p^* = \frac{1 + \text{interest rate} - \text{downside change}}{\text{upside change} - \text{downside change}} = \frac{1.0100 - (0.879)}{1.1376 - (0.879)} = 0.5063$$

- We can check that, if there is a 50.63% chance of a rise of 13.76% and a 49.37% chance of a fall of 12.10%, then the expected return must be equal to the 1.00% risk-free rate: (0.5063 × 13.76) + (0.4937 × −12.10) = 1.00%

Option Value in Month 3

- Suppose that by the end of three months, the stock price has risen to \$2,081.81. In that case, investors know that when the option finally matures in month 6, the option value will be either \$0 or \$538.26. Use risk-neutral probabilities to calculate the expected option value at month 6.

Example: The Two-Step Binomial Method

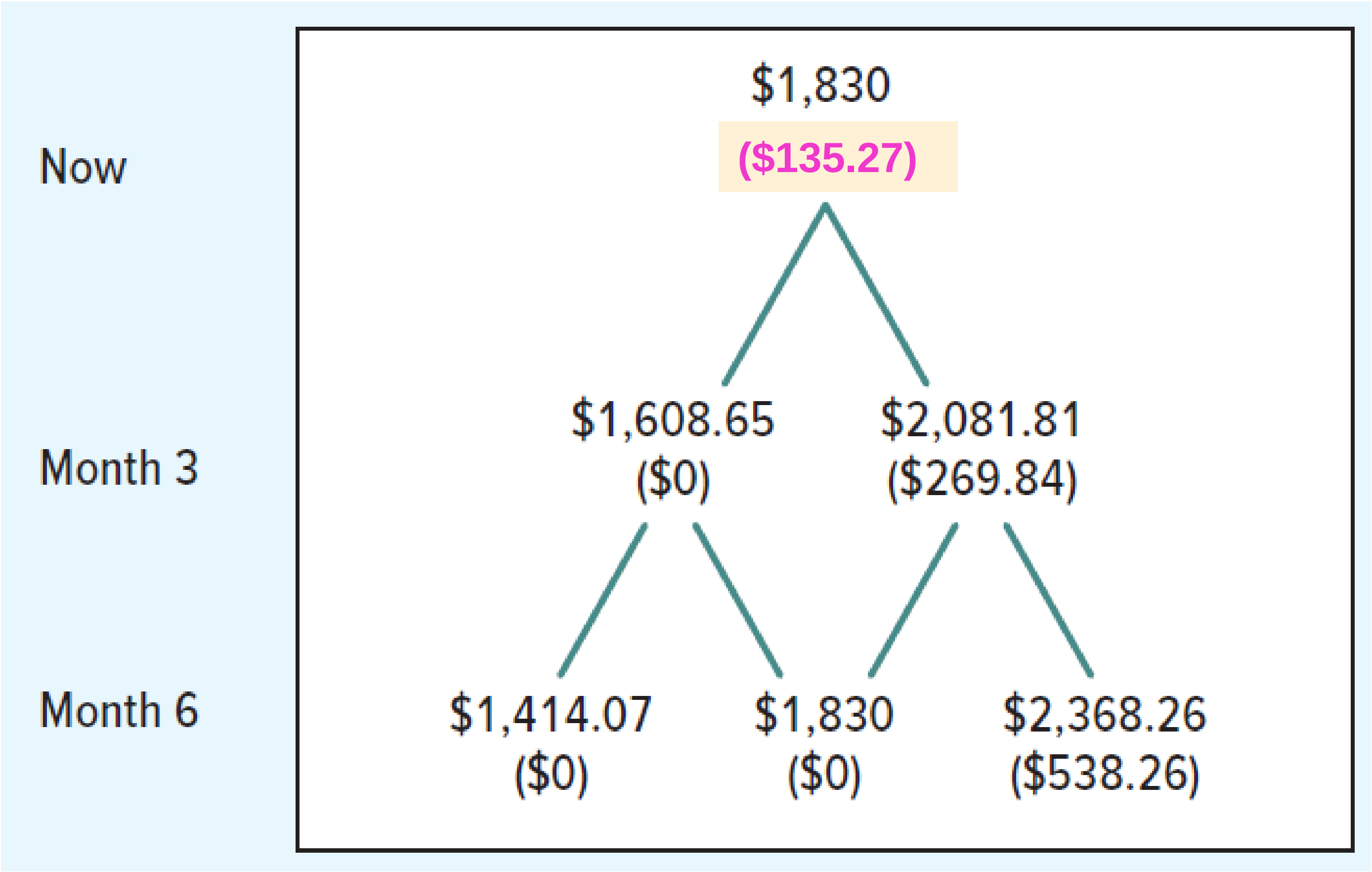
- *Expected value of call in Month 6* = (p^* × 538.26) + ((1 − p^*) × 0)
  = (0.5063 × 538.26) + (0.4937 × 0)
  = $272.52
- Its value in **month 3** is 272.52/1.01 = $269.84. What if the stock price falls to $1,608.65 by month 3? Option is bound to be worthless at maturity ⇒ expected value = 0, and its value at month 3 = 0.

