Simulating the Formation of the Oort Cloud by the Grand Tack

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December 2020

Abstract

The formation of the Oort Cloud is one of the remaining open questions concerning the solar system. In this work, the effect of a grand tack scenario on this formation was studied. For an initial planetesimal disk with 100 test particles, four different cases of the early solar system were evolved up to 100 Myr. Results show that the simulation including a Milky Way potential and a Grand Tack event gives the highest probability of 2.3% for a particle to become an Oort cloud object, as compared to the other cases where this probability is about 0.9%. Furthermore, the scattering rate during the Grand Tack is investigated, and the initial location of perturbed particles is found to always be close to the giant planets.

1 Introduction

As of today, multiple theories concerning the formation of the Oort and Hills clouds exist. Examples are Hills (1981), where nearby passing stars perturb the orbits of objects in the Hills cloud in such a way that they form the Oort cloud, and Levison et al. (2010), where over 90% of the Oort cloud objects were captured by the sun from around other stars in its birth cluster. However, none of these theories is considered confirmed, so that the formation of the clouds remains a mostly unanswered question.

This report will test another hypothesis concerning the formation of the Oort cloud, namely that the Oort cloud was formed due to the migration of Jupiter and Saturn in the first 5 Myr after formation of the Solar System. To test this hypothesis, several simplified Solar Systems with test particles were simulated and the percentage of test particles that have promising paths towards the current Oort cloud was recorded. The simulated Solar Systems differ in their early evolutionary paths as they take different gravitational effects into account, allowing for a comparison.

The formation of the Oort Cloud is one of the remaining mysteries in our Solar System. Solving this mystery would directly influence and improve our understanding of the early, as well as the present Solar System. On larger scales, this understanding can teach us about the probability and possibility of similar structures in other planetary systems and the potential influence that they may have. As the Oort cloud has still not been directly observed, learning about its origin might in turn also give us stronger constraints on its structure.

The report starts off with a brief recap of the main protagonists of the remainder of the report, being the Oort cloud and the Grand tack. Then, the methods used for simulating the different cases is discussed in Section 3. Results on the potential Oort cloud object-percentages and their initial locations are presented separately for each case in the subsections of Section 4. These results are compared to other but similar simulations in the literature in Section 5. The report is closed off by a short conclusion in Section 6.

2 Background

2.1 The Oort Cloud

The Oort cloud is proposed to be an isotropic sphere with predominantly icy planetesimals surrounding the Sun at distances ranging from 2.000 to 200.000 au (Morbidelli 2005). As of today, this is still a very mysterious structure, since it has never been directly observed. Its existence was inferred in 1950 by Dutch Astronomer Jan Hendrik Oort, who used observations of 22 then newly detected long period comets (Oort 1950). Some 30 years later, Jack G. Hills proposed the existence of an inner and outer Oort cloud, the former now known as the Hills cloud (Hills 1981). While objects from the Oort cloud fall into the planetary orbits at a steady rate, objects from the Hills cloud only enter when a nearby star passes by the sun at a sufficiently close distance (approximately the semi-major axis of the orbit of the comets), causing a short but intense infall of objects.

2.2 Grand Tack Model

Whereas initially it was thought that the orbits of the planets had never evolved much from their initial states, this belief has recently been challenged; the so-called 'Grand Tack Hypothesis' postulates that in about the first 5 My years of the Solar System, a fully formed Jupiter moved inwards from 3.5 au towards the sun, as close as some 1.5 au (Walsh et al. 2011). Saturn, a short while later, also moved inward but faster than Jupiter, until it was captured in a 2:3 resonance with Jupiter. When this happened, both planets started migrating outward to their present day orbits, pushing Uranus and Neptune outwards with them. During these movements, the giants crossed through the protoplanetary disk of planetesimals still present around the sun, destabilizing their orbits. Our hypothesis originates from the idea that these planetesimals were scattered outwards to form the Oort cloud. Simulations were already performed in 2004 by Dones et al. (2004), who showed that test-particle comets near Jupiter up until the Kuiper belt would be scattered outwards and reach a nearly isotropic inclination distribution with semi-major axes of 3.000 - 200.000 au after 4 Gyr, which is in line with current understandings of the Oort cloud.

