SLUGGER: Lossless Hierarchical Summarization of Massive Graphs (Supplemental Document)

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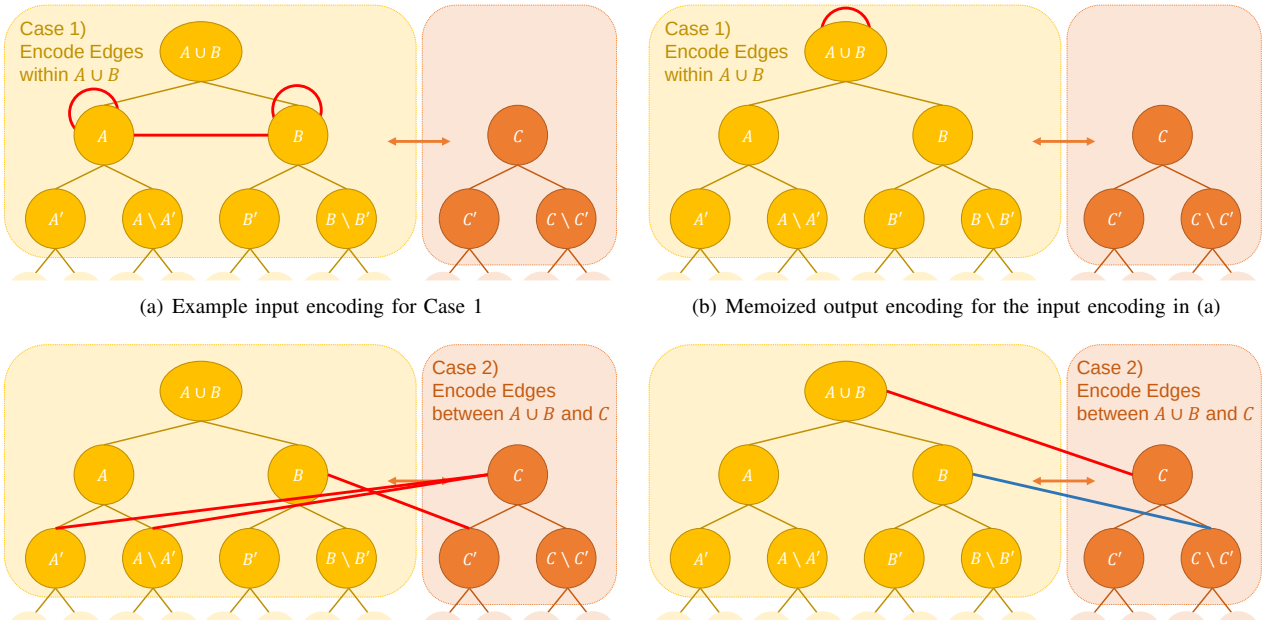

(a) Example input encoding for Case 1

(b) Memoized output encoding for the input encoding in (a)

(c) Example input encoding for Case 2

(d) Memoized output encoding for the input encoding in (c)

Fig. 1. Example input encodings and the memoized output encodings for the inputs where p-edges are in red, n-edges are in blue, h-edges are in yellow or in orange. Note that the number of p- and n-edges is reduced from 3 in (a) to 1 in (b), and it is reduced from 3 in (c) to 2 in (d).

TABLE I
(EXTENSION OF FIGURE 5(A) IN THE MAIN PAPER). RELATIVE SIZE OF OUTPUTS AND ABSOLUTE NUMBER OF TOTAL OUTPUT EDGES. THE PRESENTED VALUES ARE THE MEANS AND STANDARD DEVIATIONS OVER FIVE RUNS.

