Computer Vision
Assignment 1

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1 Calibrated vs. Uncalibrated Reconstruction

Exercise 1

Given the image projection

$$\lambda \mathbf{x} = P\mathbf{X}$$

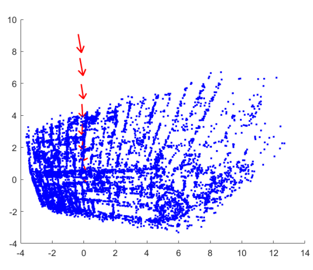
To create a new solution from the unknown transformation T, we let $\tilde{P} = PT$ and $\tilde{\mathbf{X}} = T^{-1}\mathbf{X}$, a new solution can be found to be

$$\lambda \mathbf{x} = PTT^{-1}\mathbf{X} = PT\tilde{\mathbf{X}}$$

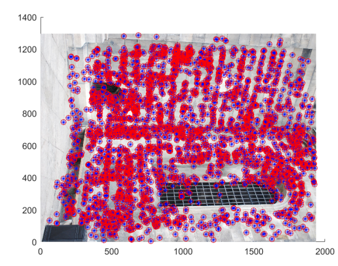
where PT is a valid camera matrix.

Compute Exercise 1

The 3D reconstruction and projection onto an image and is seen in figure 1. As can be seen in the left image does the projection appear to be distorted, i.e. the 3D reconstruction do not preserve parallel lines or angles. However, as can be seen in the right image in 1 the points look close to each other after the projection.



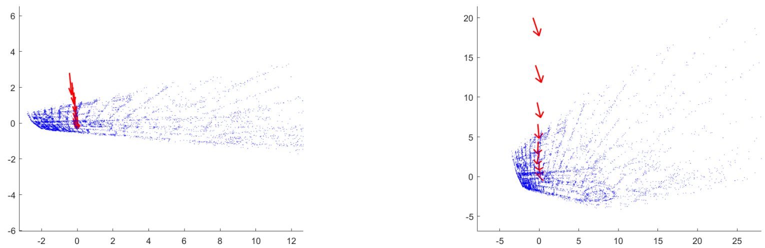
(a) Cameras and points in X



(b) Points from X projected on image 1 with camera 1

Figure 1: Plots of the points with original camera matrix and projection on image 1, the blue points are the original and the red the transformed points.

Transforming the points above with the transformations T_1 and T_2 can be seen in 2. Once again does the projections is distorted.



(a) Transformation with T_1 (b) Transformation with T_2

Figure 2: Points after transformations, note that these are zoomed in plots of all the points. The original have points much further away.

The projected points on the image is seen in 3. When projecting the points onto an image again they still look close to the original points, this is is an example of the the concept in exercise 1, i.e. the same projections as before.

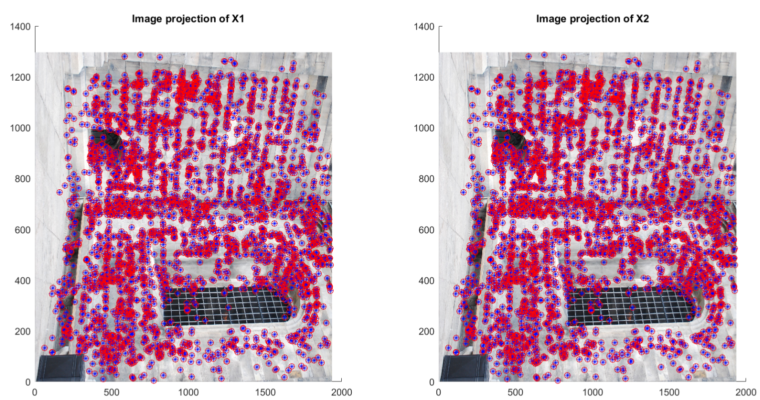


Figure 3: Image projections after transforming with T_1 and T_2

Exercise 2

If we have a calibrated camera, instead of the opposite as in the previous exercise, there are two requirement: preserve parallel lines and angle between points, also known as a similarity transform, thus we do not get the distortion.

