DATA 605: Computational Mathematics Homework 3

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Library

```
# Data load
data(cars)
# libraries
library(dplyr)
library(ggplot2)
library(ggthemes)
library(lmtest)
```

Problem 1

Transportation Safety

Task

Using the cars dataset in R, perform the following steps:

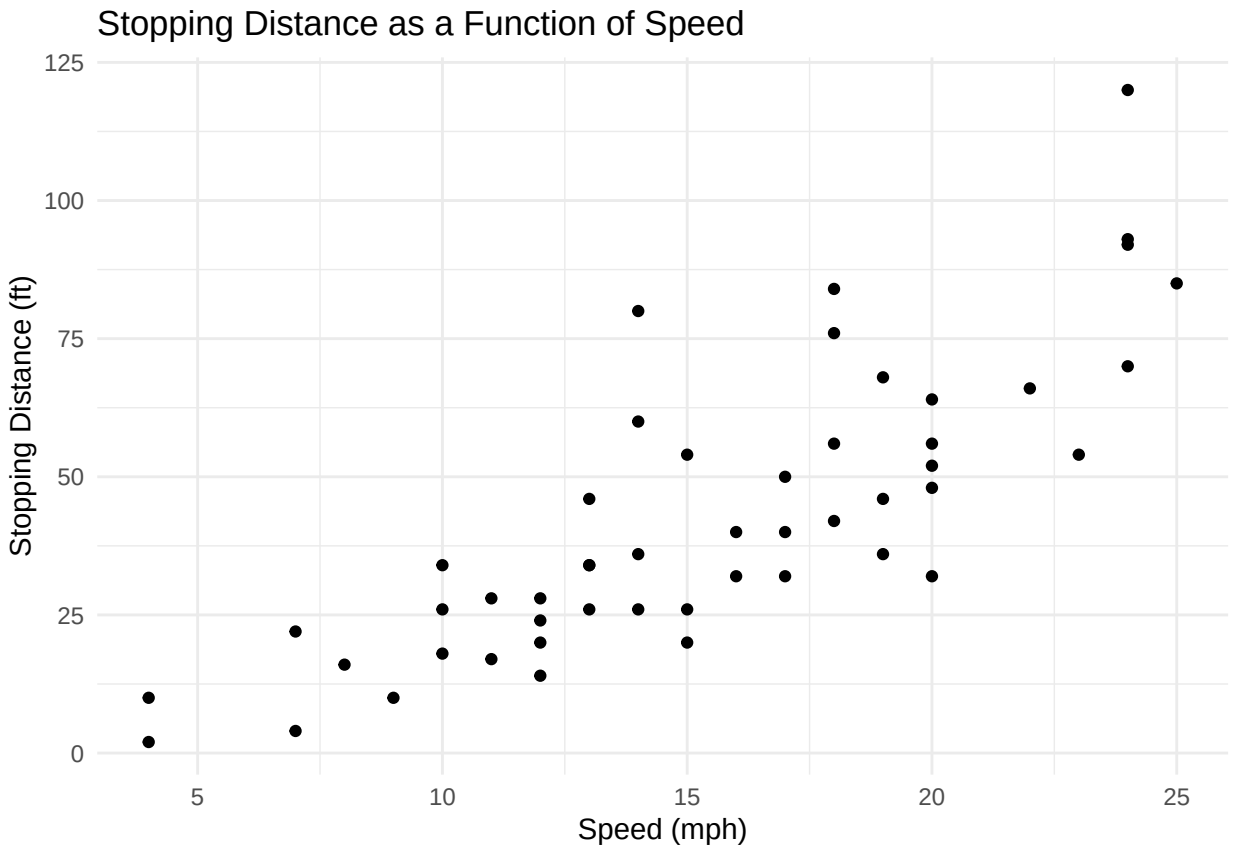
1

Data Visualization

i

Create a scatter plot of stopping distance (dist) as a function of speed (speed).

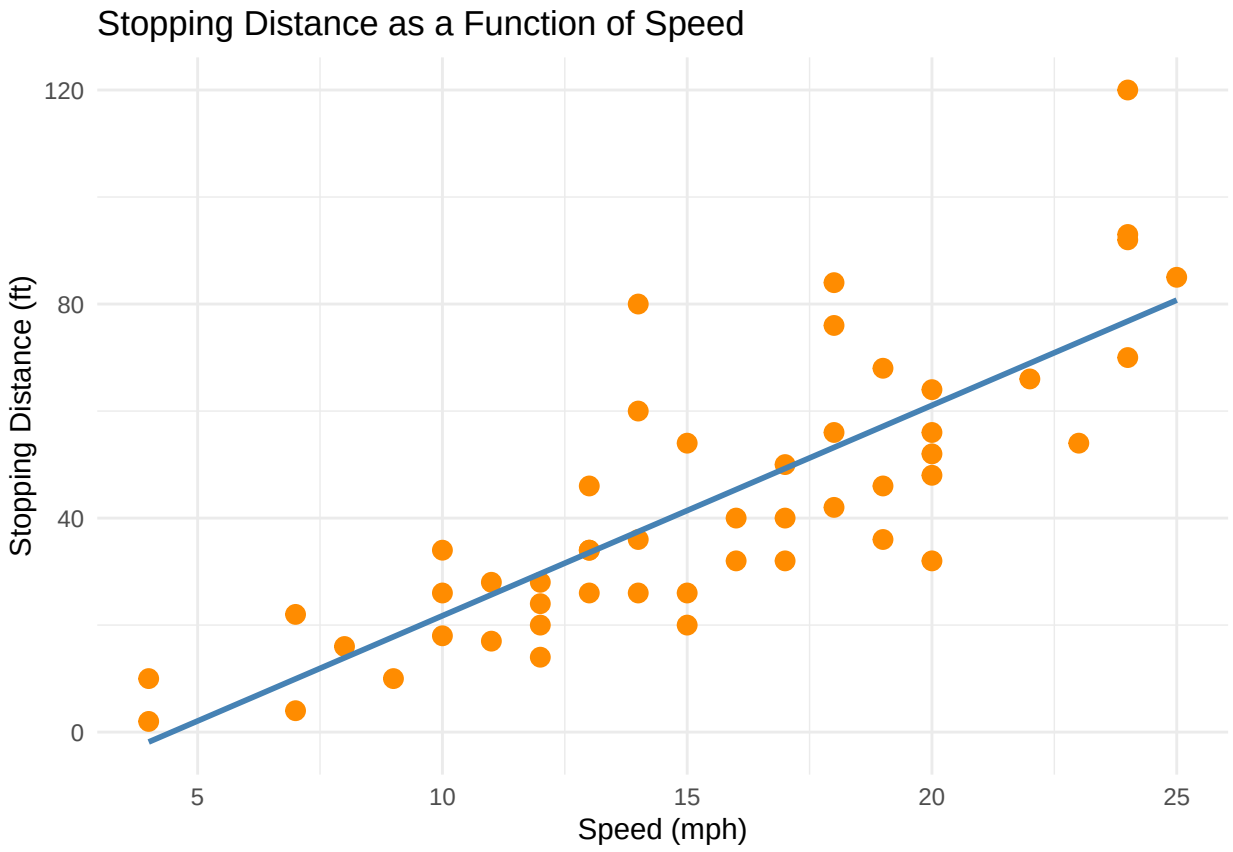
```
ggplot(cars, aes(x = speed, y = dist)) +
  geom_point() +
  labs(
    title = "Stopping Distance as a Function of Speed",
    x = "Speed (mph)",
    y = "Stopping Distance (ft)"
  ) +
  theme_minimal()
```



ii

Add a regression line to the plot to visually assess the relationship.

```
ggplot(cars, aes(x = speed, y = dist)) +
  geom_point(color = "darkorange", size = 3) +
  geom_smooth(method = "lm", color = "steelblue", se = FALSE) +
  labs(
    title = "Stopping Distance as a Function of Speed",
    x = "Speed (mph)",
    y = "Stopping Distance (ft)"
  ) +
  theme_minimal()
```



2

Build a Linear Model

i

Construct a simple linear regression model where stopping distance (dist) is the dependent variable and speed (speed) is the independent variable.

```
cars_model <- lm(dist~speed, data = cars)
```

ii

Summarize the model to evaluate its coefficients, R-squared value, and p-value.

```
summary(cars_model)
```

```
##
## Call:
## lm(formula = dist ~ speed, data = cars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -29.069  -9.525  -2.272   9.215  43.201
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -17.5791     6.7584  -2.601   0.0123 *
```

```
## speed         3.9324     0.4155   9.464 1.49e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15.38 on 48 degrees of freedom
## Multiple R-squared:  0.6511, Adjusted R-squared:  0.6438
## F-statistic: 89.57 on 1 and 48 DF,  p-value: 1.49e-12
```

- **Intercept & Speed** of -17.5791 just indicates the stopping distance at 0 mph. The value is not important only because logically a vehicle not in motion (0mph) does not require stopping distance. The value of 3.9324 indicates for every 1 mph 3.9324 ft is needed to stop. Therefore a minimum speed of ≈ 4.48 mph is needed before any distance is actually needed to stop.
- **R-Squared** value of 0.6511 indicates $\approx$ 65% of variance is captured by this relationship, so its a moderate indicator, and there may be several more and potentially stronger factor that can be used to predict the distance needed to stop.
- **P-Value** of $1.49e^{12}$ indicates that the relationship is statistically significant.

