Ex. 3 $\quad K = \begin{pmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix} \quad K^{-1} = \begin{pmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{pmatrix}$

$$KK^{-1} = \begin{pmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -x_0 + x_0 \\ 0 & 1 & -y_0 + y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$$A = \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{pmatrix}$$

$$= K^{-1}$$

$$K = \begin{pmatrix} 320 & 0 & 320 \\ 0 & 320 & 240 \\ 0 & 0 & 1 \end{pmatrix} \qquad K^{-1} = \begin{pmatrix} 1/320 & 0 & -1 \\ 0 & 1/320 & -3/4 \\ 0 & 0 & 1 \end{pmatrix}$$

Normalization

$$K^{-1} \begin{pmatrix} 0 \\ 240 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3/4 - 3/4 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$K^{-1} \begin{pmatrix} 640 \\ 240 \\ 1 \end{pmatrix} = \begin{pmatrix} 2-1 \\ 3/4 - 3/4 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

the angle between

Angle

$$(-1,0,1) \cdot (1,0,1) = 0 = \|(-1,0,1)\| \, \|(1,0,1)\| \cos\alpha$$

$$\cos\alpha = 0 \iff \alpha = \pi/2 + n \cdot \pi \quad , \; n = 0, \pm 1, \pm 2, \ldots$$

$\Rightarrow$ The viewing rays are orthogonal

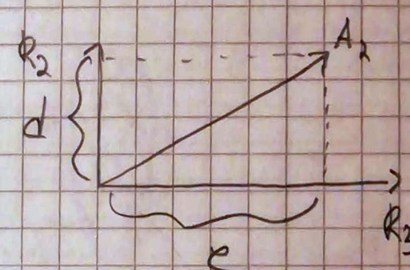
Ex. 5 $\quad K = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \qquad R = \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix}$

$$KR = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix} = \begin{pmatrix} aR_1^T + bR_2^T + cR_3^T \\ dR_2^T + eR_3^T \\ fR_3^T \end{pmatrix}$$

$$P = \left(\underbrace{\begin{matrix} 800/\sqrt{2} & 0 & 2400/\sqrt{2} \\ -700/\sqrt{2} & 1400 & 700/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{matrix}}_{A} \quad \begin{matrix} 4000 \\ 4900 \\ 3 \end{matrix} \right)$$

① $\quad fR_3 = A_3 \qquad f = \|A_3\| \qquad R_3 = \frac{1}{f} A_3$

$$R_3 = \begin{pmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} \qquad f = 1 \qquad \|R_3\| = 1$$

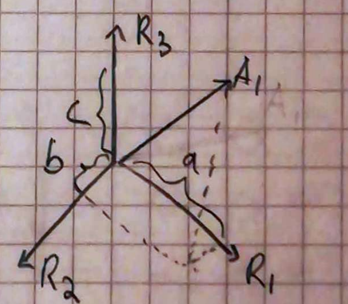


② $\quad e = A_2^T \cdot R_3 = 700\left(-1/\sqrt{2} \;\; 2 \;\; 1/\sqrt{2}\right)\begin{pmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

$$= 700\left(\frac{1}{2} + \frac{1}{2}\right) = 700$$

$$dR_2 = A_2 - eR_3 = 700\begin{pmatrix} -1/\sqrt{2} \\ 2 \\ 1/\sqrt{2} \end{pmatrix} - 700\begin{pmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1400 \\ 0 \end{pmatrix}$$

$$d = 1400 \qquad R_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$



③ $\quad A_1^T = aR_1^T + bR_2^T + cR_3^T$

$$c = A_1^T \cdot R_3 = 800\left(1/\sqrt{2} \;\; 0 \;\; 3/\sqrt{2}\right)\begin{pmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = 800\left(-1/2 + 3/2\right) = 800$$

$$b = A_1^T \cdot R_2 = 800\left(1/\sqrt{2} \;\; 0 \;\; 3/\sqrt{2}\right)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

$$aR_1 = A_1 - bR_2 - cR_3 = 800\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 3/\sqrt{2} \end{pmatrix} - 800\begin{pmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = 800\begin{pmatrix} \sqrt{2} \\ 0 \\ \sqrt{2} \end{pmatrix}$$

$$a = 800 \cdot 2 = 1600 \qquad R_1 = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

RQ factorisation of A is

$$A = \underbrace{\begin{pmatrix} 1600 & 0 & 800 \\ 0 & 1400 & 700 \\ 0 & 0 & 1 \end{pmatrix}}_{K} \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}$$

- $f = 1400$
- Skew $= 0$
- aspect ratio $= \frac{1600}{1400} = \frac{8}{7}$
- Principal point : $(800, 700)$