Riemannian Local Mechanism for SPD Neural Networks

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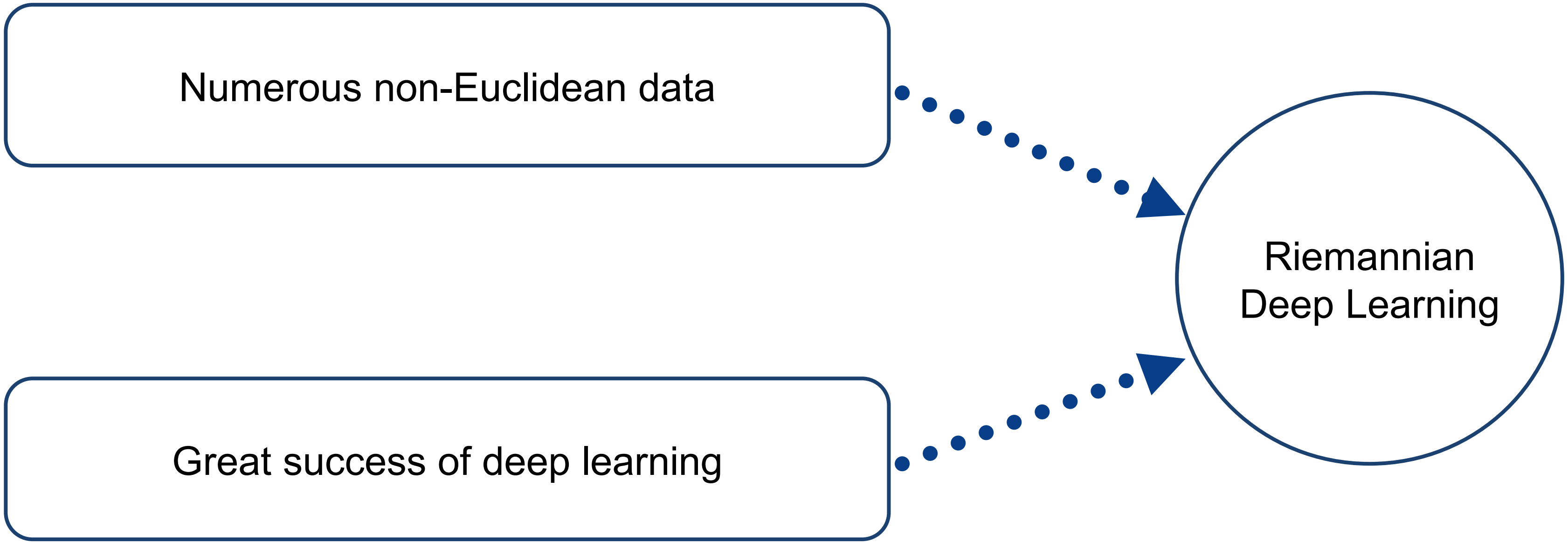
BACKGROUND



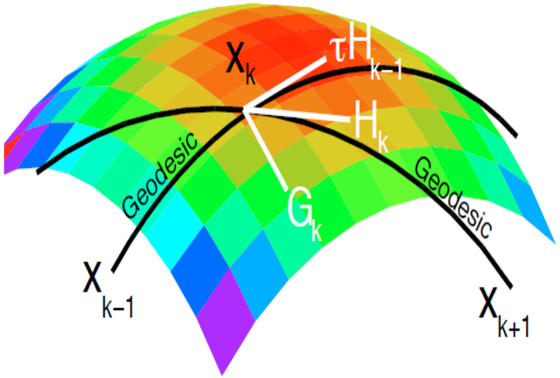
Numerous non-Euclidean data
Great success of deep learning
Riemannian Deep Learning