3 Method

As mentioned in the Introduction, several simulations involving different mechanisms were performed. Explicitly, a comparison is made between the following four cases:

1. Vanilla: The Milkyway (MW) does *not* exert force on the Solar System and *no* Grand Tack takes place

2. MW: The solar system *is* subject to the gravitational potential of the MW but the Grand Tack does *not* take place

3. Vanilla_Tack: The Solar System is *not* subject to the gravitational potential of the MW, but *does* have a Grand Tack

4. MW_Tack: The Solar System *does* undergo a Grand Tack and *is* affected by the gravitational potential of the MW.

In all cases, the Solar System is simulated as a 5-body problem. Only the gas giants and the Sun are considered, in order to speed up calculations by having fewer force-exerting bodies and by keeping the typical distance larger, which allows for the time step of the simulation to be larger. The planets start out with their post-tack orbits in the so called 'Vanilla' and 'MW' runs, while starting with pre-tack orbits for the 'Grand Tack' and 'Grand Tack & MW' runs (for more details, see Section 3.2).

To answer our main question, test particles with zero mass were used to represent comets in the Solar System. These particles were randomly distributed throughout the Solar System, by randomly drawing from a uniform distribution with different ranges. They were given a semi-major axis between 4 and 40 au, an eccentricity between 0 and 0.05, an inclination between -5 and 5 degrees, and a true anomaly, argument of periapsis and longitude of the ascending node between 0 and 360 degrees. The resulting orbits are in the same general rotational direction as the planets and the rotation of the Sun, but starting at different positions. An example of a resulting initial setup is shown in Figure 1.

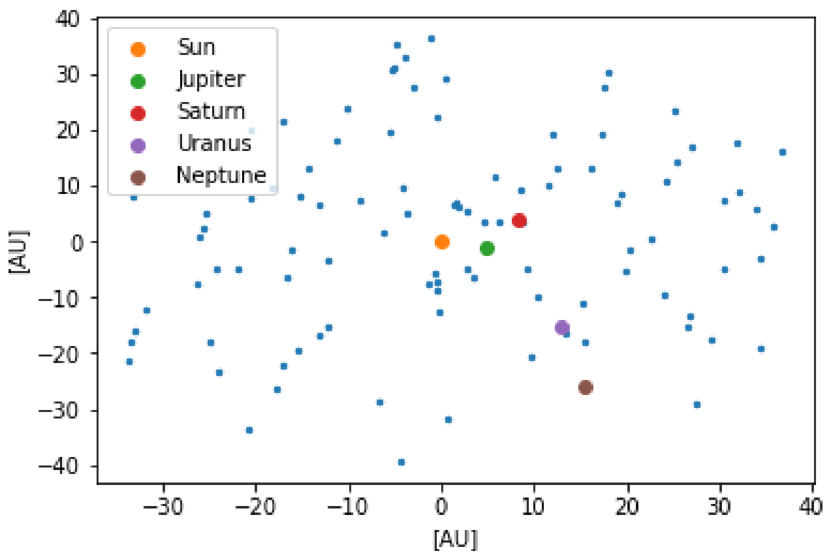

Figure 1: Example of the top view of an initial setup of the Solar System plus test particles. (Object sizes not to scale)

The choice for a 5- instead of N-body simulation is a necessary assumption; as N will be very large, having to calculate N-1 forces for all N bodies will take an extensively large amount of time. Calculating 5 forces for N bodies decreases this time by a factor of at least $\frac{N-1}{5}$ (depending on the N-body solving algorithm). This does come at the cost of a loss in accuracy, but taking into account that the current Mass-estimate for the complete Oort Cloud is about 5 Earth masses (Morbidelli 2005) of isotropically distributed planetesimals, we expect this loss in accuracy to be negligible.

Furthermore, we will neglect passing stars, since it has been shown that these will only perturb the motion of comets almost directly on their path, a relatively small tunnel (with radius 450 au for a 1 $M_\odot$ star with velocity 20 km s^{-1}) compared to the whole cloud (Weissman 1996). In a future study, the effect of passing stars on the randomization of the Oort cloud could be investigated. Molecular cloud encounters are rare and therefore also neglected.

3.1 Gravitational Potential

To implement the MW potential, we used the bridge function that AMUSE offers. This allows to solve for the movements of the objects, where the objects feel the pull of the potential, but the potential does not feel the objects.

To simulate the gravitational potential of the MW on an object at position (x,y,z), the following potential, consisting of three components is used:

$$\Phi = \Phi_{bulge} + \Phi_{disk} + \Phi_{halo}. \tag{1}$$

The bulge component corresponds to a Hernquist spheroid

$$\Phi_{bulge}(r) = -\frac{GM_{bulge}}{\sqrt{r^2 + b_1^2}}, \tag{2}$$

with the scalelength $b_1 = 0.3873$ kpc.

