Assignment2

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February 2022

Exercise 1

X are the estimated 3D points which fulfills $\lambda x = PX$ and T is a projective transformation. By multiplying with $I = TT^{-1}$ we get

$$\lambda x = PTT^{-1}X = PT\tilde{X} = \tilde{P}\tilde{X}$$

we find that $\tilde{P} = PT$ is a new valid camera and $\tilde{X} = T^{-1}X$ is a new solution that satisfies the original equation.

Computer Exercise 1

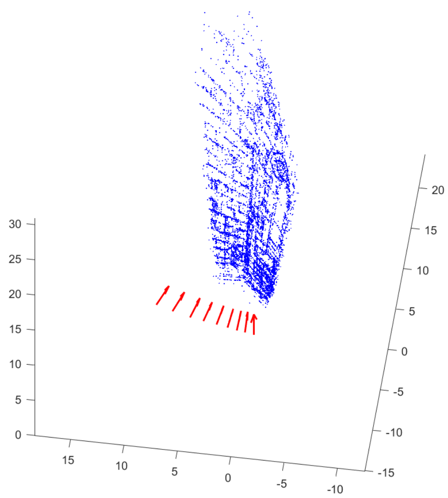

Figure 1: The given 3D points of the reconstruction plotted together with the cameras.

The reconstruction looks reasonable although a bit skewed, probably because angles and parallel lines are not preserved. When projecting the 3D points into the camera they come very close to their corresponding image points.

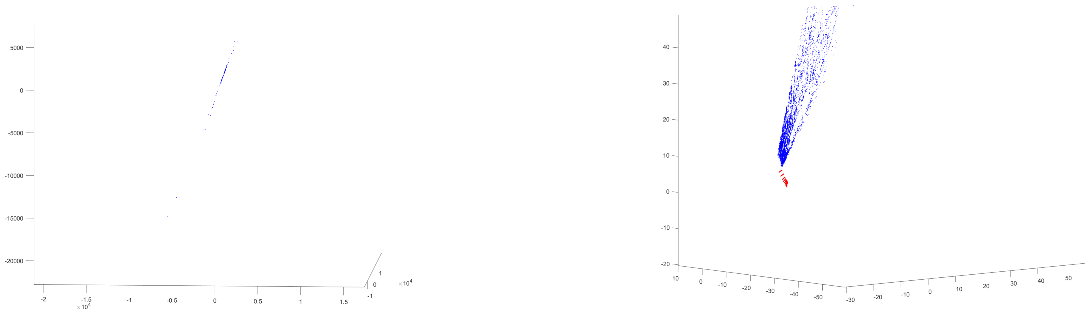

Figure 2: 3D plot of the new solution $\tilde{X}_1 = T_1^{-1}X$. The right plot is a zoomed in version of the left.

Figure 3: 3D plot of the new solution $\tilde{X}_2 = T_2^{-1}X$. The right plot is a zoomed in version of the left.

The new solutions both looks to be placed on a plane. Both the new solutions looks better when zoomed in, but the problem is that some of the new points are placed very far away from the scene. They are probably vanishing points (or at least have a very small last coordinate) and when using the *pflat* function they are placed far away. These reconstructions are not as good as the original one. When projecting the new solutions into one of the new cameras, $\tilde{P}_i = PT_i$, it's identical to the original image points since $TT^{-1} = I$.

Exercise 2

When using calibrated cameras the parameter K is known. Normalization $\tilde{x} = K^{-1}x$ gives $\lambda\tilde{x} \sim [R \quad t]X$. In order to get a new valid solution only similarity transforms can be applied. This results in less projective distortions since similarity transforms preserve angels and parallel lines.
Corresponding statement is

$$\lambda_{ij}\tilde{x}_{ij} = [R_i \quad t_i]X_j$$

where R_i is a $3x3$ rotation matrix and $\tilde{x}_{ij} = K^{-1}x_{ij}$.

