FINC2012
CORPORATE FINANCE 2
Semester 2 2024
Lecture

Options - I: Understanding Options

BMAE Ch.21

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THE UNIVERSITY OF SYDNEY

TOPICS COVERED

- Calls and Puts
- Relevance to Corporate Finance
- Exercise and Option Payoff Diagrams
- Put call Parity
- What Determines Option Value

Options

- ***A contract between two parties that gives one party, (the buyer of the option), the right but <u>not</u> the obligation to buy or sell commodity or asset, at a predetermined price, on or before a specified date in the future.***
- Options – a *derivative* contract.
- A *derivative* is an instrument whose value today or at some future date depends on the values of other more basic underlying variables.
- Value is <u>derived</u> from the value of another asset (or group of other assets), known as ***underlying asset*** (assets).
- Other key derivative instruments:

  Forwards

  Futures

  Swaps

Options

EXAMPLE 1:

- ***The right to buy 100 CBA shares for $140 per share from today until expiration on December 15 2024***

EXAMPLE 2:

- ***The right to sell CHF 10 million for USD1.15 per CHF on (and only on) the expiration date December 15 2024.***

Option Terminology

- ***Call option***: gives the holder the right, but not the obligation, to *buy* a certain asset by a certain date for a certain price (the strike or exercise price)
- ***Put option***: gives the holder the right, but not the obligation, to *sell* a certain asset by a certain date for a certain price (the strike or exercise price)
- ***American option***: can be exercised at any time during its life: any time up to and including the expiration date.
- ***European option***: can be exercised only at maturity or expiration date.
- ***Exchange-Traded Options***: Standardised option contracts that trade on organised exchanges in accordance with rules & regulations stipulated by the exchange.
- ***Over-The-Counter (OTC) Traded Options***: Option contracts whose terms and conditions are tailored to the specific needs of the two parties involved. Trade takes place in private and essentially unregulated market.

Option Terminology

- ***Option Seller or Writer***

- The party who sells the option contract;
  the party that sells the right to buy or sell the underlying asset; has gone "*short*" the option contract or holds a *short position*.

- Depending upon the type of option contract, option seller is obliged to buy or sell the underlying asset according to the contract terms, if and when the option is exercised by the option buyer.

- Receives payment (option premium) for selling the option contract.

Option Terminology

- ***Option Buyer or holder***

- The party who buys the option contract; purchases the right to buy or sell the underlying asset; has gone "long" the option contract or holds a long position.

- Option buyer decides whether or not to exercise the option contract.

- Has the right, but not the obligation, to buy or sell the underlying asset in accordance with the terms of the option contract.

- Pays a price (option premium) for buying option contract.

Option Obligations

	Buyer	Seller
Call option	Right to buy asset	Obligation to sell asset
Put option	Right to sell asset	Obligation to buy asset

Option Terminology

- ***Exercise of Option***

- The process of enforcing the right that has been purchased; the act of buying or selling the underlying asset in accordance with the terms in the option contract.

- Action which can only be taken by the buyer or holder of option contract.

- ***Exercise Price (Strike price)***

- The predetermined price in the option contract at which option buyer/holder can buy or sell the underlying asset should they choose to exercise the option contract.

Option Terminology

- ***Option Premium***
  - The sum of money paid by the option buyer to the option seller in order to obtain the option contract. Arrived at by negotiation between option buyers and sellers.
  - Paid by buyer/kept by seller irrespective of whether or not option contract is “exercised” by option buyer.
- ***Option Expiration Date***
  - The date after which the option contract cannot be exercised. (The date after which the option holder’s right to buy or sell the asset will no longer be valid).

Amazon Stock

Table 21.1 Selected prices of put and call options on Amazon.com shares in January 2020, when the closing share price was about $1,830.

Maturity Date	Exercise Price	Price of Call Option	Price of Put Option
April 2020	$1,700	$182.80	$ 33.32
	1,750	149.42	46.37
	1,830	90.28	81.53
	1,900	63.80	113.93
	1,950	49.75	135.79
July 2020	$1,700	$208.45	$53.23
	1,750	200.14	68.50
	1,830	146.20	107.34
	1,900	106.95	135.36
	1,950	83.87	168.70
January 2021[a]	$1,700	$296.78	$110.35
	1,750	263.02	126.13
	1,830	200.28	165.75
	1,900	175.54	195.67
	1,950	147.73	217.05

OPTION EXERCISE VALUE

- The value of an option at expiration is a function of the asset price S and the exercise price X.

- Call Exercise value at expiration = Max (0, S – X)
- Put Exercise value at expiration = Max (0, X – S)

OPTION EXERCISE VALUE

- Value at expiration is a function of stock price and exercise price

  Payoff: Call = Max [0, S-X]; Put = Max [0, X-S]

- *Example: Option values given exercise price, X, of $80*

Stock Price	$60	70	80	90	100	110
Call value	0	0	0	10	20	30
Put value	20	10	0	0	0	0

Option Quotes

			--Call--		--Put--	
Option/Strike		**Exp.**	**Vol.**	**Last**	**Vol.**	**Last**
IBM	162.5	Jul	21	5.85	145	0.89
167.3	165	Jul	108	3.95	150	1.48
167.3	170	Jul	212	1.25	335	3.95
167.3	170	Aug	1449	3.15	1518	6.75
167.3	170	Sep	156	3.91	264	7.71
167.3	175	Sep	101	2.18	78	11.05

Option Quotes

This option has a strike price of $170;

			--Call--		--Put--	
Option/Strike		**Exp.**	**Vol.**	**Last**	**Vol.**	**Last**
IBM	162.5	Jul	21	5.85	145	0.89
167.3	165	Jul	108	3.95	150	1.48
167.3	170	Jul	212	1.25	335	3.95
167.3	170	Aug	1449	3.15	1518	6.75
167.3	170	Sep	156	3.91	264	7.71
167.3	175	Sep	101	2.18	78	11.05

recent price for the stock is $167.30;

July is the expiration month

Option Quotes

This makes a call option with this exercise price out-of-the-money.

			--Call--		--Put--	
Option/Strike		**Exp.**	**Vol.**	**Last**	**Vol.**	**Last**
IBM	162.5	Jul	21	5.85	145	0.89
167.3	165	Jul	108	3.95	150	1.48
167.3	170	Jul	212	1.25	335	3.95
167.3	170	Aug	1449	3.15	1518	6.75
167.3	170	Sep	156	3.91	264	7.71
167.3	175	Sep	101	2.18	78	11.05

Puts with this exercise price are in-the-money by \$2.70 = \$170 - \$167.30.

Option Quotes

			--Call--		--Put--	
Option/Strike		**Exp.**	**Vol.**	**Last**	**Vol.**	**Last**
IBM	162.5	Jul	21	5.85	145	0.89
167.3	165	Jul	108	3.95	150	1.48
167.3	170	Jul	212	1.25	335	3.95
167.3	170	Aug	1449	3.15	1518	6.75
167.3	170	Sep	156	3.91	264	7.71
167.3	175	Sep	101	2.18	78	11.05

On this day, 212 call options with this exercise price were traded.

Option Quotes

CALL option with a strike price of $170 is trading for $1.25.

			--Call--		--Put--	
Option/Strike		**Exp.**	**Vol.**	**Last**	**Vol.**	**Last**
IBM	162.5	Jul	21	5.85	145	0.89
167.3	165	Jul	108	3.95	150	1.48
167.3	170	Jul	212	1.25	335	3.95
167.3	170	Aug	1449	3.15	1518	6.75
167.3	170	Sep	156	3.91	264	7.71
167.3	175	Sep	101	2.18	78	11.05

Since the option is on 100 shares of stock, buying this option would cost $125 plus commissions.

Option Quotes

			--Call--		--Put--	
Option/Strike		**Exp.**	**Vol.**	**Last**	**Vol.**	**Last**
IBM	162.5	Jul	21	5.85	145	0.89
167.3	165	Jul	108	3.95	150	1.48
167.3	170	Jul	212	1.25	335	3.95
167.3	170	Aug	1449	3.15	1518	6.75
167.3	170	Sep	156	3.91	264	7.71
167.3	175	Sep	101	2.18	78	11.05

On this day, 335 put options with this exercise price were traded.

Option Quotes

The PUT option with a strike price of $170 is trading for $3.95.

			--Call--		--Put--	
Option/Strike		**Exp.**	**Vol.**	**Last**	**Vol.**	**Last**
IBM	162.5	Jul	21	5.85	145	0.89
167.3	165	Jul	108	3.95	150	1.48
167.3	170	Jul	212	1.25	335	3.95
167.3	170	Aug	1449	3.15	1518	6.75
167.3	170	Sep	156	3.91	264	7.71
167.3	175	Sep	101	2.18	78	11.05

Since the option is on 100 shares of stock, buying this option would cost $395 plus commissions.

OPTIONS AND CORPORATE FINANCE

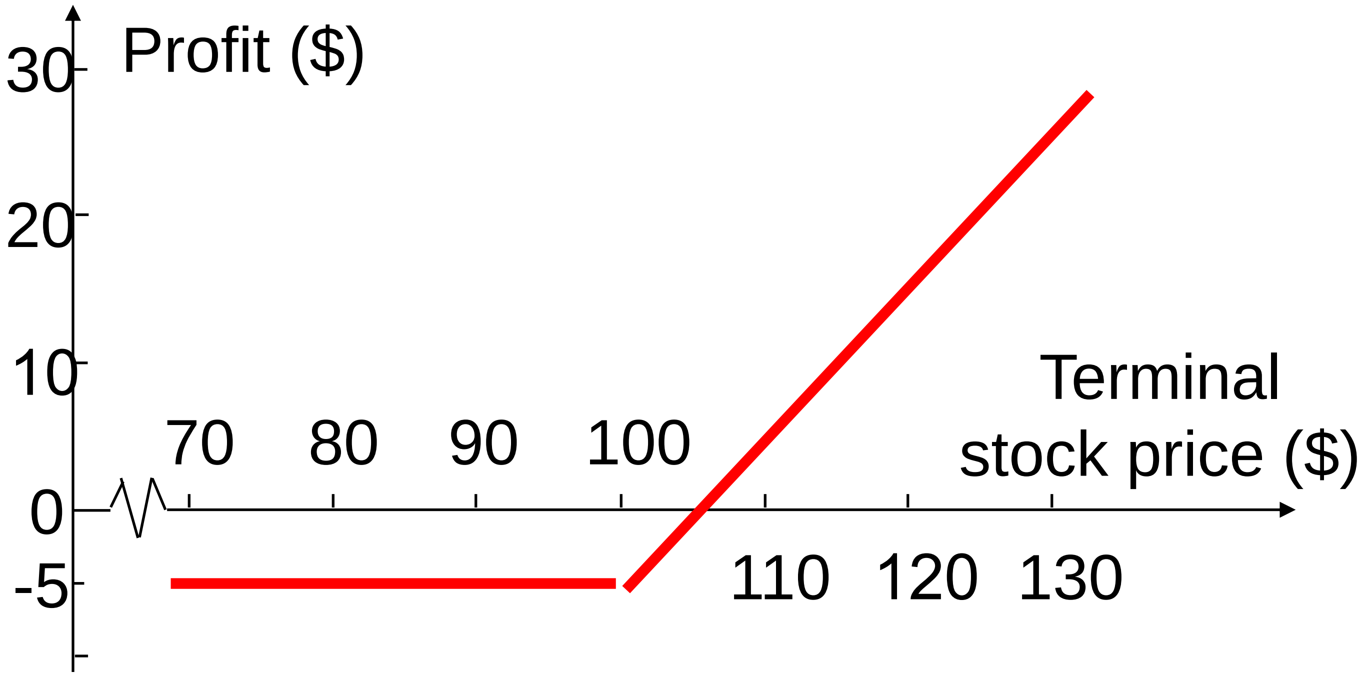
- **Risk Management:** Companies regularly use commodity, currency, and interest- rate options to reduce risk.
- **Real Options are embedded in real investment project**: Many capital investments include an embedded option: *Option to expand, Option to delay/postpone, Option to abandon, Option to alter nature of inputs Option to alter nature of outputs*
- **Companies issue options, or securities with embedded options, to raise finance:** Options often tacked on to an issue of corporate securities and so provide the investor or the company with the flexibility to change the terms of the issue.
- **Shares in levered firm can be regarded as an option on the assets of the firm**

Option Positions

- Long call
- Short call
- Long put
- Short put

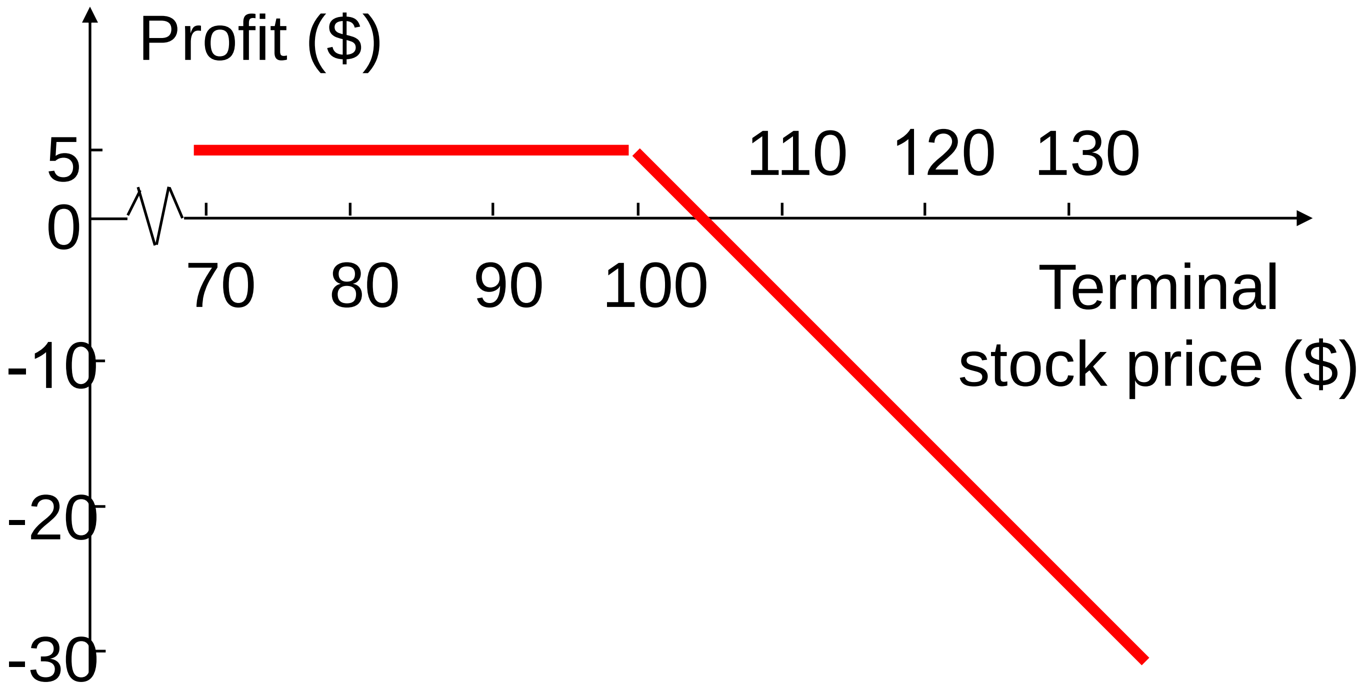
Profit Profile of a Long Call on Microsoft

Profit from buying a Microsoft European call option: option price = $5, strike price = $100, option life = 2 months



