

$$z^{-1} = e^{-Ts}$$

$$E(z) = Z[e^*(t)] = E^*(s) \Big|_{z=e^{Ts}} = \sum_{n=0}^{\infty} e(nT) \cdot z^{-n}$$

## 7.4.2 Methods of z-Transform

- By the definition.
- Partial fraction expansion.

## Properties of z-Transform

1. linear property  $Z[a \cdot e_1^*(t) \pm b \cdot e_2^*(t)] = a \cdot E_1(z) \pm b \cdot E_2(z)$
2. Real shifting theorem 
$$\begin{cases} \text{Lag} & Z[e(t - nT)] = z^{-n} E(z) \\ \text{Lead} & Z[e(t + nT)] = z^n \left[ E(z) - \sum_{k=0}^{n-1} e(kT) \cdot z^{-k} \right] \end{cases}$$
3. Complex shifting theorem  $Z[e(t) \cdot e^{\mp at}] = E(z \cdot e^{\pm aT})$
4. Initial-value theorem  $\lim_{n \rightarrow 0} e(nT) = \lim_{z \rightarrow \infty} E(z)$
5. Final-value theorem  $\lim_{n \rightarrow \infty} e(nT) = \lim_{z \rightarrow 1} (z - 1) \cdot E(z)$
6. Convolution theorem  $c^*(t) = e^*(t) * g^*(t) \Rightarrow C(z) = E(z) \cdot G(z)$

## 7.4.4 Inverse z-Transform

$$Z^{-1}[X(z)] = x(nT)$$

**Tips:**

Inverse Z-transform can only provide discrete-time signal  $x^*(t)$ , instead of continuous signal  $x(t)$ 。

{	Long Division (长除法)	Expansion of $\frac{E(z)}{z}$
	Partial-Fraction expansion	
	Residue (留数法)	

$$e(nT) = \sum \text{Res} [E(z) \cdot z^{n-1}]$$

# **Chapter 7 Analysis and Design of Linear Discrete-Time System (Sampled-data System)**

## **7.1 Introduction**

## **7.2 The Sampling Process and Sampling Theorem**

## **7.3 Signal Recovery and Zero-Order Hold**

## **7.4 Z-Transform and Inverse Z Transform**

## **7.5 Mathematical Models of Discrete-Time Systems**

## **7.6 Performance Analysis of Discrete-Time Systems**

## **7.7 Digital Control Design for Discrete-Time Systems**

## 7.5 Mathematical Models of Discrete-Time Systems

- Difference Equation
- Impulse Transfer function

### 7.5.1 Linear Time-Invariant Difference Equations

(1) Definition of difference  $e(kT) = e(k)$

$$\text{Forward difference} \left\{ \begin{array}{ll} \text{First-order} & \Delta e(k) = e(k+1) - e(k) \\ \text{Second-order} & \Delta^2 e(k) = \Delta e(k+1) - \Delta e(k) \\ & = e(k+2) - 2e(k+1) + e(k) \\ \vdots & \\ \text{nth-order} & \Delta^n e(k) = \Delta^{n-1} e(k+1) - \Delta^{n-1} e(k) \end{array} \right.$$

$$\dot{e}(t) = \frac{de(t)}{dt} = \lim_{T \rightarrow 0} \frac{\Delta e(k)}{T} \approx \frac{\Delta e(k)}{T}$$

$$\text{Backward difference} \left\{ \begin{array}{ll} \text{First-order} & \nabla e(k) = e(k) - e(k-1) \\ \text{Second-order} & \nabla^2 e(k) = \nabla e(k) - \nabla e(k-1) \\ & = e(k) - 2e(k-1) + e(k-2) \\ \vdots & \\ \text{nth-order} & \nabla^n e(k) = \nabla^{n-1} e(k) - \nabla^{n-1} e(k-1) \end{array} \right.$$

$$\dot{e}(t) = \frac{de(t)}{dt} = \lim_{T \rightarrow 0} \frac{\nabla e(k)}{T} \approx \frac{\nabla e(k)}{T}$$

## (2) Difference equation

The equation of the input, output and their higher order differences.

The (forward) difference equation of nth-order linear time-invariant discrete system.

$$\begin{aligned} c(k+n) + a_1 c(k+n-1) + a_2 c(k+n-2) + \cdots + a_{n-1} c(k+1) + a_n c(k) \\ = b_0 r(k+m) + b_1 r(k+m-1) + \cdots + b_{m-1} r(k+1) + b_m r(k) \end{aligned}$$

$$\text{or} \quad c(k+n) = -\sum_{i=1}^n a_i c(k+n-i) + \sum_{j=0}^m b_j r(k+m-j)$$

The (backward) difference equation of n-order linear time-invariant discrete system.

$$\begin{aligned} c(k) + a_1 c(k-1) + a_2 c(k-2) + \cdots + a_{n-1} c(k-n+1) + a_n c(k-n) \\ = b_0 r(k-n+m) + b_1 r(k-n+m-1) + \\ \cdots + b_{m-1} r(k-n+1) + b_m r(k-n) \end{aligned}$$

$$\text{or} \quad c(k) = -\sum_{i=1}^n a_i c(k-i) + \sum_{j=0}^m b_j r(k-j)$$

(3) To solve difference equations:  $\left\{ \begin{array}{l} \text{Iteration method (迭代)} \\ \text{Z-transform method} \end{array} \right.$

**Example 1** The **differential equation** of a continuous system is: 
$$\begin{cases} \ddot{e}(t) - 4\dot{e}(t) + 3e(t) = r(t) = 1(t) \\ e(t) = 0 \quad (t \leq 0) \end{cases}$$

Obtain the corresponding forward difference equation and its solution.

Solution.

$$\dot{e}(t) \approx \frac{\Delta e(k)}{T} = \frac{e(k+1) - e(k)}{T} \stackrel{T=1}{=} e(k+1) - e(k)$$

$$\ddot{e}(t) \approx \frac{\Delta^2 e(k)}{T^2} = \frac{\Delta e(k+1)/T - \Delta e(k)/T}{T} \stackrel{T=1}{=} e(k+2) - 2e(k+1) + e(k)$$

$$\begin{array}{l} e(k+2) - 2e(k+1) + e(k) \\ - 4 [ \quad \quad \quad e(k+1) - e(k) ] \\ + 3 [ \quad \quad \quad e(k) ] \\ \hline e(k+2) - 6e(k+1) + 8e(k) = 1(k) \end{array} \quad \begin{cases} e(k+2) - 6e(k+1) + 8e(k) = 1(k) \\ e(k) = 0 \quad (k \leq 0) \end{cases}$$



**Solution I of the difference equation — Iteration method**

$$\begin{cases} e(k+2) - 6e(k+1) + 8e(k) = 1(k) \\ e(k) = 0 \quad (k \leq 0) \end{cases}$$

