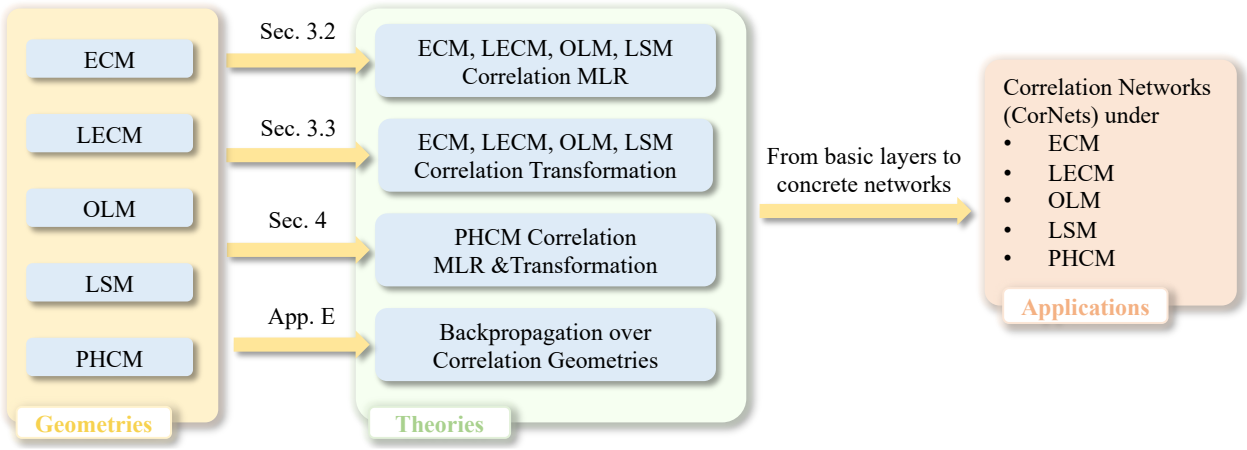

Figure A. Overview of our theoretical derivations. ECM, LECM, OLM, LSM, and PPC are the abbreviations of different correlation geometries, which will be introduced in Secs. 2 and 4.

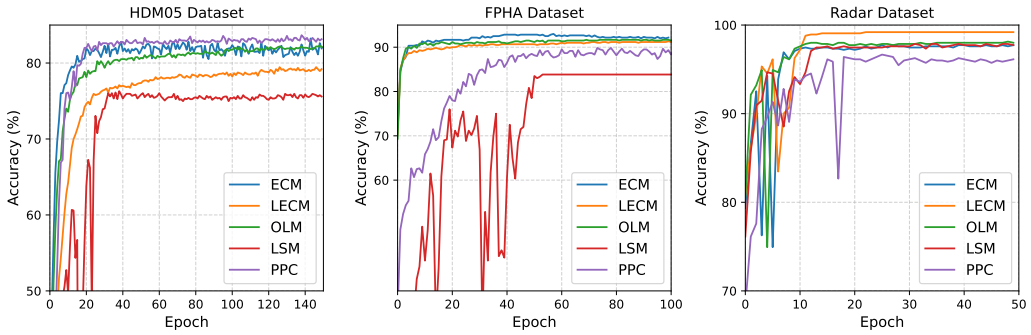

Figure B. Testing accuracy curves of CorNet under different metrics.

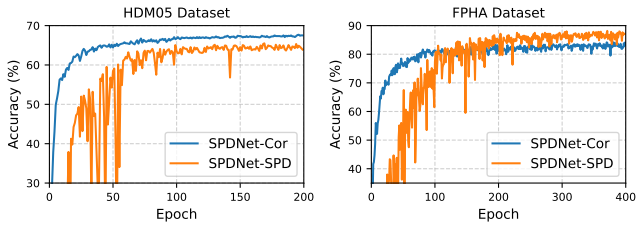

Figure C. @Reviewer kkhR: Teaser figure from Tab. 1: Comparison of SPDNet with SPD covariance or correlation inputs, denoted as SPDNet-SPD and SPDNet-Cor, respectively. SPDNet-Cor performs better on HDM05 yet worse on FPHA. This mixed performance arises because correlation matrices have distinct geometries from SPD matrices, whereas SPDNet is specifically designed for the SPD geometry. This mismatch motivates us to develop Riemannian networks based on correlation geometries to uncover the potential of correlation embeddings.

