HW2

2020年10月6日 星期二 09:55

Problem 1.

① $\nabla f(x^*) = [1, 1]^T \neq 0$, and $x^* = [1, 2]^T$, $\{\Omega : x_2 \geq 1\}$

it is definitively not a local minimizer

② $x^* = [1, 2]^T$ and $\{\Omega : x_1 \geq 1, x_2 \geq 2\}$, $\nabla f(x^*) = [1, 0]^T$

FONC: $d_1 \geq 0$, $d_2 \geq 0$, $\nabla f^T(x^*) \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \geq 0$

it is a definitively local minimizer

③ $\nabla f(x^*) = [0\ 0]^T$. x^* is a interior point, $F(x^*) > 0$

SOSC: it is definitively a local minimizer

④ $x^* = [1, 2]^T$ and $\{\Omega : x_1 \geq 1, x_2 \geq 2\}$. $\nabla f(x^*) = [1, 0]^T$

SONC: $d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\nabla f^T(x^*) \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = 0$

$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = -1 < 0$

so it it definitively not a local minimizer

Problem 2.

① $f(x) = [x_1\ x_2] \begin{bmatrix} 2 & 5 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [x_1\ x_2] \begin{bmatrix} 3 \\ 4 \end{bmatrix} + 7$

$= 2x_1^2 + 4x_1x_2 + x_2^2 + 3x_1 + 4x_2 + 7$

$\nabla f(x) = \left[\frac{\partial f}{\partial x_1}\ \frac{\partial f}{\partial x_2}\right]^T = \begin{bmatrix} 4x_1 + 4x_2 + 3 \\ 2x_2 + 4x_1 + 4 \end{bmatrix}$

$\nabla f^T(x)\Big|_{x=[0,1]^T} d = [7\ 6] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 7$

② FSOC: $\begin{matrix} 4x_1 + 4x_2 + 3 = 0 \\ 2x_2 + 4x_1 + 4 = 0 \end{matrix} \Rightarrow \begin{matrix} x_1 = -\frac{5}{4} \\ x_2 = \frac{1}{2} \end{matrix}$, $x^* = \begin{bmatrix} -\frac{5}{4} \\ \frac{1}{2} \end{bmatrix}$, a interior point

$H(x^*) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 2 \end{bmatrix}$, not positive, not satisfy SONC.

so f doesn't have a local minimizer

Problem 3

$\nabla f(x) = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$ $H(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. $\{\Omega = x_1 + x_2^2 \leq 2\}$

① $x^* = [2, 0]^T$, feasible direction $d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$, $d_1 \leq 0$, d_2 arbitrary

$\nabla f^T(x^*) \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = -3d_1 \geq 0$

so. it satisfy the FONC

② $H(x^*) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}^T \geq 0$, so it satisfy the SONC

③ $x^* = [2\ 0]^T$ is a local minimizer

Problem 4.

$\nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 4(x_1 - x_2)^3 + 2x_1 - 2 \\ -4(x_1 - x_2)^3 - 2x_2 + 2 \end{bmatrix}$

FONC: $\frac{\partial f}{\partial x_1} = 0$, $\frac{\partial f}{\partial x_2} = 0 \Rightarrow x^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$H(x^*) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}\Bigg|_{x=x^*} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$, not positive

so it not satisfy SONC

Problem 5.

$$\left.\begin{aligned} f(x) &= \frac{1}{P} \sum_{i=1}^{P} \| \hat{x} - x^{(i)} \|^2 \\ \frac{\partial f}{\partial x_i} &= \frac{2}{P} \sum_{k=1}^{P} (x_i - x_i^{(k)}) \\ \frac{\partial^2 f}{\partial x_i \partial x_j} &= 0 \quad (i \neq j) \\ \frac{\partial^2 f}{\partial x_i^2} &= 2 \end{aligned}\right\} \Rightarrow H(x) = \begin{bmatrix} 2 & & & \\ & 2 & & \\ & & \ddots & \\ & & & 2 \end{bmatrix}$$

so $H(x)$ is positive and satisfy SOSC.