Option Value Today

- There is a 50.63% chance that the stock price will rise in the first three months in which case the option will be worth $269.84. And there is a 49.37% chance that the stock price will fall, in which case the option will be valueless.
- ***Expected value of call in Month 3***
  = (0.5063 × 269.84) + (0.4937 × 0) = $136.62

- The ***value today*** is 136.62/1.01 = **$135.27**

Example: The Two-Step Binomial Method



Present and possible future prices of Amazon stock. Figures in parentheses show the corresponding values of a six-month call option with an exercise price of $1,830.

Binomial Pricing

The prior example can be generalized as the binomial model and can be shown as follows.

$$\text{Probability up } = p^{*} = \frac{(a-d)}{(u-d)}$$

$$\text{Probability down} = 1 - p^{*}$$

$$a = e^{rh}$$

$$d = e^{-\sigma\sqrt{h}}$$

$$u = e^{\sigma\sqrt{h}}$$

$$h = \Delta t = \text{time interval as \% of year}$$

r = continuously compounded interest rate p.a.

e = base for natural logarithms = 2.718

σ = standard deviation of (continuously compounded) stock returns

h = interval as fraction of a year

u = "upside" multiplicative factor; $u > 1$

$u \equiv 1$ + upside % change

d = "downside" multiplicative factor; $d < 1$

$d \equiv 1$ + downside % change; $d = 1/u$

Binomial Pricing

- When we said that Amazon's stock price could either rise by 20% or fall by 16.667% over six months ($h = 0.5$), our figures were consistent with a figure of 25.784% for the standard deviation of annual stock returns:

$$1 + \text{upside change (6-month interval)} = u = e^{0.25784\sqrt{(0.5)}} = 1.20$$

$$1 + \text{downside change} = d = 1/u = 1/1.2 = 0.833$$

- To work out the equivalent upside and downside changes when we divide the period into two three-month intervals ($h = 0.25$), we use the same formula:

$$1 + \text{upside change (3-month interval)} = u = e^{0.25784\sqrt{(0.25)}} = 1.1376$$

$$1 + \text{downside change} = d = 1/u = 1/1.1376 = 0.879$$

- To find the standard deviation given u, we turn the formula around:
  $\sigma = \ln(u)/\sqrt{h}$ where ln = natural logarithm.
  In our example, $\sigma = \ln(1.2)/\sqrt{0.5} = 0.1823/\sqrt{0.5} = 0.25784$

Binomial method

Another Example: Pricing a 90-day call option; three 30-day intervals

- Price (S) = 36
- Exercise Price (X) = 40
- $\sigma = 0.40$
- $t = 90/365$
- $h = \Delta t = 30/365$
- $r = 10\%$ p.a.

$a = 1.0083$
$u = 1.1215$
$d = 0.8917$
$p^* = 0.5075$
$1 - p^* = 0.4925$

$a = e^{(r) \times (h)} = e^{(0.10) \times (30/365)} = 1.0083;$

$u = e^{(\sigma) \times (\sqrt{h})} = e^{(0.40) \times (\sqrt{30/365})} = 1.1215;$

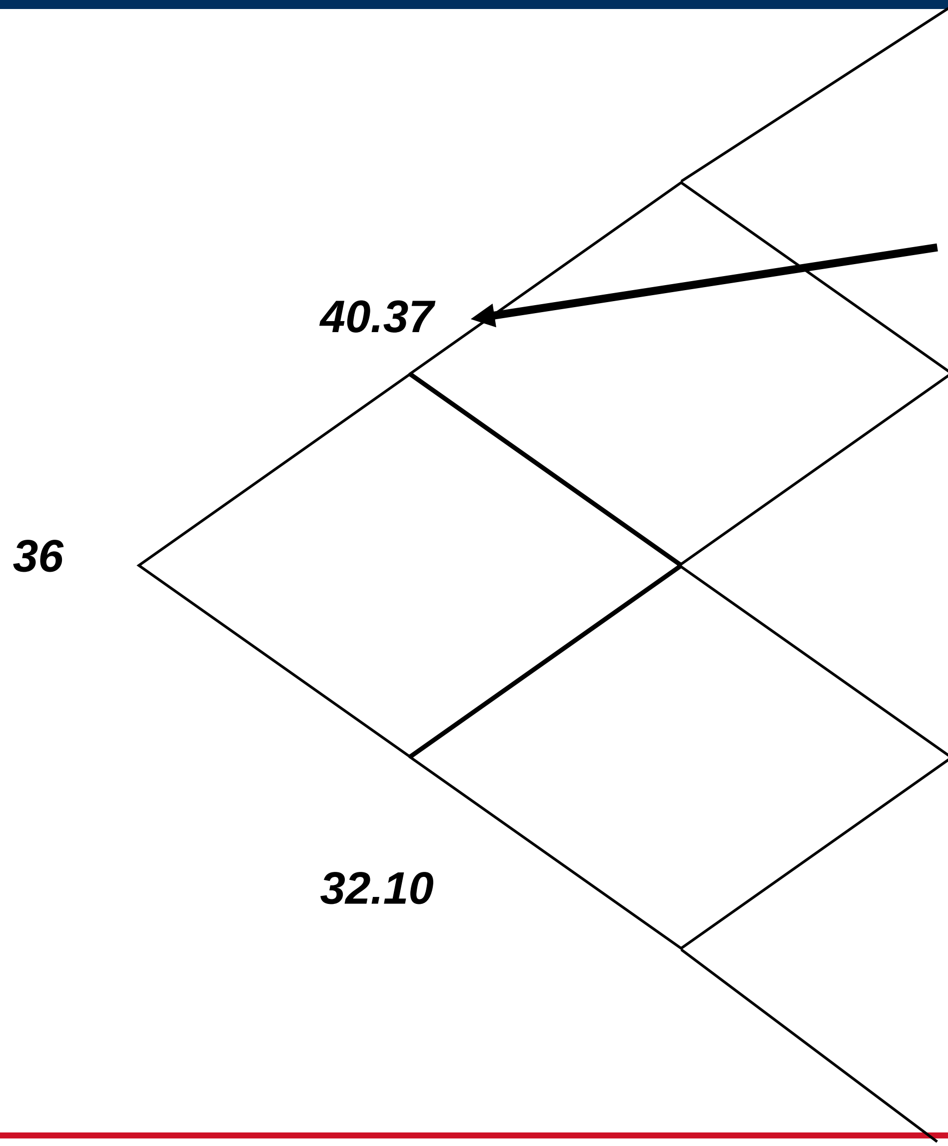
$d = e^{-(\sigma) \times (\sqrt{h})} = e^{-(0.40) \times (\sqrt{30/365})} = 0.8917;$