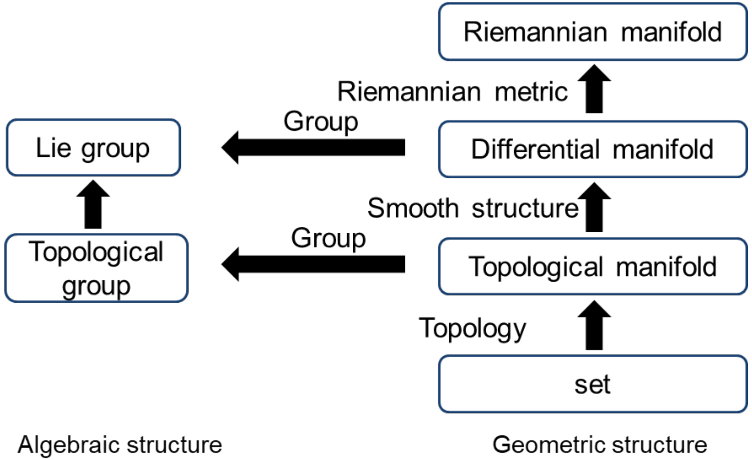
PRELIMINARIES

What is manifold

Mani·fold

Most importantly, locally Euclidean

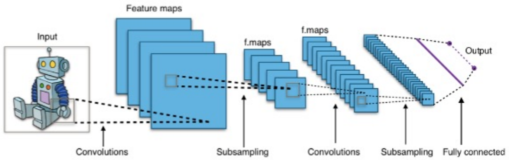




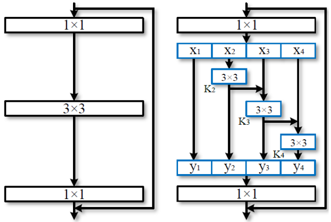
MOTIVATION & CONTRIBUTION

Motivation

- Local mechanisms in Euclidean paradigms



Typical CNNs



(a) Bottleneck block (b) Res2Net module

Res2Net

Local patterns in Riemannian manifolds?

- Missing local mechanisms in Riemannian algorithms;

Contribution

- Successfully identify the local patterns in manifolds
- Design the specific submanifold blocks for SPD neural networks

MAIN MATH TOOLS

- Category theory offers a universal view to consider diverse mathematical objects.

Definition 1. A category $\mathcal{C}$ consists of a collection of elements, called objects, denoted by $\mathrm{Obj}(\mathcal{C})$, and a set $\mathrm{Mor}(A, B)$ of elements, called morphisms from A to B, for any two objects $A, B \in \mathrm{Obj}(\mathcal{C})$. Morphisms should satisfy the below three axioms:

- composition: given any $f \in \mathrm{Mor}(A, B)$ and $g \in \mathrm{Mor}(B, C)$, the composition $h = g \circ f \in \mathrm{Mor}(A, C)$ is well-defined.
- identity: for each object A, there is an identity morphism $1_A \in \mathrm{Mor}(A, A)$ such that for any $f \in \mathrm{Mor}(A, B)$ and $g \in \mathrm{Mor}(B, A)$,

$$f \circ 1_A = f, 1_A \circ g = g; \tag{1}$$

- associative: for $f \in \mathrm{Mor}(A, B), g \in \mathrm{Mor}(B, C)$, and $h \in \mathrm{Mor}(C, D)$,

$$h \circ (g \circ f) = (h \circ g) \circ f. \tag{2}$$

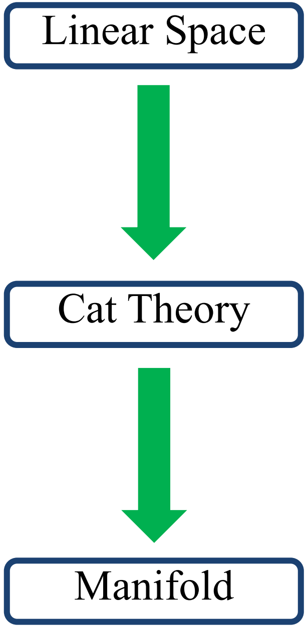
The set of all the morphisms in $\mathcal{C}$ is denoted as $\mathrm{Mor}(\mathcal{C})$.

- Regular submanifolds inherit multiple geometric properties from manifolds.

Definition 2. A subset $\mathcal{S}$ of a smooth manifold $\mathcal{N}$ of dimension n is a regular submanifold of dimension k if for every $p \in \mathcal{S}$ there is a coordinate neighbourhood $(U, \phi) = (U, x^1, \ldots, x^n)$ of p in the maximal atlas of $\mathcal{N}$ such that $U \cap \mathcal{S}$ is defined by the vanishing of $n - k$ of the coordinate functions.

