# Distance Regularized Level Sets Evolution

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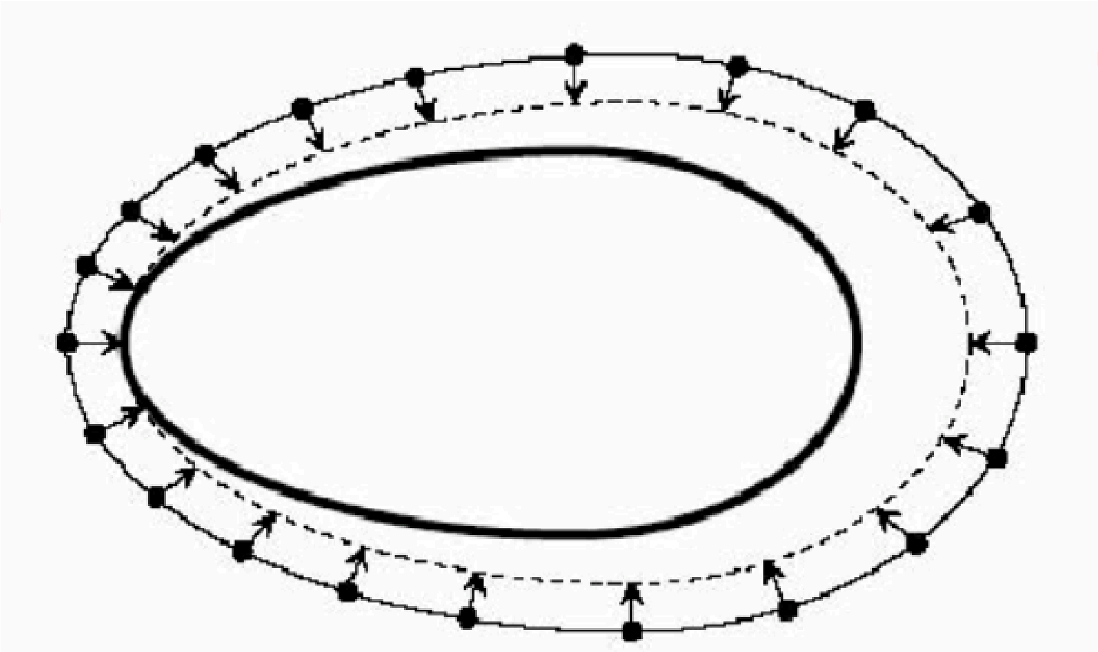
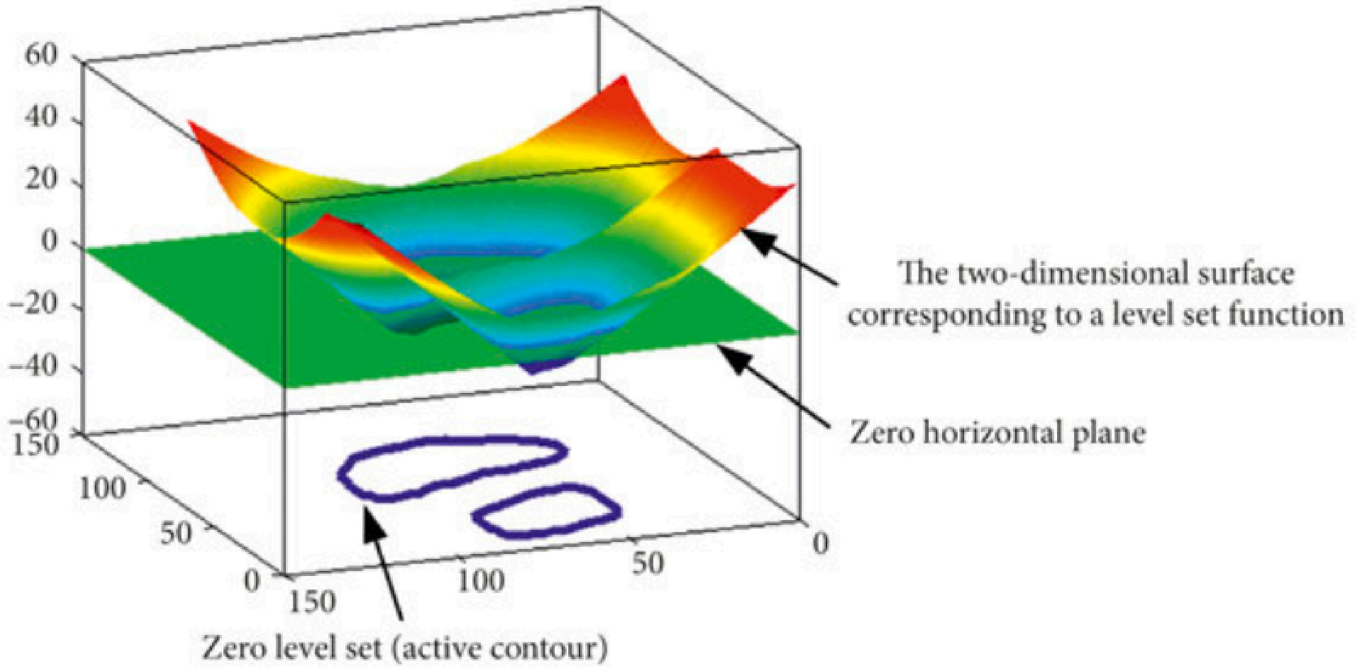
# Introduction

