

(1) 列表:

$X(k)$	$u(k)$	$X(k+1) = X(k) + u(k)$	$J = J_0 + V(X(k+1), k+1)$	$V = \min J$
$X(3)$				
2.0				4.00
1.5				2.25
1.0				1.00
0.5				0.25
0.0				0.00
$X(2)$	$u(2)$	$X_3 = X(2) + u(2)$	$J = X(2)^2 + u(2)^2 + V(X(3), 3)$	$V(X(2), u_2)$
2.0	-0.5 0.0 0.5	1.5 2.0 2.5	$(2.0)^2 + (0.5)^2 + (1.5)^2$ $(2.0)^2 + (0.0)^2 + (2.0)^2$ 非容许控制	$V(2, -0.5) = 6.5$
1.5	-0.5 0.0 0.5	1.0 1.5 2.0	$(1.5)^2 + (0.5)^2 + (1.0)^2$ $(1.5)^2 + (0.0)^2 + (1.5)^2$ $(1.5)^2 + (0.5)^2 + (2.0)^2$	$V(1.5, -0.5) = 3.5$
1.0	-0.5 0.0 0.5	0.5 1.0 1.5	$(1.0)^2 + (0.5)^2 + (0.5)^2$ $(1.0)^2 + (0.0)^2 + (1.0)^2$ $(1.0)^2 + (0.5)^2 + (1.5)^2$	$V(1.0, -0.5) = 1.5$
0.5	-0.5 0.0 0.5	0.0 0.5 1.0	$(0.5)^2 + (0.5)^2 + (0.0)^2$ $(0.5)^2 + (0.0)^2 + (0.5)^2$ $(0.5)^2 + (0.5)^2 + (1.0)^2$	$V(0.5, -0.5) = 0.5$ $V(0.5, 0.0) = 0.5$
0.0	-0.5 0.0 0.5	-0.5 0.0 0.5	非容许控制 $(0.0)^2 + (0.0)^2 + (0.0)^2$ $(0.0)^2 + (0.5)^2 + (0.5)^2$	$V(0.0, 0.0) = 0.0$

$X(k)$	$u(k)$	$X(k+1) = X(k) + u(k)$	$J = J_0 + V(X(k+1), k+1)$	$V = \min J$
$X(1)$	$u(1)$	$X_2 = X(1) + u(1)$	$J = X(1)^2 + u(1)^2 + V(X(2), 2)$	$V(X(1), u(1))$
2.0	-0.5 0.0 0.5	1.5 2.0 2.5	$(2.0)^2 + (0.5)^2 + 3.5$ $(2.0)^2 + (0.0)^2 + 6.5$ 非容许控制	$V(2, -0.5) = 7.75$
1.5	-0.5 0.0 0.5	1.0 1.5 2.0	$(1.5)^2 + (0.5)^2 + 1.5$ $(1.5)^2 + (0.0)^2 + 3.5$ $(1.5)^2 + (0.5)^2 + 6.5$	$V(1.5, -0.5) = 4$
1.0	-0.5 0.0 0.5	0.5 1.0 1.5	$(1.0)^2 + (0.5)^2 + 0.5$ $(1.0)^2 + (0.0)^2 + 1.5$ $(1.0)^2 + (0.5)^2 + 3.5$	$V(1, -0.5) = 1.75$
0.5	-0.5 0.0 0.5	0.0 0.5 1.0	$(0.5)^2 + (0.5)^2 + 0.0$ $(0.5)^2 + (0.0)^2 + 0.5$ $(0.5)^2 + (0.5)^2 + 1.5$	$V(0.5, -0.5) = 0.5$
0.0	-0.5 0.0 0.5	-0.5 0.0 0.5	非容许控制 $(0.0)^2 + (0.0)^2 + 0.0$ $(0.0)^2 + (0.5)^2 + 0.5$	$V(0.0, 0.0) = 0.0$