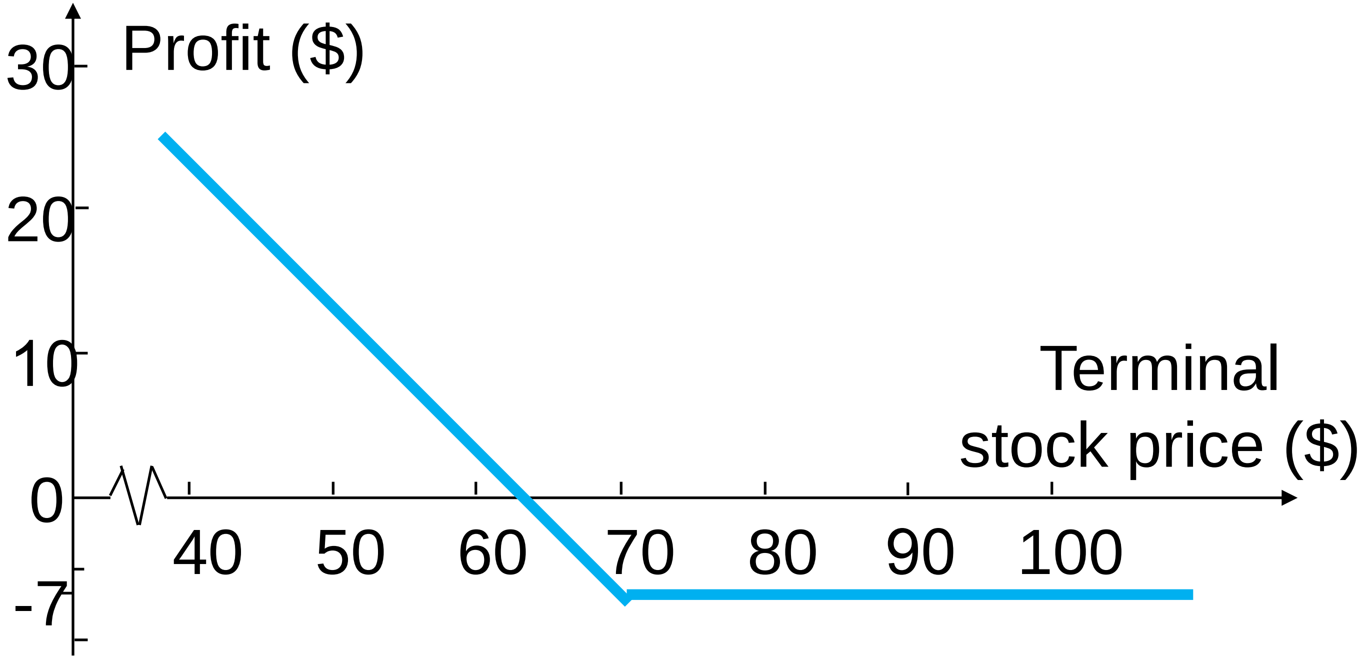
Profit Profile of a Short Call on Microsoft

Profit from writing a Microsoft European call option: option price = $5, strike price = $100



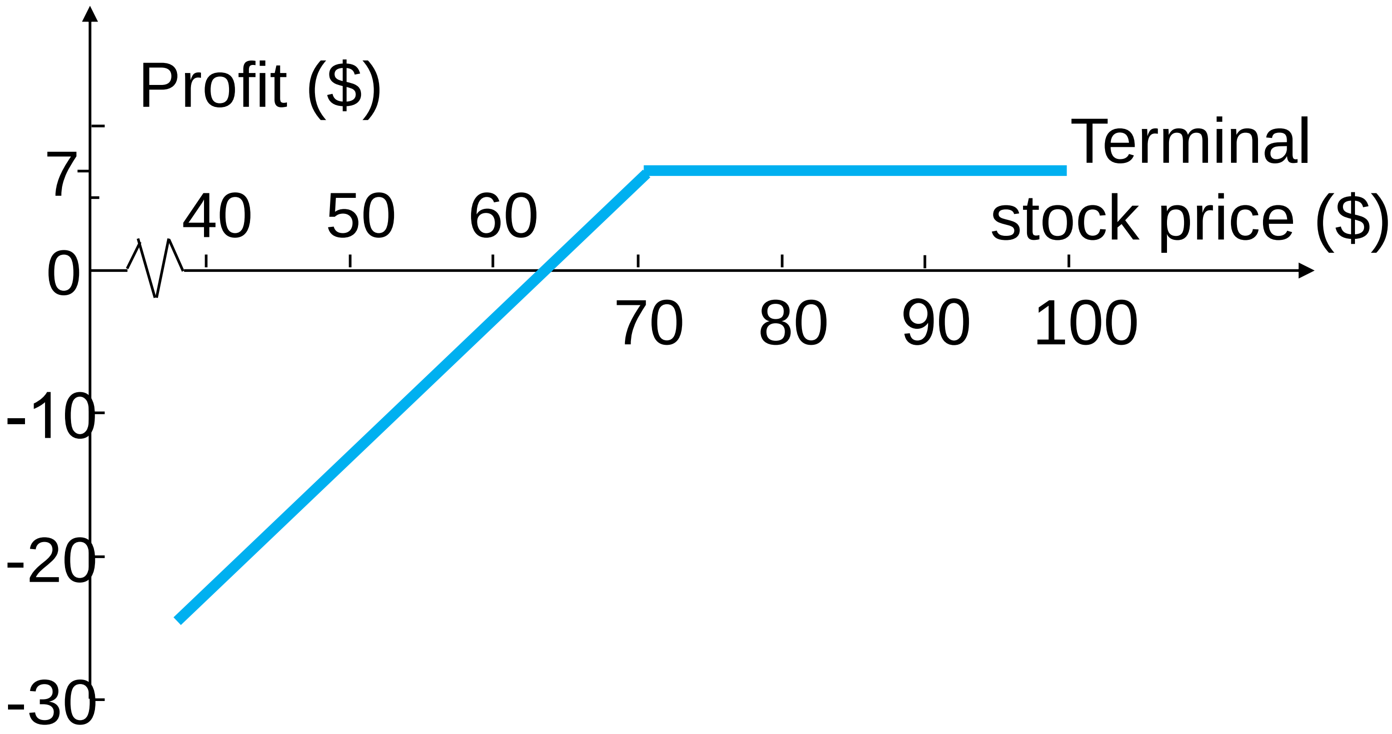
Profit Profile of a Long Put on Oracle

Profit from buying an Oracle European put option: option price = $7, strike price = $70



Profit Profile of a Short Put on Oracle

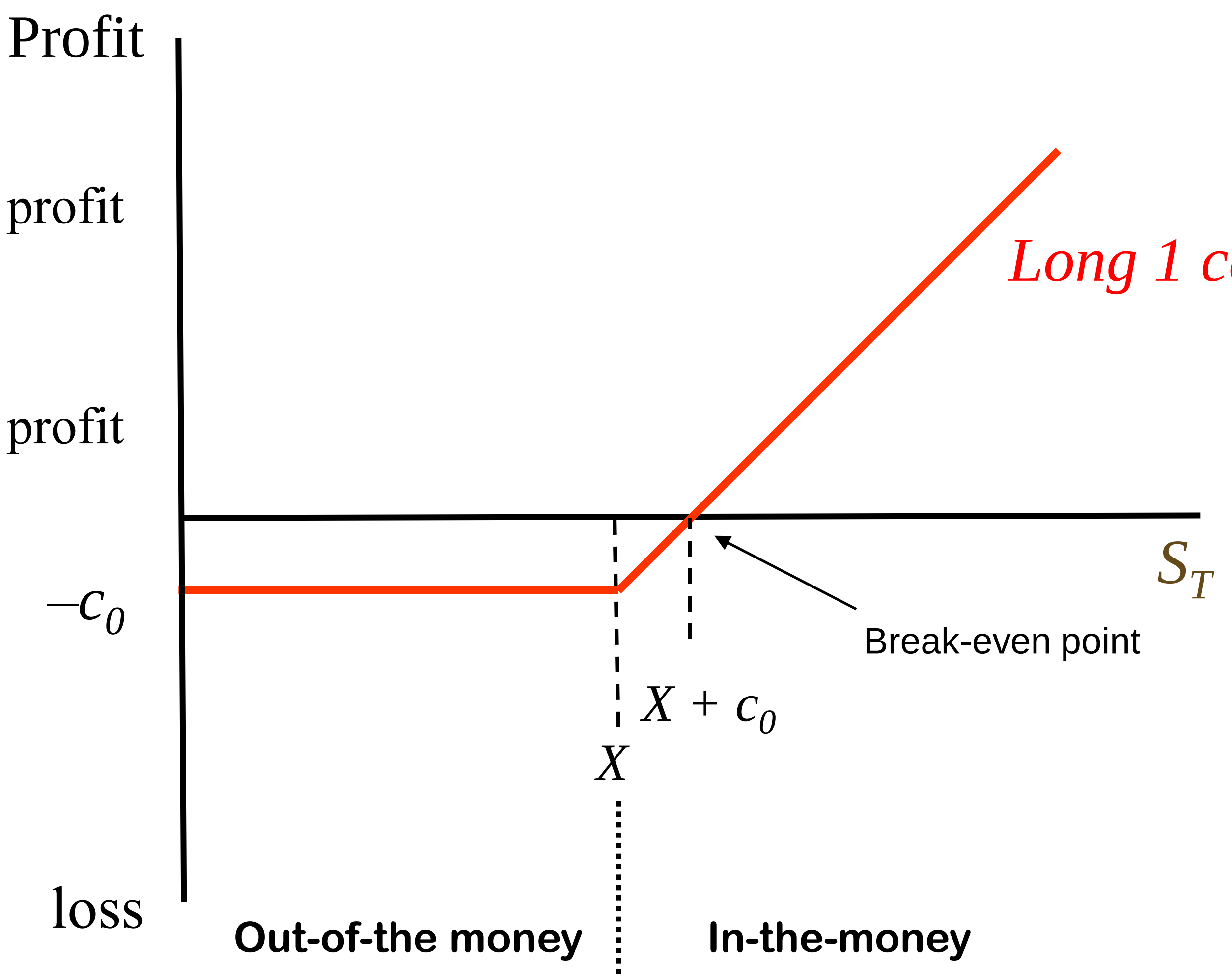
Profit from writing an Oracle European put option: option price = $7, strike price = $70



Long Call Option Profit Profile

If $S_T > X$, call is exercised, & buyer's profit is $\pi = (S_T - X) - c_0$.

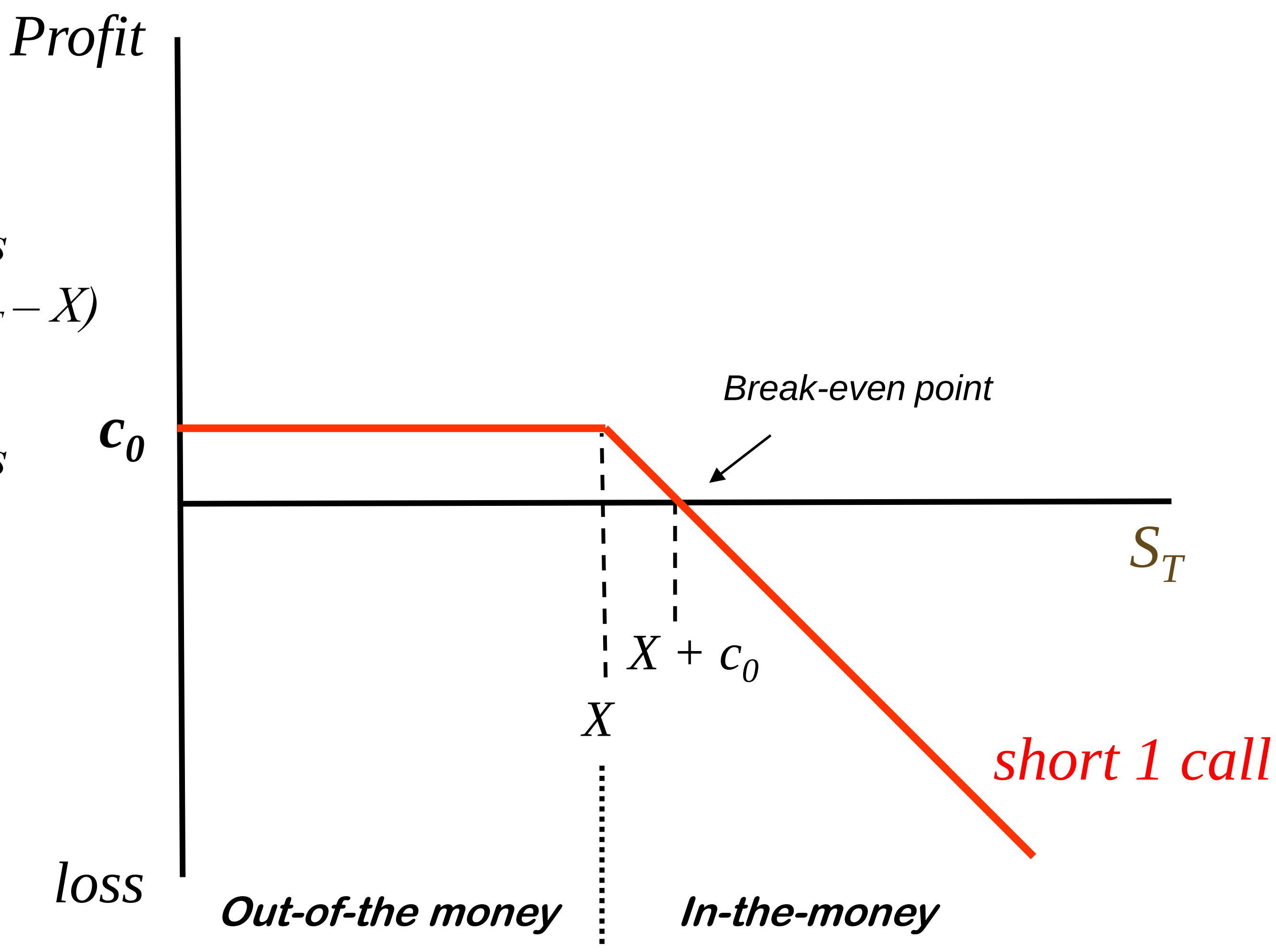
If $S_T < X$, call is not exercised, & buyer's profit is $\pi = - c_0$



Short Call Option Profit Profile

If $S_T > X$, call is exercised, & writer's profit is $\pi. = c_0 - (S_T - X)$

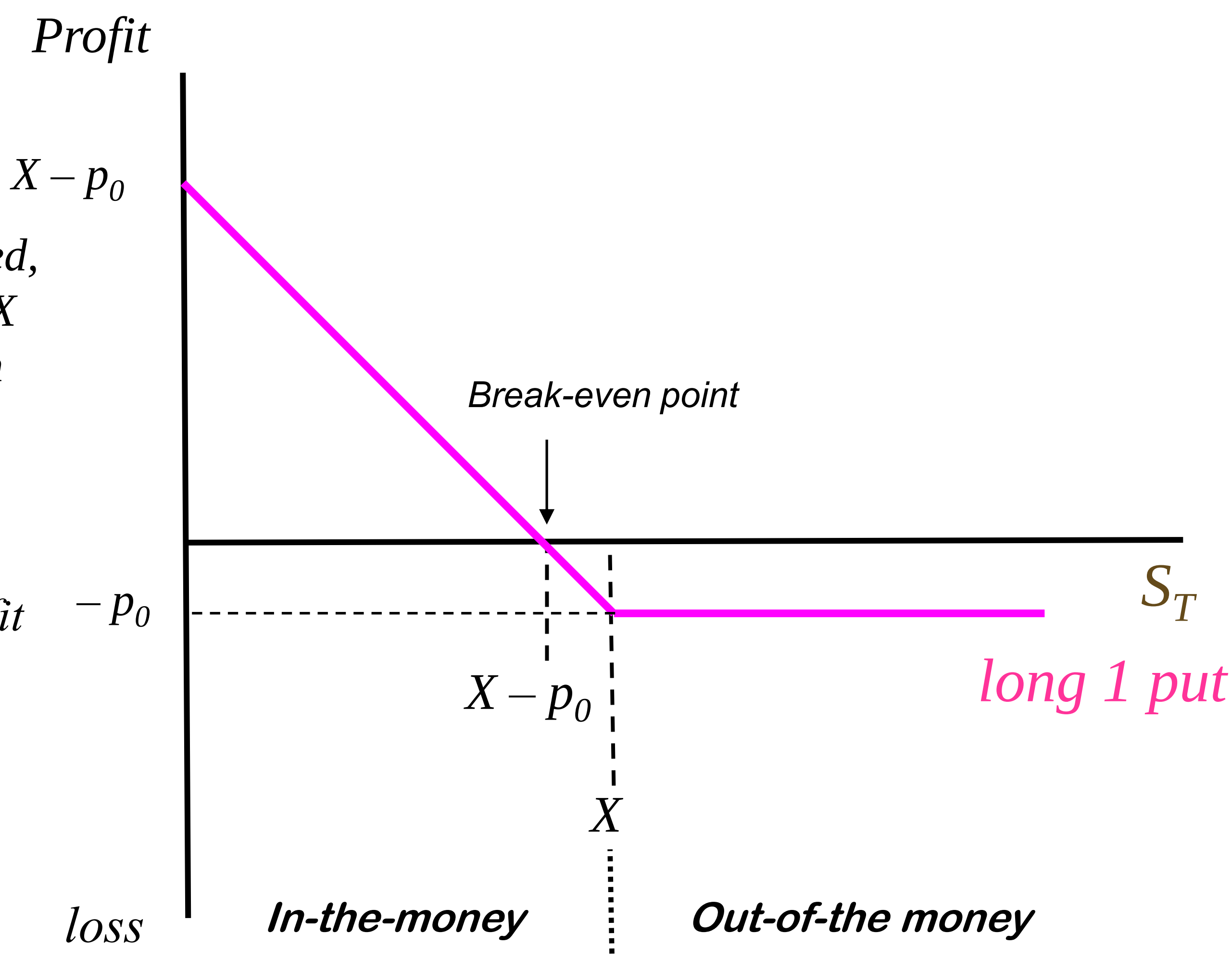
If $S_T < X$, call is not exercised, & writer's profit $\pi = c_0$



