# Corporate Finance 2: Option Valuation Models (Lecture Notes Summary)

This lecture explores the valuation of options, focusing on both theoretical models and practical examples.

## 1. Option Valuation Challenges

- **Standard discounted cash flow (DCF) methods do not work for option valuation.**
- **More rigorous techniques are needed:**
    - **Option Equivalents Approach (Replicating Portfolio):** Creating a portfolio of stocks and borrowing/lending to replicate the option's payoff structure.
    - **Risk-Neutral Method:** Assuming all investors are indifferent to risk and calculating the expected future value of the option, discounted back at the risk-free rate.
    - **Binomial Method:** A discrete-time approach that uses the replicating portfolio or risk-neutral valuation to price options across multiple time periods.

## 2. Option Equivalents Approach (Replicating Portfolio)

- **Goal:** Construct an investment strategy that matches the payoff of an option.
- **Key Components:**
    - **Option Delta (δ):** The ratio of the spread of possible option prices to the spread of possible share prices. Represents the number of shares needed to replicate one option.
    - **Borrowing/Lending:** To adjust the investment strategy and replicate the option's payoff in all scenarios.

- **Value of Option = Value of Replicating Portfolio**

**Example:** A six-month call option on Amazon shares with an exercise price of $1,830. The stock price is also $1,830. The risk-free rate is 2% for six months.

- **Scenario 1:** Amazon stock price rises to $2,196.
- **Scenario 2:** Amazon stock price falls to $1,525.

The call option payoff is $366 in scenario 1 and $0 in scenario 2. To replicate this:

- **Calculate Option Delta (δ):** (366 - 0) / (2,196 - 1,525) = 0.54545
- **Buy 0.54545 shares of Amazon.**
- **Borrow the present value of $831.82 (calculated to match the option's payoff in both scenarios).**

The replicating portfolio will have the same payoff as the call option in both scenarios. Therefore, the option's value today is equal to the cost of this portfolio.

## 3. Risk-Neutral Valuation

- **Concept:** Instead of using actual probabilities, assume all investors are risk-neutral and expect a return equal to the risk-free rate.

- **Steps:**
    - **Calculate Risk-Neutral Probabilities:** Use a formula based on the risk-free rate, the "upside change," and the "downside change" in the stock price.
    - **Calculate Expected Future Value:** Use the risk-neutral probabilities to calculate the expected future value of the option in all scenarios.
    - **Discount Back to Present Value:** Use the risk-free rate to discount the expected future value to today's value.

**Example (Continuing with the Amazon call option):**

- *Risk-Neutral Probability of a rise (p):** (1.02 - 0.8333) / (1.2 - 0.8333) = 0.50909
- **Expected Future Value:** (0.50909 x $366) + (0.49091 x $0) = $186.33
- **Present Value:** $186.33 / 1.02 = $182.67

This is the same value obtained using the replicating portfolio approach.

## 4. Binomial Method

- **Extension of the Risk-Neutral Valuation:** Divides the time period into shorter intervals, allowing for multiple "up" and "down" movements in the stock price.
- **Key Steps:**
    - **Define Time Intervals:** Divide the option's life into multiple periods.
    - **Calculate Upside and Downside Movements:** Determine the "upside" and "downside" factors (u and d) for each period.
    - **Calculate Risk-Neutral Probabilities:** Use the risk-free rate and the "upside" and "downside" factors to calculate the probability of an "up" move (p*) for each period.
    - **Work Backwards:** Calculate the option's value at each time interval, using the risk-neutral probabilities and the discounted value of the option's expected future value.

**Example (Two-step binomial method for the Amazon call option):**

- The time period is divided into two three-month intervals.
- **Calculate the option's value at each time interval and work backwards to find the current value.**

## 5. Black-Scholes Option Pricing Model (BSOPM)

- **A continuous-time model that provides an explicit solution for pricing European options.**
- **Assumptions:**
    - Perfect capital markets (all investors have access to the same risk-free rate).
    - Risk-free rate is known and constant.
    - No transaction costs or taxes.
    - No restrictions on short-selling.
    - Underlying asset is efficiently priced and follows a random walk.
    - Underlying asset pays no dividends.
    - Options are European.
    - The underlying asset's rate of return has a known and constant variance.

**BSOPM Formula for a European Call Option:**

```
c = S * N(d1) - Xe^(-rT) * N(d2)
```

where:

- c = price of the European call option
- S = current market price of the underlying asset (stock)
- X = exercise price of the option
- T = Time to expiration of the option (measured in years)
- rf = risk-free rate of interest (p.a.)
- σ = instantaneous standard deviation of the annual return to holding the underlying asset (per year)
- N(d) = probability that a standardized, normally distributed random variable will be less than or equal to d.

**Key Insights:**

- The BSOPM formula reflects the probability of the option expiring in the money and the present value of the exercise price.
- The value of a European put option can be determined using put-call parity.
- For a non-dividend paying stock, the BSOPM formula also applies to American call options.

**Limitations of BSOPM:**

- Assumes continuous trading, which is not always the case in real markets.
- Assumes a constant volatility, which can be unrealistic.
- The model is less effective for options with early exercise features, such as American put options on dividend-paying stocks.

## 6. Summary

The lecture provides a comprehensive overview of option valuation models, highlighting the use of replicating portfolios, risk-neutral valuation, and the binomial method. The Black-Scholes model offers an explicit solution for pricing European options, but its assumptions and limitations should be considered.

This lecture provides a solid foundation for understanding option pricing and its complexities. Further exploration of advanced pricing models and real-world applications will enhance your understanding of this essential concept in corporate finance.