
Performance Evaluation of MIMO Techniques

With an SDR-Based Prototype

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Abstract

There is tremendous pressure on modern wireless communication systems. First, there are more people than ever using wireless communications as their primary source of information and communication. Second, the data demand per user has exploded thanks to new mobile applications such as Facebook and Uber. Of note, data demand is being driven especially by streaming video. As a result, there has never been a greater need for finding creative solutions to improving networks' performance, reliability, and efficiency. This paper examines the use of several different algorithms for wireless data transmission that can result in greater reliability and throughput based on the use of multiple channels -- MIMO diversity and spatial division multiplexing techniques. The contribution of this work is to show through simulations and experiments that the reliability or throughput of a communication system can be significantly improved by using a combination of multiple channels based on open source and programmable hardware and software, and to bridge the gap between theory and practice.

I. INTRODUCTION

"When wireless is perfectly applied ... We shall be able to communicate with one another instantly, irrespective of distance. Not only this, but through television and telephony we shall see and hear one another as perfectly as though we were face to face, despite intervening distances of thousands of miles; and the instruments through which we shall be able to do this will be amazingly simple compared with our present telephone. A man will be able to carry one in his vest pocket."

Nikola Tesla, 1926

Closer than ever to Nikola Tesla's vision, as articulated more than 90 years ago of a global wireless communications system, the next step in bringing wireless communication systems closer to that ideal vision is the deployment of a fifth generation – 5G – of wireless telephony and data communications networks.

As wireless devices have become ubiquitous, congestion across available low frequency bands has increased, and signal performance in certain areas and for certain applications has begun to degrade. Ever-increasing demand for high data throughput rates and greater user capacity have made clear the need to use available spectrum more efficiently.

A combination of Multiple Input Multiple Output (MIMO) and Orthogonal Frequency-Division Multiplex (OFDM) techniques have been implemented in recent wireless standards (i.e 802.11, LTE, 5G) to help meet this growing demand for data (Helmut,2006; Hemanth,2002). Because MIMO utilizes antennas arrays, beamforming adoption is a good candidate to improve the received Signal to Noise Ratio (SNR) and throughput mostly in rural and remote areas (Annisa,2017), which in turn reduces the Bit Error Rate (BER).

Within the Software Defined Radio (SDR), in many instances, signal processing is implemented in software making it easy to debug, to use, or to modify. Open source software for SDRs offers the ability to use both simulations and experiments on the same tool.

This paper provides a practical assessment activity on real-world hardware making use of well-known MIMO algorithms, and presents results and relevant issues. Implementations and discussion are built upon three diversity techniques, which are Alamouti Code Space Time Block Coding (AC-STBC), Maximum Ratio Combining (MRC), and Selection Combining (SC); and upon two spatial division multiplexing techniques using Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) algorithms (Wolniansky et al.,1998).

II. OVERVIEW

Utilization of wireless communication systems is primarily dictated by the wireless channel environment. Wireless channels are dynamic and unpredictable (Feng et al.,2007), which makes the analysis of the wireless communication system often difficult. As a result, the optimization of the wireless communication system has become critical with the rapid growth of mobile communication services. The

past decades have seen tremendous growth in use of radio frequency spectrum, particularly by commercial cellular operators. The use of smartphones, computer laptops, and tablets is the primary driver behind an ever-increasing utilization of spectrum (Cordeiro,2005). The anticipated future result is a shortage of radio spectrum to meet bandwidth demands of users.

MIMO wireless systems can serve as a part of the solution for improving wireless access. MIMO is a key technology for next generation cellular communications such as LTE, as well as Wireless LAN 802.11ax (Abbas,2012). MIMO enables more than one data stream, and therefore increases the throughput by using multiple antennas on transmitters and receivers (Lee,2012). With respect to how data is transmitted across the given channel, MIMO communication links can be categorized in three groups: MIMO designs for diversity techniques, MIMO for spatial multiplexing techniques, and multi-stream beamforming.

Diversity means that the same data has traveled through multiple paths. Such techniques increase communications reliability and improves the BER performance (Siavash,1998;Salz et al.,1994; Jack,1998). Diversity techniques require the use of multiple antennas at the receiver and are commonly used to overcome the effect of fading, which causes severe degradation in the achievable data throughput of wireless communication systems.