Long Put Profit Profile

If $S_T < X$, put is exercised, & buyer's profit is $\pi = (X - S_T) - p_0$. The maximum profit is $(X - p_0)$

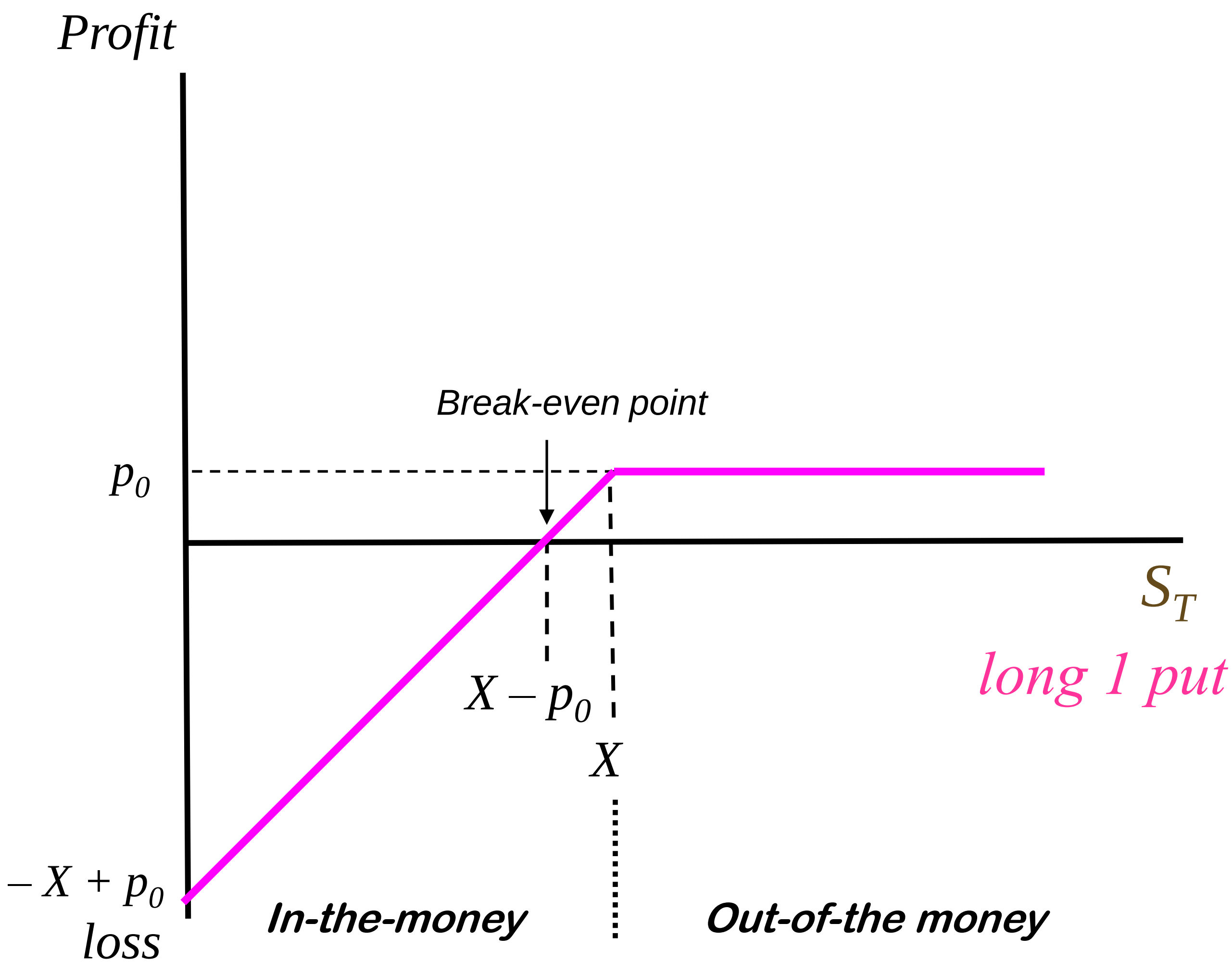
If $S_T > X$, put is not exercised & buyer's profit is $\pi = -p_0$.



Short Put Profit Profile

If $S_T < X$ *, put is exercised & seller's profit is*
$\pi = p_0 - (X - S_T)$.
The maximum loss is
$-X + p_0$

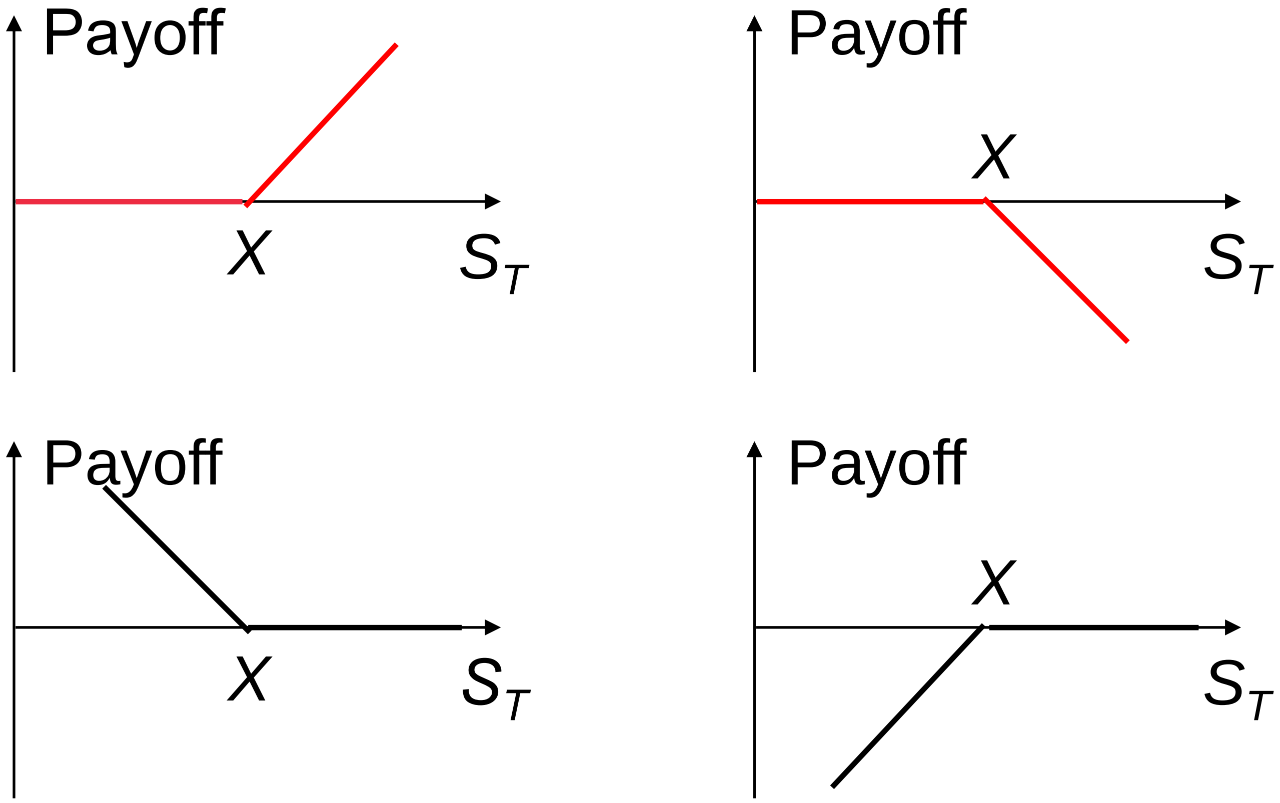
If $S_T > X$*, put is not exercised & seller's profit is*
$\pi = p_0$



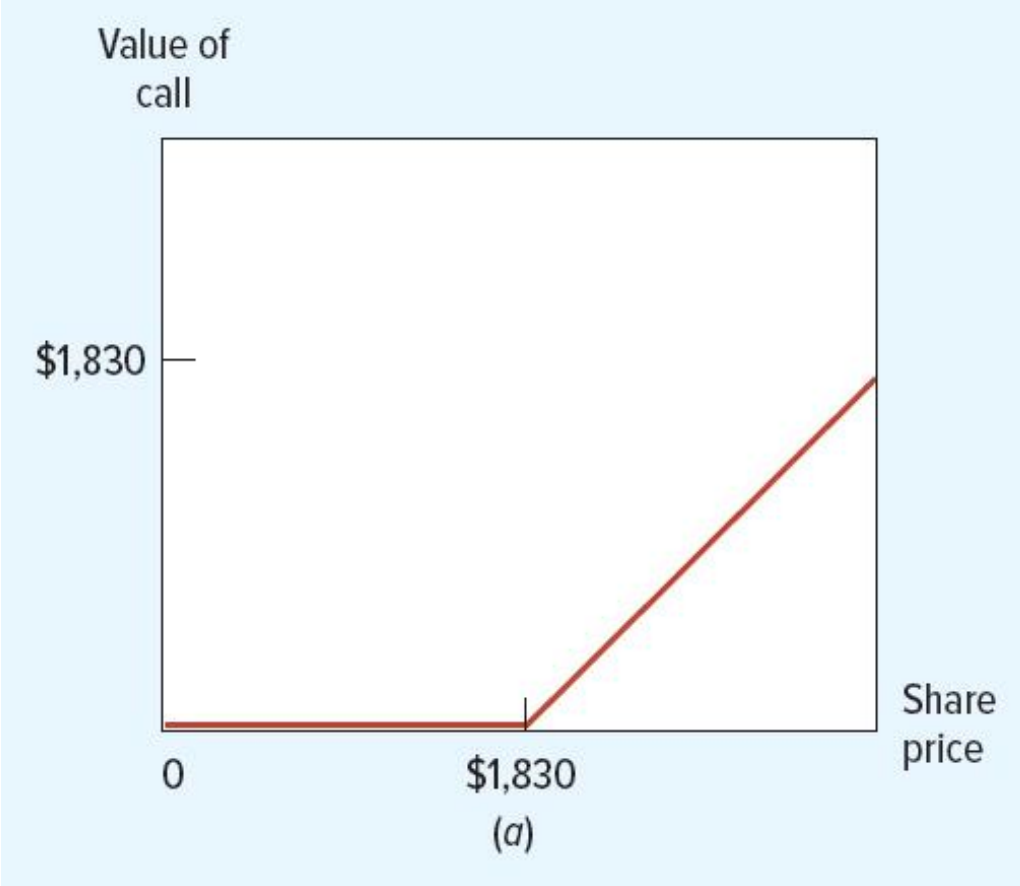
Payoffs (not Profits) from Options

What is the Option Position in Each Case?

X = Strike price, S_T = Price of asset at maturity

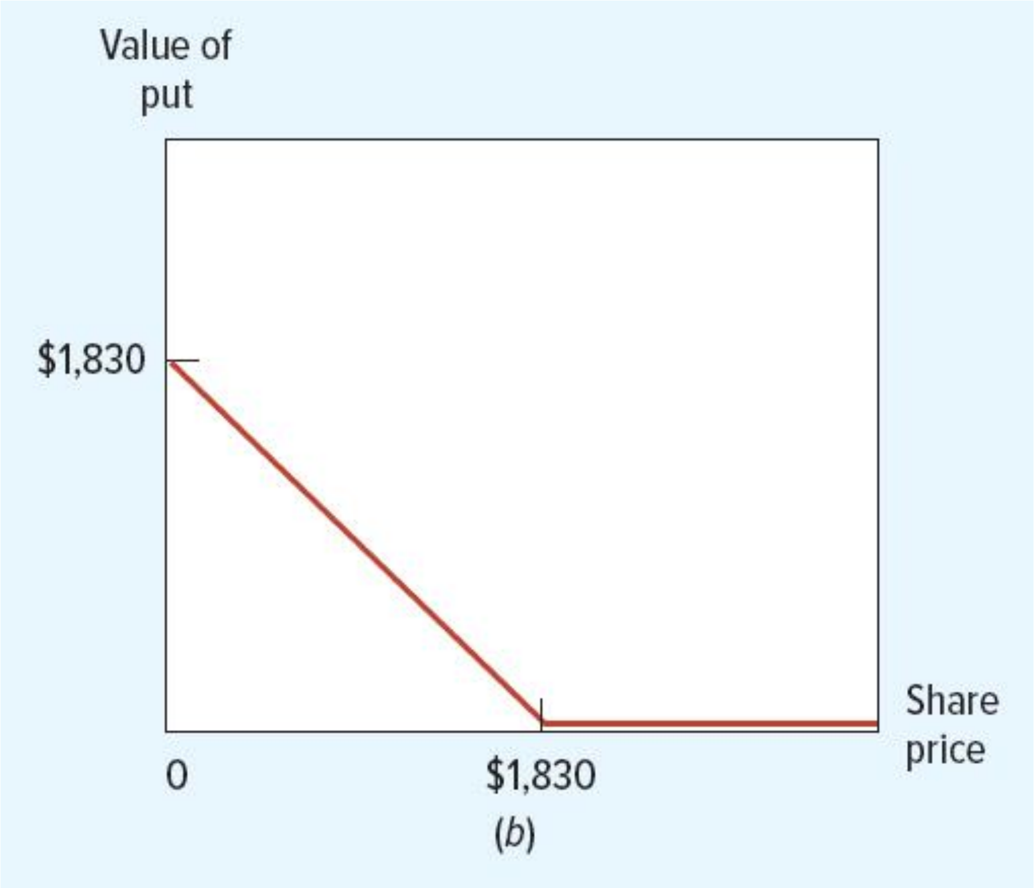


Payoff to Buyer of a Amazon Call Option



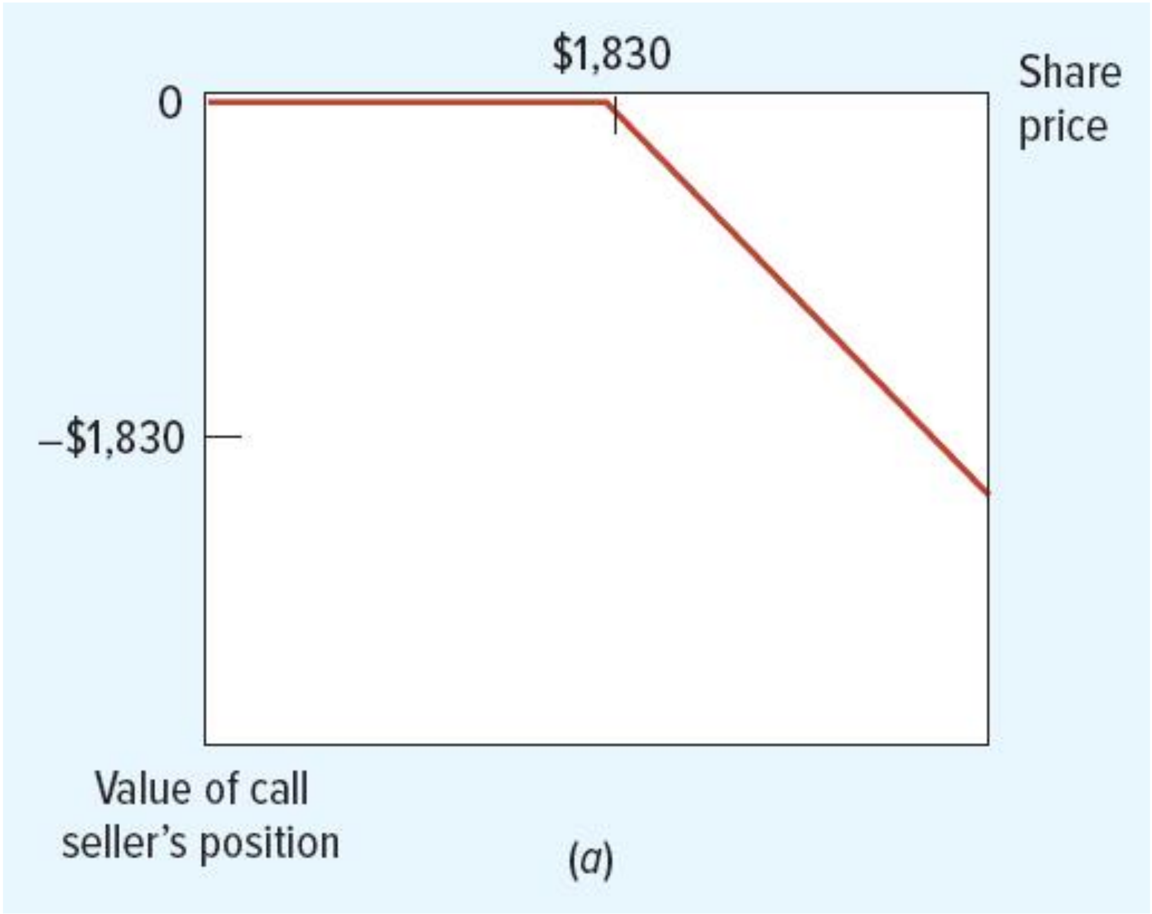
Position diagram showing payoffs from buying an Amazon call option with exercise price at $1,830.