The corresponding statement compared to that in exercise 1 is:

$$\lambda_{ij}\tilde{\mathrm{x}}_{ij} = [R_i \quad t_i]\mathbf{X}_j$$

where $\tilde{\mathrm{x}} = K^{-1}\mathrm{x}$ and R_i is a 3×3 rotation matrix

2 Camera Calibration

Exercise 3

Consider

$$K = \begin{pmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

We can verify that the inverse of K equals

$$K^{-1} = \begin{pmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{pmatrix}$$

by simply multiplying the two

$$K{\cdot}K^{-1} = \begin{pmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} f \cdot 1/f & 0 & -x_0/f \cdot f + x_0 \\ 0 & f \cdot 1/f & -y_0/f \cdot f + y_0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

The matrix K^{-1} can also be factorized into the parts (which can be verified by simply multiplying them).

$$K^{-1} = \underbrace{\begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{pmatrix}}_{B}$$

Where A can be seen as a rescaling of an image by a factor of $1/f$, e.g. a transformation from pixels to an length unit, and B move the points around the principle point.

The transformation when the inverse of K is applied can be interpreted as a point going from the image plane to the projective plane in camera coordinates, see figure 4 for a visual representation.

The principal point will after the transformation end up at (0, 0, 1) and a point with a distance of f to it will end up at distance one to the principal point.

The points are normalized by $K^{-1}p$, where the points have an additional one at the end.

1) $(0, 240) \rightarrow (-1, 0, 1)$

2) $(640, 240) \rightarrow (1, 0, 1)$

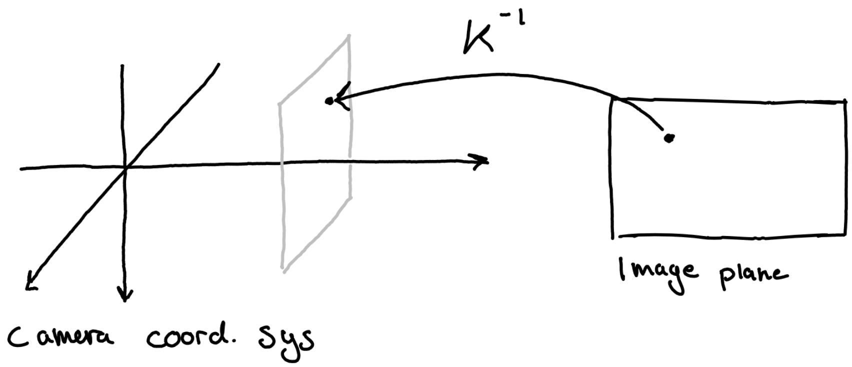


Figure 4: Transformation with K^{-1}

The dot product of the two points results in 0, thus they are separated by $\pi/2$ radians.

The camera center is found by calculating the nullspace of the camera matrices, i.e. $Pv = 0$. If the first matrix is multiplied with K^{-1} then the equation would be equal.

$$K^{-1}K[R \;\; t]v = [R \;\; t]v = 0$$

The last row of K will preserve the last row of the matrix that it multiplies with. In the camera matrices, the principal axis is the last row, so both of them will be unchanged.

Exercise 4

The K matrix can be created with the values given in the exercise description, the normalized points are then found by $K^{-1}p$, with an additional 1 in the point, similar as in the previous exercise.

1) $(0, 0) \rightarrow (-0.5, -0.5, 1)$
2) $(0, 1000) \rightarrow (-0.5, 0.5, 1)$
3) $(1000, 0) \rightarrow (0.5, -0.5, 1)$
4) $(1000, 1000) \rightarrow (0.5, 0.5, 1)$
5) $(500, 500) \rightarrow (0, 0, 1)$

3 RQ Factorization and Computation of K

Exercise 5

Consider the matrices

$$K = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \text{and} R = \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix}.$$

We can compute

$$KR = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix} = \begin{pmatrix} aR_1^T + bR_2^T + cR_3^T \\ dR_2^T + eR_3^T \\ fR_3^T \end{pmatrix},$$

where R is an orthogonal matrix, and $||R_n|| = 1$. If

$$P = \begin{pmatrix} \frac{800}{\sqrt{2}} & 0 & \frac{2400}{\sqrt{2}} & 4000 \\ -\frac{700}{\sqrt{2}} & 1400 & \frac{700}{\sqrt{2}} & 4900 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 3 \end{pmatrix} = [A \quad t_a], A = \begin{pmatrix} A_1^T \\ A_2^T \\ A_3^T \end{pmatrix}$$

what is R_3 and f. The matrix A is a 3×3 matrix and t_a is a 3×1 vector.