3

Model Quality Evaluation

i

Calculate and interpret the R-squared value to assess the proportion of variance in stopping distance explained by speed.

- **R-Squared** as indicated above this means $\approx$ 65% of the variance is captured, making this a moderate factor to stopping distance.
- **R-Squared Adjusted** Frankly, there is only 2 attributes to this table, so adjusting according to the number of predictors of the model makes little difference, therefore the variance of 0.6438 or $\approx$ 64% is relatively the same to the original, and the remaining factors would include attributes not collected in our dataset.

ii

Perform a residual analysis to check the assumptions of the linear regression model, including linearity, homoscedasticity, independence, and normality of residuals.

In order to do this without repeating the steps for Section 4 we leverage the various functions in R through the package lmtest

```
cars_resid_values <- resid(cars_model)
cars_fitted_values <- fitted(cars_model)
```

```
summary(cars_resid_values)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -29.069  -9.525  -2.272   0.000   9.215  43.201
```

```
lmtest::bptest(cars_model)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  cars_model
## BP = 3.2149, df = 1, p-value = 0.07297
```

```
cor(cars_fitted_values, cars$dist)
```

```
## [1] 0.8068949
```

```
dwtest(cars_model)
```

```
##
##  Durbin-Watson test
##
## data:  cars_model
## DW = 1.6762, p-value = 0.09522
## alternative hypothesis: true autocorrelation is greater than 0
```

```
ks.test(cars_resid_values, "pnorm", mean=mean(cars_resid_values, sd = sd(cars_resid_values)))
```

```
##
##  Asymptotic one-sample Kolmogorov-Smirnov test
##
## data:  cars_resid_values
## D = 0.49833, p-value = 3.283e-11
## alternative hypothesis: two-sided
```

- We have an mean of 0 which is ideal
- A correlation of 0.8068949 suggests strong linearity since it is close to 1.
- the $p-value > 0.05$ through the *Breusch-Pagan test* (**bptest()**) which it is at 0.7297 indicates homoscedasticity.
- the $p-value$ through the *Durbin-Watson test* (**dwtest()**) of 0.09522 indicated residuals are independent or has no significant autocorrelation in residuals.
- the $p-value$ through the *Kolmogorav-Smirnov test* (**ks.test()**) of $3.283 \times 10^{-}11/$ indicates its not normally distributed.

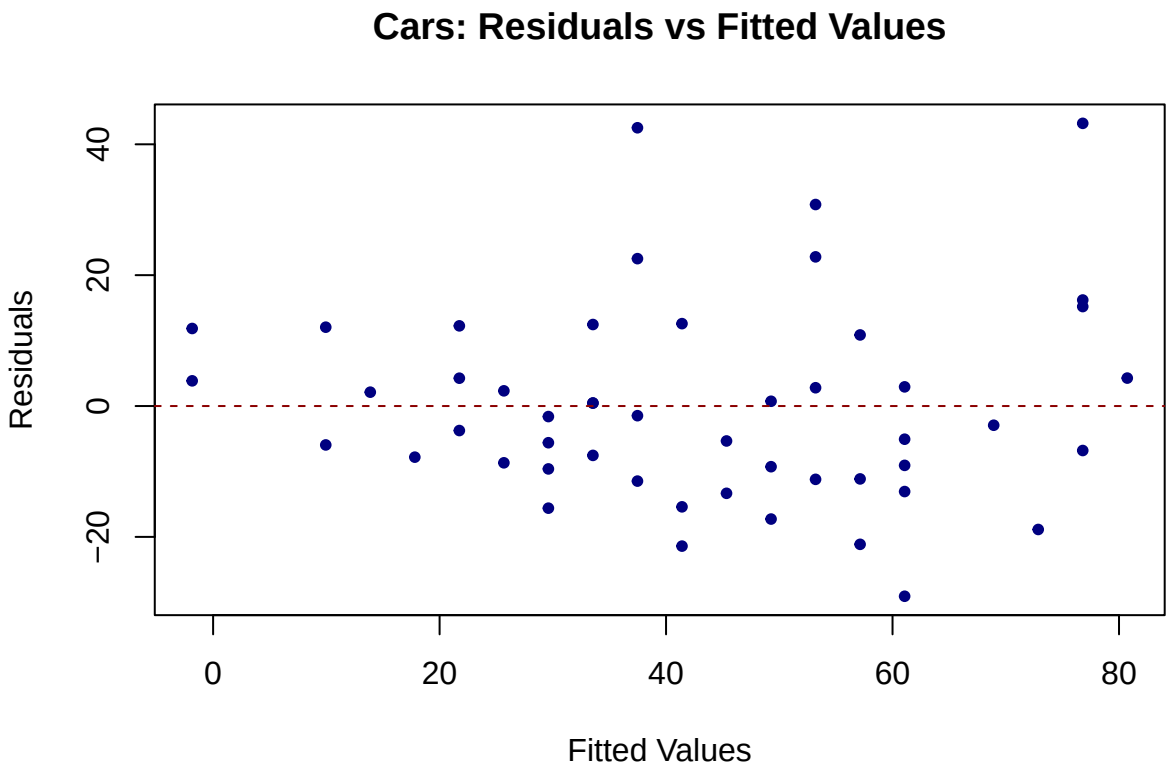
4

Residual Analysis

i

Plot the residuals versus fitted values to check for any patterns.

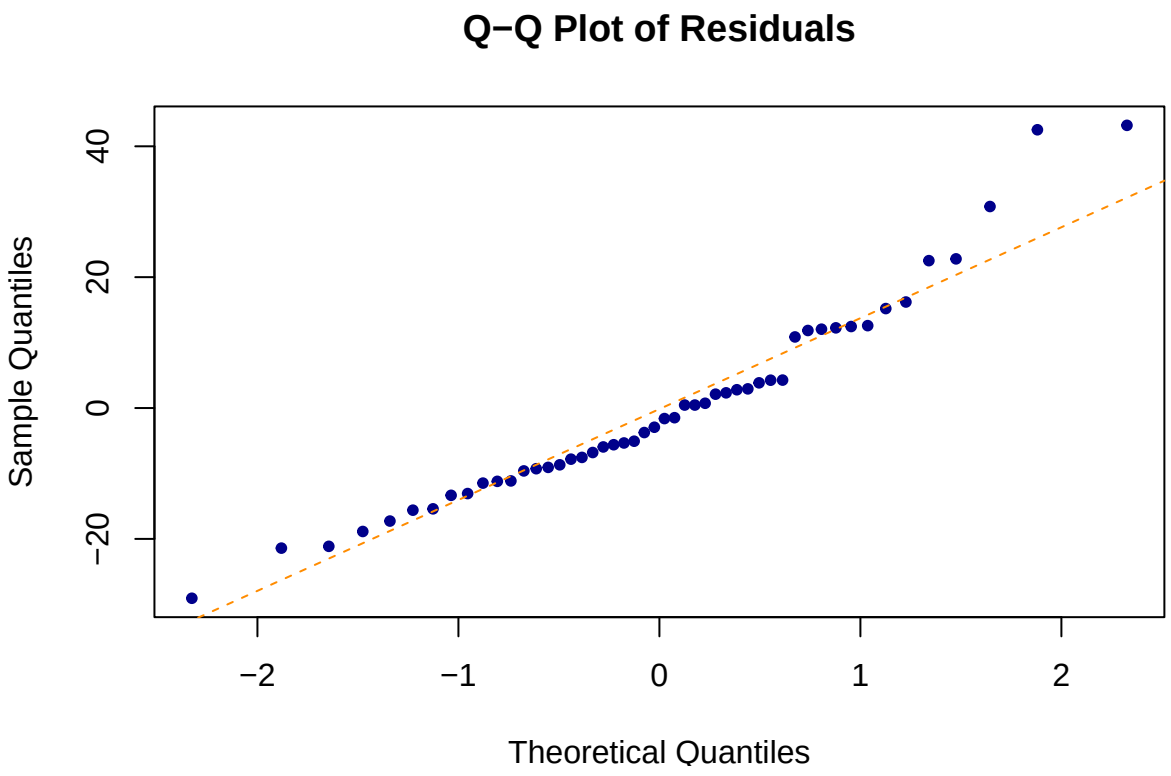
```
plot(cars_fitted_values, cars_resid_values,
     xlab = "Fitted Values",
     ylab = "Residuals",
     main = "Cars: Residuals vs Fitted Values",
     pch = 20,
     col = "navyblue")
abline(h = 0, col = "darkred", lty = 2)
```



ii

Create a Q-Q plot of the residuals to assess normality.

```
qqnorm(cars_resid_values,
       main = "Q-Q Plot of Residuals",
       pch = 20,
       col = "darkblue")
qqline(cars_resid_values, col = "darkorange", lty = 2)
```



iii

Perform a Shapiro-Wilk test for normality of residuals.

