Principle of Automatic Control II
（自动控制原理II）

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学时：**48**学时

考试：英文 闭卷

References

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Chapter 1. Introduction

Chapter 2. Mathematical Models of Linear Systems

Chapter 3. Analyses in Time Domain for Linear Systems

Chapter 4. Root Locus

Chapter 5. Analyses in Frequency Domain for Linear Systems

Chapter 6. Controller Design for Linear Systems

- Classify:
  - Classic Control Theory
    - Routh-Hurwitz Stability Criterion, Bode Diagram, Nyquist plot/curve, Nichols plot/curve, Root Locus…
  - Modern Control Theory
    - State Space, Controllability, Observability …
  - Postmodern control theory
    - Large Scale System, Robust Control, Adaptive Control, Nonlinear Control, Intelligent Control, …

Modern Control Theory

Chapter 7. Linear Discrete-Time System

Chapter 8. Nonlinear System

Chapter 9. State Space Method

Chapter 7 Linear Discrete-Time System: Analysis and Design（Sampled-data System）

7.1 Introduction

7.2 The Sampling Process and Sampling Theorem

7.3 Signal Recovery and Zero-Order Hold

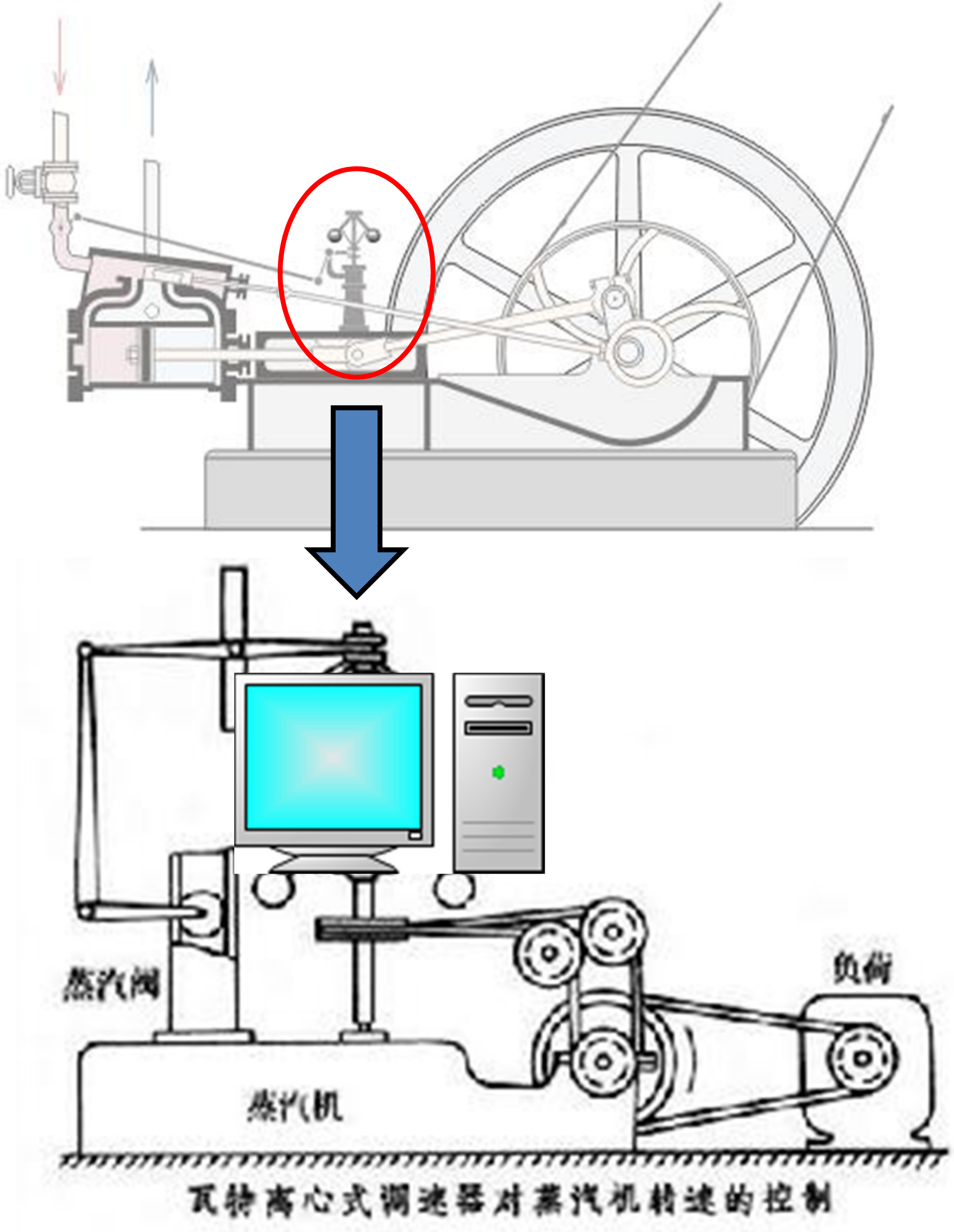
7.4 Z-Transform and Inverse Z Transform

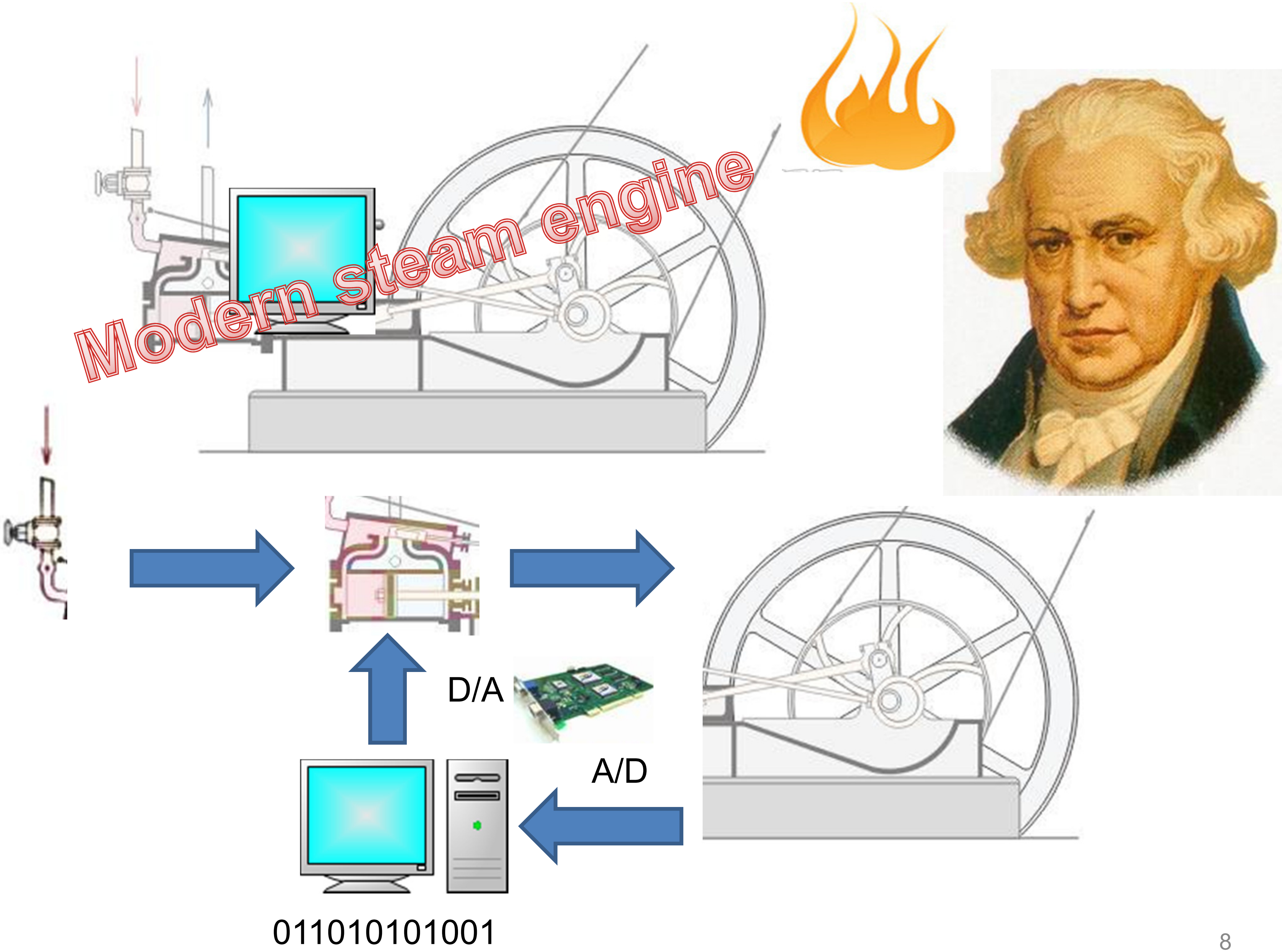
7.5 Mathematical Models of Discrete-Time Systems

