Wigner-Ville分布

复杂信号用的

WVD

双线性时频分布

Wigner-Ville 分布

Eugene Wigner (November 17, 1902 − January 1, 1995) A Hungarian American theoretical physicist and mathematician.
Nobel Prize in Physics in 1963 "for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles"

Jean-Andre Ville (June 24, 1910 – January 22, 1989) French mathematician

http://www.jehps.net/juin2009.html

Wigner-Ville 分布

定义

The Wigner-Ville (and all of Cohen’ s class of distribution) uses a variation of the autocorrelation function where time remains in the result, called instantaneous autocorrelation function 瞬时自相关

$$s^*\left(t-\frac{1}{2}\tau\right)s\left(t+\frac{1}{2}\tau\right)$$

信号

Where τ is the time lag and * represents the complex conjugate of the signal $s(t)$

Wigner-Ville 分布

- 定义

时域

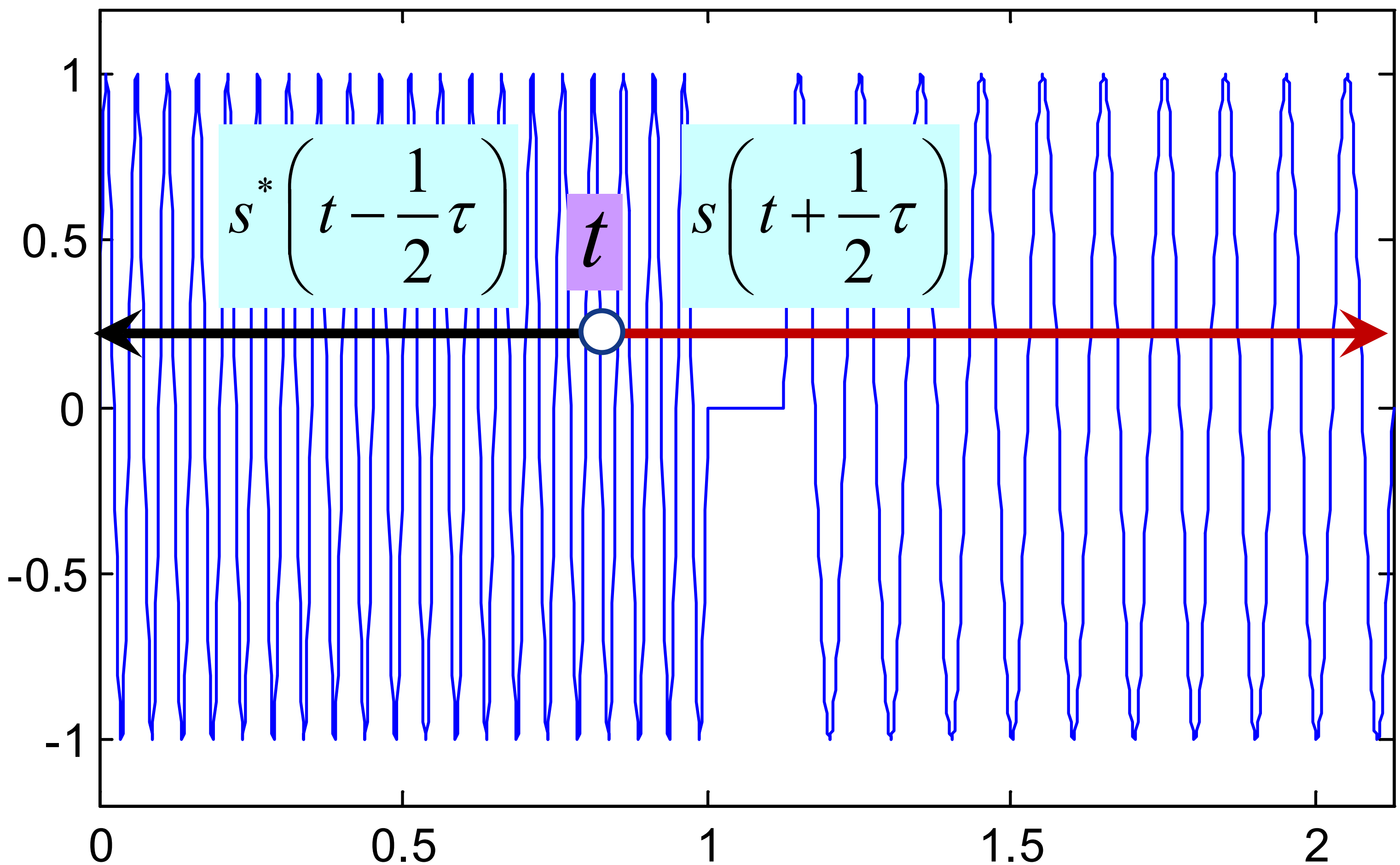
$$W_s(t,\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}s^*\left(t-\frac{1}{2}\tau\right)s\left(t+\frac{1}{2}\tau\right)e^{-j\tau\omega}d\tau$$

频域

$$W_s(t,\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}S^*\left(\omega-\frac{1}{2}\theta\right)S\left(\omega+\frac{1}{2}\theta\right)e^{j\theta t}d\tau$$

