

Lecture 1: 16 lectures

4 assignments (mandatory)

6 Q&A-sessions (Optional)

→ { optional  
exercises gives  
extra points  
to the exam.

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↖ { if stuck on an exercise in the  
Assignment mail Carl

Assignments can be done in groups but  
it must be done individually.

Latex or by hand.

Assignment 1: warm up

2: lots of work } DON'T START  
3: — 4 — } TOO LATE!

4: moderate

The assignment must be finished before  
April 1!

# What is Computer vision?

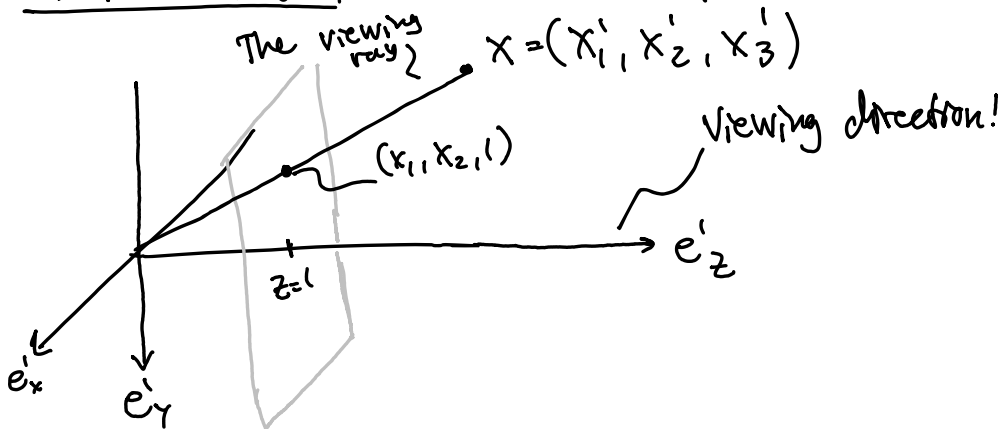
Computer graphics, comp. vision

3D  $\rightarrow$  2D

2D  $\rightarrow$  3D.

Main goal: multiview reconstruction.

<sup>(camera)</sup>  
Pinhole model, mathematical model:



The viewing ray:

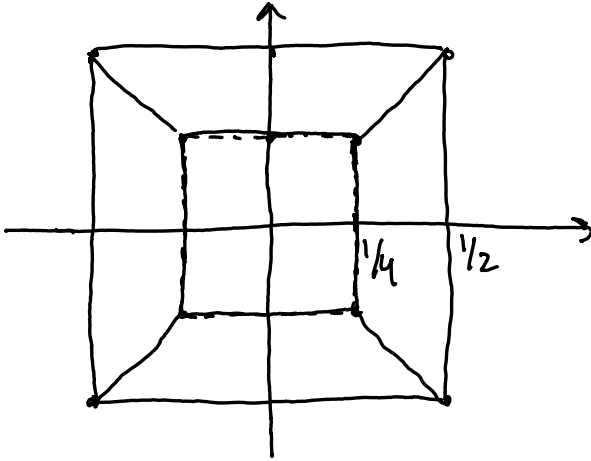
$$(0, 0, 0) + S((x'_1, x'_2, x'_3) - (0, 0, 0))$$

$$Sx'_3 = 1 \quad \Rightarrow \quad S = 1/x'_3$$

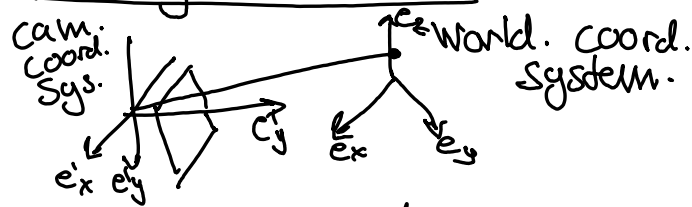
$$(x_1, x_2) = \left( x'_1/x'_3, x'_2/x'_3 \right)$$

