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Stabilization fuzzy control of inverted pendulum systems

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Abstract

A new fuzzy controller for stabilization control of inverted pendulum systems is presented based on the Single Input Rule Modules (SIRMs) dynamically connected fuzzy inference model. The fuzzy controller has four input items, each with a SIRM and a dynamic importance degree. The SIRMs and the dynamic importance degrees are designed such that pendulum angular control has priority over cart position control. It is made clear that the fuzzy controller performs the pendulum angular control and the cart position control in parallel, and switching between the two controls is realized by automatically tuning the dynamic importance degrees according to control situations. The simulation results show that the proposed fuzzy controller has a high generalization ability to stabilize completely a wide range of the inverted pendulum systems within 9.0 s for an initial angle up to 30.0°. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Adaptive control; Fuzzy control; Intelligent system; Pendulum; Single Input Rule Module; Stabilization

1. Introduction

As a typical unstable nonlinear system, inverted pendulum system is often used as a benchmark for verifying the performance and effectiveness of a new control method because of the simplicities of the structure. Since the system has strong nonlinearity and inherent instability, the variable structure control system [14] had to linearize the mathematical model of the object near upright position of the pendulum. Recently, a lot of researches on stabilization control of inverted pendulum systems by using fuzzy inference have been done.

Berenji [1] built the action evaluation neural network and the action selection fuzzy network which were both tuned by reinforcement learning, to control an inverted pendulum to a neighborhood of the upright position. Lin [5] proposed a method for adjusting the membership functions of a fuzzy rule base by adaptive sliding mode and applied it to the angular control of an inverted pendulum. Lu [7] used the genetic algorithm to automatically generate fuzzy rules and scaling factors for inverted pendulum control. Margaliot [8] showed a new approach to determining the structure of fuzzy controller for inverted pendulum by fuzzy Lyapunov synthesis. Mikukcic [10] extracted fuzzy rules for inverted pendulum control by fuzzy clustering method. Saez [15] utilized the generalized predictive controller to determine the parameters of the Takagi–Sugeno fuzzy model for controlling an inverted pendulum. Wong [17] adopted the genetic algorithm to tune all the membership functions of a fuzzy system in order to keep an inverted pendulum upright. Yamakawa [18] designed a high-speed fuzzy controller hardware system and used only seven fuzzy rules to control the angle of an inverted pendulum. Although stabilization control of an inverted pendulum system should also include the position control of the cart besides the angular control of the pendulum because of limit length of the rail, the above stated approaches only took into consideration the angular control of the pendulum.

To control both the angle of the pendulum and the position of the cart, Kandadai [2] modified the structure of Berenji [1] to a hierarchical controller and enabled it to generate fuzzy knowledge base automatically. It took more than 12.0 s, however, to asymptotically stabilize an inverted pendulum system with some offset besides its structure complexity. Based on the variable structure systems theory and the trajectory of linearized dynamic equation of an inverted pendulum on phase plane, Kawaji [3] constructed a fuzzy controller consisting of two simple rule modules. One module was for the magnitude of the manipulated variable, and the other for the sign of the manipulated variable. Since the control of the cart position can be regarded as a disturbance to the pendulum angle, the information of the cart position was first changed into a virtual target angle, and the virtual target angle was then

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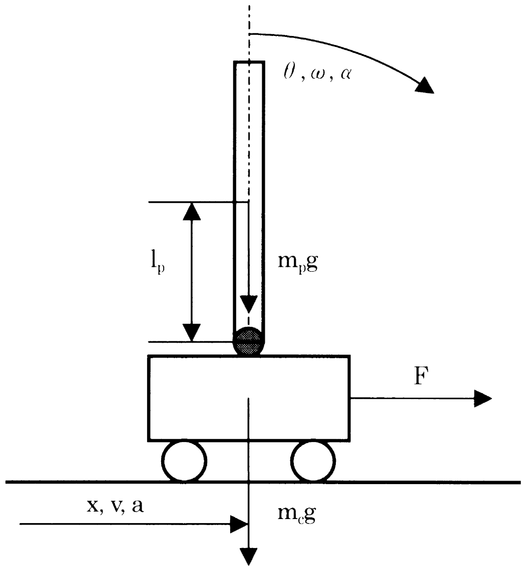

Fig. 1. Configuration of the inverted pendulum system.

imbedded into the control of the pendulum angle. Although this method was rather simple, it was difficult to completely stabilize an inverted pendulum system within a short time interval. Kyung [4] presented a fuzzy controller, whose rule base was derived from three neural networks. Although the fuzzy controller can stabilize an inverted pendulum system in about 8.0 s, it needed 396 rules even after a smoothing procedure and a logical reduction procedure. Matsuura [9] and Yasunobu [19] both used the information of the cart to build a set of 49 fuzzy rules for conducting the virtual target angle, and then used the virtual target angle and the information of the pendulum to construct another set of 49 fuzzy rules for total stabilization. Sakai [16] applied a nonlinear optimization method to train a fuzzy controller for stabilization, however, the controller spent more than 200.0 s on stabilizing an inverted pendulum system.

In the stabilization control of an inverted pendulum system, four input items are necessary in order to cover the angular controls of the pendulum and the position control of the cart. Since the angular control of the pendulum should be done first from intuition, the priority orders of the two controls have to be discriminated clearly. The conventional fuzzy inference model which puts all of the input items into the antecedent part of each fuzzy rule, however, needs many fuzzy rules and has poor ability to express the control priority orders. On the other hand, in the Single Input Rule Modules (SIRMs) dynamically connected fuzzy inference model [20], a SIRM and a dynamic importance degree are defined for each input item. Each SIRM takes the corresponding input item as the only antecedent variable. The dynamic importance degree indicates explicitly the significance of the corresponding input item and changes automatically with control situations. The output of the model is obtained by summarizing the products of the fuzzy inference result of each SIRM and its corresponding dynamic importance degree. Because the input items can be processed dispersedly by the SIRMs and the control priority orders can be represented definitely by the dynamic importance degrees, the model has been successfully applied to trajectory tracking control [21].

