

# Resonance

## FROM Physics Course (Great Courses) - Lecture 39

The resonant frequency is that frequency in which current is the greatest, with the shape and magnitude of the curve dependent on the relationship between capacitance and inductance, with the sharpness of the resonance dependent on the resistance of the circuit.

• Sharpness of resonance depends on resistance

$\omega_0$   
Frequency,  $\omega$



### RLC Circuit

At what driving frequency  $\omega_0$  will current be greatest?  $V_s \sin \omega_s t$

Series circuit: same current in  $R, L, C$

Inductor voltage leads current by  $90^\circ$

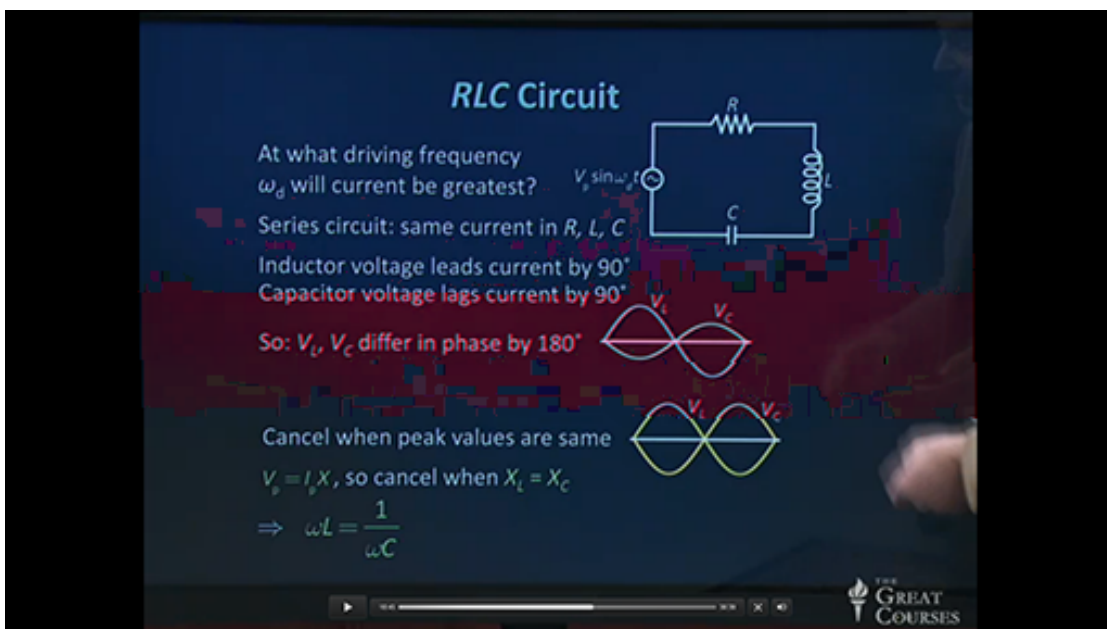
Capacitor voltage lags current by  $90^\circ$

So:  $V_L, V_C$  differ in phase by  $180^\circ$

Cancel when peak values are same

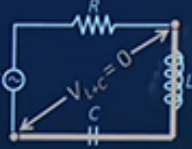
$V_p = I_p X$ , so cancel when  $X_L = X_C$

$\Rightarrow \omega L = \frac{1}{\omega C}$

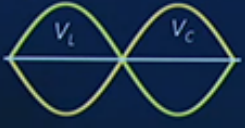


### RLC Circuit

At what driving frequency  $\omega_d$  will current be greatest?



$V_L = V_C$



$$\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

Natural frequency:  $\omega_0 = \frac{1}{\sqrt{LC}}$


Maximum current, determined entirely by resistance:  $I_p = V_p/R$

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One application is in tuning radio frequencies. It is the relationship between Capacitance (C), Inductance (L), and frequency (f) to determine the resonant frequency.

### Example: FM Radio

LC tuning circuit in FM receiver  
 $L = 200 \text{ nH}$   
 $f = 88 \text{ MHz} - 108 \text{ MHz}$



What range of variable capacitor is needed?

$$\omega = 1/\sqrt{LC} \Rightarrow \omega^2 = 1/LC \Rightarrow C = 1/L\omega^2$$

$$\omega = 2\pi f \Rightarrow C = 1/L(2\pi f)^2$$

Do the numbers:

$$C_{88 \text{ MHz}} = 1/\left[(200 \times 10^{-9} \text{ H})(2\pi(88 \times 10^6 \text{ Hz}))^2\right]$$

$$= 1.6 \times 10^{-11} \text{ F} = 16 \text{ pF}$$

$$C_{108 \text{ MHz}} = 11 \text{ pF}$$

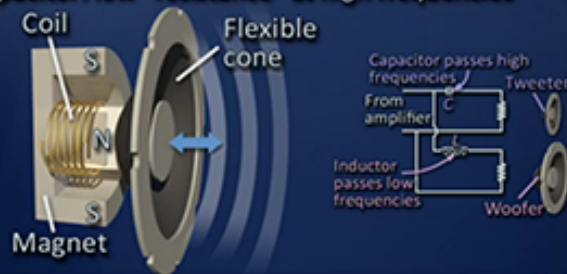
Need: 11 – 16 pF capacitor

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## LC Application: Loudspeaker System

Crossover network “steers” high frequencies to tweeter, low frequencies to woofer

- Inductor: low “resistance” at low frequencies
- Capacitor: low “resistance” at high frequencies

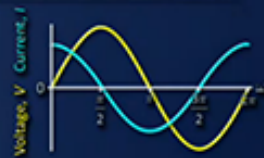


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## Summary

AC voltage and current are characterized by

- Amplitude (peak or rms)
- Frequency ( $\omega$  or  $f$ )
- Phase



Capacitors and inductors introduce phase differences

- Current leads in capacitors
- Voltage leads in inductors

Complementary behavior of C and L leads to

- Resonance in RLC circuits
- Natural oscillations in LC circuits

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