The unknowns can be found using the Gram Schmidt algorithm. The first step projects R_3 on A_3.

$$A_3 = fR_3 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \; ||A_3|| = 1, \iff f = 1, R_3 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

The next equation $A_2 = dR_2 + eR_3$ is a linear combination of R_2 and R_3. We also know that $R_2 \perp R_3$. Therefore e can be found by:

$$e = A_2^T R_3 = \begin{pmatrix} -\frac{700}{\sqrt{2}} & 1400 & \frac{700}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = 700.$$

With this we can rewrite the equation to

$$A_2 = dR_2 + eR_3 \iff dR_2 = A_2 - eR_3 = \begin{pmatrix} -\frac{700}{\sqrt{2}} \\ 1400 \\ \frac{700}{\sqrt{2}} \end{pmatrix} - 700 \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1400 \\ 0 \end{pmatrix},$$

because $||R_2|| = 1$ we know that $d = 1400$ and $R_2 = (0 \;\; 1 \;\; 0)^T$

Lastly, we utilise $A_1 = aR_1^T + bR_2^T + cR_3^T$ to find the last unknowns, b and c can be found using a similar method as before.

$$c = A_1^T R_3 = \begin{pmatrix} \frac{800}{\sqrt{2}} & 0 & \frac{2400}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = 800$$

$$b = A_1^T R_2 = \begin{pmatrix} \frac{800}{\sqrt{2}} & 0 & \frac{2400}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

Now, the last R can be calculated

$$aR_1 = A_1 - dR_2 - cR_3 = \begin{pmatrix} \frac{800}{\sqrt{2}} \\ 0 \\ \frac{2400}{\sqrt{2}} \end{pmatrix} - 0 - 800 \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = 1600 \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix},$$

i.e. $a = 1600$ and $R_1 = (\frac{1}{\sqrt{2}} \quad 0 \quad \frac{1}{\sqrt{2}})^T$.

We now have all the element in K and R:

$$K = \begin{pmatrix} 1600 & 0 & 800 \\ 0 & 1400 & 700 \\ 0 & 0 & 1 \end{pmatrix}, \quad R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

To find the focal length, skew, aspect ratio and principal point, we remember that K can be written as

$$K = \begin{pmatrix} \gamma f_l & s f_l & x_0 \\ 0 & f_l & y_0 \\ 0 & 0 & 1 \end{pmatrix},$$

where f_l is the focal length, γ is the aspect ratio, s is the skew and (x_0, y_0) is the principal point. Thus we have

$$\begin{cases} f_l & = 1400 \\ s & = 0 \\ \gamma & = 1600/1400 = 8/7 \\ (x_0, y_0) & = (800, 700) \end{cases}$$

Computer Exercise 2

Using the first camera, the resulting transformation after $T1$ and $T2$.

$$K1 = \begin{pmatrix} 2393.95 & 0 & 932.38 \\ 0 & 9592.47 & 628.26 \\ 0 & 0 & 1 \end{pmatrix}$$

and

$$K2 = \begin{pmatrix} 2393.95 & 0 & 932.38 \\ 0 & 2398.12 & 628.26 \\ 0 & 0 & 1 \end{pmatrix}.$$

4 Direct Linear Transformation DLT

Exercise 7

The formulae to compute P from $\tilde{P}$ would be $P = N^{-1}\tilde{P}$.

Computer Exercise 3

After normalization of the points in `x`, they are plotted in figure 5. A visual inspection suggests that the points are centered around the origin for both the plots, with a mean distance of 1.