In this paper, a new fuzzy controller for the stabilization control of inverted pendulum systems is presented based on the SIRMs dynamically connected fuzzy inference model. The fuzzy controller takes the angle and angular velocity of the pendulum and the position and velocity of the cart as its input items, and the driving force as its output item. Each input item is given with a SIRM and a dynamic importance degree. The SIRMs of the four input items have the same rule setting. The four dynamic importance degrees all select the absolute value of the pendulum angle as the antecedent variable. The dynamic importance degrees are set up such that the angular control of the pendulum takes priority over the position control of the cart when the pendulum is still not balanced upright. By using the SIRMs and the dynamic importance degrees, the fuzzy controller realizes smoothly the angular control of the pendulum and the position control of the cart in parallel with totally only 24 fuzzy rules. Switching between the angular control of the pendulum and the position control of the cart is smoothly realized by automatically adjusting the dynamic importance degrees according to control situations. The simulation results show that the proposed fuzzy controller has a high generalization ability to stabilize completely a wide range of the inverted pendulum systems within 9.0 s for an initial angle up to 30.0°.

2. Inverted pendulum system

The inverted pendulum system considered here is shown in Fig. 1, which consists of a straight-line rail, a cart, a pendulum, and a driving unit. The cart can move left or right on the rail freely. The pendulum is hinged on the center of the top surface of the cart and can rotate around the pivot in the same vertical plane with the rail. Given that no friction exists in the system, the dynamic equation of the inverted pendulum system can be expressed as

$$\alpha = \frac{(m_c + m_p)g \sin\theta - \{F + m_p l_p \omega^2 \sin\theta\}\cos\theta}{\{4/3(m_c + m_p) - m_p(\cos\theta)^2\}l_p}, \tag{1}$$

$$a = \frac{4/3\{F + m_p l_p \omega^2 \sin\theta\} - m_p g \sin\theta\cos\theta}{\{4/3(m_c + m_p) - m_p(\cos\theta)^2\}}. \tag{2}$$

Here, the parameters m_c and m_p are, respectively, the mass of the cart and the mass of the pendulum in the unit [kg], and $g = 9.8\ \mathrm{m/s^2}$ is the gravity acceleration. The parameter l_p is the length from the center of the pendulum to the pivot in the unit [m] and equals to the half length of the pendulum. The variable F means the driving force in the unit [N] applied horizontally to the cart. The variables θ, ω, α represent, respectively, the angle of the pendulum from upright position, its angular velocity, its angular acceleration, and the clockwise direction is positive. The variables x, v, a denote the position of the cart from the rail origin, its velocity, its acceleration, and right direction is positive.

The variables θ, ω, x, v are the four state variables to describe the dynamic system. In the following simulations, the Euler approximation method is adopted in order to obtain the values of the four state variables.

3. SIRMs dynamically connected fuzzy inference model

Before presenting the stabilization fuzzy controller, let us describe the SIRMs dynamically connected fuzzy inference model [20] briefly for systems of n input items and 1 output item. The model can be easily extended for multi-output systems.

As is well known, the conventional fuzzy inference model, which puts all the input items into the antecedent part of each fuzzy rule, causes the total number of possible fuzzy rules to increase exponentially with the number of the input items and has difficulty in setting up each rule. To solve the problems, the SIRMs dynamically connected fuzzy inference model first sets up a SIRM separately for each input item as

$$\text{SIRM-i}: \ \{R_i^j: \text{ if } x_i = A_i^j \text{ then } f_i = C_i^j\}_{j=1}^{m_i}. \tag{3}$$

Here, SIRM-i denotes the SIRM of the ith input item, and R_i^j is the jth rule in the SIRM-i. The ith input item x_i is the only variable in the antecedent part, and the consequent variable f_i is an intermediate variable corresponding to the output item f. A_i^j and C_i^j are the membership functions of the x_i and f_i in the jth rule of the SIRM-i. Further, $i = 1, 2, \ldots, n$ is the index number of the SIRMs, and $j = 1, 2, \ldots, m_i$ is the index number of the rules in the SIRM-i.

The inference result f_i^0 of the consequent variable f_i can be calculated by using the min–max-gravity method or the product–sum-gravity method or the simplified inference method [11]. Since all the consequent variables of the SIRMs correspond to the output item, the simplest way to obtain the value of the output item is just summing up the inference results of all the SIRMs. But this does not work well because each input item usually plays an unequal role in system performance. Among the input items, some may contribute significantly while the contribution of the others may be relatively small. Some input items may improve system performance more if their roles are strengthened, while others may not have a positive influence on system performance if emphasized.

To express clearly the different role of each input item on system performance, the SIRMs dynamically connected fuzzy inference model defines a dynamic importance degree w_i^{D} independently for each input item x_i ($i = 1, 2, \ldots, n$) as

$$w_i^{\mathrm{D}} = w_i + B_i \cdot \Delta w_i^0. \tag{4}$$

On the right side of Eq. (4), the first term and the second term separate the base value and the dynamic value. The base value w_i guarantees the necessary function of the corresponding input item through a control process. The dynamic value, defined as the product of the breadth B_i and the inference result Δw_i^0 of the dynamic variable Δw_i, plays a role in tuning the degree of the influence of the input item on system performance according to control situation changes. The base value and the breadth are control parameters, and the dynamic variable can be described by fuzzy rules. Because the inference result of the dynamic variable is limited in [0.0, 1.0], the dynamic importance degree will vary between $[w_i, w_i + B_i]$.

Suppose that each dynamic importance degree w_i^{D} and the fuzzy inference result f_i^0 of each SIRM are already calculated. Then, the SIRMs dynamically connected fuzzy inference model obtains the value of the output item f by

$$f = \sum_{i=1}^{n} w_i^{\mathrm{D}} \cdot f_i^0, \tag{5}$$

as the summation of the products of the fuzzy inference result of each SIRM and its dynamic importance degree for all the input items.

As shown in Eq. (5), the model output is linear to the inference result of each SIRM. If the inference result of each SIRM is identical, then the contribution of one input item to the model output is controlled by its dynamic importance degree. Therefore, the input items with larger importance degrees will contribute more to the model output, while the input items with smaller importance degrees contribute less to the model output.

