Computer Vision
Assignment 1

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1 Points in Homogeneous Coordinates

Exercise 1

The 2D Cartesian coordinates of the points x_1, x_2 and x_3 are:

$$x_1 \sim \begin{pmatrix} 4/2 \\ -2/2 \\ 2/2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \implies (2, -1)$$

$$x_2 \sim \begin{pmatrix} 3/-1 \\ -2/-1 \\ -1/-1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \implies (3, 2)$$

$$x_3 \sim \begin{pmatrix} 4\lambda/2\lambda \\ -2\lambda/2\lambda \\ 2\lambda/2\lambda \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \implies (2, -1)$$

The interpretation of x_4 is a point infinitely far away in the $(4, -2)$–direction, this is called vanishing points.

Computer Exercise 1

The matlab function `pflat` and plots are provided below.

```
function h_points = pflat(points)
    h_points = points(:, :) ./ points(end, :);
end
```

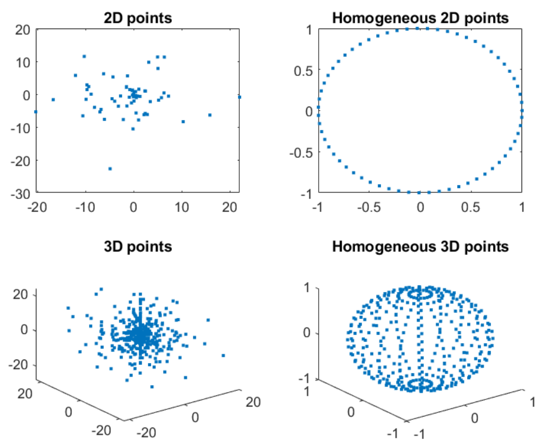

Figure 1: Homogeneous plots

2 Lines

Exercise 2

Let us say that the intersection between the lines

$$l_1 = \begin{pmatrix}1\\1\\1\end{pmatrix} \text{ and } l_2 = \begin{pmatrix}3\\2\\1\end{pmatrix}$$

is the point $\mathrm{x} \sim (x, y, z)$. To find the point we solve the following system.

$$\begin{cases} l_1\mathrm{x} = 0 \\ l_2\mathrm{x} = 0 \end{cases} \iff \begin{cases} x + y + z & = 0 \\ 3x + 2y + z & = 0 \end{cases} \iff \begin{cases} x = t \\ y = -2t \quad , t \neq 0 \\ z = t \end{cases} \tag{1}$$

By dividing the result with the last coordinate, the corresponding homogeneous coordinates (i.e. in $\mathbb{P}^2$) is found.

$$\mathrm{x} \sim \begin{pmatrix} t/t \\ -2t/t \\ t/t \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

The corresponding point in $\mathbb{R}^2$ is then simply $(1, -2)$.

Now, let us take a look at the second pair of lines and their intersection point.

$$l_3 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ and } l_4 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

The same procedure, as above, is done once again

$$\begin{cases} l_3\mathrm{x} = 0 \\ l_4\mathrm{x} = 0 \end{cases} \iff \begin{cases} x + 2y + 3z & = 0 \\ x + 2y + z & = 0 \end{cases} \iff \begin{cases} x = -2t \\ y = t \\ z = 0 \end{cases}, t \neq 0.$$

The intersection in $\mathbb{R}^2$ can be interpreted as a point infinitely far away in the direction $(-2, 1)$.

Lastly, the line between the two point x_1 and x_2. The line, $l = (a, b, c)$, can be found by the following equation

$$l^T [x_1 \quad x_2] = 0$$

We can do this by turning both the point into homogeneous points, this is done by adding a one to the end. The line is then found to be

$$l = (t, -2t, t)$$

Which is he same exact line as in the first part of this exercise.