Payoff to Buyer of an Amazon Put Option



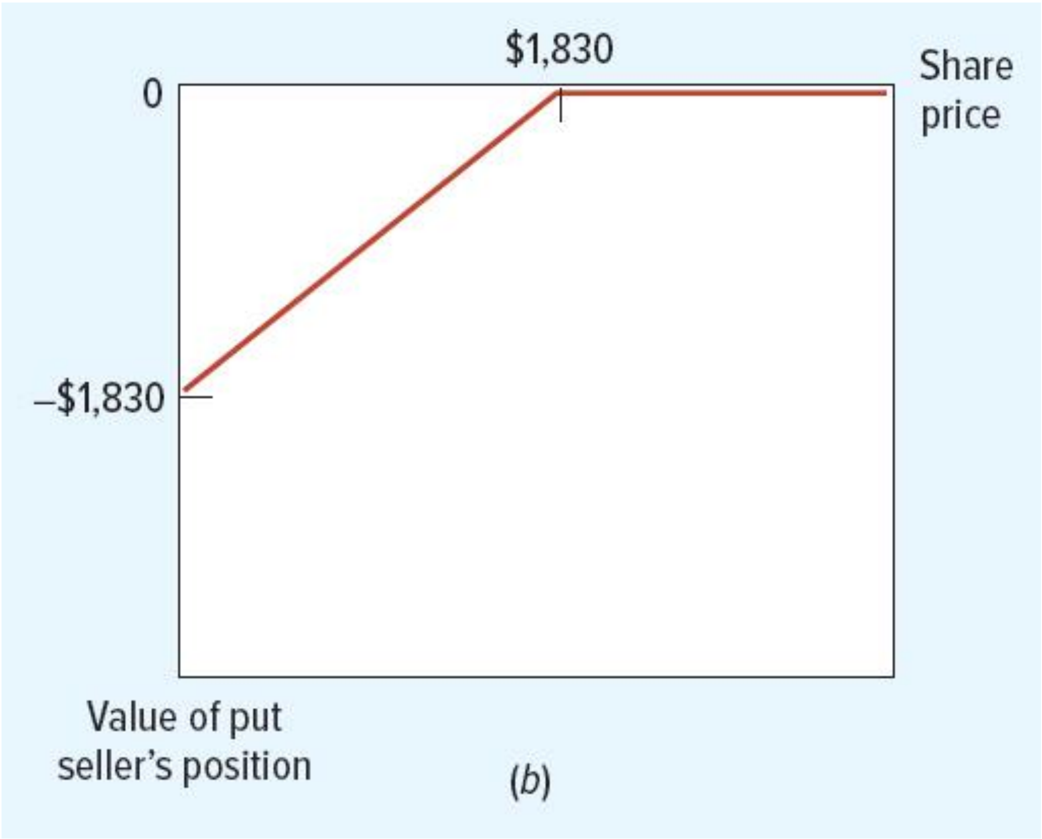
Position diagram showing payoffs from buying an Amazon put option with exercise price at $1,830.

Payoff to Seller of an Amazon Call Option



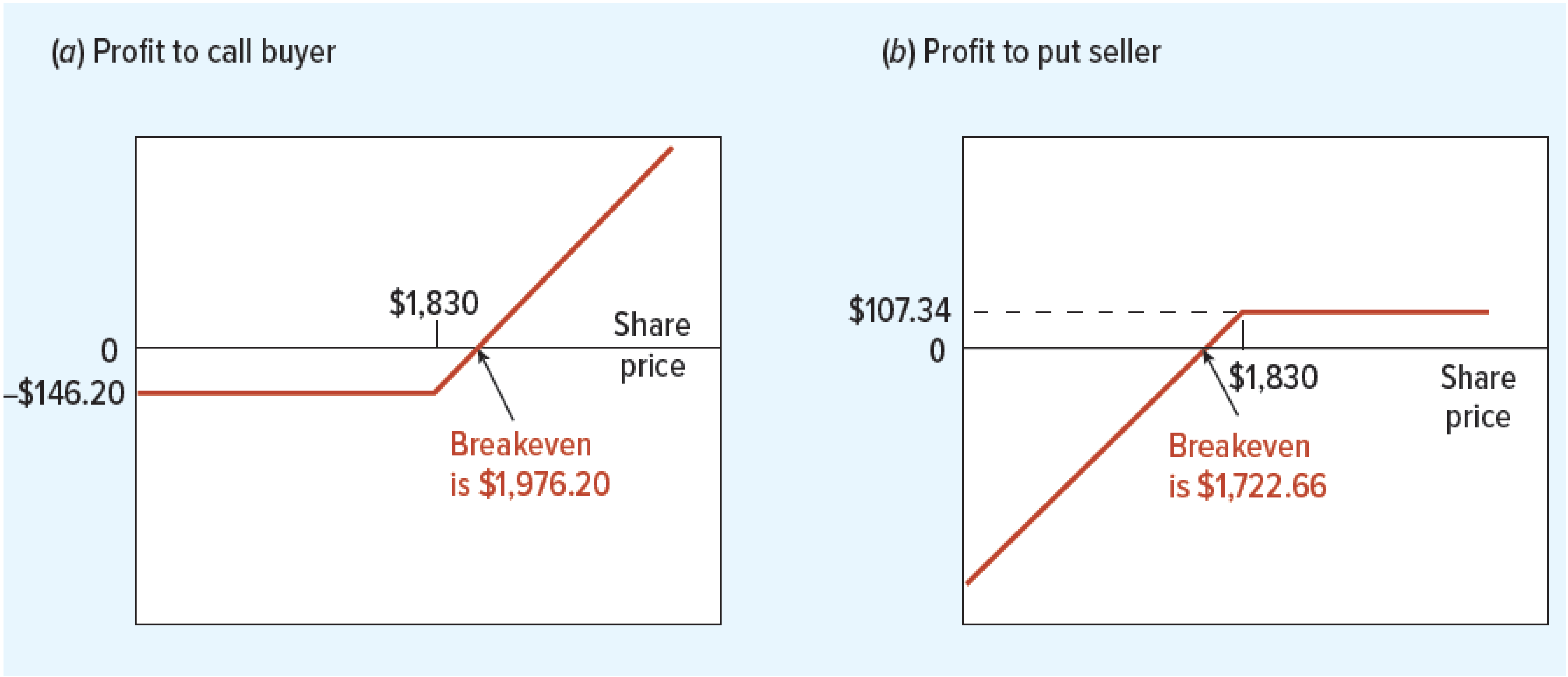
Position diagram showing payoffs from selling an Amazon call option with exercise price at $1,830.

Payoff to Seller of an Amazon Put Option



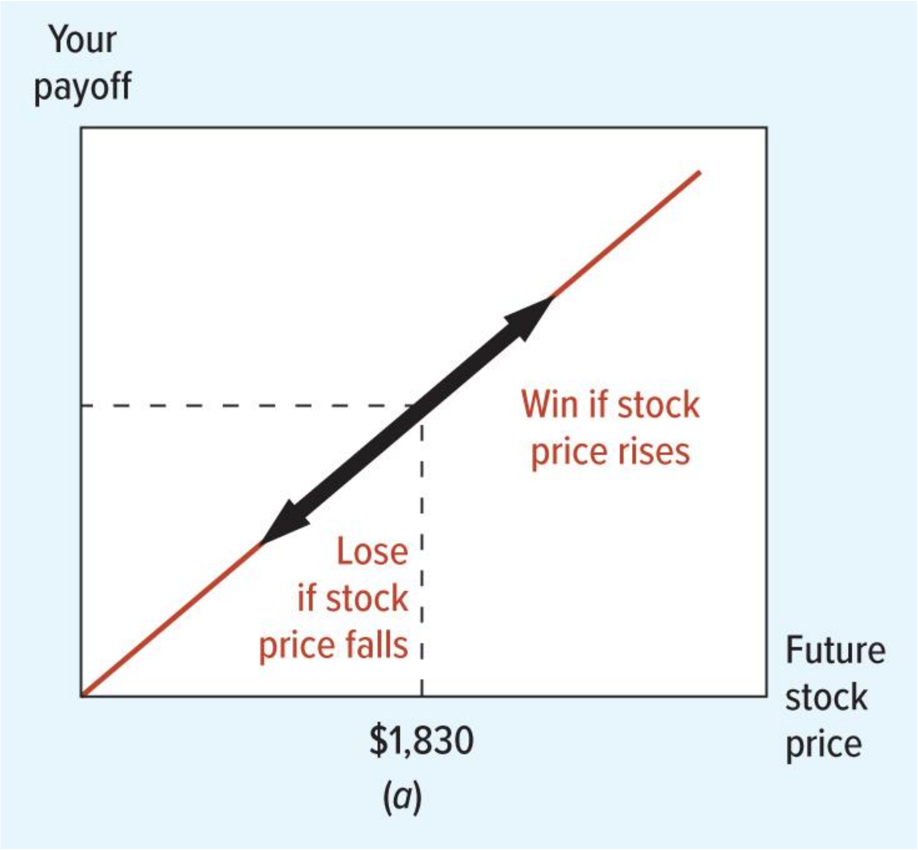
Position diagram showing payoffs from selling an Amazon put option with exercise price at $1,830.

Payoff Diagrams Are Not Profit Diagrams



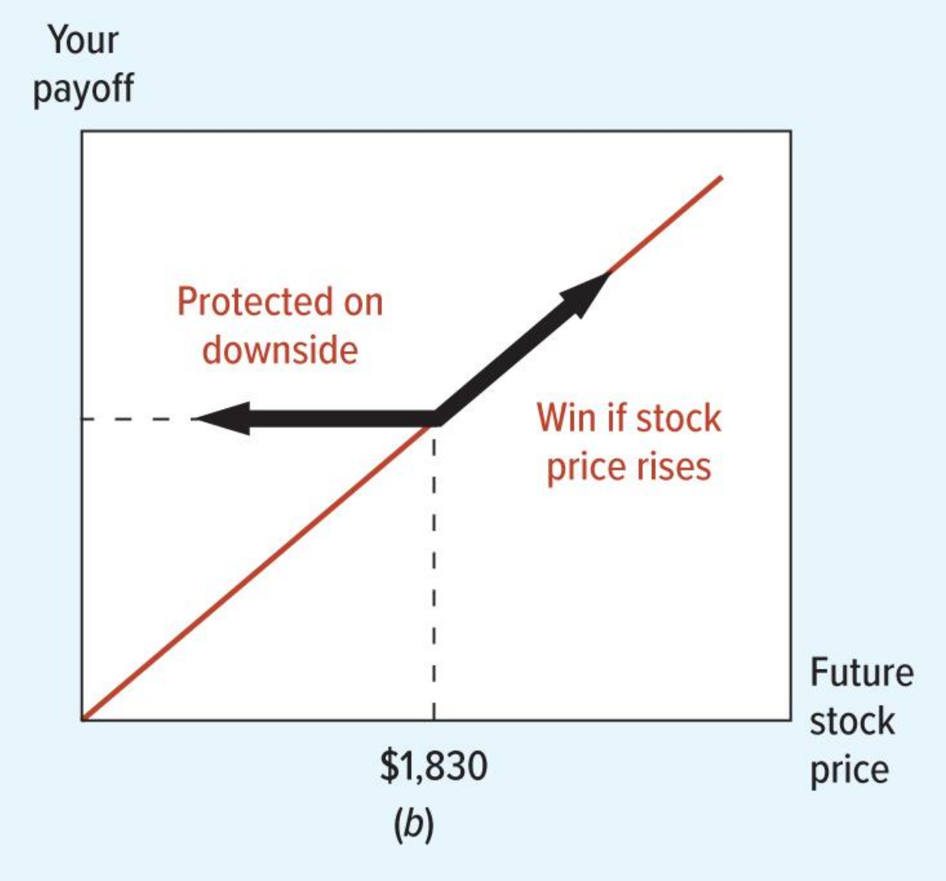
Profit diagrams incorporate the costs of buying an option or the proceeds from selling one. In panel (a), we subtract the $146.20 cost of the Amazon call from the payoffs plotted in Figure 21.1a. In panel (b), we add the $107.34 proceeds from selling the Amazon put to the payoffs in Figure 21.2b.

Figure 21.4 (a) Amazon



Payoff at the end of six months of investment strategy involving purchase of one share Amazon for $1,830 (Long 1 Amazon share). You gain dollar-for-dollar if the share price goes up and you lose dollar-for-dollar if it falls (45-degree line).

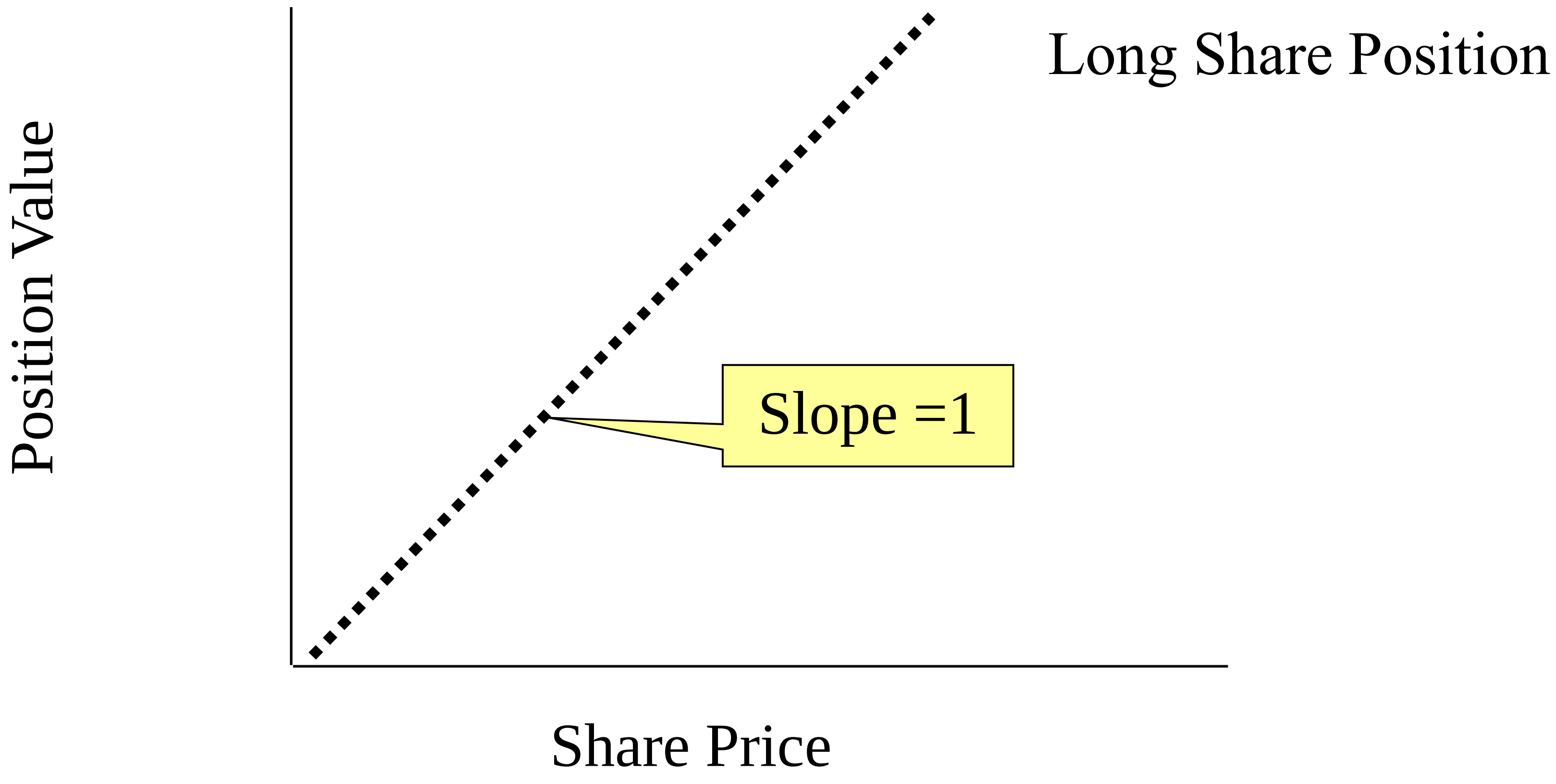
Figure 21.4 (b) Amazon



Payoffs at the end of six months payoffs from an investment strategy that retains the upside potential of Amazon stock but gives complete downside protection. Your payoff stays at $1,830 even if the Amazon share price falls to $1,700, $1,000, or zero.