**Solution**  $e(k+2) = 6e(k+1) - 8e(k) + 1(k)$

$$k = -1: e(1) = 6e(0) - 8e(-1) + 1(-1) = 0$$

$$k = 0: e(2) = 6e(1) - 8e(0) + 1(0) = 0 - 0 + 1 = 1$$

$$k = 1: e(3) = 6e(2) - 8e(1) + 1(1) = 6 - 0 + 1 = 7$$

$$k = 2: e(4) = 6e(3) - 8e(2) + 1(2) = 6 \times 7 - 8 \times 1 + 1 = 35$$

$$\vdots \quad \quad \quad \vdots$$

$$e^*(t) = \delta(t-2) + 7\delta(t-3) + 35\delta(t-4) + \dots$$

## Solution II of difference equation — Z-transform method

$$e(k+2) - 6e(k+1) + 8e(k) = 1(k)$$

$$\begin{aligned} Z : \quad & z^2 [E(z) - e(0)z^0 - e(1)z^{-1}] \\ & - 6 \cdot z [E(z) - e(0)z^0] \\ & + 8 [E(z)] \end{aligned}$$

$$\begin{cases} e(k+2) - 6e(k+1) + 8e(k) = 1(k) \\ e(k) = 0 \quad (k \leq 0) \end{cases}$$

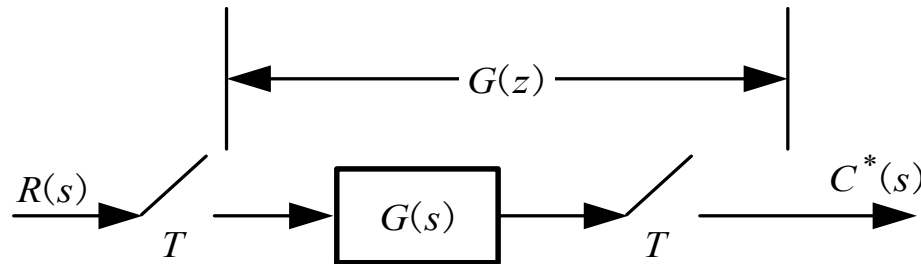
$$\frac{(z^2 - 6z + 8)E(z) = Z[1(k)] = \frac{z}{z-1}}{(z-1)(z-2)(z-4)} \quad E(z) = \frac{z}{(z-1)(z-2)(z-4)}$$

$$Z^{-1} : e(n) = \sum \text{Res} [E(z) \cdot z^{n-1}]$$

$$= \lim_{z \rightarrow 1} \frac{z \cdot z^{n-1}}{(z-2)(z-4)} + \lim_{z \rightarrow 2} \frac{z \cdot z^{n-1}}{(z-1)(z-4)} + \lim_{z \rightarrow 4} \frac{z \cdot z^{n-1}}{(z-1)(z-2)} = \frac{1}{3} - \frac{2^n}{2} + \frac{4^n}{6}$$

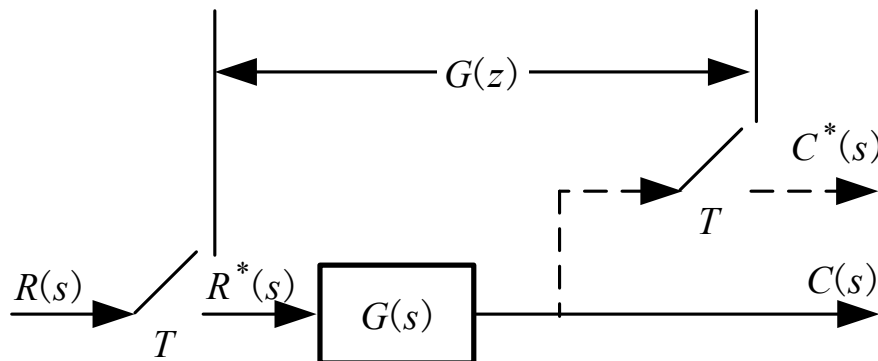
$$e^*(t) = \sum_{n=0}^{\infty} e(nT) \cdot \delta(t - nT) = \sum_{n=0}^{\infty} \left( \frac{1}{3} - \frac{2^n}{2} + \frac{4^n}{6} \right) \cdot \delta(t - nT)$$

## 7.5.2 Mathematical Models in Complex Domain — Impulse Transfer Function (脉冲传递函数)

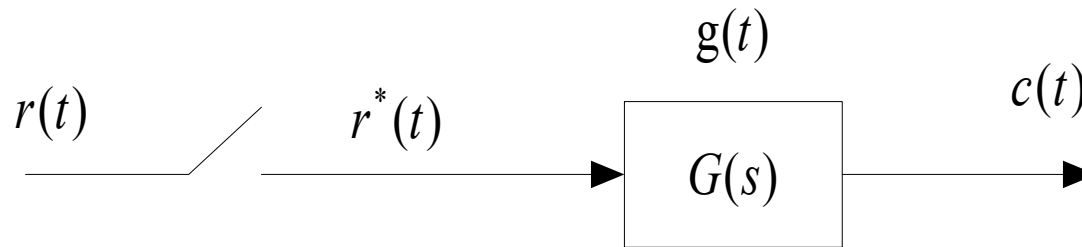


### 1. Definition

The ratio of the z-T. of the output to the z-T. of the input under zero initial condition.



$$G(z) = \frac{C(z)}{R(z)} = \frac{\sum_{k=0}^{\infty} c(kT)z^{-k}}{\sum_{k=0}^{\infty} r(kT)z^{-k}}$$



for a LTI system:

$$\left. \begin{array}{l} \text{if : } r(nT) = \delta(nT), \text{ then : } c(nT) = g(nT) \\ \text{if : } r(nT) = \delta[(n-k)T], \text{ then : } c(nT) = g[(n-k)T] \end{array} \right\} \text{Weighted sequence}$$

**Thus:**

$$\begin{aligned} r^*(t) &= \sum_{n=0}^{\infty} r(nT) \delta(t - nT) \\ &= r(0) \delta(t) + r(T) \delta(t - T) + \cdots + r(nT) \delta(t - nT) + \cdots \end{aligned}$$