The disk component is a Miyamoto-Nagai disk (Miyamoto & Nagai 1975):

$$\Phi_{disk}(R, z) = -\frac{GM_{disk}}{\sqrt{R^2 + (a_2 + \sqrt{z^2 + b_2^2})^2}}, \tag{3}$$

with scalelengths $a_2 = 5.31$ kpc and $b_2 = 0.25$ kpc.

Lastly, the dark matter halo component is taken from (Allen & Santillan 1991) and given by

$$\Phi_{halo}(r) = -\frac{GM_{halo}}{a_3}\frac{d_1^{1.02}}{1 + d_1^{1.02}} - \frac{GM_{halo}}{1.02a_3}\left(\frac{-1.02}{c} + \ln(c) + \frac{1.02}{1 + d_1^{1.02}} - \ln(1 + d_1^{1.02})\right), \tag{4}$$

where $a_3 = 12.0$ kpc, $d_1 = r/a_3$ and $c = 1 + (\text{cutoff}/a_3)^{1.02}$, the cutoff is set at 100 kpc.

3.2 Setting up the Grand Tack

For the Grand Tack scenario, the planets start off differently from their current day orbits. For simplicity all orbits were assumed to be circular and in the ecliptic plane. The pre-tack semi-major axes were set at 3.5 au, 4.5 au, 6 au and 8 au for Jupiter, Saturn, Uranus and Neptune respectively, while the three angular orbital parameters were randomised. In order to allow the gas giants to move inward, and later outward through the Solar System, the planets were slowed down and sped up with fictional tidal friction exerting a force on them. As we were unable to both implement this tidal friction and get the desired movement, we were forced to manually change the semi-major axis of the migrating planets at each timestep to create a similar effect. Additionally the three orbital parameter angles were kept at the same value when such a change occurred. In combination with small time steps, this approximated a continuous inward and outward migration. The semi-major axis was changed in such a way that it simulated the following movement:

$$v(t) = v_0 + \Delta v_{in}[t/t_{end}], \tag{5}$$

where v_0 is the original velocity and t_{end} is the time at which inward migration ends, and an outward velocity following

$$v(t) = v_{in} + \Delta v_{out}[1 - exp(-t/\tau)], \tag{6}$$

where v_{in} is the velocity at the end of the inward migration and τ is the adopted migration timescale ($5 \cdot 10^5$ yr). Both of these formulae were adopted from Walsh et al. (2011). Jupiter's inward movement starts at $t = 0$ and ends at $t = 0.1$ Myr, while Saturn's inward movement starts at $t = 0.1$ Myr and ends at $t = 0.1025$ Myr. This is directly followed by outward migration of Jupiter and Saturn, and once Saturn gets in 9:5 resonance with Uranus and 5:2 resonance with Neptune, these start migrating outward in turn as well. The outward migration ends at $t = 0.6$ Myr. After the outward movement has stopped, the final post-tack orbits of the planets are at 5.4 au, 7.1 au, 10.5 au and 13 au respectively. The resulting movements are shown in Figure 2, where the eccentricity and inclination of their orbits are shown as well, to show the stability of the orbits.

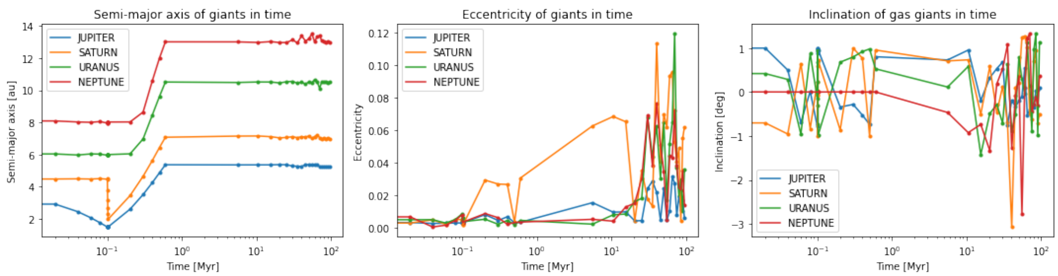

Figure 2: Orbital elements of planets' orbits around the Sun, plotted in time, for a random Vanilla Grand Tack run.

3.3 Ejection

In order to evaluate the amount of objects leaving the Solar System, we first need to define when the orbit of objects is no longer similar to their original orbits. Objects start with an original semi-major axis of $a \leq 40$ au, so we define perturbed orbits as having $a > 100$ au. This value is chosen since it is significantly larger than the original value, and it is nice and round. Moreover, at present times, Oort cloud comets orbit with semi-major axes of at least approx. 2.000 au, so their orbits need to be significantly perturbed.