	Relative Size of Outputs					Absolute Number of Output Edges				
	Slugger	SWeG	MoSSo	Randomized	SAGS	Slugger	SWeG	MoSSo	Randomized	SAGS
PR	0.094±0.0006	0.134±0.0011	0.152±0.0009	0.157±0.0025	0.346±0.0447	13754.4±145.95	19544.8±160.052	22148.6±128.481	22900.4±366.495	50557.8±6529.62
EM	0.743±0.0007	0.867±0.0004	0.899±0.0009	0.867±0.0013	0.921±0.0018	136537.4±268.455	159293.4±68.376	165223.0±164.373	159465.8±237.624	169353.0±328.773
AM	0.700±0.0001	0.767±0.0001	0.791±0.0002	0.788±0.0001	0.814±0.0003	1709651.6±316.74	1874265.0±165.401	1933112.8±379.424	1925484.0±190.945	1988457.4±738.293
DB	0.678±0.0001	0.775±0.0001	0.837±0.0006	0.792±0.0002	0.806±0.0006	712294.8±159.084	813241.2±130.435	878400.4±617.164	831704.4±233.059	846681.2±679.724
HO	0.422±0.0002	0.567±0.0002	0.586±0.0001	out of time	0.619±0.0005	48313317.4±23264.4	64949439.8±17778.3	67050002.8±10659.5	out of time	70927316.4±57478.8
FA	0.429±0.0015	0.526±0.0004	0.565±0.0122	0.548±0.0026	0.592±0.0014	37879.4±69.349	46406.2±37.957	49853.2±1078.84	48390.6±228.607	52268.0±122.967
YO	0.917±0.0003	1.014±0.0003	0.999±0.0003	0.990±0.0017	1.020±0.0003	2740180.0±290.493	3028638.6±963.088	2985537.0±932.299	2957932.8±5107.55	3047720.0±1016
ES	0.718±0.0007	0.790±0.0007	0.817±0.0004	out of time	0.874±0.0008	15203198.4±11859.1	16743709.2±13865.4	17306850.8±8392.24	out of time	18512651.2±15915.1
LJ	0.744±0.0001	0.833±0.0000	0.839±0.0001	0.848±0.0001	0.890±0.0006	25787399.0±1604.45	28891149.6±1532.51	29091762.8±4200.39	29413695.4±3717.88	30871010.6±19734.6
CA	0.835±0.0010	0.929±0.0007	0.927±0.0020	0.916±0.0174	0.984±0.0060	44559.0±102.413	49594.6±37.293	49470.4±104.361	48893.6±927.268	52541.8±320.642
SK	0.542±0.0004	0.628±0.0004	0.649±0.0006	0.653±0.0008	0.710±0.0030	6018137.2±3665.37	6965689.4±4626.19	7195481.0±7040.29	7242379.4±8753.51	7875292.4±32890.7
CN	0.216±0.0003	0.249±0.0002	0.301±0.0045	0.269±0.0036	0.365±0.0051	591144.2±731.872	681679.0±520.699	823187.4±12237.8	737583.4±9812.7	998554.6±13996.6
EU	0.187±0.0007	0.253±0.0004	0.278±0.0007	0.273±0.0012	0.370±0.0029	3018421.0±3685.78	4079028.4±5832.04	4491985.2±11972.8	4400871.0±20056.6	5976099.4±47379.5
IC	0.101±0.0002	0.127±0.0003	0.158±0.0007	out of time	0.230±0.0012	15235678.0±30341.1	19207769.0±38374.8	23877230.2±106554.2	out of time	34690454.2±179803
U2	0.142±0.0001	0.170±0.0000	0.268±0.0001	out of time	0.291±0.0015	37097478.4±6826.24	44605172.4±12026.3	70247704.6±28318.4	out of time	76217765.0±381003.8
U5	0.108±0.0000	0.128±0.0000	out of memory	out of time	out of memory	84753556.8±25189.8	99968601.0±23705	out of memory	out of time	out of memory
AVG. (-U5)	0.498±0.0004	0.575±0.0003	0.604±0.0015	out of time	0.656±0.0044	10443975.4±2253.68	12807641.4±1568.18	15077863.4±9473.82	out of time	16818448.2±29149.83
AVG.	0.474±0.0005	0.547±0.0003	out of memory	out of time	out of memory	15088324.2±1993.50	18255201.4±2721.78	out of memory	out of time	out of memory

TABLE II

(EXTENSION OF FIGURE 5(B) IN THE MAIN PAPER). THE AVERAGE RUNNING TIME. THE PRESENTED VALUES ARE THE MEANS AND STANDARD DEVIATIONS OVER FIVE RUNS.