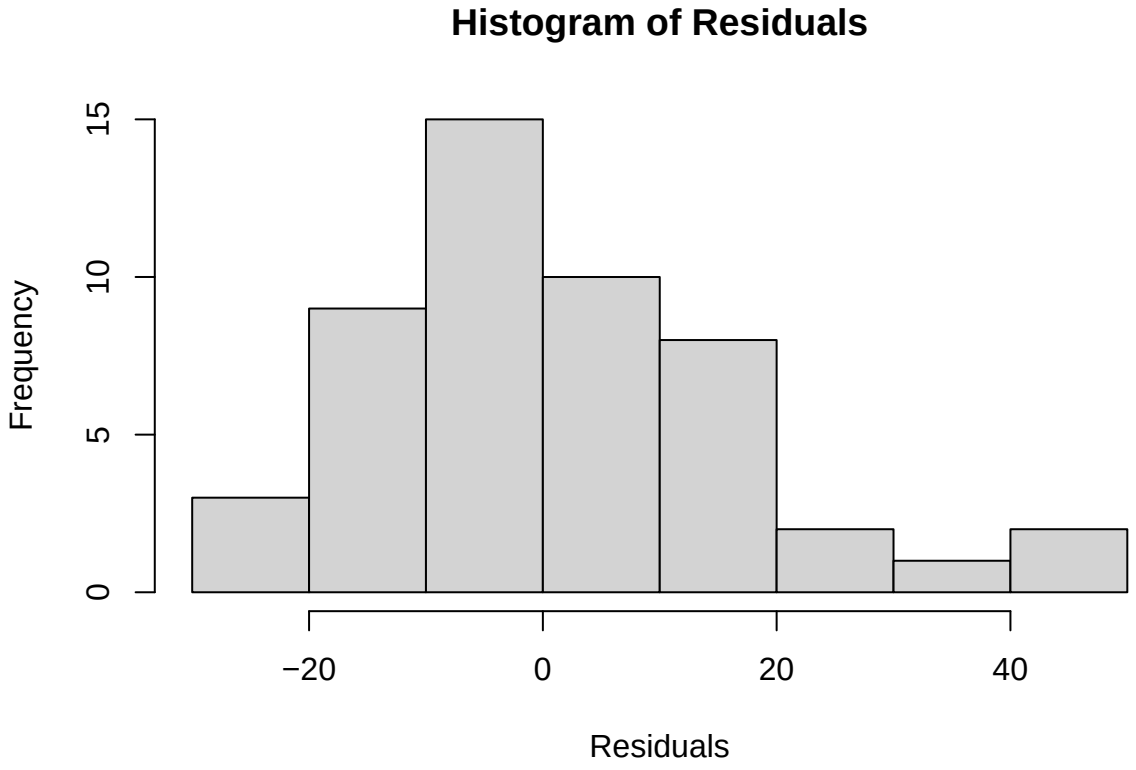
```
cars_shapiro_test <- shapiro.test(cars_resid_values)
cars_shapiro_test
```

```
##
##  Shapiro-Wilk normality test
##
## data:  cars_resid_values
## W = 0.94509, p-value = 0.02152
```

iv

Plot a histogram of residuals to further check for normality.

```
hist(cars_resid_values,
     breaks = 10, # Adjust the number of bins as needed
     col = "lightgrey",
     main = "Histogram of Residuals",
     xlab = "Residuals",
     ylab = "Frequency",
     border = "black")
```



Conclusion

Based on the model summary and residual analysis, determine whether the linear model is appropriate for this data. Discuss any potential violations of model assumptions and suggest improvements if necessary.

The model for this data set is reasonably appropriate, as it demonstrates moderate explanatory power ($R^2 \approx 0.65$). While the Q-Q plot and histogram suggest residuals indicate near-normal residuals, the Shapiro-Wilk test ($p = 0.02152$) suggests mild non-normality. Additionally, the residuals vs fitted plot reveals 3 outliers that may impact the models validity. To improve the model, we can consider removing or adjusting for these outliers or testing a non-linear regression model to capture potential curvature in the data.

Problem 2

Health Policy Analyst

As a health policy analyst for an international organization, you are tasked with analyzing data from the World Health Organization (WHO) to inform global health policies. The dataset provided (who.csv) contains crucial health indicators for various countries from the year 2008. The variables include:

- Country: Name of the country
- LifeExp: Average life expectancy for the country in years
- InfantSurvival: Proportion of those surviving to one year or more
- Under5Survival: Proportion of those surviving to five years or more
- TBFree: Proportion of the population without TB
- PropMD: Proportion of the population who are MDs
- PropRN: Proportion of the population who are RNs
- PersExp: Mean personal expenditures on healthcare in US dollars at average exchange rate
- GovtExp: Mean government expenditures per capita on healthcare, US dollars at average exchange rate

- TotExp: Sum of personal and government expenditures

Your analysis will directly influence recommendations for improving global life expectancy and the allocation of healthcare resources.

```
who_df <- read.csv("who.csv", fileEncoding = "UTF-8")%>%
              select(-LifeExp.1,-X)
```

Question 1

Initial Assessment of Healthcare Expenditures and Life Expectancy

Task

Create a scatterplot of LifeExp vs. TotExp to visualize the relationship between healthcare expenditures and life expectancy across countries. Then, run a simple linear regression with LifeExp as the dependent variable and TotExp as the independent variable (without transforming the variables).

- Provide and interpret the F-statistic, R-squared value, standard error, and p-values.
- Discuss whether the assumptions of simple linear regression (linearity, independence, homoscedasticity, and normality of residuals) are met in this analysis.

```
str(who_df)
```

```
## 'data.frame':    42 obs. of  10 variables:
##  $ Country       : chr  "Afghanistan" "Albania" "Algeria" "Andorra" ...
##  $ LifeExp       : int  42 71 71 82 41 73 75 69 82 80 ...
##  $ InfantSurvival: num  0.835 0.985 0.967 0.997 0.846 0.99 0.986 0.979 0.995 0.996 ...
##  $ Under5Survival: num  0.743 0.983 0.962 0.996 0.74 0.989 0.983 0.976 0.994 0.996 ...
##  $ TBFree        : num  0.998 1 0.999 1 0.997 ...
##  $ PropMD        : num  2.29e-04 1.14e-03 1.06e-03 3.30e-03 7.04e-05 ...
##  $ PropRN        : num  0.000572 0.004614 0.002091 0.0035 0.001146 ...
##  $ PersExp       : int  20 169 108 2589 36 503 484 88 3181 3788 ...
##  $ GovtExp       : int  92 3128 5184 169725 1620 12543 19170 1856 187616 189354 ...
##  $ TotExp        : int  112 3297 5292 172314 1656 13046 19654 1944 190797 193142 ...
```

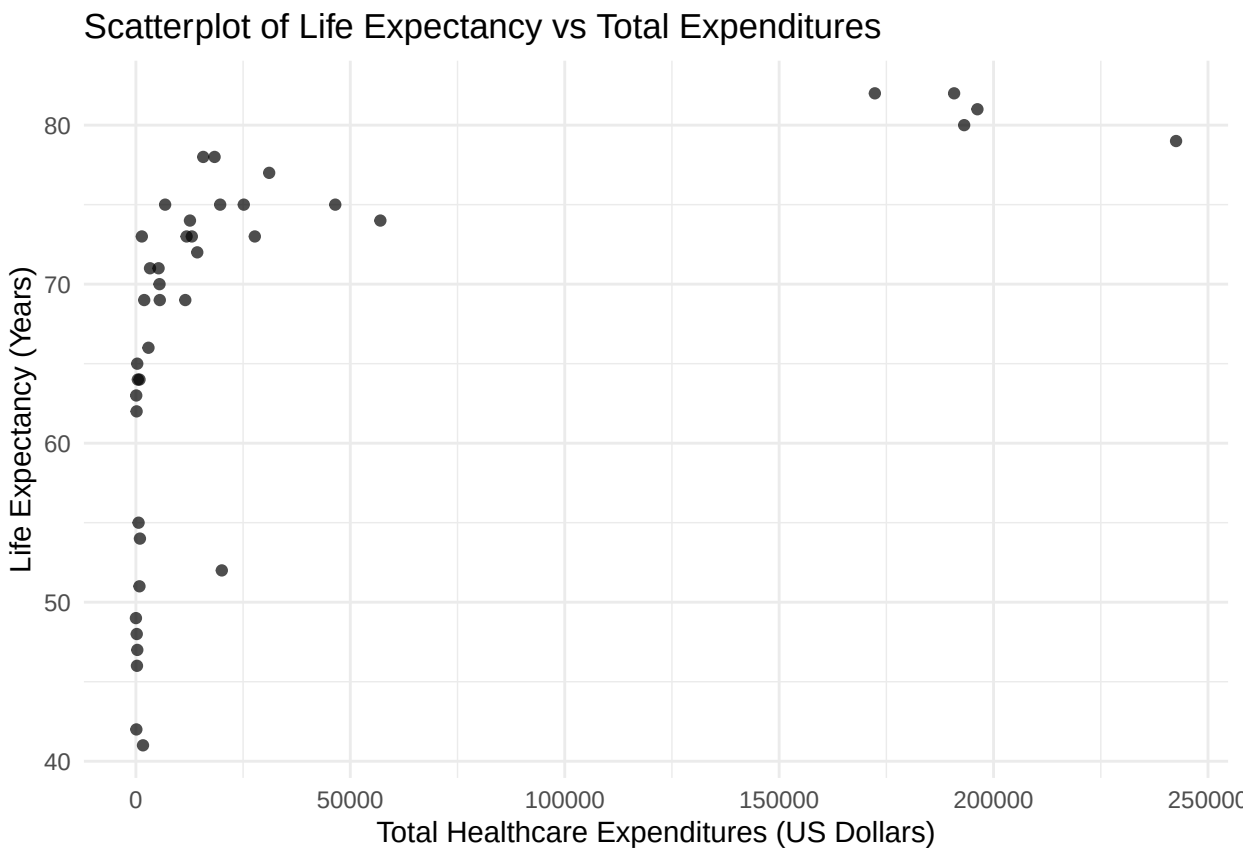
```
summary(who_df)
```

```
##    Country             LifeExp      InfantSurvival   Under5Survival
##  Length:42          Min.   :41.00   Min.   :0.8350   Min.   :0.7400
##  Class :character   1st Qu.:62.00   1st Qu.:0.9270   1st Qu.:0.9110
##  Mode  :character   Median :71.00   Median :0.9810   Median :0.9790
##                     Mean   :66.76   Mean   :0.9561   Mean   :0.9357
##                     3rd Qu.:75.00   3rd Qu.:0.9900   3rd Qu.:0.9880
##                     Max.   :82.00   Max.   :0.9970   Max.   :0.9960
##                     NA's   :1       NA's   :1        NA's   :1
##      TBFree           PropMD              PropRN             PersExp
##  Min.   :0.9929   Min.   :0.0000245   Min.   :0.0001649   Min.   :   3.0
##  1st Qu.:0.9973   1st Qu.:0.0001719   1st Qu.:0.0007410   1st Qu.:  36.0
##  Median :0.9993   Median :0.0010471   Median :0.0019340   Median : 198.0
##  Mean   :0.9982   Mean   :0.0012641   Mean   :0.0032899   Mean   : 603.5
##  3rd Qu.:0.9997   3rd Qu.:0.0014286   3rd Qu.:0.0046694   3rd Qu.: 484.0
##  Max.   :1.0000   Max.   :0.0047587   Max.   :0.0140792   Max.   :3788.0
##  NA's   :1        NA's   :1           NA's   :1           NA's   :1
##     GovtExp           TotExp
##  Min.   :    10   Min.   :    13
##  1st Qu.:   780   1st Qu.:   833
##  Median :  5394   Median :  5574
```

```
##  Mean   : 32547   Mean   : 33150
##  3rd Qu.: 19604   3rd Qu.: 20035
##  Max.   :239105   Max.   :242556
##  NA's   :1        NA's   :1
```