7.6 Performance Analysis of Discrete-Time Systems

7.7 Digital Control Design for Discrete-Time Systems

Mr. J. Watt

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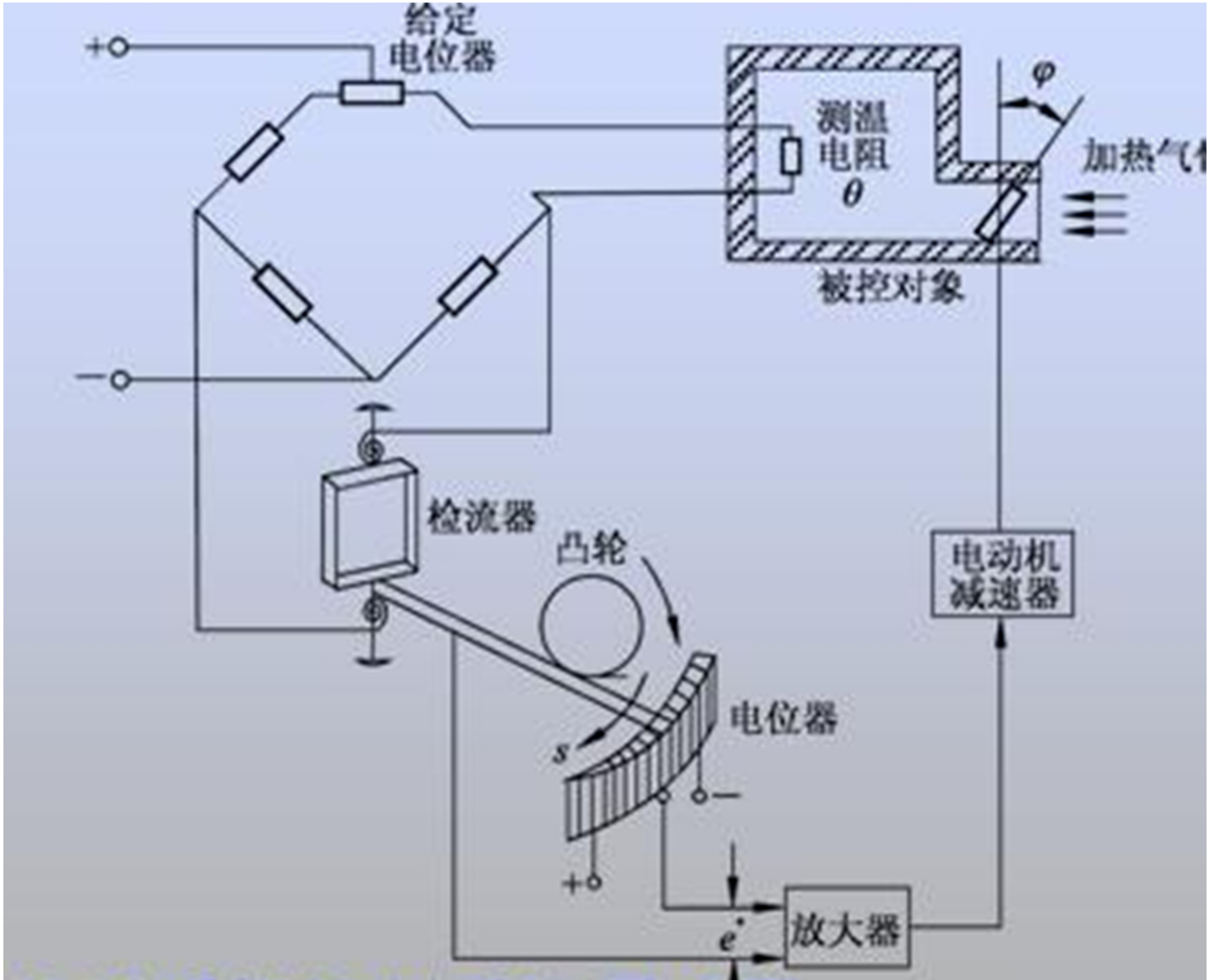


炉温采样控制系统原理图

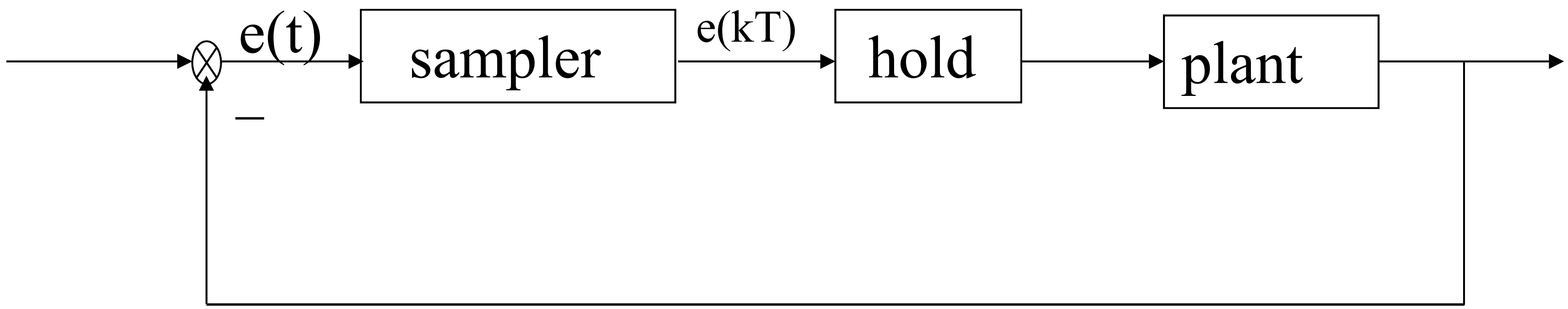
7.1 Introduction

Discrete-Time Systems:

Types:
- Sampling systems: Discrete Time, Continuous Value
- Digital systems: Discrete Time, Quantized Value

Sampled-Data System: a system that is continuous except for one or more sampling operations.

Digital System: There is one or more impulse series or digital signals in the system.