Exercise 3

The nullspace of a matrix is defined as the set of vectors where

$$N(A) = \{\mathrm{x} \in \mathbb{R}^n;\ A\mathrm{x} = 0\},\ \mathrm{x} \sim \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

In this exercise the null space of the matrix below is examined.

$$M = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Note that this matrix can be expressed using the lines in *Exercise 2* by

$$M = \begin{pmatrix} l_2^T \\ l_1^T \end{pmatrix}$$

Thus the equation to solve to find the nullspace is

$$0 = M\mathrm{x} = \begin{pmatrix} l_2^T \\ l_1^T \end{pmatrix} \mathrm{x} \iff \begin{cases} l_1^T \mathrm{x} = 0 \\ l_2^T \mathrm{x} = 0 \end{cases}$$

This equation is already solved in the previous exercise (however with different rotations of l and x), i.e. the intersection of the two lines. Thus the nullspace vector is

$$\mathrm{x} = \begin{pmatrix} t \\ -2t \\ t \end{pmatrix}.$$

If we consider this vector as in $\mathbb{P}^2$ (i.e. $\mathrm{x} \sim (1, -2, 1)$) then there would be no other solution other than the intersection point. If there would be another point than it would mean that equation (1) should have yielded anther solution, which it does not do.

Computer Exercise 2

The distance between the first line and the intersection of the other two were found to be around 8.195. The plot is found in figure 2.

As can be seen in the figure (and by the fact that the distance is not zero) the lines do not intersect at the same point, this either means that the lines are not parallel in 3D or that the image have been altered or warped in some way. Lines that are parallel in 3D should intersect in 2D.

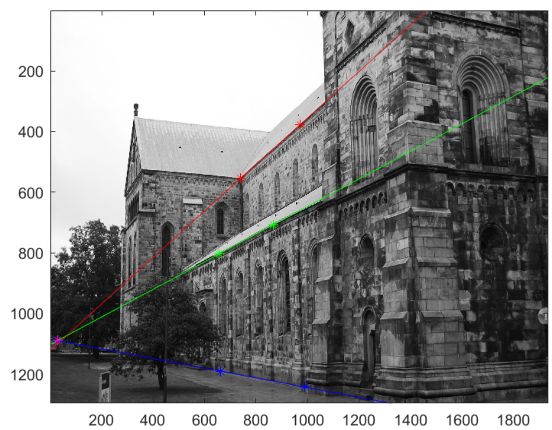

Figure 2: Points and lines for Computer Exercise 2

3 Projective Transformations

Exercise 4

Start by calculating $\mathrm{y}_1 \sim H\mathrm{x}_1 = (1, 0, 0)^T$ and $\mathrm{y}_2 \sim H\mathrm{x}_2 = (1, 1, 1)^T$. The technique of finding the line between two points have already been found previously in this assignment, we reuse these methods to find the line between x_1, x_2 and y_1, y_2.

$$l_1 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \text{ and } l_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

Then $(H^{-1})^T l_1 = (0, -1, 1)^T$ is found to be same as l_2.

The proof that projective lines preserves lines are as follows. Assume x lies on the line l, i.e. $I^T\mathrm{x} = 0$, and that $\mathrm{y} \sim H\mathrm{x}$, it follows that y lies on the line $\hat{l} = (H^{-1})^T I$ because

$$0 = l^T\mathrm{x} = \underbrace{l^T H^{-1}}_{\text{target line}} \underbrace{H\mathrm{x}}_{\mathrm{y}} \sim (H^{-T}l)^T\mathrm{y} = \hat{l}^T\mathrm{y}.$$

Computer Exercise 3

Using the matrices the transformations in figure 3 are created, where both the original grid and the warped are plotted. The properties of the different projections are provided in table 1.

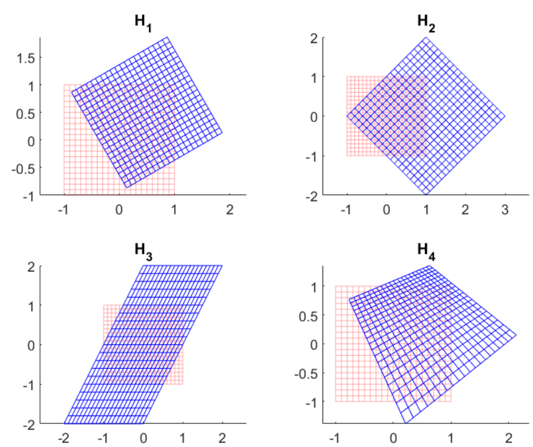

Figure 3: The transformation (blue) and original grid (red)

4 The Pinhole Camera

Exercise 5

The projection of the points $\mathbf{X}_1, \mathbf{X}_2$ and $\mathbf{X}_3$ in the camera matrix P can be solved by simply computing $P\mathbf{X}_n$ and then dividing with the last coordinate to

Table 1: Properties of the different projective mappings

	H_1	H_2	H_3	H_4
Preserves lengths	x			
Preserves angles	x	x		
Parallel to parallel	x	x	x	
Euclidian	x			
Similarity	x	x		
Affine	x	x	x	
Projective	x	x	x	x

find the projection on the image plane. Note that in this case the K matrix is equal the identity matrix, which means that it does not need to be factored out.

