



Figure A. Overview of our theoretical derivations, where L is a Cholesky matrix, *i.e.*, a lower triangular matrix with positive diagonals. Starting from existing metric on the Cholesky manifold, we reveal the product structure (Sec. 3.1), propose two new metrics (Thms. 3.3-3.4), analyze their deformations (Sec. 3.3) and gyro structures (Sec. 3.4), and finally pull them back via Cholesky decomposition to obtain the Power Cholesky Metric (PCM) and Bures-Wasserstein Cholesky Metric (BWCM) on the SPD manifold (Sec. 4).

Table A. Failure probabilities (%) of geodesics under different metrics with small eigenvalues in $L \in \mathcal{L}_{++}^3$. An output matrix containing any INF or NaN is considered a failure. Here, DLM denotes the diagonal log metric. Experiments follow Tab. 2 in the main paper.

ϵ	DLM	$\theta = 1.5$		$\theta = 0.5$		$\theta = 0.25$		$\theta = 0.15$	
		θ -DPM	(θ, \mathbb{M}) -DBWM	θ -DPM	(θ, \mathbb{M}) -DBWM	θ -DPM	(θ, \mathbb{M}) -DBWM	θ -DPM	(θ, \mathbb{M}) -DBWM
$1e^{-15}$	100	0	0	0	0	0	0	0	0
$1e^{-16}$	100	0	0	0	0	0	0	0	0
$1e^{-17}$	100	0	0	0	0	0	0	0	0
$1e^{-18}$	100	0	0	0	0	0	0	0	0
$1e^{-19}$	100	0	0	0	0	0	0	0	0
$1e^{-20}$	100	0	0	0	0	0	0	0	0.002
$1e^{-21}$	100	0	0	0	0	0	0	0	0.015
$1e^{-22}$	100	0	0	0	0	0	0	0	0.245
$1e^{-23}$	100	0	0	0	0	0	0	0	2.285
$1e^{-24}$	100	0	0	0	0	0	0	0	22.775
$1e^{-25}$	100	0	0	0	0	0	0	0	86.336
$1e^{-30}$	100	0	0	0	0	0	0	0	100