Simulating the Formation of the Oort Cloud by the Grand Tack

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Abstract

The formation of the Oort Cloud is one of the remaining open questions concerning the solar system. In this work, the effect of a grand tack scenario on this formation was studied. For an initial planetesimal disk with 100 test particles, four different cases of the early solar system were evolved up to 100 Myr. At the moment of writing, results are still being collected. However, initial results show that the probability of becoming an Oort cloud object is 2% after 100 Myr for a model that includes the Solar System only, and 5% for a model that includes the Solar System and a Milky Way potential. This suggests that the Milky Way has a significant effect on the capture of comets in the Oort cloud.

1 Introduction

As of today, multiple theories concerning the formation of the Oort and Hills clouds exist. Examples are Hills (1981), where nearby passing stars perturb the orbits of objects in the Hills cloud in such a way that they form the Oort cloud, and Levison et al. (2010), where over 90% of the Oort cloud objects were actually captured by the sun from around other stars in its birth cluster. However, none of these theories is considered confirmed, so that the formation of the clouds remains a mostly unanswered question. This report will therefore test another hypothesis concerning the formation of the Oort cloud: "The Oort cloud was formed due to the migration of Jupiter and Saturn in the first 5 Myr after formation of the Solar System". To test this hypothesis, we will simulate several simplified Solar Systems with many test particles and count the percentage of test particles that have promising paths towards the current Oort cloud distance from the center of the Solar System. The simulated Solar Systems will have different evolution paths and take different gravitational effects into account, allowing for a comparison.

The formation of the Oort Cloud is one of the remaining mysteries in our Solar System. Solving this mystery would directly influence and improve our understanding of the early, as well as the present Solar System. On larger scales, this understanding can teach us about the probability and possibility of similar structures in other planetary systems and the potential influence that they may have. As the Oort cloud has still not been directly observed, this project might in turn also give us stronger constraints on its structure. This would allow for better preparation of interstellar missions, as these will eventually all have to cross through the cloud.

2 Background

2.1 The Oort Cloud

The Oort cloud is proposed to be an isotropic sphere with predominantly icy planetesimals surrounding the Sun at distances ranging from 2.000 to 200.000 au (Morbidelli 2005). As of today, this is still a very mysterious structure, since it has never been directly observed. Its existence was inferred in 1950 by Dutch Astronomer Jan Hendrik Oort, who used observations of 22 then newly detected long period comets (Oort 1950). Some 30 years later, Jack G. Hills proposed the existence of an inner and outer Oort cloud, the former now known as the Hills cloud (Hills 1981). While objects from the Oort cloud fall into the planetary orbits at a steady rate, objects from the Hills cloud only enter when a nearby star passes by the sun at a sufficiently close distance (approximately the semi major axis of the orbit of the comets), causing a short but intense infall of objects.

2.2 Grand Tack Model

Whereas initially it was thought that the orbits of the planets had never evolved much from their initial states, this belief has recently been challenged; the so-called 'Grand Tack Hypothesis' postulates that in about the first 5 My years of the Solar System, a fully formed Jupiter moved inwards from 3.5 au towards the sun, as close as some 1.5 au (Walsh et al. 2011). Saturn, a short while later, also moved inward but faster than Jupiter, until it was captured in a 2:3 resonance with Jupiter. When this happened, both planets started migrating outward to their present day orbits, pushing Uranus and Neptune outwards with them. During these movements, the giants crossed through the protoplanetary disk of planetesimals still present around the sun, destabilizing their orbits. Our hypothesis originates from the idea that these planetesimals were scattered outwards to form the Oort cloud. Simulations were already performed in 2004 by Dones et al. (2004), who showed that test-particle comets near Jupiter up until the Kuiper belt would be scattered outwards and reach a nearly isotropic inclination distribution with semi-major axes of 3000 - 200000 au after 4 Gyr, which is in line with current understandings of the Oort cloud.