## Active Contour Segmentation with Level Sets

- **Goal**: Detect object boundaries in images
- **Traditional methods (snakes) :** Explicit contour representation with points
- **Level Set approach:** Implicit representation via function ϕ(x, y )
  - Contour defined as zero-level set of ϕ
  - Naturally handles topological changes (splitting/merging)
  - Evolution governed by partial differential equations (PDEs)



**Traditional active contours**



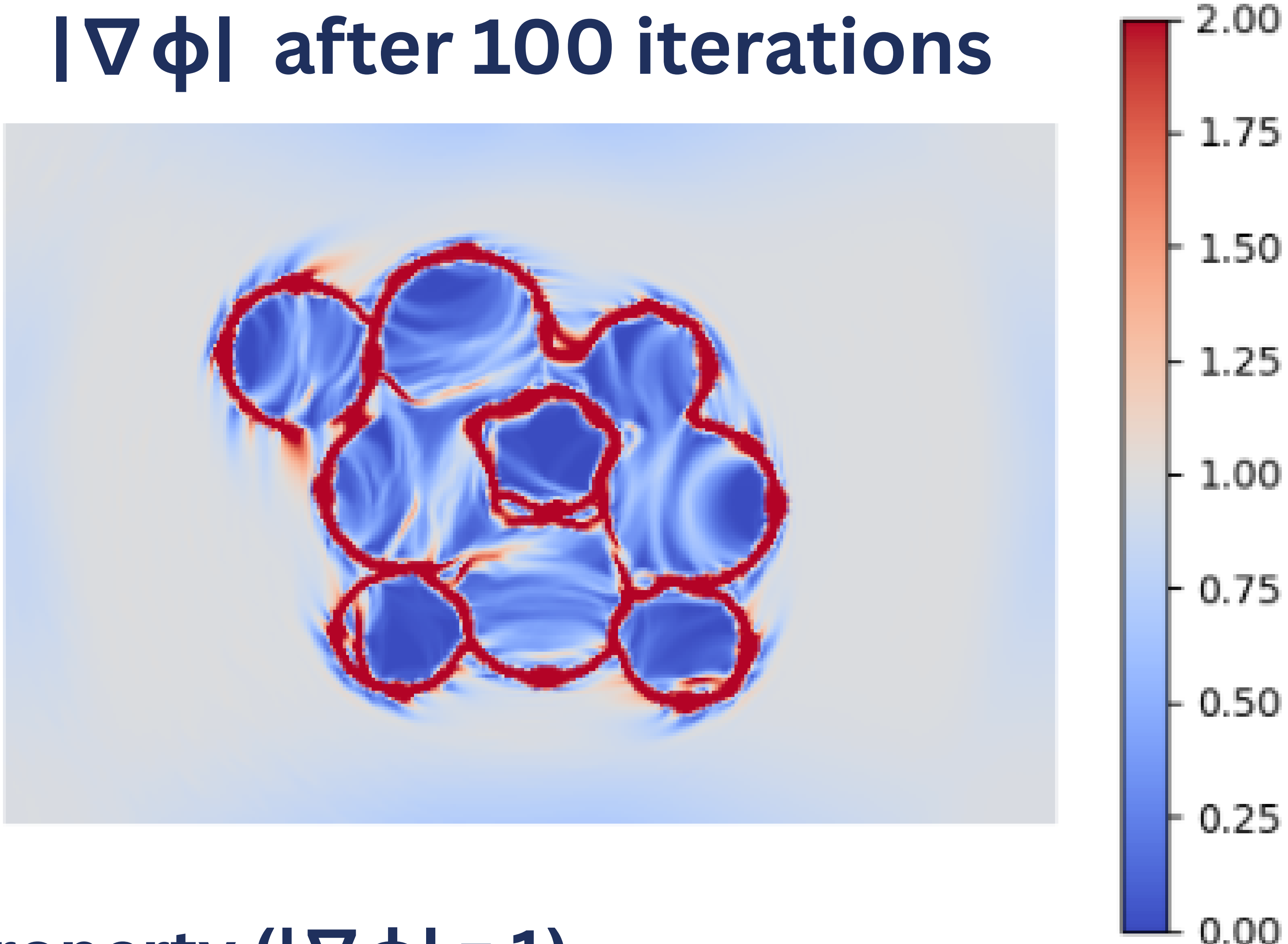

**Level set contour**

# Fundamental Level Set Method

## Principles

- Contour C defined where $\phi(x, y, t) = 0$
- $\phi$ is a signed distance function
- Evolution: $\partial\phi/\partial t = -F\,|\nabla\phi|$
- Speed function: $F = g \cdot \kappa + \nabla g \cdot \nabla\phi$
  - $g$ = edge detector function
  - $\kappa$ = curvature

$|\nabla\phi|$ after 100 iterations




## Key Challenge

- Level set function loses its signed distance property ($|\nabla\phi| = 1$)
- Results in:
  - Numerical instabilities
  - Irregular, unwanted contours
  - Computational inefficiency