Exercise 3

- The interpretation of A is scaling by a factor one over the focal length. The points are transformed from pixels to meters.
- The interpretation of B is negative translation by the principipal point which now centers the points around $(0, 0)$.
- By multiplying the image points with K^{-1} they are transformed into points on the projective plane in the camera coordinate system.
- The principal point (x_0, y_0) ends up at $(0, 0, 1)$
- A point with distance f to the principal point end up with distance 1 to the principal point after normalization.

$$P_1 = K[R \;\; t]$$
$$P_2 = [R \;\; t]$$

The camera centers are found by calculating the null space of P_1 and P_2.

$$P_1 \begin{bmatrix} C_1 \\ 1 \end{bmatrix} = K[R \;\; t] \begin{bmatrix} C_1 \\ 1 \end{bmatrix} = 0$$

$$P_2 \begin{bmatrix} C_2 \\ 1 \end{bmatrix} = [R \;\; t] \begin{bmatrix} C_2 \\ 1 \end{bmatrix} = 0$$

If both sides of the first equation are multiplied by K^{-1} (normalized) then we see that they are the same equation. This results in $C_1 = C_2$ and thus both cameras will have the same center.
The principal axis is the third row of the matrix R. K has the form

$$\begin{bmatrix} \gamma f & sf & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

which results in the third row of R being preserved when multiplied by K (because the third row of K is $(0, 0, 1)$ which does not change the third row of R). This results in that the viewing direction of P_1 and P_2 are the same.

Exercise 4

$$K^{-1}P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1170/1351 & 1/2 & 0 \\ 0 & -1/2 & 1170/1351 & 1 \end{bmatrix}$$

The normalization of the corners $(1-4)$ and center (5) are

1. $(0,0) \longrightarrow (-1/2,-1/2,1)$
2. $(0,1000) \longrightarrow (-1/2,1/2,1)$
3. $(1000,0) \longrightarrow (1/2,-1/2,1)$
4. $(1000,1000) \longrightarrow (1/2,1/2,1)$
5. $(500,500) \longrightarrow (0,0,1)$

Computer Exercise 2

RQ-factorization of the first camera in both of the two new solutions in *Computer Exercise 1* obtained by calculating PT_1 and PT_2 respectively.

$$K_1 = \begin{bmatrix} 2394.0 & 0 & 932.4 \\ 0 & 9592.5 & 628.3 \\ 0 & 0 & 1 \end{bmatrix} \quad K_2 = \begin{bmatrix} 2394.0 & 0 & 932.4 \\ 0 & 2398.1 & 628.3 \\ 0 & 0 & 1 \end{bmatrix}$$

The cameras both have the same rotation matrix, $R = I$.

Exercise 7

- $P = N^{-1}\tilde{P}$

Computer Exercise 3

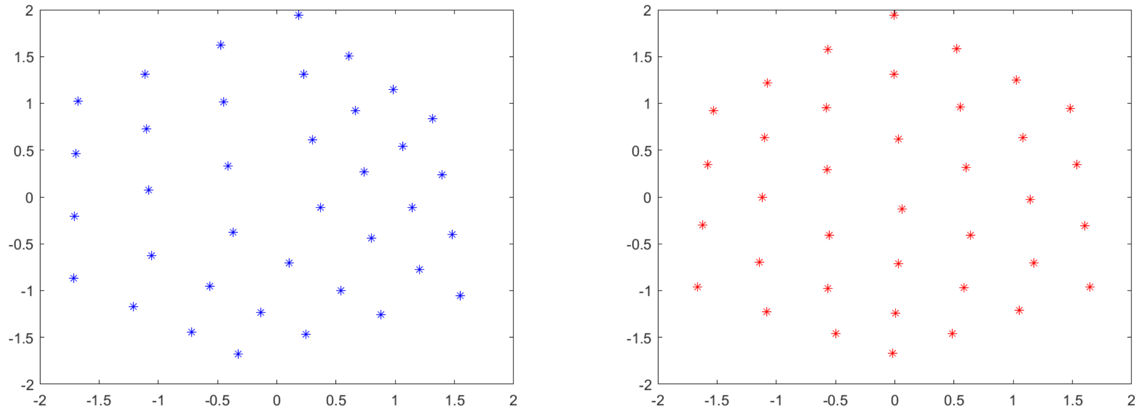

Figure 4: Normalization by subtracting mean and scaling with standard deviation of the measured projections in image 1 (left plot) and image 2 (right plot). The points look centered around $(0, 0)$.