3 Method

As mentioned in the Introduction, several simulations involving different mechanisms were performed. Explicitly, a comparison is made between the following four cases: One where the Solar System undergoes a Grand Tack and is affected by the gravitational potential of the Milky Way (MW), one where it is only subject to the gravitational potential of the MW but the Grand Tack does not take place, one where the Solar System is not subject to the gravitational potential of the MW, but does have a Grand Tack, and one where the MW does not exert force on the Solar System and no Grand Tack takes place.

In all cases, the Solar System is simulated as a 5-body problem. Only the gas giants and the Sun are considered, in order to speed up calculations by having fewer force-exerting bodies and by keeping the typical distance larger, which allows for the time step of the simulation to be larger. The planets start out with their present-day orbits in the so called 'Vanilla' and 'MW' runs, while starting with different orbits for the 'Grand Tack' and 'Grand Tack & MW' runs (for more details, see section 3.2).

To answer our main question, test particles with zero mass were used to represent asteroids in the Solar System. These particles were randomly distributed throughout the Solar System, by randomly drawing from a uniform distribution with different ranges. They were given a semi-major axis between 4 and 40 au, an eccentricity between 0 and 0.05, an inclination between -5 and 5 degrees, and a true anomaly, argument of periapsis and longitude of the ascending node between 0 and 360 degrees. The resulting orbits are in the same general rotational direction as the planets and the rotation of the Sun, but starting at different positions. The resulting initial setup is shown in Figure 1.

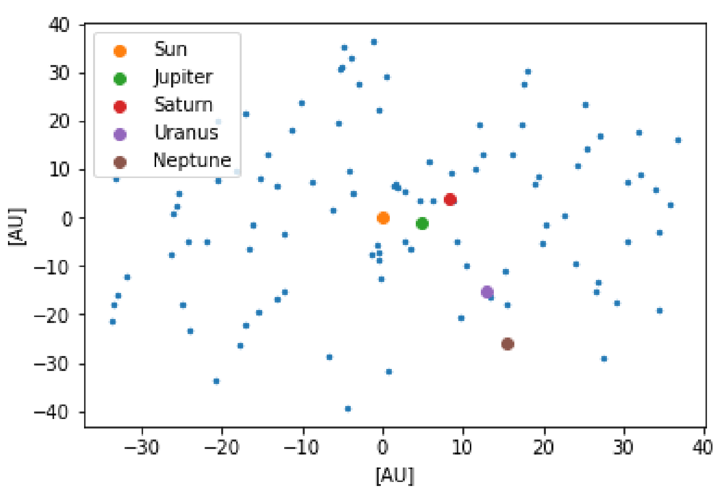

Figure 1: Example of the top view of an initial setup of the Solar System plus test particles. (Object sizes not to scale)

The choice for a 5- instead of N-body simulation is a necessary assumption; as N will be very large (At least on the order of 10^2), having to calculate N-1 forces for all N bodies will take an extensively large amount of time. Calculating 5 forces for N bodies decreases this time by a factor of at least $\frac{N-1}{5}$ (depending on the N-body solving algorithm). This does come at the cost of a loss in accuracy, but taking into account that the current Mass-estimate for the complete Oort Cloud is about 5 Earth masses (Morbidelli 2005) of isotropically distributed planetesimals, we expect this loss in accuracy to be negligible.

Furthermore, we will neglect passing stars, since it has been shown that these will only perturb the motion of comets almost directly on their path, a relatively small tunnel (with radius 450 au for a 1 $M_\odot$ star with velocity 20 km s^{-1}) compared to the whole cloud (Weissman 1996). In a future study, the effect of passing stars on the randomization of the Oort cloud could be investigated. Molecular cloud encounters are rare and therefore also neglected.