The three multiplications

$$P\mathbf{X}_1 = \begin{pmatrix}1\\2\\4\end{pmatrix} \sim \begin{pmatrix}1/4\\1/2\end{pmatrix},\ P\mathbf{X}_2 = \begin{pmatrix}1\\1\\2\end{pmatrix} \sim \begin{pmatrix}1/2\\1/2\end{pmatrix},\ P\mathbf{X}_3 = \begin{pmatrix}1\\1\\0\end{pmatrix}$$

The geometric interpretation of the prjection of $\mathbf{X}_3$ is a point infinitely far away in the direction $(1, 1)$.

The method of finding the camera's center position is based on calculating the nullspace of the corresponding camera matrix. The method in this report utilises that P can be written as $[R \ \ t]$ were R is a 3×3 rotational matrix and t is a 3×1 vector.

$$C = -R^T t = \begin{pmatrix}0\\0\\-1\end{pmatrix}$$

The viewing direction is the third row of the matrix R.

$$R_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

Computer Exercise 4

The camera centers of the two cameras were

$$C_1 = \begin{pmatrix}0\\0\\0\end{pmatrix} \text{ and } C_2 = \begin{pmatrix}6.6352\\14.8460\\-15.0691\end{pmatrix}$$

and the principle axis were

$$\text{PA}_1 = \begin{pmatrix} 0.3129 \\ 0.9461 \\ 0.0837 \end{pmatrix} \text{ and } \text{PA}_2 = \begin{pmatrix} 0.0319 \\ 0.3402 \\ 0.9398 \end{pmatrix}.$$

The axis were normalized using the second norm.

The 3D image with the camera center and principal axis is seen in figure 4.

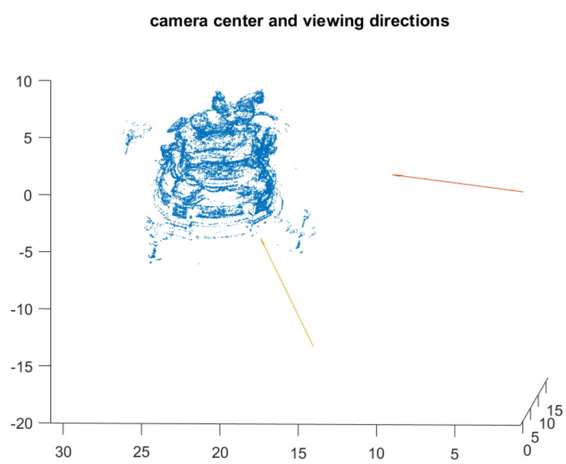

Figure 4: 3D plot

The projection plot is presented in figure 5. The projection seems to be reasonable, the points fall withing the objects (statue, lamppost etc.) in the images.

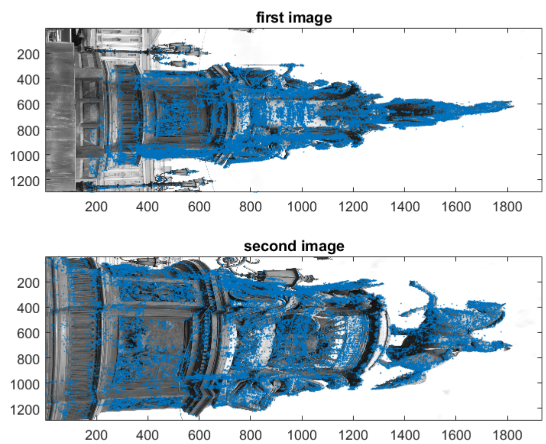

Figure 5: Projection of matrix `U`