Since the dynamic importance degrees change with control situations to adjust the model output, Eq. (4) is in effect an adaptive control law. Moreover, each SIRM has only one antecedent variable, and each dynamic importance degree usually handles only one or two antecedent variable(s). Therefore, the SIRMs dynamically connected fuzzy inference model has a simple structure and is suitable for real applications. Although the adaptive membership function scheme of Lotfi [6] tuned the parameters of the membership functions based on a generalized neural network, the obtained parameters were fixed through a control process and training patterns were also necessary. For a real control system, collecting training patterns usually is difficult, and the training patterns directly affect the performance of the controller based on such a scheme. On the other hand, the fuzzy model reference learning controller of Moudgal [12] had an ability to modify on-line the rule base of a fuzzy controller, making the fuzzy controller more flexible. It needed, however, to design a reference model and a fuzzy inverse model for each plant.

Table 1
SIRM for each input item

Antecedent variable x_i $(i = 1, 2, 3, 4)$	Consequent variable f_i $(i = 1, 2, 3, 4)$
NB	−1.0
ZO	0.0
PB	1.0

Considering the complexity of its structure, the control performance indicated little improvement.

4. Stabilization fuzzy controller

In this paper, the stabilization control of the inverted pendulum system means to balance upright the pendulum and put the cart back to the origin of the rail in a short time. The desired position of the cart can be set to any reasonable point on the rail, however, the rail origin is selected here as the desired position of the cart without losing generality. Since the desired values of the state variables are all zeros, the control problem can be regarded as a regulator design problem. Here, the fuzzy controller for the stabilization of the inverted pendulum system is constructed based on the SIRMs dynamically connected fuzzy inference model. The four state variables (pendulum angle, pendulum angular velocity, cart position, and cart velocity) after normalization by their scaling factors are chosen in this order as the input items x_i $(i = 1, 2, 3, 4)$, and the driving force after normalization by its scaling factor is selected as the output item f.

4.1. Setting the SIRMs

As stated in Section 3, each input item is given with a SIRM and a dynamic importance degree in the SIRMs dynamically connected fuzzy inference model. The SIRMs of the four input items in the stabilization control of the inverted pendulum system are considered here first.

In setting up the SIRMs for the angle and the angular velocity of the pendulum, it is enough to utilize the relation of the input items with the whole control performance from experience and intuition. When the angle and the angular velocity of the pendulum are positive, the driving force should be positive from the sign definition in Fig. 1 so that the cart moves toward the right direction. As a result, the pendulum will rotate counterclockwise toward the upright position, and the angle and the angular velocity will tend to decrease toward zero. In the same way, when the angle and the angular velocity are negative, the driving force should become negative to make the cart move toward the left direction. Consequently, the pendulum will turn clockwise toward the upright position, and the angle and the angular velocity will come to increase toward zero.

As is well known in the stabilization control, the cart position can be considered as a disturbance to the pendulum angle. In setting up the SIRMs for the position and the velocity of the cart, the position control of the cart can then be indirectly realized by intentionally putting the pendulum a little down to the direction opposite to the cart position. If the position and the velocity of the cart are positive, then the driving force of positive value should be executed on the cart so that the cart moves further toward the right direction, causing the pendulum down counterclockwise to the negative direction deliberately. If the position and the velocity are negative, then the driving force of negative value should be given to drive the cart further toward the left direction, causing the pendulum down clockwise to the positive direction deliberately. Since the pendulum falls down toward the rail origin, the driving force will then change to move the cart toward the rail origin if the angular control of the pendulum takes priority over the position control of the cart. As a result, the pendulum is balanced upright and the cart is returned to the rail origin.

From the above analysis, the fuzzy rules for each SIRM can all be set up to Table 1. Here, membership functions NB, ZO, PB of the antecedent variable of each SIRM are defined in Fig. 2 as triangles or trapezoids. The consequent variable f_i is an intermediate variable corresponding to the output item f of the fuzzy controller. Because the simplified reasoning method is adopted here, real numbers are assigned as singleton membership functions to the consequent variable of each SIRM.

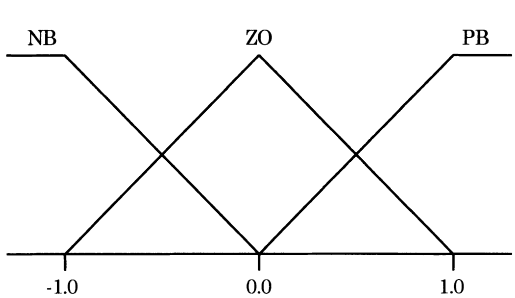

Fig. 2. Membership functions for each SIRM.

4.2. Setting the dynamic variables of the dynamic importance degrees

From Table 1, each SIRM directly corresponds to the output item. To obtain the value of the output item by Eq. (5), each input item is given with a dynamic importance degree defined by Eq. (4). As is well known in the stabilization control of the inverted pendulum system, if the position control of the cart takes priority over the angular control of the pendulum, then the angular control of the pendulum will fail. Therefore, the angular control of the pendulum should have priority over the position control of the cart when the pendulum does not stand up yet, and the position control of the cart should be done after the pendulum is almost balanced upright. The dynamic importance degree shows the influence strength on system performance and hence

Table 2
Fuzzy rules for the two dynamic variables of the pendulum

Antecedent variable $\lvert x_1 \rvert$	Consequent variable Δw_i $(i = 1, 2)$
DS	0.0
DM	0.5
DB	1.0

can signify the priority orders. The bigger the value of a dynamic importance degree is, the higher the priority order of the corresponding input item becomes.

The priority order of the pendulum depends on the two dynamic importance degrees of the angle and angular velocity of the pendulum, and the priority order of the cart depends on the two dynamic importance degrees of the position and velocity of the cart. Because each dynamic importance degree consists of two control parameters and one dynamic variable, the fuzzy rules of the dynamic variables are established here.