$p^* = \frac{(a-d)}{(u-d)} = \frac{(\mathbf{1.00853-0.89165})}{(\mathbf{1.121617-0.89165})} = 0.5075;$

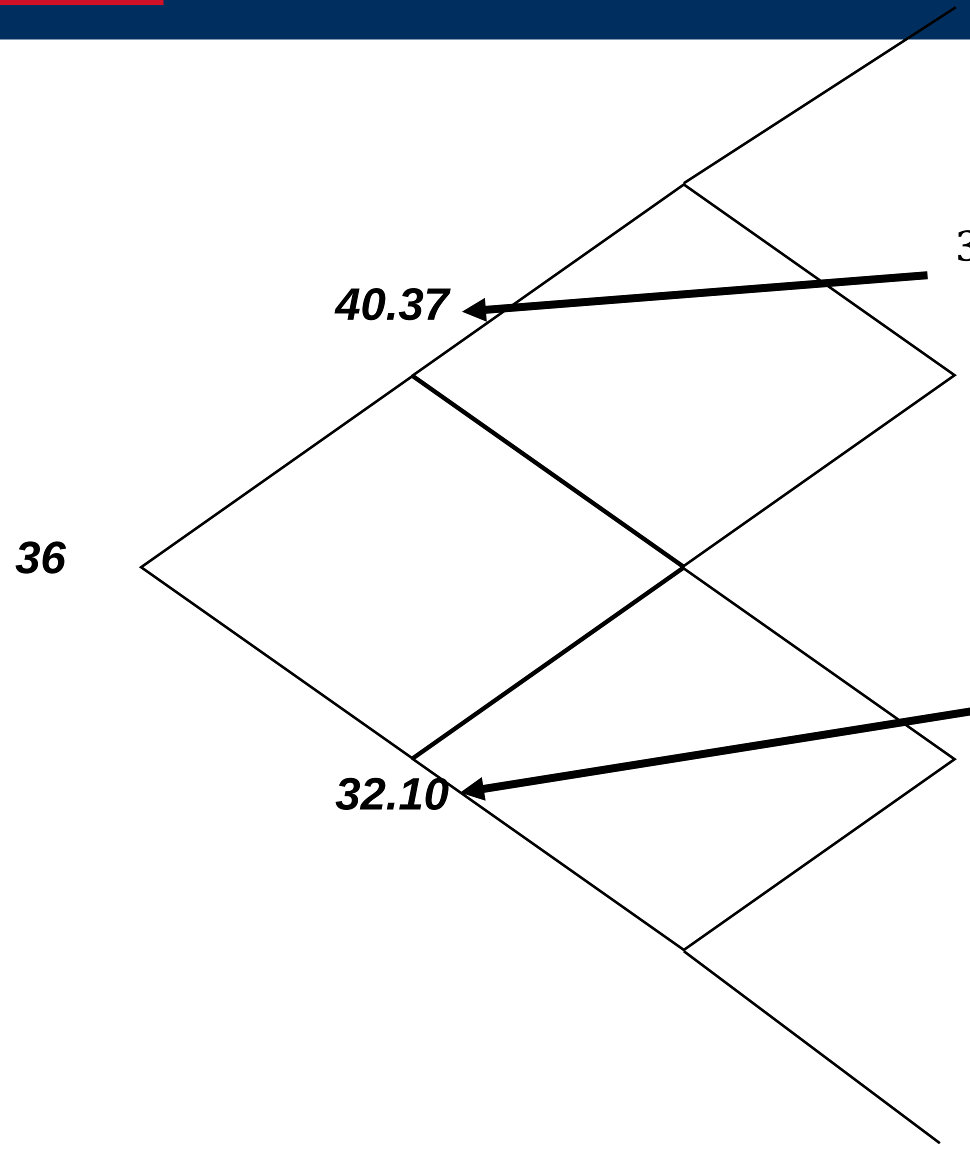
$1 - p^* = 0.4925$

Binomial method

$$S_0 \times u = S_{u1}$$
$$36 \times 1.1215 = 40.37$$



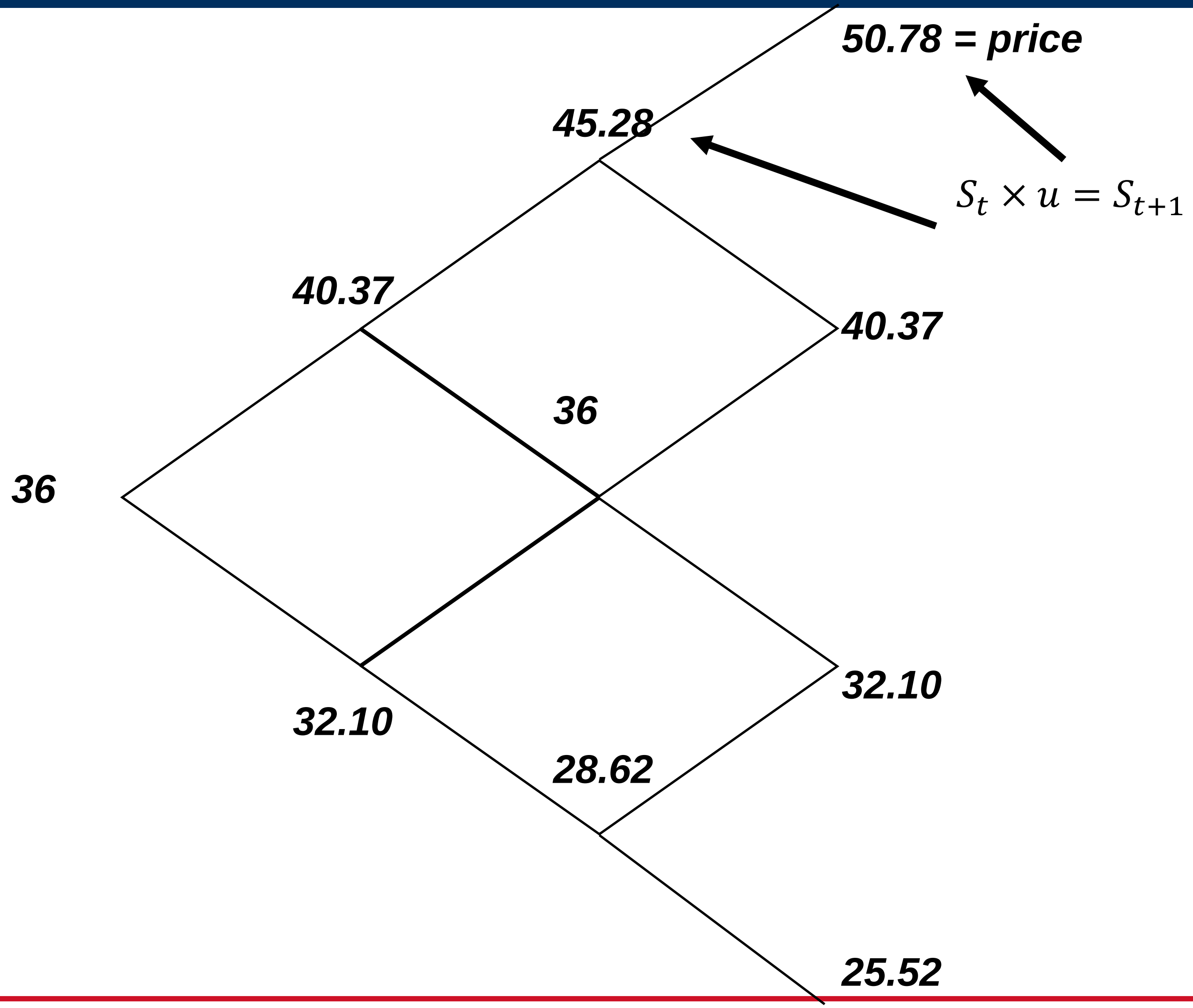
Binomial method



