Assignment 1

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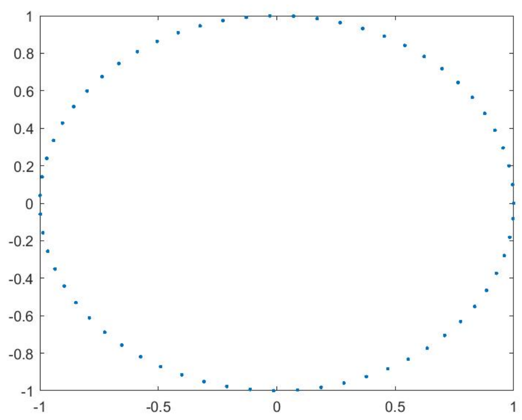
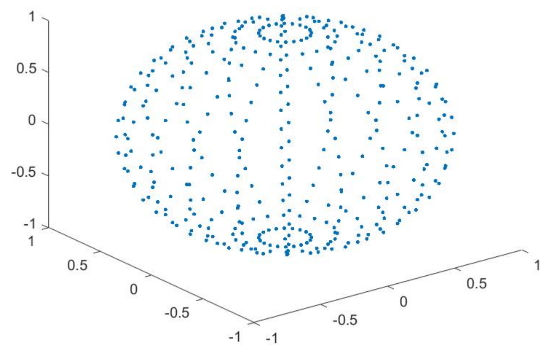
Exercise 1

- $x_1 = (2, -1)$
- $x_2 = (-3, 2)$
- $x_3 = (2, -1)$
- The interpretation of x_4 is a vanishing point infinitely far away in the $(4, -2)$ direction.

Computer Exercise 1



(a) pflat applied to $x2D$



(b) pflat applied to $x3D$

Ex 2. $l_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad l_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$$\begin{cases} l_1^T x = 0 \\ l_2^T x = 0 \end{cases} \Leftrightarrow \begin{cases} x + y + z = 0 \\ 3x + 2y + z = 0 \end{cases} \Leftrightarrow \begin{cases} x + y + z = 0 \\ 2x + y = 0 \\ z = t \end{cases}$$

$$\Leftrightarrow \begin{cases} x - 2x + z = 0 \\ y = -2x \\ z = t \end{cases} \Leftrightarrow \begin{cases} x = z \\ y = -2x \\ z = t \end{cases} \Leftrightarrow \begin{cases} x = t \\ y = -2t \\ z = t \end{cases}$$

Intersection $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

Corresponding point in $\mathbb{R}^2$ (by dividing with 3rd coord) is $(1, -2)$

$l_3 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\quad l_4 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$$\begin{cases} l_3^T x = 0 \\ l_4^T x = 0 \end{cases} \Leftrightarrow \begin{cases} x + 2y + 3z = 0 \\ x + 2y + z = 0 \end{cases} \Leftrightarrow \begin{cases} 2z = 0 \\ x + 2y + z = 0 \end{cases} \Leftrightarrow \begin{cases} x = -2t \\ y = t \\ z = 0 \end{cases}$$

Intersection $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

The Geometric interpretation is that l_3 and l_4 does not intersect in $\mathbb{R}^2$

$$X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad X_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

l_1 and l_2 can be seen as homogenous coordinates to x_1 and x_2 due to duality.

The line l will hence be the intersection of l_1 and l_3.

From earlier calculations: $l = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

Ex. 3 When solving for the intersection point we look for a point that satisfies the equation of both lines.

A line in $\mathbb{P}^2$ has the equation $ax + by + cz = 0$

$$\Longrightarrow \begin{cases} l_1: 3x + 2y + z = 0 \\ l_2: x + y + z = 0 \end{cases} \Leftrightarrow \begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

Therfore the soluton $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is the null space to the matrix. The intersection is then in the null space.

Null space : $t \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$. All the vectors in the null space represent the same point in $\mathbb{P}^2$ since they are equal up to scale. So there is only one point (the intersection point).