3.1 Gravitational Potential

To simulate the gravitational potential of the MW on an object at position (x,y,z), the following potential, consisting of three components is used:

$$\Phi = \Phi_{bulge} + \Phi_{disk} + \Phi_{halo}. \tag{1}$$

The bulge component corresponds to a Hernquist spheroid

$$\Phi_{bulge}(r) = -\frac{GM_{bulge}}{\sqrt{r^2 + b_1^2}}, \tag{2}$$

with the scalelength $b_1 = 0.3873$ kpc.

The disk component is a Miyamoto-Nagai disk (Miyamoto & Nagai 1975):

$$\Phi_{disk}(R, z) = -\frac{GM_{disk}}{\sqrt{R^2 + (a_2 + \sqrt{z^2 + b_2^2})^2}}, \tag{3}$$

with scalelengths $a_2 = 5.31$ kpc and $b_2 = 0.25$ kpc.

Lastly, the dark matter halo component is taken from (Allen & Santillan 1991) and given by

$$\Phi_{halo}(r) = -\frac{GM_{halo}}{a_3}\frac{d_1^{1.02}}{1 + d_1^{1.02}} - \frac{GM_{halo}}{1.02a_3}\left(\frac{-1.02}{c} + \ln(c) + \frac{1.02}{1 + d_1^{1.02}} - \ln(1 + d_1^{1.02})\right), \tag{4}$$

where $a_3 = 12.0$ kpc, $d_1 = r/a_3$ and $c = 1 + (\text{cutoff}/a_3)^{1.02}$, the cutoff is set at 100 kpc.

3.2 Setting up the Grand Tack

For the Grand Tack scenario, the planets start off differently from their current day orbits. For simplicity all orbits were assumed to be circular and in the ecliptic plane. The semi major axes were set at 3.5 au, 4.5 au, 6 au and 8 au for Jupiter, Saturn, Uranus and Neptune respectively, while the three angular orbital parameters were randomised. In order to allow the gas giants to move inward, and later outward through the Solar System, the planets were slowed down and sped up with fictional tidal friction exerting a force on them. This involves a tidal friction coefficient c_{tf}, which is relevant to AMUSE and controls the resulting speed of the inward/outward movement. An attempt was made to match c_{tf} to give an inward velocity following

$$v(t) = v_0 + \Delta v_{in}[t/t_{end}], \tag{5}$$

where v_0 is the original velocity and t_{end} is the time at which inward migration ends ($\sim 10^5$ yr), and an outward velocity following

$$v(t) = v_{in} + \Delta v_{out}[1 - exp(-t/\tau)], \tag{6}$$

where v_{in} is the velocity at the end of the inward migration and τ is the adopted migration timescale (also $\sim 10^5$ yr). Both of these formulae were adopted from Walsh et al. (2011).

3.3 Ejection

In order to evaluate the amount of objects leaving the Solar System, we first need to define when the orbit of objects is no longer similar to their original orbits. Objects start with an original semi-major axis of $a \leq 40$ au, so we define perturbed orbits as having $a > 100$ au. This value is chosen since it is significantly larger than the original value, and it is nice and round. Moreover, at present times, Oort cloud comets orbit with semi-major axes of at least approx. 2.000 au, so their orbits need to be significantly perturbed.

Besides the fact that orbits can be perturbed, particles can also be ejected. This also requires a definition. If we say an object is ejected, we mean it is no longer orbiting the Sun. To find out if this is the case for a particle, multiple methods can be used. First of all, one can look at the current velocity of the particle v_{rel}, which is the velocity with respect to the Sun, and see if this value exceeds the escape velocity $v_{esc} = \sqrt{\frac{2GM_{\odot}}{r_{obj}}}$ from the Sun. A second approach is to determine the distance from the Sun, and (a) determine if this is further away than the typical distance to another stellar system ($\sim 300.000au$, however, we use 250.000au to be sure), or (b) determine if the attraction of the MW has a bigger impact on the particle than the attraction of the Sun. This distance is defined by the Hill radius of the Sun, which lays at over 7 million au, so we can neglect this method. The third method to determine if a particle is still orbiting the Sun is by determining the eccentricity e of the particle's orbit and evaluating this value. If $e > 1$, the particle has a hyperbolic orbit and is therefore no longer orbiting the Sun. All of these methods will be explored in the results, since it is very important to know if the comets have the potential to become Oort cloud objects.