First consider the dynamic variables of the angle and the angular velocity of the pendulum. In the case where the absolute value of the pendulum angle is big, the pendulum will fall down if it is not controlled to rotate toward the upright position at once. Therefore, in order to balance the pendulum upright in this case, the dynamic importance degrees of the angle and the angular velocity of the pendulum should be set up to rather large values to strengthen their influences on system performance. In the case where the absolute value of the pendulum angle is near zero, meaning that the pendulum already stands almost upright, the almost balanced state will be destroyed if the angular control of the pendulum is emphasized too much. In order to achieve and keep the balanced state in this situation, it is necessary to reduce the importance degrees of the angle and the angular velocity to weaken their influences. Since the dynamic variables bring about changes in the dynamic importance degrees, the fuzzy rules for the dynamic variables of the dynamic importance degrees of the angle and the angular velocity are given in Table 2. Here, the absolute value of the pendulum angle after normalization is selected as the only antecedent variable. Membership functions DS, DM, DB are defined in Fig. 3 as trapezoids or triangles.

As stated above, the angular control of the pendulum must be done with priority over the position control of the cart in order to achieve complete stabilization of the whole inverted pendulum system. When the pendulum is not located in the upright position yet, the angular control of the pendulum is first executed so that the pendulum is balanced almost upright. While the balanced state of the pendulum is kept, starting the position control of the cart is allowable. This fact means that in the stabilization control of the inverted pendulum system, the angular control of the pendulum is more important than the position control of the cart. Therefore, the angle and the angular velocity of the pendulum should be assigned with larger importance degrees, and the dynamic variables of the position and the velocity of the cart should be determined based on the angle of the pendulum. When the pendulum is far from the upright position, the stabilization control will be impossible if the angular control of the pendulum is not done immediately. In this case, the importance degrees of the position and the velocity of the cart should be reduced, and the control focus should be brought into the angular control of the pendulum. When the pendulum is almost balanced upright, the importance degrees of the position and the velocity of the cart can be increased to some extent so that the angular control of the pendulum is maintained and at the same time the position control of the cart is started. Therefore, the dynamic variables of the dynamic importance degrees of the position and the velocity of the cart can be inferred by using the fuzzy rules of Table 3 which also takes the absolute value of the pendulum angle as the antecedent variable. The membership functions of the antecedent variable are the same as the ones shown in Fig. 3.

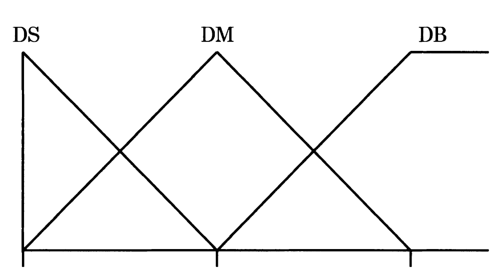

Fig. 3. Membership functions for each dynamic variable.

4.3. Setting the control parameters of the dynamic importance degrees

Since the SIRMs and the dynamic variables of the dynamic importance degrees all have been determined, the structure of the proposed fuzzy controller becomes clear. As stated above, however, each dynamic importance degree also has two control parameters, i.e. the base value and the breadth. The rule setting of the dynamic variables only does not guarantee the necessary control priority orders. If the control parameters of the cart are bigger than the control parameters of the pendulum, for example, the cart position control will take priority over the pendulum angular control even though the dynamic variables are settled as given in Tables 2 and 3. Therefore, the control parameters also have to be adequately set up.

To determine the base value and the breadth of the dynamic importance degree of each input item, a typical inverted pendulum system [2,15,17] is selected. The mass of the pendulum is $m_p = 0.1$ kg, the half length of the pendulum is $l_p = 0.5$ m, and the mass of the cart is $m_c = 1.0$ kg. The angle of the pendulum is limited to $[-30.0°, +30.0°]$, and the moving range of the cart is limited to $[-2.4\ \text{m}, +2.4\ \text{m}]$ [1]. If the pendulum angle or the cart

Table 3
Fuzzy rules for the two dynamic variables of the cart

Antecedent variable $\lvert x_1 \rvert$	Consequent variable Δw_i ($i = 3, 4$)
DS	1.0
DM	0.5
DB	0.0

position gets out of the above range, then the stabilization control is regarded as a failure.

Although the maximum angular velocity of the pendulum and the maximum velocity of the cart are unknown, in this paper the scaling factors of the four input items to the fuzzy controller are set to 30.0°, 100.0°/s, 2.4 m, and 1.0 m/s, respectively. Since the fuzzy controller does not take the mass of the pendulum and the cart into consideration, the scaling factor of the output item should include a factor reflecting the mass of the objects. Control simulations reveal that if the scaling factor of the output item is selected to just 10 times as large as the total mass of the pendulum and the cart, the stabilization control can be performed satisfactorily. Resultantly, the scaling factor of the output item is set up to 10 times the total mass of the pendulum and the cart.

It is understood from the above discussion that the two importance degrees of the pendulum have to be larger than the two importance degrees of the cart in order to stabilize the whole pendulum system. That is, the base values and the breadths of the two importance degrees of the pendulum should be bigger than the base values and the breadths of the two importance degrees of the cart. To cover all the control situations, the importance degree of the pendulum angle when the pendulum angle is big should almost be the same as the importance degree of the angular velocity when the angular velocity is big. Similarly, the importance degree of the cart position when the positive is big should almost be the same as the importance degree of the cart velocity when the velocity is big. Therefore, the base value and the breadth of the pendulum angle should almost equal to the base value and the breadth of the angular velocity, and the base value and the breadth of the cart position should almost equal the base value and the breadth of the cart velocity.

The final set of the base values and the breadths tuned by trial and error is given in Table 4. Because the control parameters of the pendulum are about 10 times as large as the control parameters of the cart, the two dynamic importance degrees of the pendulum get bigger when the pendulum is not balanced yet. In this way, the priority order of the angular control of the pendulum over the position control of the cart is guaranteed. The control parameters of Table 4 and the scaling factors of the input items are fixed in the following simulations.