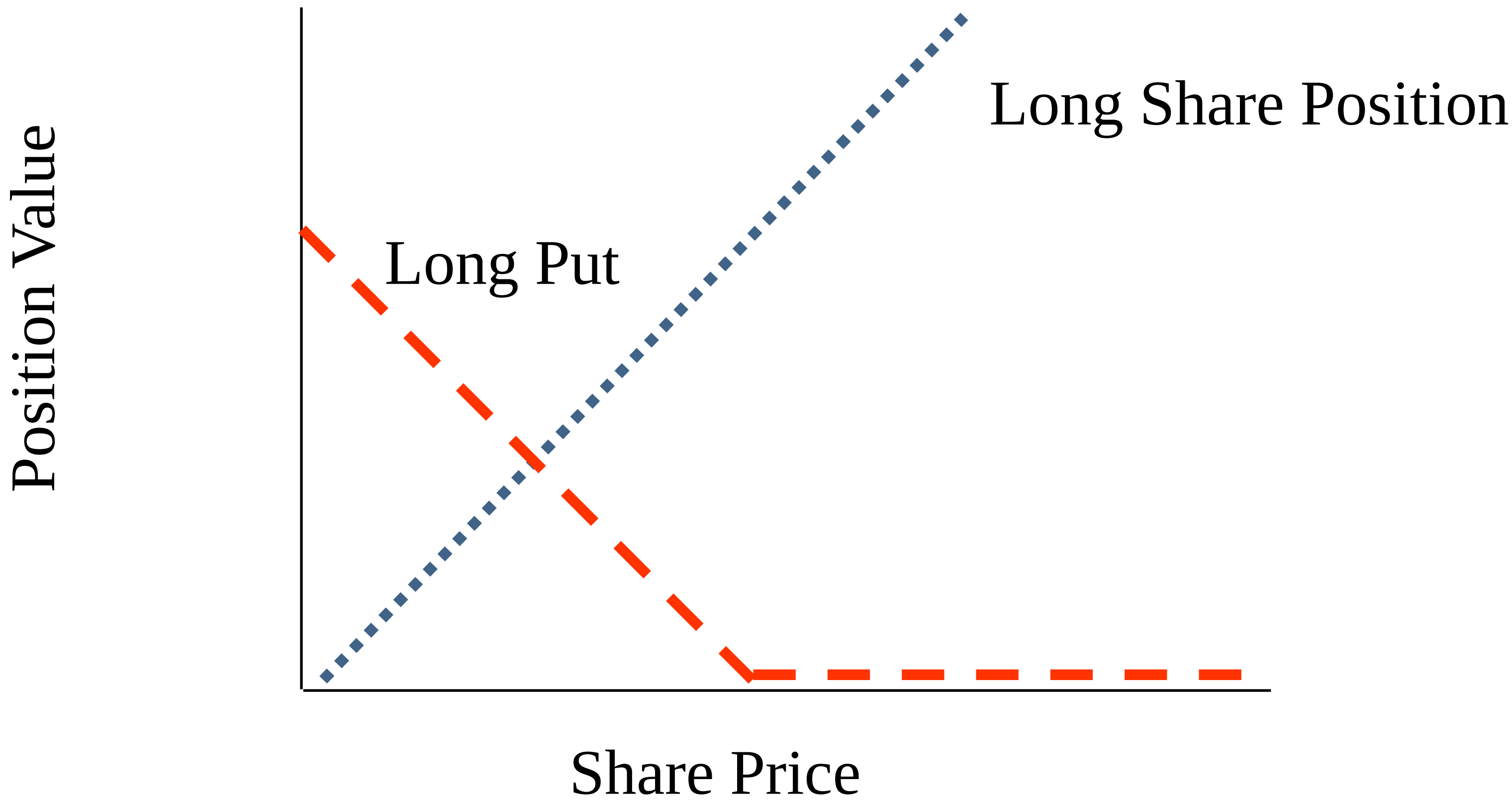
Panel b's payoffs are clearly better than panel a's. If a financial alchemist could turn panel a into panel b, you'd be willing to pay for the service. Such a transformation could be achieved with options.

LONG SHARE POSITION PAYOFF



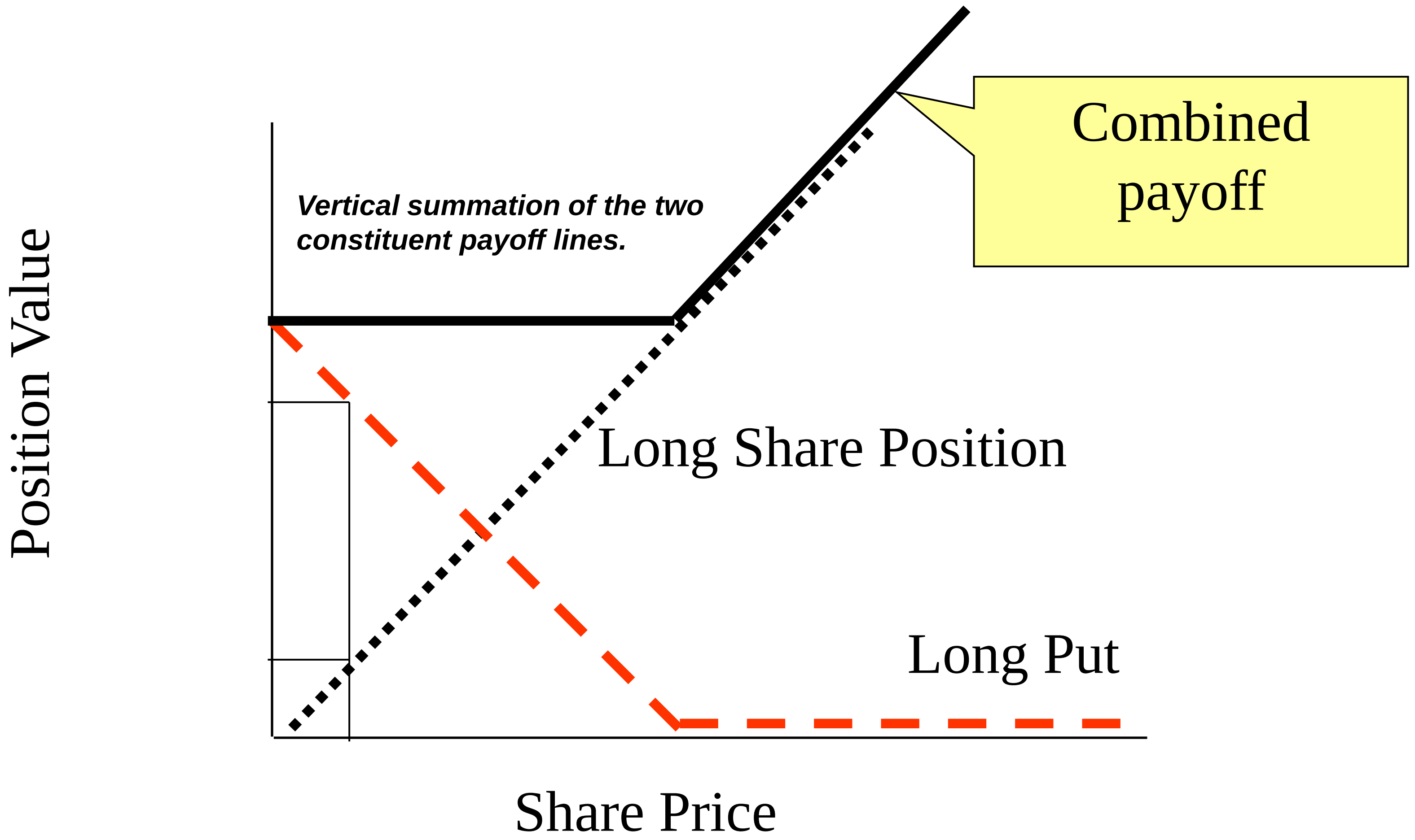
LONG SHARE PLUS LONG PUT PAYOFF

To put floor under the share price: Buy the share and a put option - **Protective Put**

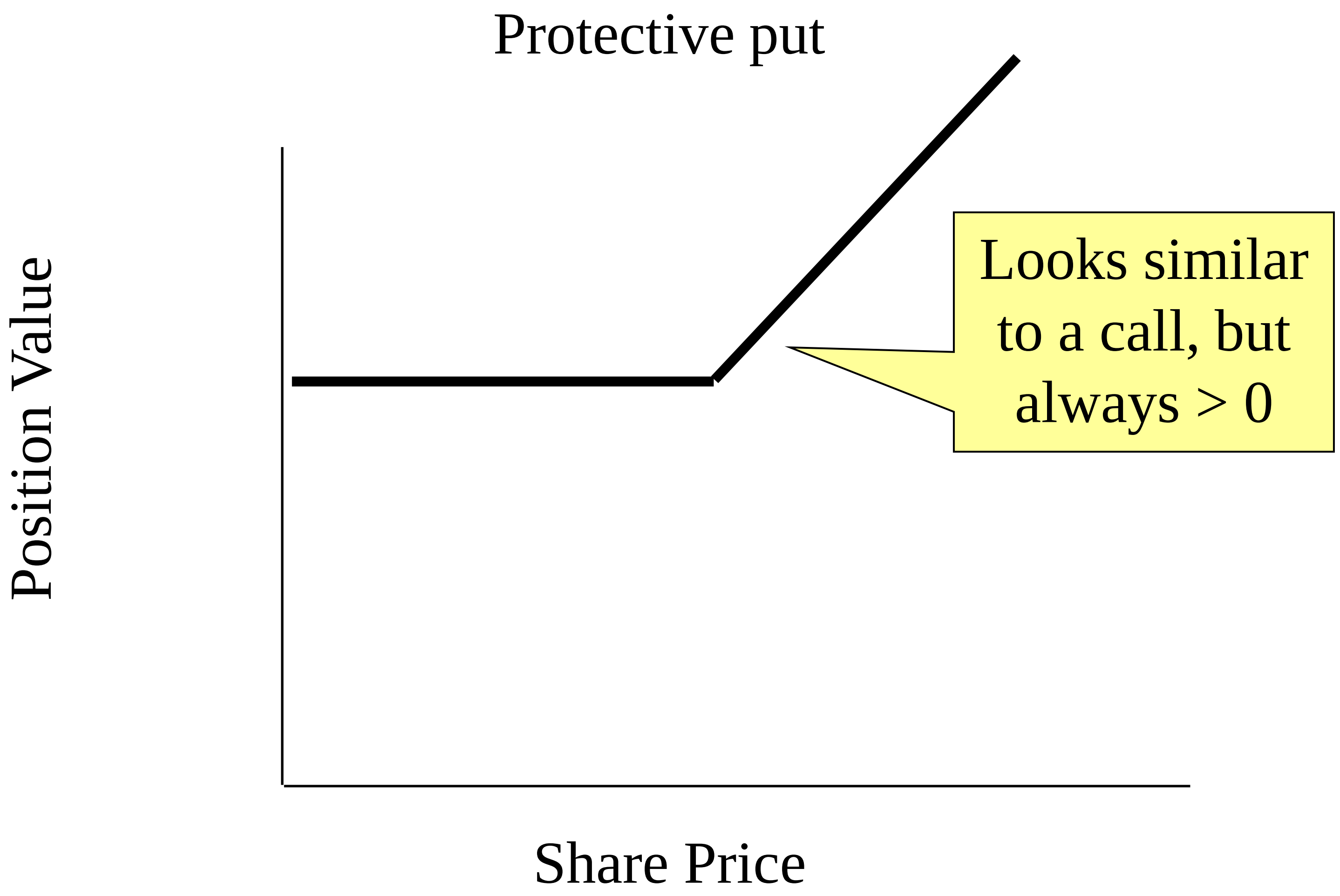


LONG SHARE PLUS LONG PUT PAYOFF

Protective Put - Buy the share and buy a put option

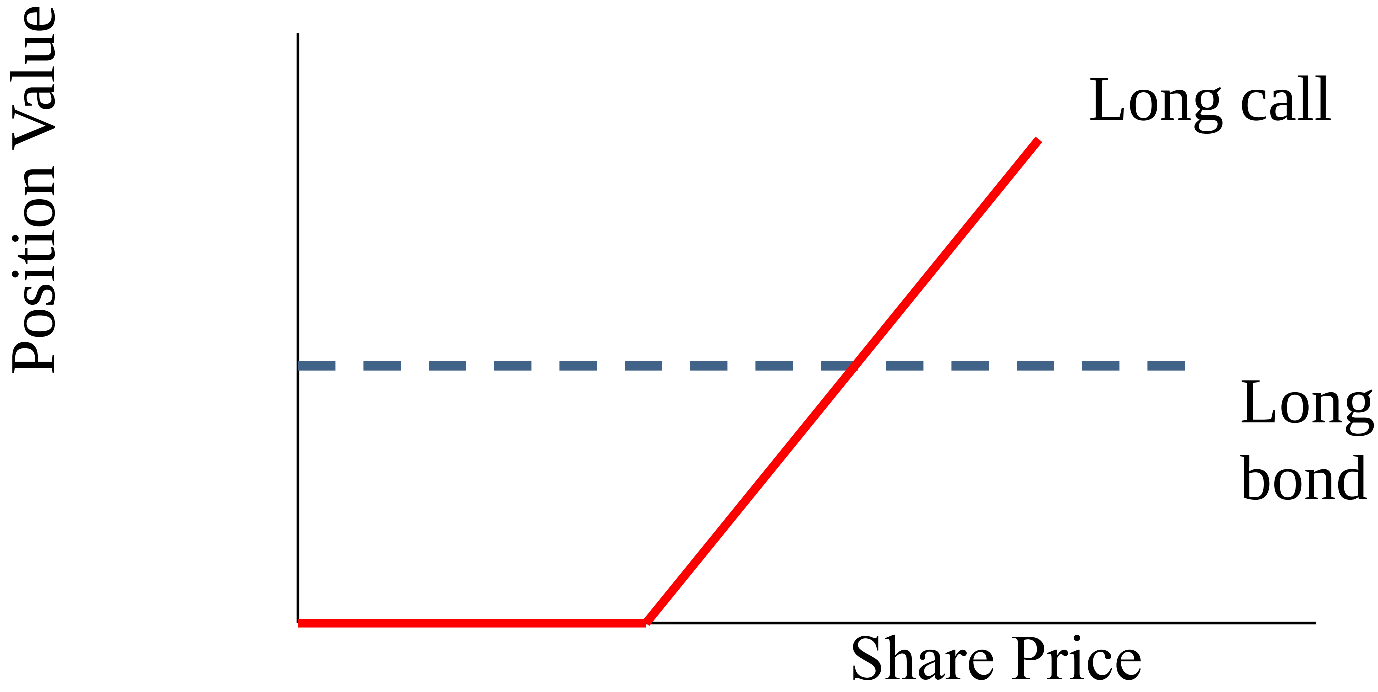


PROTECTIVE PUT PAYOFF



SHARE PLUS BOND

Buy call and buy a bond with value = PV(Exercise)



SHARE PLUS BOND

Long call plus long bond with value PV(Exercise)