Sampled-data control system

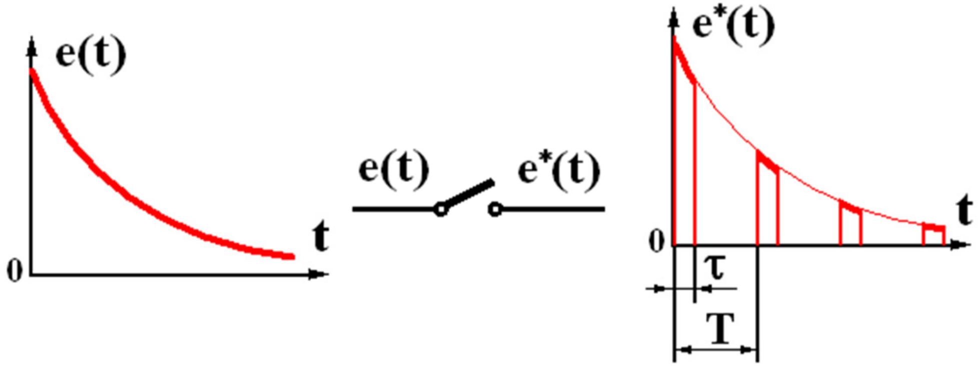
e(kT) is obtained by sampling a continuous signal e(t).

Sampler: continuous to discrete signal / AD

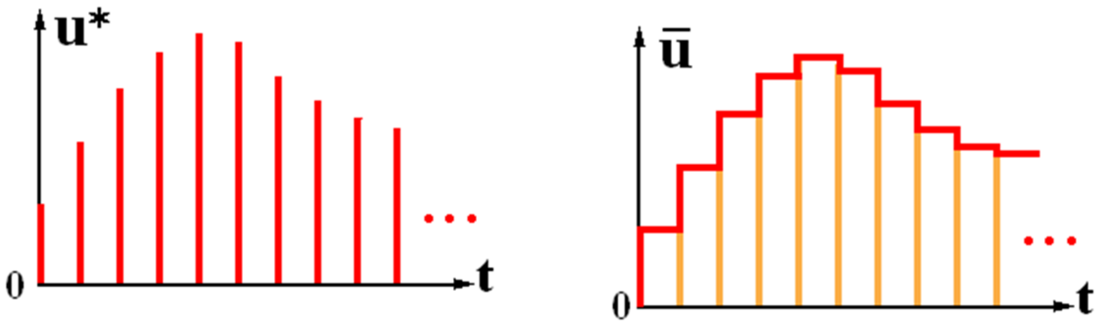
Holder : discrete to continuous signal / DA

Sampling process /AD

- **Sampling** — Time sampled
- **Quantization** — Value quantized



Holding process / DA



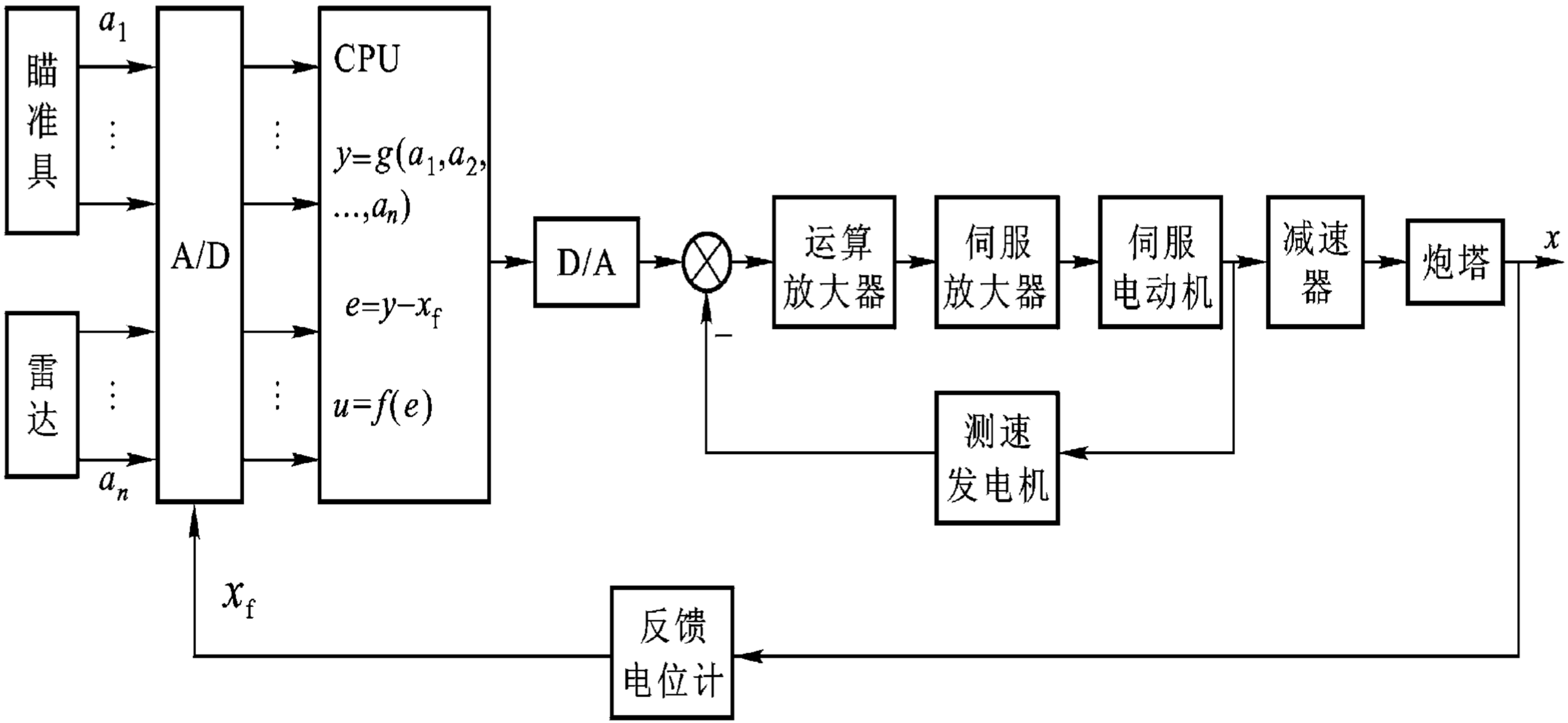


Fig 7-1 机载火力控制系统原理

History of Discrete-time system (p. 196-197）

DDC-Direct Digital Control （直接数字控制系统 ）

SCC- Surveillance Computer Control System(计算机监督控制系统)

TDE- Total and Distributed Control(集散控制系统)

Advantages and Disadvantages

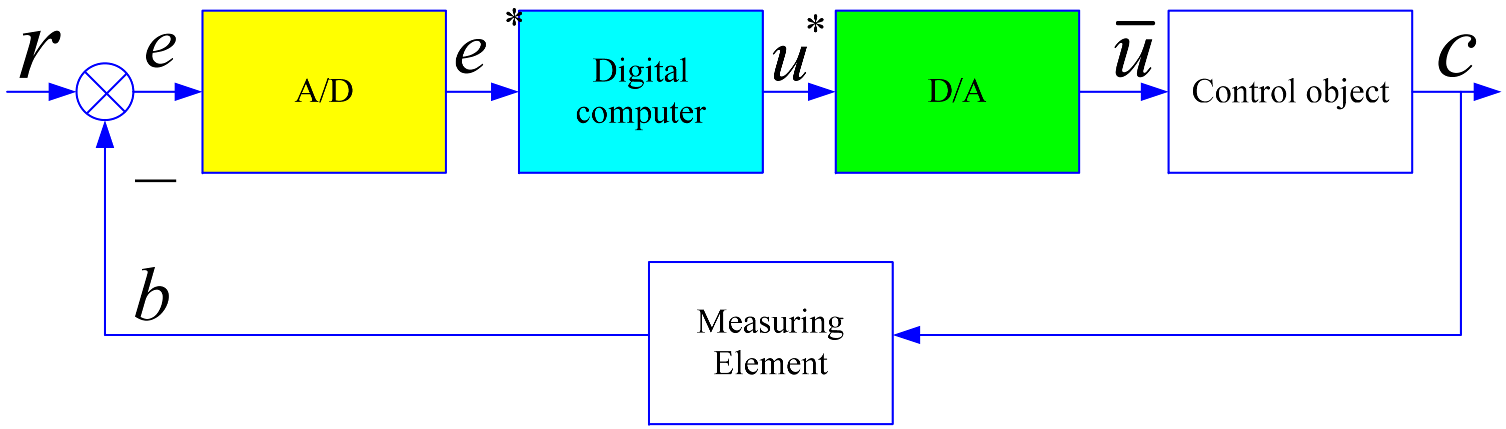
Computer Control System

- (1) Calculations are performed in the software. Easy for modification.
  (2) Complex control laws easily realized;
  (3) Reduced sensitivity to noise;
  (4) One computer for multi-tasks, high utilization ；
  (5) Network for process automation, macro-management and remote control.