Table 4
The base value and breadth of the input items

Input item	Base value	Breadth
Pendulum angle	2.00	2.50
Angular velocity	1.50	1.00
Cart position	0.15	0.20
Cart velocity	0.15	0.20

4.4. Block diagram of the fuzzy controller

The block diagram of the fuzzy controller for the stabilization control of the inverted pendulum system is shown in Fig. 4. The state variables θ, ω, x, v of the pendulum system is fed back and compared with the desired values. Because the desired values are all zeros in the stabilization control, the variables are reversely inputted into the Norm block. The Norm block normalizes the state variables by their scaling factors each and creates the input items x_1, x_2, x_3, x_4 from θ, ω, x, v, respectively. Each input item x_i ($i = 1, 2, 3, 4$) is then guided to the SIRM-i block, where the fuzzy inference of the SIRM corresponding to the input item x_i is done. All the Dynamic Importance Degree (DID) blocks take the absolute value of the input item x_1 as their antecedent variable. The DID-i block calculates the value of the dynamic importance degree of the input item x_i. After the output of each SIRM-i block is multiplied by the output of the DID-i block, summing them for all the input items gives the output value of the output item f of the fuzzy controller. The OSF (Output Scaling Factor) block finally multiplies the output value of the output item of the fuzzy controller by its scaling factor to generate the actual driving force F to the cart. Because each of the SIRM blocks and the DID blocks has only three 1-input 1-output fuzzy rules, the proposed fuzzy controller has a simple structure and is easy to realize in hardware.

As shown in Table 1 for the four input items, the consequent variable in the fuzzy rules of each SIRM corresponds to the same output item, and the consequent part of each SIRM has the same setting. According to Eq. (5), each SIRM with its dynamic importance degree contributes directly to the output item, and the angular control of the pendulum and the position control of the cart are treated in parallel. Because the two SIRMs for the position and the velocity of the cart are built such that the cart position is controlled indirectly by putting down the pendulum intentionally, the information about the position and the velocity of the cart is changed essentially into part of the angle of the pendulum. Although similar to the virtual target angle method [3,9,19], the proposed fuzzy controller executes completely in parallel the pendulum angular control and the cart position control to directly obtain the driving force without inferring a virtual target angle.

Moreover, it is clear that the setting order of the real number membership functions in the consequent part of Table 3 for the position and the velocity of the cart is just reverse to that of Table 2 for the angle and the angular velocity of the pendulum. By this setting, the inference results of Tables 2 and 3 become complementary with

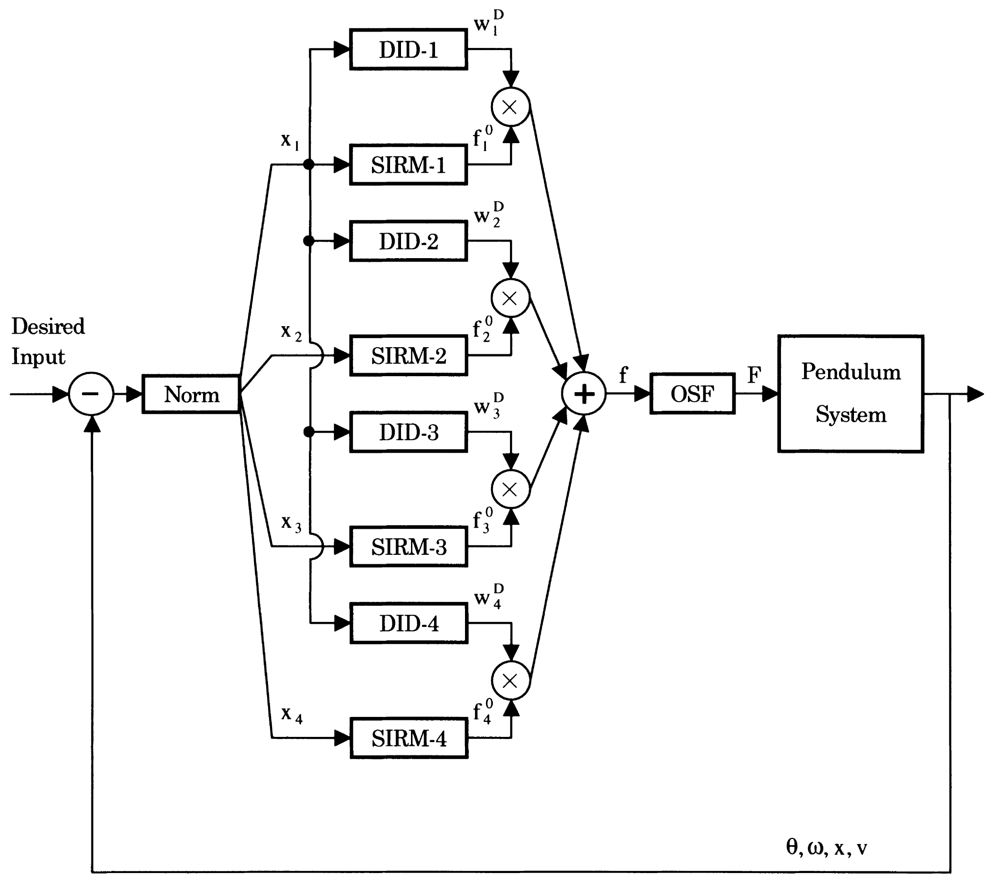

Fig. 4. Block diagram of the stabilization fuzzy controller.

each other. If the absolute value of the pendulum angle is large, the dynamic variables of the angle and the angular velocity of the pendulum take large values, while the dynamic variables of the position and the velocity of the cart can only take small values. Because the control parameters of the pendulum are bigger than the control parameters of the cart, the two dynamic importance degrees of the pendulum become much bigger than the two dynamic importance degrees of the cart. Consequently, the angular control of the pendulum takes priority over the position control of the cart and makes the pendulum rotate toward the upright position. On the contrary, if the pendulum is almost balanced upright, the dynamic variables of the pendulum become small, while the dynamic variables of the cart become large. In this case, the two dynamic importance degrees of the pendulum decrease almost to their base values and the two dynamic importance degrees of the cart increase almost to the sums of their base values and breadths. Although either of the base values of the pendulum is still larger than the sum of the base value and the breadth of either the dynamic importance degrees of the cart, the inference result of the SIRM corresponding to the pendulum angle becomes almost zero. As a result, the contribution from the cart in Eq. (5) will exceed the contribution from the pendulum so that the position control of the cart is started while the pendulum is kept balanced. In this way, the pendulum angular control and the cart position control are switched smoothly by adjusting automatically the dynamic importance degrees according to control situations, and then the stabilization control of the inverted pendulum system is realized.

