Ex. 3 When solving for the intersection point we look for a point that satisfies the equation of both lines.

A line in $\mathbb{P}^2$ has the equation $ax + by + cz = 0$

$$\Longrightarrow \begin{cases} l_1: 3x + 2y + z = 0 \\ l_2: x + y + z = 0 \end{cases} \Longleftrightarrow \begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

Therefore the solution $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is the null space to the matrix. The intersection is then in the null space.

Null space: $t \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$. All the vectors in the null space represent the same point in $\mathbb{P}^2$ since they are equal up to scale. So there is only one point (the intersection point).