HW4

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Problem 1.

$A=\begin{bmatrix}1&2\\1&1\\3&2\end{bmatrix}$, $b=\begin{bmatrix}6\\4\\5\end{bmatrix}$, $X=\begin{bmatrix}a\\b\end{bmatrix}$

$X^*=(A^TA)^{-1}A^Tb=\begin{bmatrix}11&9\\9&9\end{bmatrix}^{-1}\begin{bmatrix}32\\26\end{bmatrix}=\begin{bmatrix}-\frac{1}{2}\\61/18\end{bmatrix}$

Problem 2.

$\begin{bmatrix}1\\2\\8\end{bmatrix}=\begin{bmatrix}0&1\\1&1\\2&1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}$

$\begin{bmatrix}a\\b\end{bmatrix}^*=(A^TA)^{-1}A^Tb=\begin{bmatrix}5&3\\3&3\end{bmatrix}^{-1}\begin{bmatrix}18\\11\end{bmatrix}=\begin{bmatrix}7/2\\1/6\end{bmatrix}$

Problem 3.

minimize $\frac{1}{2}X^TX-X^Tb \iff$ minimize $\frac{1}{2}X^TX-X^Tb+\frac{1}{2}b^Tb$

$=\frac{1}{2}\|X-b\|^2$

consider minimize $\frac{1}{2}\|X-b\|^2$, $X\in R(A)$, $X=Ay$

$\therefore$ minimize $\frac{1}{2}\|Ay-b\|^2 \Rightarrow y^*=(A^TA)^{-1}A^Tb$

$X^*=A(A^TA)^{-1}A^Tb$

Problem 4.

$f(x,\lambda)=x_1^2+(x_1+3)^2+x_3^2+\lambda(x_1+x_2+x_3-1)$

$\frac{\partial f}{\partial x}=\begin{bmatrix}2x_1+\lambda\\2x_2+6+\lambda\\2x_3+\lambda\end{bmatrix}=0$, $\frac{\partial f}{\partial \lambda}=x_1+x_2+x_3-1=0$

$\therefore$ $x_1=-\frac{\lambda}{2}$ $x_2=\frac{-6-\lambda}{2}$ $x_3=-\frac{\lambda}{2}$

$\frac{\partial f}{\partial \lambda}=-\frac{1}{2}\lambda-3=1 \Rightarrow \lambda=-8$

$X^*=\begin{bmatrix}4\\-1\\4\end{bmatrix}$

Problem 5.

suppose CX^* is not the minimizer of $\|By-b\|^2$, so exists $\hat{y}$

$\|B\hat{y}-b\|^2<\|By-b\|^2=\|BCX^*-b\|^2=\|AX^*-b\|^2$

since C is full rank, so exists $\hat{x}=C^+\hat{y}$

$\|A\hat{x}-b\|^2=\|AC^+\hat{y}-b\|^2=\|BCC^+\hat{y}-b\|^2=\|B\hat{y}-b\|^2<\|AX^*-b\|^2$

so it contradicts

Problem 6.

(a) minimize $-2x_1-x_2$

subject to $x_1+y_1=2$

$x_1+x_2+y_2=3$

$x_1+2x_2+y_3=5$

$x_1,x_2,y_1,y_2,y_3\geq 0$

(b) $\forall u\in R^n, \exists u_1,\dots,u_n : \|x\|=u_1\cdot u_1+\dots+u_n\cdot u_n$

minimize $C_1(u_1+v_1)+C_2(u_2+v_2)+\dots+C_n(u_n+v_n)$

subject to $A(u-v)=b$

$u_1,v_1,u_2,u_3\dots u_n,v_n\geq 0$

let $z=[u_1,u_2\dots u_n,v_1,v_2\dots v_n]$

$\Rightarrow$ minimize $(C^T,-C^T)z$

subject to $(A,-A)z=b$

$z\geq 0$

Problem 7.

A→C x_1 B→C x_2

A→D x_3 B→D x_4

minimize $x_1+2x_2+3x_3+4x_4$

subject to $x_1+x_3=50$

$x_2+x_4=60$

$x_1+x_2\leq 70$

$x_3+x_4\leq 80$

$x_1,x_2,x_3,x_4\geq 0$

$\Rightarrow$

Standard form:

minimize $x_1+2x_2+3x_3+4x_4$

subject to $x_1+x_3=50$

$x_2+x_4=60$

$x_1+x_2+x_5=70$

$x_3+x_4+x_6=80$

$x_1,x_2,x_3,x_4,x_5,x_6\geq 0$

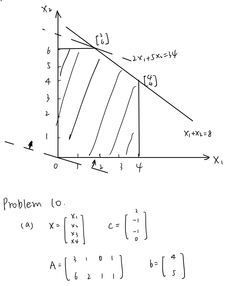
Problem 8.