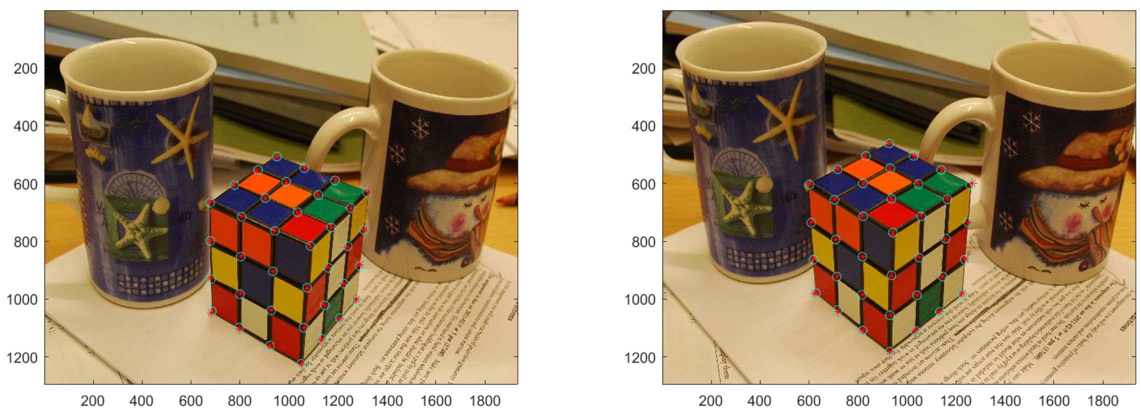

Figure 5: A plot of the projection of the model points (*) into the cameras together with the measured image points (o) from the images. Camera 1 and image 1 to the left and camera 2 and image 2 to the right.

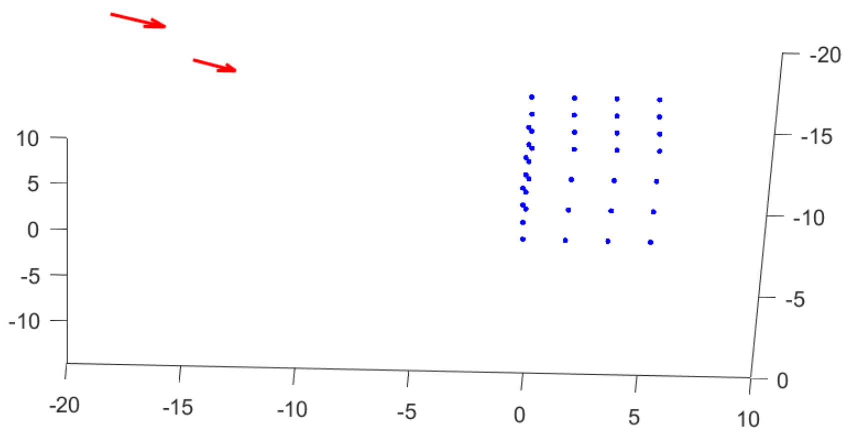

Figure 6: A plot of the model points together with the camera centers and viewing directions.

Inner parameters

$$K_1 = \begin{bmatrix} 2449 & -18 & 960 \\ 0 & 2447 & 676 \\ 0 & 0 & 1 \end{bmatrix} \quad K_2 = \begin{bmatrix} 2389 & -25 & 814 \\ 0 & 2401 & 791 \\ 0 & 0 & 1 \end{bmatrix}$$

From these camera matrices we find that

- $f_1 = 2447 \quad f_2 = 2401$
- $s_1 = -0.0074 \quad s_2 = -0.0104$
- $\gamma_1 = 1.0008 \quad \gamma_2 = 0.995$
- $x_0^1 = 960 \quad x_0^2 = 814$
- $y_0^1 = 676 \quad y_0^2 = 791$

Since these are calibrated cameras the parameters are the true ones.

Computer Exercise 5

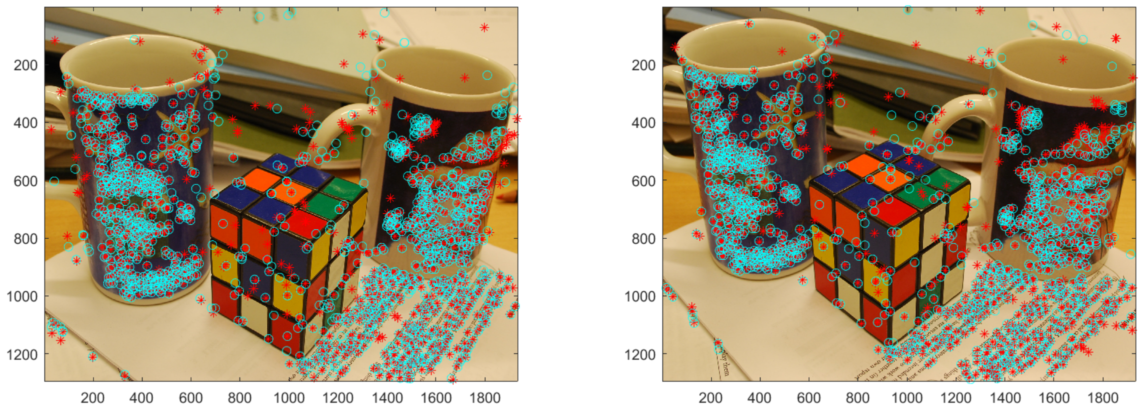

Figure 7: Projected points (red) calculated with DLT without normalization plotted together with the measured SIFT points (cyan).