Create a scatterplot of LifeExp vs. TotExp

```
ggplot(who_df, aes(x = TotExp, y = LifeExp)) +
  geom_point(alpha = 0.7) +
  labs(
    title = "Scatterplot of Life Expectancy vs Total Expenditures",
    x = "Total Healthcare Expenditures (US Dollars)",
    y = "Life Expectancy (Years)"
  ) +
  theme_minimal()
```



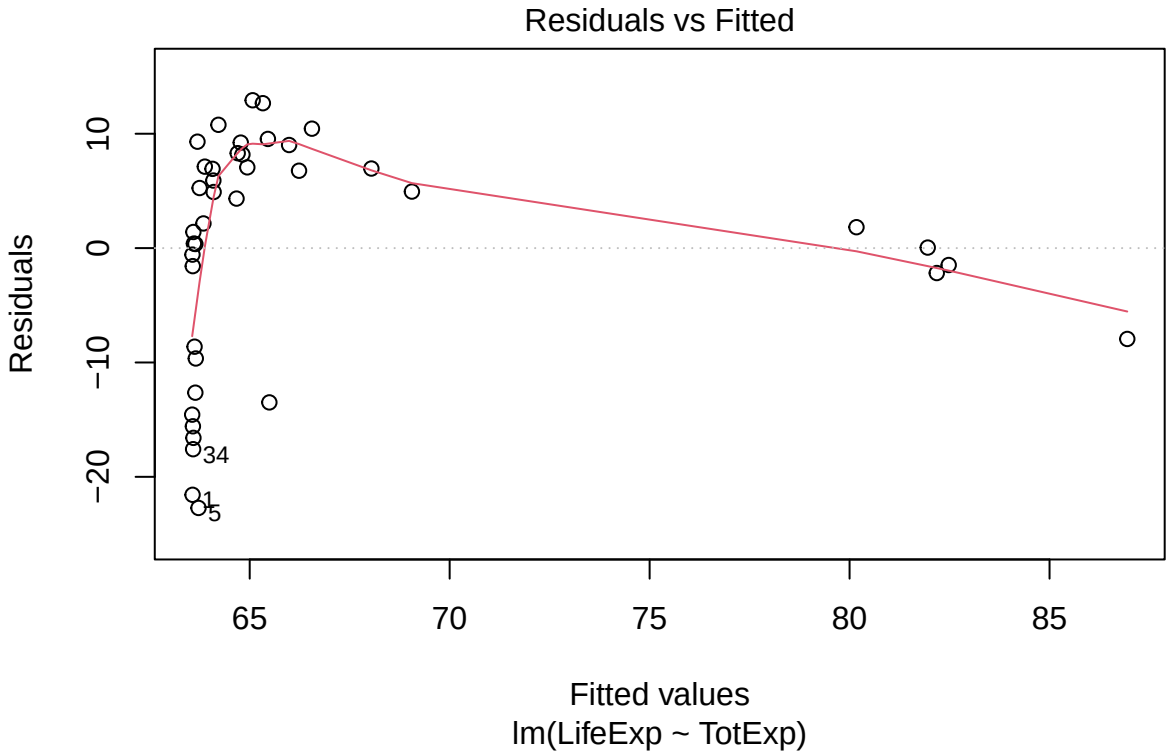

```
who_model <- lm(LifeExp ~ TotExp, data = who_df)
```

```
summary(who_model)
```

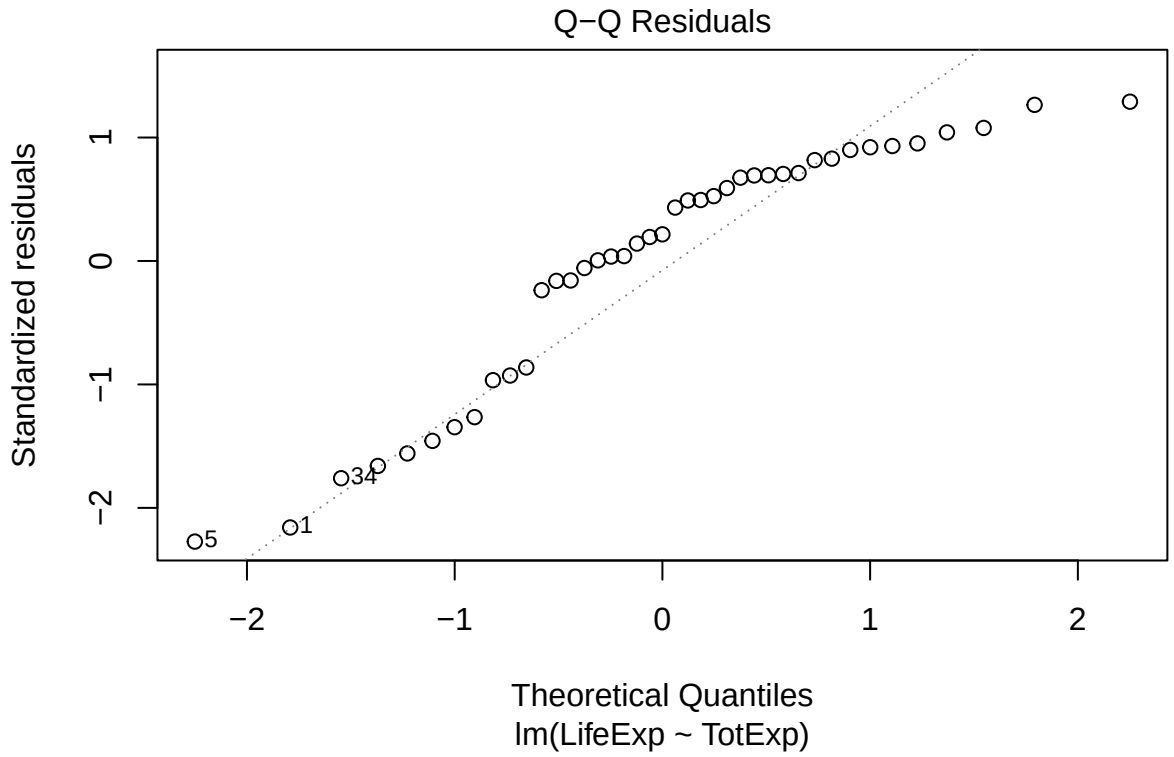
```
##
## Call:
## lm(formula = LifeExp ~ TotExp, data = who_df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -22.720  -7.944   2.157   7.122  12.926
##
## Coefficients:
```

```
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6.356e+01  1.788e+00  35.553  < 2e-16 ***
## TotExp      9.641e-05  2.493e-05   3.867 0.000406 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.15 on 39 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.2772, Adjusted R-squared:  0.2587
## F-statistic: 14.96 on 1 and 39 DF,  p-value: 0.0004064
```