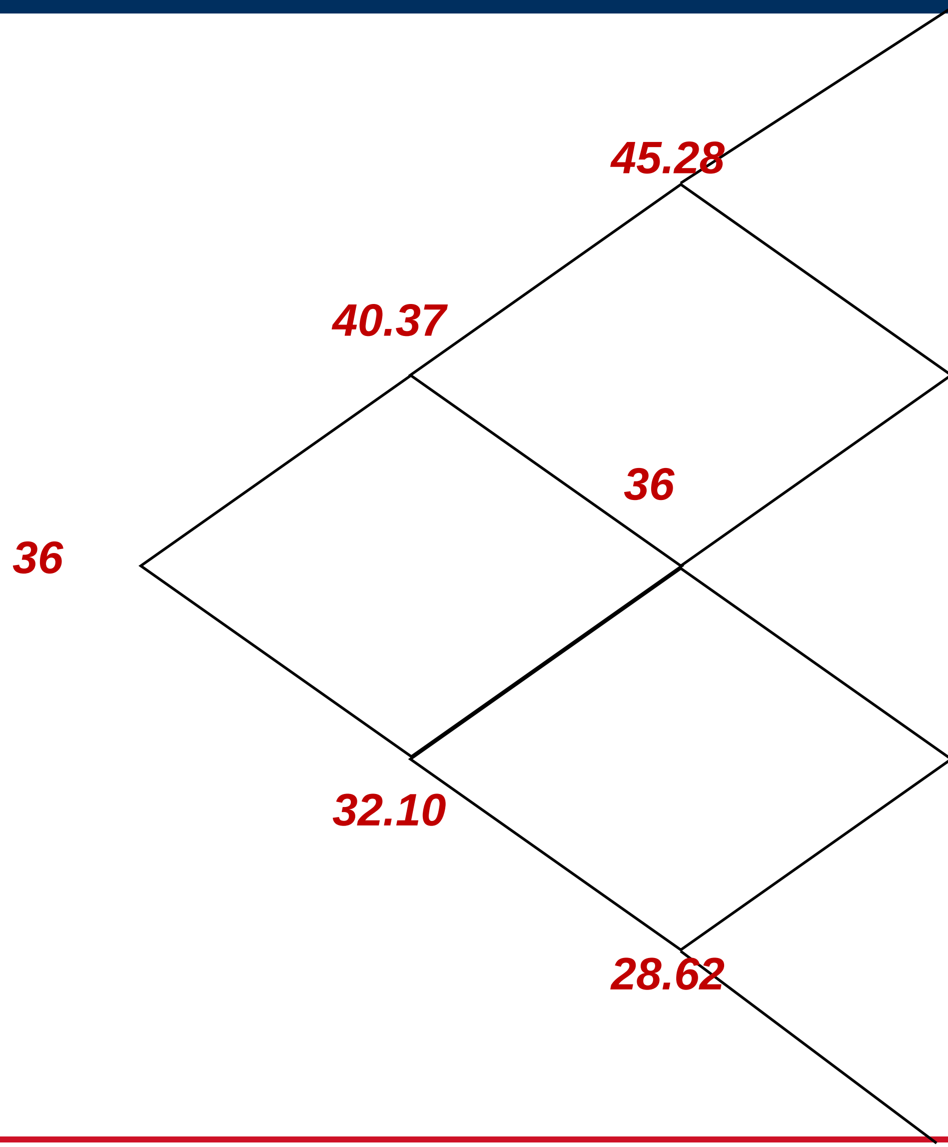
$$S_0 \times u = S_{u1}$$
$$36 \times 1.1215 = 40.37$$

$$S_0 \times d = S_{d1}$$
$$36 \times .8917 = 32.10$$

Binomial method for valuing options



Binomial method for valuing options



50.78 = price

10.78 = intrinsic value

Max[0, (50.78 – 40)]