In spatial multiplexing, each spatial channel carries different data, and therefore increases the data rate, which is similar to OFDM techniques where frequency sub-channels carry different parts of the modulated data (Kinjal,2016). By multiplexing the signals, data throughput is greatly improved, and therefore presents a high spectral efficiency (Hui et al.,1998). A new component incorporated in MIMO techniques, beamforming, is a technique for generating a custom radiation pattern based on channel information knowledge that provides antenna gains in a specific direction. In beamforming, amplitude and phase are combined to efficiently adjust side lobe levels and steer nulls (Likhitkar,2015).

A MIMO throughput and transmission rate system's performance depends on the condition of the channel information matrix. The channel information matrix represents the relationship between the received signals in a MIMO system, and the transmitted signal. Entries in the channel information matrix are an alteration coefficient acting on the transmitted signal amplitude and phase (Smita et al.,2005). Based on the channel conditions, MIMO systems will yield an increase in the channel capacity according to the modified Shannon equation given by (Espineira et al.,2008):

$$C_{MIMO} \approx \sum_{i=1}^m \log_2 \left(1 + \frac{\rho}{N} d_i^2 \right) \text{ Hz} \quad (1)$$

Where $m = \min(M, N)$, N is the number of antennas element at the transmitter side, M is the number of antennas element at the receiver side, ρ is the Signal to Noise Ratio, d_i^2 is the element of the transmission matrix, and C_{MIMO} is the channel capacity. There is a linear increase in the channel capacity when the number of antenna elements from transmitter and receiver are increased, with the assumption that the multipath channels are not correlated (Paulraj et al.,1993). Therefore, the design and use of compact antennas type and arrays systems with low correlation between elements of the array is very critical in order to achieve the high data rates expected (Ertug et al.,2015). Modern wireless communication systems use spatial multiplexing to improve data throughput within the system in a multipath rich environment. Transmitting multiple data streams through channels requires that a set of precoding and combining weights be derived from the channel matrix (Mohammad et al.,2010). Then, each datum can be independently recovered. Designated weights comprise both magnitude and phase terms and are typically applied in the digital domain.

Data exchange techniques in wireless communication systems are mainly grouped into four categories with respect to number of antennas in the transmitter and receiver. First, Single Input Single Output (SISO) signifies one transmitter antenna and one receiver antenna; Single Input Multiple Output (SIMO) refers to one transmitter antenna and multiple antennas at the receiver; Multiple Input Single Output (MISO) refers to multiple transmitter antennas and one receiver antenna; and, finally, Multiple Input Multiple Output (MIMO) refers to multiple transmitter antennas and multiple receiver antennas.

In the next sections, the testing environment, the implementation of MIMO diversity techniques using AC-STBC, MRC, and SC algorithm; and MIMO spatial multiplexing using ZF and MMSE algorithms are evaluated.

III. TESTING ENVIRONMENT

The test bed utilizes an open source, GNU Radio software environment, and the Ettus Research Universal Software Radio Peripheral (USRP N210) for performing SDR functionality, which reduces the cost and complexity of the system development. Using GNU Radio provides the benefit from an active community, a graphical editor to set transceivers, a graphical output to visualize the signal processing in real-time, and a large variety of supported radio platforms (Bastian et al.,2017). MIMO cables from Ettus Research hardware are used to interconnect radios in the implementation of diversity techniques and spatial

multiplexing technique. Omnidirectional antennas are also deployed. Components used to build the experiment can be found in Table 1.

The same setup is maintained throughout each case study, and only the case study block is updated. A dedicated decoder is used for receiver respectively for SC, MRC, AC-STBC, ZF, and MMSE implementations.

Note that all MIMO core encoders-decoders have been extracted from the GNU Radio repository ([Mueller,2018](#)). MIMO core encoders-decoders have been implemented by Luca Moritz Schmid ([Schmid,2018](#)) for the Google Summer of Code (GSoC) 2018 with GNU Radio. The author of this paper assisted Schmid for performance testing of MIMO algorithms implemented with hardware, USRPs, and software loopbacks.

The Diversity Combiner and Space Time Multiplexing (STM) decoder blocks offer the choice to select a specific algorithm such as SC or MRC for Diversity Combiner, and ZF or MMSE for Spatial Division Multiplexing.