PROPOSED METHOD

How to describe the local pattern theoretically? Take convolution as an example



Proposition 1. *For a given linear space V of dimension $d \times d$, n linear subspaces $V_1, V_2, \cdots, V_n$ of dimension $k \times k$ are selected and a linear function $f_i(\cdot) : V_i \rightarrow M_i$ is performed in each of them to extract local linear information. The resulting linear spaces $M_1, \cdots, M_n$ are combined into a final linear space M by direct sum, i.e.,$M = M_1 \oplus \cdots \oplus M_n$.*

Proposition 2. *In a category $\mathcal{C}$, for an object $A \in Obj(\mathcal{C})$, we extract n isomorphic sub-objects from A, denoted by $A_1, A_2, \ldots, A_n$. Then morphism $f_i \in \mathrm{Hom}\,(A_i, B_i)$ is applied to each sub-object to map it into a resulting object B_i. The resulting objects $B_1, \cdots, B_i$ are combined into a final object $B \in Obj(\mathcal{C})$ according to certain principles.*

Proposition 3. *For a manifold $\mathcal{M}$, we extract n isomorphic submanifolds $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_n$ and map each one of them by a smooth maps $f_i(\cdot) : \mathcal{M}_i \rightarrow \mathcal{M}'_i$. The resulting manifolds $\mathcal{M}'_i$ are aggregated into a final manifold $\mathcal{M}'$ according to certain principles.*

PROPOSED METHOD

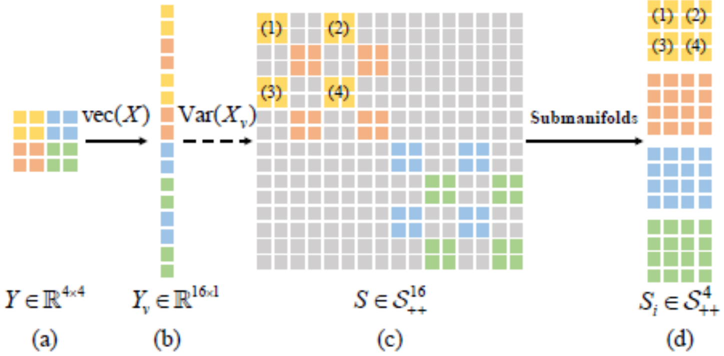
How about in SPD manifolds?

- Isomorphic principal submatrices

Theorem 1. *For an SPD manifold $\mathcal{S}^n_{++}$, the set of principal submatrices $(\mathcal{S}^n_{++})_{\mathcal{I}}$ is an SPD manifold and can be embedded into the $\mathcal{S}^n_{++}$ as a regular submanifold. In addition, for any proper indices $\mathcal{I}$ and $\mathcal{J}$ satisfying $|\mathcal{I}| = |\mathcal{J}|$, $(\mathcal{S}^n_{++})_{\mathcal{I}}$ is isomorphic to $(\mathcal{S}^n_{++})_{\mathcal{J}}$.*

The number of submatrices is combinatorial
How to select?