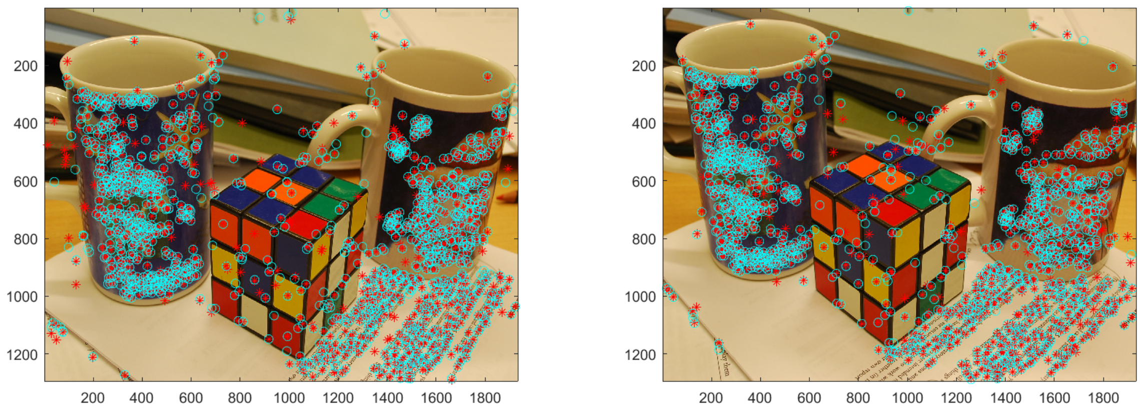

Figure 8: Projected points (red) calculated with DLT where the measured SIFT points (cyan) and cameras were normalized.

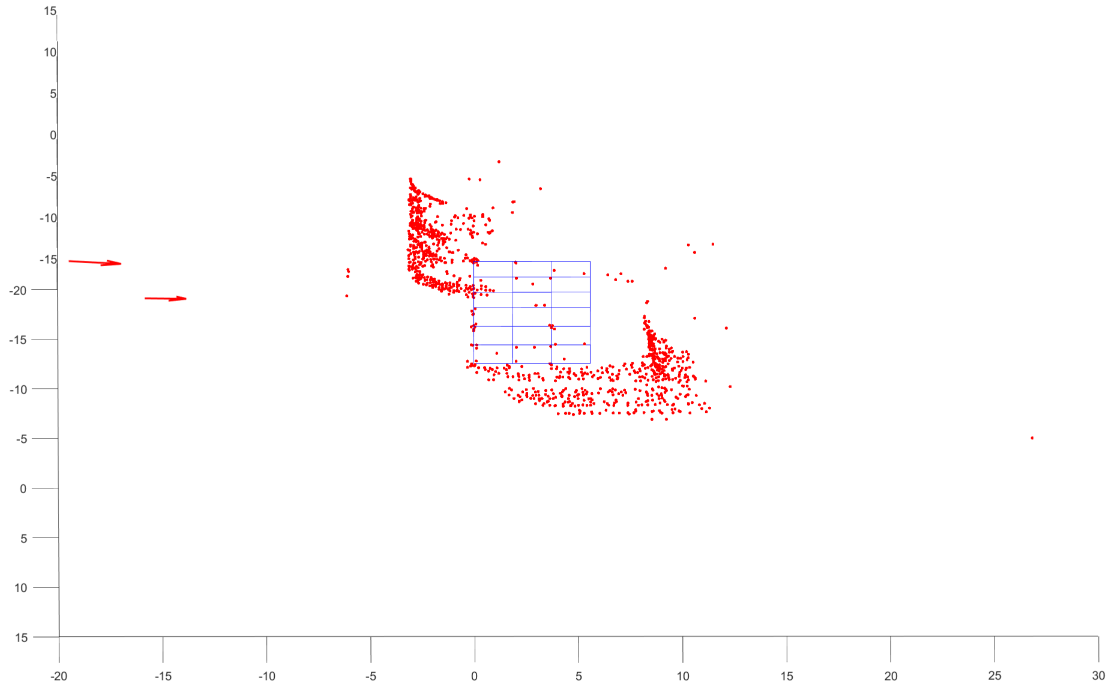

Figure 9: 3D plot of the points where the errors between the projected 3D points and the SIFT points are less than 3 pixels. The cameras and the cube model are also plotted.