Riemannian Local Mechanism for SPD Neural Networks

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Introduction

The Symmetric Positive Definite (SPD) matrices have received wide attention for data representation in many scientific areas. Although there are many different attempts to develop effective deep architectures for data processing on the Riemannian manifold of SPD matrices, very few solutions explicitly mine the local geometrical information in deep SPD feature representations. Given the great success of local mechanisms in Euclidean methods, we argue that it is of utmost importance to ensure the preservation of local geometric information in the SPD networks. We first analyse the convolution operator commonly used for capturing local information in Euclidean deep networks from the perspective of a higher level of abstraction afforded by category theory. Based on this analysis, we define the local information in the SPD manifold and design a multi-scale submanifold block for mining local geometry. Experiments involving multiple visual tasks validate the effectiveness of our approach.

Reanalysis of Euclidean Local Mechanisms

linear Spaces

Proposition 1. *For a given linear space V of dimension $d \times d$, n linear subspaces $V_1, V_2, \cdots, V_n$ of dimension $k \times k$ are selected and a linear function $f_i(\cdot) : V_i \rightarrow M_i$ is performed in each of them to extract local linear information. The resulting linear spaces $M_1, \cdots, M_n$ are combined into a final linear space M by direct sum, i.e.,$M = M_1 \oplus \cdots \oplus M_n$.*

Cat Theory

Proposition 2. *In a category $\mathcal{C}$, for an object $A \in Obj(\mathcal{C})$, we extract n isomorphic sub-objects from A, denoted by $A_1, A_2, \ldots, A_n$. Then morphism $f_i \in \mathrm{Hom}(A_i, B_i)$ is applied to each sub-object to map it into a resulting object B_i. The resulting objects $B_1, \cdots, B_i$ are combined into a final object $B \in Obj(\mathcal{C})$ according to certain principles.*

Manifolds

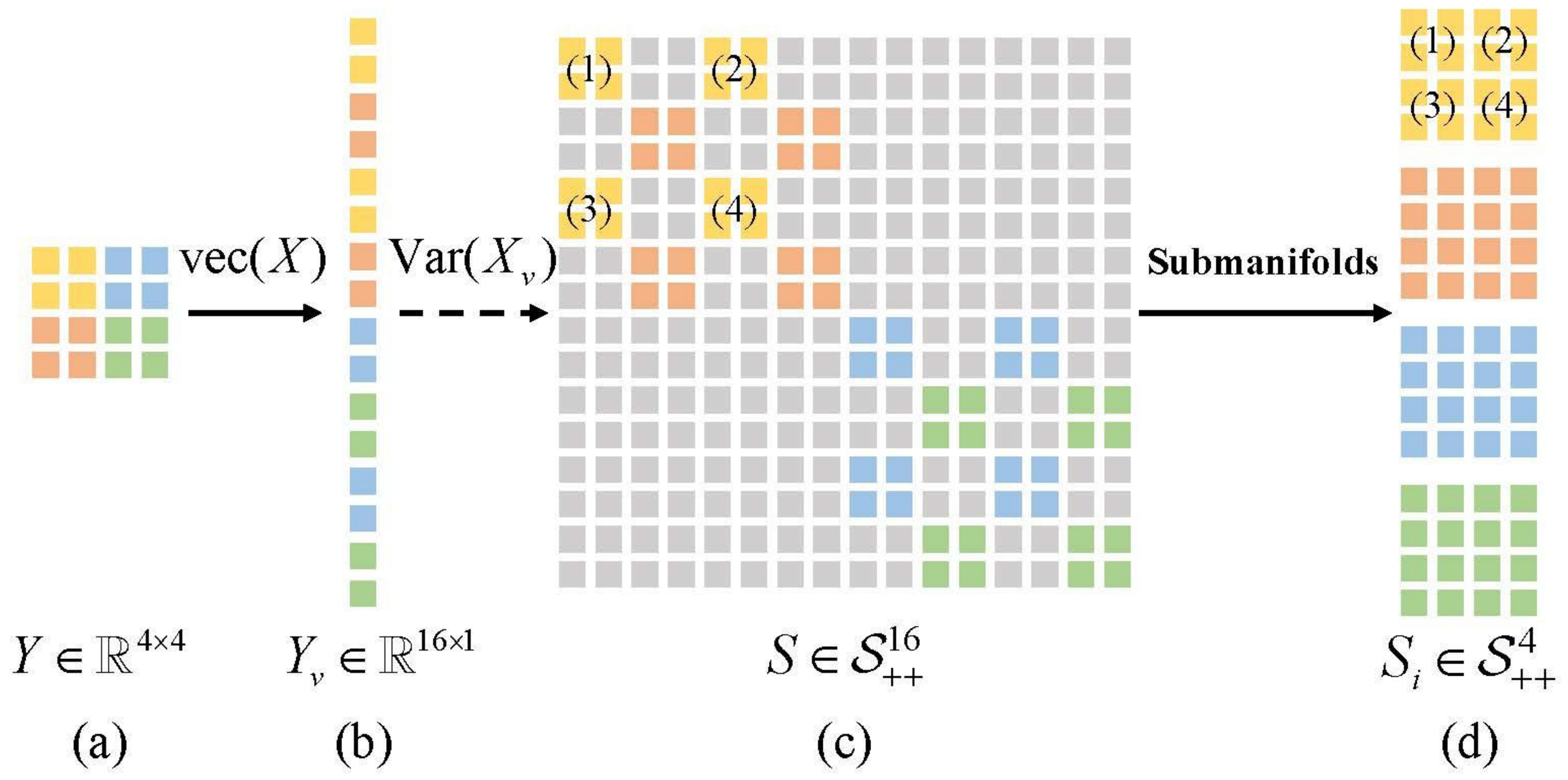
Proposition 3. *For a manifold $\mathcal{M}$, we extract n isomorphic submanifolds $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_n$ and map each one of them by a smooth maps $f_i(\cdot) : \mathcal{M}_i \rightarrow \mathcal{M}'_i$. The resulting manifolds $\mathcal{M}'_i$ are aggregated into a final manifold $\mathcal{M}'$ according to certain principles.*