Besides the fact that orbits can be perturbed, particles can also be ejected. This also requires a definition. If we say an object is ejected, we mean it is no longer orbiting the Sun. To find out if this is the case for a particle, multiple methods can be used. First of all, one can look at the current velocity of the particle v_{rel}, which is the velocity with respect to the Sun, and see if this value exceeds the escape velocity $v_{esc} = \sqrt{\frac{2GM_\odot}{r_{obj}}}$ from the Sun. This is equivalent to having a hyperbolic orbit with eccentricity $e > 1$. A second approach is to determine the distance from the Sun, and (a) determine if this is further away than the typical distance to another stellar system ($\sim 300.000au$), or (b) determine if the attraction of the MW has a bigger impact on the particle than the attraction of the Sun. This distance is defined by the Hill radius of the Sun, which lays at over 7 million au, so we can neglect this method. The methods of velocity and distance will be explored in the results, since it is very important to know if the comets have the potential to become Oort cloud objects. This gives the following condition for being a potential Oort cloud object: $a > 100$ au, $v_{rel} < v_{esc}$ and $r < 300.000$ au.

4 Results

In this section, the main results for the four different cases will be shown and compared. Since the two cases with MW-potential were calculated using different gravity solvers than without MW-potential (The Vanilla runs used a symplectic solver, while the MW runs used a non-symplectic one), one should keep in mind that any difference in results could be caused by this. Each run contains 100 new randomly generated test particles. The amount of runs used for each model was limited by time, and is 8 for Vanilla, 8 for Vanilla GT, 10 for MWG and 9 for MWG GT. In the analysis, the non-tack runs were considered every 5 Myr, while for the tack runs the tack is resolved with a higher resolution.

4.1 The significance of the Grand Tack on creating potential Oort cloud objects

The main goal of this paper is to find out whether more particles have the potential to become Oort cloud objects in the case of a Grand Tack, or without it. To quantify this, particles were classified as being perturbed, but not ejected (see Section 3.3). To start, the amount of perturbed particles is investigated.

One of the difficulties with showing results lies with the duration of our simulation. The simulation lasts for 100 Myr, instead of the 4.5 Gyr which - according to the hypothesis - has passed since the Grand Tack happened. Therefore, we cannot assume the results at the end of our simulation are definitive. Instead, we show results of the progress throughout time. As a result, a prediction can be made of how the results of the simulation will evolve at later and current times.

4.1.1 Perturbation percentage

To start, particles that were classified as potential Oort cloud objects required to have perturbed orbits, which we defined as having an orbit with semi-major axis $a > 100$ au. The amount of particles deleted from the simulation due to collisions is insignificant, while the amount deleted due to getting out of the 250.000 au range is significant and also added, since their orbit is in another way perturbed. Figure 3 shows the percentage of particles meeting this condition for each timestep. It shows the mean value over all runs, with the standard error as error bars.

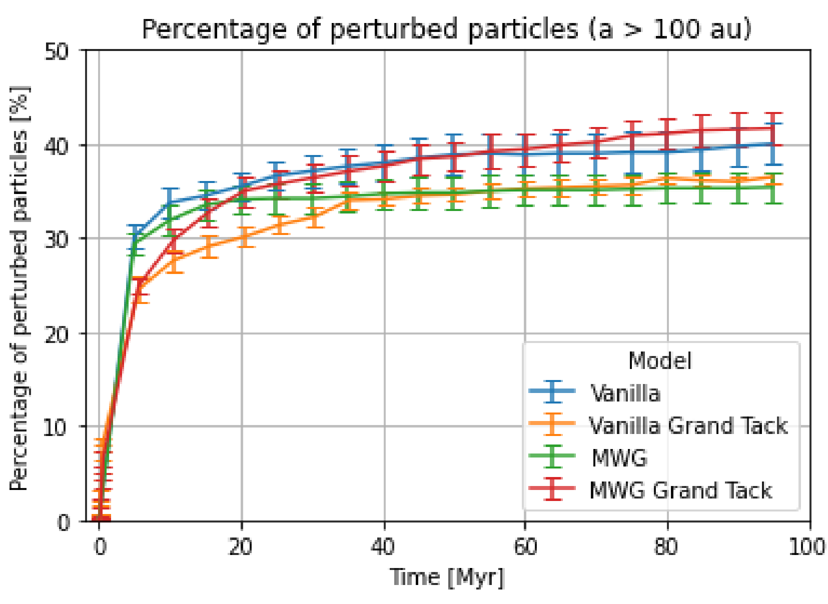

Figure 3: Percentage of perturbed particles as a function of time for each model, averaged over all runs.