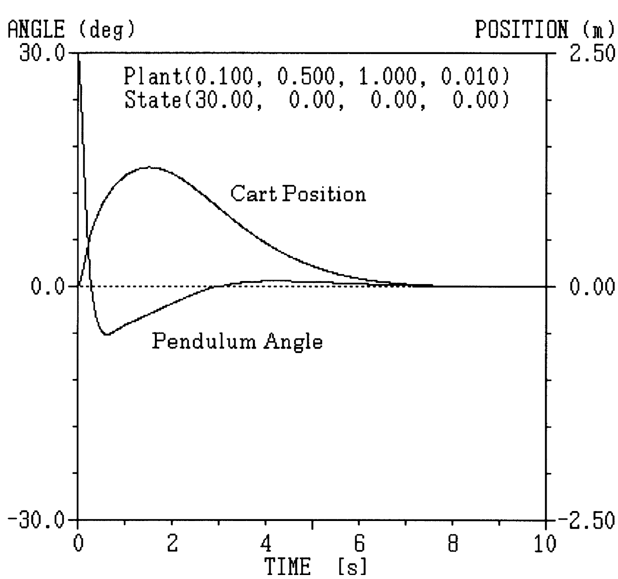

Fig. 5. Control result of 1.0 m pendulum system for initial angle 30.0°.

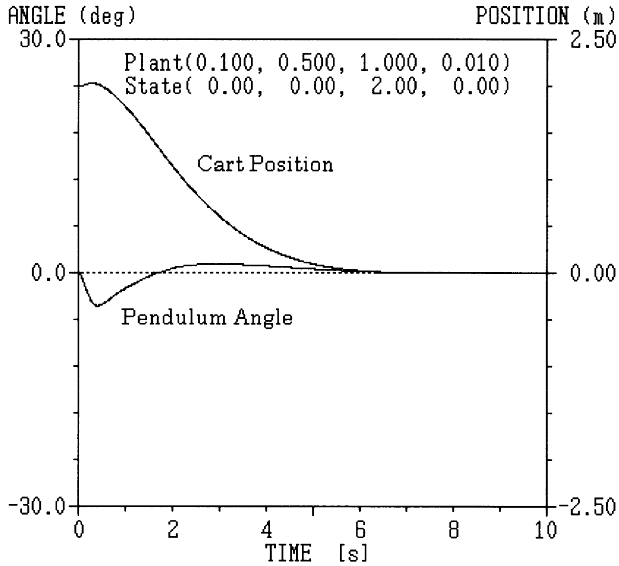

Fig. 6. Control result of 1.0 m pendulum system for initial position 2.0 m.

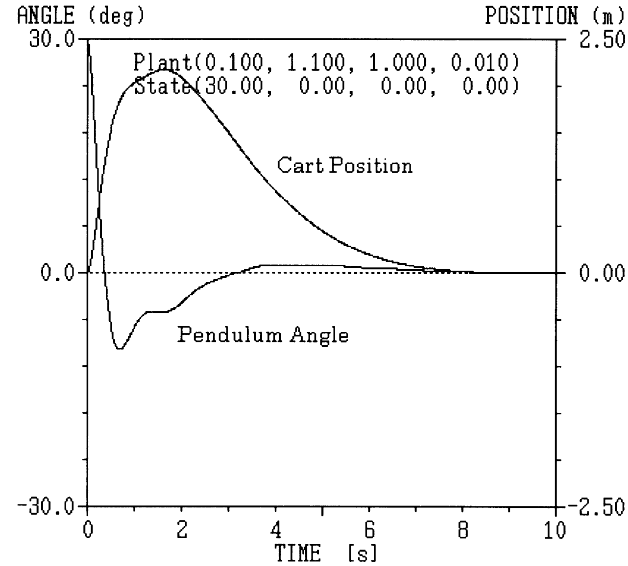

Fig. 8. Control result of 2.2 m pendulum system for initial angle 30.0°.

5. Stabilization control simulations

To verify the effectiveness of the proposed fuzzy controller, stabilization control simulations are first done for the inverted pendulum system given in Section 4.3. Fig. 5 shows the control result, where the initial angle of the pendulum is 30.0° and the other initial values are all zeros. The left axis and the right axis indicate, respectively, the pendulum angle and the cart position, and the horizontal axis indicates the control time. The numbers in Plant (0.100, 0.500, 1.000, 0.010) denote the pendulum mass, the pendulum half length, the cart mass, and the sampling period in this order, respectively. The sampling period is set up to 0.01 s [8,10,14,15,17,19]. The numbers in State (30.00, 0.00, 0.00, 0.00) mean, respectively, the initial values of the pendulum angle, the angular velocity, the cart position, and the cart velocity in order.

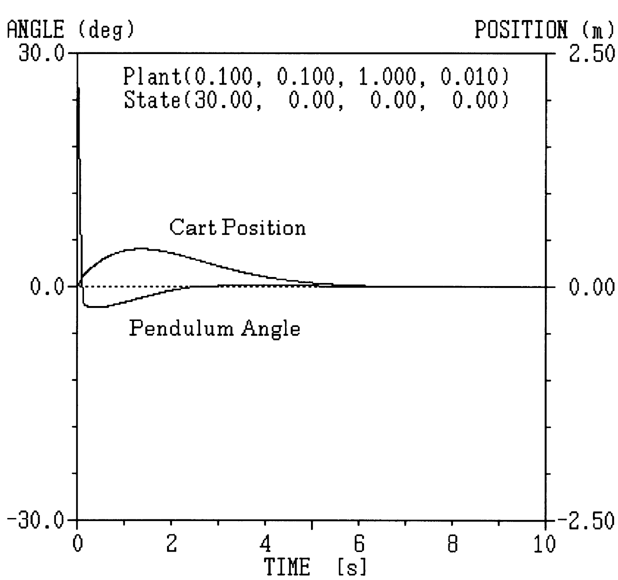

Fig. 7. Control result of 0.2 m pendulum system for initial angle 30.0°.