$$c(t) = r(0)g(t) + r(T)g[t - T] + \cdots + r(nT)g[t - nT] + \cdots$$

$$c(kT) = r(0)g(kT) + r(T)g[(k-1)T] + \cdots + r(nT)g[(k-n)T] + \cdots$$

$$= \sum_{n=0}^{\infty} r(nT)g[(k-n)T]$$

$$c(kT) = \sum_{n=0}^{\infty} r(nT)g[(k-n)T]$$

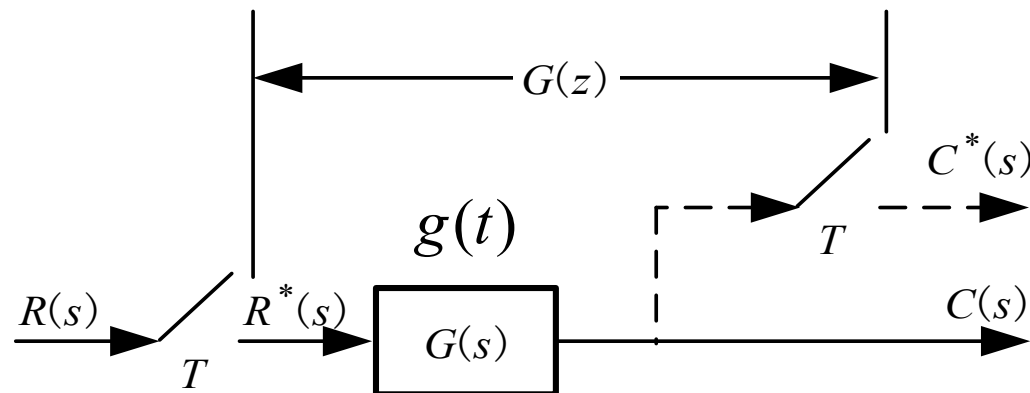
$$C(z) = \sum_{k=0}^{\infty} c(kT)z^{-k} = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} r(nT)g[(k-n)T]z^{-k}$$

$$= \sum_{n=0}^{\infty} r(nT)z^{-n} \sum_{k=0}^{\infty} g[(k-n)T]z^{-(k-n)}$$

The z-transform of  
unity impulse  
response sequence

$$\therefore G(z) = \frac{C(z)}{R(z)} = \sum_{k=n}^{\infty} g[(k-n)T]z^{-(k-n)} = \sum_{j=0}^{\infty} g(jT)z^{-j}$$

$$G(Z) = Z[g(t)] = Z[G(s)]$$



**For a difference equation:**

$$c(kT) + a_1 c(kT - T) + \cdots + a_{n-1} c(kT - (n-1)T) + a_n c(kT - nT) \\ = b_0 r(kT - (n-m)T) + \cdots + b_{m-1} r(kT - (n-1)T) + b_m r(kT - nT)$$

$$\longleftrightarrow c(kT) = -\sum_{i=1}^n a_i c[(k-i)T] + \sum_{j=0}^m b_j r[(k-j)T]$$

$$\longleftrightarrow \mathbf{z-T.} \quad C(z) = -\sum_{i=1}^n a_i C(z) z^{-i} + \sum_{j=0}^m b_j R(z) z^{-j}$$

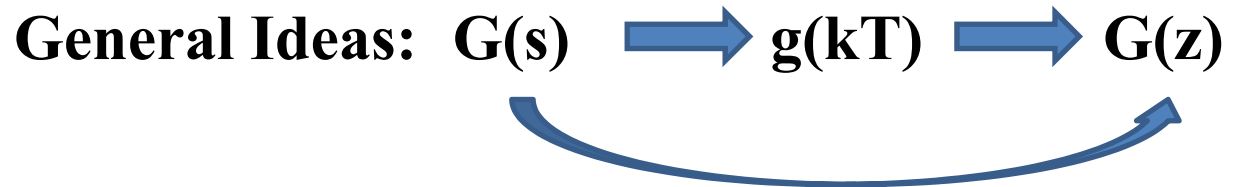
$$\longleftrightarrow G(z) = \frac{C(z)}{R(z)} = \frac{\sum_{j=0}^m b_j z^{-j}}{1 + \sum_{i=1}^n a_i z^{-i}}$$

**Example 1** Consider the discrete system shown in the figure with

$$G(s) = \frac{1}{s(0.1s + 1)}$$

**Obtain the impulse-transfer function  $G(z)$ .**

**Solution:**



**Method I.** The impulse response is:

$$G(s) = \frac{1}{s(0.1s + 1)} \xrightarrow{\quad} \begin{aligned} g(t) &= (1 - e^{-10t}) & (t > 0) \\ g(kT) &= 1 - e^{-10kT} \end{aligned}$$

Then the impulse transfer function is:

$$\begin{aligned} G(z) &= \sum_{k=0}^{+\infty} g(kT)z^{-k} = \sum_{k=0}^{+\infty} (1 - e^{-10kT})z^{-k} \\ &= \frac{z}{z-1} - \frac{z}{z-e^{-10T}} = \frac{z(1 - e^{-10T})}{(z-1)(z-e^{-10T})} \end{aligned}$$

**Method II.** Because  $G(s) = \frac{1}{s} - \frac{1}{s+10}$

Then by  $G(z) = Z[g(t)] = Z[G(s)]$ , it derives

$$G(z) = \frac{z}{z-1} - \frac{z}{z-e^{-10T}} = \frac{z(1 - e^{-10T})}{(z-1)(z-e^{-10T})}$$

### **The properties of impulse transfer function:**

- (1)  $G(z)$  is a complex function of complex variable  $z$ ;**
- (2)  $G(z)$  depends only on the structure and parameters of the system;**
- (3)  $G(z)$  has a relation with the difference equation of the system;**
- (4)  $G(z)$  is equal to  $Z[ k^*(t) ]$ ;**
- (5)  $G(z) \sim$  zero-pole location in  $z$  plane.**

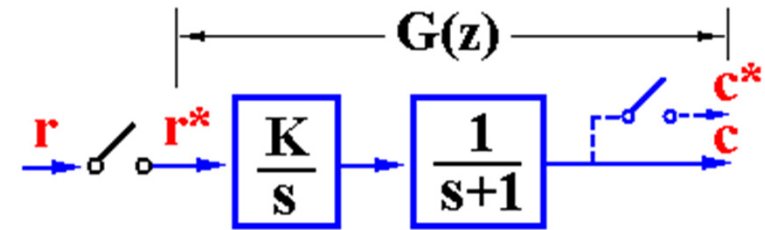
### **The limitation of impulse-transfer functions**

- (1) It can not reflect the full information of the system response under non-zero initial conditions;**
- (2) It is only for SISO discrete systems;**
- (3) It is only for LTI (linear time-invariant) difference equations;**



**Example 2** Consider the discrete system shown in the figure ( $T=1$ ). Obtain

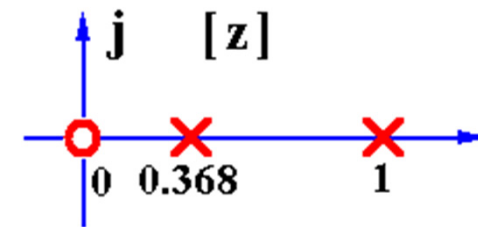
- (1) Impulse-transfer function of the system
- (2) Zero-poles location in  $z$  plane;
- (3) Difference equation of the system.