- (1) Information between samples is lost. Compared with continuous system in the similar condition, the performance is reduced;
  (2) Needs A/D and D/A conversion devices.

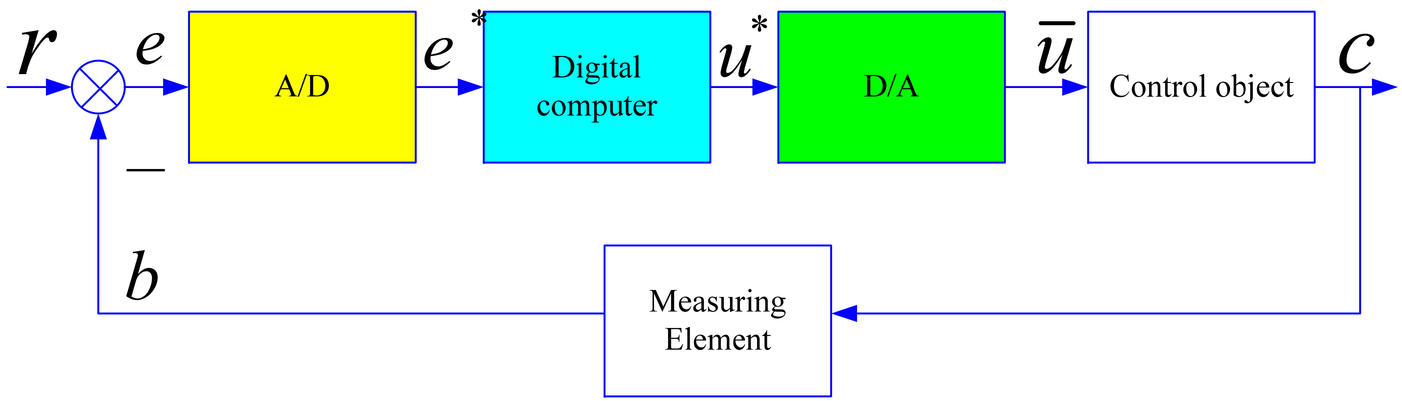
7.2 The Sampling Process and Sampling Theorem

7.2.1 The Sampling Process

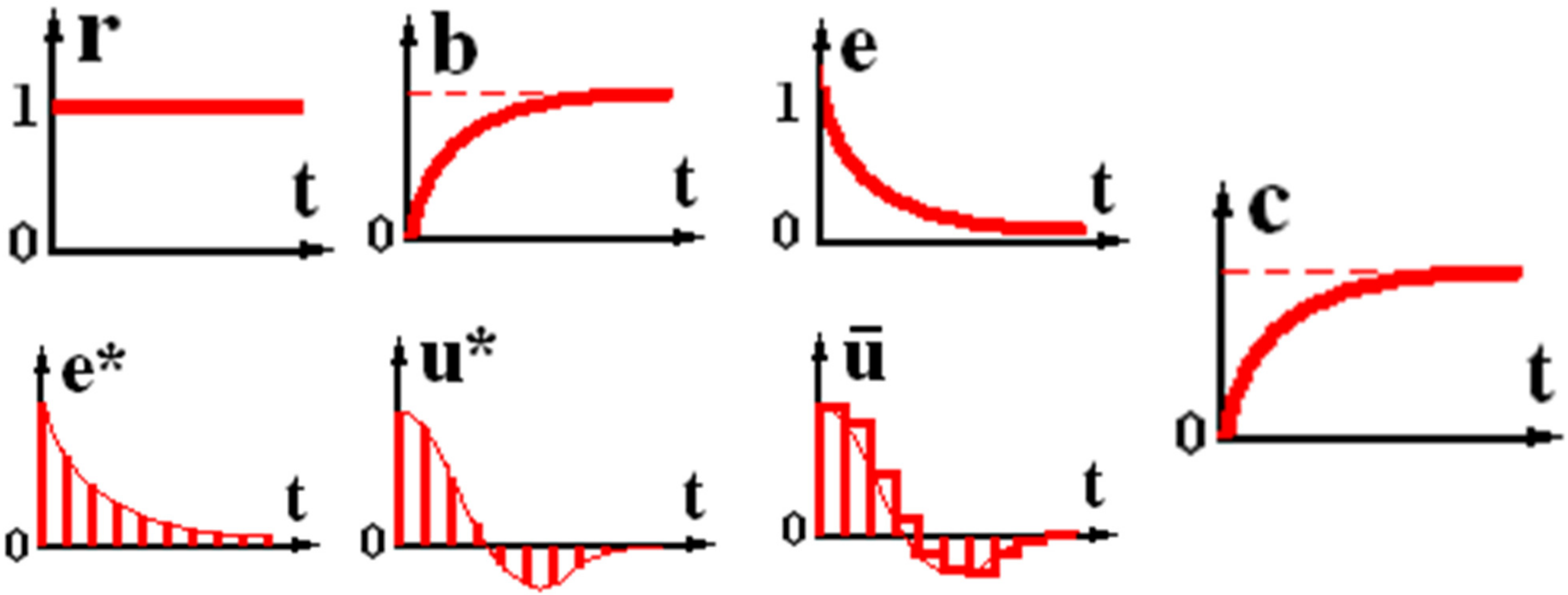


Computer Control System

Question: In the above computer control system, which signals are discrete, which signals are continuous?

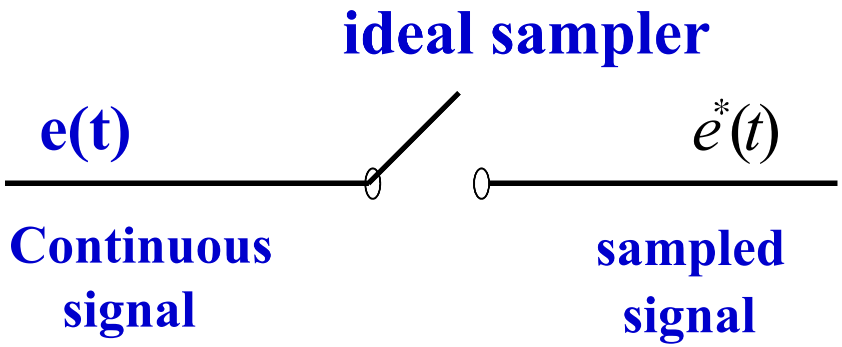


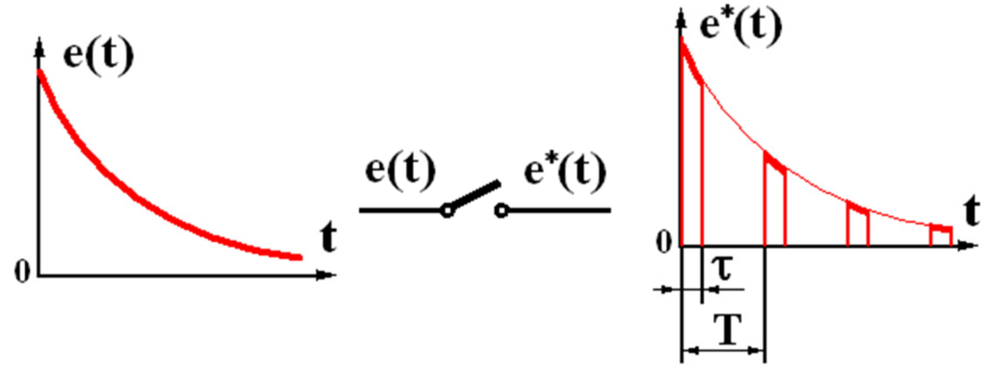
Computer Controlled Systems



- **Sampling Process：Continuous signal → Discrete Signal**
- **Holding Process：Discrete Signal → Continuous Signal.**
- **The two are inverse process to each other.**

Sampler: A switch which closes every T seconds for one instant of time.