$X(k)$	$u(k)$	$X(k+1) = X(k) + u(k)$	$J = J_0 + V(X(k+1), k+1)$	$V = \min J$
$X(0)$	$u(0)$	$X_1 = X(0) + u(0)$	$J = X(0)^2 + u(0)^2 + V(X(1), 1)$	$V(X(0), u(0))$
2.0	-0.5 0.0 0.5	1.5 2.0 2.5	$(2.0)^2 + (0.5)^2 + 4$ $(2.0)^2 + (0.0)^2 + 7.75$ 非容许控制	$V(2.0, -0.5) = 8.25$
1.5	-0.5 0.0 0.5	1.0 1.5 2.0	$(1.5)^2 + (0.5)^2 + 1.75$ $(1.5)^2 + (0.0)^2 + 4$ $(1.5)^2 + (0.5)^2 + 7.75$	$V(1.5, -0.5) = 4.25$
1.0	-0.5 0.0 0.5	0.5 1.0 1.5	$(1.0)^2 + (0.5)^2 + 0.5$ $(1.0)^2 + (0.0)^2 + 1.75$ $(1.0)^2 + (0.5)^2 + 4$	$V(1, -0.5) = 1.75$
0.5	-0.5 0.0 0.5	0.0 0.5 1.0	$(0.5)^2 + (0.5)^2 + 0.0$ $(0.5)^2 + (0.0)^2 + 0.5$ $(0.5)^2 + (0.5)^2 + 1.75$	$V(0.5, -0.5) = 0.5$
0.0	-0.5 0.0 0.5	-0.5 0.0 0.5	非容许控制 $(0.0)^2 + (0.0)^2 + 0.0$ $(0.0)^2 + (0.5)^2 + 0.5$	$V(0.0, 0.0) = 0.0$

查表可得对于 $X(0) = 0.0$, $\{u(0) = 0.0, u(1) = 0.0, u(2) = 0.0\}$ 使 J 最小为 0对于 $X(0) = 0.5$, $\{u(0) = -0.5, u(1) = 0.0, u(2) = 0.0\}$ 使 J 最小为 0.5对于 $X(0) = 1.0$, $\{u(0) = -0.5, u(1) = -0.5, u(2) = 0.0\}$ 使 J 最小为 1.75对于 $X(0) = 1.5$, $\{u(0) = -0.5, u(1) = -0.5, u(2) = -0.5 \text{ 或 } u(2) = 0.0\}$ 使 J 最小为 4.25对于 $X(0) = 2.0$, $\{u(0) = -0.5, u(1) = -0.5, u(2) = -0.5\}$ 使 J 最小为 8.25

(2)

$$V(f) = 0$$

$$F: V(F) = J(F, f) + V(f) = 12 + 0 = 12$$

$$\phi(F) = f$$

$$G: V(G) = \min(J(G, F) + V(F), J(G, f) + V(f)) = \min(8 + 12, 15) = 15$$

$$\phi(G) = f$$

$$H: V(H) = \min(J(H, G) + V(G), J(H, f) + V(f)) = \min(12 + 15, 21) = 21$$

$$\phi(H) = f$$

$$C: V(C) = J(C, F) + V(F) = 20 + 12 = 32$$

$$\phi(C) = F$$

$$E: V(E) = \min(J(E, G) + V(G), J(E, H) + V(H)) = \min(17 + 15, 12 + 21) = 32$$

$$\phi(E) = G$$

$$D: V(D) = \min(J(D, C) + V(C), J(D, F) + V(F), J(D, G) + V(G), J(D, E) + V(E)) = \min(10 + 32, 23 + 12, 19 + 15, 16 + 32) = 34$$

$$\phi(D) = G$$

$$B: V(B) = \min(J(B, D) + V(D), J(B, E) + V(E)) = \min(15 + 34, 18 + 32) = 49$$

$$\phi(B) = D$$

$$A: V(A) = \min(J(A, C) + V(C), J(A, D) + V(D), J(A, B) + V(B)) = \min(12 + 32, 11 + 34, 18 + 49) = 44$$

$$\phi(A) = C$$

$$S: V(S) = \min(J(S, A) + V(A), J(S, B) + V(B)) = \min(10 + 44, 15 + 49) = 64$$

$$\phi(S) = A \text{ 或 } B$$

最短路径为 $S \rightarrow A \rightarrow C \rightarrow F \rightarrow f$ 或 $S \rightarrow B \rightarrow D \rightarrow G \rightarrow f$