TABLE 1: components of our SDR setup

| Component | Type |
|------------------|-----------------------|
| Operating System | Ubuntu 16.04 |
| Gnu Radio | Version 3.7.12 |
| Ettus UHD | 3.11 |
| SDR | Ettus Research N210r4 |
| Antenna | VERT2450 Dual band |
| MIMO Cable | 783076-01 Sync Cable |
| Daughterboard | SBX V5 |
| Frequency | 2.45 GHz |
| Bandwidth | 20 MHz |

To run an algorithm, its decoder is hooked to the outputs of the USRP source for further received signal processing. Each receiver has its own transmitter algorithm, which is not discussed in this paper.

IV. MIMO DIVERSITY TECHNIQUES

MIMO diversity techniques are a powerful communication method that offers wireless link improvements at fairly low cost. Diversity coding techniques are deployed to mitigate the effect of fading ([Charles,1995](#)), thereby achieving an increase in SNR. Output sub-channels can be combined to produce a useful signal that is resilient against multipath effects.

In wireless transmission and reception, channel impairments include noise, i.e. undesirable signals, path loss such as loss in power as the radio signal propagates,

fading signals such as multipath effects and reflector, shadowing phenomenon due to the presence of fixed obstacles in the radio path, and others.

In MIMO diversity techniques, a communication channel is provided with single or multiple transmitting antennas and multiple or single receiving antennas. The same signal is transmitted or received through multiple paths. It provides one or more inputs at the receiver such that the fading phenomena among inputs are uncorrelated. As a result, the probability that all replicated signals will fade simultaneously is reduced dramatically ([Siavash,1998](#)). The main purpose is to provide a correction to wireless channel impairments, and to converge to the theoretical Shannon channel capacity without disregarding the complexity and feasibility for practical use cases. If one radio path undergoes a deep fade, another independent path may have a strong signal at that input to improve receiver signal quality.

Diversity techniques can be deployed at the base station and mobile receivers. In the following sections, implementation of three different techniques to illustrate the robustness of diversity techniques are presented.

A. Case Study One: AC-STBC Alamouti Code

In 1998, Siavash Alamouti, the father of STBC, proposed a transmission scheme for MIMO systems to achieve transmit diversity, while at the same time working as a simple decoding scheme for MIMO wireless systems ([Kshetrimayum,2017](#)). It explores spatial and temporal aspects of diversity schemes and achieves full diversity order. STBC Alamouti Code (AC-STBC) began with the purpose of providing transmit diversity for MIMO systems. It does not require a feedback link from receiver to transmitter to provide information regarding some parameters of the channel states. AC-STBC is a technique used in wireless communication to enable the transmission of multiple copies of a data stream across a number of antennas. It exploits the various received versions of the data to improve the reliability of data transfer. AC-STBC merges all the duplicates of the received signals in an optimum way to extract as much information from each of them as possible. Such a technique helps compensate for any channel problems such as fading, reflection, scattering, refraction, or thermal noise. Though there is redundancy, some data copies may arrive corrupted at the receiver. Figure 1 illustrates the Alamouti Code receiver configuration in GNU Radio. The same code basis is used for Diversity Combiner or V-BLAST. The Error Rate block is used in the implementation to capture the BER. Additional blocks are used for the configuration and are clearly shown on the figures. Figure 2 shows a two by two transmitter-receiver run and the BER recorded.

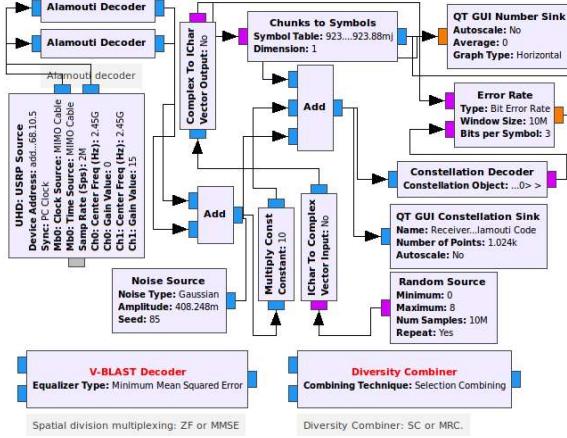


Fig. 1: Configuration AC-STBC receiver in GNU Radio, including specific case study blocks: V-BLAST and Diversity decoders.