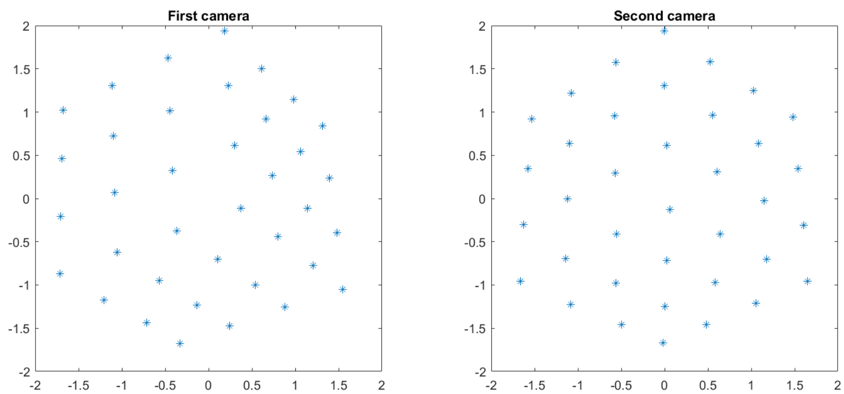


Figure 5: The normalized point plotted.

After both of the M metricise for both the cameras were found, the singular value decomposition of it is computed to find both the v and smallest eigenvalues. The smallest eigenvalues where 0.0151 respectively 0.0122, it were also found that $||Mv||$ equal the smallest eigenvalue.

The model points are projected into the images, see figure 6. The model points and the projected ones are very close to each other, however there is a small error.

In figure 7 the model cube, camera centers and viewing directions are present. The plot is, in my opinion, reasonable because the cameras both points to the cube but from slightly different centers.

$$K1 = \begin{pmatrix} 2448.60 & -18.09 & 959.80 \\ 0 & 2446.85 & 675.89 \\ 0 & 0 & 1 \end{pmatrix}$$

$$K2 = \begin{pmatrix} 2388.55 & -24.50 & 814.24 \\ 0 & 2400.93 & 790.48 \\ 0 & 0 & 1 \end{pmatrix}$$