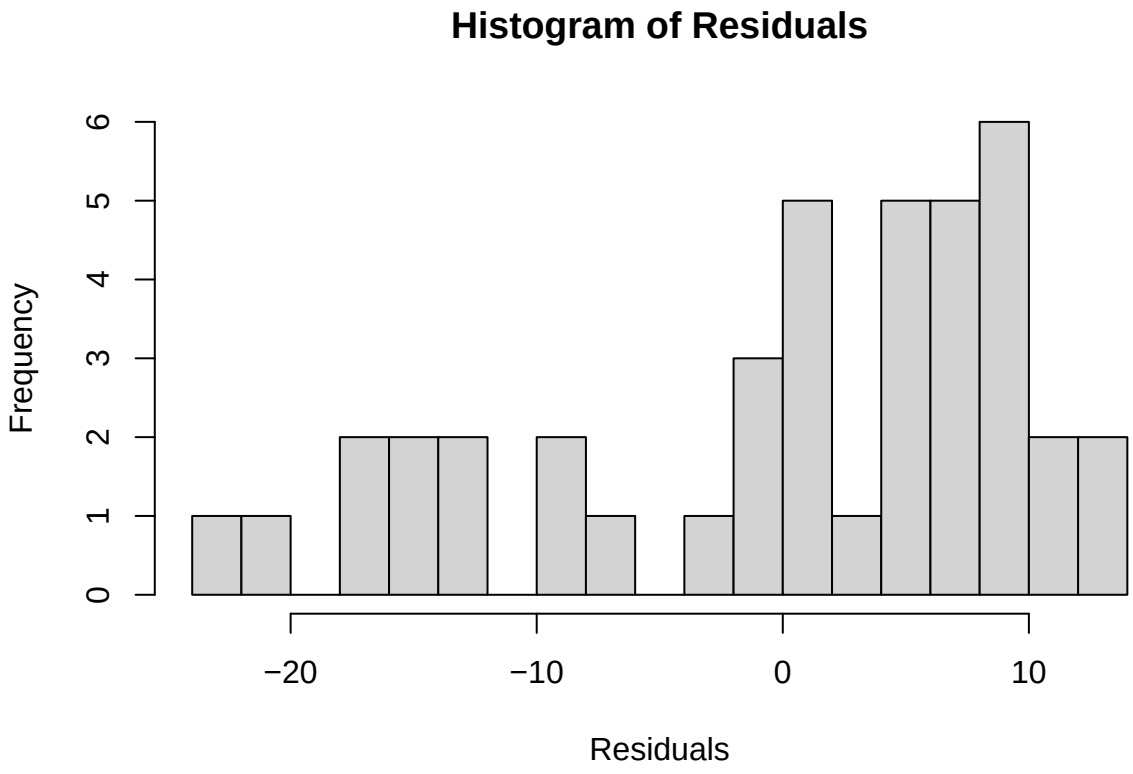
```r
#Residuals vs Fitted
plot(who_model, which = 1)
```




```r
# Q-Q plot
plot(who_model, which = 2)
```




```
# Histogram of residuals
hist(residuals(who_model), breaks = 20, main = "Histogram of Residuals", xlab = "Residuals")
```



Discussion

i.

Consider the implications of your findings for health policy. Are higher healthcare expenditures generally associated with longer life expectancy?

The data shows a moderate positive relationship between healthcare expenditures and life expectancy.

- $F-statistic$ of 14.96 and $p-value \approx 0.0004$ indicates statistical significance, suggesting the variables have a meaningful relationship with life expectancy. However, the models explains limited variability in life expectancy.
- Standard Error (SE) of 10.15 suggests a relatively good fit,providing reasonable precision of the estimating regression coefficient such as slope or intercept.
- A positive coefficient 9.641×10^{-5} implies as healthcare expenditure increases, life expectancy improves. However, the magnitude suggests a small incremental effect.
- The residuals deviates significantly from normality, as shown by the Q-Q plot and supported by the Histogram of Residuals.
- There are no evident signs of homoscedasticity. The points do not show randomness or form a funnel or fan like shape.
- There is a clear pattern with the residuals, which do not appear random, with a visible curvature. This indicates a non-linear relationship between the variables, and makes it innappropriate to assume independence of residuals.

ii.

What do the assumptions of the regression model suggest about the reliability of this relationship? $R^2 = 0.2772$ suggests the relationship is no particularly strong, implying that while healthcare expenditures explain some variation in life expectancy, the model does not capture all the influencing factors.

Question 2

Transforming Variables for a Better Fit

Task

Recognizing potential non-linear relationships, transform the variables as follows

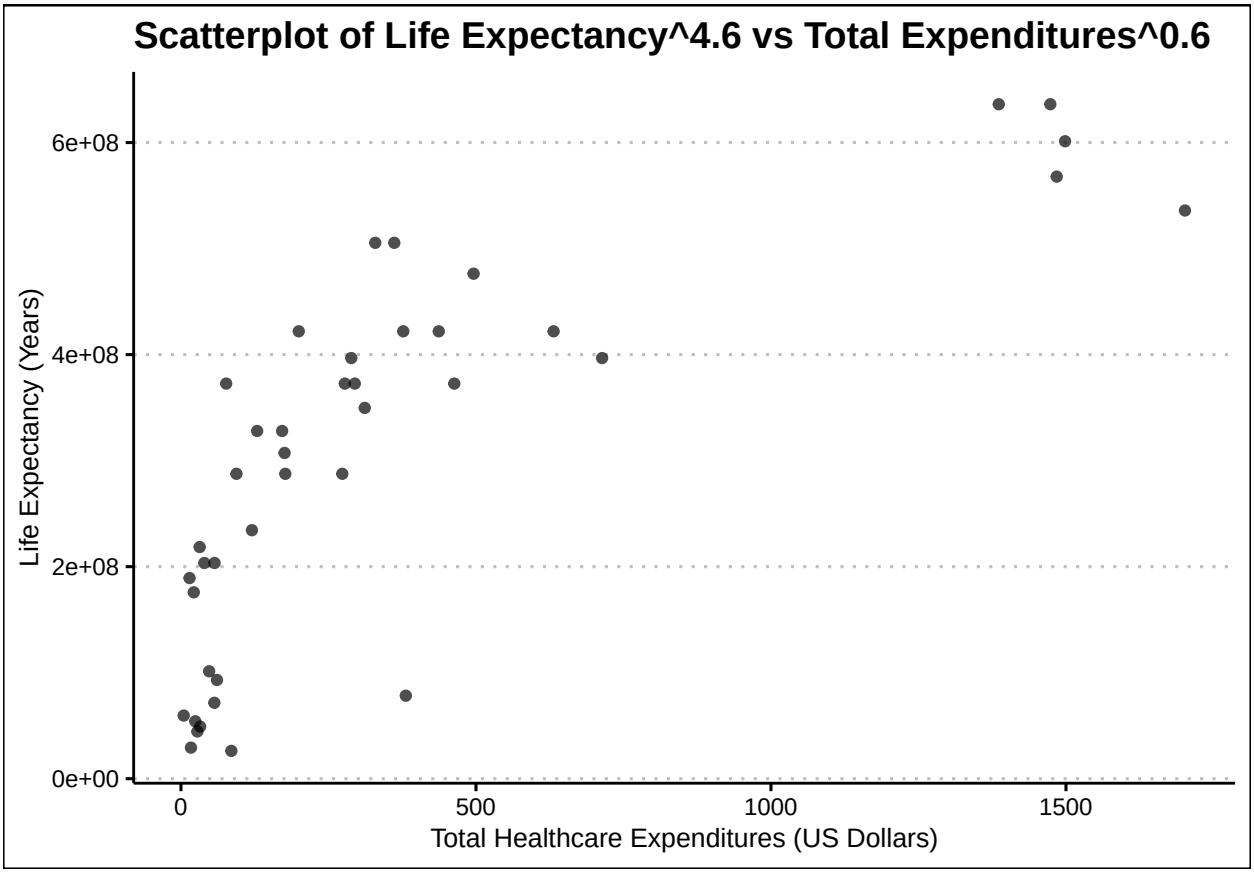
- Raise life expectancy to the 4.6 power (LifeExp^4.6).
- Raise total expenditures to the 0.06 power (TotExp^0.06), which is nearly a logarithmic transformation.

Create a new scatterplot with the transformed variables and re-run the simple linear regression model.