Computer Exercise 2

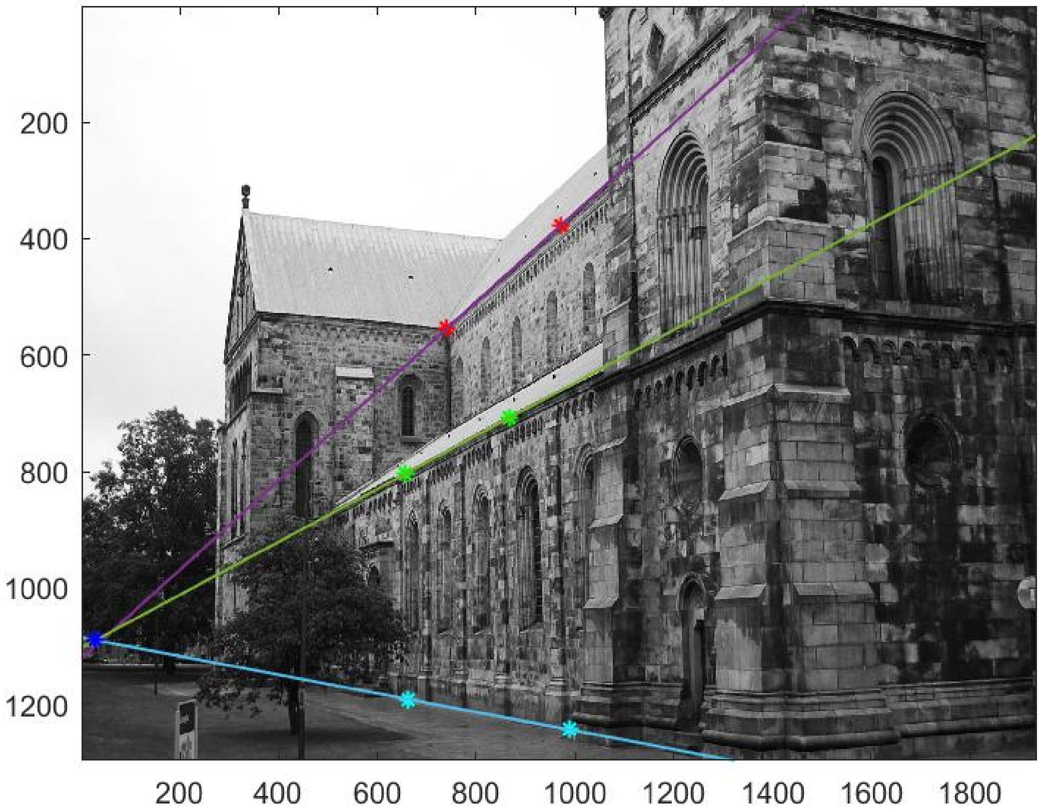


Figure 2: Plot of the points in $p1$ (red), $p2$ (green), $p3$ (cyan) together with the lines that goes through them. The intersect (blue) is also plotted.

- The lines look to be parallel in 3D and that's why they should intersect in the image.
- $d = 8.1950$ which is quite close given the scaling of the picture. All 3 lines almost intersect at the same point in this picture. This is because they are parallel in $\mathbb{R}^3$. There could be some noise which explains why they all do not intersect perfectly.

Exercise 4

- $y_1 = (1, 0, 0)^T$
- $y_2 = (1, 1, 1)^T$
- $l_1 = (-1, -1, 1)^T$
- $l_2 = (0, -1, 1)^T$
- $(H^{-1})^T = l_2$
- $y \sim Hx$

$0 = l_1^T x = l_1^T H^{-1} H x = l_1^T H^{-1} y = ((H^{-1})^T l_1)^T y = l_2^T y$

Computer Exercise 3

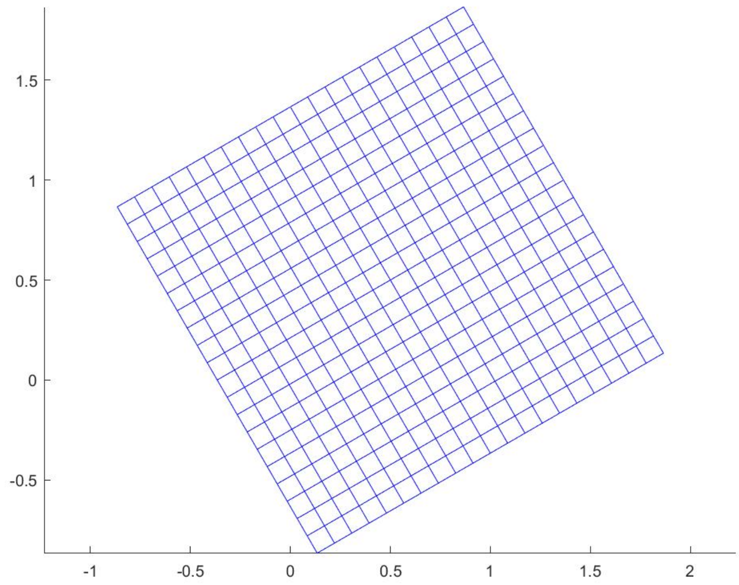


Figure 3: H_1 is an euclidean transformation since the original points have only been rotated. Both angles, parallel lines and distance between points are preserved. The digit in the right corner is not 1, but factorizing out a 2 solves the problem and shows that the matrix has the right form. Added *axis equal* when plotted in order to clearly see that angles are preserved.