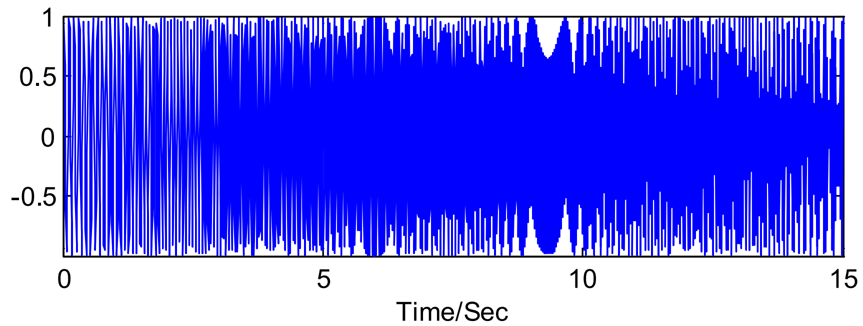
Wigner-Ville 分布

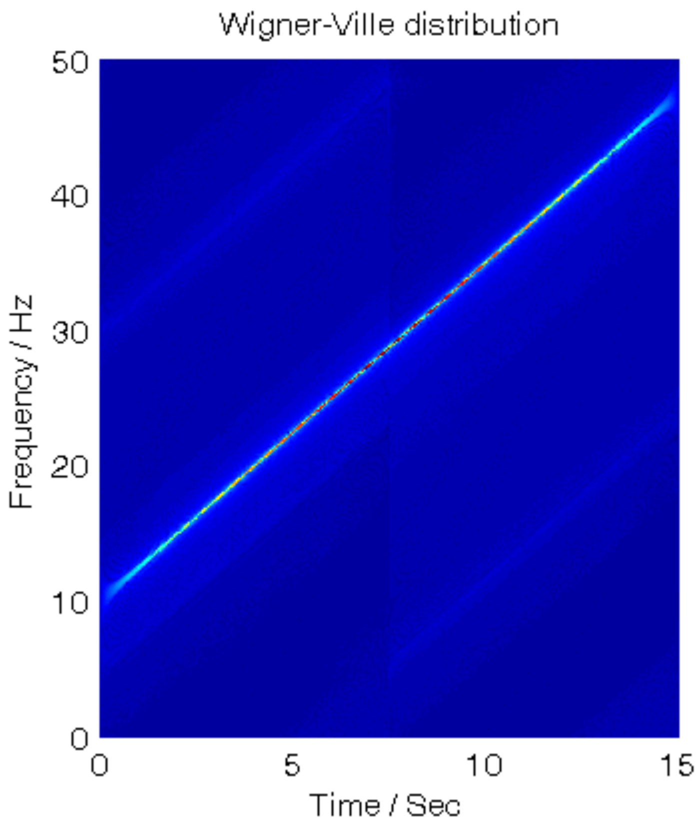
- 计算

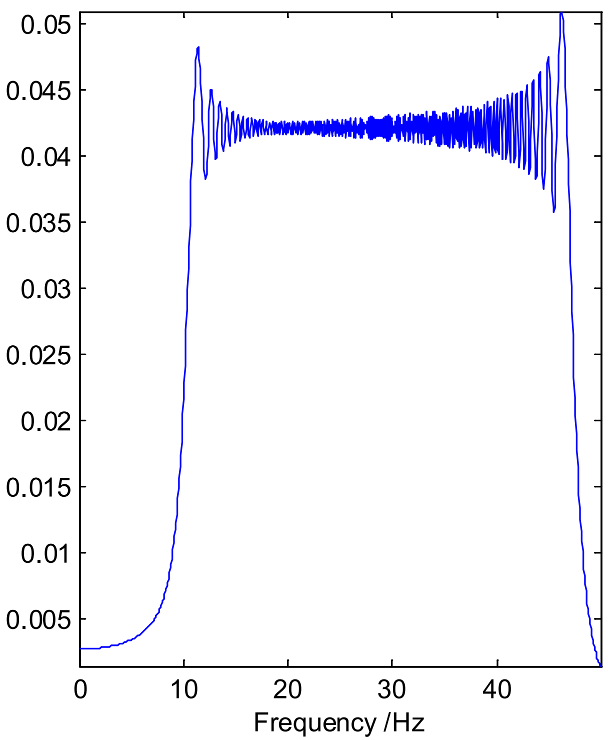


Wigner-Ville 分布

- 示例







Wigner-Ville 分布

典型信号

Constant signal

$$W_x(t,f) = \int_{-\infty}^{\infty} e^{-i2\pi\tau f}\, d\tau = \delta(f).$$

Harmonic signal

$$\begin{aligned} W_x(t,f) &= \int_{-\infty}^{\infty} e^{i2\pi k(t+\tau/2)} e^{-i2\pi k(t-\tau/2)} e^{-i2\pi\tau f}\, d\tau \\ &= \int_{-\infty}^{\infty} e^{-i2\pi\tau(f-k)}\, d\tau \\ &= \delta(f-k)\ . \end{aligned}$$

Wigner-Ville 分布

- 典型信号

Linear Chirp signal

$$x(t) = e^{i2\pi kt^2}$$

$$\begin{aligned}
W_x(t,f) &= \int_{-\infty}^{\infty} e^{i2\pi k(t+\tau/2)^2} e^{-i2\pi k(t-\tau/2)^2} e^{-i2\pi\tau f}\, d\tau \\
&= \int_{-\infty}^{\infty} e^{i4\pi kt\tau} e^{-i2\pi\tau f}\, d\tau \\
&= \int_{-\infty}^{\infty} e^{-i2\pi\tau(f-2kt)}\, d\tau \\
&= \delta(f-2kt)\ .
\end{aligned}$$

Wigner-Ville 分布

- 典型信号

Delta signal

$$
\begin{aligned}
W_x(t,f) &= \int_{-\infty}^{\infty} \delta(t+\tau/2)\delta(t-\tau/2)e^{-i2\pi\tau f}\,d\tau \\
&= 4\int_{-\infty}^{\infty} \delta(2t+\tau)\delta(2t-\tau)e^{-i2\pi\tau f}\,d\tau \\
&= 4\delta(4t)e^{i4\pi t f} \\
&= \delta(t)e^{i4\pi t f} \\
&= \delta(t).
\end{aligned}
$$

Wigner-Ville 分布

- 性质

➢ Energy conservation: by integrating the WVD of all over the time-frequency plane, the energy of *s* is obtained

$$E_s = \iint W_s(t,\omega)dtd\omega$$

➢Real-valued: the WVD is real-valued across time and frequency

$$W_s(t,\omega) \in \mathbb{R}, \qquad \forall\, t,\omega$$

Wigner-Ville 分布

性质

➢ **Marginal properties:** the energy spectral density and the instantaneous power can be obtained as marginal distributions of WVD

$$\int W_s(t,\omega)dt = |S(\omega)|^2 \qquad \int W_s(t,\omega)d\omega = |s(t)|^2$$

➢ **Moment properties**

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} t^n W_s(t,f)\,dtdf = \int_{-\infty}^{\infty} t^n \left|s(t)^2\right| dt$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f^n W_s(t,f)\,dtdf = \int_{-\infty}^{\infty} f^n \left|S(f)^2\right| df$$

Wigner-Ville 分布

- 性质

➢ Translation covariance: the WVD is time- and frequency covariant

时频移动同步

$$x(t) = \underline{s(t - t_0)} \;\Rightarrow\; W_x(t, \omega) = \underline{W_s(t - t_0, \omega)}$$

$$x(t) = s(t)e^{-j\omega_0 t} \;\Rightarrow\; W_x(t, \omega) = W_s(t, \omega - \omega_0)$$

➢ Dilation covariance: the WVD also preserves dilation

$$x(t) = \sqrt{k}\, s(kt)\;;\; k > 0 \;\Rightarrow\; W_x(t, \omega) = W_s\left(kt, {}^{\omega}/_{k}\right)$$

Wigner-Ville 分布

- 性质

➢**Compatibility with filterings:** it expresses the fact that if a signal x is the convolution of s and h, the WVD of is the time-convolution between the WVD of s and the WVD of h

$$x(t) = \int s(\tau)\, h(t-\tau) d\tau \;\Rightarrow$$

$$W_x(t,\omega) = \int W_s(\tau,\omega) W_h(t-\tau,\omega) d\tau$$

Wigner-Ville 分布

- 性质

➢Compatibility with modulations : this is the dual property of the previous one : if x is the modulation of s by a function m, the WVD of x is the frequency-convolution between the WVD of s and the WVD of m.

$$x(t) = s(t)m(t) \;\Rightarrow$$

$$W_x(t,\omega) = \int W_s(\tau,\theta) W_m(t,\omega-\theta)\,d\theta$$

Wigner-Ville 分布

性质

➢ **Wide-sense support conservation :** if a signal has a compact support in time (respectively in frequency), then its WVD also has the same compact support in time (respectively in frequency). This is also called weak finite support.