Isomorphic Regular Submanifolds in SPD Manifolds

Principal submatrices as submanifolds.

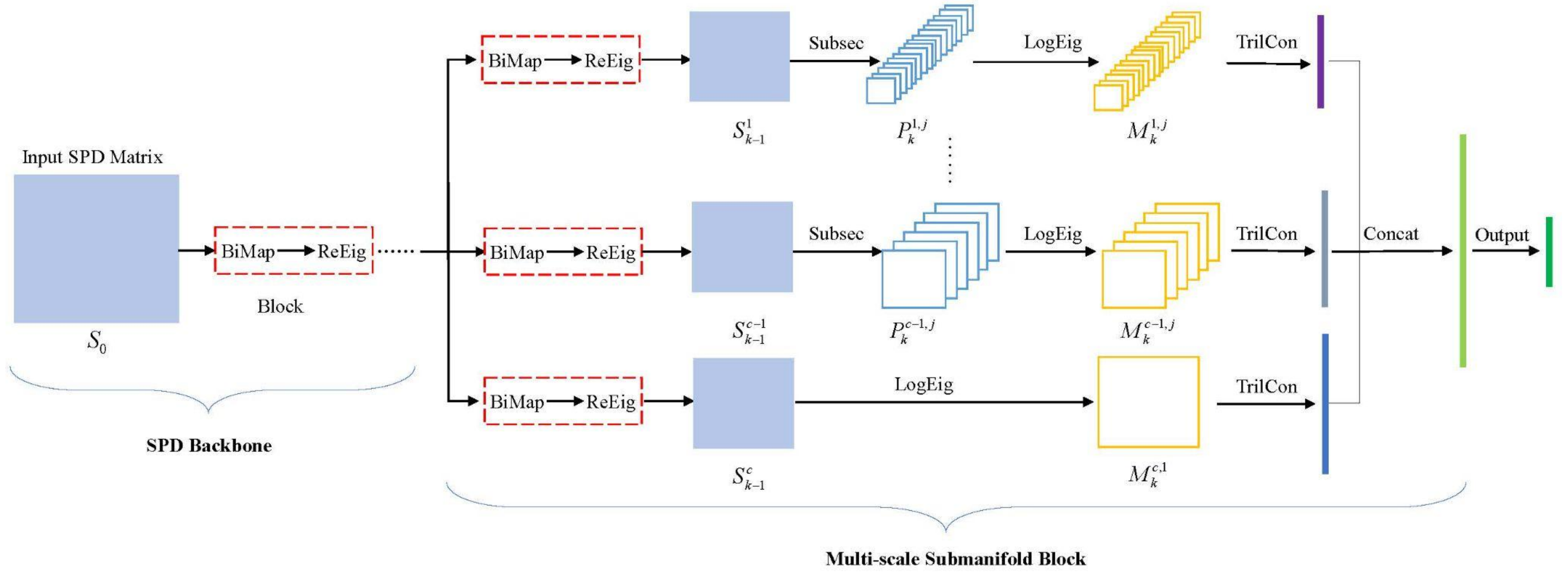
Theorem 1. *For an SPD manifold $\mathcal{S}^n_{++}$, the set of principal submatrices $(\mathcal{S}^n_{++})_\mathcal{I}$ is an SPD manifold and can be embedded into the $\mathcal{S}^n_{++}$ as a regular submanifold. In addition, for any proper indices $\mathcal{I}$ and $\mathcal{J}$ satisfying $|\mathcal{I}| = |\mathcal{J}|$, $(\mathcal{S}^n_{++})_\mathcal{I}$ is isomorphic to $(\mathcal{S}^n_{++})_\mathcal{J}$.*