As can be seen from Fig. 5, since the initial pendulum angle is positively big, the cart is first driven from the rail origin to the right side such that the pendulum is put a little down in the negative direction. Then by moving the cart back toward the rail origin, the pendulum is balanced upright and the cart is returned to the rail origin. Apparently from control beginning until complete stabilization, the cart makes only one round trip and the pendulum swings around the upright position only once a time except the initial part. Here, the complete stabilization means that all the state variables converge, respectively, to 0.1°, 0.1°/s, 0.01 m, and 0.01 m/s. In this example, the complete stabilization time is 8.24 s, the maximum driving force is about 50.0 N, and the maximum velocity of the cart is about 2.5 m/s.

Fig. 6 depicts the control result of the same pendulum system, where the initial position of the cart is 2.0 m and the other initial values are all zeros. Since at the control beginning the cart position is big and the pendulum stands up, the fuzzy controller starts the cart position control first. The cart is moved further toward the right direction such that the pendulum is inclined to about −5.0°. After that, the pendulum angular control takes priority over the cart position control, and the fuzzy controller moves the cart toward the rail origin so that the pendulum is rotated toward the upright position. As a result, the pendulum system is completely stabilized in 7.16 s. The maximum driving force is only about 3.0 N, and the maximum velocity of the cart is about 0.7 m/s.

To check the generalization ability of the proposed fuzzy controller, the length of the pendulum is changed while the other parameters are all fixed. Fig. 7 draws the simulation result where the pendulum is 0.2 m long. Since the

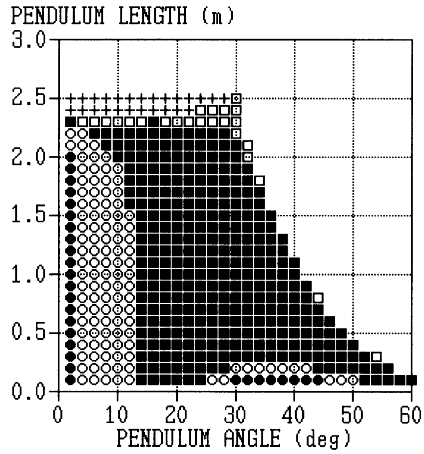

Fig. 9. Relation of the pendulum length with controllable initial angle.

pendulum is rather short, a small amount of the cart moving can cause the pendulum to rotate because the pendulum has a high natural frequency. Therefore, even though the cart moves a little, the pendulum is balanced upright in a short time. The complete stabilization time in this case is 6.65 s. Fig. 8 depicts an example where the pendulum has a length of 2.2 m. Since the pendulum is long, the pendulum has a small natural frequency and a big momentum. To balance the pendulum, then the cart has to move for a long distance. As a result, the cart moves from the rail origin to almost the margin of the rail and returns back to the rail origin. At the same time, the pendulum is rotated to the negative direction and then stood up. The complete stabilization time of this example is 9.04 s. By the way, the maximum driving forces in these two examples are both about 50.0 N.

From Figs. 5–8, it is found that the control result changes largely with the pendulum length. When the pendulum length is short, the pendulum will be controlled first to the reverse direction sharply and then balanced smoothly. When the pendulum is long enough, it will be rotated at first to the reverse direction rather smoothly and then stood up with some fluctuation. Moreover, as the pendulum gets longer, the cart is moved further away from the rail origin in order to put the pendulum a little down toward the rail origin.

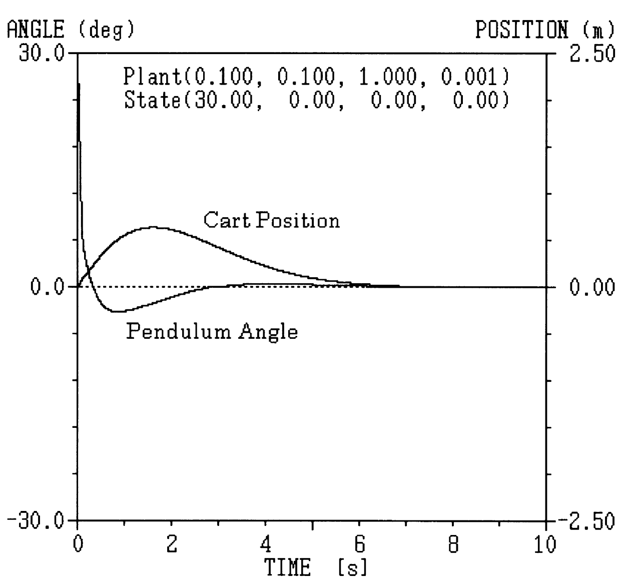

Fig. 10. Control result when the sampling period is 0.001 s.

Fig. 9 shows the relation of the pendulum length with the pendulum initial angle. Here, the horizontal axis and the vertical axis stand for the initial angle of the pendulum and the full length of the pendulum, respectively. The initial angle of the pendulum is selected every 2.0° from 2.0° to 60.0°, and the initial values of the other state variables are all fixed to zeros. The full length of the pendulum is changed every 0.1 m from 0.1 m to 3.0 m. The symbols X, W, B, A, + mean that the complete stabilization time is within 5.0 s, or within 7.0 s, or within 9.0 s, or within 11.0 s, or exceeds 11.0 s, respectively, for the initial states where the symbols are located. Further, the failure limit of the angle of the pendulum is extended to 60.0°. No need to say that the symmetric result can be obtained for negative values of the initial angle of the pendulum.

As can be seen from Fig. 9, the proposed fuzzy controller can completely stabilize the pendulum system within 7.0 s for the initial angle of up to 10.0° if the pendulum is shorter than 2.0 m. If the pendulum is shorter than 2.2 m, the fuzzy controller can stabilize the pendulum system entirely within 9.0 s for the initial angle of up to 30.0°. The maximum initial angle, which can be stabilized by the fuzzy controller, decreases almost linearly as the pendulum gets longer. If the pendulum is only 0.1 m long, even the initial angle of 60.0° can be stabilized.

If the sampling period is bigger than 0.01 s, it may give a bad influence on the stabilization control of a short pendulum system because of the high natural frequency of the pendulum. In fact, if the sampling period is 0.02 s, the fuzzy controller will fail to stabilize the pendulum system when the pendulum is only 0.1 m long. Because a short pendulum responds sharply to the cart moving, a small sampling period is effective to stabilize smoothly a short pendulum system with a quick control action. Fig. 10 illustrates such a control result of the pendulum system used in Fig. 7, where the sampling period is changed from 0.01 s to 0.001 s. Although the moving distance of the cart from the rail origin to the right side increases, the pendulum is smoothly rotated from its initial position to the negative direction and then smoothly balanced upright. In this case, the complete stabilization time is 7.411 s, almost equal to that shown in Fig. 7.