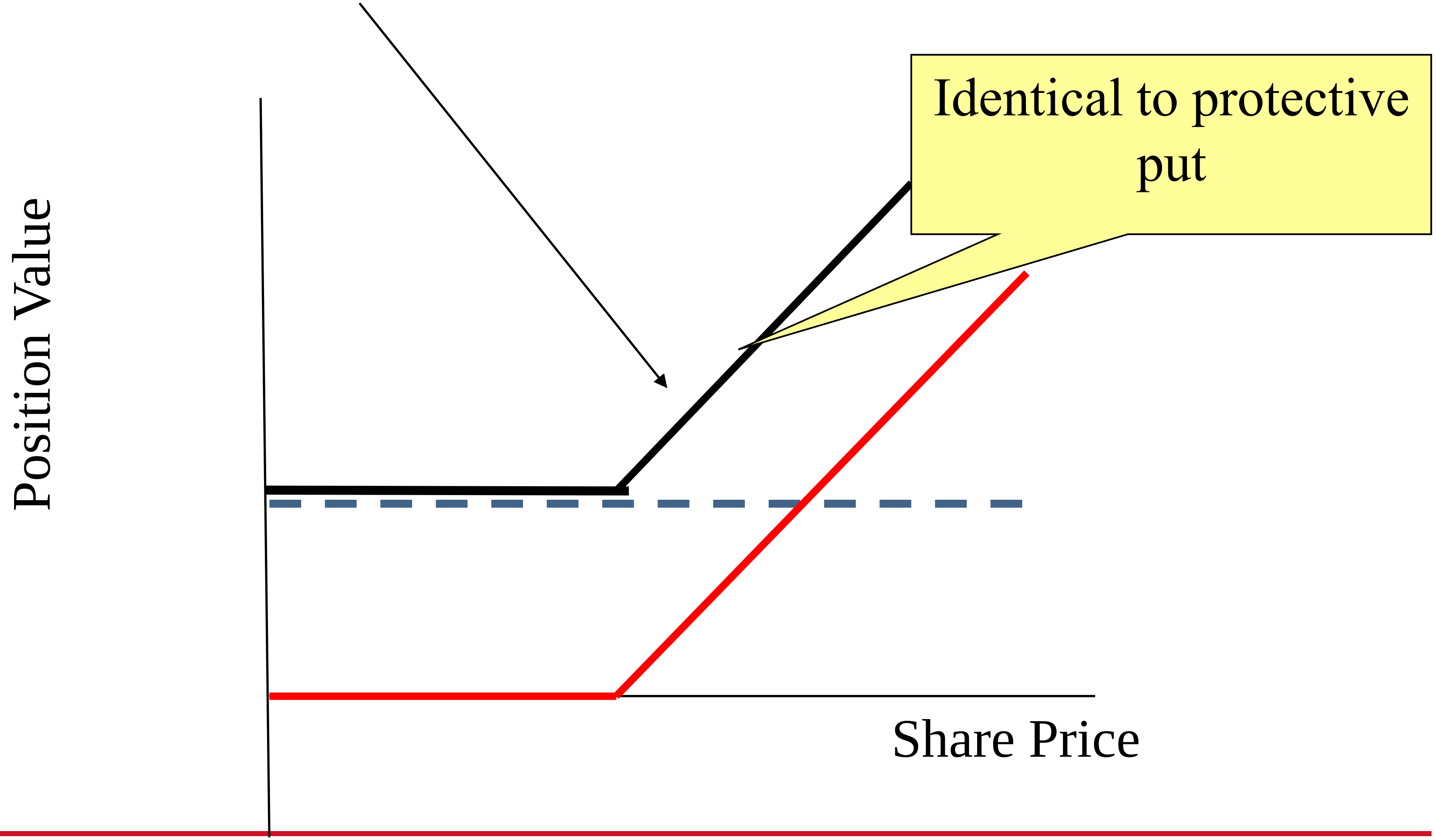
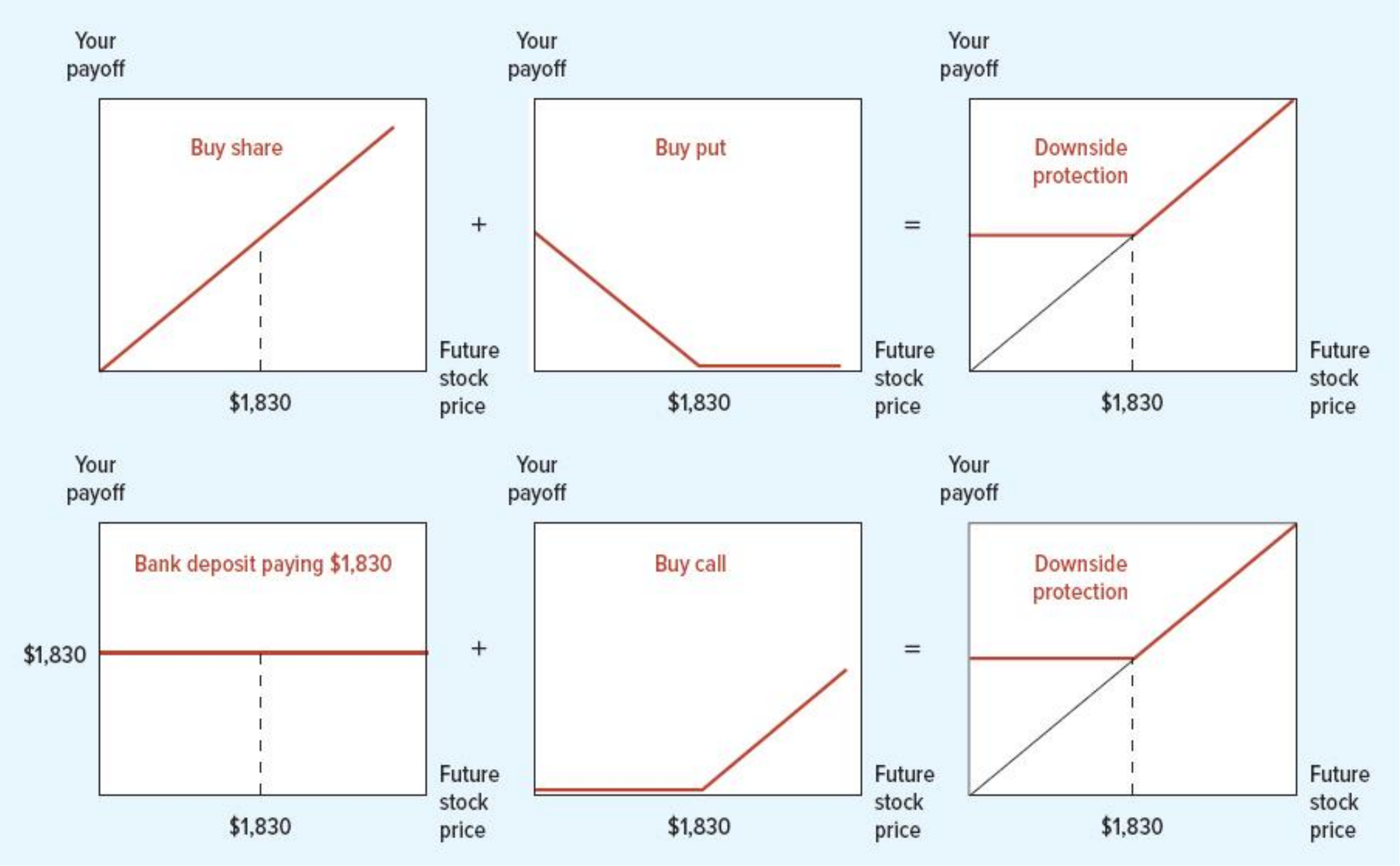


FIGURE 21.5 PROTECTION STRATEGIES
Financial Alchemy With Options



Each row in the figure shows a different way to create a strategy where you gain if the share price rises but are protected on the downside (strategy b in Figure 21.4).

EQUIVALENCE OF PAYOFFS

The conclusion from the previous payoff diagrams:

If you:

i. *buy the share, &*

ii. *buy a put option to sell the share at maturity for the exercise price,*

you will have the <u>same</u> payoff as if you had:

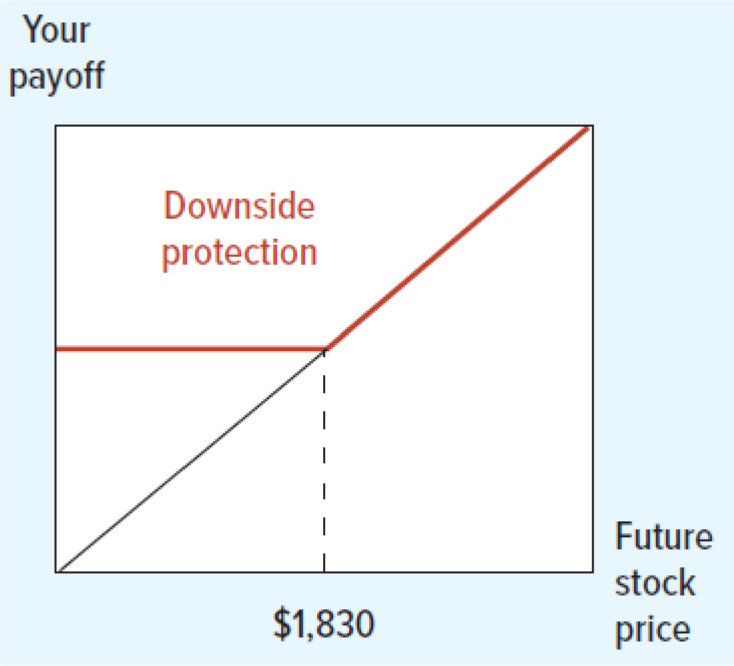
i. *bought a call option with an identical maturity, &*

ii. *invested sufficient money to pay for the share (at the exercise price) at maturity*

Payoff of ***[Buy put, buy share]*** = Payoff of ***[Buy call, invest present value of exercise price in safe asset]***

IDENTICAL OUTCOMES

- In both positions the same outcome results:
  - S > X end up holding the share
  - S < X end up with cash equal to X
  - S = X share and cash of X are equivalent



PUT CALL PARITY

- Since the payoffs of the two positions are identical, the value of the two positions must also be identical. Therefore:
  - *Value of call + present value of exercise price*

    = Value of put + share price

- This relationship is called **put call parity**.

Put-Call Parity; No Dividends

- Relationship between the price of European put and call options, possessing the same maturity and same exercise price and written on the same underlying share of stock.
- Important for valuation of European options: if we know the value of a European call option, put-call parity relationship also allows us to determine value of corresponding put option.

Put-Call Parity; No Dividends

- Consider the following 2 portfolios:
  - *Portfolio A: European call on a stock + zero-coupon bond that pays* X *at time* T
  - *Portfolio C: European put on the stock + the stock*

Values of Portfolios

		$S_T > X$	$S_T < X$
Portfolio A	Call option	$S_T - X$	0
	Zero-coupon bond	X	X
	Total	S_T	X
Portfolio C	Put Option	0	$X - S_T$
	Share	S_T	S_T
	Total	S_T	X

***Value*$_{\textit{Portfolio A}}$ = *Value*$_{\textit{Portfolio C}}$** in both states of nature

The Put-Call Parity Result

- Both are worth max(S_T , X) at the maturity of the options
- They must therefore be worth the same today. This means that:

Portfolio A ***Portfolio C***

[Value of call + present value of exercise price] = [value of put + share price]

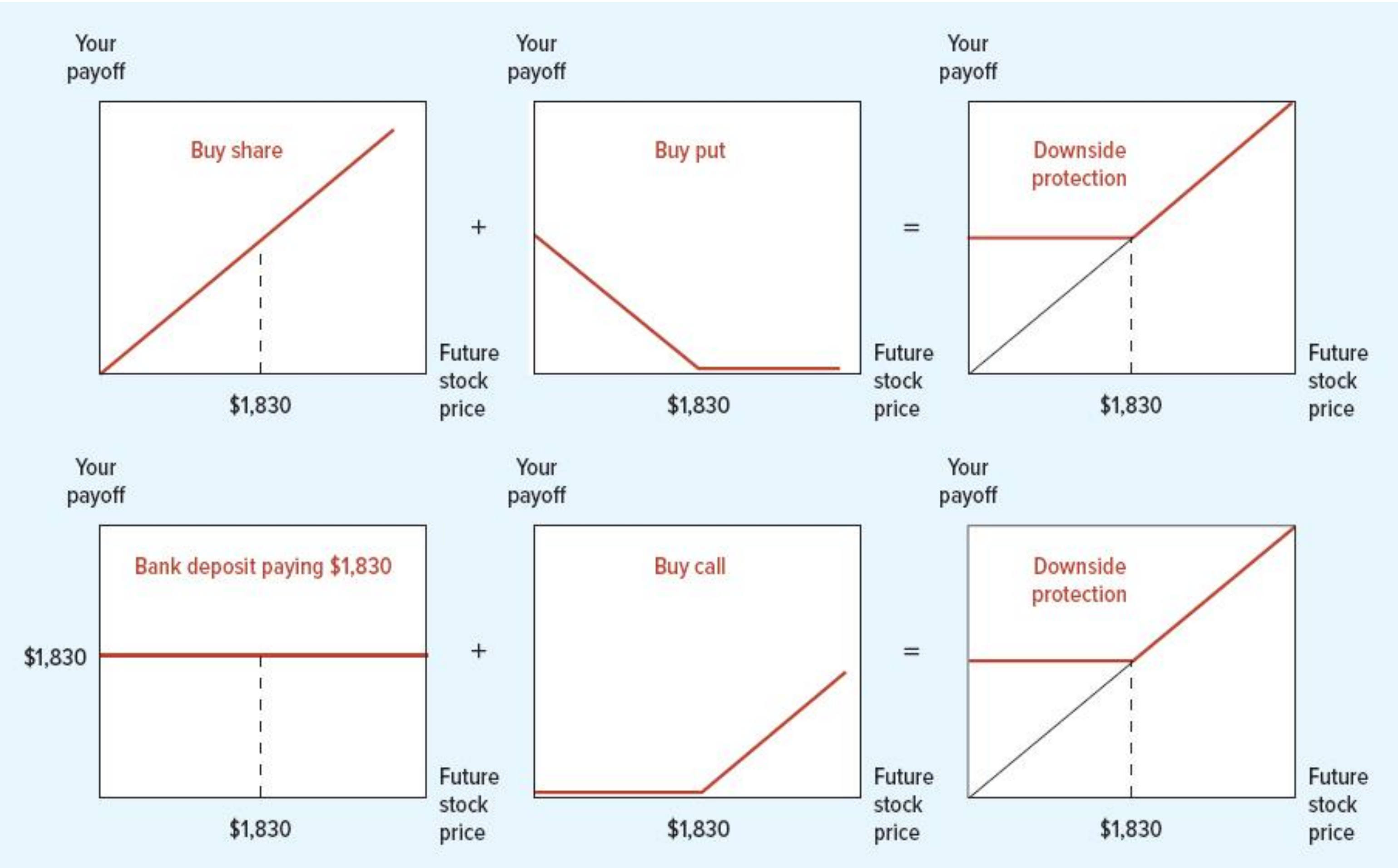
$$c + PV(X) = p + S_0$$

$$c + Xe^{-rT} = p + S_0 \qquad (\textit{continuous compounding})$$

$$c + X(1+r)^{-T} = p + S_0 \qquad (\textit{discrete compounding})$$

The Put-Call Parity Result

$$c + Xe^{-rT} = p + S_0$$



$S_0 + p$

$c + Xe^{-rT}$

Equivalent Versions of Put-Call Parity

Share price + value of put - value of call = present value of exercise price

Why?

Payoff from [***buy share, buy put, sell call***] is identical to [***invest PV of exercise price in safe asset***]

$$S_0 + p - c = PV(X)$$

$$S_0 + p - c = Xe^{-rT}$$

$$S_0 + p - c = X(1+r)^{-T}$$

Equivalent Versions of Put-Call Parity

Value of call - value of put = share price - present value of exercise price

Why?

Because strategies [***buy call, sell put***] & [***buy share, borrow PV of exercise price***] have identical payoffs.

$$c - p = S_0 - PV(X)$$

$$c - p = S_0 - Xe^{-rT}$$

$$c - p = S_0 - X(1+r)^{-T}$$

Equivalent Versions of Put-Call Parity

Value of put = value of call - share price + present value of exercise price

Why?

Payoff from [***buy put***] is identical to payoff from [***buy call, sell share, invest PV of exercise price***]

$$p = c - S_0 + PV(X)$$

Synthetic long put position

$$p = c - S_0 + Xe^{-rT}$$

$$p = c - S_0 + X(1+r)^{-T}$$

Equivalent Versions of Put-Call Parity

Share price = Value of call + present value of exercise price - value of put

Why?

Because strategies [***buy call, invest PV of exercise price in safe asset and sell put***] & [***buy share***] have identical to payoffs.

$$S_0 = c + PV(X) - p$$

Synthetic long share position

$$S_0 = c + Xe^{-rT} - p$$

$$S_0 = c + X(1+r)^{-T} - p$$

Equivalent Versions of Put-Call Parity

Value of call = Share price + value of put - present value of exercise price

Why?

Because strategies [***buy share, borrow PV of exercise price and buy put***] & [***buy call***] have identical to payoffs.

$$c = S_0 + p - \mathrm{PV}(X)$$

$$c = S_0 + p - Xe^{-rT}$$

$$c = S_0 + p - X(1+r)^{-T}$$

Synthetic long call position

Determinants of Option Premiums

- The value or price of an option (option premium) consists of 2 basic components:
- ***Intrinsic Value***
- ***Time Value***

Option Premium	=	Intrinsic Value	+	Time Value

Intrinsic Value

- Relationship between current market price of underlying asset and exercise price of the option ie the "money-ness" of the option.
- The payoff that could be obtained if option was exercised immediately.
- Greater the intrinsic value of option, the higher would be the value of the option, and hence the greater the option premium.
  - If option is "out-of-the-money" $\Rightarrow$ intrinsic value is zero.
- Intrinsic value cannot be less than zero.