- Provide and interpret the F-statistic, R-squared value, standard error, and p-values for the transformed model.
- Compare this model to the original model (from Question 1). Which model provides a better fit, and why?

```
who_df$LifeExp_t <- who_df$LifeExp^4.6
who_df$TotExp_t <- who_df$TotExp^0.6
```

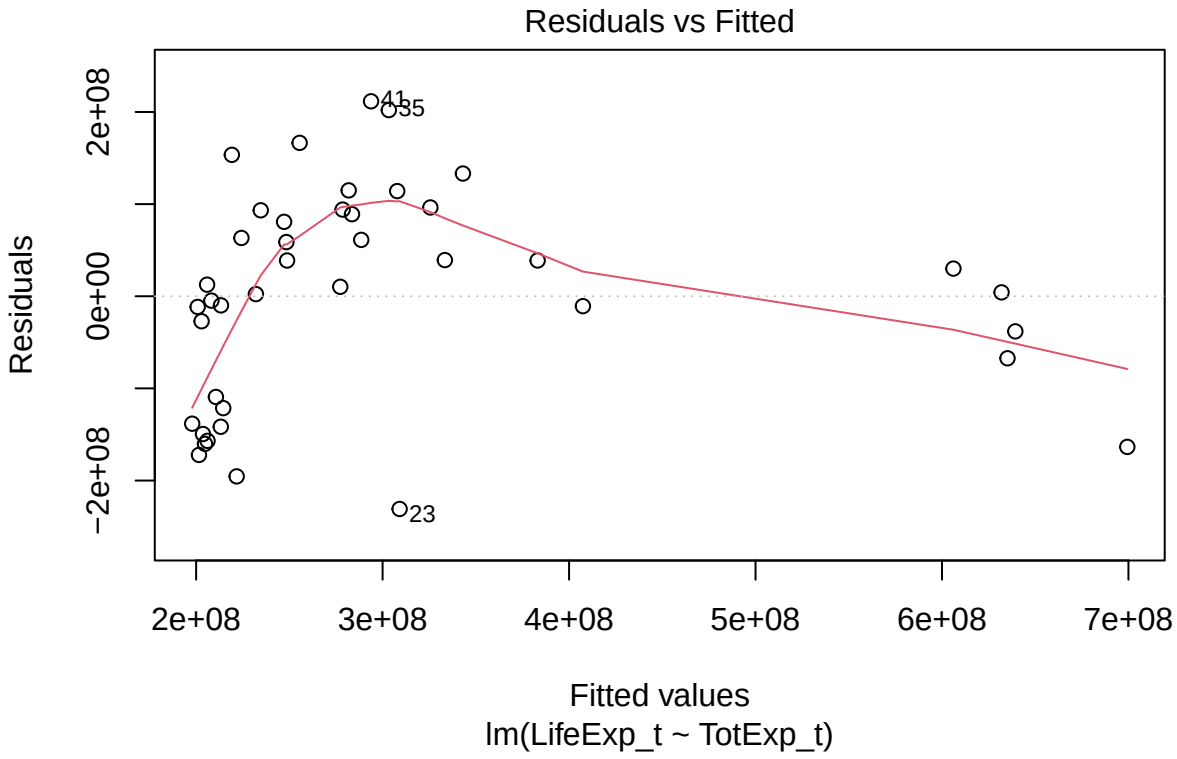
```
ggplot(who_df, aes(x = TotExp_t, y = LifeExp_t)) +
  geom_point(alpha = 0.7) +
  labs(
    title = "Scatterplot of Life Expectancy^4.6 vs Total Expenditures^0.6",
    x = "Total Healthcare Expenditures (US Dollars)",
    y = "Life Expectancy (Years)"
  ) +
  theme_clean()
```



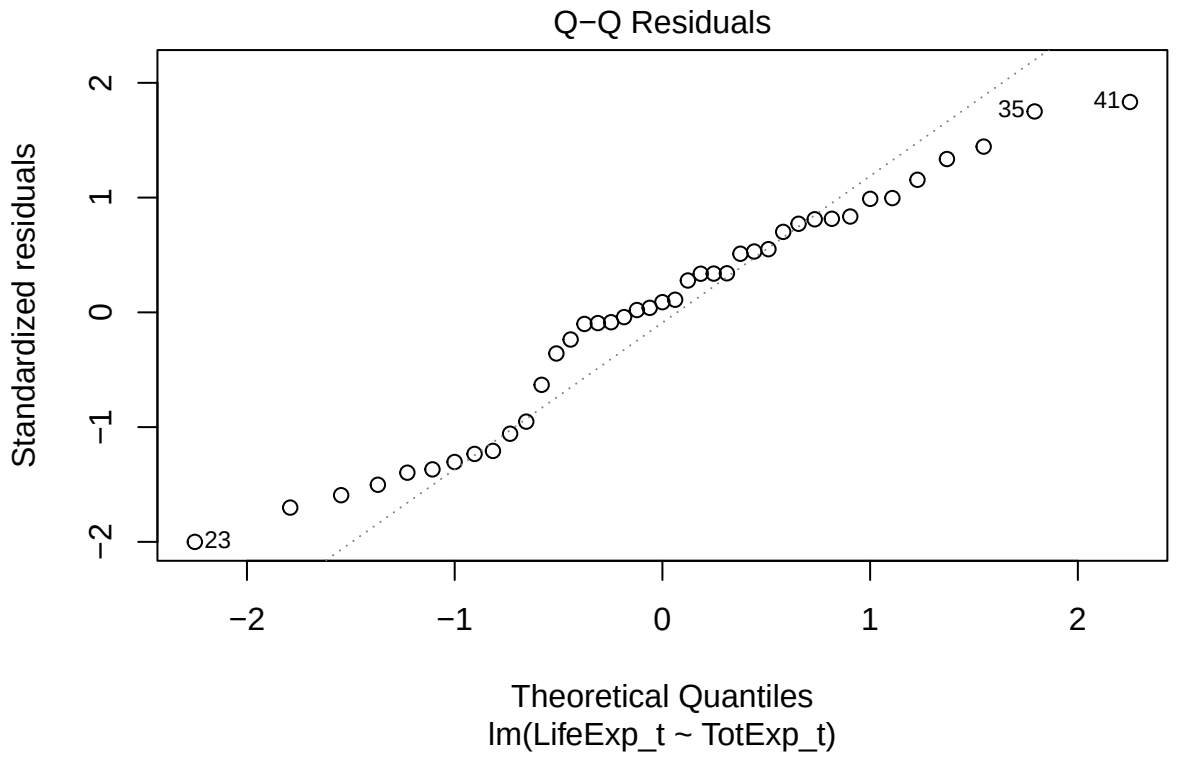

```r
who_model_t <- lm(LifeExp_t ~ TotExp_t, data = who_df)
summary(who_model_t)
```

```
##
## Call:
## lm(formula = LifeExp_t ~ TotExp_t, data = who_df)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -230855907 -109289555   10242350   89067282  211587690
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 196498750   23177824   8.478 2.21e-10 ***
## TotExp_t       295518      39460   7.489 4.61e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 116900000 on 39 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.5898, Adjusted R-squared:  0.5793
## F-statistic: 56.09 on 1 and 39 DF,  p-value: 4.61e-09
```

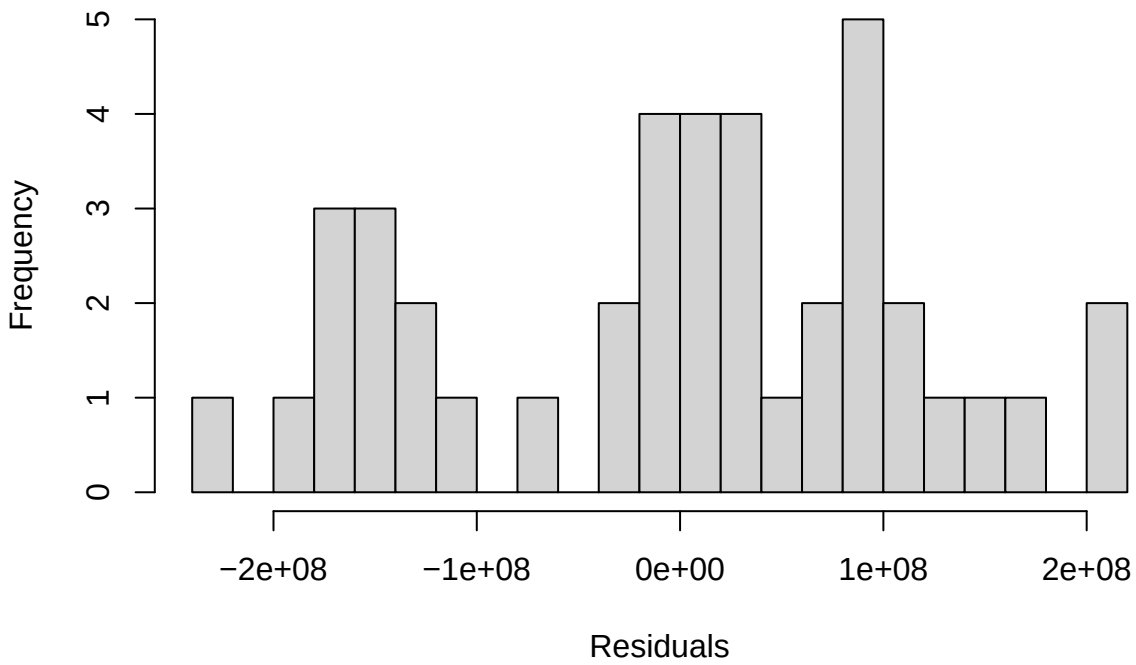
```r
#Residuals vs Fitted
plot(who_model_t, which = 1)
```




```r
# Q-Q plot
plot(who_model_t, which = 2)
```




```r
# Histogram of residuals
hist(residuals(who_model_t), breaks = 20, main = "Histogram of Residuals", xlab = "Residuals")
```



- $F-statistic$ of 56.09 and $p-value = 4.61\times10^{9}$ indicates the transformed model is statistical significance and captures greater proportion of variability.
- Standard Error (SE) of $116,900,000$ suggests a worse fit with the precision of the estimated regression coefficient. However, this is more likely a result of the greater magnitude of the transformed variables rather than an actual decline in performance.
- Positive coefficient $295,518$ implies as healthcare expenditure increases so does life expectancy still, reflecting a stronger relationship compared to the original model.
- The residuals appear more normalized, as shown with the Histogram of Residuals and Q-Q plot. Deviations are still present, however they are less severe than the original model.
- No homoscedasticity appears with the transformation.
- Residual patterns still do not support independence between the variables.