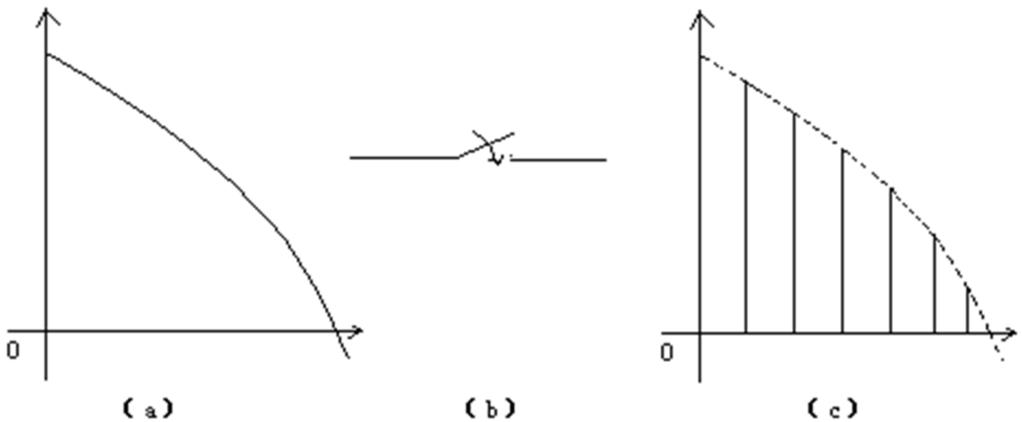
Where T is called the sampling period.

Ideal sampling process:

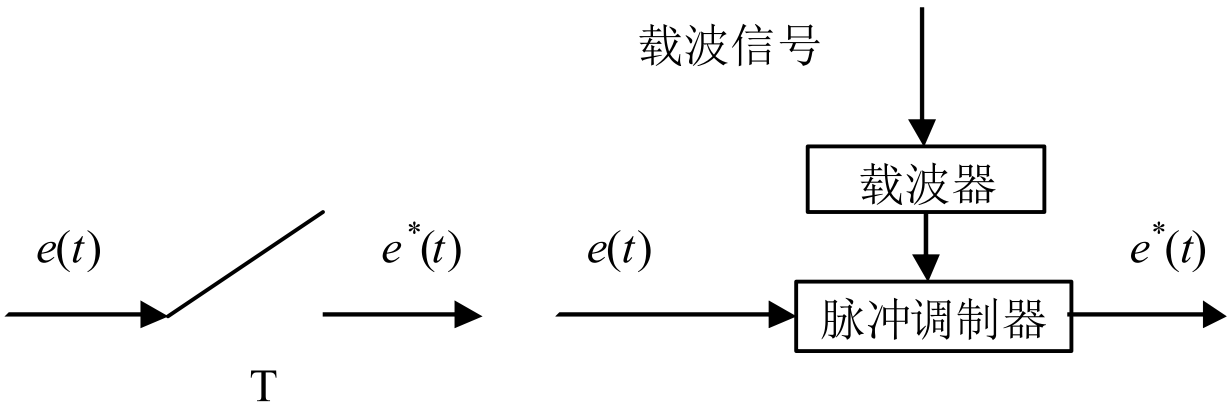
(1) $\tau \ll T$. The sampling process is completed instantaneously

(2) Word Length is enough, for quantization, thus $e^*(KT)=e(KT)$

Types of Samplers: ideal, periodical, random,…



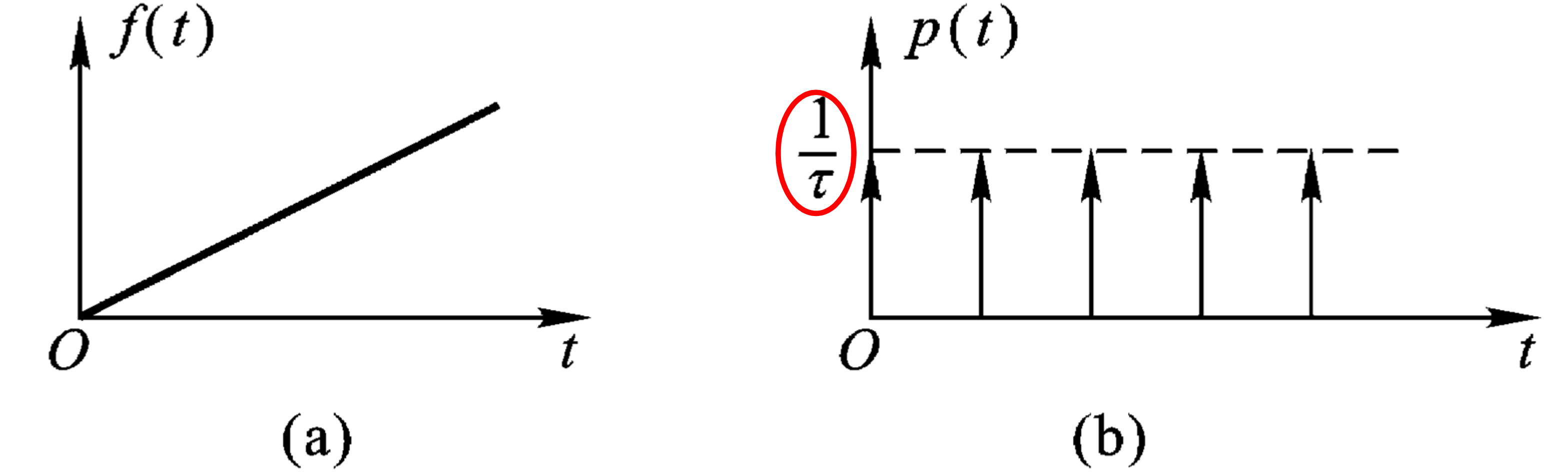
Sampling Process

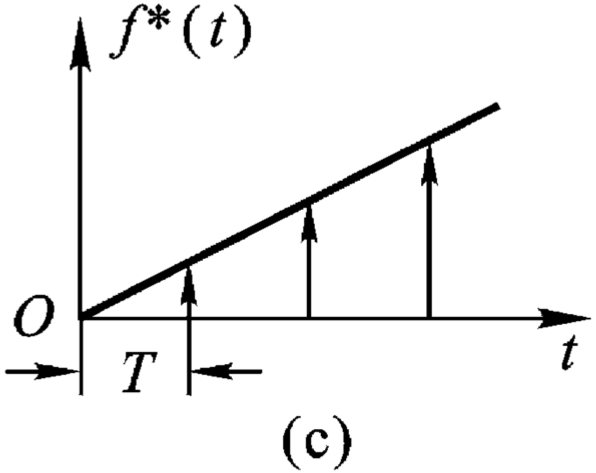


7.2.2 Mathematical Model for sampling Signals

1、Some ideal assumptions

- The sampling process is completed instantaneously ；
- $\tau \ll T$, that is $\tau \to 0$;
- The signals in and out the sampler have no difference;
- Output of the sampler is constant when it shuts down;
- Sample Period T is a constant。