Solution. (1)  $G(z) = \frac{C(z)}{R(z)} = Z\left[\frac{K}{s(s+1)}\right] = K \cdot Z\left[\frac{1}{s} - \frac{1}{s+1}\right]$

$$= K\left[\frac{z}{z-1} - \frac{z}{z-e^{-T}}\right] = \frac{(1-e^{-T})Kz}{(z-1)(z-e^{-T})} = \frac{(1-e^{-T})Kz}{z^2 - (1+e^{-T})z + e^{-T}}$$

$$= \frac{0.632Kz^{-1}}{1 - 1.368z^{-1} + 0.368z^{-2}}$$

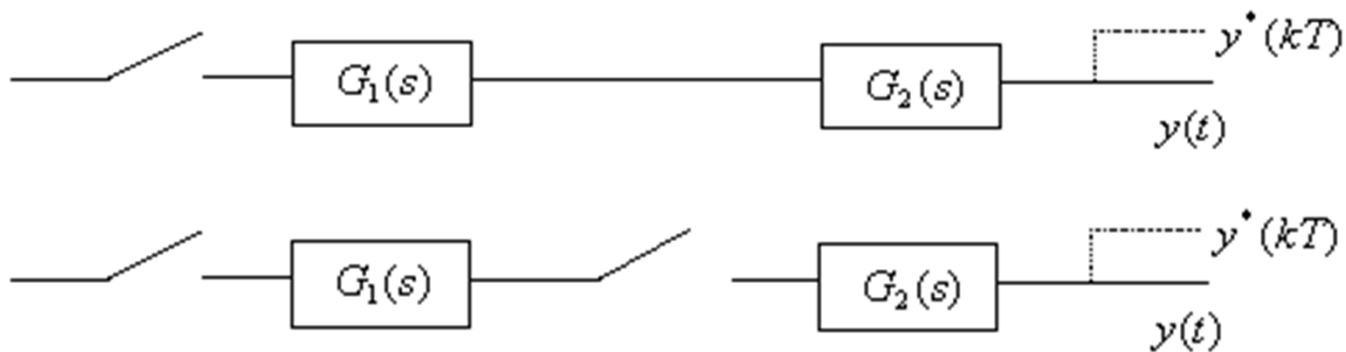


(2) Zero-poles location in  $z$  plane

(3)  $(1 - 1.368z^{-1} + 0.368z^{-2})C(z) = 0.632Kz^{-1}R(z)$

$$c(k) - 1.368c(k-1) + 0.368c(k-2) = 0.632Kr(k-1)$$

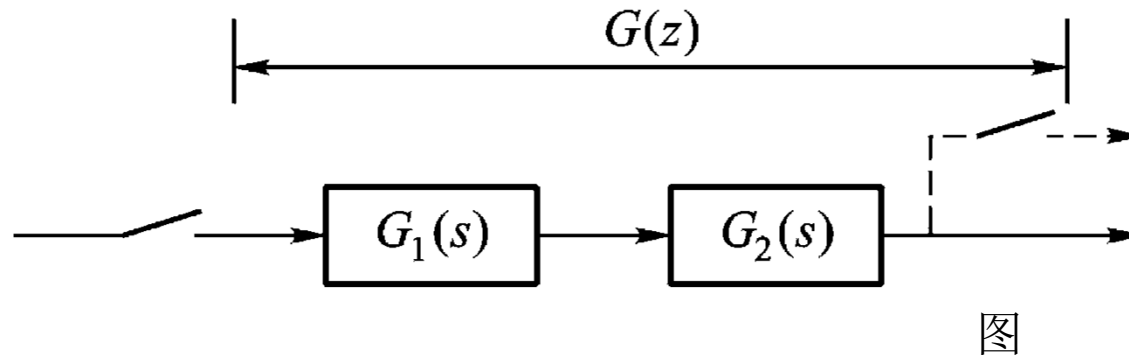
### 7.5.3 Impulse transfer function of Open-Loop Systems



**(1)** There is no sampler/switch between two components

$$G(s) = G_1(s)G_2(s)$$

$$G(z) = Z[G_1(s)G_2(s)] = G_1G_2(z)$$



**Example 3** Consider the discrete system shown in the above figure , where

$$G_1(s) = \frac{1}{s + a} \quad G_2(s) = \frac{1}{s + b}$$

**Obtain the open-loop impulse transfer function.**

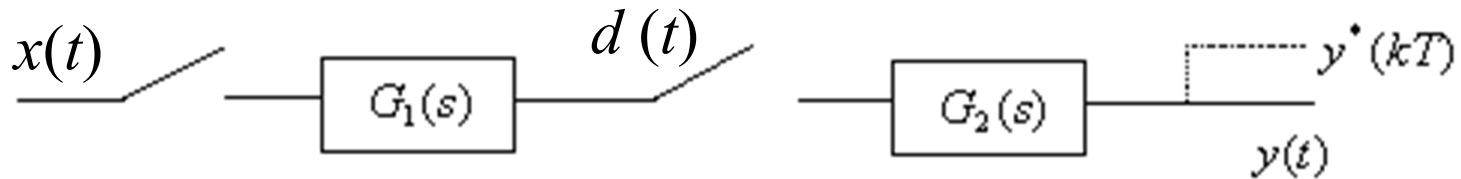
**solution:**

$$G_1(s)G_2(s) = \frac{1}{b-a} \left[ \frac{1}{s+a} - \frac{1}{s+b} \right]$$

$$G(z) = G_1G_2(z)$$

$$= \frac{1}{b-a} \left[ \frac{z(e^{-aT} - e^{-bT})}{(z - e^{-aT})(z - e^{-bT})} \right]$$

(2) There is a sampler/switch between two components



$$D(z) = G_1(z)X(z)$$

$$Y(z) = G_2(z)D(z) = G_1(z)G_2(z)R(z)$$

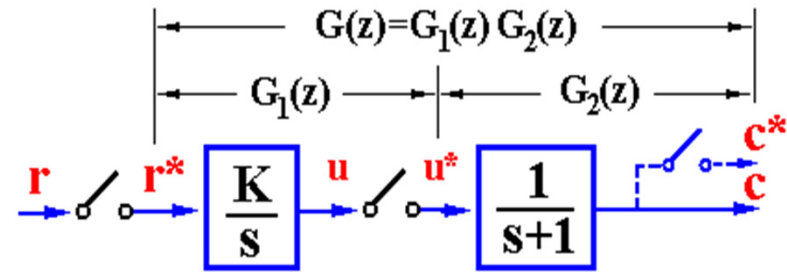
$$\therefore G(z) = G_1(z)G_2(z)$$

注  $G_1(z)G_2(z) \neq G_1G_2(z)$

### (1) Switch between factors

$$G(z) = G_1(z) G_2(z) = Z\left[\frac{K}{s}\right] \cdot Z\left[\frac{1}{s+1}\right]$$

