FFR135 - Artificial Neural Networks - Homework 2 Task 1

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Question 1

For the first question we have 2^n different inputs. For each Boolean function of the inputs we can either have 0 or 1 as an output. This leads to the amount of different Boolean functions being $2^{(2^n)}$. Where n is the dimension. In our case n=3 and thus:

$$2^{(2^3)} = 256$$

Question 2

In question 2 we want to find the amount of unique functions that maps 4 of the possible input patterns to 1. Several functions can be found but a lot of them will be symmetries or mirrors of other functions and thus they will still only count as one unique function. In total 6 such functions were found. In order to come up with this the different possible functions that were not symmetries of each other were drawn up and the resulting functions can be seen in 1

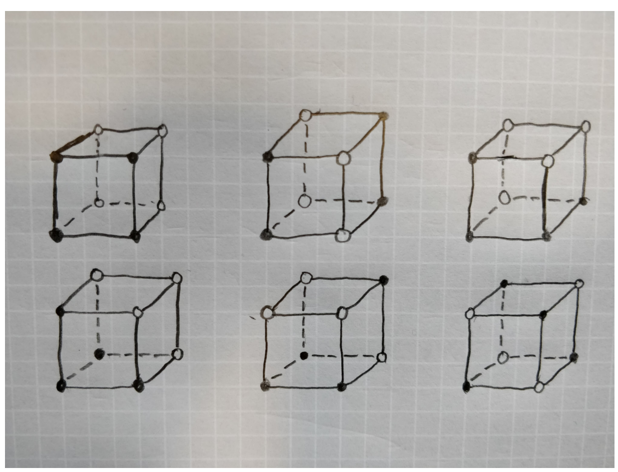

Figure 1: 3-Dimensional Boolean functions that are not symmetries of each other

Question 3

In question 3 we want to see how many linearly separable 3-dimensional Boolean functions there are. For the function to be linearly separable it needs to be able to be separated with a plane. For this task the easiest way was to start with looking at k=1, this means where one of the possible inputs map to 1. The amount of such functions and symmetries were counted and then I moved on to k=2. The process and the different functions/symmetries can be seen in 2. While solving this task I only worked with k = 0-4 since I noticed that k = 5-8 is just mirrors of k=0-3. The total amount of symmetries/functions for each k can be found below:

	Total amount of functions/symmetries
k = 0	1
k = 1	8
k = 2	12
k = 3	24
k = 4	6+8

The reason why k = 6+8 is because of it having two different linearly separable functions with 6/8 symmetries for each. At first I almost forgot about k=0 and k=8 but with those the total amount of linearly separable functions I found was: $2 * (1 + 8 + 12 + 24) + 6 + 8 = 104$

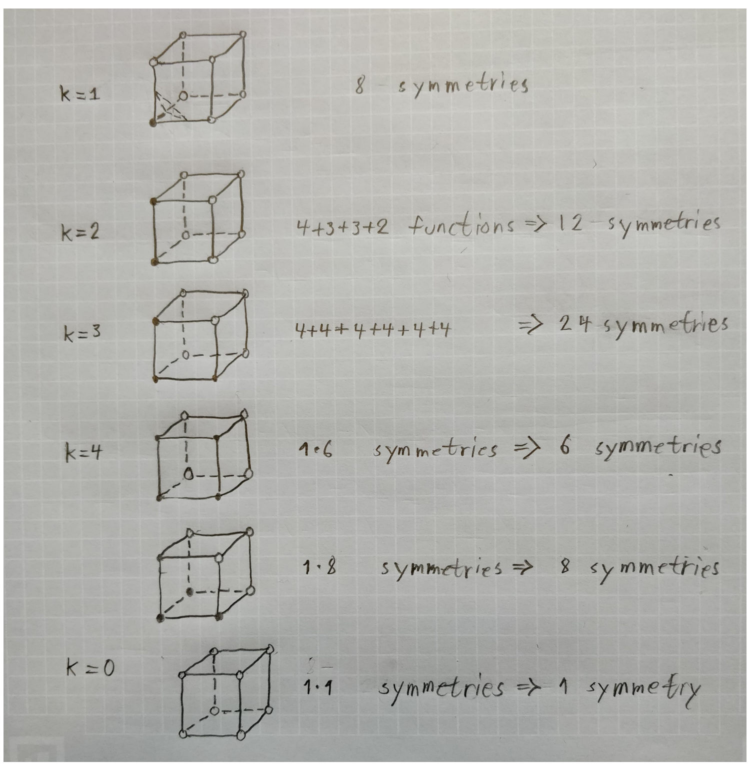

Figure 2