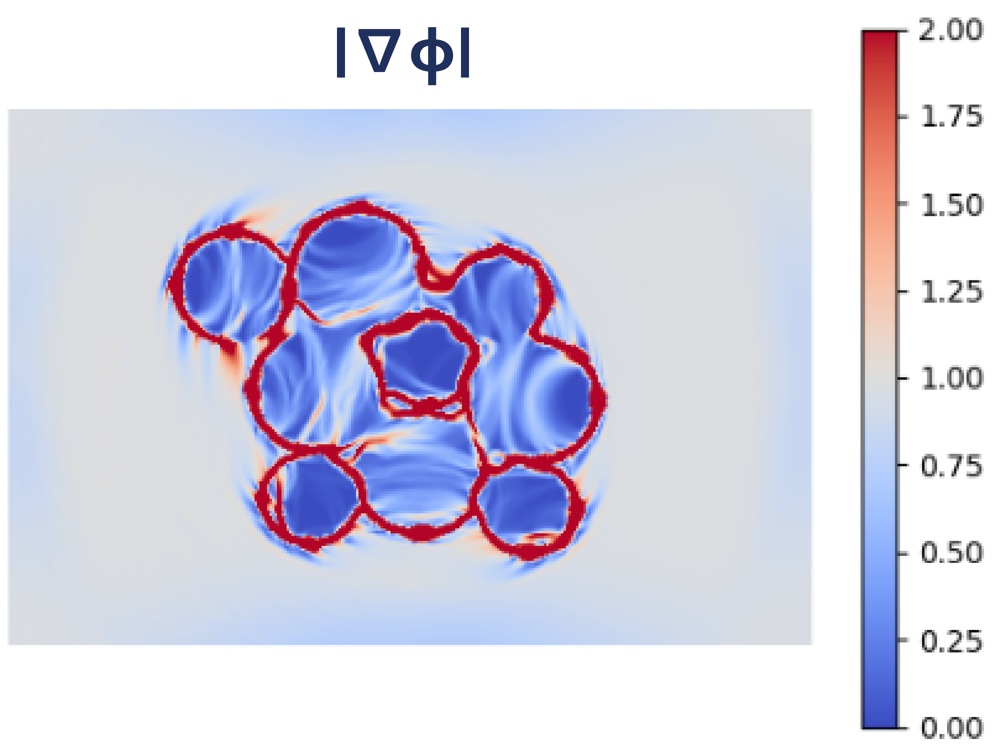
# Reinitialization

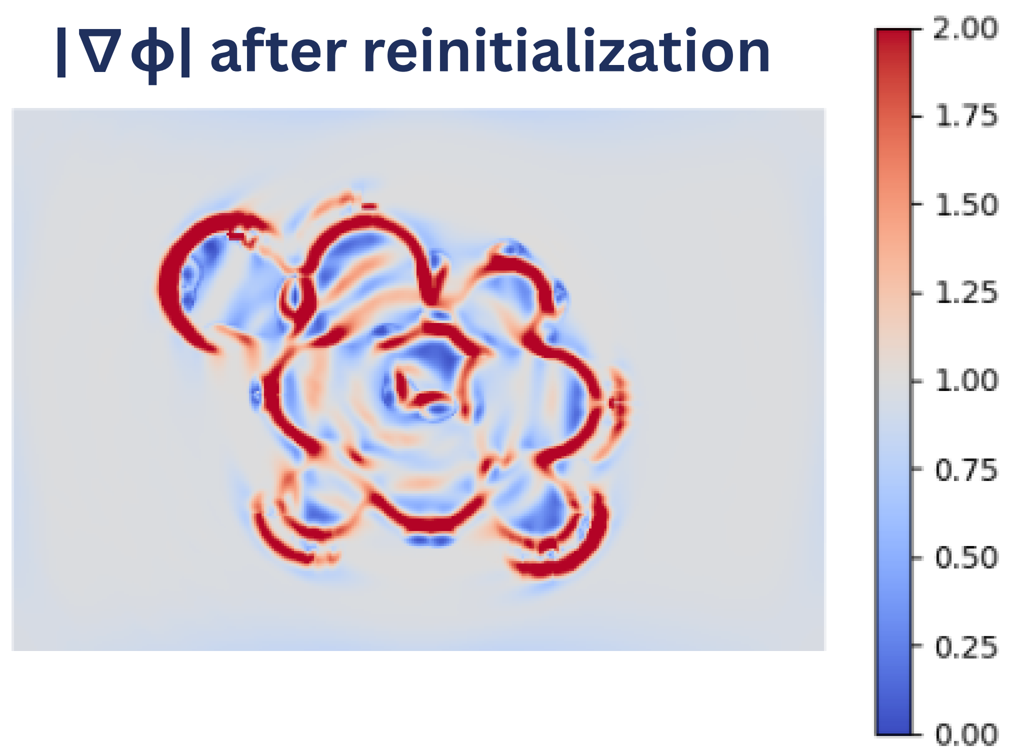
## The Approach

- **Periodically reshape ϕ to restore signed distance property → Improved numerical stability, Prevents development of irregularities**
- **Solves:** $\partial\phi/\partial\tau = \mathrm{sign}(\phi 0)(1 - |\nabla\phi|)$
- **Preserves zero-level set location while regularizing function**

## Disadvantages

- **Computationally expensive additional step**
- **LSF struggles to transform into accurate distance function**
- **Disrupts the natural evolution process**



# DRLSE

## Distance Regularized Level Set Evolution (Li et al., 2010)

- **Key Innovation:** Incorporate distance regularization directly into the energy functional

$$E(\phi) = \mu R_p(\phi) + \lambda L_g(\phi) + \alpha A_g(\phi)$$

**Where:**

- **$R_p(\phi)$:** Distance regularization term → **maintains signed distance property**
- **$L_g(\phi)$:** Length term weighted by edge indicator → **smooths the contour**
- **$A_g(\phi)$:** Area term weighted by edge indicator → **apply shrinking (balloon force)**