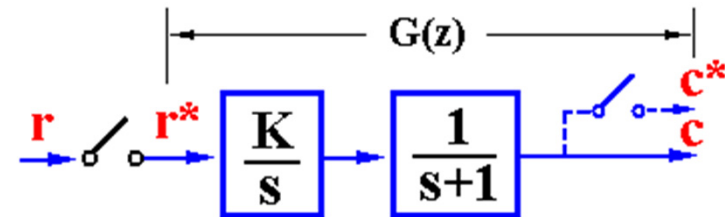
$$= \frac{Kz}{z-1} \cdot \frac{z}{z-e^{-T}} = \frac{Kz^2}{(z-1)(z-e^{-T})}$$



### (2) No switch between factors

$$G(z) = Z[G_1(s) \cdot G_2(s)] = G_1 G_2(z)$$

$$= K \left[ \frac{z}{z-1} - \frac{z}{z-e^{-T}} \right] = \frac{(1-e^{-T})Kz}{(z-1)(z-e^{-T})}$$



**Note: the zeros of  $G(z)$ , the poles of  $G(z)$ .**

**Exercise:** Consider  $G_1(s) = \frac{1}{s}$ ,  $G_2(s) = \frac{10}{s+10}$ , obtain  $G(z)$ .

**Solution:**

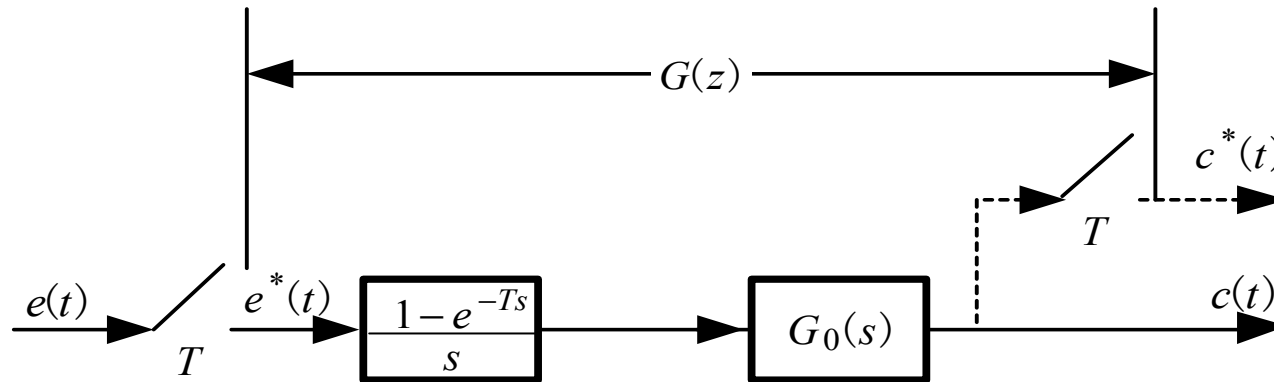
**If there is no switch between the components,**

$$G(z) = G_1 G_2(z) = Z\left[\frac{10}{s(s+10)}\right] = \frac{z(1 - e^{-10T})}{(z-1)(z - e^{-10T})}$$

**If there is a sampler between the components,,**

$$\begin{aligned} G(z) &= G_1(z)G_2(z) = Z\left[\frac{1}{s}\right]Z\left[\frac{10}{s+10}\right] \\ &= \frac{z}{z-1} \frac{10z}{z - e^{-10T}} = \frac{10z^2}{(z-1)(z - e^{-10T})} \end{aligned}$$

### (3) ZOH in the system



$$C(z) = Z \left[ \frac{1 - e^{-Ts}}{s} G_0(s) \right] R(z) = Z \left[ \frac{1}{s} G_0(s) - \frac{e^{-Ts}}{s} G_0(s) \right] R(z)$$

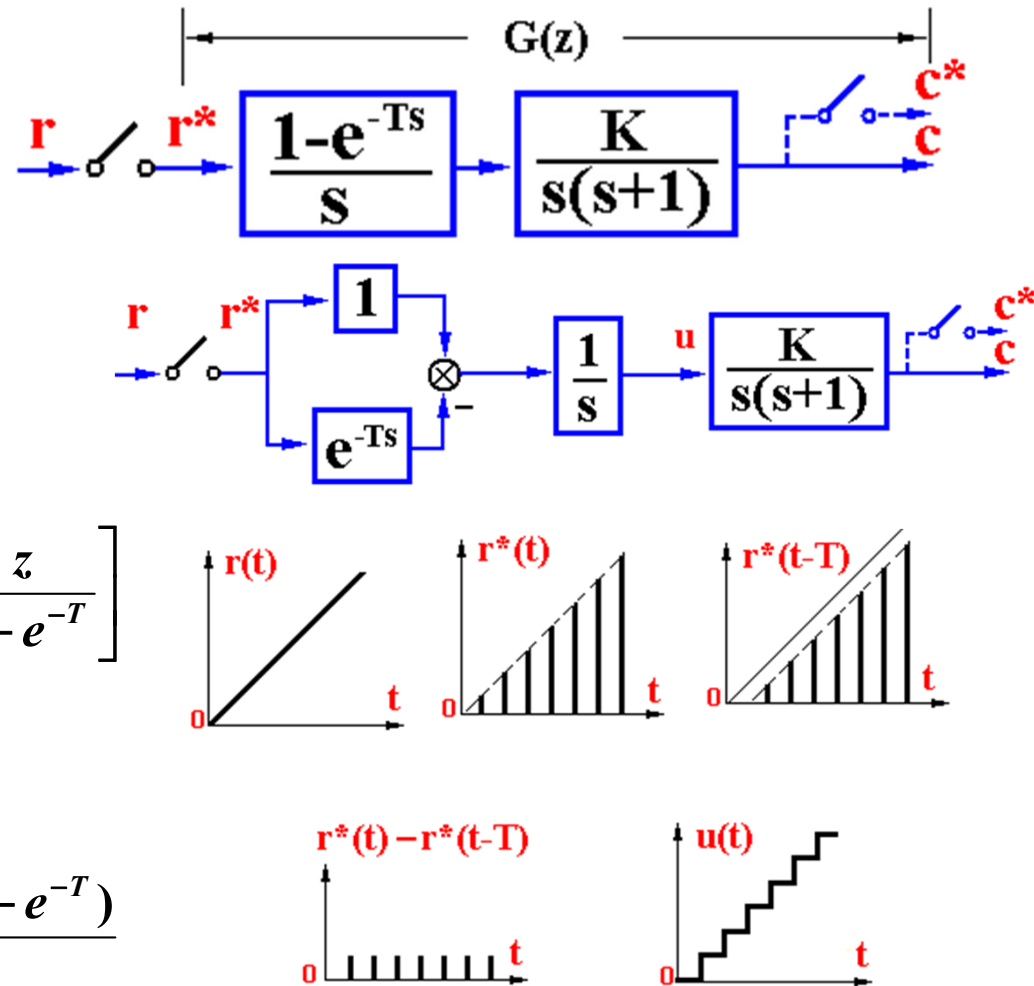
$$Z \left[ \frac{e^{-Ts}}{s} G_0(s) \right] = z^{-1} Z \left[ \frac{G_0(s)}{s} \right] \quad C(z) = (1 - z^{-1}) Z \left[ \frac{G_0(s)}{s} \right] R(z)$$