40.37

.37 Max[0, (40.37– 40)] = 0.37

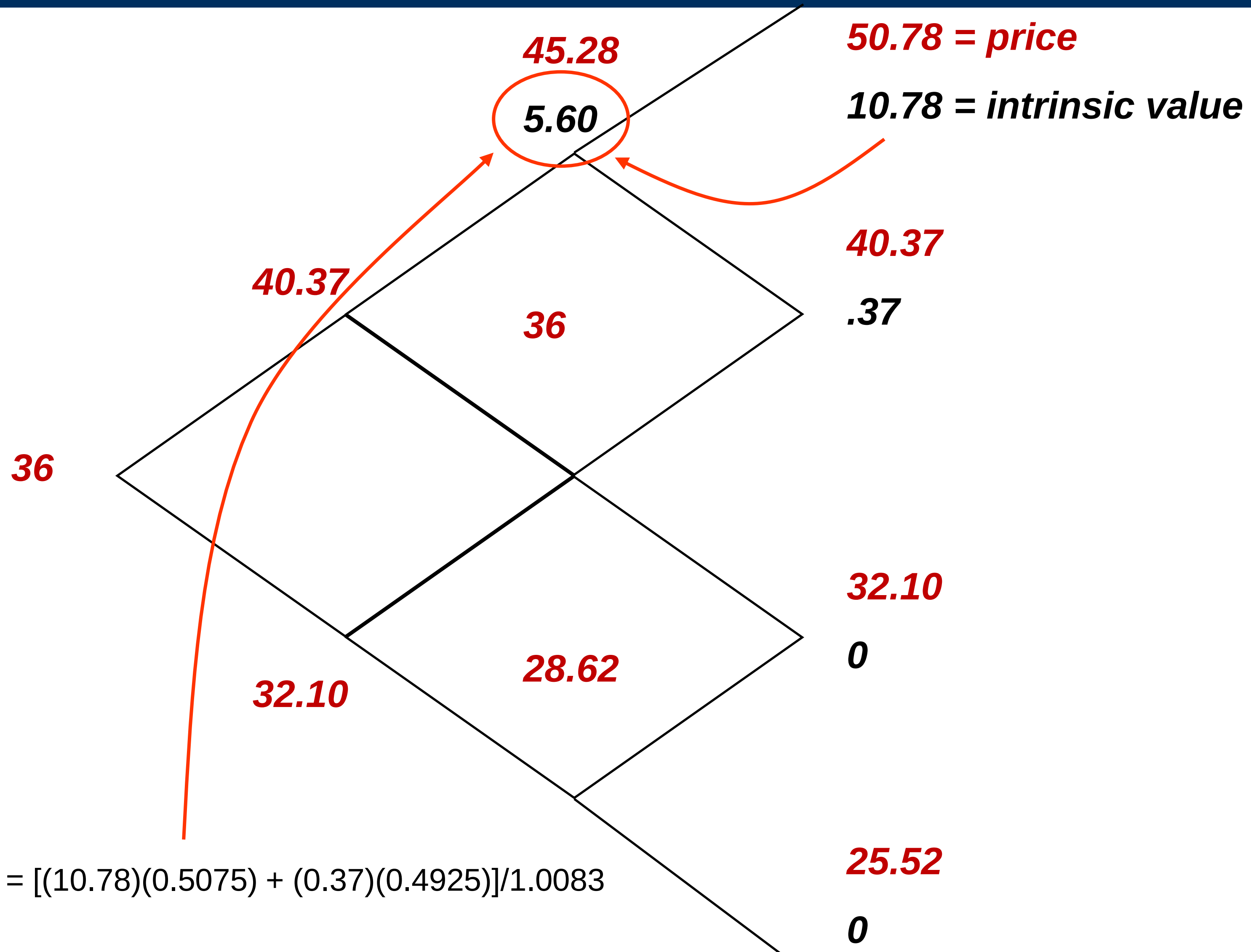
32.10

0 Max[0, (32.10 – 40)] = 0

25.52

0 Max[0, (25.52 – 40)] = 0

