Next Up Previous Contents
Next: Proving that a Language **Up:** PUSH-DOWN AUTOMATA **Previous:** Deterministic PDAs
Contents

Non-Deterministic CFLs

The important point then is that:
not every context-free language is determinstic

To see this, consider the language $\{x^n y^n \mid n > 0\} \cup \{x^n y^{2n} \mid n > 0\}$ We can easily show that this context-free by giving its CFG. However, we will show that it is not *deterministic* context-free.

Proof: *(by contradiction)*

Suppose that this language *is* deterministic context-free; then it has a corresponding deterministic PDA.

Let us create two copies of this PDA called M_1 and M_2. Call any two states ''cousins'' if they are copies of the same state in the original PDA. Now we construct a new PDA as follows:

- The states of the new PDA is the union of the states in M_1 and M_2, where
  - the start state of M_1 is the new start state
  - the final states of M_2 are the new final states
- The transition relation is that of M_1 and M_2 with the following alterations:
  - Change any transition originating from a final state in M_1 so that it now goes to its ''cousin'' state in M_2
  - Change all those 'y' transitions which cause a move into some state from M_2 into 'z' transitions

This is a PDA over the alphabet $\{x, y, z\}$. To see what language it recognises, consider its actions on an input of $x^k y^k z^k$ for some fixed $k \geq 0$.

Initially it will move from the start state to a final state of M_1 while consuming the input $x^k y^k$. Because it is deterministic, there is no other state which it could reach while consuming this input. But we know that by its construction M_1 can now also go on to accept k more copies of 'y'; therefore if we run the new PDA on the rest of the input, it will consume k more copies of 'z' as it moves through M_2.

Thus the constructed PDA accepts the language $\{x^n y^n z^n \mid n \geq 0\}$ - but this is impossible, as this language is not context free. Thus our assumption that the PDA for the original language could be deterministic is contradicted.
QED

Next Up Previous Contents
Next: Proving that a Language **Up:** PUSH-DOWN AUTOMATA **Previous:** Deterministic PDAs
Contents
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