The centroid of p point defines as: $\frac{x^{(1)} + x^{(2)} + \cdots + x^{(P)}}{P} = \begin{bmatrix} \frac{x_1^{(1)} + x_1^{(2)} + \cdots + x_1^{(N)}}{P} \\ \vdots \\ \frac{x_i^{(1)} + x_i^{(2)} + \cdots + x_i^{(N)}}{P} \\ \vdots \end{bmatrix}$

$\hat{x}$ is calculate by $\frac{\partial f}{\partial x_i} = 0 = \frac{2}{P} \sum_{k=1}^{P} (x_i - x_i^{(k)})$

$\Rightarrow x_i = \frac{x_i^{(1)} + x_i^{(2)} + \cdots + x_i^{(P)}}{P}$

So $\hat{x}$ is the centroid

Problem 6.

$\nabla f(x) = Qx - b$

① $\Rightarrow$ x^* satisfy FONC. $\nabla f(x^*) = 0 \Rightarrow Qx^* = b$

$H(x^*) = Q$, Q is positive

according to SOSC, x^* is a local minimizer

in fact, x^* is a global minimizer

so x^* minimize f

② $\Leftarrow$ x^* minimize f. x^* is a interior point

for any direction d

$f(x^* + d) \geq f(x^*)$, Taylor expension

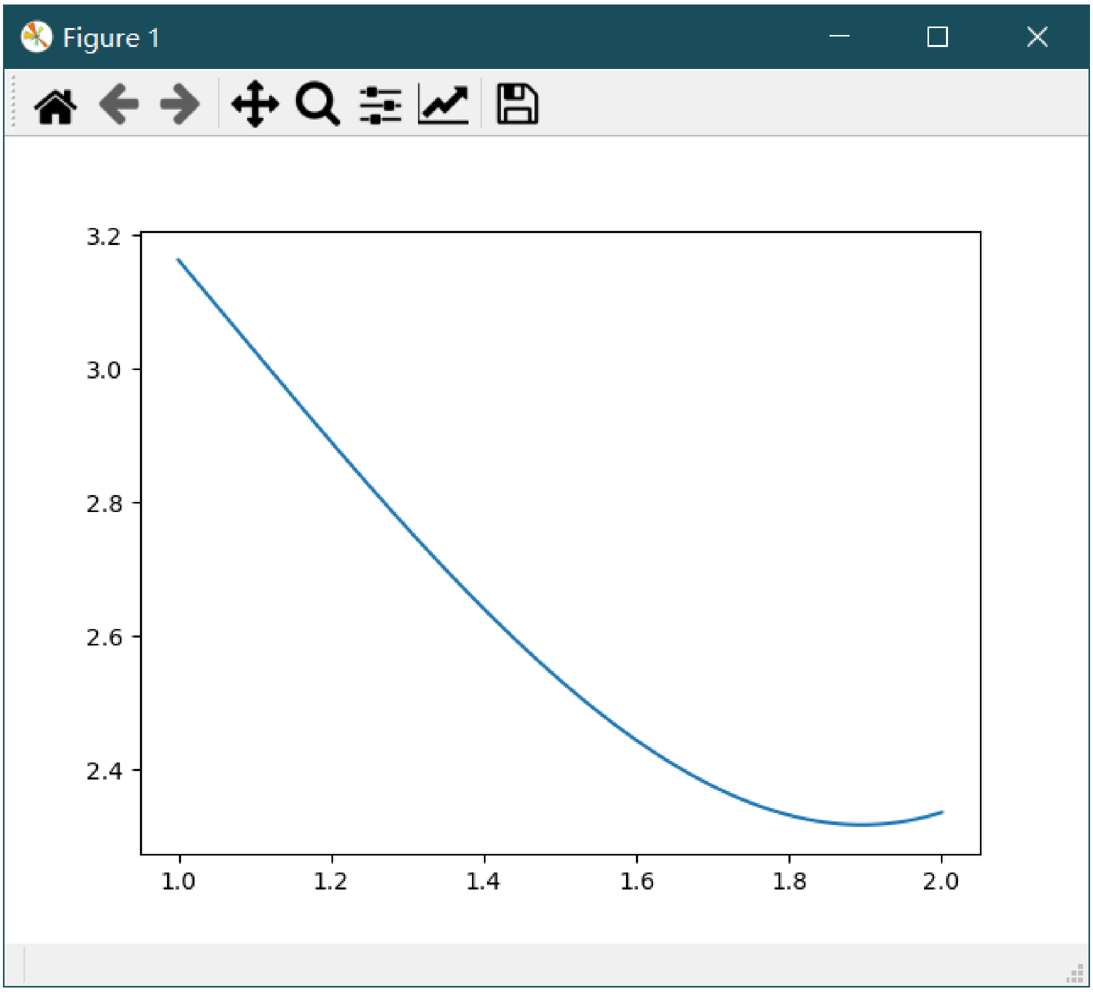
$f(x^* + d) = f(x^*) + d^T \nabla f(x^*) + o(\|d\|^2) \geq f(x^*)$

So $d^T \nabla f(x^*) \geq 0 \Rightarrow$ FONC

Problem 7.