Submanifolds Selection in SPD manifolds



↑ Illustration of submanifolds seletion.

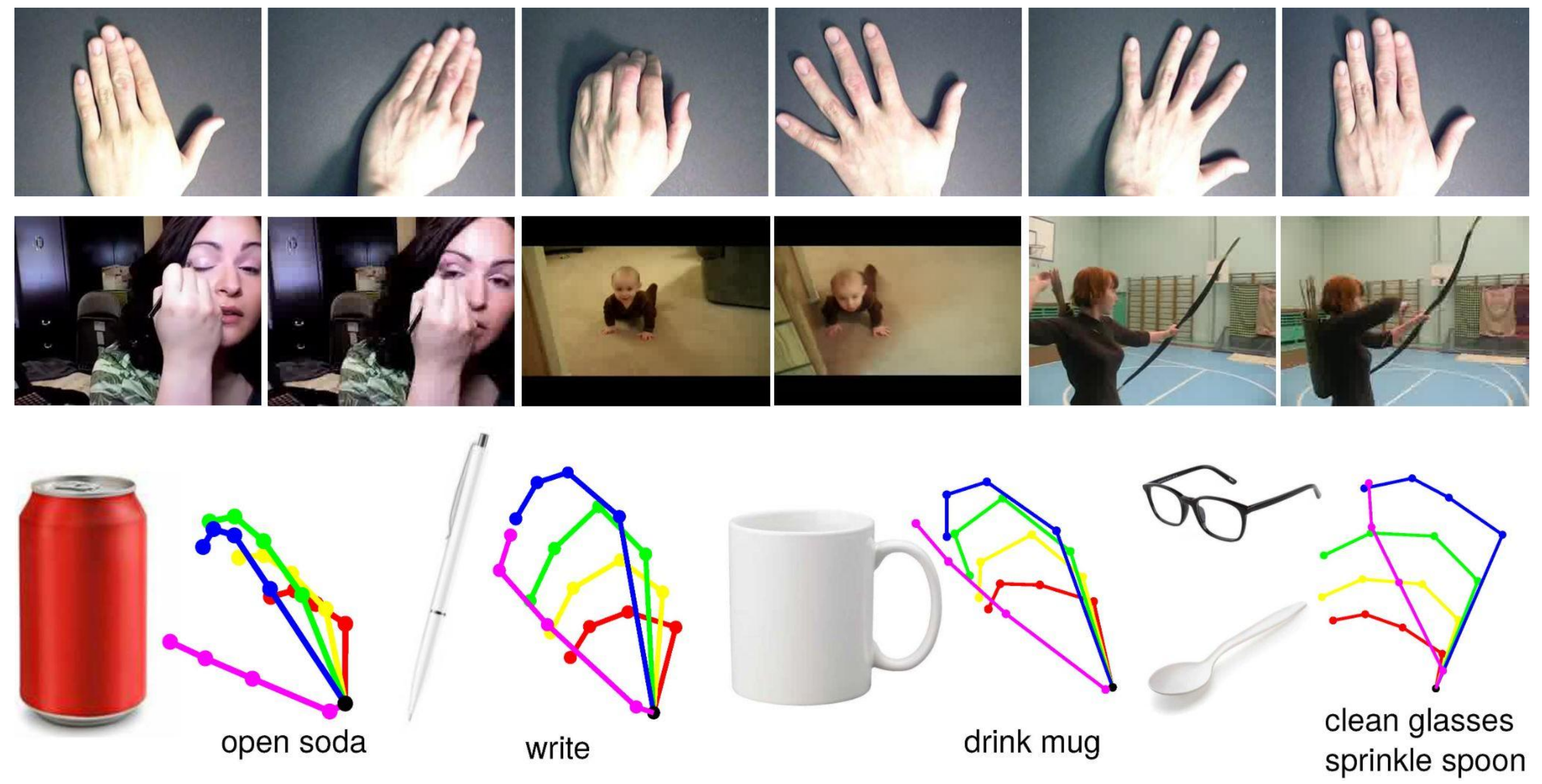
Pipeline



↑ Illustration of the proposed Multi-scale Submanifold Network (MSNet). We apply our local mechanism in to SPDNet.