$$G(z) = \frac{C(z)}{R(z)} = (1 - z^{-1}) Z \left[ \frac{G_0(s)}{s} \right]$$



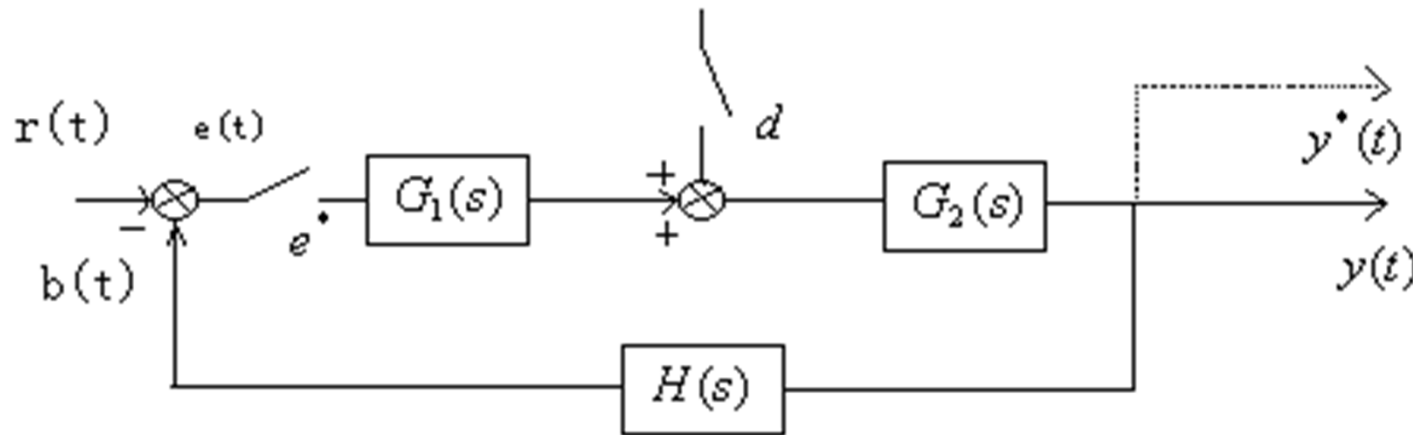
**Example 4** Consider the discrete system shown in the following figure, obtain its impulse transfer function.

$$\begin{aligned}
 G(z) &= Z \left[ \frac{1 - e^{-Ts}}{s} \cdot \frac{K}{s(s+1)} \right] \\
 &= K(1 - z^{-1}) Z \left[ \frac{1}{s^2(s+1)} \right] \\
 &= K \frac{z-1}{z} Z \left[ \frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} \right] \\
 &= K \frac{z-1}{z} \left[ \frac{Tz}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-T}} \right] \\
 &= K \left[ \frac{T}{z-1} - 1 + \frac{z-1}{z-e^{-T}} \right] \\
 &= K \frac{(T-1+e^{-T})z + (1-Te^{-T}-e^{-T})}{(z-1)(z-e^{-T})}
 \end{aligned}$$

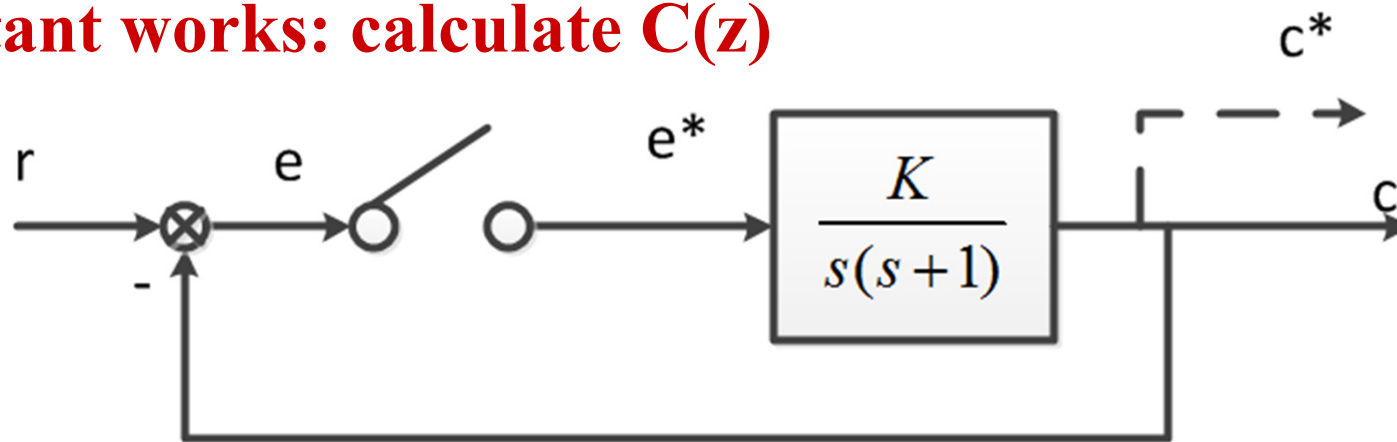


**ZOH does not change the system order and O.-L. poles but changes the O.-L. zeros.**

### 7.5.4、 Impulse transfer function of Closed-Loop Systems



**Instant works: calculate  $C(z)$**



## (1) Impulse Transfer Function for input to output

$$d = 0$$

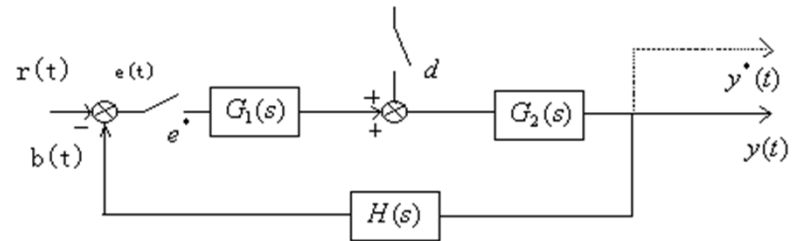
$$Y(z) = G_1 G_2(z) E(z)$$

$$e(t) = r(t) - b(t)$$

$$\Rightarrow E(z) = R(z) - B(z)$$

$$B(z) = G_1 G_2 H(z) E(z)$$

$$\left. \begin{array}{l} E(z) = R(z) - B(z) \\ B(z) = G_1 G_2 H(z) E(z) \end{array} \right\} \Rightarrow E(z) = \frac{R(z)}{1 + G_1 G_2 H(z)}$$