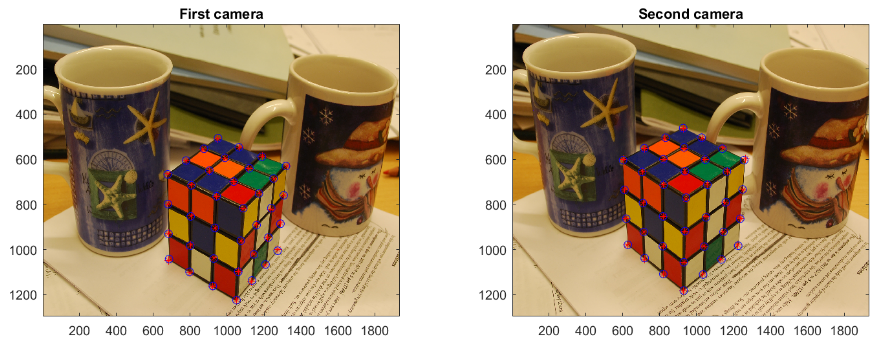


Figure 6: Model points on the images

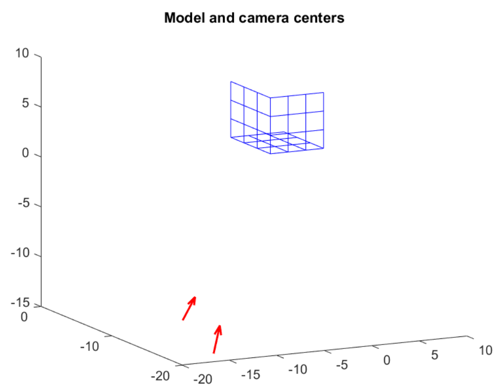


Figure 7: Camera centers and model

5 Triangulation using DLT

Computer Exercise 5

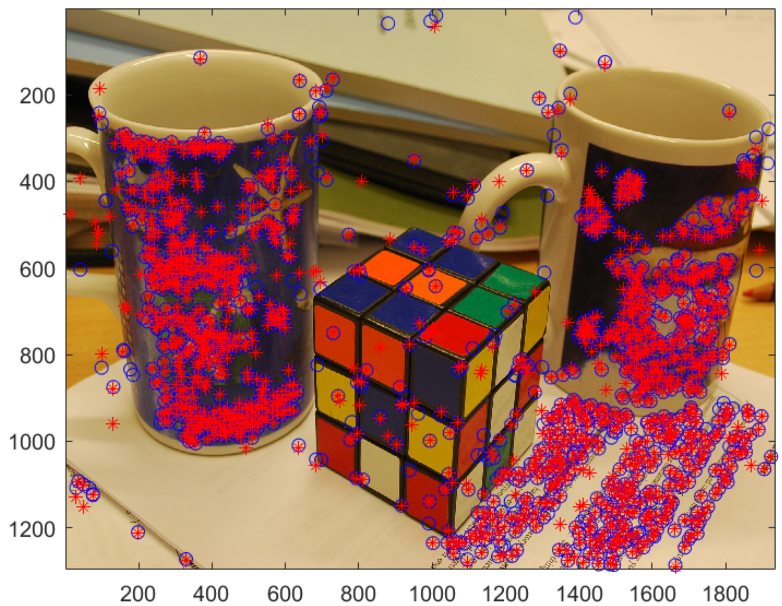


Figure 8: First camera, blue are model and red are projected

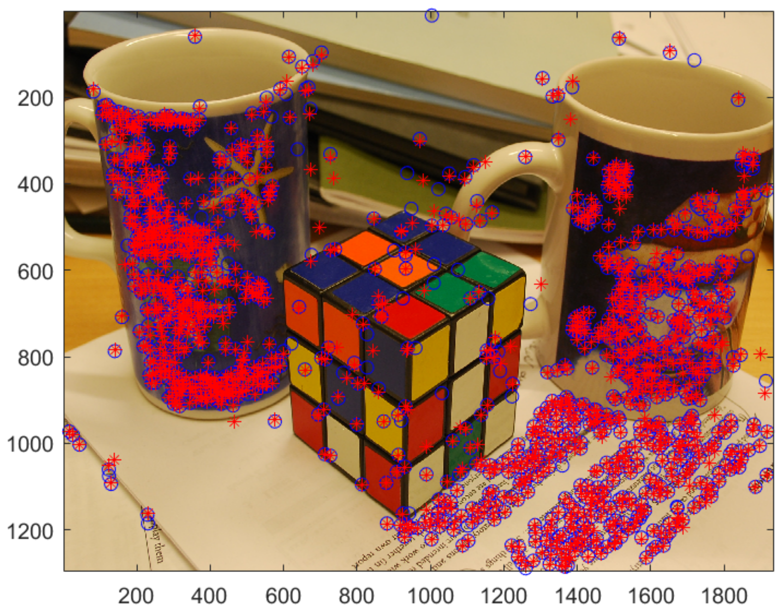


Figure 9: Second camera, blue are model and red are projected

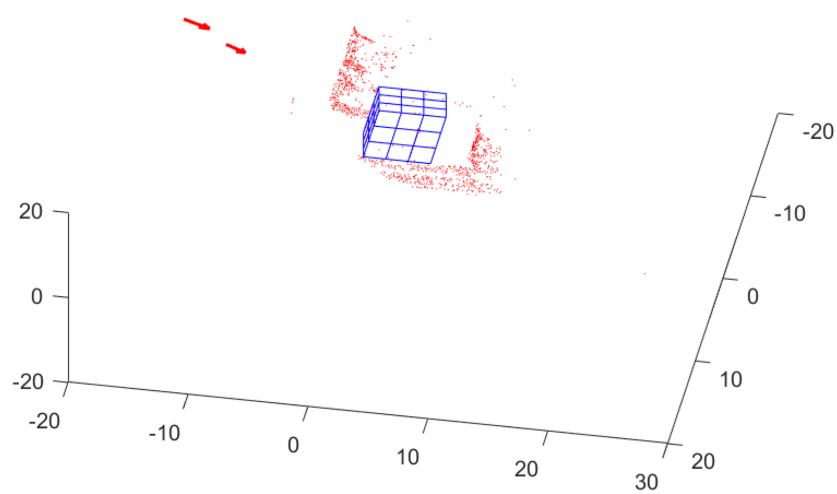


Figure 10: Point calibration, a contour of the cup can be seen to the left of the cube and the paper surface to the right of the cube