	Slugger	SWeG	MoSSo	Randomized	SAGS
PR	1.458±0.253	2.303±0.377	5.618±0.107	7.215±0.219	0.456±0.020
EM	6.946±0.215	4.594±0.261	9.983±0.19	107.67±2.455	0.982±0.088
AM	34.384±0.296	32.769±0.845	108.12±1.418	260.91±15.12	10.259±0.126
DB	13.356±0.219	14.790±0.47	41.086±0.54	61.222±0.813	5.422±0.046
HO	13611.4±194.1	3849.9±52.27	12315.8±73.79	out of time	600.86±12.29
FA	1.826±0.141	2.588±0.373	4.382±0.144	5.448±0.074	0.688±0.026
YO	139.62±2.955	59.644±0.388	226.15±7.144	11827.9±753.2	20.104±0.668
ES	1605.6±19.41	443.67±5.361	4576.2±172.2	out of time	182.77±21.45
LJ	1745.0±17.83	713.64±8.158	1741.5±30.03	66401.0±761.4	151.36±2.510
CA	1.934±0.078	1.440±0.118	4.124±0.169	29.487±3.250	0.627±0.050
SK	320.97±3.208	176.96±2.475	773.26±15.21	38939.6±969.7	104.46±8.998
CN	23.456±0.699	27.464±0.889	112.14±1.985	1291.4±36.7	17.078±3.179
EU	182.63±2.486	175.17±1.54	708.82±11.23	33345.9±448.6	105.28±11.77
IC	1953.6±34.33	2291.1±47.6	5199.1±73.8	out of time	599.60±55.71
U2	2239.7±22.95	2627.8±70.01	9853.3±108.1	out of time	626.08±11.05
U5	12892.5±533.3	24311.2±1035.7	out of memory	out of time	out of memory
AVG. (-U5)	1458.79±17.22	694.921±4.618	2378.64±19.40	out of time	161.734±3.880
AVG.	2173.40±22.04	2170.94±66.96	out of memory	out of time	out of memory

TABLE III

ELAPSED TIME COMPARISON FOR PARTIAL DECOMPRESSION WITH SWEG. WE REPORT THE AVERAGE TIME TAKEN FOR RETRIEVING THE NEIGHBORS OF EACH NODE IN THE SUMMARY GRAPHS OBTAINED BY SLUGGER AND SWEG. WE USED ALGORITHM 1 IN THE SWEG PAPER FOR SWEG. NOTABLY, THE TIME TAKEN FOR SLUGGER WAS STRONGLY CORRELATED WITH THE AVERAGE DEPTH OF LEAF NODES IN HIERARCHY TREES. THE PEARSON COEFFICIENT BETWEEN THEM WAS ABOUT 0.82. THE PRESENTED VALUES ARE THE MEANS AND STANDARD DEVIATIONS OVER FIVE RUNS.

	Avg. Depth per Leaf Node	Elapsed Time (Microseconds)	
		Slugger	SWeG
PR	1.75±0.050	10.08±0.445	2.65±0.093
EM	0.83±0.006	3.47±0.652	0.92±0.056
AM	1.37±0.001	3.26±0.029	0.78±0.011
DB	0.78±0.001	2.36±0.062	0.53±0.040
HO	1.74±0.007	11.38±0.569	5.40±0.089
FA	2.50±0.030	14.46±1.102	3.65±0.124
YO	0.23±0.001	1.03±0.040	0.40±0.035
ES	1.68±0.015	6.07±0.349	1.91±0.216
LJ	0.83±0.000	2.97±0.145	1.06±0.028
CA	0.48±0.003	2.24±0.174	0.72±0.076
SK	1.14±0.004	2.04±0.055	0.76±0.020
CN	0.93±0.007	2.41±0.049	0.57±0.012
EU	1.61±0.048	3.54±0.021	1.10±0.017
IC	1.33±0.006	3.18±0.116	0.90±0.004
U2	1.28±0.002	2.23±0.168	0.78±0.027
U5	1.36±0.005	3.20±0.221	3.12±0.118

TABLE IV

(EXTENSION OF TABLE III IN THE MAIN PAPER WITH STANDARD DEVIATIONS). THE EFFECTS OF THE ITERATION NUMBER T IN SLUGGER. AS T INCREASED, THE OUTPUTS BECAME CONCISE AND THE EXECUTION TIME OF SLUGGER INCREASED. THE COMPRESSION RATES (I.E., EQ. (10)) ALMOST CONVERGED AFTER 40 ITERATIONS. THE PRESENTED VALUES ARE THE MEANS AND STANDARD DEVIATIONS OVER FIVE RUNS.