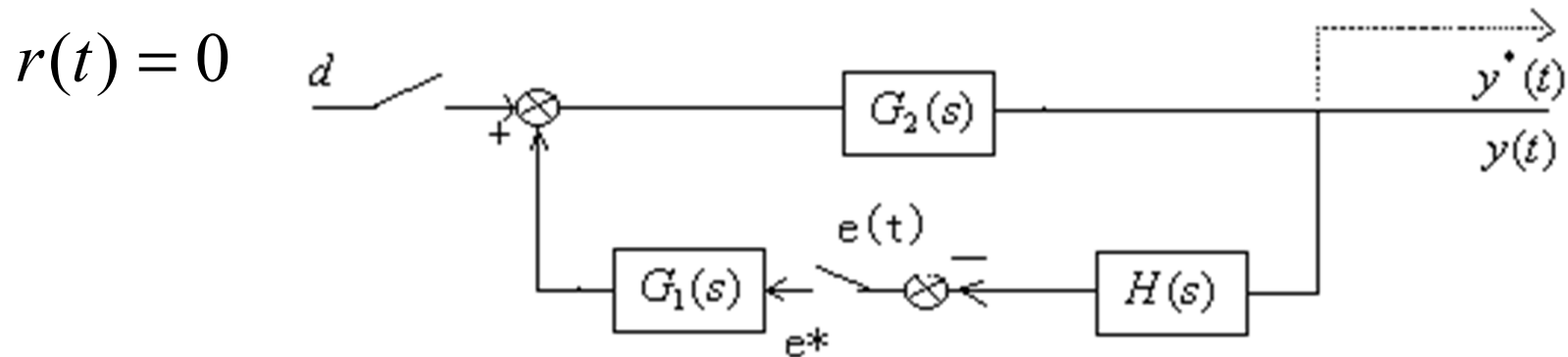
Error impulse transfer function (误差脉冲传递函数):

$$G_e(z) = \frac{E(z)}{R(z)} = \frac{1}{1 + G_1 G_2 H(z)}$$

$$\Rightarrow Y(z) = G_1 G_2(z) \frac{R(z)}{1 + G_1 G_2 H(z)}$$

$$\therefore \Phi(z) = \frac{Y(z)}{R(z)} = \frac{G_1 G_2(z)}{1 + G_1 G_2 H(z)}$$

## (2) Impulse Transfer Function for disturbance to output



$$Y(z) = G_2(z)D(z) + G_1G_2(z)E(z)$$

$$E(z) = -[G_2H(z)D(z) + G_1G_2H(z)E(z)]$$

$$\Rightarrow E(z) = -\frac{G_2H(z)}{1 + G_1G_2H(z)}D(z)$$

$$\therefore Y(z) = G_2(z)D(z) - \frac{G_1G_2(z)G_2H(z)}{1 + G_1G_2H(z)}D(z)$$

$$\Rightarrow \Phi_d(z) = \frac{Y(z)}{D(z)} = G_2(z) - \frac{G_1G_2(z)G_2H(z)}{1 + G_1G_2H(z)}$$

**E(z) :**

**D(z) passing through  
G<sub>2</sub>(z);**

**Loop of E(z) itself.**

**There is no switch/sampler for the error signal  $e(t)$**

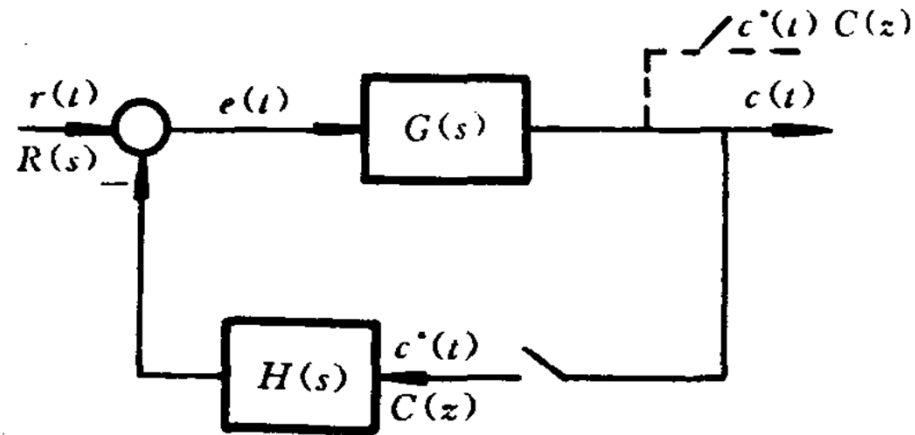


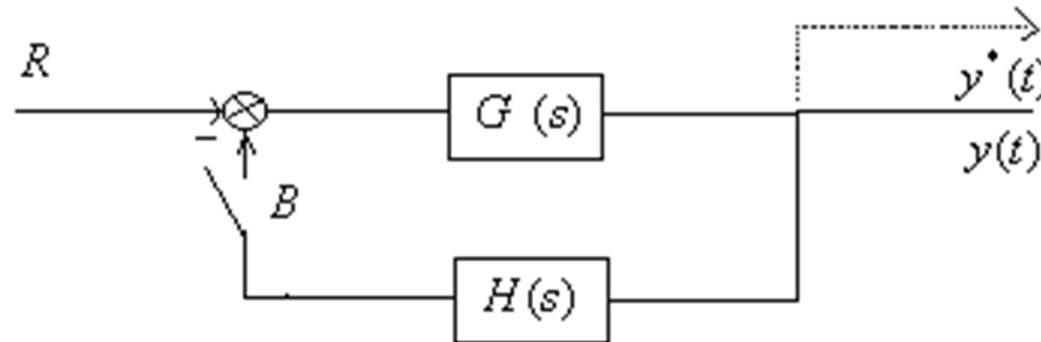
图 7-35 闭环离散系统

$$C(s) = G(s)R(s) - G(s)H(s)C^*(s)$$

$$C(z) = GR(z) - GH(z)C(z) \quad \Rightarrow \quad C(z) = \frac{GR(z)}{1 + GH(z)}$$

**Then, for this system, there exists no impulse transfer function.**

**Example** Consider the discrete-time system as shown in the figure, find the z-transform of the output  $y(t)$ .



**Solution:**

$$Y(z) = GR(z) - G(z)B(z)$$

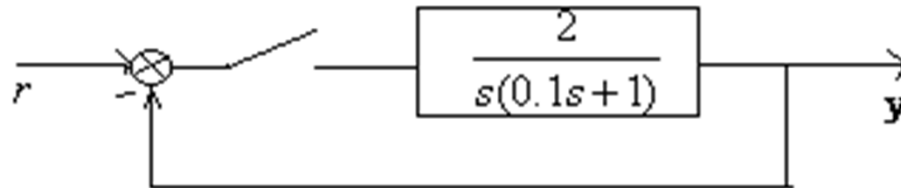
$$B(z) = GHR(z) - GH(z)B(z)$$

$$\therefore B(z) = \frac{GHR(z)}{1 + GH(z)}$$

$$\therefore Y(z) = GR(z) - \frac{G(z)GHR(z)}{1 + GH(z)}$$

**There exists no impulse transfer function.**

**Example** Consider the discrete-time system as shown in the figure, for  $T=0.1$ , find the **unit step response** of the system.