$A=\left[\begin{array}{cccc|c}2&-1&2&-1&1&14\\1&2&3&1&0&5\\1&0&-2&0&5&-10\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&-0.5&1&-0.5&0.5&7\\0&2.5&2&1.5&-0.5&-3\\0&1&-5&1&7&-34\end{array}\right]$

$\rightarrow\left[\begin{array}{cccc|c}1&0&-2&0&5&-10\\0&1&-4&1&7&-34\\0&0&17&-1&-9&83\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&0&0&-\frac{2}{17}&\frac{47}{17}&\frac{4}{-17}\\0&1&0&\frac{8}{17}&\frac{2}{17}&\frac{-89}{-17}\\0&0&1&-\frac{1}{17}&-\frac{9}{17}&\frac{83}{17}\end{array}\right]$

Columns	Basic Solutions
1, 2, 3	$[-\frac{4}{17},\frac{-89}{17},\frac{83}{17},0,0]^T$
1, 2, 4	$[-10,49,0,-83,0]^T$
1, 2, 5	$[\frac{107}{51},\frac{25}{51},0,0,\frac{83}{51}]^T$
1, 3, 4	$[-\frac{12}{11},0,\frac{49}{11},\frac{-89}{11},0]^T$
1, 3, 5	$[\frac{109}{55},0,\frac{247}{55},0,\frac{83}{55}]^T$
1, 4, 5	$[\frac{61}{78},0,0,\frac{247}{78},\frac{49}{78}]^T$
2, 3, 4	$[0,-6,5,2,0]^T$
2, 3, 5	$[0,-\frac{109}{53},\frac{147}{53},0,\frac{4}{53}]^T$
2, 4, 5	$[0,13,0,-21,2]^T$
3, 4, 5	$[0,0,\frac{83}{149},\frac{-100}{149},\frac{12}{149}]^T$

Problem 9.



Problem 10.

(a) $X=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$ $C=\begin{bmatrix}1\\-1\\-1\\0\end{bmatrix}$

$A=\begin{bmatrix}3&1&0&1\\6&2&1&1\end{bmatrix}$ $b=\begin{bmatrix}4\\5\end{bmatrix}$

(b)

$\left[\begin{array}{cccc|c}3&1&1&0&4\\3&1&0&1&1\end{array}\right]$

(c) $X^*=\begin{bmatrix}0\\0\\4\\1\end{bmatrix}$, $C^TX^*=-4$

Problem 11.

minimize $-2X_1-X_2-0X_3-0X_4-0X_5$

subject to $X_1+X_3=5$

$X_2+X_4=7$

$X_1+X_4+X_5=9$

$X_1,X_2,X_3,X_4,X_5\geq 0$

Simplex method.

a_1	a_2	a_3	a_4	a_5	b
1	0	1	0	0	5
0	1	0	1	0	7
1	1	0	0	1	9

$x=\begin{bmatrix}0\\0\\5\\7\\9\end{bmatrix}$, $z=0$.

$r_1=c_1-z_1=-2-(c_1y_{11}+c_2y_{21}+c_3y_{31})=-2<0$

$r_2=c_2-z_2=-1-(c_1y_{12}+c_2y_{22}+c_3y_{32})=-1<0$

$\min\left\{\frac{y_{10}}{y_{1p}}\right\}=\min\left\{5,9\right\}\Rightarrow 5 \Rightarrow i=1$ $\left(a_1\Rightarrow\begin{pmatrix}1\\0\end{pmatrix}\right)$

$\rightarrow\left[\begin{array}{ccccc|c}1&0&1&0&0&5\\0&1&0&1&0&7\\0&1&-1&0&1&4\end{array}\right]$ $x=\begin{bmatrix}5\\0\\0\\7\\4\end{bmatrix}$, $z=-10$

$r_2=c_2-z_2=-1-(c_1y_{12}+c_2y_{22}+c_3y_{32})=-1$

$r_1=c_1-z_1=0-(c_1y_{11}+c_2y_{21}+c_3y_{31})=2$

$r_4=0-\min\left\{\frac{y_{13}}{y_{23}}\right\}=4\Rightarrow i=3$ $\left(a_3\Rightarrow\begin{pmatrix}0\\1\end{pmatrix}\right)$

$\rightarrow\left[\begin{array}{ccccc|c}1&0&1&0&0&5\\0&1&0&0&0&7\\0&0&-1&-1&1&-3\end{array}\right]$ $x=\begin{bmatrix}5\\7\\0\\0\\-3\end{bmatrix}$ $z=-17$

$r_1=c_1-z_1=0-(c_1y_{11}+c_2y_{21}+c_3y_{31})=2$

$r_4=c_4-z_4=0-(c_1y_{14}+c_2y_{24}+c_3y_{34})=0$

$X^*=\begin{bmatrix}5\\7\\0\\0\\-3\end{bmatrix}$, $z=-17$

$\left[\begin{array}{ccccc|c}1&0&1&0&0&5\\0&1&0&1&0&7\\0&1&-1&0&1&4\end{array}\right]$ $z=-14$ $\begin{bmatrix}5\\4\\0\\3\\0\end{bmatrix}$

$r_4=c_3-z_3=0-(c_1y_{13}+c_2y_{23}+c_3y_{33})=0-(-1)\cdot 1=2$

$r_2=c_2-z_2=0-(c_1y_{12}+c_2y_{22}+c_3y_{32})=0-0=0$