4 Results

In this section, the main results for the four different cases will be discussed on a case by case basis. Before we start with this, an interesting but troubling result should be mentioned; in all the Vanilla and MW runs performed so far, the 5-body system becomes unstable during the run. After only about 10^7 years, one of the planets is either kicked out of the solar system or obtains such a large eccentricity that it enters the orbit of one of the other planets, creating a chaotic system. As today we observe a stable solar system and have no reason to believe that this will change in the coming time, this is most likely an error in our algorithm. However, as comets on an escaping or large semi-major axis trajectory will not notice any of these instabilities (only noticing the gravitational attraction from the Solar System as a whole), we can indeed analyse all objects that were on such trajectories before the instability occurs. Hopefully, in the coming month, we will have fixed this problem.

4.1 Vanilla and MW-only runs

Starting with the two models where **no Grand Tack** takes place, for the 100 cometary objects in three similar runs (with different random initial values) for each model, Figure 2 shows the perturbation percentage (defined in Section 3.3) at each timestep. This includes particles that were deleted from the particle set due to having $a > 250.000$au ánd particles that were deleted due to collisions. The amount of particles deleted due to collisions is very small, but the number was added, since their orbit is in another way perturbed.

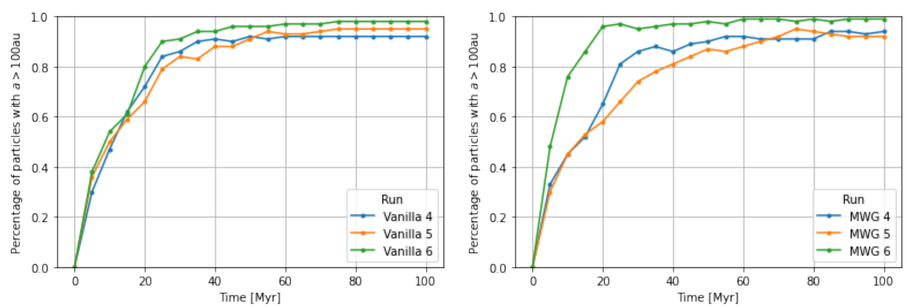

Figure 2: Perturbation percentage (percentage of particles with $a > 100$au) at each timestep, for three runs of the [Left] Vanilla model, [Right] MW-only model. This includes particles that were deleted due to having $a > 250.000$au ánd particles that were deleted due to collisions. Note that the number of the run has no special meaning; each run was performed with the exact same algorithm, only with different randomised initial conditions as mentioned in section 3

Furthermore, the amount of ejected particles was evaluated, using each of the methods described in Section 3.3. First of all, the percentage of particles with $v_{rel} > v_{esc}$ at each timestep is shown in Figure 3.

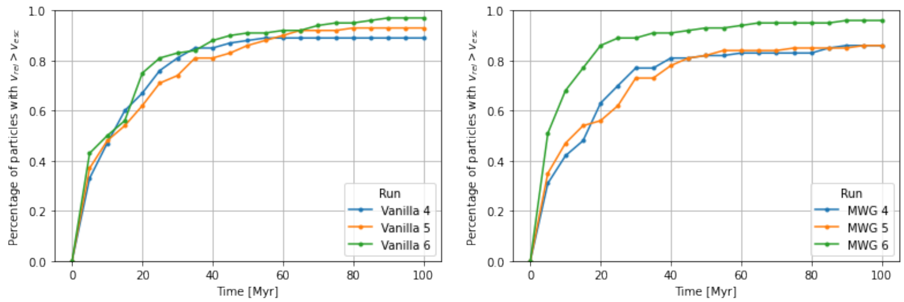

Figure 3: Percentage of particles having a relative velocity to the Sun v_{rel} that is higher than the escape velocity v_{esc} from the Sun at their current distance from it, for three runs of the [Left] Vanilla model, [Right] MW-only model. Particles that were later deleted are also added here, as they were on escaping trajectories.