T	Relative Size of Outputs						Running Time (Seconds)					
	1	5	10	20	40	80	1	5	10	20	40	80
PR	0.147±0.001	0.098±0.001	0.095±0.001	0.094±0.001	0.093±0.000	0.093±0.000	0.91±0.037	1.25±0.210	1.19±0.098	1.46±0.253	1.57±0.172	1.66±0.215
EM	0.842±0.004	0.805±0.002	0.760±0.002	0.743±0.001	0.736±0.001	0.734±0.001	1.84±0.173	3.58±0.180	4.95±0.580	6.95±0.215	10.92±0.383	16.87±0.883
AM	0.776±0.000	0.731±0.000	0.708±0.000	0.700±0.000	0.697±0.000	0.696±0.000	10.81±0.144	19.45±0.345	23.88±0.489	34.38±0.296	47.17±0.307	79.48±1.584
DB	0.734±0.001	0.703±0.000	0.687±0.000	0.678±0.000	0.676±0.000	0.675±0.000	5.22±0.087	8.25±0.247	9.50±0.180	13.36±0.219	18.43±0.268	32.32±0.095
HO	0.572±0.003	0.485±0.001	0.445±0.000	0.422±0.000	0.412±0.000	0.408±0.000	2309.3±179.7	5677.0±166.8	8730.8±166.4	13611±194.1	21324±401.7	33797±511.3
FA	0.523±0.006	0.456±0.002	0.437±0.001	0.429±0.002	0.427±0.001	0.426±0.001	0.68±0.068	1.49±0.191	1.47±0.062	1.83±0.141	2.24±0.106	2.37±0.103
YO	0.962±0.001	0.951±0.000	0.934±0.001	0.917±0.000	0.909±0.000	0.905±0.000	20.77±0.756	49.68±1.082	74.86±3.268	139.62±2.955	239.39±2.55	456.13±11.2
ES	0.847±0.004	0.789±0.000	0.747±0.001	0.718±0.001	0.705±0.001	0.701±0.001	234.03±20.6	576.41±25.7	971.44±56.5	1605.6±19.41	2803.5±39.1	4965.2±55.0
LJ	0.831±0.000	0.795±0.000	0.762±0.000	0.744±0.000	0.736±0.000	0.734±0.000	273.46±5.78	643.16±15.7	1007.1±23.7	1745.0±17.83	3046.5±28.5	5514.1±68.1
CA	0.916±0.009	0.873±0.003	0.850±0.001	0.835±0.001	0.827±0.001	0.823±0.001	0.77±0.075	1.30±0.111	1.38±0.139	1.93±0.078	2.70±0.127	4.16±0.128
SK	0.698±0.003	0.586±0.000	0.556±0.000	0.542±0.000	0.537±0.000	0.535±0.000	63.46±5.011	150.29±2.86	200.53±5.03	320.97±3.208	473.48±14.5	778.85±19.3
CN	0.299±0.005	0.226±0.000	0.219±0.000	0.216±0.000	0.215±0.000	0.214±0.000	8.58±0.393	16.66±0.723	18.16±0.274	23.46±0.699	30.46±0.532	44.63±0.990
EU	0.283±0.002	0.206±0.000	0.194±0.000	0.187±0.001	0.183±0.000	0.182±0.000	55.35±0.972	116.00±3.50	134.04±2.91	182.63±2.486	234.41±4.68	331.22±6.13
IC	0.155±0.001	0.107±0.000	0.102±0.000	0.101±0.000	0.100±0.000	0.100±0.000	648.65±49.2	1666.3±55.8	1745.4±306.7	1953.6±34.33	2156.3±14.4	2773.3±68.5
U2	0.210±0.000	0.148±0.000	0.144±0.000	0.142±0.000	0.141±0.000	0.141±0.000	695.46±12.3	1576.2±20.2	1730.5±27.0	2239.7±22.95	2772.1±27.5	4163.9±19.3
U5	0.156±0.001	0.113±0.000	0.110±0.000	0.108±0.000	0.108±0.000	0.107±0.000	5752.5±89.9	11046±250.7	11170±177.3	12893±533.3	14111±339.9	19379±516.5

TABLE V

(EXTENSION OF TABLE IV IN THE MAIN PAPER WITH STANDARD DEVIATIONS). THE PRUNING STEP IN SLUGGER IS EFFECTIVE. EVERY SUBSTEP SUCCESSFULLY DECREASED THE SIZE OF OUTPUT REPRESENTATIONS, PROVING THE BENEFITS OF OUR DESIGN CHOICES. THE PRESENTED VALUES ARE THE MEANS AND STANDARD DEVIATIONS OVER FIVE RUNS.