$$f_\tau^*(t) = p(t) \cdot f(t)$$

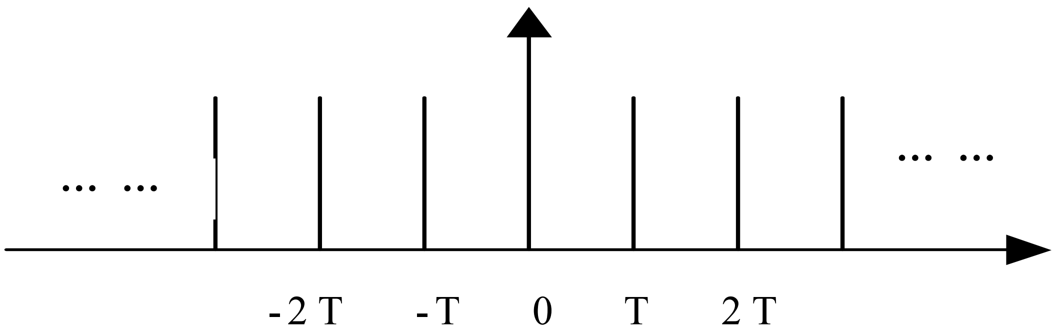
Fig 7—3 Sampling Process

2、Unit Impulsive Signal $\delta(t)$

$$\delta(t)=\begin{cases}1 & t=0\\ 0 & t\neq 0\end{cases}$$

3、Unit Impulse Sequence (Unit Impulse Train)

$$\delta_T(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT)=\cdots+\delta(t+T)+\delta(t)+\delta(t-T)+\cdots+\delta(t-kT)+\cdots$$

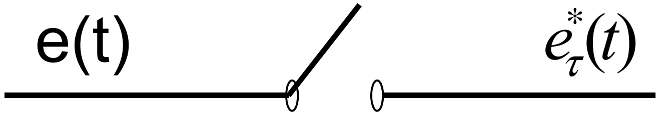


Unit Impulse sequence

4、Sampling Signal

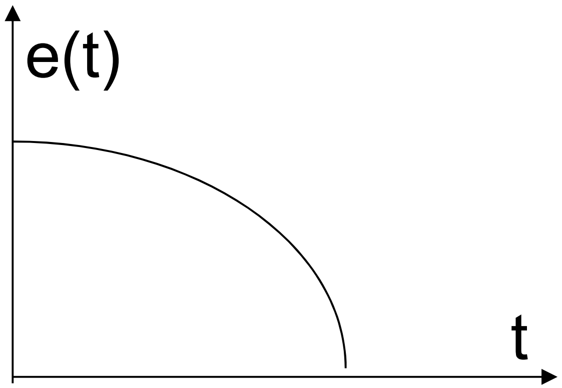
$$e^*(t)=\sum_{k=-\infty}^{\infty}e(t)\delta(t-kT)$$ or $$e^*(t)=\sum_{k=-\infty}^{\infty}e(kT)\delta(t-kT)$$

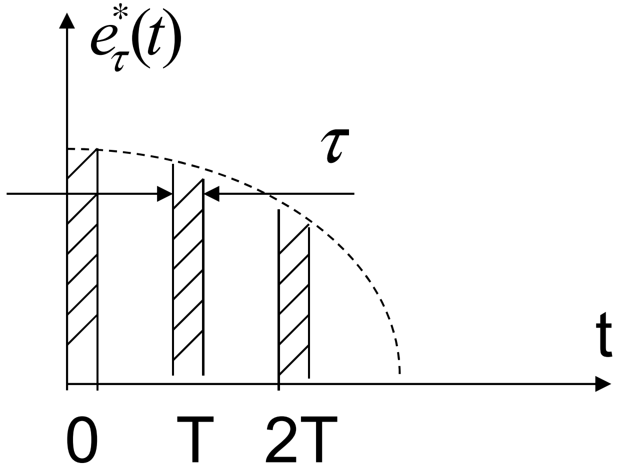
for real sampler

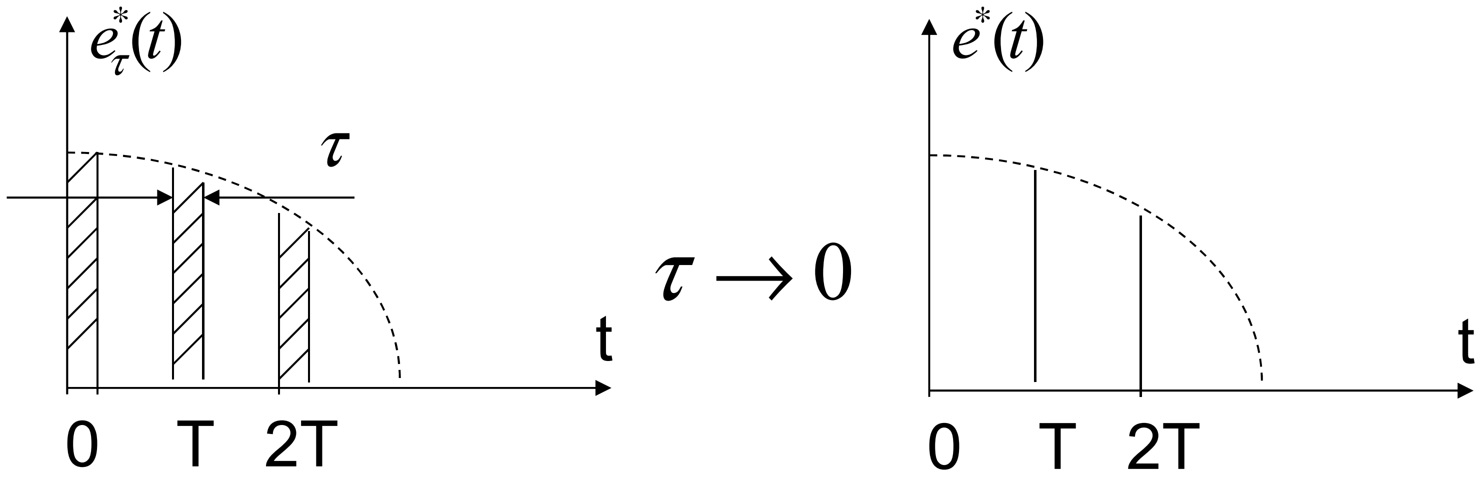


Initial working time:

$t \geq 0$







So the sampling operation can be expressed as

$$e^*(t) = \sum_{k=0}^{+\infty} e(kT) \cdot \delta(t - kT)$$

or

$$e^*(t) = e(t) \cdot \sum_{k=0}^{\infty} \delta(t - kT)$$

or

$$e^*(t) = e(t) \cdot \delta_T(t)$$

where

$$\delta_T(t) = \sum_{k=0}^{\infty} \delta(t - kT)$$

Laplace Transformation

(6) 位移定理(Transposition Property)：

a. 实域中的位移定理，若原函数在时间上延迟 τ ，则其象函数应乘以$e^{-\tau\cdot s}$

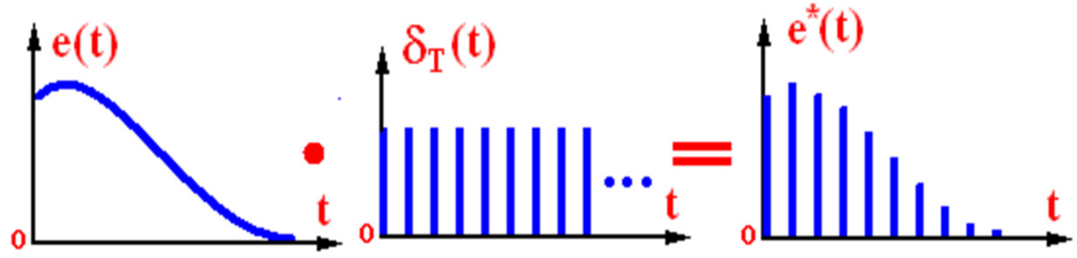
$$L[f(t-\tau)] = e^{-\tau\cdot s}F(s)$$

b. 复域中的位移定理，象函数的自变量延迟a，原函数应乘以 e^{at} ，即

$$L[e^{at}f(t)] = F(s-a)$$

Ideal sampling sequence

$$\delta_T(t) = \sum_{k=0}^{\infty} \delta(t - kT)$$



$$e^*(t) = e(t) \cdot \delta_T(t)$$

$$= e(t) \cdot \sum_{k=0}^{\infty} \delta(t - kT) \qquad = \sum_{k=0}^{\infty} e(kT) \cdot \delta(t - kT)$$