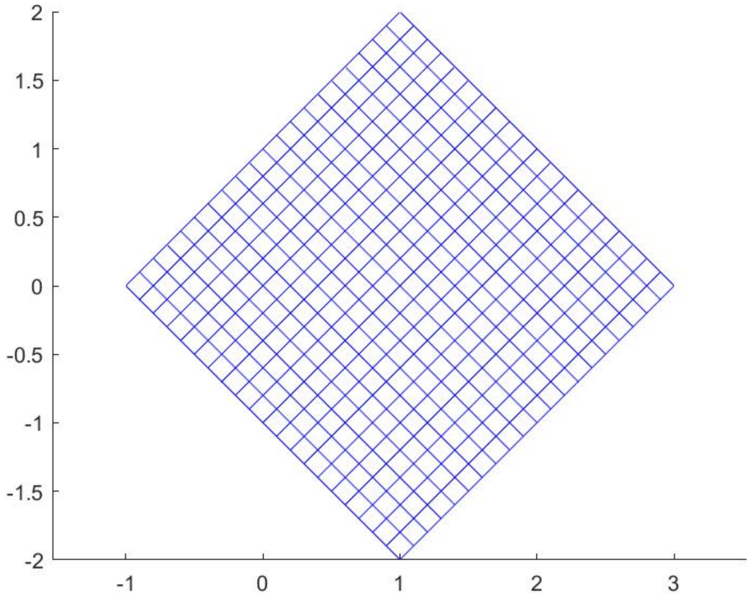


Figure 4: H_2 has both rotated and scaled the points. Angles between lines are preserved so this is an similarity transformation. Parallel lines are also preserved, but lengths are not. Added *axis equal* when plotted in order to get a better view of the conservation of angels.

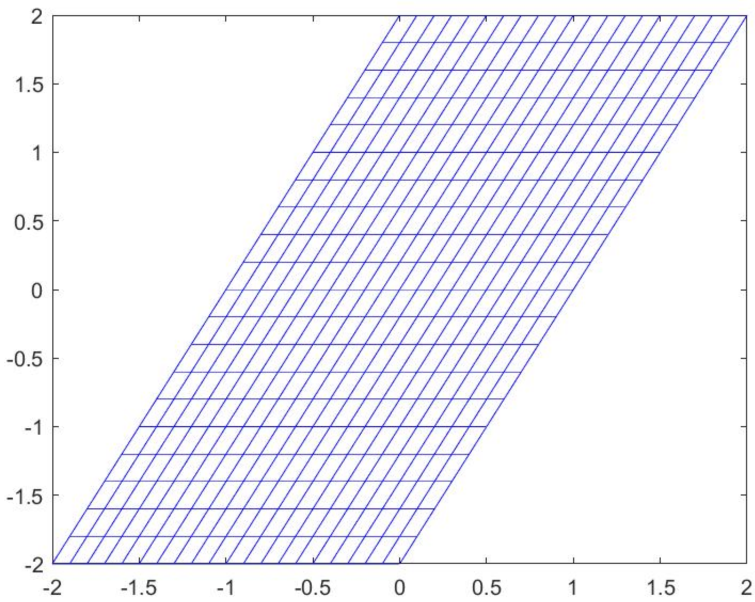


Figure 5: H_3 is an affine transform which map parallel lines to parallel lines. Angles are not preserved and lengths between points are not preserved.

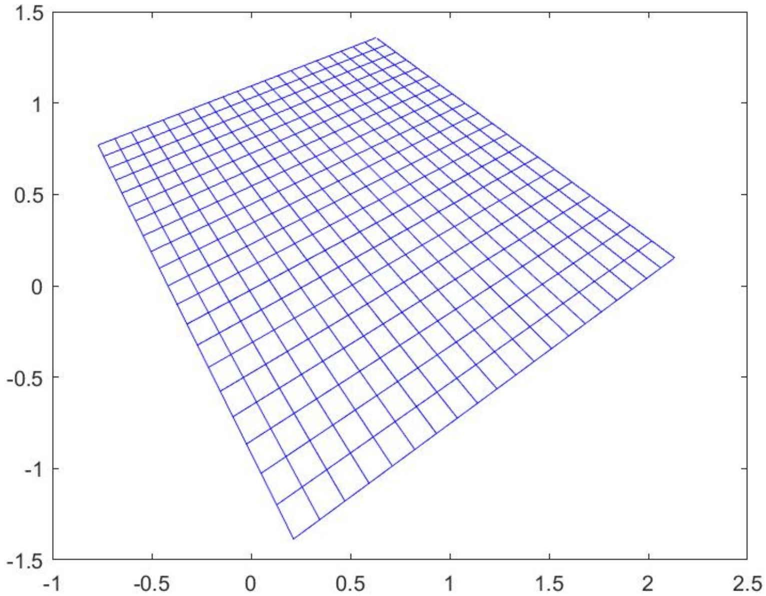


Figure 6: H_4 is a projective transformation. This can be seen upon inspecting the last row of the matrix which does not match the affine form. Parallel lines, angles and distance between points are not preserved.