From the figure, one can conclude the value converges to a constant value around 40%, where it is unclear whether the Grand Tack has a significant influence. In order to investigate this further, we looked at the ejection rate for each run.

4.1.2 Ejection rate

The Ejection rate at time t is defined as follows: it is the percentage of ejected particles between timestep t and $t - \Delta t$, divided by Δt, where the condition for a particle being ejected is that the it has reached a velocity v_{rel} larger than the escape velocity v_{esc} at time t, as described in Section 3.3. We use ejected particles instead of perturbed here, because particles that escape are rarely present in the following time step and therefore not double-counted, whereas a perturbed but not escaping particle will probably still be present in the particle set in the following time step.

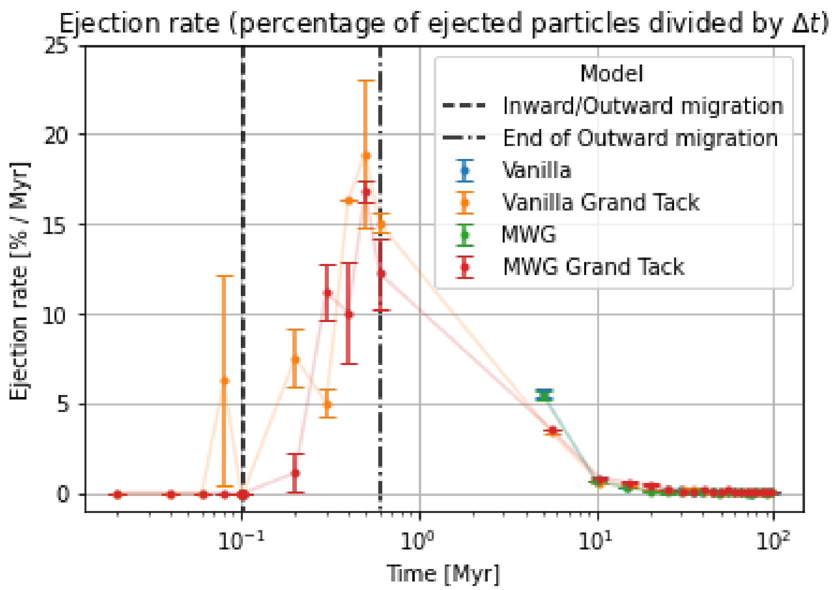

Figure 4: Ejection rate of particles for each model, averaged over all runs. The first dashed line marks the end of the inward migration of the Grand Tack and the start of the outward migration. The second dash-dotted line marks the end of the outward migration.

Figure 4 shows the averaged results for each model. We see that, during the outward migration part of the Grand Tack, most particles are ejected per unit time. This is especially true for the end of this phase, where all giants are migrating.

4.1.3 Potential Oort cloud particles percentage

To get an answer to our main question, whether the Grand Tack has a significant effect on the amount of particles becoming Oort cloud objects, we needn't look at particles that were perturbed or ejected alone, but we want to combine these results into one. Figure 5 shows just that: for each model the percentage of particles that were perturbed, but not ejected is shown at each time step (averaged over all runs). Figure 5 will be discussed in Section 5, however to gain more insight into the result, we will look at the initial locations of particles that were classified as potential Oort cloud objects.

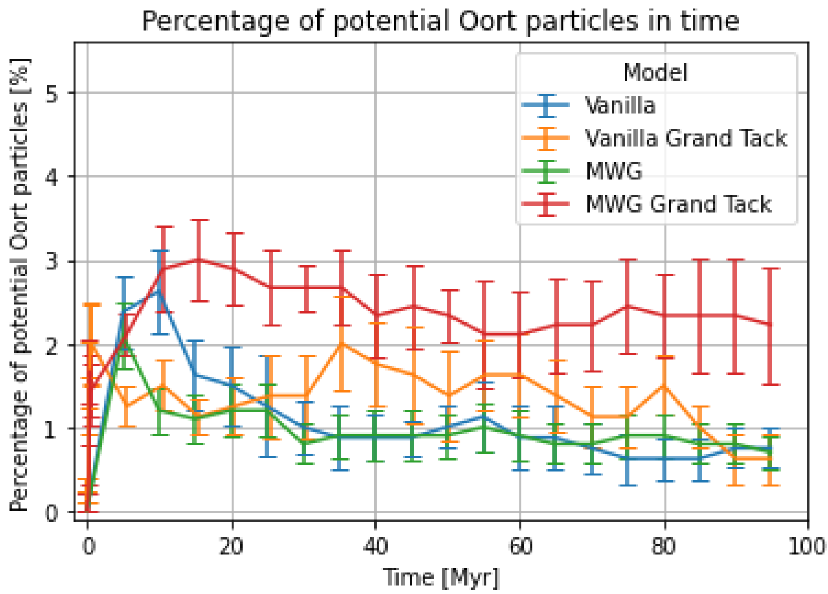

Figure 5: Percentage of particles classified as potential Oort cloud objects as a function of time, averaged over all runs.