	Avg. Max Height				Avg. Depth per Leaf Node				Relative Size of Outputs			
	0	1	2	3	0	1	2	3	0	1	2	3
PR	50.4±3.362	9.2±1.095	9.0±1.414	9.0±1.414	4.572±0.125	1.904±0.050	1.847±0.046	1.746±0.050	0.115±0.001	0.097±0.001	0.097±0.001	0.094±0.001
EM	12.6±0.548	6.2±0.447	6.2±0.447	6.2±0.447	1.231±0.010	0.848±0.006	0.835±0.007	0.826±0.006	0.773±0.001	0.745±0.001	0.745±0.001	0.743±0.001
AM	10.0±0.000	6.2±0.447	6.2±0.447	6.2±0.447	1.705±0.001	1.436±0.001	1.413±0.001	1.370±0.001	0.720±0.000	0.705±0.000	0.703±0.000	0.700±0.000
DB	27.8±5.070	6.2±0.447	6.2±0.447	6.2±0.447	1.417±0.002	0.827±0.001	0.799±0.001	0.783±0.001	0.746±0.000	0.686±0.000	0.683±0.000	0.679±0.000
HO	183.0±0.000	14.8±0.837	14.8±0.837	14.8±0.837	4.702±0.014	2.048±0.006	2.047±0.005	1.739±0.007	0.439±0.000	0.430±0.000	0.430±0.000	0.422±0.000
FA	9.6±0.894	8.8±1.304	8.8±1.304	8.8±1.304	2.837±0.037	2.539±0.027	2.536±0.028	2.504±0.030	0.434±0.002	0.430±0.001	0.430±0.001	0.429±0.001
YO	17.6±2.302	6.8±0.837	6.8±0.837	6.8±0.837	0.385±0.002	0.239±0.001	0.228±0.001	0.227±0.001	0.936±0.000	0.919±0.000	0.918±0.000	0.917±0.000
ES	25.4±1.517	11.0±1.000	11.0±1.000	11.0±1.000	2.650±0.018	1.723±0.015	1.720±0.015	1.685±0.015	0.728±0.000	0.720±0.001	0.720±0.001	0.718±0.001
LJ	65.6±12.641	12.0±0.000	12.0±0.000	12.0±0.000	1.001±0.000	0.872±0.000	0.848±0.000	0.834±0.000	0.752±0.000	0.747±0.000	0.745±0.000	0.744±0.000
CA	18.8±3.493	4.8±0.447	4.8±0.447	4.6±0.548	1.501±0.025	0.506±0.002	0.485±0.003	0.484±0.003	0.950±0.001	0.837±0.001	0.836±0.001	0.836±0.001
SK	22.6±1.342	11.4±0.548	11.4±0.548	11.4±0.548	2.029±0.011	1.231±0.005	1.159±0.005	1.142±0.004	0.577±0.001	0.547±0.001	0.544±0.001	0.542±0.000
CN	44.2±1.924	9.6±0.894	9.6±0.894	9.6±0.894	2.774±0.013	0.998±0.005	0.945±0.007	0.928±0.007	0.259±0.000	0.219±0.000	0.218±0.000	0.216±0.000
EU	202.0±73.607	9.2±0.837	9.2±0.837	9.2±0.837	4.259±0.130	1.793±0.030	1.728±0.044	1.610±0.048	0.221±0.001	0.197±0.001	0.196±0.001	0.187±0.001
IC	502.2±3.033	12.0±0.707	12.0±0.707	12.0±0.707	4.196±0.043	1.433±0.004	1.379±0.005	1.330±0.006	0.126±0.000	0.104±0.000	0.104±0.000	0.101±0.000
U2	488.8±25.044	12.4±0.548	12.4±0.548	12.4±0.548	4.034±0.006	1.400±0.001	1.317±0.001	1.281±0.002	0.177±0.000	0.145±0.000	0.144±0.000	0.142±0.000
U5	499.8±0.837	13.6±1.517	13.6±1.517	13.6±1.517	5.014±0.010	1.492±0.004	1.395±0.004	1.356±0.005	0.136±0.000	0.110±0.000	0.110±0.000	0.108±0.000