B. Case Study Two: Diversity Combiner SC and MRC

Received signals in the different antennas can be combined by various techniques. These combining techniques include Selection Combining (SC) and Maximum Ratio Combining (MRC). SC is the simplest diversity combining scheme. In SC, only the received signal with the highest SNR among received copies is selected for decoding (Cho et al., 2010). MRC, invented by Leonard R Kahn (Leonard, 1954), requires as many radio frequency chains at the receiver as the number of receiving antennas. With increased hardware complexity, MRC can provide optimum performance in terms of probability of error among all the diversity combining schemes (Kshetrimayum, 2017; Cho et al., 2010). In MRC, different proportionality constants are used for each channel, the signals from each channel are added together, and the gain of each channel is then made proportional to the Root Mean Square signal level, and inversely proportional to the mean square noise level in that channel. After implementing the diversity combiner, measurements are made using one antenna from the transmitter and two antennas from the receiver to distinguish the algorithm responsiveness.

V. MIMO SPATIAL MULTIPLEXING

The objective of spatial division multiplexing (SDM) is to increase the throughput of the system. Incoming data streams from the transmitter are converted from serial to parallel for transmission. The parallel streams of information obtained after the conversion are transmitted simultaneously from multiple sub-channels or antennas available on the transmitter side (Wolniansky, et al., 1998; Espineira et al., 2008; Foschini et al., 1998). Consequently, two effects can be distinguished on the resulting of wireless communication systems. First, the bandwidth requirement will decrease if the data are transmitted simultaneously using the same rate at which they are generated. Second, the effective data rate is proportional to the minimum

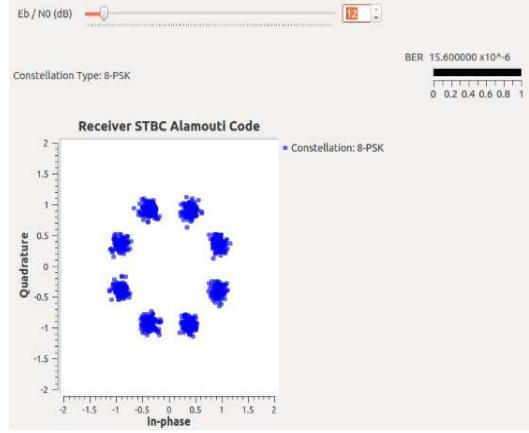


Fig. 2: AC-STBC BER measured

number of transmitting and receiving antennas. Therefore, the overall rate of communication can be increased.

A. Case study: V-BLAST using ZF and MMSE

Vertical Bell Laboratories Layered Space-Time, or V-BLAST, exploits the rich scattering wireless channel system, and is theoretically capable of providing enormous capacity, assuming the multipath scattering is sufficiently rich and appropriately exploited (Peter et al., 1998). V-BLAST is a simplified architecture version of D-BLAST, which seeks to reduce its computational complexity (Hoon et al., 2002). All symbols of a certain stream are transmitted through the same antenna since each stream is tied to its antenna. In this technique, space-time codes are vertically layered, which is where V-BLAST's name is derived from (Peter et al., 1998). In this implementation, two popular approaches are used: Zero-Forcing and the Minimum Mean Square Error equalizer (Bhojak et al., 2013).

The main purpose of Zero-Forcing is to cancel inter-symbol interference and multiple access interference by designing a Nyquist pulses, or an equalization (Wong et al., 2002). MMSE refers to estimation via a quadratic loss function, which minimizes the mean square error (Balanis, 2005). MMSE is similar to ZF but maximizes the output Signal to Interference plus Noise Ratio (SINR). It optimizes the tradeoff between noise enhancement and interference cancellation (Elnashar, 2018; Xie et al., 1994).

V-BLAST is an effective technique for achieving high spectral efficiency (Bhojak et al., 2013). The V-BLAST algorithm requires a feedback link or channel state information from receiver to transmitter to provide information regarding the channel states, or some parameters of channel matrix.