4.2 Initial locations of later potential Oort cloud objects

Figure 6 shows the distribution of initial semi-major axes of particles that were later classified as potential Oort cloud objects, and Figure 7 shows the same result for all particles that had perturbed orbits at some point in time. From the second figure it is clear that the cause for perturbation is scattering by giant planet, because only particles that orbit near them get their orbits perturbed, because particles with initial semi-major axis $22 \lesssim a < 40$ au are not present in these graphs. There is no indication in the data for a preference in eccentricity.

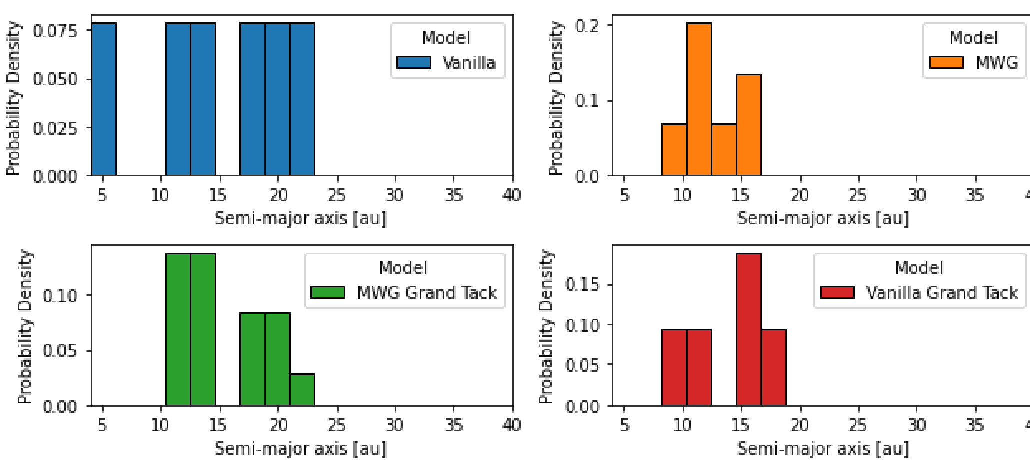

Figure 6: Distribution of initial semi-major axes for all particles that were later classified as potential Oort cloud objects.

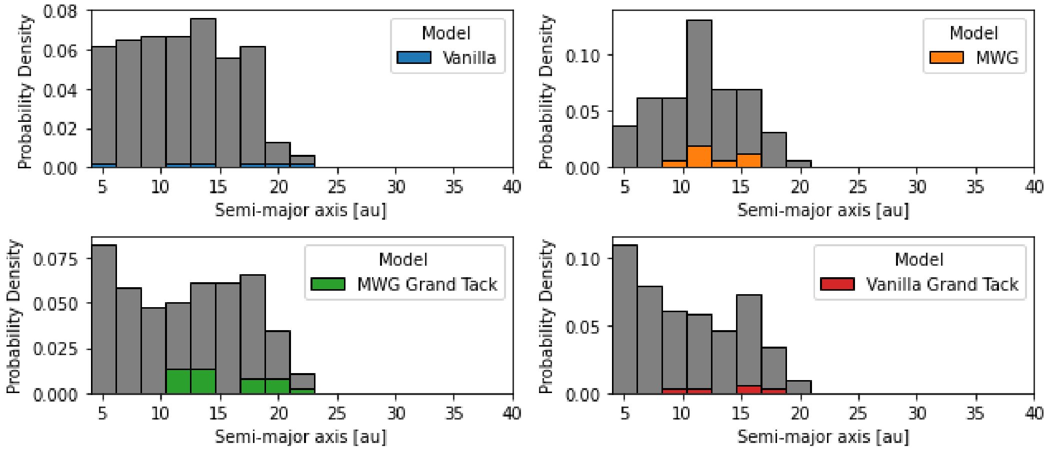

Figure 7: Distribution of initial semi-major axes for all particles whose orbits later were perturbed. Colored segments show particles that ended up classified as potential Oort cloud objects, as in Figure 6.

