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最优化方法

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Problem 1.

$$f(x_1,x_2,x_3) = x_2^2 = [x_1\ x_2\ x_3]\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix},\quad Q=\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix} \rightarrow \text{positive semi-define}$$

$$f(x_1,x_2,x_3) = x_1^2+2x_2^2-x_1x_3 = \vec{x}^T\begin{bmatrix}1&0&-1\\0&2&0\\0&0&0\end{bmatrix}x,\quad Q_0=\begin{bmatrix}1&0&-\frac{1}{2}\\0&2&0\\-\frac{1}{2}&0&0\end{bmatrix} \rightarrow \lambda_1=-0.207\quad \lambda_2=1.207\quad \lambda_3=2$$

it is indefine.

$$f(x_1,x_2,x_3) = x_1^2+x_3^2+2x_1x_2+2x_1x_3+2x_2x_3 = x^T\begin{bmatrix}1&1&0\\1&0&1\\0&1&1\end{bmatrix}x,\quad Q=\begin{bmatrix}1&1&0\\1&0&1\\0&1&1\end{bmatrix} \rightarrow \lambda_1=-1\quad \lambda_2=1\quad \lambda_3=2$$

it is indefine

Problem 2.

$$f(\lambda x^{(1)}+(1-\lambda)x^{(2)}) = \|\lambda x^{(1)}+(1-\lambda)x^{(2)}\| = \sqrt{\lambda^2 x^{(1)2}+2\lambda(1-\lambda)x^{(1)}x^{(2)}+(1-\lambda)^2x^{(2)2}}$$

$$(\lambda f(x^{(1)})+(1-\lambda)f(x^{(2)}))^2 = \lambda^2 f^2(x^{(1)})+(1-\lambda)^2f^2(x^{(2)})+2\lambda(1-\lambda)f(x^{(1)})\cdot f(x^{(2)}) = \lambda^2\|x^{(1)}\|^2+(1-\lambda)^2\|x^{(2)}\|^2+2\lambda(1-\lambda)\|x^{(1)}\|\|x^{(2)}\|$$

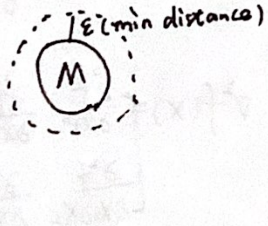
$$\because \lambda^2x^{(1)2}+2\lambda(1-\lambda)x^{(1)}x^{(2)}+(1-\lambda)^2x^{(2)2} \le \lambda^2x^{(1)2}+2\lambda(1-\lambda)\|x^{(1)}\|\|x^{(2)}\|+(1-\lambda)^2x^{(2)2}$$

$$\therefore f(\lambda x^{(1)}+(1-\lambda)x^{(2)}) \le \lambda f(x^{(1)})+(1-\lambda)f(x^{(2)})$$

so it is convex.

Problem 3.

I don't know how to prove that. Could you please upload the solution to the canvas! thx.



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Problem 7: $f(x) = x_1^4 + 2x_1^2x_2^2 + x_2^4$ $\quad \nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 4x_1^3 + 4x_1x_2^2 \\ 4x_2^3 + 4x_1^2x_2 \end{bmatrix}$

$$\nabla^2 f(x) = \begin{bmatrix} 12x_1^2 + 4x_2^2 & 8x_1x_2 \\ 8x_1x_2 & 12x_2^2 + 4x_1^2 \end{bmatrix}$$

$$f(x) = 4 + \begin{bmatrix} 8 \\ 8 \end{bmatrix}^T \begin{bmatrix} x_1 - 1 \\ x_2 - 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x_1 - 1 \\ x_2 - 1 \end{bmatrix}^T \begin{bmatrix} 16 & 8 \\ 8 & 16 \end{bmatrix} \begin{bmatrix} x_1 - 1 \\ x_2 - 1 \end{bmatrix}$$

$$= 8x_1^2 + 8x_2^2 - 16x_1 - 16x_2 + 8x_1x_2 + 12$$

Problem 8.

$$\max \sum_{i=1}^{4} a_i x_i \quad , \quad a_1 = 200,\ a_2 = 100,\ a_3 = 150,\ a_4 = 200$$

$$\text{s.t.} \quad 12x_1 + 10x_2 + 25x_3 + 20x_4 \le 6000$$

$$8x_1 + 6x_2 + 2x_3 + 4x_4 \le 2000$$

$$x_i \ge 0,\ i = 1, 2, 3, 4.$$

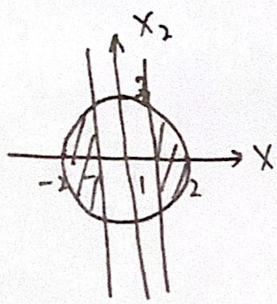
Problem 9.

$x_a = [0.5, 0.5]^T$ is feasible, on the boundary

$x_b = [1, 1]^T$ is infeasible.

$x_c = [-0.5, 0]^T$ is infeasible

Problem 10.



min x_1 is -2, $x_2 = 0$.