Because the scaling factor of the output item is set as 10 times the total mass of the pendulum and the cart, the influence of the pendulum mass or the cart mass is basically absorbed by the scaling factor of the output item. Fig. 11 displays a simulation result of the pendulum system used in Fig. 5, where only the pendulum mass is changed from 0.1 to 0.5 kg. Compared with Fig. 5, the control result is very similar to that of Fig. 5, and the complete stabilization time

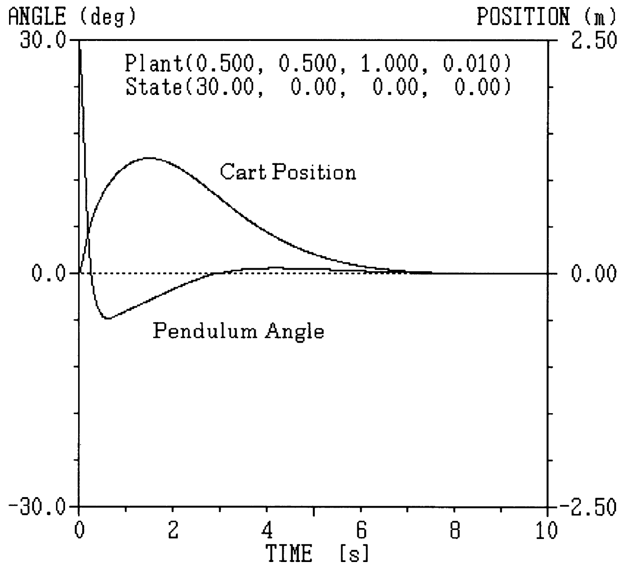

Fig. 11. Control result when the pendulum mass is 0.5 kg.

is 8.18 s. The result indicates that the pendulum mass has little influence on the control performance. In fact, if the pendulum length is within [0.2 m, 2.2 m], the pendulum mass is equal to or larger than 0.001 kg, the cart mass is equal to or larger than 0.002 kg, and the ratio of the pendulum mass to the cart mass is within [0.005, 0.500], then the pendulum system can be stabilized when the initial angle is within [$-30.0°$, $+30.0°$] or the initial position is within [-2.25 m, $+2.25$ m]. Note that the upper limits of the pendulum mass and the cart mass are not given here because they relate with the driving force. If there is no limit to the magnitude of the driving force, then the stabilization control is theoretically possible no matter how heavy the pendulum and the cart are. In the control simulations, the inverted pendulum system, where the pendulum mass is 5.0 kg and the cart mass is 10.0 kg, is successfully stabilized. Since the controllable parameter range covers almost all the inverted pendulum systems reported till now [2,4,5,7,8,10,13–15,17], the proposed fuzzy controller is shown to have a high generalization ability to stabilize a wide range of the inverted pendulum systems.

On the other hand, if the pendulum is shorter than 0.2 m or longer than 2.2 m, the stabilization control may fail since the scaling factor of the output item is given as a function of only the pendulum mass and the cart mass without consideration of the pendulum length. If the pendulum is rather short, small driving force is enough. For a short pendulum that has a high natural frequency, a big driving force may cause the pendulum to rotate sharply to fall down. On the other hand, if the pendulum is rather long, a bigger driving force is necessary. For a long pendulum that has a low natural frequency and a big momentum, the stabilization control becomes impossible if the driving force is not big enough. Therefore, it would be ideal for the scaling factor of the output item to be a function of the pendulum length also. For example, the pendulum system, whose pendulum is only 0.04 m long, can be stabilized if the scaling factor of the output item is reduced to half. The stabilization control of the pendulum system, whose pendulum has a length of 3.0 m, is possible if the scaling factor of the output item is enlarged three times.

As a comparison, Kandadai [2] took more than 12.0 s to realize asymptotic stabilization with some offset left. Kawaji [3] found that it was difficult to achieve complete stabilization even after 20.0 s. Kyung [4] needed about 8.0 s to stabilize approximately an inverted pendulum system by using 396 fuzzy rules. Pan [14] used about 8.0 s to finish complete stabilization by a variable structure control system. Sakai [16] stabilized completely an inverted pendulum system in more than 200.0 s. Although having a simple structure, the proposed fuzzy controller can stabilize completely a wide range of the inverted pendulum systems with 9.0 s.

6. Conclusions

A new fuzzy controller based on the SIRMs dynamically connected fuzzy inference model is proposed for the stabilization control of the inverted pendulum systems. A SIRM and a dynamic importance degree are assigned for each input item. The dynamic importance degrees are built such that the angular control of the pendulum takes priority over the position control of the cart. It is made clear that by using the SIRMs and the dynamic importance degrees, the pendulum angular control and the cart position control are executed in parallel, and switching between the two controls is realized by automatically adjusting the dynamic importance degrees according to control situations. The simulation results show that the proposed fuzzy controller has a high generalization ability to stabilize completely a wide range of the inverted pendulum systems within 9.0 s for an initial angle up to 30.0°.

Moreover, the proposed fuzzy controller has a simple and intuitively understandable structure. It is also possible to implement in hardware easily by the look-up-table approach because each SIRM or each dynamic importance degree only requests a small amount of memory.

For systems with more input items and interactions between the input items, the SIRMs dynamically connected fuzzy inference model is still effective. Each SIRM handles separately only one input item, and each dynamic importance degree takes one or two input items as its antecedent variables. The interactions between the input items are imbedded into the fuzzy rules of the dynamic variables. For instance, consider a series-type double inverted pendulum system, which has six input items. The control priorities of the lower pendulum and the cart are affected by both the upper pendulum angle and the lower pendulum angle. To stabilize the double inverted pendulum system, then it is necessary to use the absolute values of the two angles as the antecedent variables of the dynamic variables of the lower pendulum and the cart. Such a stabilization fuzzy controller is being developed right now.

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