TABLE VI

(EXTENSION OF TABLE V IN THE MAIN PAPER WITH STANDARD DEVIATIONS). THE EFFECTS OF THE HEIGHT OF HIERARCHY TREES. AS THE UPPER BOUND H_b OF THE HEIGHT INCREASED, THE AVERAGE DEPTH OF LEAF NODES INCREASED, AND THE RELATIVE SIZE OF OUTPUTS DECREASED. NOTABLY, THE AVERAGE DEPTH OF LEAF NODES WAS MUCH LOWER THAN THE UPPER BOUND H_b. THE PRESENTED VALUES ARE THE MEANS AND STANDARD DEVIATIONS OVER FIVE RUNS.

	Avg. Depth per Leaf Node					Relative Size of Outputs				
	2	5	7	10	∞	2	5	7	10	∞
PR	0.94±0.033	1.28±0.037	1.42±0.031	1.57±0.035	1.75±0.050	0.194±0.001	0.112±0.000	0.103±0.001	0.099±0.001	0.094±0.001
EM	0.70±0.002	0.80±0.005	0.80±0.004	0.80±0.004	0.83±0.006	0.757±0.001	0.743±0.001	0.743±0.001	0.743±0.001	0.743±0.001
AM	1.14±0.001	1.36±0.002	1.37±0.001	1.37±0.002	1.37±0.001	0.722±0.000	0.704±0.000	0.704±0.000	0.704±0.000	0.700±0.000
DB	0.67±0.000	0.75±0.001	0.75±0.002	0.76±0.001	0.78±0.001	0.722±0.000	0.682±0.000	0.680±0.000	0.679±0.000	0.679±0.000
HO	1.12±0.000	1.48±0.002	1.67±0.002	1.85±0.002	1.74±0.007	0.503±0.000	0.446±0.000	0.437±0.000	0.433±0.000	0.422±0.000
FA	1.50±0.005	2.26±0.012	2.42±0.043	2.46±0.072	2.50±0.030	0.463±0.001	0.433±0.000	0.433±0.002	0.432±0.002	0.429±0.001
YO	0.21±0.000	0.23±0.001	0.23±0.001	0.23±0.001	0.23±0.001	0.924±0.000	0.919±0.000	0.918±0.000	0.918±0.001	0.917±0.000
ES	1.22±0.006	1.47±0.007	1.56±0.010	1.63±0.018	1.68±0.015	0.742±0.001	0.725±0.001	0.722±0.000	0.721±0.000	0.718±0.001
LJ	0.71±0.000	0.82±0.001	0.82±0.000	0.83±0.001	0.83±0.000	0.755±0.000	0.747±0.000	0.746±0.000	0.746±0.000	0.744±0.000
CA	0.44±0.003	0.47±0.007	0.48±0.008	0.48±0.002	0.48±0.003	0.886±0.002	0.845±0.002	0.839±0.001	0.837±0.002	0.836±0.001
SK	0.84±0.001	1.07±0.001	1.12±0.004	1.14±0.004	1.14±0.004	0.579±0.000	0.547±0.000	0.545±0.000	0.545±0.000	0.542±0.000
CN	0.69±0.001	0.84±0.006	0.88±0.009	0.87±0.004	0.93±0.007	0.306±0.002	0.231±0.001	0.223±0.001	0.218±0.000	0.216±0.000
EU	1.10±0.007	1.45±0.012	1.55±0.024	1.62±0.012	1.61±0.048	0.285±0.001	0.206±0.000	0.200±0.001	0.197±0.000	0.187±0.001
IC	0.89±0.000	1.16±0.003	1.27±0.005	1.33±0.003	1.33±0.006	0.202±0.000	0.119±0.000	0.110±0.000	0.106±0.000	0.101±0.000
U2	0.91±0.000	1.13±0.001	1.20±0.001	1.24±0.001	1.28±0.002	0.241±0.000	0.158±0.000	0.149±0.000	0.146±0.000	0.142±0.000
U5	0.96±0.000	1.19±0.000	1.26±0.010	1.31±0.003	1.36±0.005	0.210±0.000	0.125±0.000	0.116±0.000	0.112±0.000	0.108±0.000