## How DRLSE Fixes Prior Issues

- Eliminates re-initialization: Distance regularization term intrinsically **maintains $|\nabla\phi| \approx 1$ near the contour**
- Improves stability: Allows larger time steps
- Reduces computational cost: **No separate re-initialization step**
- Prevents contour shifts: No arbitrary perturbations to zero level set
- Better accuracy: More consistent evolution behavior

# DRLSE - Iterative algorithm

**The evolution equation becomes : $\partial\phi/\partial t = -\partial E/\partial\phi$**

**Converting the continuous equation to a discrete update scheme :**

$$\phi^{n+1} = \phi^n + \Delta t \cdot \left(-\frac{\partial E}{\partial \phi}\right)$$

**The Distance Regularization Term**

$$R_p(\phi) = \iint p(|\nabla\phi|)\, dxdy$$

**Double-Well Potential Function**

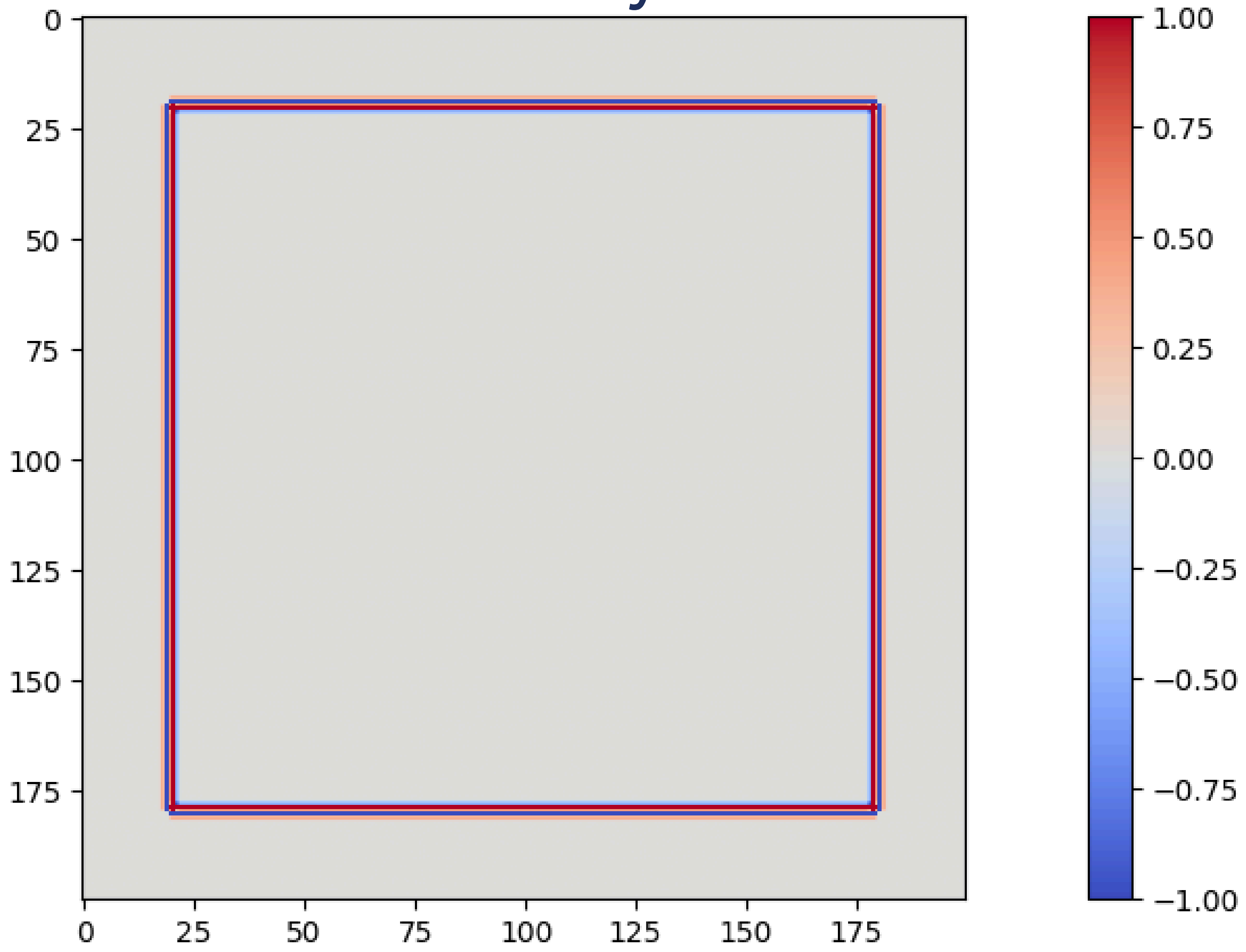
$$p_2(s) = \begin{cases} \frac{\sin(2\pi s)}{2\pi} & \text{if } s \leq 1 \\ (s-1) & \text{if } s > 1 \end{cases}$$