(2) *L* :

$$E^*(s) = L\left[e^*(t) \right]$$

$$= L\left[\sum_{k=0}^{\infty} e(kT) \cdot \delta(t - kT) \right] \qquad = \sum_{k=0}^{\infty} e(kT) \cdot e^{-kTs}$$

Example 7-1 $e(t)=1(t)$ to find $E^*(s)$

$$E^*(s)=\sum_{k=0}^{\infty} e(kT)\cdot e^{-kTs}$$

Solution $E^*(s)=\sum_{k=0}^{\infty} 1\cdot e^{-kTs}$

$$=1+e^{-Ts}+e^{-2Ts}+\cdots=\frac{1}{1-e^{-Ts}}=\frac{e^{Ts}}{e^{Ts}-1}$$

Example 7-2 $e(t)=e^{-at}$ to find $E^*(s)$

Solution $E^*(s)=\sum_{k=0}^{\infty} e^{-akT}\cdot e^{-kTs}=\sum_{n=0}^{\infty} e^{-(s+a)kT}$

$$=\frac{1}{1-e^{-(s+a)T}}=\frac{e^{Ts}}{e^{Ts}-e^{-aT}}$$

Instant work in 10 min:

If $e(t)=e^{-t}-e^{-2t}$ **and** $e(t)=te^{-at}$

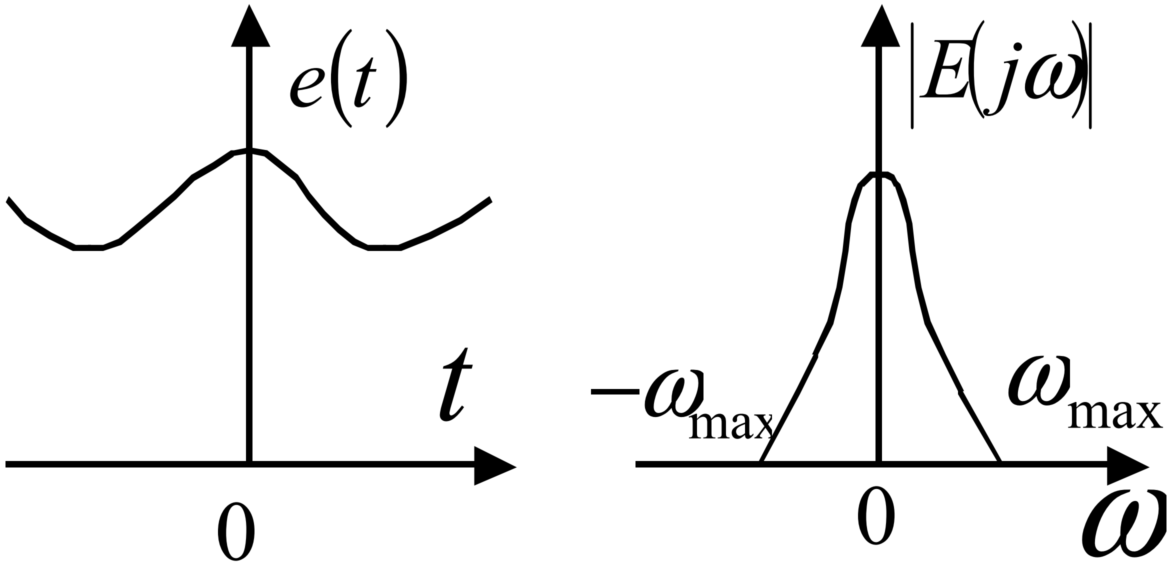
Try to find $E^{*}(s)$ **respectively.**

Consulting examples 7-1, 7-2

for roll call (点名) …

7.2.3 Frequency Spectrum Analysis of Sampled Signal

Consider a continuous signal and its amplitude spectrum are:



The Fourier-series expansion of $\delta_T(t)$ is

$$\delta_T(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{jk\omega_s t}$$

So the sampled signal is

$$e^*(t) = \frac{1}{T}\cdot\sum_{k=-\infty}^{\infty} e(t)\cdot e^{jk\omega_s t}$$

which Laplace transform is $E^*(s) = \frac{1}{T}\sum_{k=-\infty}^{\infty} E(s+jk\omega_s)$

$$E^*(\mathrm{j}\omega) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} E\left[j\left(\omega + k\omega_s\right)\right]$$

where the operator s is replaced by $j\omega$

$$E^*(s)=\sum_{n=0}^{\infty} e(nT)\cdot \mathrm{e}^{-nTs}$$

① shows the value relation of E*(s) and e(t) on the sampling point;

② can be written into the closed form;

③ can be used to obtain the time response and the Z transform of e*(t)

$$E^*(s)=\frac{1}{T}\sum_{n=-\infty}^{\infty} E(s+jn\omega_s)$$

① shows relationship of E*(s) and E(s);

② can not be written as close form;

③ can be used for the frequency spectrum analysis of e*(t).

The frequency spectrum analysis of continuous signal e(t) and discrete signal e*(t)

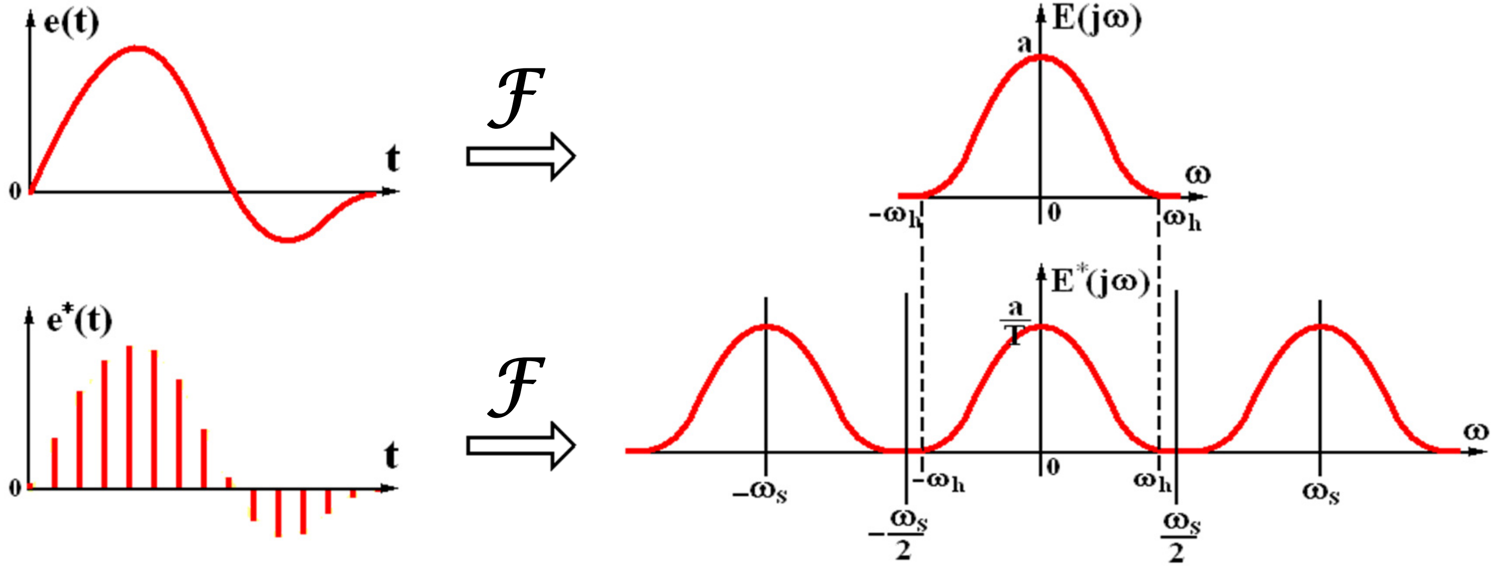
Frequency spectrum — Frequency expansion of the signal