Second, the percentage of particles that are considered ejected due to being at a position $r_{obj} \gtrsim 250.000au$ was explored. Since we are filtering out particles that cannot possibly become Oort cloud objects, we can simply consider particles that were deleted from the particle set due to having this value $r_{obj} \gtrsim 250.000au$ ór due to colliding with a gas giant. Besides, the amount of particles deleted from the particle set due to a collision is insignificant. The resulting graph therefore shows a subset of the particles with a perturbation, and is shown in Figure 4.

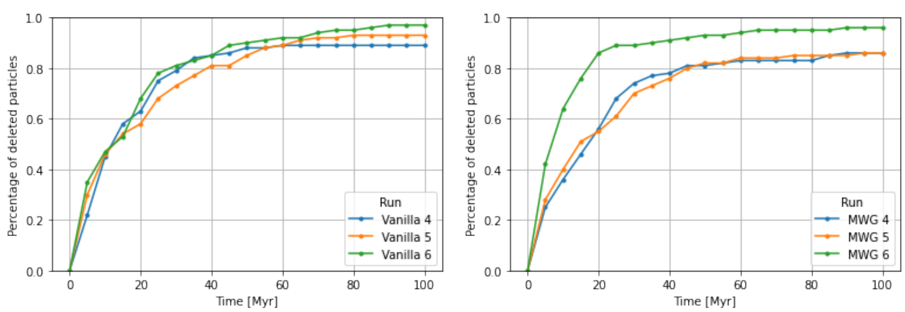

Figure 4: Percentage of particles deleted from the particle set due to having a position $r_{obj} > 250.000au$ or due to colliding with a gas giant at each timestep, for three runs of the [Left] Vanilla model, [Right] MW-only model.

Lastly, the percentage of particles with a hyperbolic orbit ($e > 1$) was calculated. We noticed that these percentages are **exactly** the same as the ones calculated from the condition $v_{rel} > v_{esc}$, so in the final report we will have a look at the theory to confirm if this is indeed the same condition.

Finally, to find a result for the percentage of particles that have potential to become Oort-cloud comets, we need to combine above results. As a reminder, potential Oort-cloud objects have the criterion that their orbit is perturbed ($a > 100$au), but not “destroyed”, meaning $v_{rel} < v_{esc}$, $r < 250.000$au and $e < 1$. Figure 5 shows the resulting percentage of potential Oort-cloud objects that meet above criteria. From further analysis of the data, it is clear that some particles meet the criteria at some timesteps, but not at others, indicating that the particle has some strange movement or orbital parameters. An example of two particles’ semi-major axes were plotted against their eccentricities at different times, shown in Figure 6. The particles were chosen for having an eccentricity below 1 at some point after having an eccentricity above 1, and having a large semi-major axis. From this plot it is visible that the particles get ‘stuck’ at a certain value of $40.000au < a < 60.000$au and slightly below $e = 1$.

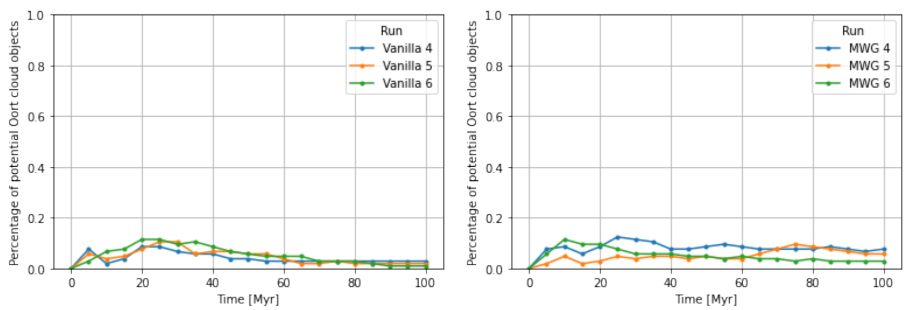

Figure 5: Percentage of particles that were classified as having potential to become Oort-cloud comets, at each timestep, for three runs of the [Left] Vanilla model, [Right] MW-only model.