4.3 Direction of scattering w.r.t. MWG center

Lastly, in the simulations that include a Milky Way Galaxy potential, the question arose whether there is a preferred direction of ejection with respect to the MWG center. Perhaps the Milky Way causes particles that were ejected in the direction opposite the MWG center to be pulled back into the Solar System, after they had escaped. Figure 8 shows the distribution of the angular deviation from each ejected object's trajectory towards the MWG center, where an angle of 0 degrees means straight towards the MWG center, and 180 degrees means in the opposite direction of the MWG center. In both cases, no significant preference for ejection direction is visible.

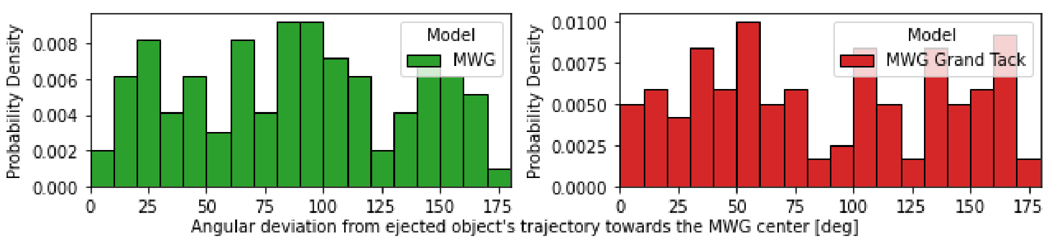

Figure 8: Angular deviation from the MWG center, for the trajectories of all ejected test particles in all runs. 0 degrees indicates a trajectory straight towards the MWG center, 180 degrees exactly opposite.

5 Discussion

This Section will interpret the figures from Section 4, and compare these to our expectations and results of similar simulations done by others. The reader should keep in mind that only a limited amount of simulations was done, causing statistics to be sparser than wished for. Furthermore, the simulations involving a MWG potential were performed using different gravity solvers, which caused larger errors in the energy conservation. Lastly, whenever the energy errors in the code caused the giant planets to change their orbits too much undesirably (i.e. when the semi-major axis changed by more than 5% in 10^3 years), the orbits were redefined to their original state (either post tack or pre tack, depending on the moment in the simulation) to regain the desired orbital elements. This causes some irregular movement, so could have influenced the results as well.

Figure 3 tells us that roughly 40% of the particles get their orbits perturbed in all cases. Judging from Figure 7, it is visible that this 40% consists of nearly all particles with semi-major axis $4 < a \lesssim 22$ au. Almost all particles near the planets being perturbed might sound like a lot, especially in the non-GT cases, since from our current Solar System we know that there are asteroids orbiting in between the giant planets. The Solar System initially had a very dense planetary disk. Only a few of the asteroids were required to remain in place in order to form the asteroid belts the Solar System has today. Furthermore, considering the lengthy timescale at which the simulation took place, it is natural that almost all test particles near the planets get perturbed.

Figure 4 shows the development of the ejection rate throughout time. The graph shows that during the outward migration of the giant planets during the Grand Tack, most particles are ejected per unit time. This makes sense, since the giant planets basically sweep the whole covered area of the Solar System they move through. One odd thing is that the inward migration of the MWG model does not seem to have any scattering, whereas the vanilla model does. We cannot think of a reason for this result. As a final remark, it makes sense that the ejection rate goes down to zero %/Myr, as the total number of particles available around the planets reduces to zero as well. This means a comparison between the non-GT and GT models is difficult to make without using the same time steps for both models. However, this remark strengthens the argument that the outward migration is more effective in scattering particles.

As far as Oort cloud object creation goes, making the comparison between the non-GT and GT models is easier. Figure 5 shows our results regarding this quantity, and, in the MWG case, shows a significantly higher percentage of potential Oort cloud objects for the GT ($\sim 2.3\%$) than the non-GT model ($\sim 0.9\%$), consistently larger over all large time values. In the models without MWG potential, this is not the case. According to these results, the gravitational effect of the Milky Way has a significant impact on the formation of the Oort cloud, assuming the Grand Tack takes place. This is in line with several sources in the literature (e.g. Dones et al. (2004)), stating that the MW perturbs the Oort cloud even nowadays. Furthermore, our fractions of potential Oort cloud particles are relatively low compared to Shannon et al. (2019), who find that about 5% of all particles located near the giant planets become Oort cloud objects after 4.5Gyr. However, similarly to us they do not find particles far outside the giants' orbits to become Oort cloud objects. Levison et al. (2004) find that after 4 Gyr, 5-9% of the initial comet population around Uranus and Neptune ends up in the Oort cloud, while only 2% of the objects around the orbits of Jupiter and Saturn ends up there. Our finds from figure 6 do not clearly show a difference in efficiency, but we do find that particles are scattered even beyond Neptune's orbit at about 14 au, indicating Neptune still has the potency to affect these particles' orbits.