$$s(t) = 0, |t| > T \;\Rightarrow\; W_s(t,\omega) = 0, |t| > T$$

$$S(\omega) = 0, |\omega| > B \;\Rightarrow\; W_s(t,\omega) = 0, |\omega| > B$$

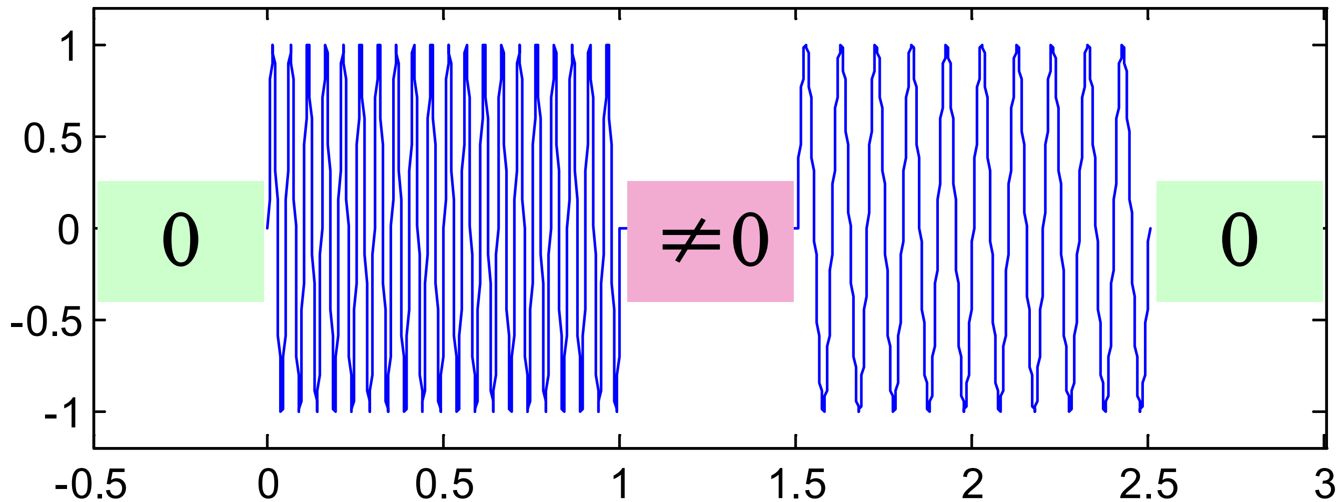
However, the WVD does not have strong finite support

Wigner-Ville 分布

- 性质

$$s(t) = 0, |t| > T \Rightarrow W_s(t,\omega) = 0, |t| > T$$

$$S(\omega) = 0, |\omega| > B \Rightarrow W_s(t,\omega) = 0, |\omega| > B$$



Wigner-Ville 分布

性质

➢Unitarity： also known as the Moyal formula. It proves that the Wigner-Ville transform is unitary, which implies energy conservation properties.

$$\left| \int s(t)x^*(t)dt \right|^2 = \iint W_s(t,\omega)W_x{}^*(t,\omega)dtd\omega$$

Wigner-Ville 分布

性质

- **Instantaneous frequency and group delay :** The instantaneous frequency characterizes a local frequency behaviour as a function of time. In a dual way, the local time behaviour as a function of frequency is described by the group delay.

- In order to introduce these terms, the concept of analytic signal must be defined first

$$s_a(t) = s(t) + jHT(s(t))$$

s_a is called the analytic signal associated to signal s.

Wigner-Ville 分布

- 性质

➢ **Instantaneous frequency:** the instantaneous frequency of a signal can be recovered from the WVD as its first order moment (or center of gravity) in frequency

$$f_s(t) = \frac{\int \omega W_{s_a}(t,\omega)d\omega}{\int W_{s_a}(t,\omega)d\omega}$$

➢ **Group delay:** the group delay of a signal can be recovered from the WVD as its first order moment (or center of gravity) in time.

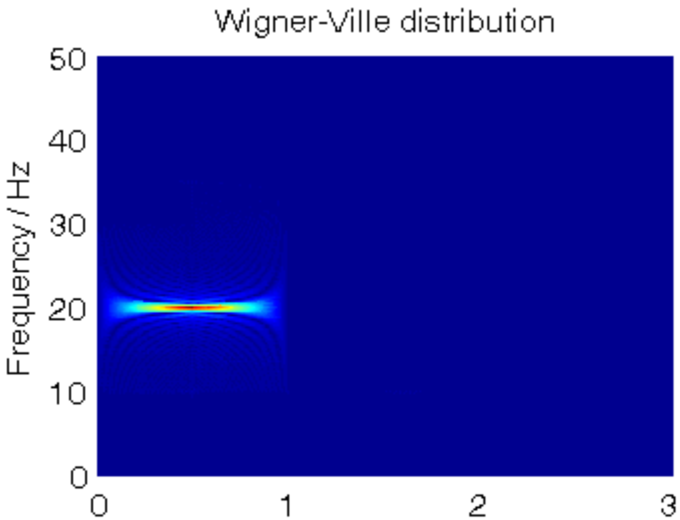
$$t_s(\omega) = \frac{\int t W_{s_a}(t,\omega)dt}{\int W_{s_a}(t,\omega)dt}$$