Discussion

i

How do the transformations impact the interpretation of the relationship between healthcare spending and life expectancy?

$R^2 = 0.5898$ suggests the relationship is strong, particularly when compare to the predecessor.

ii

Why might the transformed model be more appropriate for policy recommendations?

The stronger relationship is highlighted after the transformation, which better accounts for the complex, non-linear relationship between healthcare expenditures and life expectancy. This approach reduces risk in model specification and captures the diminishing return on healthcare spending, that has less of an impact after a certain threshold.

Question 3

Forecasting Life Expectancy Based on Transformed Expenditures

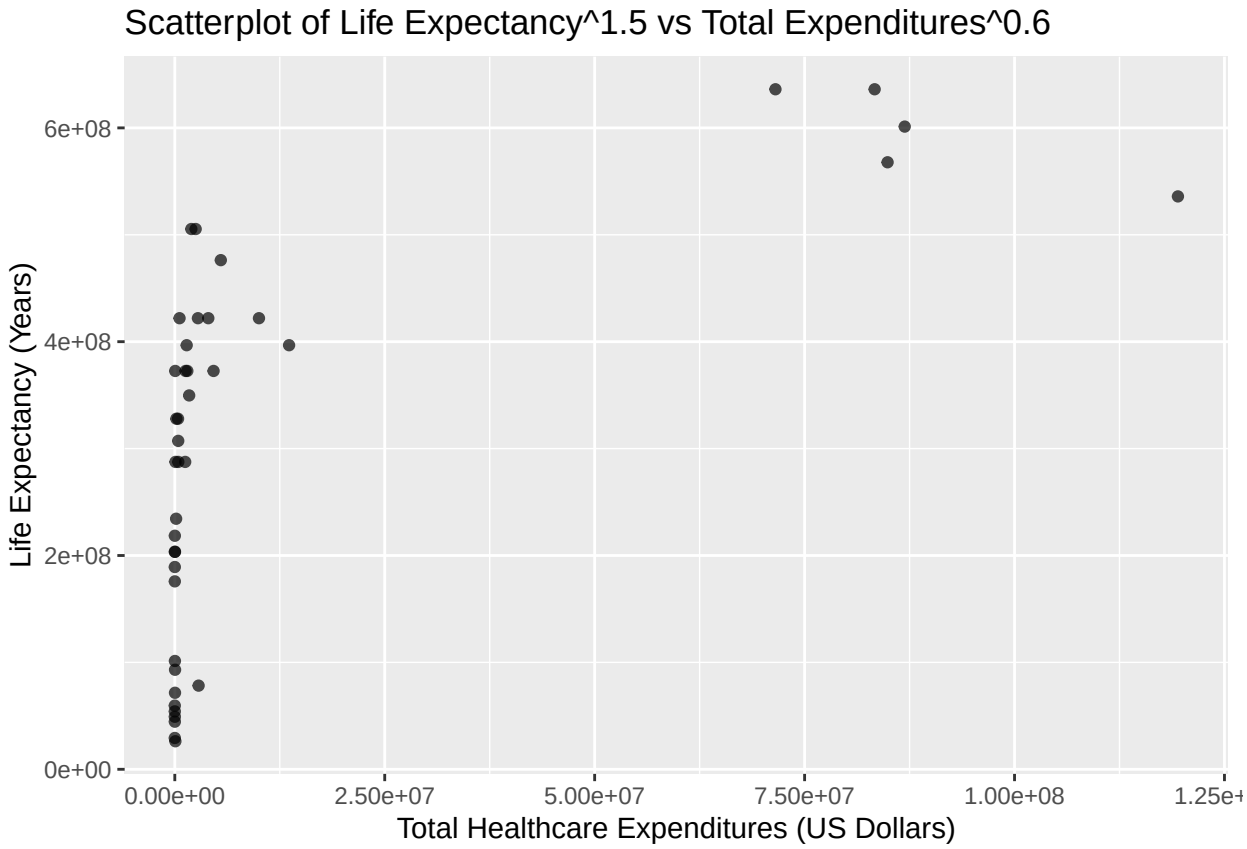
Task

Using the results from the transformed model in Question 2, forecast the life expectancy for countries with the following transformed total expenditures ($TotalExp^{0.06}$):

- When $TotExp^{0.06} = 1.5$
- When $TotExp^{0.06} = 2.5$

```
who_df$TotExp_t15 <- who_df$TotExp^1.5
who_df$TotExp_t25 <- who_df$TotExp^2.5
```

```
ggplot(who_df, aes(x = TotExp_t15, y = LifeExp_t)) +
  geom_point(alpha = 0.7) +
  labs(
    title = "Scatterplot of Life Expectancy^1.5 vs Total Expenditures^0.6",
    x = "Total Healthcare Expenditures (US Dollars)",
    y = "Life Expectancy (Years)"
  )
```

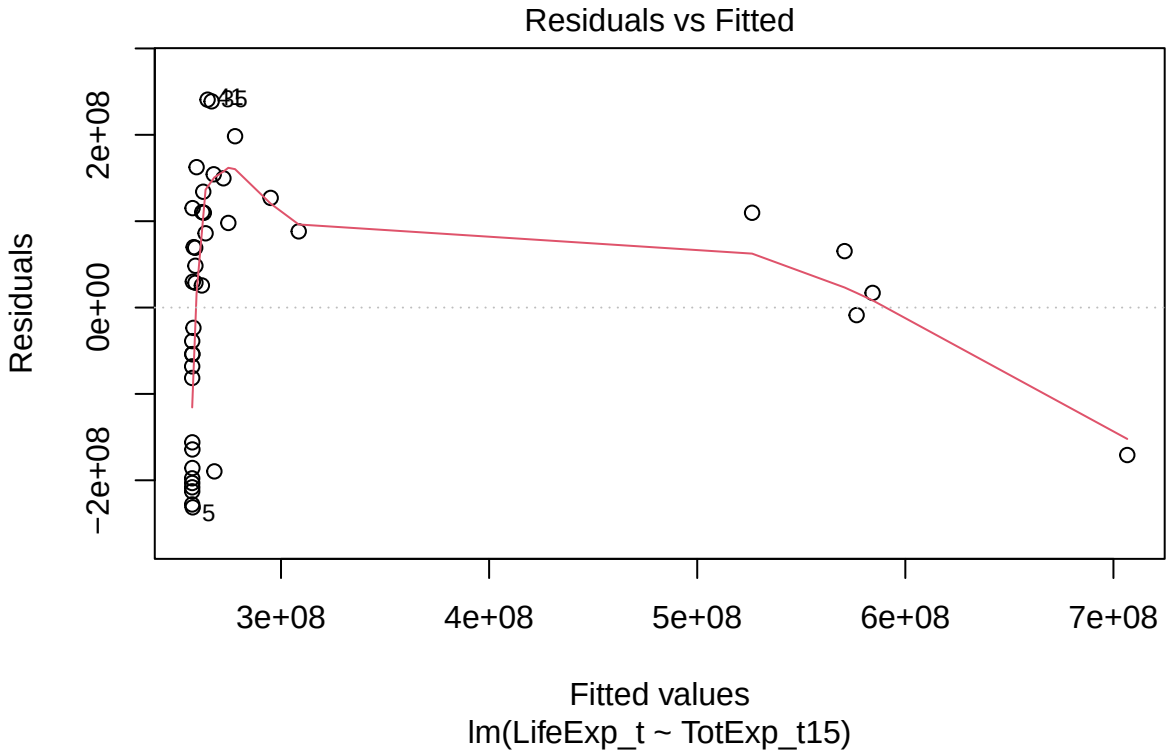



```
who_model_t15 <- lm(LifeExp_t ~ TotExp_t15, data = who_df)
summary(who_model_t15)
```