Binomial method for valuing options



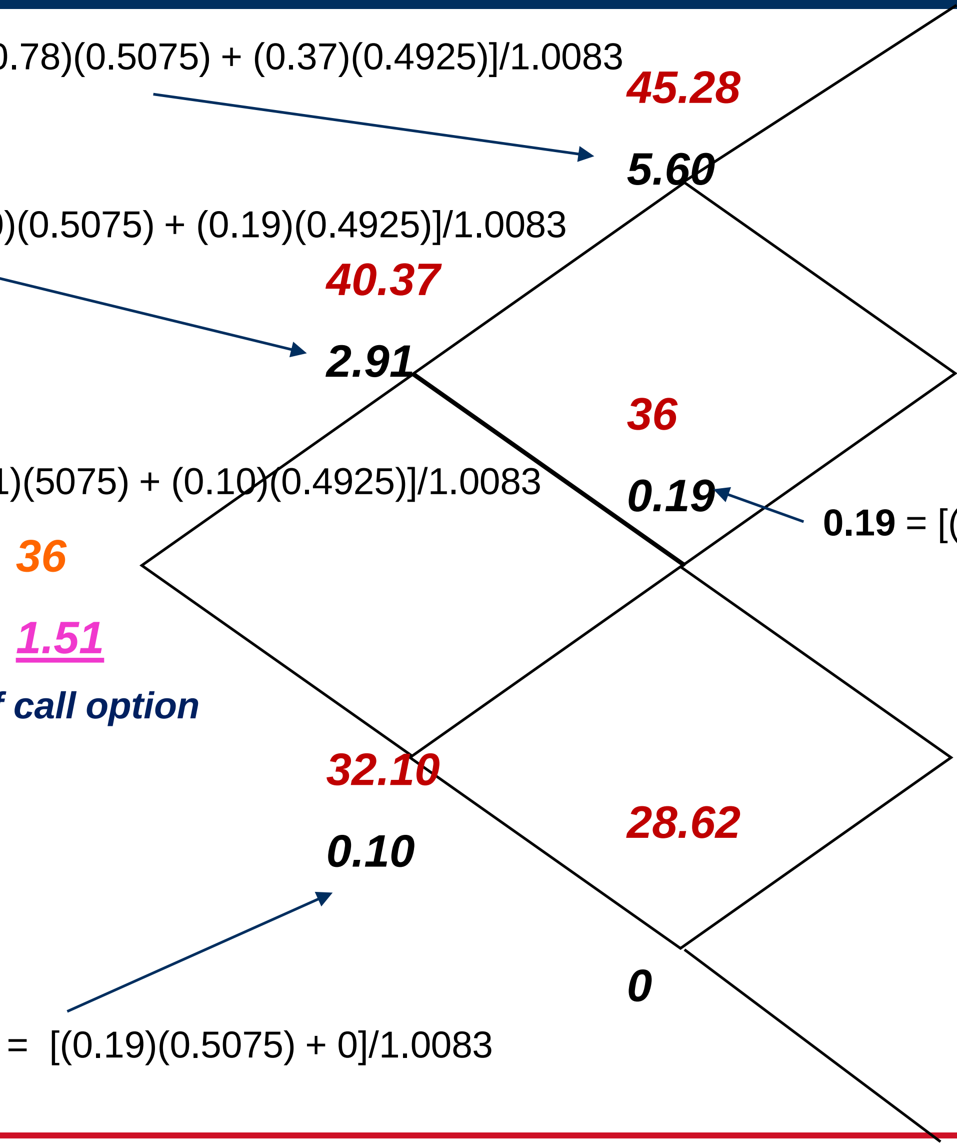
5.60 = [(10.78)(0.5075) + (0.37)(0.4925)]/1.0083