Wigner-Ville 分布

- 交叉项 – 示例一

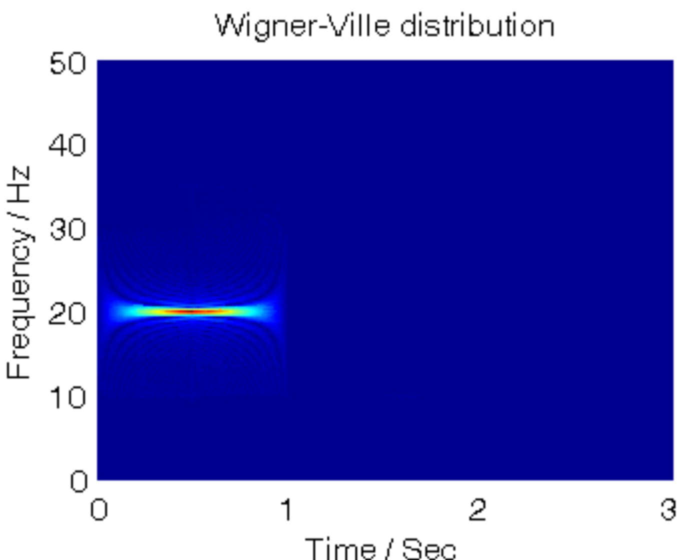
单分量信号

x_1



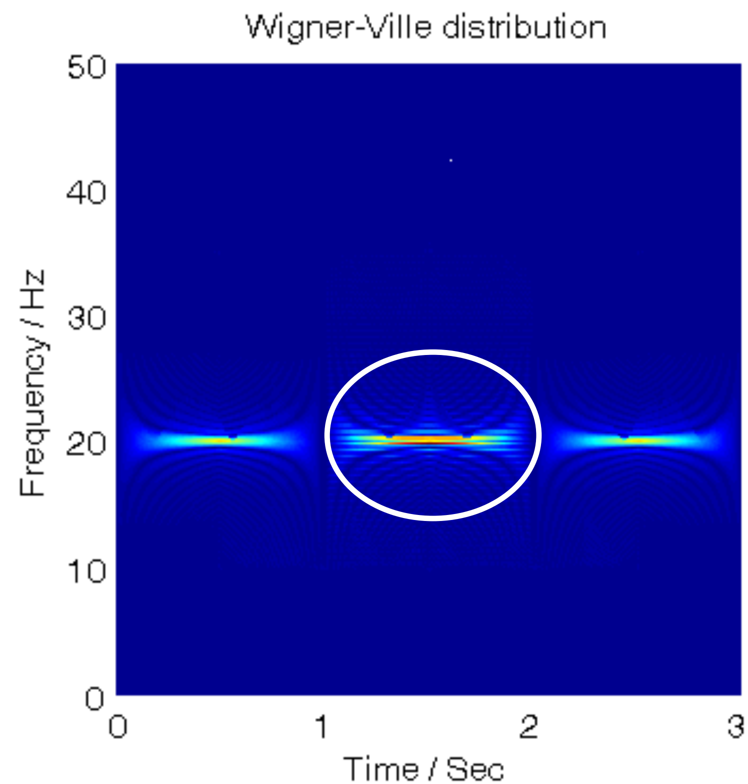
单分量信号

x_2



多分量信号

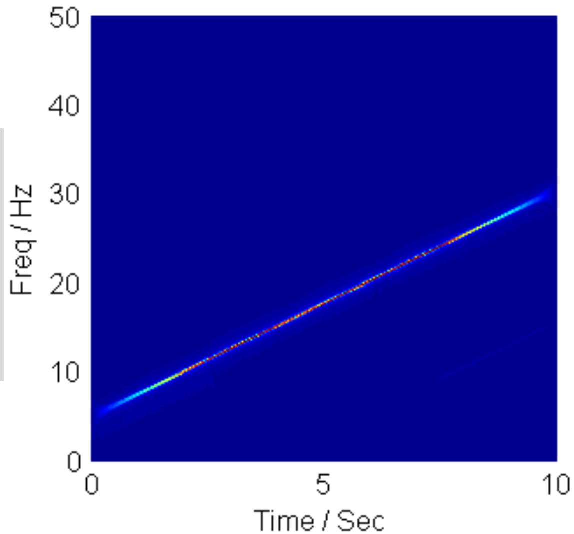
$x=x_1+x_2$



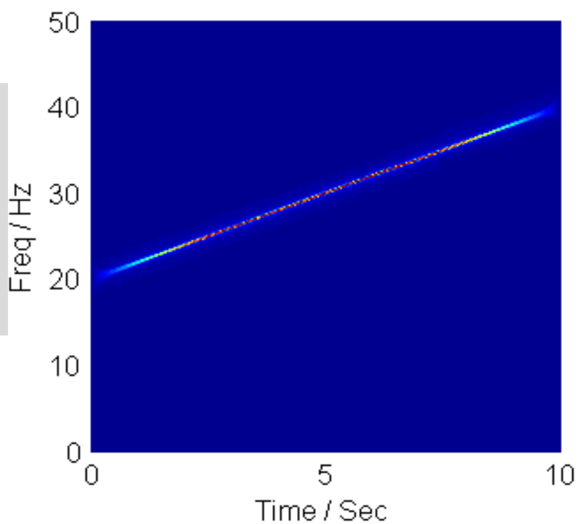
Wigner-Ville 分布

- 交叉项 – 示例二

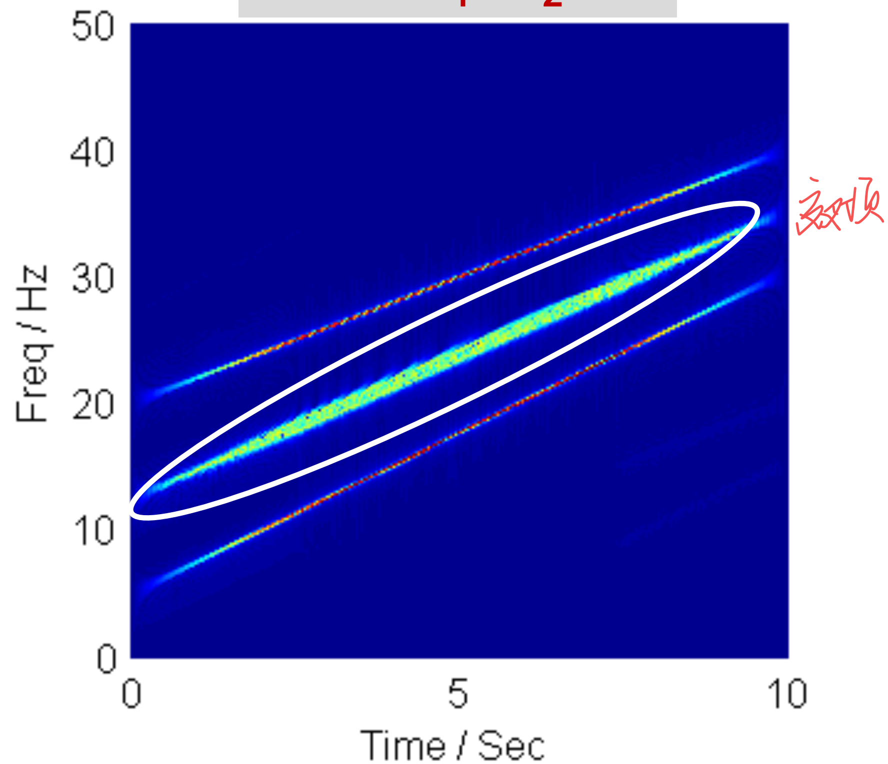
单分量信号
x_1



单分量信号
x_2

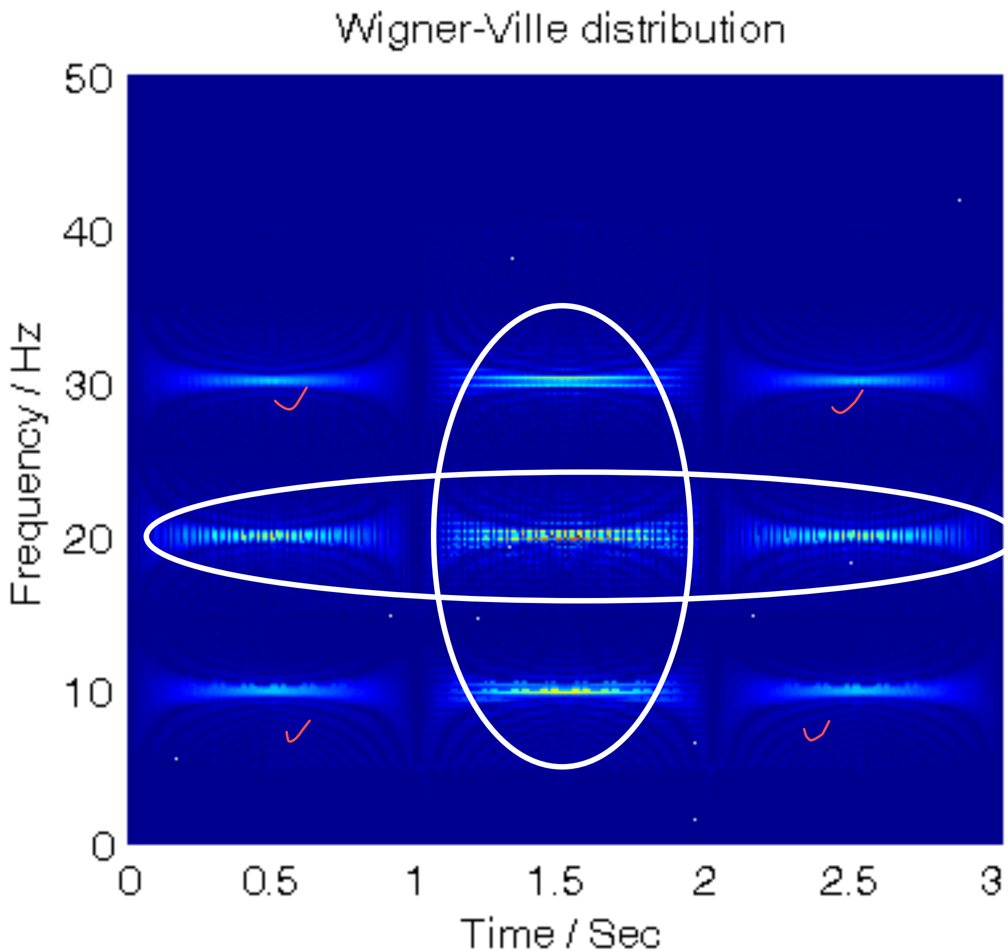


多分量信号
$x=x_1+x_2$



Wigner-Ville 分布

- 交叉项 – 示例三



4个成分

Wigner-Ville 分布

- 交叉项

双线性

➤As the WVD is a bilinear function of the signal , the quadratic superposition principle applies

$$W_{s+x}(t,\omega) = W_s(t,\omega) + W_x(t,\omega) + 2\Re\{W_{s,x}(t,\omega)\}$$