Because spatial division multiplexing emphasizes increasing data throughput, the data rate and their corresponding BER in ZF and MMSE are measured. The

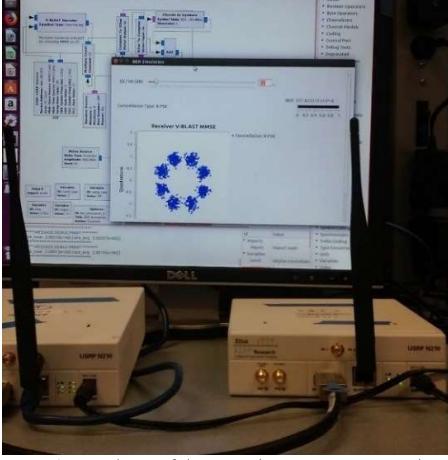


Fig. 3: Photo of the experiment setup – Receiver

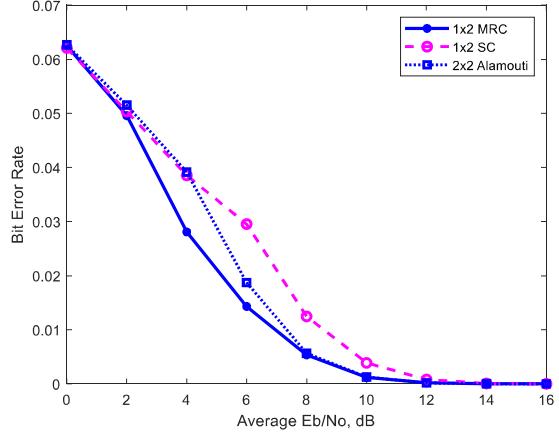


Fig. 4: MRC, SC and Alamouti Code BER measured

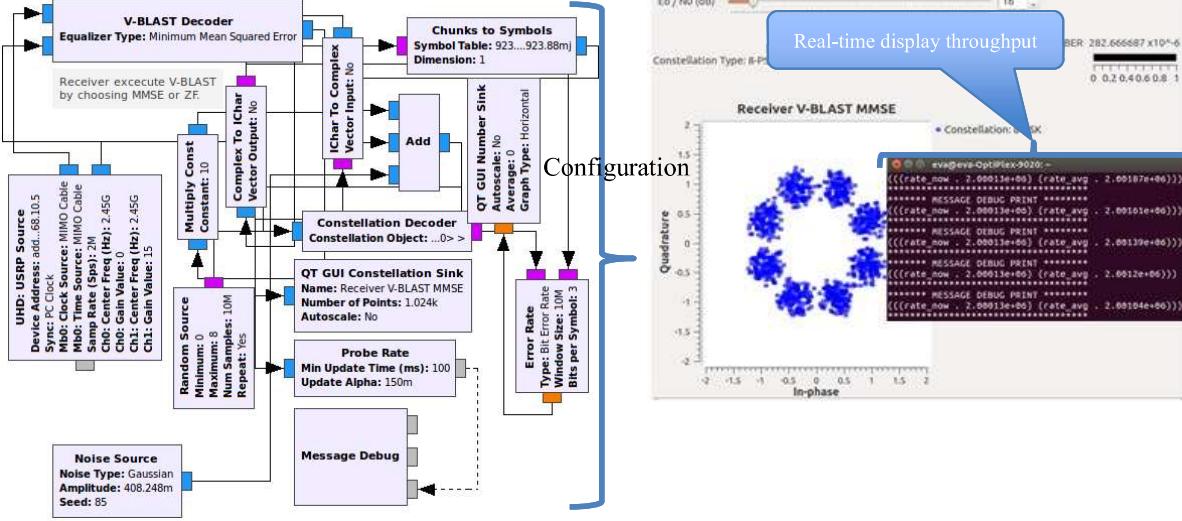


Fig. 5: V-BLAST receiver structure, BER, and throughput measured

throughput measurement is possible thanks to the Probe Rate block, which is hooked to V-BLAST block output, and directed the Probe Rate output to Message Debug block.

VI. EXPERIMENT RESULTS AND PERFORMANCES ANALYTICAL

All implementations and testing are done within the Technology, Cybersecurity and Policy wireless lab, Discovery Learning Center at the University of Colorado. As a result, the power azimuth spectrum tends to be uniform. In other words, scattered components will be received from all directions with the same power. Another consideration is that a channel tends to be static due to the low mobility of radio USRPs inside the lab [Fig. 3]. As a result, the wireless channel is multipath fading free ([Gerardo, 2017](#)). The same higher-order modulation

scheme, 8-PSK, is used through all implementations. Running the experiment, the received signals are decoded and displayed in the form of a constellation diagram. The computation of BER value includes the first received frames, while some of the frames in the 8-PSK receiver have not converged yet. As a result, the overall BER results are highly affected during the first ten seconds by high BER values at the beginning of the simulation. Once the transient period is over, the receiver is able to estimate the transmitted frames, and at that time, the BER level dramatically improves. Increasing the transmission duration lowers the BER values.