$e(t)$

continuous signal

$e^*(t)$

discrete signal



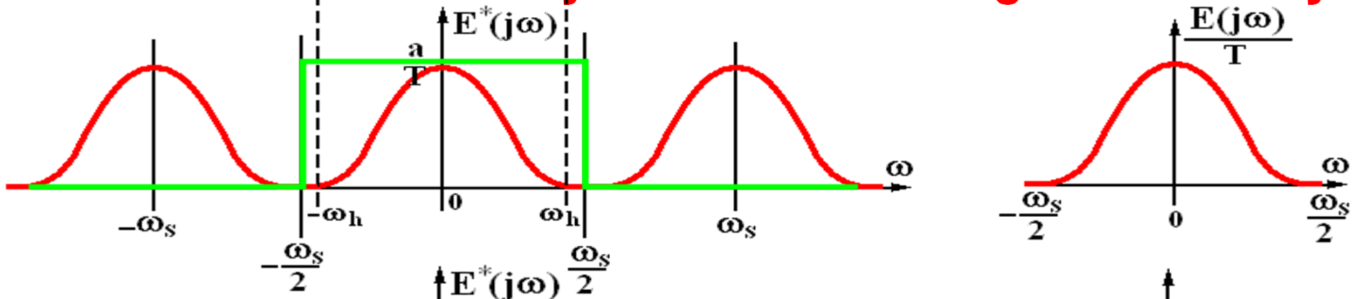
$$E^*(s)=\sum_{n=0}^{\infty} e(nT)\cdot e^{-nTs}$$

$$E^*(s)=\frac{1}{T}\sum_{n=-\infty}^{\infty} E(s+jn\omega_s)$$

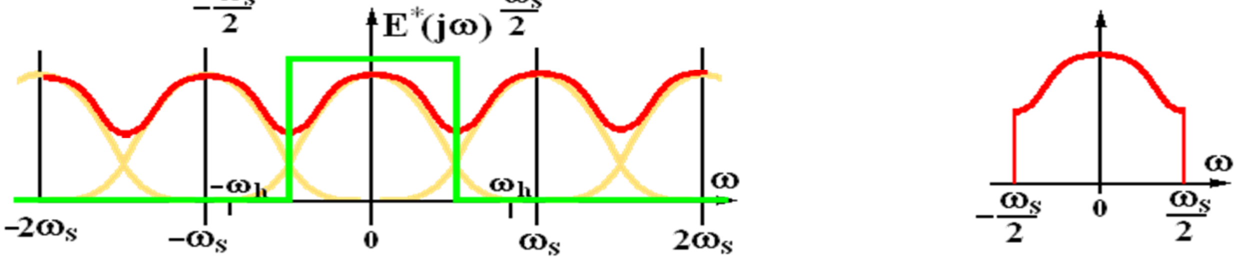
Shannon Sampling Theorem—

The Necessary Condition for signal recovery

$$\omega_s = \frac{2\pi}{T} > 2\omega_h$$

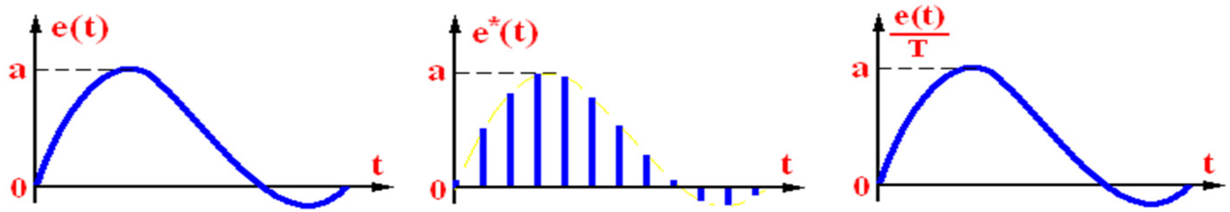


$$\omega_s = \frac{2\pi}{T} < 2\omega_h$$



$$\omega_s = \frac{2\pi}{T} > 2\omega_h$$

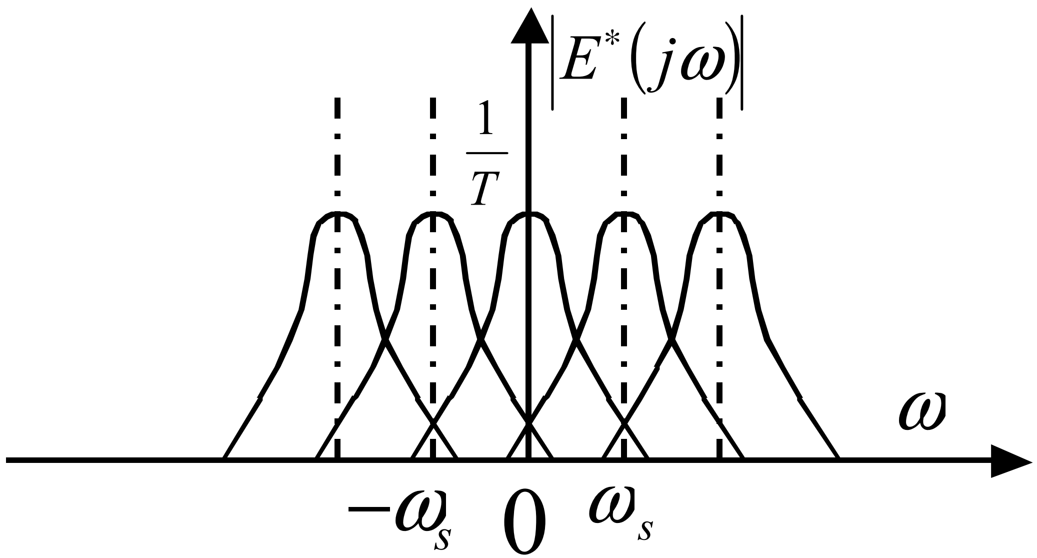
$$T < \frac{\pi}{\omega_h}$$



$$\omega_s > 2\omega_h$$

Sampling switch **Ideal filter**

there are no overlap of each component, so the input signal can be recovered approximately. The is called sampling theorem or Shannon's Theorem



In the figure the input signal can't be recovered.

History

- 采样定理是1928年由美国电信工程师H. Nyquist首先提出来的，因此称为Nyquist采样定理；
- 1933年由苏联工程师科捷利尼科夫首次用公式严格地表述这一定理，因此在苏联文献中称为科捷利尼科夫采样定理；
- 1948年信息论的创始人C.E. Shannon对这一定理加以明确地说明并正式作为定理引用，因此在许多文献中又称为Shannon采样定理或Nyquist–Shannon定理。

Homework

P236. 1(2， 3， 4)