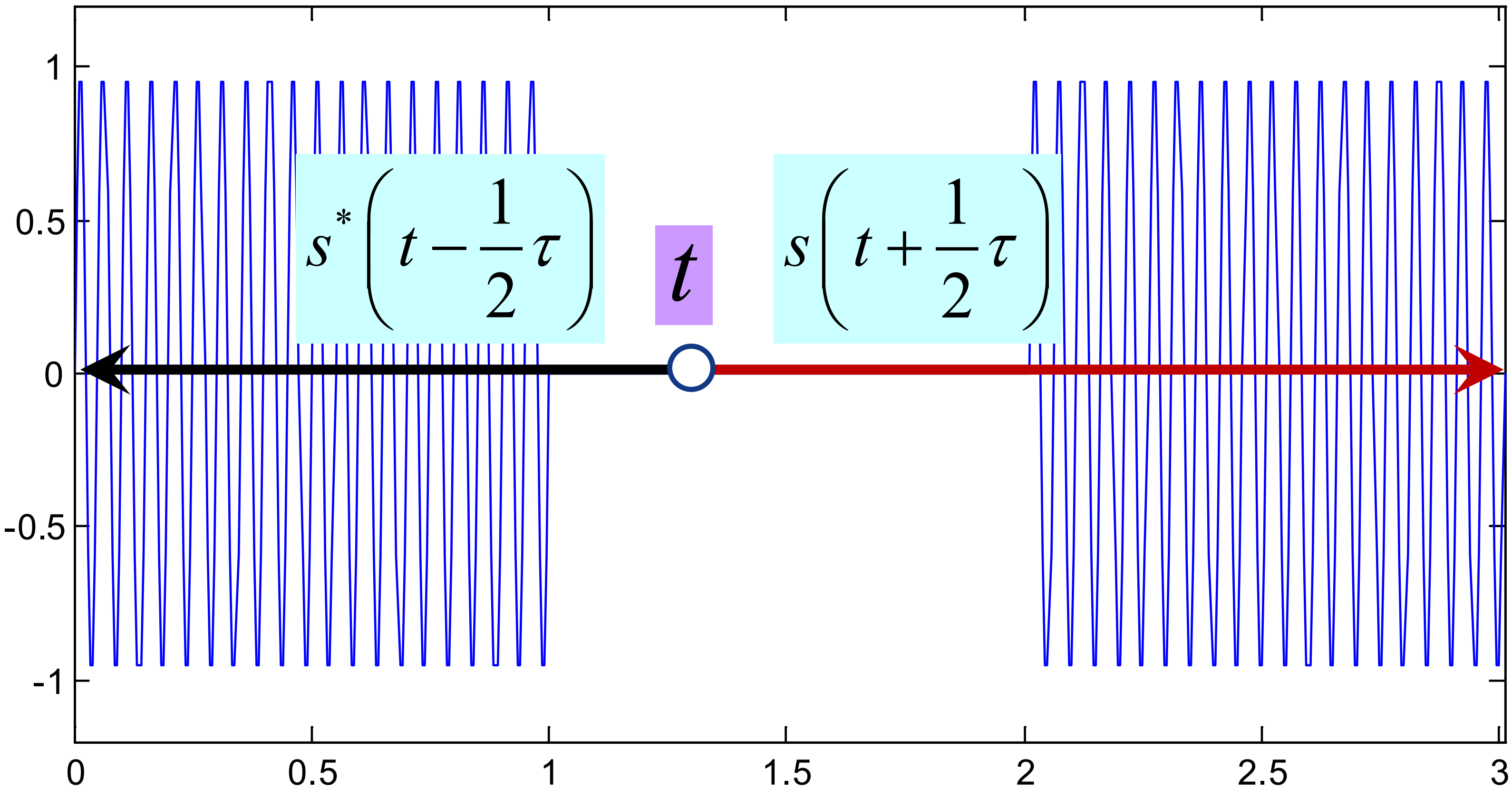
Where

$$W_{s,x}(t,\omega) = \frac{1}{2\pi}\int s(t+\tau/2)x^*(t-\tau/2)e^{-j\omega\tau}d\tau$$

is the cross-WVD of x and s.

Wigner-Ville 分布

- 交叉项



Wigner-Ville 分布

交叉项

➢These interference terms are troublesome since they may overlap with auto-terms (signal terms) and thus make itdifficult to visually interpret the WVD image.

➢ It appears that these terms must be present or the good properties of the WVD (marginal properties, instantaneous frequency and group delay, localization, unitarity, . . .) cannot be satisfied.

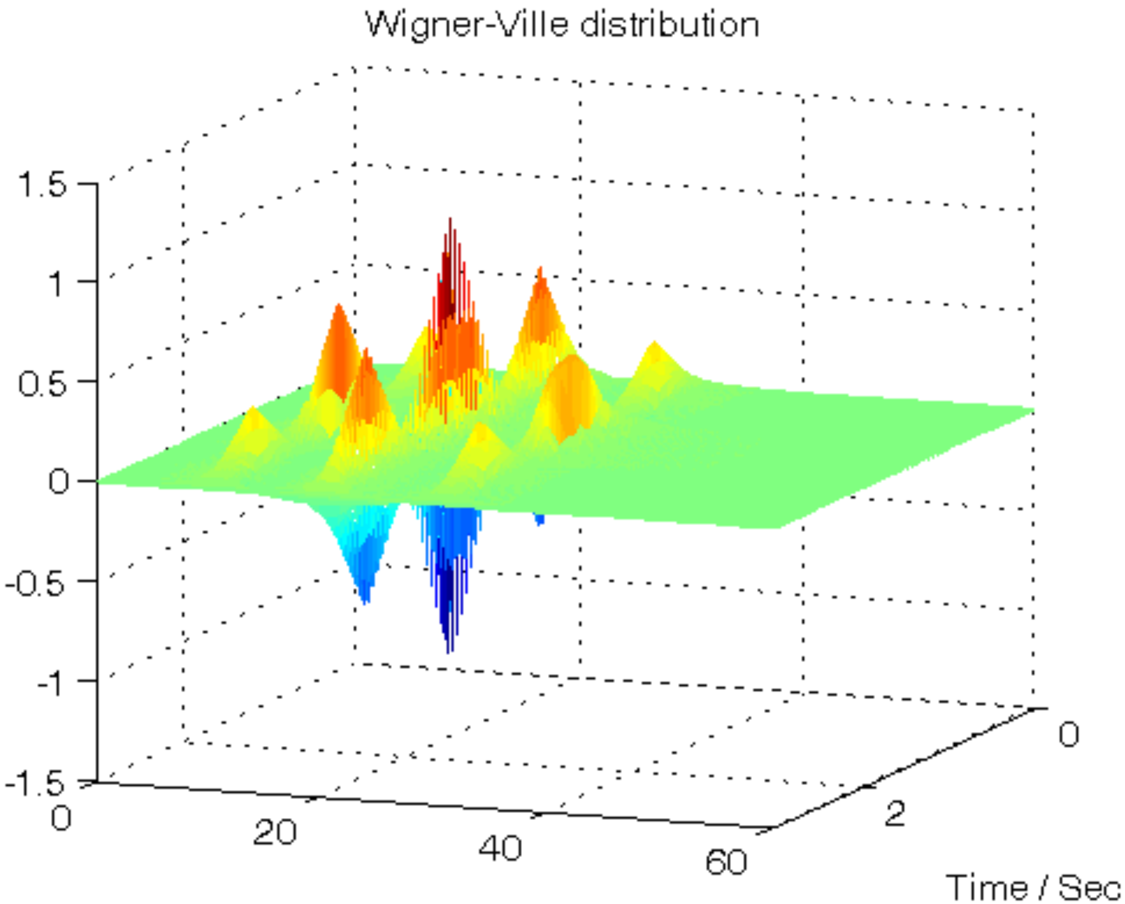
➢The sum of these terms must be zero.