TABLE VII

THE EXECUTION TIME OF FOUR GRAPH ALGORITHMS ON SUMMARIZED GRAPHS. THE PRESENTED VALUES ARE THE MEANS AND STANDARD DEVIATIONS OVER THREE RUNS. WE MEASURED THE RUNNING TIME OF 4 COMMON ALGORITHMS: DIJKSTRA'S, BFS, PAGERANK, AND TRIANGLE COUNTING, ON SUMMARIZED GRAPHS OBTAINED BY SLUGGER AND SWEG. FOR DIJKSTRA'S AND BFS, THE NODE WITH A MAXIMUM DEGREE WAS CHOSEN AS THE INITIAL SOURCE NODE, AND WE ASSUME THE WEIGHT OF EVERY EDGE AS 1. FOR PAGERANK, WE CHOSE A DAMPING FACTOR OF 0.85. DURING EXECUTION, WE PARTIALLY DECOMPRESSED THE MODEL BASED ON ALGORITHM 4 TO RETRIEVE THE NEIGHBORS OF THE NODES FOR SLUGGER AND USED ALGORITHM 1 IN THE SWEG PAPER FOR SWEG. AS SEEN IN THE NUMBERS BELOW, THERE IS A TRADE-OFF BETWEEN THE SIZE OF COMPRESSED GRAPHS AND THE SPEED OF ALGORITHMS.

Tasks	Triangle Counting			BFS			Dijkstra			PageRank		
	Original	SWeG	Slugger	Original	SWeG	Slugger	Original	SWeG	Slugger	Original	SWeG	Slugger
PR	0.197±0.001	0.532±0.069	1.065±0.099	0.013±0.006	0.019±0.001	0.054±0.014	0.012±0.005	0.020±0.001	0.047±0.010	0.107±0.010	0.332±0.014	0.741±0.057
EM	0.152±0.001	0.420±0.201	0.677±0.021	0.020±0.002	0.036±0.006	0.062±0.005	0.020±0.001	0.051±0.031	0.061±0.004	0.228±0.003	0.485±0.044	1.089±0.032
AM	0.650±0.008	1.932±0.025	3.466±0.018	0.103±0.009	0.378±0.013	0.860±0.021	0.122±0.004	0.401±0.012	0.740±0.030	0.994±0.057	4.686±0.124	10.24±0.117
DB	0.342±0.004	0.805±0.085	1.477±0.016	0.099±0.002	0.211±0.014	0.327±0.024	0.105±0.001	0.238±0.007	0.343±0.002	0.706±0.030	3.133±0.039	5.672±0.116
HO	746.2±14.51	1905.8±76.91	4641.0±41.89	1.947±0.007	8.999±0.560	24.36±0.139	1.792±0.031	8.103±0.230	18.83±0.728	36.57±0.468	153.7±3.948	390.5±10.58
FA	0.124±0.001	0.463±0.208	0.800±0.120	0.006±0.001	0.015±0.001	0.038±0.006	0.007±0.005	0.014±0.001	0.041±0.006	0.060±0.003	0.228±0.014	0.572±0.060
YO	1.521±0.082	3.518±0.176	7.529±0.055	0.289±0.034	0.597±0.072	1.149±0.031	0.315±0.013	0.698±0.086	0.923±0.010	2.886±0.109	12.06±0.328	21.89±0.246
ES	27.89±0.361	66.06±0.528	165.9±0.820	0.449±0.083	1.852±0.018	5.685±0.162	0.473±0.026	1.843±0.049	4.285±0.172	9.506±0.166	37.58±0.629	107.9±3.068
LJ	27.08±0.100	78.35±0.423	179.4±3.113	2.006±0.263	6.295±0.116	12.90±1.249	2.248±0.184	5.577±0.074	10.78±0.374	22.52±0.417	93.52±1.436	206.6±2.199
CA	0.023±0.001	0.056±0.002	0.110±0.018	0.007±0.001	0.017±0.001	0.022±0.001	0.009±0.001	0.024±0.005	0.023±0.002	0.085±0.025	0.285±0.008	0.535±0.006
SK	5.983±0.193	17.67±0.449	38.26±0.739	0.299±0.027	1.421±0.037	3.195±0.146	0.472±0.039	1.546±0.025	2.844±0.113	7.227±0.175	36.74±0.459	82.87±0.627
CN	0.741±0.009	2.391±0.155	5.466±0.350	0.058±0.008	0.218±0.014	0.430±0.040	0.068±0.003	0.239±0.011	0.414±0.010	1.741±0.051	7.120±0.112	15.68±0.138
EU	7.168±0.112	25.59±0.590	56.90±1.079	0.302±0.003	1.377±0.042	2.824±0.086	0.317±0.006	1.141±0.005	2.465±0.082	7.429±0.072	31.69±1.071	75.88±2.145
IC	916.7±14.61	5724.9±113.2	9113.0±158.7	1.992±0.262	11.60±0.219	26.77±0.173	2.242±0.061	10.30±0.257	20.14±0.228	60.89±4.812	336.5±8.746	707.5±10.39
U2	88.55±1.111	373.4±2.729	859.9±21.82	7.313±2.205	26.92±0.843	62.78±1.405	5.262±0.228	19.46±0.211	39.13±0.635	111.4±1.484	530.4±13.55	1319.8±19.33
U5	780.5±34.34	2023.6±25.24	4157.0±83.14	43.24±0.560	98.14±2.796	223.7±3.175	29.83±1.176	82.11±2.326	145.9±3.176	724.9±22.52	2110.6±33.33	4246.3±75.63