Intrinsic Value or 'Moneyness' of Option

In-the-money option

- Option is *in-the-money* if it can be exercised at a positive payoff, excluding premium costs. [*in-the-money* option has a positive intrinsic value].

At-the-money option

- Option is *at-the-money* if market price of underlying asset is equal to exercise price.

Out-of-the-money option

- Option is out-of-the-money if exercise of the option will lead to a negative payoff or loss, excluding premium costs.
  ["*out-of-the-money*" option has zero intrinsic value].

Intrinsic Value or ‘Moneyness’ of Option

The “*money-ness*” of options

Situation	**Call option**	***Put Option***
In-the-money	$S_t > X$	$S_t < X$
At-the-money	$S_t = X$	$S_t = X$
Out-of-the-money	$S_t < X$	$S_t > X$

- “*near-the-money*” ← if stock price is near exercise price
- “*deep-in-the-money*” ← for call $S_t >> X$; for put $S_t << X$

Time Value

- Before expiry, an option will usually trade for a price greater than its intrinsic value. This part of the option premium over and above its intrinsic value is called option's ***time value***.

Time value = Option price – intrinsic value

- ***Time value***: What investors are prepared to pay for the potential to gain in the future from favourable price movements.
- The amount of money option buyers are willing to pay for an option in the anticipation that over time a favourable change in the underlying price of the asset rate will cause the option to increase in value.
- The greater the time value, the greater is the chance to exercise the option at a positive payoff and hence the more valuable is the option.

Factors Affecting Price of a Stock Option

- Current Stock Price
- Exercise or Strike Price
- Time to Expiration
- Volatility of Stock Price
- Risk-free Interest Rate
- Expected Dividends on Stock

Current Stock Price

Call

- The higher the price of the underlying asset, the higher the intrinsic value of the option and hence the larger the call option’s premium.

Put

- The lower the price of the underlying asset, the higher the intrinsic value of the option and hence the larger the put option’s premium.

Exercise Price

Call

- The lower the option exercise price, the higher the intrinsic value of the option and hence the larger the call option’s premium.

Put

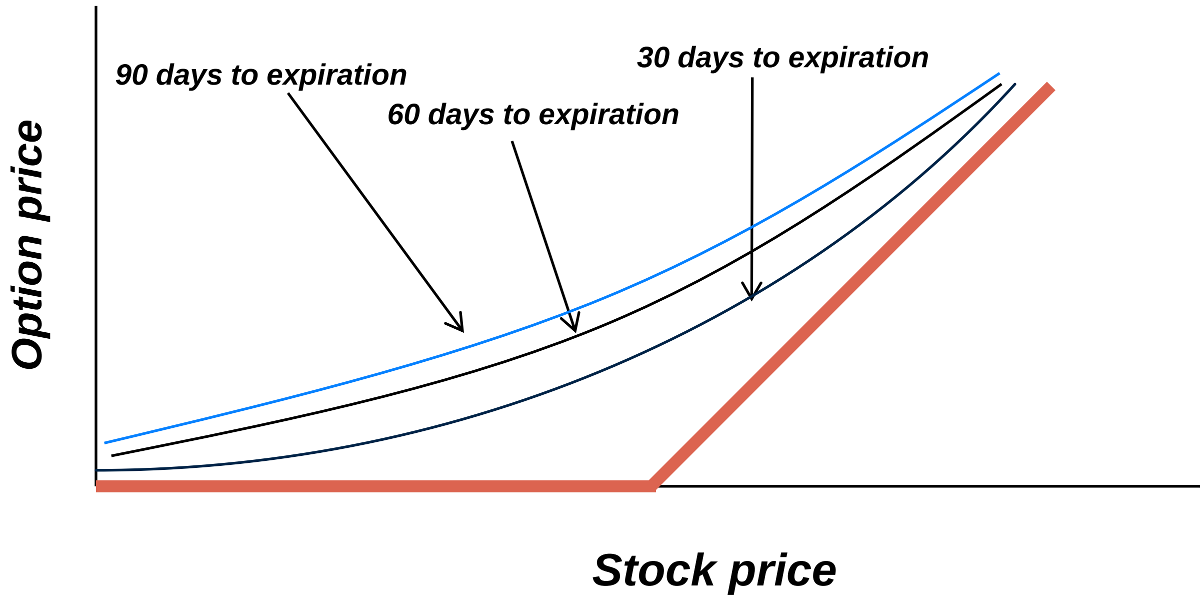
- The higher the option exercise price, the higher the intrinsic value of the option and hence the larger the put option’s premium.

Time to Expiration

- American call and put options become more valuable as time to expiration increases.
  - The longer the time to expiration, the greater is the chance that the price of the underlying asset will be above the exercise price of a call option, or below the exercise price of a put option.
- The longer the time to expiration, the greater is option's time value and hence the higher the options' premium.
- European call and put options also usually become more valuable as time to expiration increases; but not always true as it is possible for in-the-money European options to lose value as time evolves (esp. in case of options on dividend paying stock).

TIME DECAY CHART

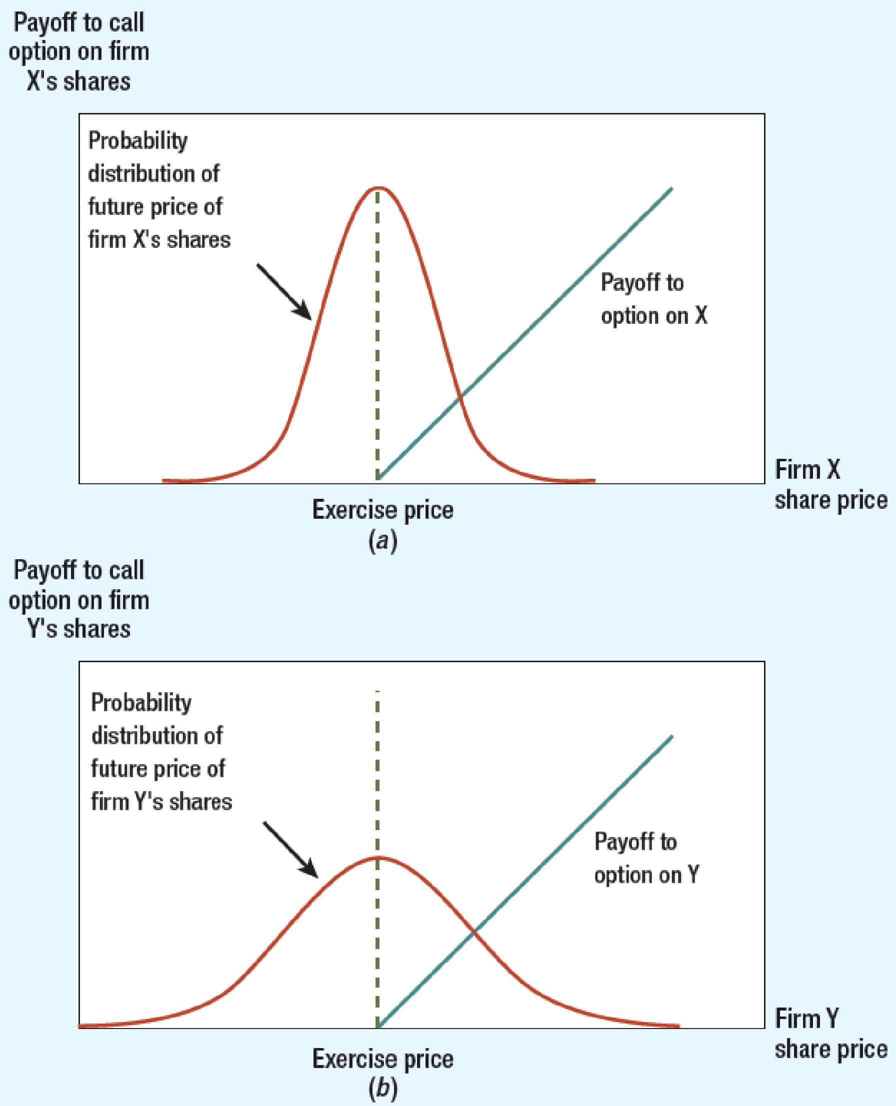
Option prices decline, ceteris paribus, when the time to expiration declines.



Volatility

- The greater the variability or volatility in price of the underlying asset, the higher is the option's time value and hence the larger the option's premium.
- The larger is volatility, the larger the size of likely stock price movements, both up and down, and hence the greater is the chance to exercise the option at favourable prices at a point in time in the future!
- Higher volatility implies increased range of future prices of the underlying asset and hence an increased likelihood that the option can be favourably exercised.

Call option values on the shares on firms X and Y



The greater the distribution of possible outcomes, relative to the final price of the stock, the higher the value of the option. This is due to the greater potential for profit. Thus, Y will have a higher option price, ceteris paribus.

In each case, the current share price equals the exercise price, so each option has a 50% chance of ending up worthless (if the share price falls) and a 50% chance of ending up "in the money" (if the share price rises).
However, the chance of a large payoff is greater for the option on firm Y's shares because Y's stock price is more volatile and therefore has more upside potential.

Risk-Free Interest rate

- Value of a **call** option is *positively* related to interest rates.
  - Buyers of call options do not pay exercise price until they exercise the option. This delayed payment is lower in present value terms when interest rates are high than when they are low. (The higher the interest rate, the lower is the present value of the call option's exercise price).
- Value of **put** option is *negatively* related to interest rates.
  - Buyers of put option will not receive revenue (exercise price) until they exercise put option. Ability to sell the asset at a fixed exercise price sometime in the future is worth less if the PV of exercise price is diminished by higher interest rate.

Dividends

- Dividends have effect of reducing the stock price on the ex-dividend date. ‘Bad news’ for call options & ‘good news’ for put options.
- Value of call option is negatively related to size of anticipated dividends.
- Value of put option is positively related to size of anticipated dividends.

Notation

c : European call option price

p : European put option price

S_0 : Stock price today

X : Strike price

T : Life of option

σ: Volatility of stock price

C : American Call option price

P : American Put option price

S_T : Stock price at option maturity, T.

D : Present value of dividends during option's life

r : continuously compounded risk-free rate for investment with maturity T.

Effect of Variables on Option Pricing

Variable	c	p	C	P
S_0	+	−	+	−
X	−	+	−	+
T	?	?	+	+
σ	+	+	+	+
r	+	−	+	−
D	−	+	−	+

American vs European Options

An American option is worth at least as much as the corresponding European option

$$C \geq c$$

$$P \geq p$$

Greater flexibility of American options as compared to European options, means that for a given exercise price, volatility and time to expiry, American options will be more valuable than European options.

Bounds on Option Values: Call

- Call price must be non-negative

$$C \geq 0 \qquad\qquad c \geq 0$$

- The option cannot have negative value as holder cannot be forced to exercise it.

- Call option value or price can never exceed price of underlying asset (stock)

$$C \leq S_0 \qquad\qquad\qquad c \leq S_0$$

- *Why pay more than S for the right to buy the underlying asset by making a further payment of X?*

- Stock price is ***upper bound*** for call option prices.

- If not true, riskless arbitrage profit from buying underlying stock and selling call option written on the stock.

Lower Bound for European Call Option Prices; No Dividends

$$c \geq S_0 - PV(X)$$

$$c \geq S_0 - Xe^{-rT}$$

Consider the following two portfolios:

i. ***Portfolio A***: One European call option plus an amount of cash equal to the PV(X) invested in a risk-free zero-coupon bond (ie provides a payoff of X at time T)

ii. ***Portfolio B***: One share of the stock

Lower Bound on European Call Option Values; No Dividends

		Portfolio value, given stock price at expiration date T	
Portfolio	**Current Value**	$\mathbf{S_T < X}$	$\mathbf{S_T \geq X}$
A = 1 European call option at price c & Xe^{-rT} cash invested in risk-free bond	$V_A = c + Xe^{-rT}$	$0 + X = X$ (Do not exercise call & receive \$X from bond at maturity)	$(S_T - X) + X = S_T$ (Exercise option & receive \$X from bond)
B = 1 share of stock at price S_0	$V_B = S_0$	S_T	S_T
Relationship between terminal values of Portfolio A and Portfolio B		$V_A = X > V_B = S_T$ $V_A > V_B$	$V_A = (S_T - X) + X = S_T$ $V_B = S_T$ $V_A = V_B$

Lower Bound on European Call Option Values

- In any state of nature, portfolio A pays an amount greater than or equal to that paid by portfolio B at option's maturity: $(V_A > V_B)$ or $(V_A = V_B)$
- In the absence of arbitrage opportunities this must also be true today, i.e. portfolio A must have a higher price than portfolio B.

$$\Rightarrow \qquad c + Xe^{-rT} \geq S_0$$

$$\Rightarrow \qquad c \geq S_0 - Xe^{-rT}$$

$$c \geq Max[(S_0 - Xe^{-rT}), 0]$$

$$c \geq Max[(S_0 - PV(X)), 0]$$

Calls: An Arbitrage Opportunity?

- Suppose that

$c = 3$ $\quad$ $S_0 = 20$

$T = 1$ $\quad$ $r = 10\%$

$X = 18$ $\quad$ $D = 0$

- Is there an arbitrage opportunity?
- Yes!
- $S_0 - Xe^{-rT} = 20 - 18e^{-0.1} = \$3.71$; this greater than quoted call price of \$3!

Quoted call option price is less than its lower bound $\Rightarrow$ call option is undervalued $\Rightarrow$ go long undervalued call option.

Calls: An Arbitrage Opportunity?

Strategy

- buy call option
- Short stock
- invest surplus cash at 10% p.a.

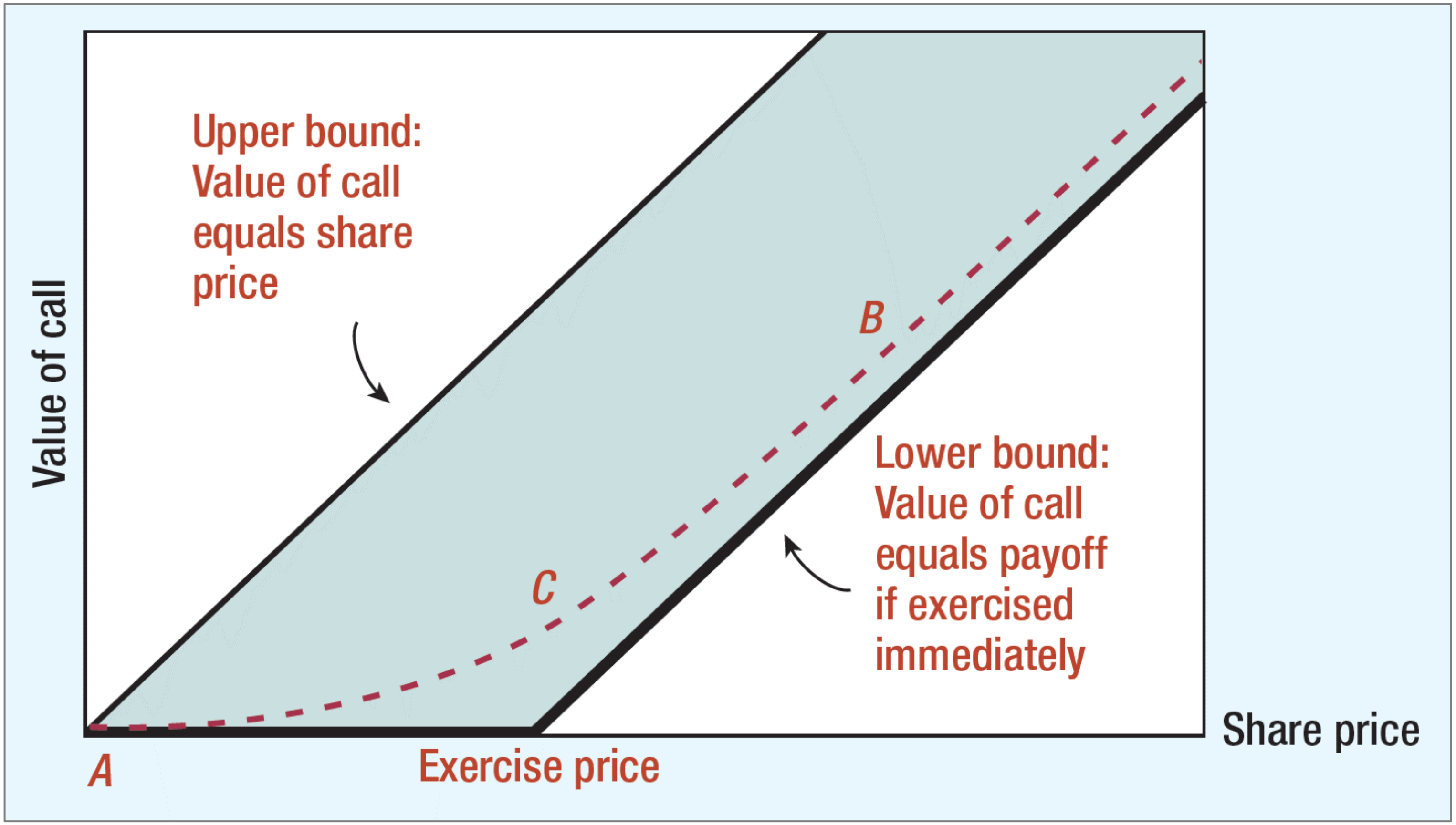
Result

Immediate cash flow of \$20 - \$3 = \$17; if invested at 10% p.a for 1 year it accumulates to $17e^{0.1}$ = \$18.79.