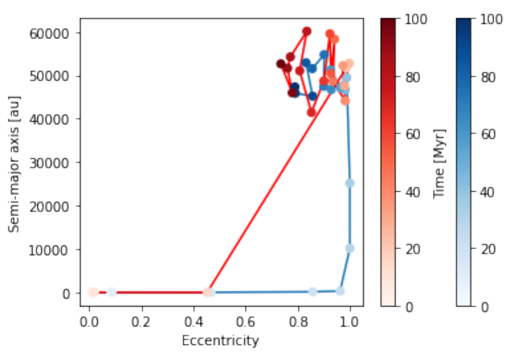

Figure 6: Example trajectory of a vs e in time, for two particles having $a >$ 20.000au at t=100 Myr and a value for e that varies around 1.

4.1.1 Initial conditions of potential Oort-cloud objects

Now that there is an understanding of the amount of particles that have the potential to become Oort-cloud objects, it is interesting to know what the initial conditions of these particles were. Therefore, the distribution of certain initial conditions for potential Oort-cloud objects is shown in the following figure. Figure 7 shows the initial semi-major axis, eccentricity and (positive) inclination distributions.

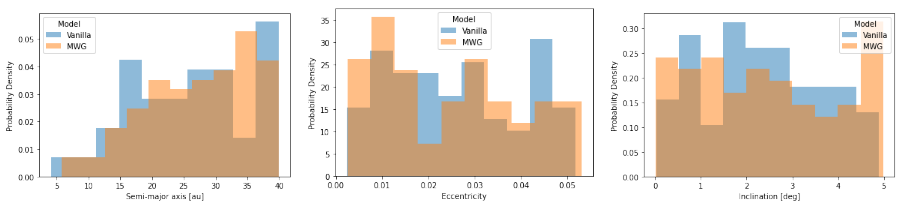

Figure 7: Distribution of initial (Left) semi-major axis, (Middle) eccentricity, and (Right) inclination; for particles that were classified as potential Oort-cloud objects, for various models. Note that the inclination value is always positive when retrieved from the binary.

From above figure, it is fairly clear that there is no significant difference in distributions for all of the orbital elements. However, an interesting observation is that objects with larger initial semi-major axis seem to have a higher chance of being perturbed while not ejected. We confirm this by showing the distribution of semi-major axes for all particles in Figure 8 below.

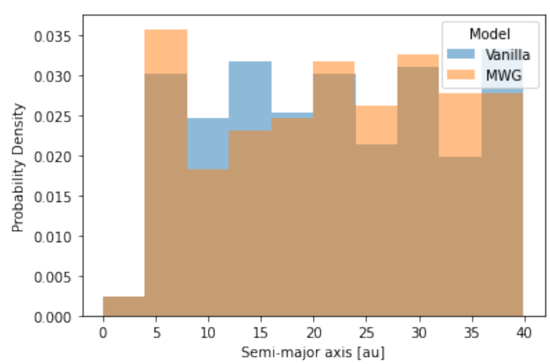

Figure 8: Distribution of initial semi-major axes of all particles, for various models.

4.2 Grand Tack only

At the moment of writing, we have not yet been able to consistently simulate a plausible Grand Tack scenario without the solar system residing to a chaotic state before Jupiter reaching it's most inward orbit at 1.5 au. Currently, the main problem is that about 10^4 years into Jupiter's migration, Saturn's eccentricity increases by such an amount that the orbit of Uranus eventually becomes unstable, causing it to move inside the orbit of Saturn, further disrupting the orbits of the three outer gas giants. Interestingly enough, Jupiter seems to ignore these chaotic movements and is able to continue on it's inward path. We hope to be able to simulate the tack more realistically by the end of this month.