$$\int\int \mathrm{Re}\left(W_{s,x}(t,\omega)\right) = 0$$

Wigner-Ville 分布

- 交叉项

$$\iint \mathrm{Re}\left(W_{s,x}(t,\omega)\right) = 0$$



Pseudo – WVD

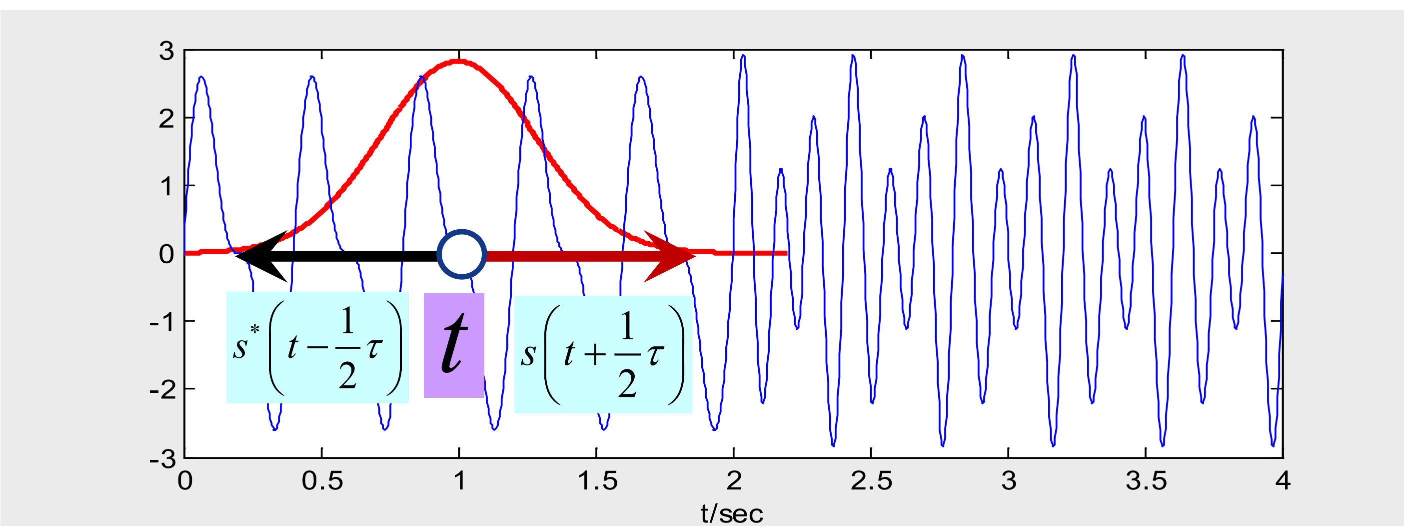
伪 WVD

交叉项抑制

加窗WVD

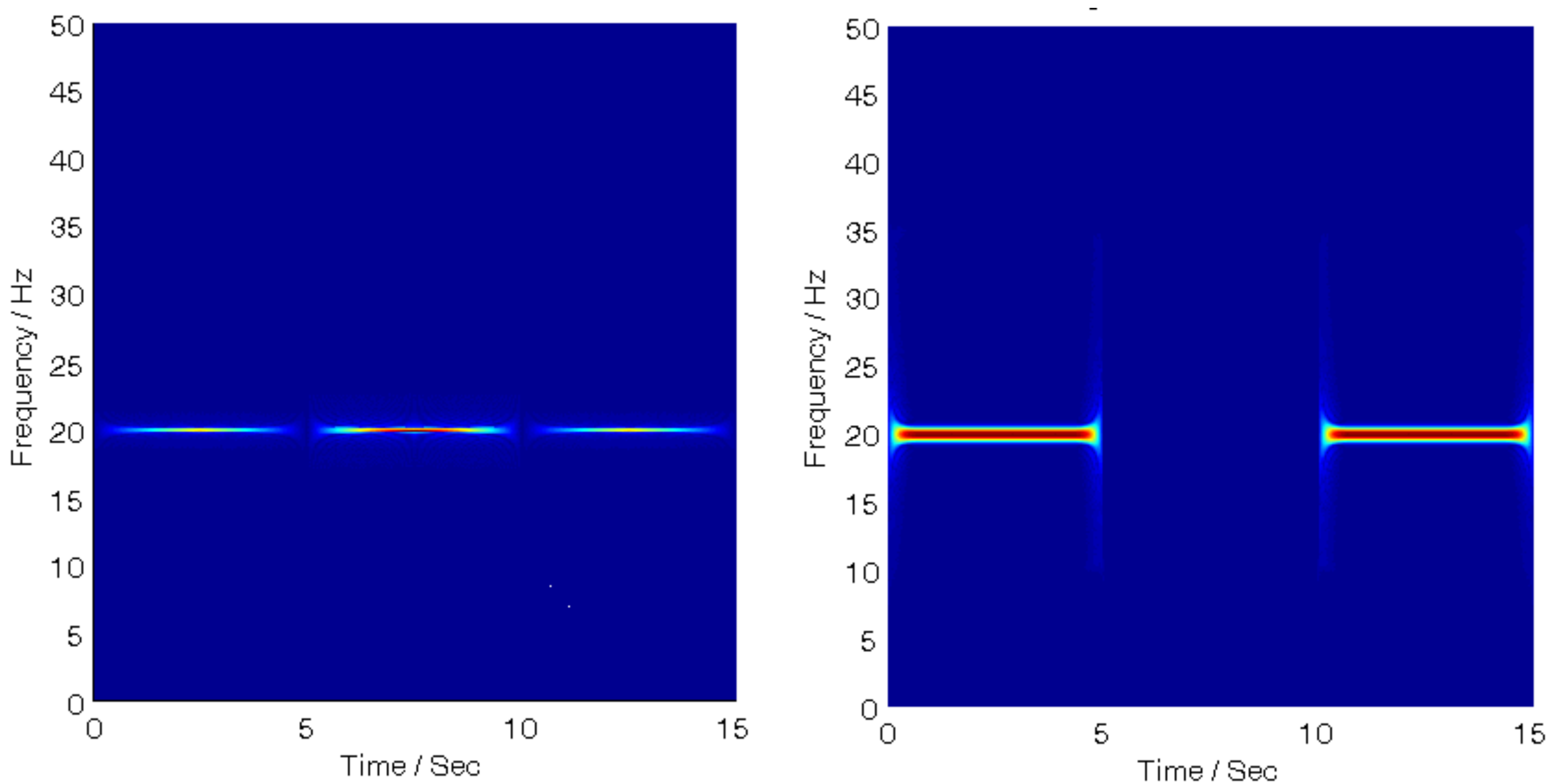
$$PW_x(t,\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}h(\tau)x^*\left(t-\frac{1}{2}\tau\right)x\left(t+\frac{1}{2}\tau\right)e^{-j\tau\omega}d\tau$$