Exercise 5

- Projection of $X_1 = (0.25, 0.5)$
- Projection of $X_2 = (0.5, 0.5)$
- The interpretation of the projection of X_3 is that it's a point at infinity (because it projects to $(1, 1, 0)$. Direction $(1, 1)$
- Camera center: $(0, 0, -1)$ (null space of P)
- Viewing direction: $(0, 0, 1)$ (element $1 - 3$ in the third row of P)

Computer Exercise 4

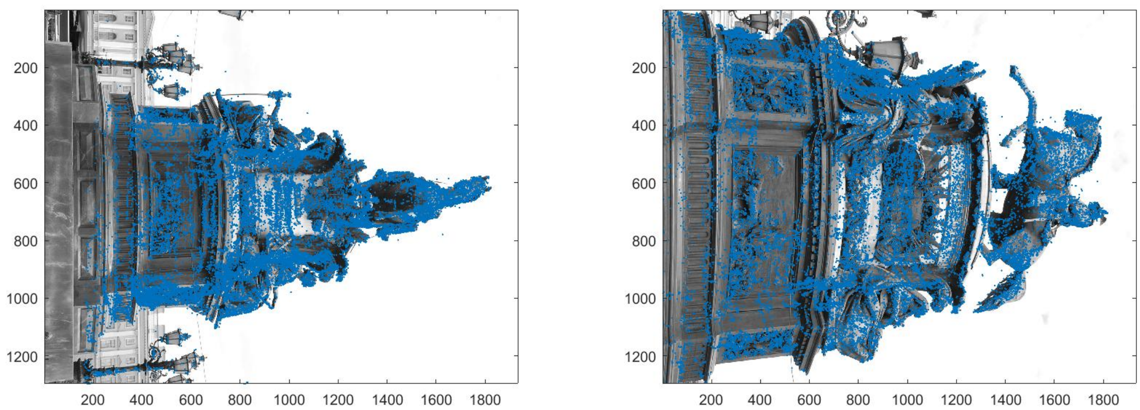


Figure 7: U projected in P1 and P2 looks very reasonable. The points seem to cover the right parts of the images.

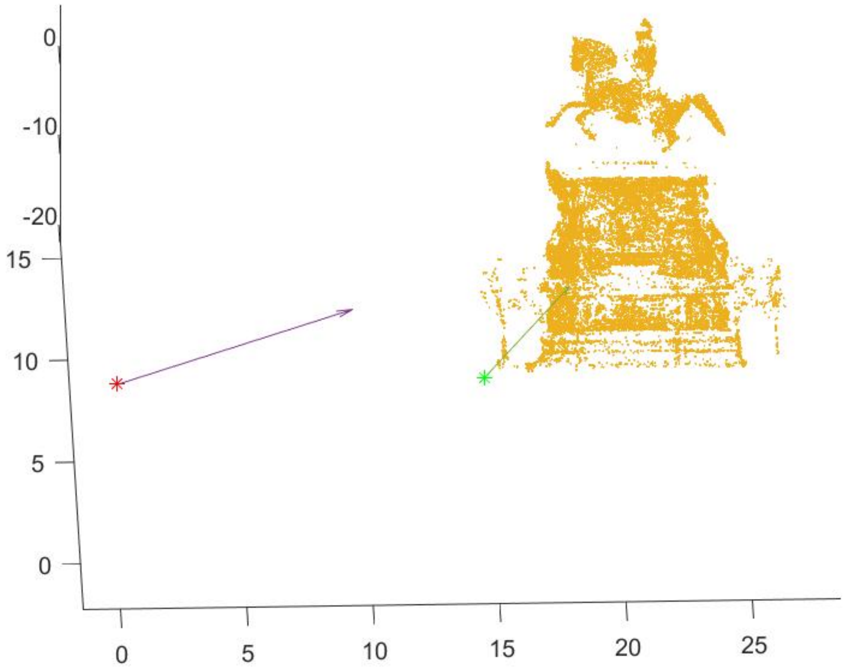


Figure 8: A plot of U, camera center of $P1$ (red) and camera center of $P2$ (green). The arrows from each center represent the viewing direction of the camera.

- Camera center for P1 = $(0, 0, 0)$
- Camera center for P2 = $(6.64, 14.85, -15.07)$
- Viewing direction of P1:$(0.313, 0.946, 0.084)$
- Viewing direction of P2: $(0.032, 0.340, 0.940)$