Binomial method for valuing options

5.60 = [(10.78)(0.5075) + (0.37)(0.4925)]/1.0083

2.91 = [(5.60)(0.5075) + (0.19)(0.4925)]/1.0083

1.51 = [(2.91)(5075) + (0.10)(0.4925)]/1.0083



50.78 = price

10.78 = intrinsic value

Max[0, (50.78 – 40)]

40.37

.37 Max[0, (40.37– 40)] = 0.37

0.19 = [(0.37)(0.5075) + 0]/1.0083

Value of call option

32.10

0 Max[0, (32.10 – 40)] = 0

0.10 = [(0.19)(0.5075) + 0]/1.0083

25.52

0 Max[0, (25.52 – 40)] = 0

Table 22.1 Black–Scholes Value of Amazon Call Option

- As the number of steps is increased, you must adjust the range of possible changes in the value of the asset to keep the same standard deviation.

- **Change per Interval (%)**

Number of Steps	Upside	Downside	Estimated Option Value
1	+20.00	−16.67	$182.67
2	+13.76	−12.10	135.27
6	+7.73	−7.17	144.95
26	+3.64	−3.51	149.07
		Black–Scholes value =	150.33

- ***Note:*** The standard deviation is $\sigma = 0.25784.$

BLACK-SCHOLES OPTION PRICING MODEL (BSOPM)

- Developed by Fischer Black and Myron Scholes (1973)
- Provides an explicit or exact solution to the problem of option pricing.
- Used to value European put and call options, as well as American call options on non-dividend paying stocks.

Assumptions of BSOPM

- *Perfect capital markets: all investors can borrow and lend at the same risk-less interest*
- *Risk-less rate of interest is known and constant over the life of the option.*
- *No transaction costs or taxes associated with buying and selling the underlying asset or option*
- *No restrictions or penalties for short-selling*
- *Underlying asset is efficiently priced and follows a random walk in continuous time.*
- *Underlying asset pays no dividends.*
- *Options are European.*
- *Variance of the underlying asset's rate of return is known and constant.*

BSOPM for European Call Options

$$c = SN(d_1) - Xe^{-r_f T} N(d_2)$$

$$d_1 = \frac{\ln(S/X) + r_f T}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2} = \frac{\ln(S/X) + (r_f + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

BSOPM for European Call Options

Variables in BSOPM Formula

- *c = price of European call option*
- *S = current market price of underlying asset (stock)*
- *X = exercise price of option*
- *T = Time to expiration of option (measured in years)*
- r_f *= risk-free rate of interest (p.a.)*
- σ *= instantaneous standard deviation of the annual return to holding the underlying asset (per year)*
- *N(d) = probability that a standardised, normally distributed random variable will be less than or equal to d.*

N(d) terms in BSOPM

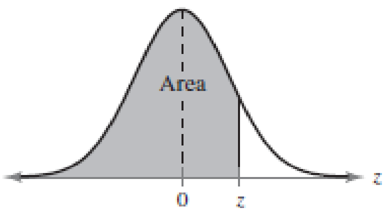
N(d) = probability that a standardised, normally distributed random variable z will be less than or equal to d.

- z is a standard normal variable ie z ~ N(0, 1)

N(-∞) = 0   N(0) = 0.5 N(+∞) = 1

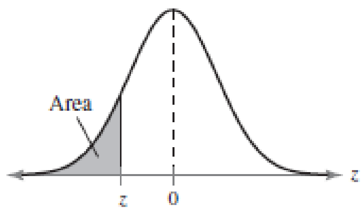
$$N(d_1) \equiv \Pr[\tilde{z} \leq d_1] \equiv \int_{-\infty}^{d_1} f(z)dz$$

Table 4—Standard Normal Distribution *(continued)*



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table 4—Standard Normal Distribution



z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
–3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
–3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
–3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
–3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
–3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
–2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0017	.0018	.0019
–2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
–2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
–2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
–2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
–2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
–2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
–2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
–2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
–2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
–1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287
–1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0352	.0359
–1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
–1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
–1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
–1.4	.0681	.0694	.0708	.0722	.0735	.0749	.0764	.0778	.0793	.0808
–1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
–1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151
–1.1	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357
–1.0	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587
–0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
–0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119
–0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420
–0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743
–0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085
–0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
–0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
–0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
–0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
–0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000

Intuitive Interpretation of BSOPM

$$c = SN(d_1) - Xe^{-r_f T} N(d_2)$$

- The value of call option is equal to the stock price, *S*, minus the present value of the exercise price, Xe^{-rT}. However each component is weighted by a probability, the N(d) terms.