Table A. @Reviewer kkhR: Comparison of SPDNet with SPD or correlation input. The best result is highlighted in **red**.

Input	HDM05	FPHA	Radar
SPD	64.57 ± 0.61	**85.59 ± 0.72**	**93.25 ± 1.10**
Correlation	**66.81 ± 0.73**	83.37 ± 0.40	89.49 ± 0.67

Table B. @Reviewer kkhR: Comparison of SPDMLR-Trivlz (with similar trivialization) on raw covariances against CorMLR on raw correlations on all three datasets. The input matrix dimensions are 93×93, 63×63, and 20×20, respectively. The best result is highlighted in **red**.

Dataset	Measurement	SPDMLR-Trivlz			CorMLR				
		LEM	LCM	AIM	ECM	LECM	OLM	LSM	PPC
HDM05	Acc	54.31 ± 1.65	45.12 ± 1.05	52.46 ± 2.44	**65.57 ± 0.62**	64.44 ± 0.63	62.86 ± 0.65	64.01 ± 0.92	62.78 ± 0.85
	Fit Time (s/epoch)	3.24	5.38	260.67	3.18	3.87	3.39	3.57	**2.73**
FPHA	Acc	84.13 ± 1.14	76.62 ± 0.43	83.25 ± 0.59	**85.37 ± 0.16**	85.24 ± 0.22	84.67 ± 0.27	80.17 ± 0.15	73.67 ± 0.32
	Fit Time (s/epoch)	0.51	0.52	18.96	0.51	0.64	0.8	0.81	**0.45**
Radar	Acc	95.47 ± 0.66	**95.55 ± 0.35**	94.87 ± 0.87	89.47 ± 0.93	87.41 ± 0.23	85.79 ± 0.83	91.63 ± 0.32	83.33 ± 1.29
	Fit Time (s/epoch)	0.65	0.63	0.99	**0.56**	0.62	0.78	0.68	0.74

Table C. @Reviewer kkhR: Ablations on mixed geometries on the HDM05 and FPHA datasets. Each row represents the metric used for the Convolution (Conv) layer, and each column denotes the metric used for the MLR layer. The diagonal entries correspond to configurations where both layers use the same metric. The best result in each row is highlighted in **red**.

(a) Results on the HDM05 dataset

Conv \ MLR	ECM	LECM	OLM	LSM	PPC
ECM	**81.35 ± 1.27**	73.38 ± 0.34	80.11 ± 0.77	78.54 ± 0.43	80.80 ± 0.54
LECM	66.49 ± 1.13	78.05 ± 1.14	**79.21 ± 1.23**	73.61 ± 0.99	58.37 ± 2.24
OLM	77.82 ± 0.48	76.56 ± 0.89	**81.46 ± 0.61**	80.77 ± 0.81	77.39 ± 1.29
LSM	68.83 ± 1.19	70.41 ± 1.57	67.56 ± 1.52	**74.89 ± 1.07**	72.69 ± 3.56
PPC	81.16 ± 0.40	80.05 ± 0.45	81.96 ± 0.51	78.28 ± 0.64	**82.26 ± 0.92**

(b) Results on the FPHA dataset

Conv \ MLR	ECM	LECM	OLM	LSM	PPC
ECM	**92.17 ± 0.49**	91.50 ± 0.21	91.67 ± 0.28	87.37 ± 1.14	91.97 ± 0.24
LECM	87.90 ± 0.57	**91.17 ± 0.32**	90.25 ± 0.25	89.63 ± 0.31	86.09 ± 0.98
OLM	92.17 ± 0.58	**92.27 ± 0.78**	91.63 ± 0.12	89.90 ± 0.67	91.83 ± 0.15
LSM	78.97 ± 2.80	75.10 ± 1.15	82.25 ± 3.38	**83.43 ± 0.65**	78.97 ± 4.97
PPC	88.30 ± 0.81	79.80 ± 0.69	87.37 ± 0.72	86.63 ± 0.27	**90.03 ± 0.63**