Ex 1:

projection of  $(\pm 1, \pm 1, 2)$  is  $(\frac{\pm 1}{2}, \frac{\pm 1}{2})$   
 ——— " ———  $(\pm 1, \pm 1, 4)$  is  $(\frac{\pm 1}{4}, \frac{\pm 1}{4})$



Moving cameras:

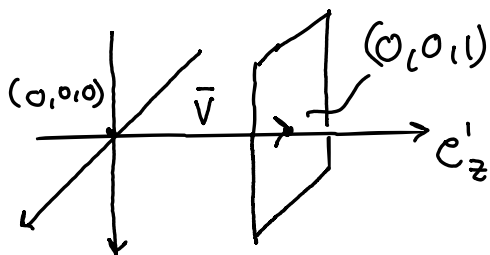


Relationship between Coord. Systems

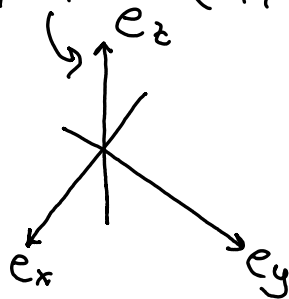
$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = R \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + t$$

$\uparrow$  rotation matrix  $3 \times 3$        $\uparrow$  vector  $3 \times 1$

Ex 2:



$$X = (x_1, x_2, x_3) = (x'_1, x'_2, x'_3)$$



$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = R \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + t$$

Computer Counter

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = RC + t \Leftrightarrow 0 = RC + t \Leftrightarrow RC = -t \Leftrightarrow C = -R^{-1}t = \underline{\underline{-R^T t}}$$

$(0,0,1)$ :

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = RX + t \Leftrightarrow X = R^T \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{\text{last row of } R = R_3} - \underbrace{R^T t}_{=C}$$

$$X = R_3 + C$$

viewing direction:  $X - C = R_3 + C - C = \underline{\underline{R_3}}$

## Camera matrice:

comp the proj. (from world coord to img. plane)

$$x'_3 \begin{pmatrix} x'_1 / x'_3 \\ x'_2 / x'_3 \\ 1 \end{pmatrix} = \begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \underset{3 \times 4}{[R \quad t]} \underset{4 \times 1}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{pmatrix}}$$

Algo:

1. comp.  $v = [R \quad t] \begin{bmatrix} x \\ 1 \end{bmatrix}$
2. Divide  $v$  with its third coordinate

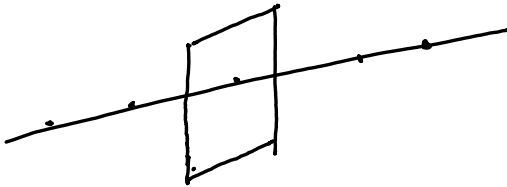
Ex 3: compute the projection of  $X = (0, 0, -1)$  in the camera

$$P = \frac{1}{3} \begin{pmatrix} 1 & 2 & 2 & 0 \\ 2 & 1 & -2 & 0 \\ -2 & 2 & 1 & 1 \end{pmatrix}$$

What other points gives the same projection?

$$\frac{1}{3} \begin{pmatrix} 1 & 2 & 2 & 0 \\ 2 & 1 & -2 & 0 \\ -2 & 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix} = v$$

Projection is  $(-1, 1)$



$$v = \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = RX + t \Leftrightarrow RX = \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} - t$$

$$X = R^T \left( \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} - t \right)$$

$$X = \frac{1}{3} \begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \left( \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{3} \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix} \right) = \lambda \underbrace{\frac{1}{3} \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}}_{\text{dir vector}} + \underbrace{\frac{1}{9} \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}}_{\text{starting point}}$$

if  $\lambda = 2/3$  we get  $\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

Depth of a Point: = z-coord. in cam.  
Coord. System.

A point is in front of the camera  
if  $\text{Depth} > 0$

A point is behind of the camera  
if  $\text{Depth} < 0$

