# 作业5

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1、 $x' = -\frac{1}{2}x + u \Rightarrow A = -\frac{1}{2}$ , $B = 1$

$$J = 5x^2(1) + \frac{1}{2}\int_0^1 [2x^2 + u^2]dt \Rightarrow H = 10,\ Q = 2,\ R = 1$$

Ricatti :

$$k' + 2 - k \cdot 1 \cdot 1 \cdot 1 \cdot k + \left(-\frac{1}{2}k\right) + (-2)k = 0$$

$$\Rightarrow \quad k' = k^2 + \frac{5}{2}k - 2$$

$$k_{tf} = 10$$

解得 $$k = \frac{\sqrt{57}\tanh\left(2\operatorname{atanh}\left(\frac{15\sqrt{57}}{19}\right) - \frac{\sqrt{57}}{2}(t-1)\right)}{4} - \frac{5}{4}$$

$$x' = \left(-\frac{1}{2} - k\right)x\ ,\quad x(0) = 2$$

解得 $$x^* = 2\tanh\left(\operatorname{atanh}\frac{15\sqrt{57}}{19} + \frac{\sqrt{57}}{4} + 2\right) \cdot e^{-\ln\left(\tanh\left(\operatorname{atanh}\frac{15\sqrt{57}}{19} - \frac{\sqrt{57}}{4}t + \frac{\sqrt{57}}{4}\right) + 1\right) - \left(\frac{\sqrt{57}}{4} - \frac{3}{4}\right)t}$$

$$p = k \cdot x = \frac{\sqrt{57}\tanh\left(2\operatorname{atanh}\frac{15\sqrt{57}}{19} - \frac{\sqrt{57}}{2}(t-1)\right) - 5}{4} \cdot 2\tanh\left(\operatorname{atanh}\frac{15\sqrt{57}}{19} + \frac{\sqrt{57}}{4} + 2\right) \cdot e^{-\ln\left(\tanh\left(\operatorname{atanh}\frac{15\sqrt{57}}{19} - \frac{\sqrt{57}}{4}t + \frac{\sqrt{57}}{4}\right) + 1\right) - \left(\frac{\sqrt{57}}{4} - \frac{3}{4}\right)t}$$

$$u^* = -p = -\frac{\sqrt{57}\tanh\left(2\operatorname{atanh}\frac{15\sqrt{57}}{19} - \frac{\sqrt{57}}{2}(t-1)\right) - 5}{4} \cdot 2\tanh\left(\operatorname{atanh}\frac{15\sqrt{57}}{19} + \frac{\sqrt{57}}{4} + 2\right) \cdot e^{-\ln\left(\tanh\left(\operatorname{atanh}\frac{15\sqrt{57}}{19} - \frac{\sqrt{57}}{4}t + \frac{\sqrt{57}}{4}\right) + 1\right) - \left(\frac{\sqrt{57}}{4} - \frac{3}{4}\right)t}$$

2、

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ u \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u \Rightarrow A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\ ,\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$J = \int_0^\infty (y^2 + ru^2)dt = \int_0^\infty (x_1^2 + ru^2)dt \Rightarrow H = 0\ ,\ Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ R = r$$

Racitti :

$$\begin{bmatrix} k_1' & k_2' \\ k_3' & k_4' \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\frac{1}{r}\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} + \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix} = 0$$

$$\begin{bmatrix} k_1'+1 & k_2' \\ k_3' & k_4' \end{bmatrix} - \frac{1}{r}\begin{bmatrix} k_2k_3 & k_2k_4 \\ k_4k_3 & k_4k_4 \end{bmatrix} + \begin{bmatrix} 0 & k_1 \\ 0 & k_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} k_1' + 1 - \frac{1}{r}k_2k_3 & k_2' - \frac{1}{r}k_2k_4 + k_2 \\ k_3' - \frac{1}{r}k_3k_4 + k_1 & k_4' - \frac{1}{r}k_4k_4 + k_2 + k_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

解得