窗函数 $h(t)=e^{-at^2/2}$ ***a*-窗宽调节参数**



Pseudo － WVD

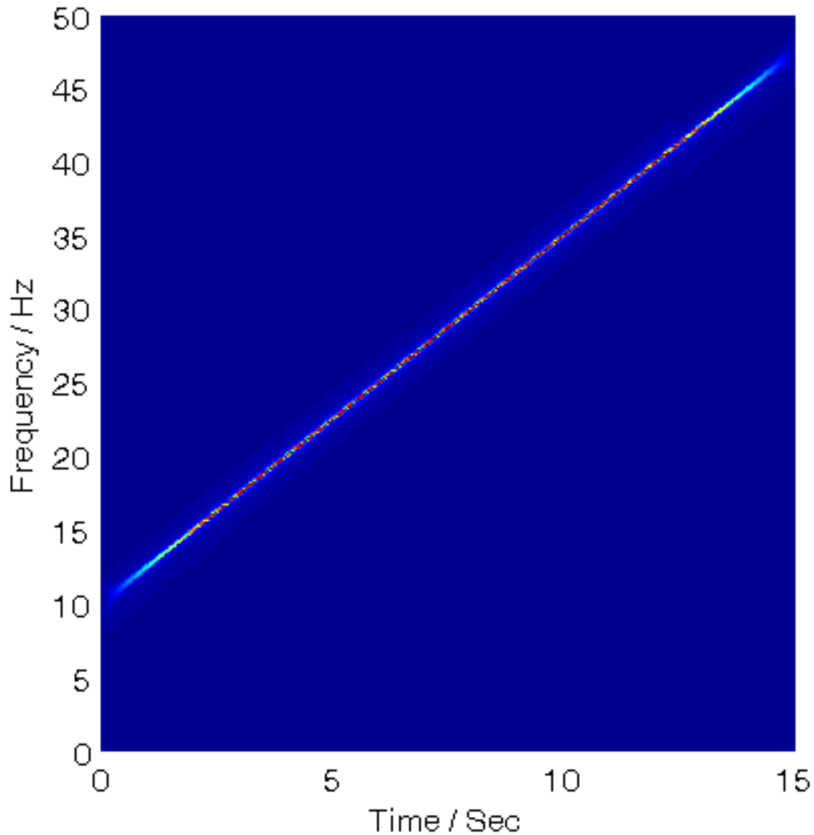
- 交叉项抑制 － 示例1

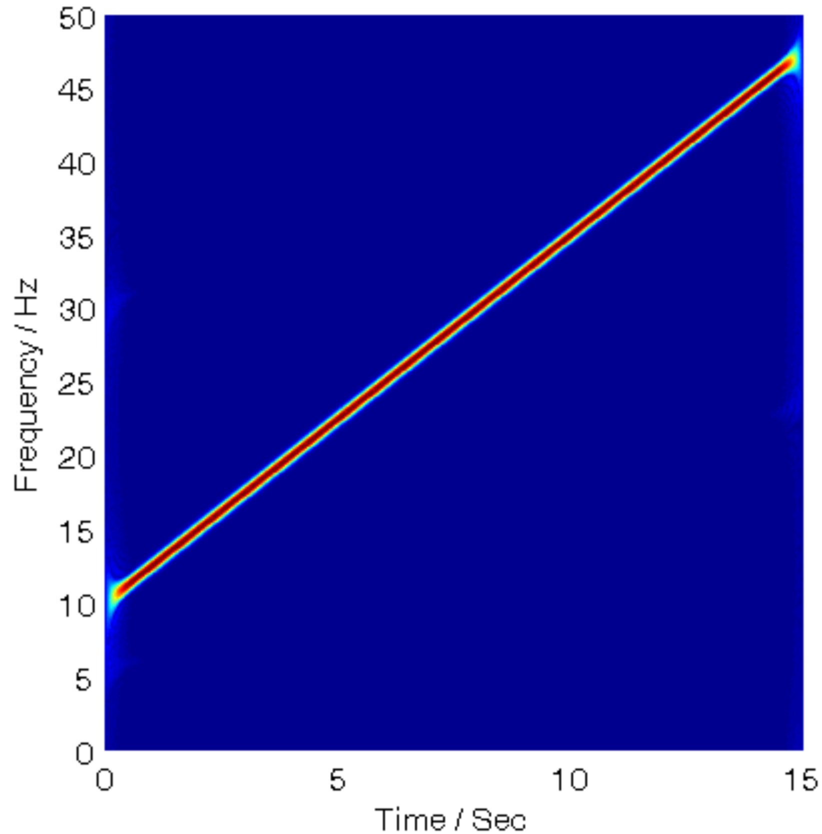


交叉项消失，但集中性变差

Pseudo － WVD

- 交叉项抑制 － 示例2





集中性变差

Pseudo － WVD

- 交叉项抑制

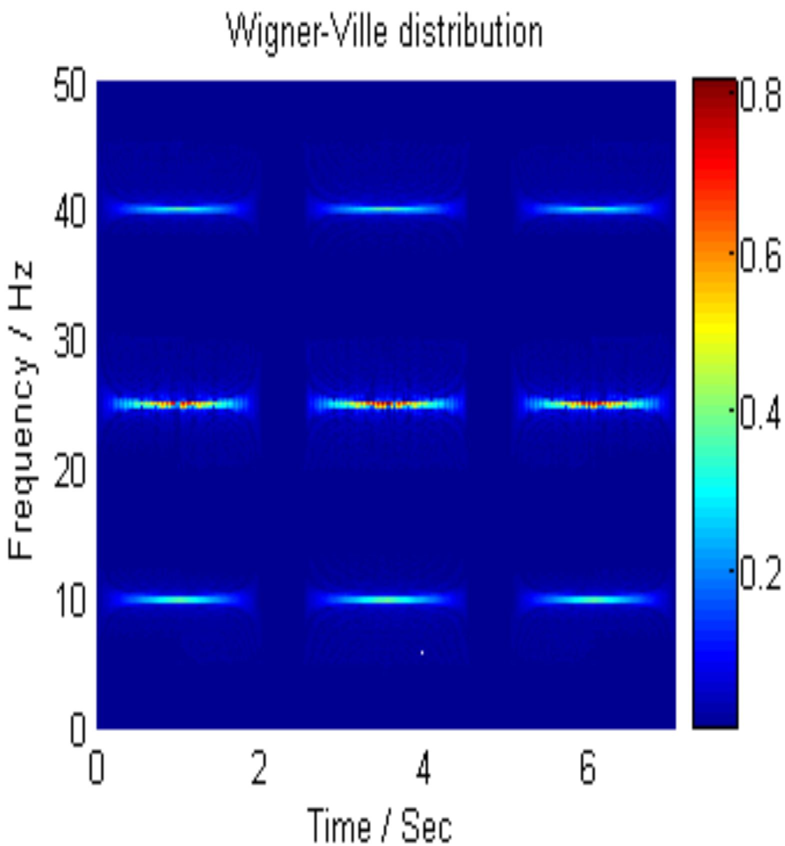
$$x(t) = A_1 e^{j\omega_1 t} + A_2 e^{j\omega_2 t}$$