**Solution:** 
$$G(z) = Z \left[ \frac{2}{s(0.1s + 1)} \right] = \frac{2z}{z-1} - \frac{2z}{1 - e^{-10T}}$$

$$= \frac{2z - 0.736z}{(z-1)(z-0.368)} = \frac{1.264z}{z^2 - 1.368z + 0.368}$$

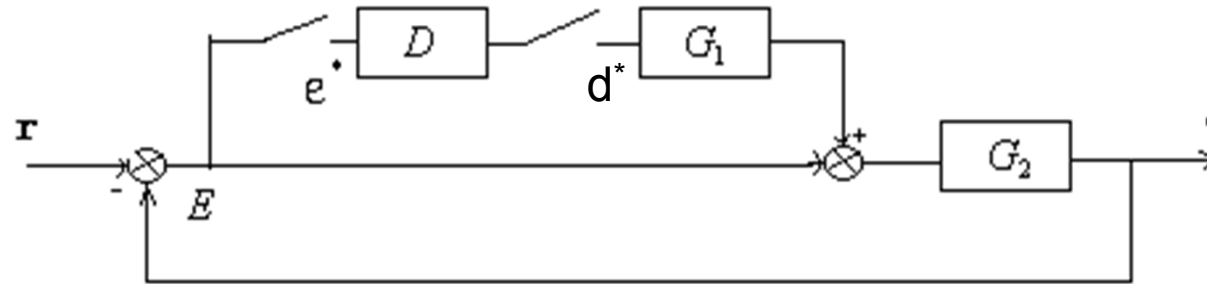
$$\therefore \Phi(z) = \frac{G(z)}{1 + G(z)} = \frac{1.264z}{z^2 - 0.104z + 0.368}$$

$$\therefore Y(z) = \Phi(z)R(z) = \Phi(z) \frac{z}{z-1}$$

$$= 1.264z^{-1} + 1.396z^{-2} + 0.945z^{-3} + 0.849z^{-4} + \dots$$

$$y^*(t) = 1.264\delta(t - 0.1) + 1.396\delta(t - 0.2) + \dots$$

**Example** Consider the discrete-time system as shown in the figure, find the expression of the output  $c$ .



**Solution:** There exist both discrete and continuous signals, then employing L-Transform firstly,

$$\begin{aligned} C &= G_1 G_2 \cdot D^* \cdot E^* + G_2 E = G_1 G_2 \cdot D^* \cdot (R - C)^* + G_2 (R - C) \\ &= G_1 G_2 \cdot D^* \cdot (R^* - C^*) + G_2 (R - C) \end{aligned}$$

$$C = \frac{G_2 R}{1 + G_2} + \frac{G_1 G_2}{1 + G_2} \cdot D^* \cdot R^* - \frac{G_1 G_2}{1 + G_2} \cdot D^* \cdot C^*$$



$$C = \frac{G_2 R}{1 + G_2} + \frac{G_1 G_2}{1 + G_2} \cdot D^* \cdot R^* - \frac{G_1 G_2}{1 + G_2} \cdot D^* \cdot C^*$$

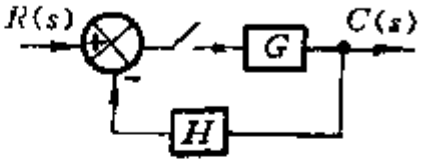
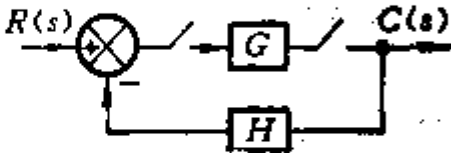
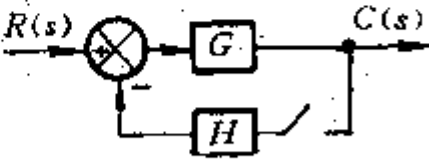
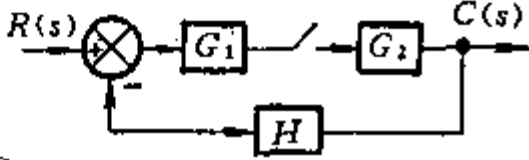
Discretize C to  $C^*$ , then

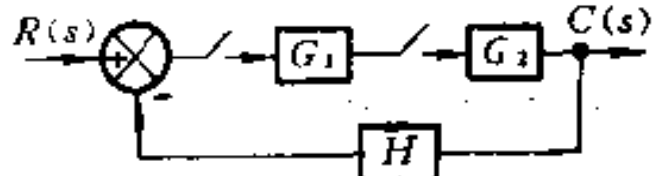
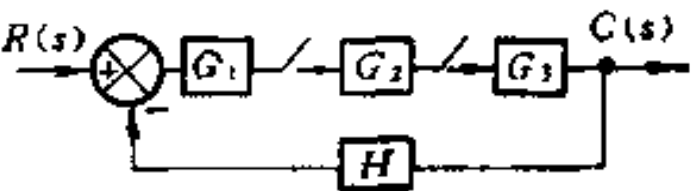
$$C^* = \left[ \frac{G_2 R}{1 + G_2} \right] + \left[ \frac{G_1 G_2}{1 + G_2} \right]^* \cdot D^* \cdot R^* - \left[ \frac{G_1 G_2}{1 + G_2} \right]^* \cdot D^* \cdot C^*$$

$$\text{PS: } \left[ G_1(s) \cdot G_2(s)^* \right]^* = G_1(s)^* \cdot G_2(s)^*$$

$$\therefore C^* = \frac{\left[ \frac{G_2 R}{1 + G_2} \right]^* + \left[ \frac{G_1 G_2}{1 + G_2} \right]^* D^* R^*}{1 + \left[ \frac{G_1 G_2}{1 + G_2} \right]^* D^*}$$

# Typical diagram of C.L.discrete-time systems

	系 统 方 框 图	$C(z)$
1		$C(z) = \frac{G(z)}{1 + HG(z)} R(z)$
2		$C(z) = \frac{G(z)}{1 + G(z)H(z)} R(z)$
3		$C(z) = \frac{RG(z)}{1 + HG(z)}$
4		$C(z) = \frac{RG_1(z)G_2(s)}{1 + G_1G_2H(z)}$

5		$C(z) = \frac{G_1(z)G_2(z)}{1 + G_1(z)HG_2(z)} R(z)$
6		$C(z) = \frac{G_2(z)G_3(z)RG_1(z)}{1 + G_3(z)G_1G_3H(z)}$

作业： p256. 7-5, 7-8