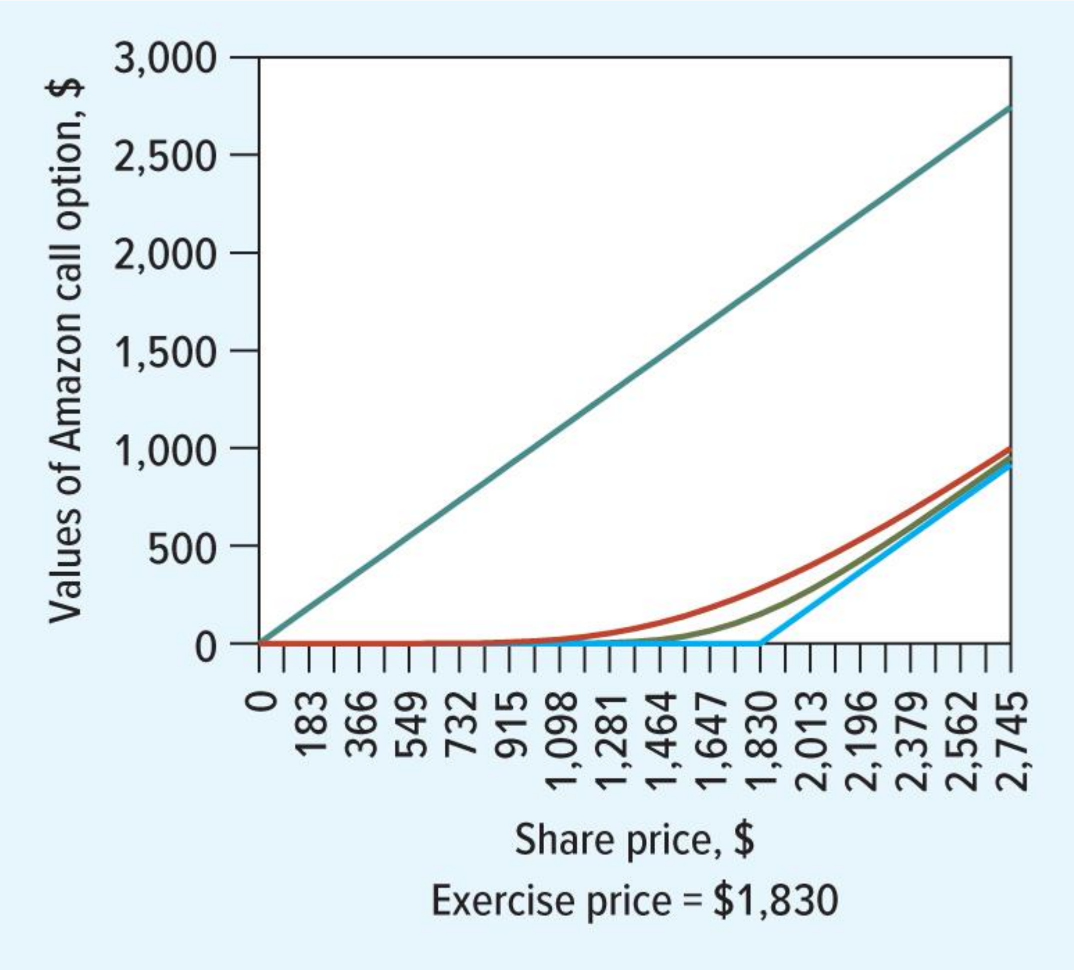
- If $S_T > 18.00$, investor exercises option & closes out short position for profit of \$0.79.
- If $S_T < 18.00$, investor buys stock in market & closes out short position for profit of [18.79 - S_T] (> \$0.79)

Option Value

Value of a call before its expiration date (dashed line). The value depends on the stock price. It is always worth more than its value if exercised now (heavy line). It is never worth more than the stock price itself.



AMAZON CALL OPTION VALUE VERSUS STOCK PRICE



- How the value of the Amazon call option increases with the volatility of the stock price.
- Each of the curved lines shows the value of the option for different initial stock prices.
- The only difference is that the upper line assumes a much higher level of uncertainty about Amazon's future stock price.

DETERMINANTS OF CALL PRICE

1. If there is an *increase* in:	The change in the call option price is:
Stock price (P)	Positive
Exercise price (EX)	Negative
Interest rate (r_f)	Positive*
Time to expiration (t)	Positive
Volatility of stock price (σ)	Positive*

2. Other properties of call options:

a. *Upper bound.* The option price is always less than the stock price.

b. *Lower bound.* The call price never falls below the payoff to immediate exercise ($P - EX$ or zero, whichever is larger).

c. If the stock is worthless, the call is worthless.

d. As the stock price becomes very large, the call price approaches the stock price less the present value of the exercise price.

***The direct effect of increases in r_f or σ on option price, given the share price. There may also be indirect effects. For example, an increase in rf could reduce share price S. This in turn could affect option price.**

Early Exercise: American Calls on Non-Dividend-Paying Stock

- Usually there is some chance that an American option will be exercised early
- An exception is an American call on a non-dividend paying stock
- This should never be exercised early.

It is never optimal to exercise an American call option on non-dividend paying stock.

Early Exercise: Calls on Non-Dividend-Paying Stock

Note: $c \geq S_0 - Xe^{-rT}$

However $C \geq c$

Hence $C \geq S_0 - Xe^{-rT}$

Given $r > 0$, it follows that:

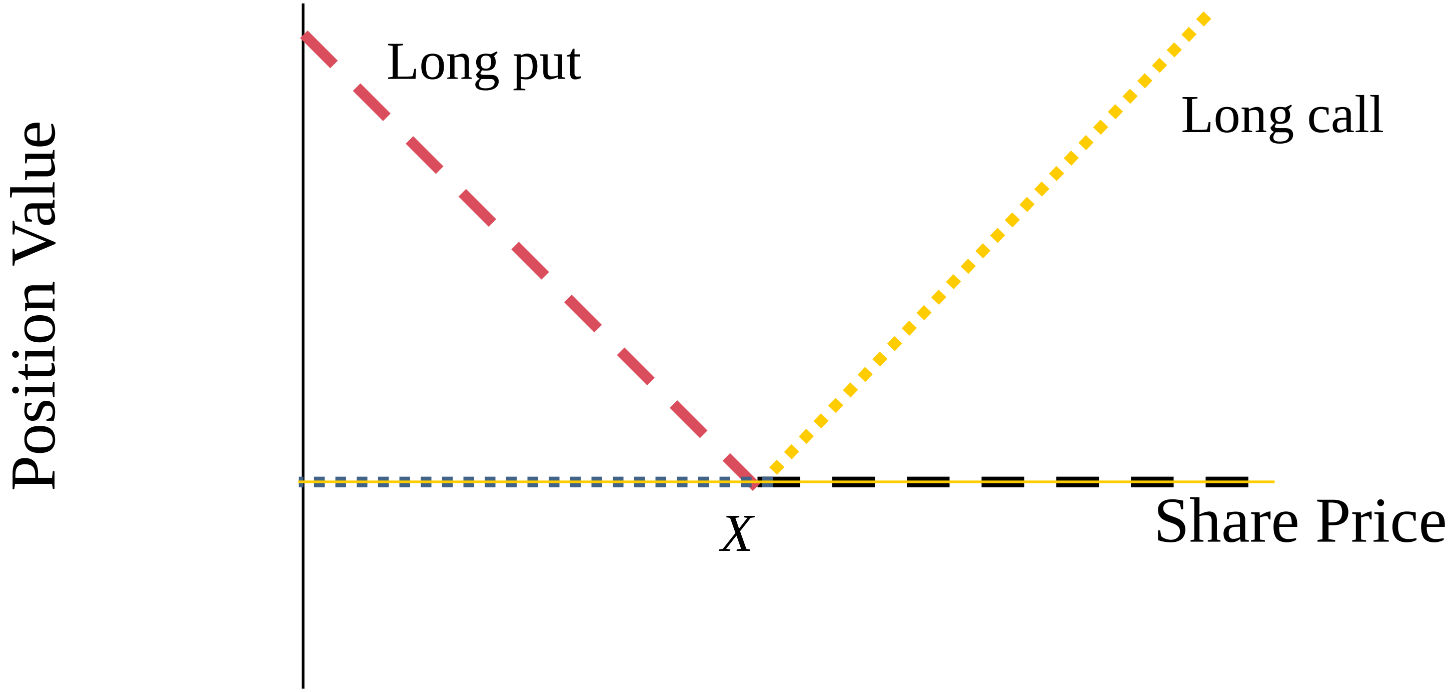
$$C \geq S_0 - Xe^{-rT} > S_0 - X$$

- If it were optimal to exercise early, then C would equal $[S_0 - X]$ the payoff/value from immediate exercise of American call.

  Holder of American call option can always do better by selling option in the market place rather than exercising it prior to expiration.

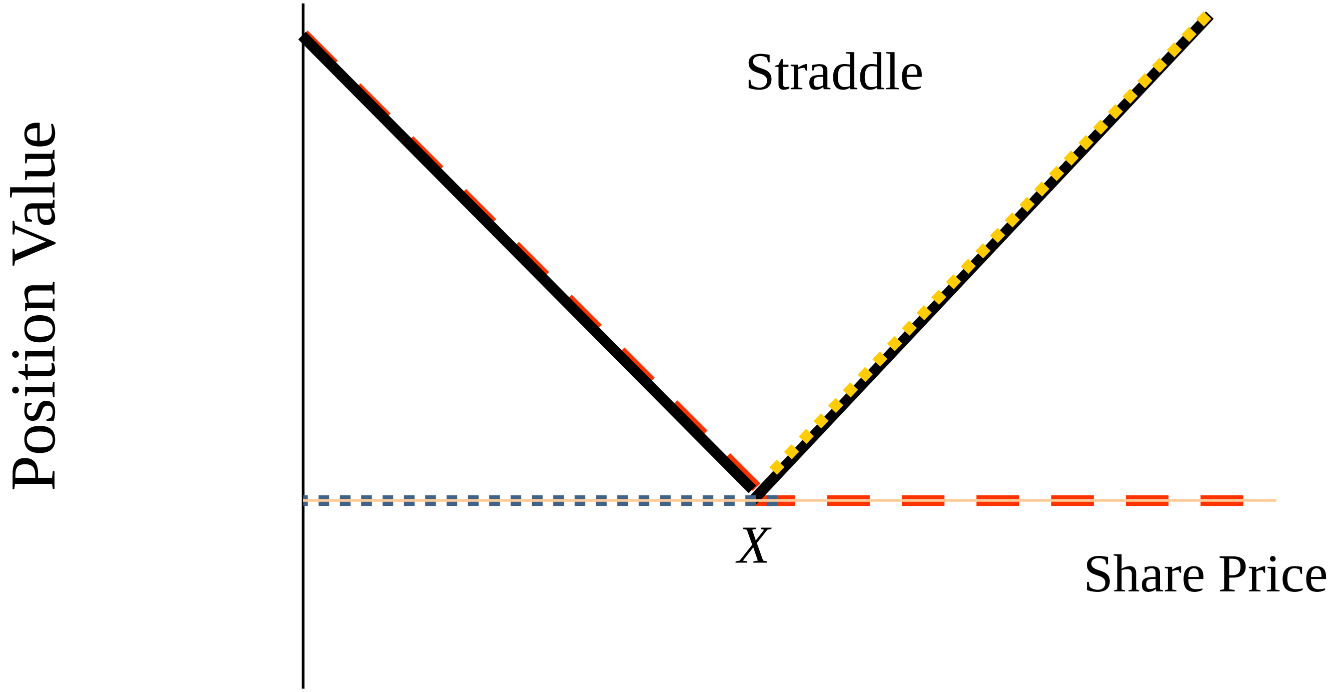
STRADDLE

Long Straddle - Strategy for payoff from high volatility; Long call and long put at same exercise price.



Payoff Profile of a Long Straddle

Long Straddle - Strategy for payoff from high volatility;
Long call and long put at same exercise price.



STRADDLE

- **Straddle**
- Call option & put option with same exercise price and same expiration date.

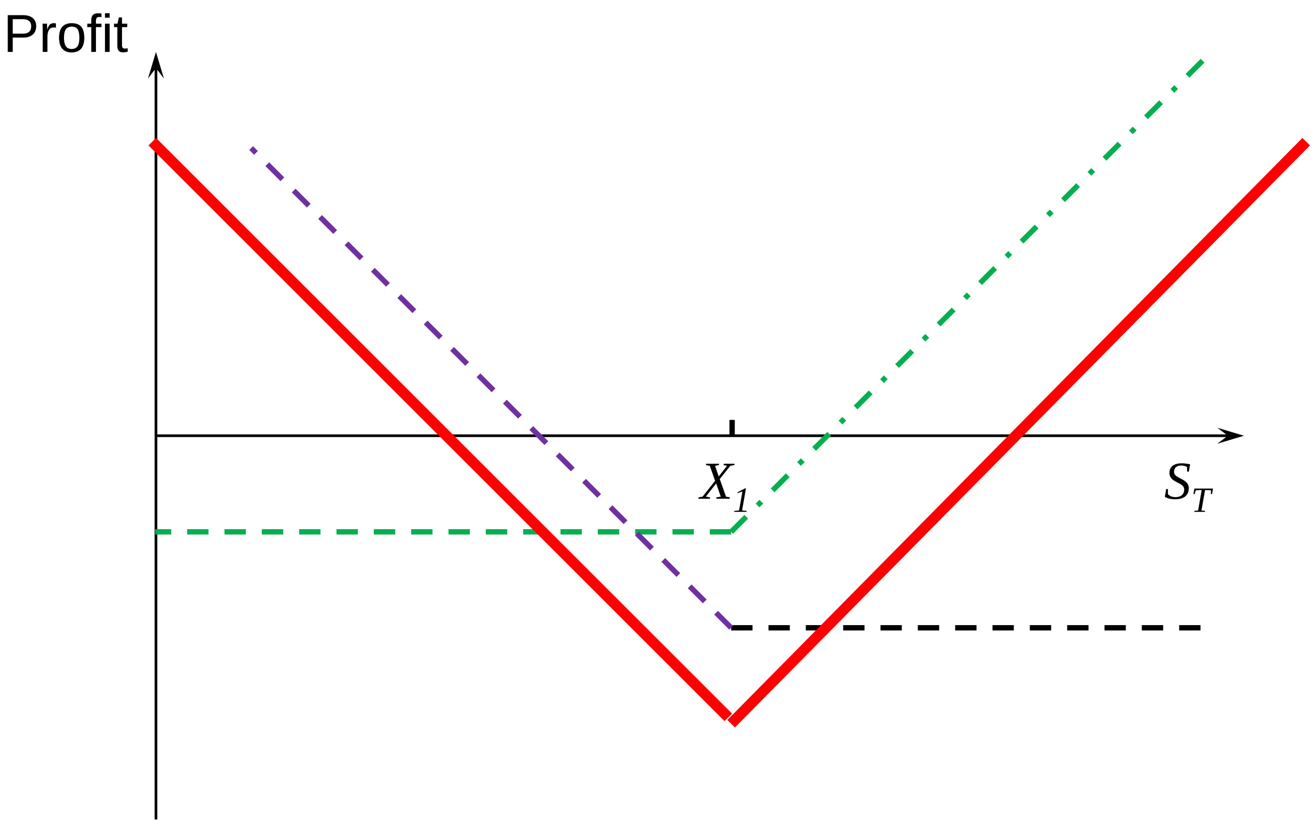
<u>Long straddle</u>

Long straddle = [long put at X_1] + [long call at X_1]

<u>Short straddle</u>

Short straddle = [short put at X_1] + [short call at X_1]

Profit Profile of a Long Straddle



STRANGLE

- **Strangle**
- Call option & put option with same expiration date but different exercise prices.
- ***Long strangle*** = [long put with out-of-the-money exercise price at X_1] + [long call with out-of-the-money exercise price at X_2] where $X_1 < X_2$.
- ***Short strangle*** = [short put with out-of-the-money exercise price at X_1] + [short call with out-of-the-money exercise price at X_2] where $X_1 < X_2$.

Profit Profile of a Long Strangle Combination