Problem 7 & Problem 8

Codes are attached.

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PS C:\Users\zzw\Desktop\hw\Optimization> python .\hw2_problem7.py
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Iteration k	ak	bk	f(ak)	f(bk)	New uncertainty interval
0	1	2	3.161209223472559	2.3354126538114306	[1, 2]
1	1.3820000000000001	1.6179999999999999	2.660630985058765	2.429179418434649	[1.3820000000000001, 2]
2	1.6179999999999999	1.763924	2.429179418434649	2.343710459527045	[1.6179999999999999, 2]
3	1.763924	1.854076	2.343710459527045	2.319573396142112	[1.763924, 2]
4	1.854076	1.909818968	2.319573396142112	2.317146396751533	[1.854076, 2]

Iteration k	xk(Newton1st)	f(xk)
0	1	3.161209223472559
1	1.2314405443006367	2.8479645498370587
2	1.4490091416772217	2.5855728845877293
3	1.6901209926731962	2.3803421599033046
4	2.092671582017154	2.3852477674664057
5	1.7603716497126456	2.3451409720508267

Iteration k	xk(Newton2nd)	f(xk)
0	1	3.161209223472559
1	-7.4727406398312315	57.33014327366494
2	14.478520982873837	208.28851659077012
3	6.935115408046103	51.275482705508004
4	16.63568412143092	274.34734843892346
5	8.34393754931617	67.7389458654033

PS C:\Users\zzw\Desktop\hw\Optimization> python .\hw2_problem8.py

Iteration k	x0	x1
Iteration 0	x0=0	x1=1
Iteration 1	x1=1	x2=6.030862576000828e-08
Iteration 2	x2=6.030862576000828e-08	x3=1.2061445465253277e-07
Iteration 3	x3=1.2061445465253277e-07	x4=0.0013021502646188051
Iteration 4	x4=0.0013021502646188051	x5=0.0018503987473188718
Iteration 5	x5=0.0018503987473188718	x6=0.002381502809472576
Iteration 6	x6=0.002381502809472576	x7=0.0027475406654164105
Iteration 7	x7=0.0027475406654164105	x8=0.0030339486001054457
Iteration 8	x8=0.0030339486001054457	x9=0.003247321447495228
Iteration 9	x9=0.003247321447495228	x10=0.0034094181887987453
Iteration 10	x10=0.0034094181887987453	x11=0.0035318503094257173
Iteration 11	x11=0.0035318503094257173	x12=0.0036249090563946602
Iteration 12	x12=0.0036249090563946602	x13=0.0036961508185400045
Iteration 13	x13=0.0036961508185400045	x14=0.0037517837212720533
Iteration 14	x14=0.0037517837212720533	x15=0.0037972734727107745
Iteration 15	x15=0.0037972734727107745	x16=0.003838895123668396
Iteration 16	x16=0.003838895123668396	x17=0.0038898895296839197
Iteration 17	x17=0.0038898895296839197	x18=0.004050296550614562
Iteration 18	x18=0.004050296550614562	x19=0.003939275554084987
Iteration 19	x19=0.003939275554084987	x20=0.003972697925030309
Iteration 20	x20=0.003972697925030309	x21=0.004057272781496471
Iteration 21	x21=0.004057272781496471	x22=0.003989755791974867
Iteration 22	x22=0.003989755791974867	x23=0.003997977735150837
Iteration 23	x23=0.003997977735150837	x24=0.004004949573289028
Iteration 24	x24=0.004004949573289028	x25=0.004003873910240935
Iteration 25	x25=0.004003873910240935	x26=0.004003937621816487
Iteration 26	x26=0.004003937621816487	x27=0.004003938289811941

Last x = 0.004003938289811941, f(x)=0.9844488981267419