TABLE VIII

THE EFFECTS OF THE MAXIMUM NUMBER OF CANDIDATE SET. AS THE MAXIMUM NUMBER OF EACH CANDIDATE SET INCREASED, THE RELATIVE SIZE OF OUTPUTS DECREASED, AND THE RUNNING TIME INCREASED. THE PRESENTED VALUES ARE THE MEANS AND STANDARD DEVIATIONS OVER FIVE RUNS.

	Relative Size of Outputs				Running Time (Seconds)			
	20	100	500	2500	20	100	500	2500
PR	0.095±0.001	0.095±0.001	0.094±0.001	0.093±0.001	0.843±0.072	1.169±0.170	1.458±0.253	1.740±0.267
EM	0.759±0.002	0.747±0.001	0.743±0.001	0.743±0.001	2.724±0.090	4.334±0.117	6.946±0.215	9.401±1.044
AM	0.702±0.000	0.700±0.000	0.700±0.000	0.700±0.000	34.345±0.484	38.912±0.792	34.384±0.296	46.208±1.760
DB	0.680±0.000	0.678±0.000	0.678±0.000	0.678±0.000	14.823±0.144	16.763±0.346	13.356±0.219	16.822±0.252
HO	0.443±0.000	0.431±0.000	0.422±0.000	0.418±0.000	1318.1±14.15	3215.5±48.85	13611±194.1	52592±2053.7
FA	0.441±0.001	0.431±0.001	0.429±0.001	0.430±0.001	1.001±0.057	1.729±0.308	1.826±0.141	1.866±0.108
YO	0.931±0.001	0.923±0.001	0.917±0.000	0.914±0.001	59.500±0.597	83.644±2.310	139.62±2.955	308.98±14.95
ES	0.741±0.000	0.724±0.001	0.718±0.001	0.713±0.000	319.26±4.984	756.61±19.27	1605.6±19.41	5357.9±267.8
LJ	0.758±0.000	0.746±0.000	0.744±0.000	0.743±0.000	610.88±2.284	1147.6±22.85	1745.0±17.84	2616.4±93.79
CA	0.858±0.002	0.847±0.002	0.835±0.002	0.831±0.001	1.140±0.051	1.327±0.101	1.934±0.078	3.047±0.516
SK	0.558±0.000	0.547±0.000	0.542±0.000	0.539±0.000	135.54±1.802	190.09±2.039	320.97±3.208	690.63±37.73
CN	0.220±0.001	0.217±0.000	0.216±0.000	0.215±0.001	16.293±0.248	19.374±0.435	23.456±0.699	57.388±2.063
EU	0.192±0.001	0.189±0.000	0.187±0.000	0.186±0.000	86.701±1.195	120.04±2.749	182.63±2.486	469.15±19.43
IC	0.105±0.000	0.102±0.000	0.101±0.000	0.100±0.000	652.12±9.228	990.46±25.88	1953.6±34.33	7318.0±234.4
U2	0.147±0.000	0.144±0.000	0.142±0.000	0.141±0.000	1343.6±8.465	1639.6±5.681	2239.7±22.95	4768.7±39.90
U5	0.112±0.000	0.109±0.000	0.108±0.000	0.108±0.000	6404.4±92.88	9355.6±220.8	12893±533.3	17350±436.4