- N(d) terms can be (loosely) viewed as risk-adjusted probabilities that the European call option will expire in the money.

Intuitive Interpretation of BSOPM

- If $N(d_1)$ and $N(d_2)$ are close to 1 $\Leftrightarrow$ there is a very high probability that the option will be exercised. $\Rightarrow$ call option value is equal to $[S - Xe^{-rT}]$; if exercise is certain, we have claim on a stock with current value S, and an obligation with present value Xe^{-rT}
- If $N(d_1)$ and $N(d_2)$ are close to 0 $\Leftrightarrow$ option almost certainly will not be exercised $\Rightarrow$ formula confirms that call option will be worthless.
- For middle range values of $N(d)$ between 0 and 1, formula indicates that call value will be present value of call option's potential payoff adjusting for the probability of in-the-money expiration

Intuitive Interpretation of BSOPM

- Note that ln(S/X) (in numerators of $N(d_1)$ and $N(d_2)$) is approx. the percentage amount by which option is currently in or out of the money. The denominator $\sigma T^{1/2}$ adjusts "moneyness" of option for volatility of stock price over the remaining life of option.
- An option in the money by a given percent is more likely to stay in the money if both stock price volatility and time to expiration is small.

$\therefore$ $N(d_1)$ and $N(d_2)$ increase with probability that the option will expire in the money

Example of BSOPM

Suppose we want to calculate value of a European call option given the following info.

$S = \$50.00$

$X = \$45.00$

$T = 9$ months (0.75)

$r_f = 9\% \ (0.09)$

$\sigma = 30\% \ (0.30)$

Example of BSOPM

- **calculate d_1**

$$d_1 = \frac{\ln(50/45) + (0.09)(0.75)}{(0.30)\sqrt{(0.75)}} + \frac{(0.30)\sqrt{(0.75)}}{2} =$$

$$\frac{0.10536 + 0.0675}{(0.30)\sqrt{(0.75)}} + 0.1299$$

$$= \frac{0.17286}{0.2598} + 0.1299 = 0.66536 + 0.1299 = 0.7952$$

- **calculate d_2**

$$d_2 = 0.7952 - 0.30\sqrt{0.75} = 0.5354$$

Example of BSOPM

- **compute N(d_1) and Nd_2)**

N(0.7952) = N(0.79) + 0.52[N(0.80) – N(0.79)]
= 0.7852 + 0.52[0.7881 – 0.7852]
= 0.7852 + 0.0015 = **0.7867**

$$N(0.7952) = \int_{-\infty}^{0} f(z)dz + \int_{0}^{0.7952} f(z)dz = 0.5 + 0.2867 = 0.7867$$

$$N(0.5354) = \int_{-\infty}^{0} f(z)dz + \int_{0}^{0.5354} f(z)dz = 0.5 + 0.2037 = 0.7037$$

N(0.5354) = N(0.53) + 0.54[N(0.54) – N(0.53)]
= 0.7019 + 0.54[0.7054 – 0.7019]
= 0.7019 + 0.0018 = **0.7037**

- **calculate value of call option**

$$c = 50(0.7867) - 45e^{-(0.09)(0.75)}(0.7037)$$

c = 39.335 – 42.064(0.7037)

c = $ 9.73

BSOPM - Value of European Put

- The above formula gives us the value of a European call option.
- How do we determine the value of a European put option? Use the put-call parity relationship discussed earlier to determine value of a European put option once we have employed BSOPM to price the European call option.

$$c - p = S_0 - Xe^{-r_f T}$$

BSOPM - Value of an American Call
(non-dividend paying stock)

- Given that an American call option written on a non-dividend paying stock will not be exercised prior to its expiration, BSOPM for European call will also determine price of American call option.

Option Menagerie

Asian (or average) option	The exercise price is equal to the *average* of the asset's price during the life of the option.
Barrier option	Option where the payoff depends on whether the asset price reaches a specified level. A knock-in option (up-and-in call or down-and-in put) comes into existence only when the underlying asset reaches the barrier. Knock-out options (down-and-out call or up-and-out put) *cease* to exist if the asset price reaches the barrier.
Bermuda option	The option is exercisable on discrete dates before maturity.
Caput option	Call option on a put option.
Chooser (as-you-like-it) option	The holder must decide before maturity whether the option is a call or a put.
Compound option	An option on an option.
Digital (binary or cash-or-nothing) option	The option payoff is zero if the asset price is the wrong side of the exercise price and otherwise is a fixed sum.
Lookback option	The option holder chooses as the exercise price any of the asset prices that occurred before the final date.
Rainbow option	Call (put) option on the best (worst) of a basket of assets.