Samples of Dataset

- CG
- UCF
- FPHA



Empirical Results

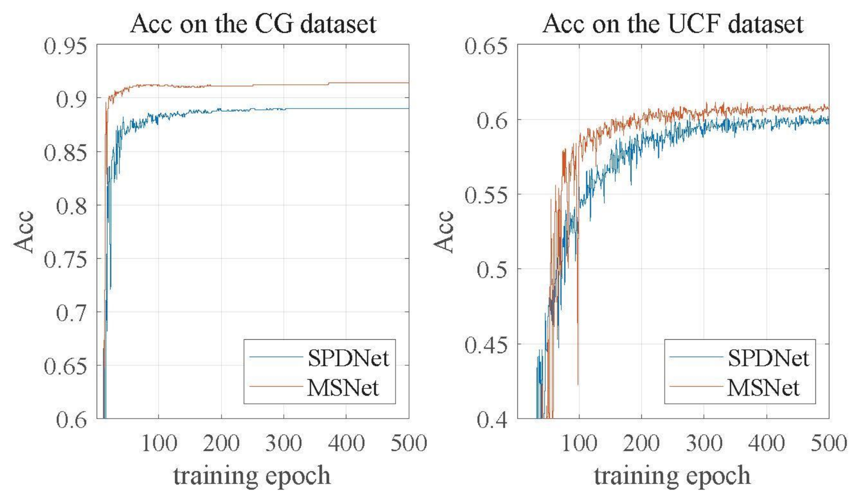
Method	CG	UCF-sub
GDA	88.68	43.67
CDL	90.56	41.53
PML	84.32	50.60
LEML	71.15	44.67
SPDML-Stein	82.62	51.40
SPDML-AIM	88.61	51.13
HERML	88.94	NA
MMML	89.92	NA
GrNet	85.69	35.80
SPDNet	89.03	59.93
SymNet	89.81	56.73
MSNet-H	89.03	58.27
MSNet-PS	90.14	57.73
MSNet-AS	NA	58.33
MSNet-S	90.14	59.40
MSNet-MS	**91.25**	**60.87**

↑ Performance (%) on the CG and UCF datasets.

Methods	Year	Colour	Depth	Pose	Acc.
Lie Group	2014	✗	✗	✓	82.69
HBRNN	2015	✗	✗	✓	77.40
JOULE	2015	✓	✓	✓	78.78
Two stream	2016	✓	✗	✗	75.30
Novel View	2016	✗	✓	✗	69.21
TF	2017	✗	✗	✓	80.69
TCN	2017	✗	✗	✓	78.57
LSTM	2018	✗	✗	✓	80.14
H+O	2019	✓	✗	✗	82.43
DARTS	2018	✗	✗	✓	74.26
FairDARTS	2020	✗	✗	✓	76.87
SPDML-AIM	2018	✗	✗	✓	76.52
HERML	2015	✗	✗	✓	76.17
MMML	2018	✗	✗	✓	75.05
SPDNet	2017	✗	✗	✓	85.57
GrNet	2018	✗	✗	✓	77.57
SymNet	2021	✗	✗	✓	82.96
MSNet-H		✗	✗	✓	85.74
MSNet-PS		✗	✗	✓	80.52
MSNet-AS		✗	✗	✓	82.26
MSNet-S		✗	✗	✓	86.61
MSNet-MS		✗	✗	✓	**87.13**

↑ Performance (%) on the FPHA datasets.

Ablation



↑ Convergence curves.

Configuration	SPDNet	MSNet
{100,60,36}	56.73	57.33
{100,80,60,36}	56.27	58.33
{100,80,50,25}	48.47	52.67
{100,80,50,25,16}	40.07	45.60
{100,80,50,25,9}	28.73	36.07

↑ Performance (%) on the UCF dataset under different configurations.

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