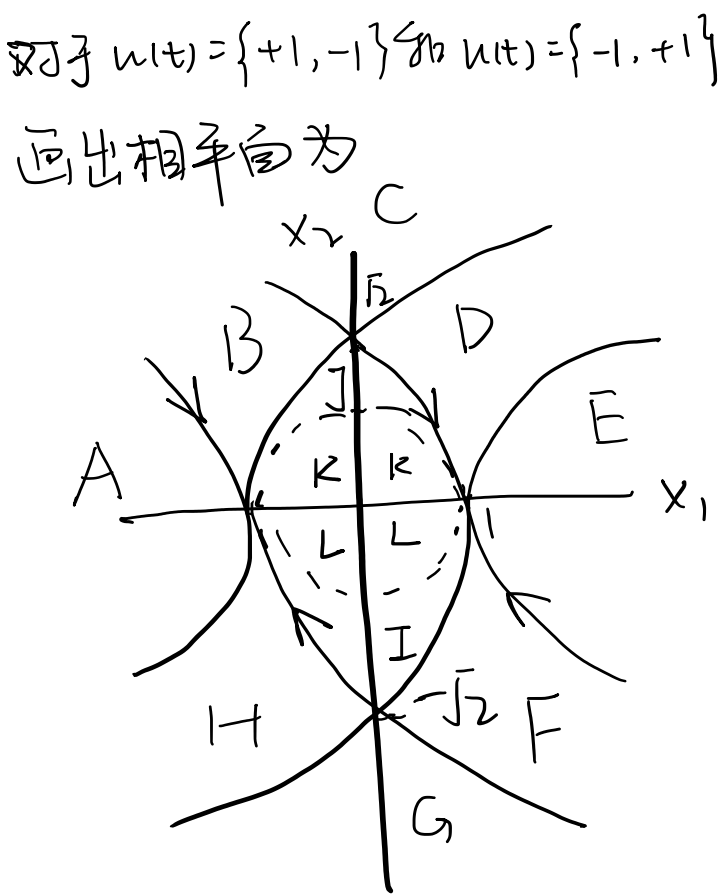


1. 时间最优控制: $J(u) = \int_{t_0}^{t_f} dt$
 $H = 1 + p_1 x_2 + p_2 u$
 $p_1' = -\frac{\partial H}{\partial x_1} = 0 \Rightarrow p_1 = c_1$
 $p_2' = -\frac{\partial H}{\partial x_2} = -p_1 \Rightarrow p_2 = -c_1 t + c_2$
 $u(t) = 1, t \in [t_0, t_f]$
 $u(t) = -1, t \in [t_0, t_f]$
 $u(t) = \begin{cases} +1 & t \in [t_0, t_1] \\ -1 & t \in [t_1, t_f] \end{cases}$
 $u(t) = \begin{cases} -1 & t \in [t_0, t_1] \\ +1 & t \in [t_1, t_f] \end{cases}$

分情况讨论
① $u(t) = \{+1\}$
 $x_2 = t + c_3$
 $x_1 = \frac{1}{2} x_2^2 + c_4$
由图像可知
 $-1 \leq c_4 \leq 1$ 可以到达目标集

② $u(t) = \{-1\}$
 $x_2 = -t + c_3$
 $x_1 = -\frac{1}{2} x_2^2 + c_5$
由图像可知
 $-1 \leq c_5 \leq 1$ 可以到达目标集

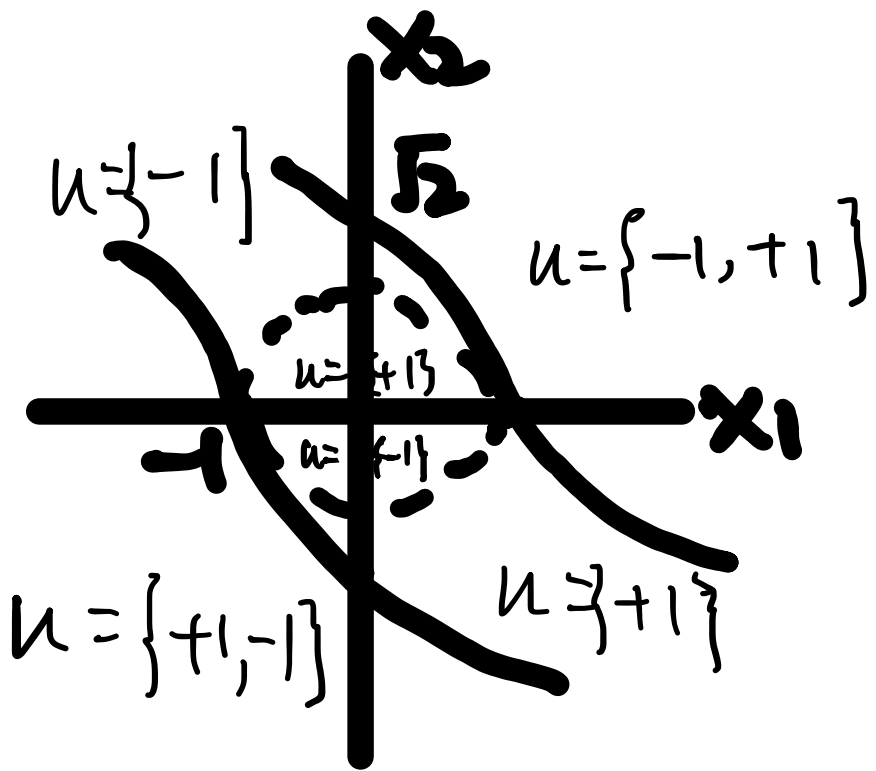


由图像可得
共分为12个区域
其中A的控制律为 $u = \{+1, -1\}$
B的控制律为 $u = \{-1\}$
C的控制律为 $u = \{-1, +1\}$
D的控制律为 $u = \{-1, +1\}$
E的控制律为 $u = \{-1, +1\}$