4.3 MW & Grand Tack

Naturally, the inability to simulate a realistic Grand Tack also causes this scenario to not have been properly tested yet. Again, we hope to have fixed this by the end of this month.

5 Discussion

Due to the lack of results for two of the four cases, the discussion will not yet be complete. However, a few things can already be concluded about the preliminary results. It should be noted that our model is not yet working properly, as mentioned in the introduction of Section 4. Because of this, instead of discussing our results in detail, we will mostly discuss what we expected in the case of a properly working model.

Since at this point we only want to compare the Vanilla and MW-only runs, it is only interesting to look at points where we expect a difference. For starters, we expect particles in the run that includes a MW potential to experience a more significant force from the MW when they get further from the Solar System. Therefore, when particles move in the direction away from the MW center, they will be pulled back towards the MW center and therefore back to the Solar System. This gives the possibility for these particles to stay bound to the Sun, whereas in the Vanilla run they will be unbound forever. This should result in a small increase in potential Oort-cloud particles for the MW-only run, compared to the Vanilla run. For times above $t \approx 40$Myr this is true, according to Figure 5. In the coming month, we will investigate upon this matter by tracking the direction of the test particles wrt. the MW center.

Secondly, we can compare the Oort cloud efficiencies with the literature. In Levison et al. (2004) for example, it is found that about 5-9 % of the initial comet population around Uranus and Neptune ends up in the Oort cloud, while only 2% of the objects around the orbits of Jupiter and Saturn ends up there. Our finds from Section 4.1.1 confirm that Jupiter and Saturn are indeed worse Oort object creators, since larger initial semi-major axes correspond with a larger probability to become an Oort-cloud object (see Figure 7 [Left]), which can be explained due to their larger masses causing a too large initial escape velocity to end up in a bound orbit around the solar system. Comparing our absolute Oort object percentages to the results of Levison et al. (2004) is not entirely realistic, as their simulation was done over a time span of 4 Gyr versus our 100 Myr and stellar encounters were indeed taken into account. However, our percentage of potential Oort cloud objects at the end of our simulation ($t = 100$Myr) is about 2% for the Vanilla runs and about 5% for the MW galaxy runs. This value is lower than the theoretical value from above named paper, and could only get lower in another 3900 Myr. If our model that includes the Grand Tack gets higher probabilities, this is an indication for the Grand Tack to be true. From Figure 21 in Morbidelli (2005) it is to be expected that there will also be comets moving to Oort cloud orbits after these first 100 Myr. The absolute value of Oort cloud efficiency was therefore also never our main goal; this was only to check the relative efficiencies for different scenarios such as the grand tack.

6 Conclusion

Since it was found that particles have a higher probability of becoming an Oort cloud object when the Milky Way potential is present (5% after 100 Myr) than when it is not present (2% after 100 Myr), it is suggested that the MW has a significant effect on the potential capture of comets into the Oort cloud.

Once the Grand Tack model is working, we can say more about its effect on the capture probability of comets into the Oort cloud when it is present.

7 Resources

For the simulations, we have decided to make use of the AMUSE-framework (Portegies Zwart et al. 2009). More specifically, we used the direct N-body code `ph4`, which is an N-body solver that can run up to 10^5-body simulations and is based on the fourth-order Hermite prediction-corrector scheme. Unfortunately, despite the zero-mass test particles, the code seemed to be affected by N. For large N, `ph4` gradually slowed down and became slower than for example the `Mercury` direct N-body solver. Therefore, when $N > 2000$, this solver was used instead.

Due to the expensive code, the simulations were run on the ALICE High Performance Computing Facility at the University of Leiden[1].

All code is publicly available on our GitHub page.

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[1]For more information, refer to https://www.universiteitleiden.nl/en/research/research-facilities/alice-leiden-computer-cluster

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