6 Conclusion

Summarising our project, our most important find is that the combination of both the Milky way potential and a Grand Tack taking place creates a larger amount of potential Oort cloud objects, namely 2.3% of the initial particles present in the protoplanetary disk. This value is slightly lower than, but not incompatible with the values found in literature ($\sim 5\%$). No preference was found in the initial locations of these potential Oort cloud objects. However, potential Oort cloud objects, as well as other scattered particles, were only perturbed if their orbits had semi-major axes similar to those of the planets. Lastly it was found that in the case of a Grand Tack, the scattering takes place almost exclusively during the outward migration.

7 Resources

For the simulations, we have decided to make use of the AMUSE-framework (Portegies Zwart et al. 2009). More specifically, we used the direct N-body code `Huayno`, which is an N-body solver that can run up to 10^5-body simulations and is based on the fourth-order Hermite prediction-corrector scheme. Unfortunately, despite the zero-mass test particles, the code seemed to be affected by N. For large N, `Huayno` gradually slowed down and became slower than for example the `Mercury` direct N-body solver.

Due to the expensive code, the simulations were run on the ALICE High Performance Computing Facility at the University of Leiden[1].

All code is publicly available on our GitHub page.

References

Allen, C. & Santillan, A. (1991), 'An improved model of the galactic mass distribution for orbit computations.', **22**, 255.

Dones, L., Weissman, P. R., Levison, H. F. & Duncan, M. J. (2004), Oort Cloud Formation and Dynamics, *in* D. Johnstone, F. C. Adams, D. N. C. Lin, D. A. Neufeeld & E. C. Ostriker, eds, 'Star Formation in the Interstellar Medium: In Honor of David Hollenbach', Vol. 323 of *Astronomical Society of the Pacific Conference Series*, p. 371.

Hills, J. G. (1981), 'Comet showers and the steady-state infall of comets from the Oort cloud.', *The Astronomical Journal* **86**, 1730–1740.

Levison, H. F., Duncan, M., Brasser, R. & Kaufmann, D. (2010), Capture of the Sun's Oort Cloud from Stars in its Birth Cluster, *in* 'AAS/Division for Planetary Sciences Meeting Abstracts #42', AAS/Division for Planetary Sciences Meeting Abstracts, p. 5.06.

Levison, H. F., Morbidelli, A. & Dones, L. (2004), 'Sculpting the Kuiper Belt by a Stellar Encounter: Constraints from the Oort Cloud and Scattered Disk', **128**(5), 2553–2563.

Miyamoto, M. & Nagai, R. (1975), 'Three-dimensional models for the distribution of mass in galaxies.', **27**, 533–543.

Morbidelli, A. (2005), 'Origin and dynamical evolution of comets and their reservoirs of water ammonia and methane', *arXiv e-prints* pp. astro–ph/0512256.

Oort, J. H. (1950), 'The structure of the cloud of comets surrounding the Solar System and a hypothesis concerning its origin', *Bulletin of the Astronomical Institutes of the Netherlands* **11**, 91–110.

Portegies Zwart, S., McMillan, S., Harfst, S., Groen, D., Fujii, M., Nualláin, B. Ó., Glebbeek, E., Heggie, D., Lombardi, J., Hut, P., Angelou, V., Banerjee, S., Belkus, H., Fragos, T., Fregeau, J., Gaburov, E., Izzard, R., Jurić, M., Justham, S., Sottoriva, A., Teuben, P.,

[1]For more information, refer to https://www.universiteitleiden.nl/en/research/research-facilities/alice-leiden-computer-cluster

van Bever, J., Yaron, O. & Zemp, M. (2009), 'A multiphysics and multiscale software environment for modeling astrophysical systems', **14**(4), 369–378.

Shannon, A., Jackson, A. P. & Wyatt, M. C. (2019), 'Oort cloud asteroids: collisional evolution, the nice model, and the grand tack', *Monthly Notices of the Royal Astronomical Society* **485**(4), 5511–5518.
URL: *http://dx.doi.org/10.1093/mnras/stz776*

Walsh, K. J., Morbidelli, A., Raymond, S. N., O'Brien, D. P. & Mandell, A. M. (2011), 'A low mass for Mars from Jupiter's early gas-driven migration', **475**(7355), 206–209.

Weissman, P. R. (1996), The Oort Cloud, *in* T. Rettig & J. M. Hahn, eds, 'Completing the Inventory of the Solar System', Vol. 107 of *Astronomical Society of the Pacific Conference Series*, pp. 265–288.