L的控制律为 $u = \{-1\}$
K的控制律为 $u = \{+1\}$
F的控制律为 $u = \{+1\}$
G的控制律为 $u = \{+1, -1\}$
H的控制律为 $u = \{+1, -1\}$
I的控制律为 $u = \{+1\}$
J的控制律为 $u = \{-1\}$

综合可得 $AHG \rightarrow \{+1, -1\}$
 $CDE \rightarrow \{-1, +1\}$
 $KFI \rightarrow \{+1\}$
 $LBJ \rightarrow \{-1\}$

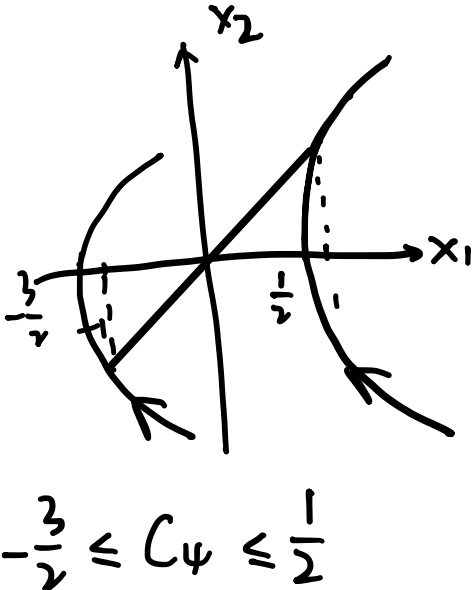
重画相平面与控制律如图



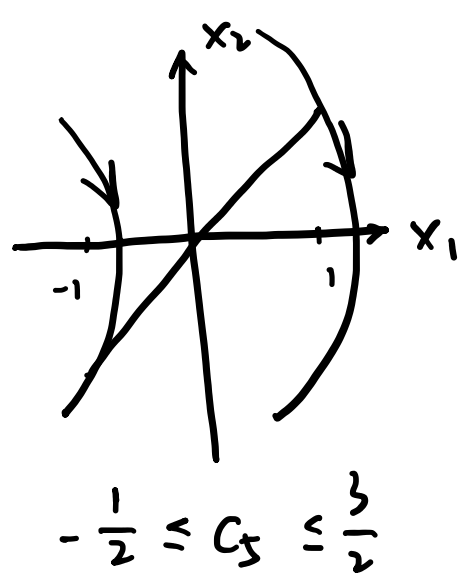
2.

目标集改为 $S = \{(x_1, x_2) | x_1 = x_2, |x_1| \leq 1\}$

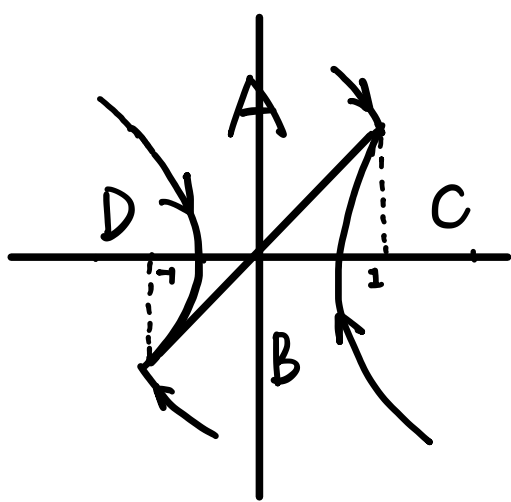
① $u(t) = \{+1\}$



② $u(t) = \{-1\}$



对于 $u(t) = \{-1, +1\}$ 和 $u(t) = \{+1, -1\}$
画出相平面为



由相平面分析
可得控制律为
A区域 $u = \{-1\}$
B区域 $u = \{+1\}$
C区域 $u = \{-1, +1\}$
D区域 $u = \{+1, -1\}$

3.

燃料最优控制

$H = |u| + p_1 x_2 + p_2 u \leq |u| + p_1 x_2 + p_2 u' \Rightarrow$ 记 $\eta(u) = \begin{cases} u(1+p_2) & u \geq 0 \\ u(-1+p_2) & u < 0 \end{cases}$

分类讨论: ① $p_2 > 1, u \geq 0$
 $\eta = 0, u = 0$ 时取

$p_2 > 1, u < 0$
极小值在 $u = -1$ 取, $\eta < 0$
 $\Rightarrow u = -1$

② $p_2 = 1, u \geq 0$
 $\eta = 0, u = 0$ 时取.

$p_2 = 1, u < 0$
 u 任取
 $\Rightarrow u \leq 0$

③ $-1 < p_2 < 1, u \geq 0, u \geq 0$
 $\eta = 0, u = 0$ 时取

$-1 < p_2 < 1, u < 0$
 $u = 0$ 时取, $\eta = 0$
 $\Rightarrow u = 0$

④ $p_2 = -1, u \geq 0$
 $\forall u \geq 0$ 取 $\eta = 0$

$p_2 = -1, u < 0$
 η 恒大于 0, $\forall u$
 $\Rightarrow u \geq 0$

⑤ $p_2 < -1, u \geq 0$
 $u = 1$ 取 η 最小.

$p_2 < -1, u < 0$
 η 恒大于 0, $\forall u$
 $\Rightarrow u = 1$

故 $u(t) = \begin{cases} -1 & p_2 > 1 \\ \leq 0 & p_2 = 1 \\ 0 & -1 < p_2 < 1 \\ \geq 0 & p_2 = -1 \\ 1 & p_2 < -1 \end{cases}$

画出 Bang-off-Bang 控制律

