作业1

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Problem 1.

$x(0)=1,\ x(1)=2$

$g(\dot{x}(t),t)=1+\dot{x}^2(t)$，不显含 x

Euler-lagrange 可简化为

$$\frac{\partial g}{\partial \dot{x}}(\dot{x}(t),t)=C_1=2\dot{x}(t)$$

积分得 $x(t)=\frac{1}{2}C_1t+C_2$

代入边界条件 $x(0)=1,\ x(1)=2$ 得

$$x(t)=t+1 \quad t\in[0,1]$$

Problem 2.

$$g(x,y,x',y')=x'^2+y'^2+2xy$$

Euler-lagrange 方程组为

$$\frac{\partial g}{\partial x}-\frac{d}{dt}\left(\frac{\partial g}{\partial x'}\right)=0 \Rightarrow 2y-\frac{d}{dt}(2x') \Rightarrow y=x''$$

$$\frac{\partial g}{\partial y}-\frac{d}{dt}\left(\frac{\partial g}{\partial y'}\right)=0 \Rightarrow 2x-\frac{d}{dt}(2y') \Rightarrow x=y''$$

消去 x 可得 $y=y^{(4)}$，积分得

$$y=C_1e^t+C_2e^{-t}+C_3\cos t+C_4\sin t$$

$$x=y''=C_1e^t+C_2e^{-t}-C_3\cos t-C_4\sin t$$

带入边界条件 $x(0)=y(0)=0,\ x(\frac{\pi}{2})=1,\ y(\frac{\pi}{2})=-1$

$C_1=C_2=C_3=0 \quad C_4=-1$

极值曲线为 $x=\sin t \quad y=-\sin t$

Problem 3.

记增广性能指标，$g=\sqrt{1+(x')^2}$，$\frac{\partial g}{\partial x'}=\frac{x'}{\sqrt{1+(x')^2}}$

$$\bar{J}(x,\lambda)=\lambda\cdot m(x_{t_f},t_f)+\int_{t_0}^{t_f}g(x',t)\,dt$$

$$=\underbrace{\lambda\cdot(x_{t_f}+t_f-2)}_{h(x_{t_f},t_f)}+\int_{t_0}^{t_f}\sqrt{1+(x')^2}\,dt$$

必要条件：

① $\frac{\partial g}{\partial x}-\frac{d}{dt}\frac{\partial g}{\partial x'}=0 \Rightarrow \frac{x'}{\sqrt{1+(x')^2}}=c_1 \Rightarrow x=at+C_2$，$a=\sqrt{\frac{C_1^2}{1-C_1^2}}$，$x'=a$

② $0=\frac{\partial h}{\partial t}\Big|_{t_f}+g\Big|_{t_f}-\frac{\partial g}{\partial x'}\cdot x'\Big|_{t_f} \Rightarrow \lambda+\sqrt{1+a^2}-\frac{a^2}{\sqrt{1+a^2}}=0$

③ $0=\frac{\partial h}{\partial x}\Big|_{t_f}+\frac{\partial g}{\partial x'}\Big|_{t_f} \Rightarrow \lambda+\frac{a}{\sqrt{1+a^2}}=0$

带入 $x(0)=1$ 得 $a=1$，$C_2=1$

$x=t+1$，$t_f=0.5$，$J^*=\int_0^{0.5}\sqrt{2}\,dt=\frac{\sqrt{2}}{2}$

Problem 4

末端时刻自由状态固定

$g(x,x',t)=2x+\frac{1}{2}(x')^2$，$\frac{\partial g}{\partial x'}=x'$

必要条件

① $0=\frac{\partial g}{\partial x}-\frac{d}{dt}\left(\frac{\partial g}{\partial x'}\right) \Rightarrow 2=x'' \Rightarrow x=t^2+C_1t+C_2$

② $0=g\Big|_{t_f}-\frac{\partial g}{\partial x'}x'\Big|_{t_f} \Rightarrow 8+\frac{1}{2}(x')^2\Big|_{t_f}-(x')^2\Big|_{t_f}=8-\frac{1}{2}(x')^2\Big|_{t_f}=8-\frac{1}{2}(2t_f+C_1)^2$

带入 $x_1=4$，$x_{t_f}=4$ 可得

$C_1=-6$ 或 2

$C_2=9$ 或 1 $\Rightarrow x=t^2-6t+9$

$t_f=5$ 或 1（舍） $t_f=5$