- Has two minima: $|\nabla\phi| = 0$ and $|\nabla\phi| = 1$
- Maintains distance property near zero level set
- Keeps LSF constant farther away

# DRLSE - Distance Regularization Term

**Its evolution equation :** $$\frac{\partial \phi}{\partial t} = \mu \, \mathrm{div}(d_p(|\nabla \phi|)\nabla \phi)$$ **Where dp(s) = p’(s)/s**

**Regularization term from the initial binary LSF**

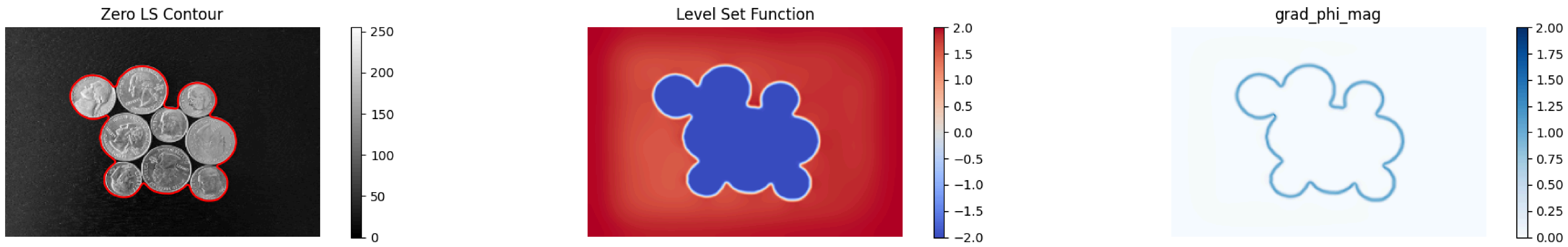



**Effects of regularization term :**

- High positive values at inner edges (LSF negative) → LSF increases
- Negative values at outer edges (LSF positive) → LSF decreases
- **Smooths abrupt jumps into gradual ramps**
- **Automatically maintains signed distance property near the contour**

# DRLSE - Testing

DRLSE Evolution after 500 iterations




## Improvements:

- **Signed distance property (|∇ϕ| = 1) :** we can see its being maintained all over the contour even after 500 iterations **-> Great stability**
- **Computational efficiency:** Elimination of reinitialization **reduces computation time by 60%**
- **Numerical stability:** Larger time steps possible (5-10x larger than basic level sets)

# Conclusion

## Conclusions:

- Level set methods provide powerful framework for image segmentation
- **significant improvement :** from basic level sets → reinitialization → DRLSE
- **DRLSE effectively addresses fundamental challenges:**
  - Eliminates reinitialization requirement
  - Maintains signed distance property intrinsically
  - Improves computational efficiency and stability

## References :


- C. Li, C. Xu, C. Gui, and M. D. Fox, ”Distance regularized level set evolution and its application to image segmentation”,  2010.
- Claude 3.7 sonnet

# Thank You