From Fig. 4, the three combining techniques shows MRC effectiveness over Alamouti code, and SC. However, at high Eb/No, the performances converge to the same BER. Because the output of the SC is equal to the signal on only one of the branches, the coherent sum of the individual

branch signal is not required (Marvin et al., 1998), and therefore, it is less CPU intensive (Dogay et al., 2017). As a result, there is a tradeoff between the performance and the complexity based on the diversity technique selected.

The decision to implement V-BLAST was to prove the theoretical prediction that high throughput can be achieved in combination with algorithms such as ZF or MMSE. Results have proven that the capacity of a SISO throughput is double using two channels. Throughput can be further improved based on how much the hardware can handle. USRPs N210 can handle up to 25 MSps, see Fig. 5. Given the same throughput, ZF receiver BER is close to MMSE receiver at high Eb/No. The received signal is estimated at every Eb/No. Figure 6 emphasizes the superiority of MMSE algorithm over ZF. The ZF technique exhibits some degradation at low SNR due to its noise enhancement shortcoming.

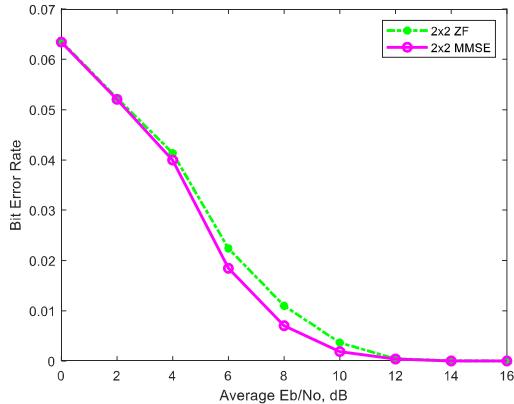


Fig. 6: ZF and MMSE BER measured

VII. CONCLUSION AND FUTURE PERFORMANCE EVALUATION

In this study, three combining techniques, and two spatial multiplexing techniques have been illustrated through simulation to experiment. Experimented results obtained can vary significantly based on the testing environment, such as signal interference, high fading due to poor multipath and bad hardware conditions. Though the quality of the hardware components may affect the communication performance – and particularly the spectral efficiency – the assessment in GNU Radio of the algorithms, SC, MRC, Alamouti Code, ZF, and MMSE results are highly encouraging, and are simply unattainable using traditional Single-Input Single-Output communication techniques. This experiment was easily conducted via GNU Radio which benefits from an active community and also of a large ecosystem.

Overall, major benefits of using antenna arrays in wireless communication networks include improved reliability and throughput, diminished service cost, and lower energy consumption. A communication system can be made more

reliable by using a combinatorial algorithm in conjunction with multiple antennas on both the transmitting and receiving ends of the system. Wireless communication system throughput and reliability can be enhanced with a densification of the number of antennas from both sides, transmitter and receiver.

The growing demand for high data rates and greater user capacity increase the need to utilize spectrum more efficiently. As a result, the next generation, 5G, wireless systems will use millimeter wave band to take advantage of its wider bandwidth.

5G systems will deploy large scale antenna arrays to alleviate severe propagation loss in the millimeter wave band. However, these configurations come with their own unique technical challenges.

Beamforming can be implemented to improve the received SNR, which in turn reduces the BER because MIMO deploys arrays. Beamforming uses several antennas to govern directions of a wave by weighting the magnitude and phase of individual antenna signals in an array of multiple antennas and will be an essential part of MIMO technology.

Beamforming (Gross, 2018) is gaining momentum in MIMO because it provides further gains when the transmitter has information regarding the channel state and the receiver location. It is a useful technique used to improve the SNR of received signals, prevent undesirable interference sources, and focus transmitted signals on a specific location.

Future experiment assessment will seek to implement beamforming existing algorithms into MIMO using GNU Radio and USRPs hardware and evaluate their effectiveness.

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