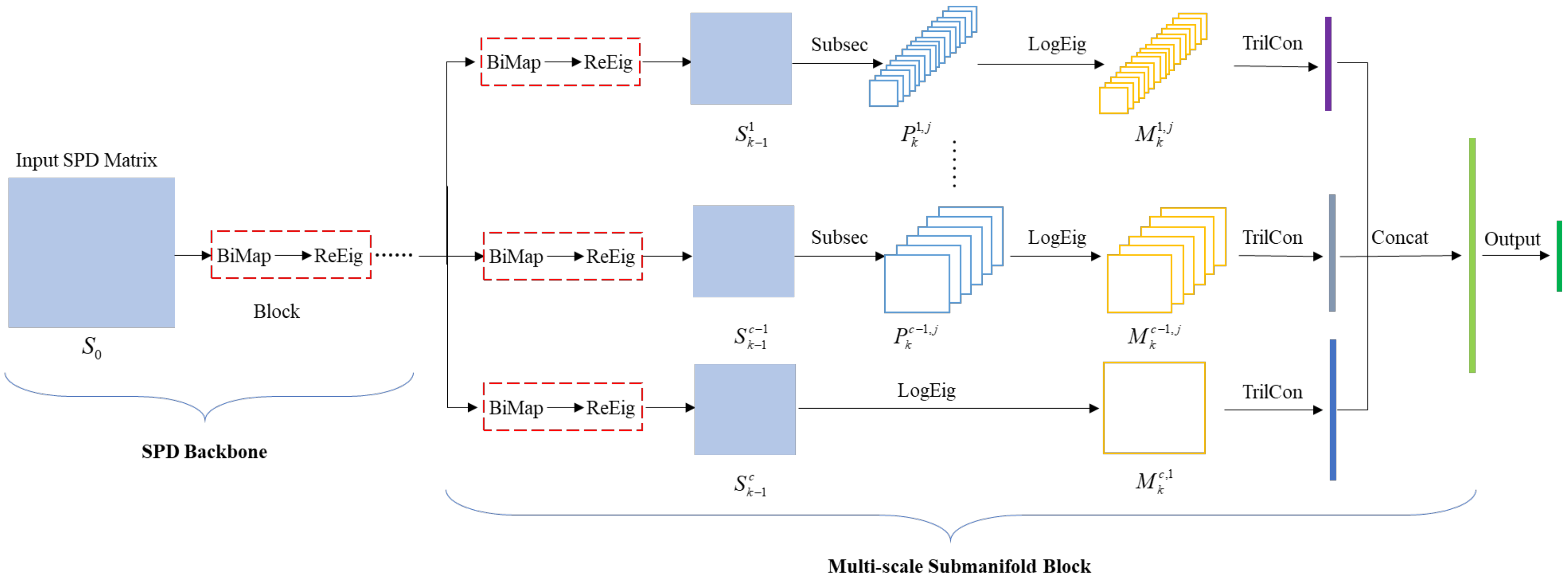
- Inspired by region covariance



PROPOSED METHOD

Muti-scale mechanisms to capture vibrant local geometry

- Both local and global geometry are effectively captured



Multi-scale Submanifolds Networks

EXPERIMENTS

Method	CG	UCF-sub
GDA	88.68	43.67
CDL	90.56	41.53
PML	84.32	50.60
LEML	71.15	44.67
SPDML-Stein	82.62	51.40
SPDML-AIM	88.61	51.13
HERML	88.94	NA
MMML	89.92	NA
GrNet	85.69	35.80
SPDNet	89.03	59.93
SymNet	89.81	56.73
MSNet-H	89.03	58.27
MSNet-PS	90.14	57.73
MSNet-AS	NA	58.33
MSNet-S	90.14	59.40
MSNet-MS	**91.25**	**60.87**

Performance (%) on the CG and UCF-sub datasets.

Methods	Year	Colour	Depth	Pose	Acc.
Lie Group	2014	✗	✗	✓	82.69
HBRNN	2015	✗	✗	✓	77.40
JOULE	2015	✓	✓	✓	78.78
Two stream	2016	✓	✗	✗	75.30
Novel View	2016	✗	✓	✗	69.21
TF	2017	✗	✗	✓	80.69
TCN	2017	✗	✗	✓	78.57
LSTM	2018	✗	✗	✓	80.14
H+O	2019	✓	✗	✗	82.43
DARTS	2018	✗	✗	✓	74.26
FairDARTS	2020	✗	✗	✓	76.87
SPDML-AIM	2018	✗	✗	✓	76.52
HERML	2015	✗	✗	✓	76.17
MMML	2018	✗	✗	✓	75.05
SPDNet	2017	✗	✗	✓	85.57
GrNet	2018	✗	✗	✓	77.57
SymNet	2021	✗	✗	✓	82.96
MSNet-H		✗	✗	✓	85.74
MSNet-PS		✗	✗	✓	80.52
MSNet-AS		✗	✗	✓	82.26
MSNet-S		✗	✗	✓	86.61
MSNet-MS		✗	✗	✓	**87.13**

Recognition Results (%) on the FPHA Dataset.

RESOURCE

Code (Released soon): https://github.com/GitZH-Chen/MSNet

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