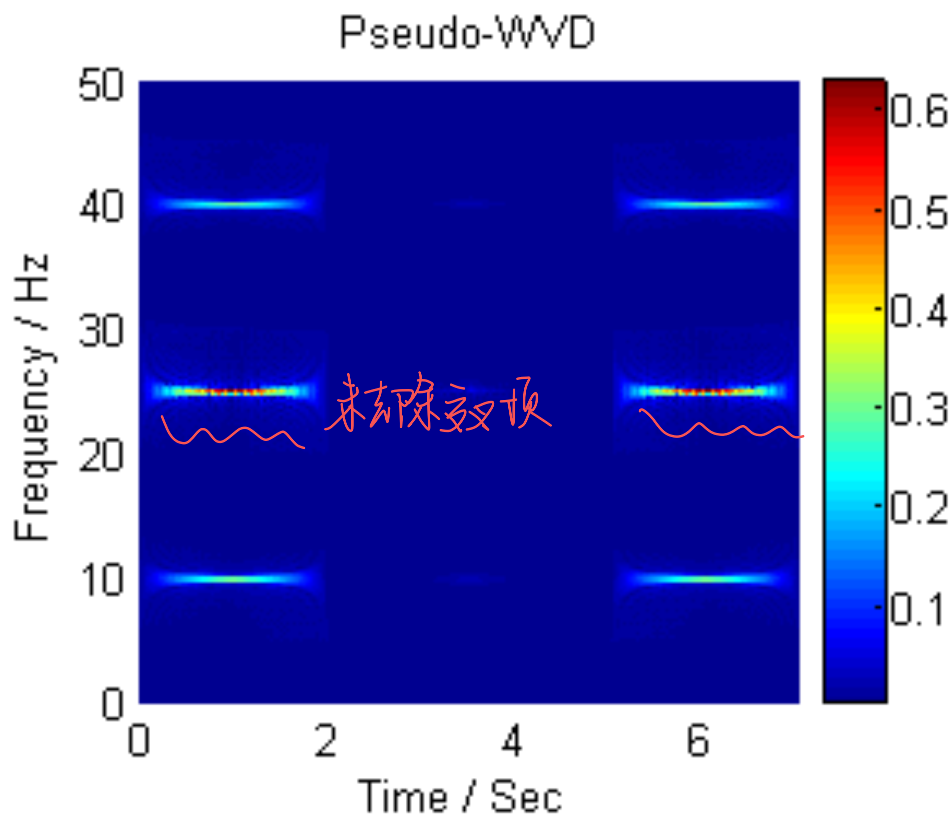
$$PW_x(t,\omega) = \frac{1}{\sqrt{2\pi a}}\left[A_1^2 e^{-(\omega-\omega_1)/(2a)} + A_2^2 e^{-(\omega-\omega_2)/(2a)}\right] + \frac{2A_1A_2}{\sqrt{2\pi a}}\cos((\omega_2-\omega_1)t)e^{-(\omega-(\omega_1-\omega_2)/2)^2/(2a)}$$

α的值增大，交叉项变小，但信号的真实分量也会变小，而且它们随α变小的速率差不多，因此Pseudo-WVD对频率轴方向的交叉项抑制效果不明显.

Pseudo － WVD

- 交叉项抑制-示例3





Smoothed-Pseudo-WVD

平滑伪WVD.

- 交叉项抑制

$$SPW_x(t,\omega)$$

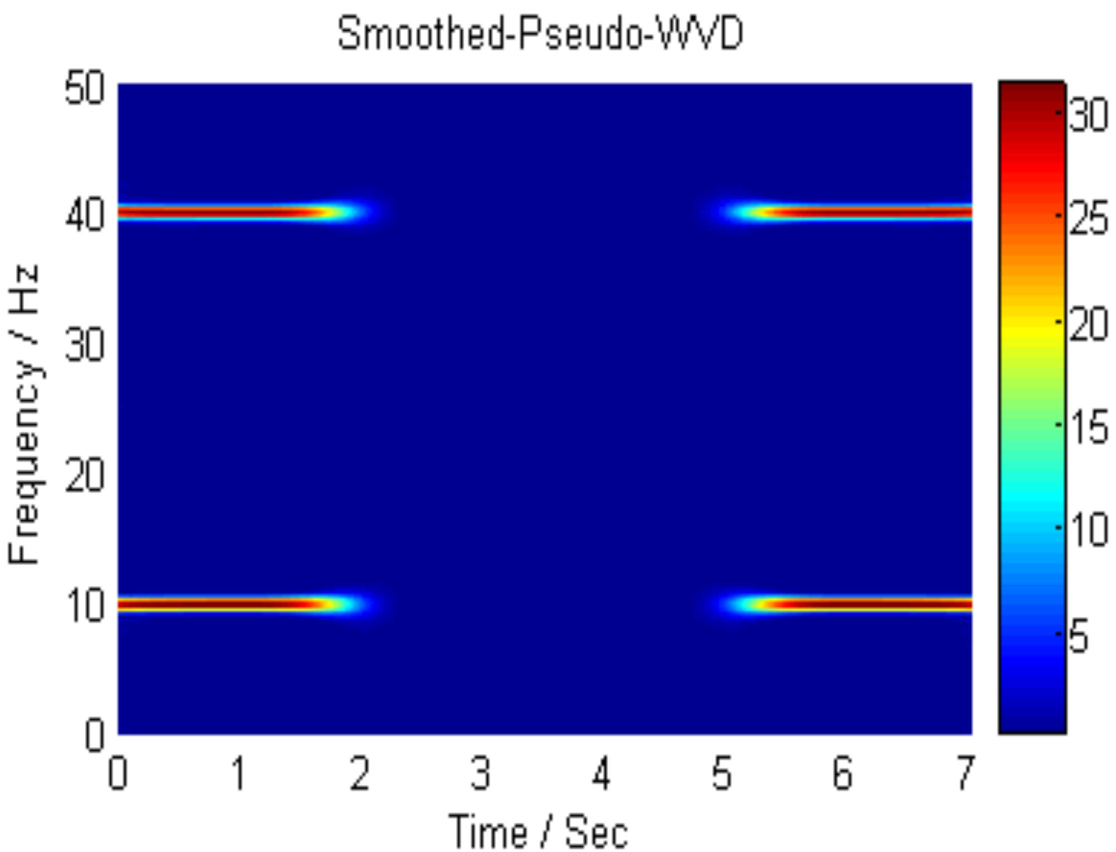
$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty} h(\tau)\left(\int_{-\infty}^{+\infty} g(s-t)x^*\left(t-\frac{1}{2}\tau\right)x\left(t+\frac{1}{2}\tau\right)ds\right)e^{-j\tau\omega}d\tau$$

窗函数 $\boldsymbol{h}$ – 抑制时间轴方向的交叉项

窗函数 g – 抑制频率轴方向的交叉项

Smoothed-Pseudo-WVD

- 交叉项抑制



谢谢聆听
欢迎交流