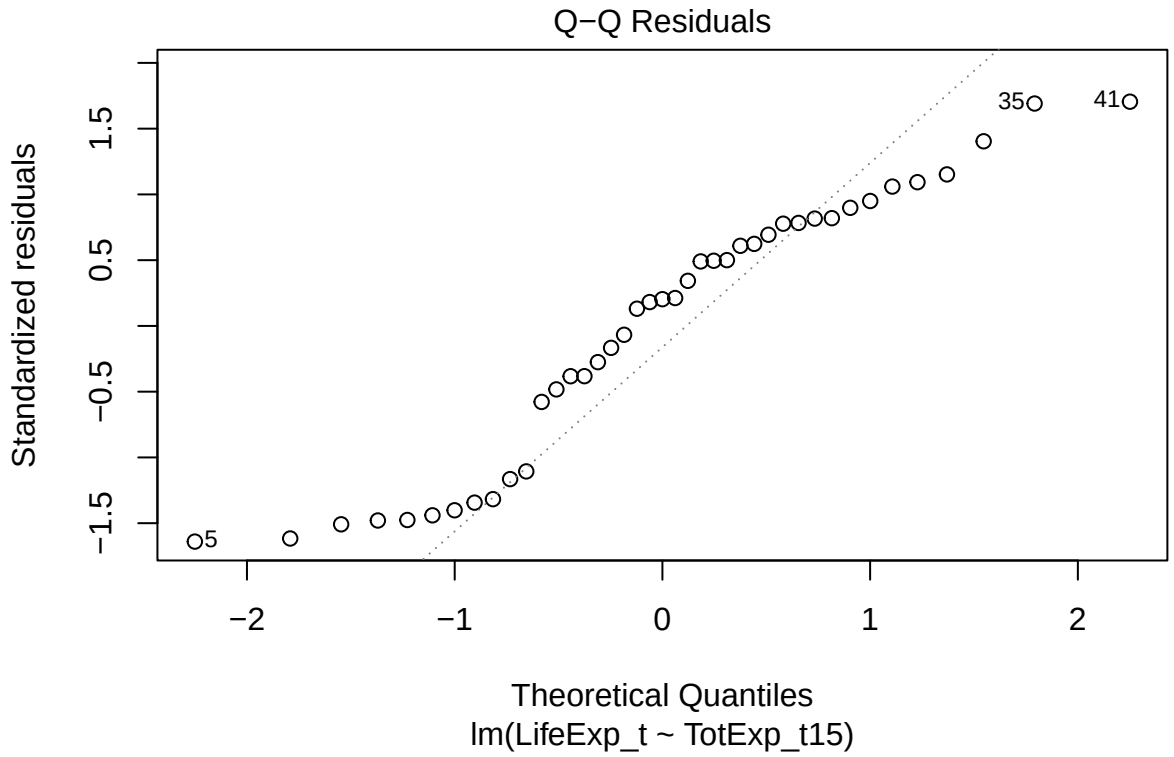
```
##
## Call:
## lm(formula = LifeExp_t ~ TotExp_t15, data = who_df)
##
```

```
## Residuals:
##        Min         1Q     Median         3Q        Max
## -231330911 -156053328   28671772  109763522  240690290
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.573e+08  2.423e+07  10.618 4.55e-13 ***
## TotExp_t15  3.761e+00  7.618e-01   4.938 1.53e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 143100000 on 39 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.3847, Adjusted R-squared:  0.3689
## F-statistic: 24.38 on 1 and 39 DF,  p-value: 1.525e-05
```

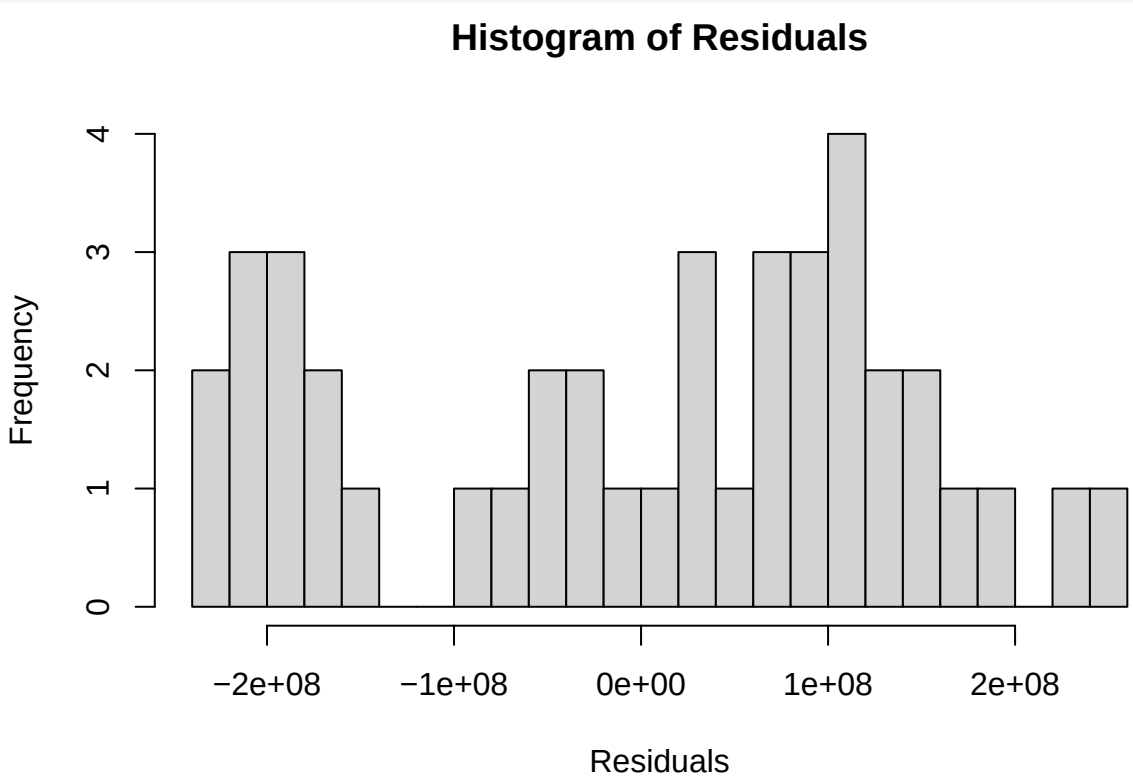
```r
#Residuals vs Fitted
plot(who_model_t15, which = 1)
```




```r
# Q-Q plot
plot(who_model_t15, which = 2)
```

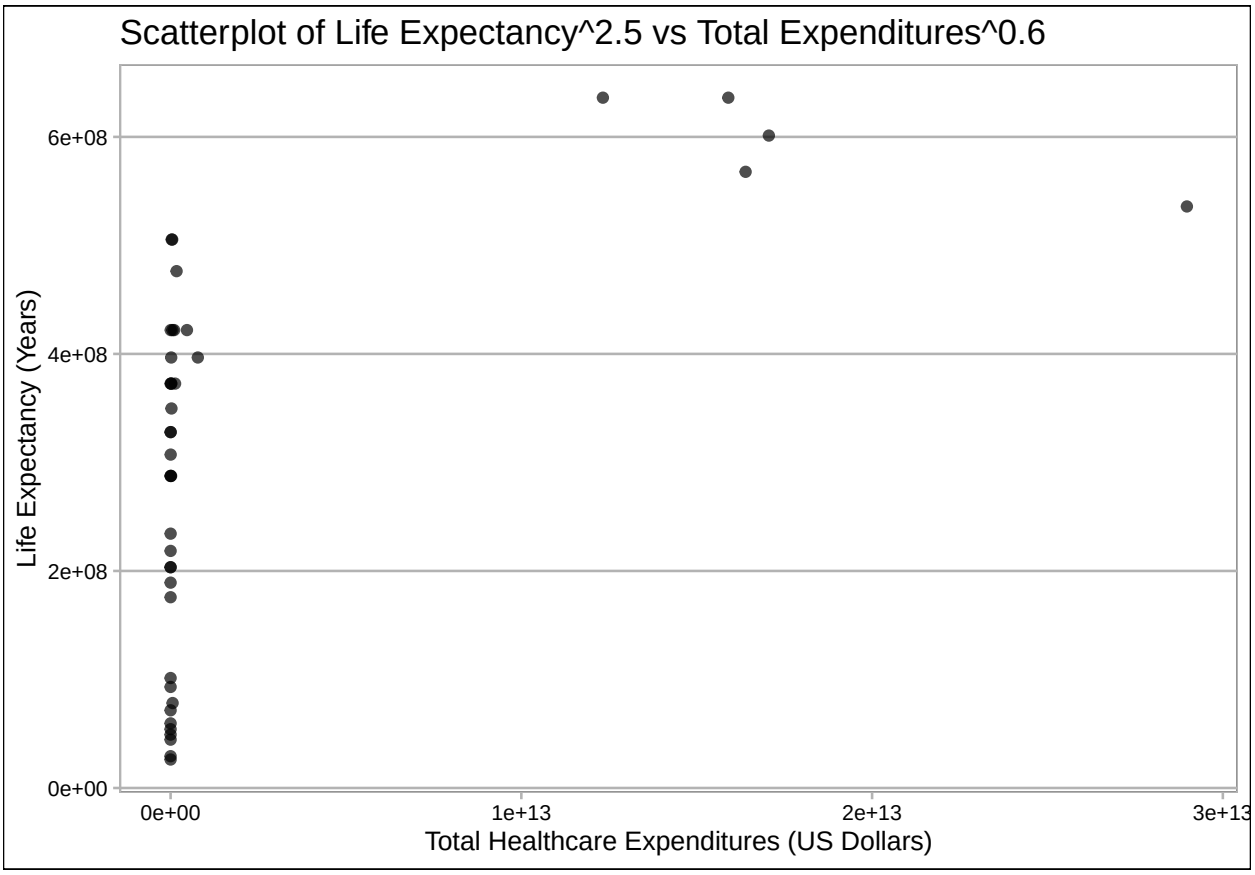



```
# Histogram of residuals
hist(residuals(who_model_t15), breaks = 20, main = "Histogram of Residuals", xlab = "Residuals")
```




```
ggplot(who_df, aes(x = TotExp_t25, y = LifeExp_t)) +
  geom_point(alpha = 0.7) +
  labs(
```

```
    title = "Scatterplot of Life Expectancy^2.5 vs Total Expenditures^0.6",
    x = "Total Healthcare Expenditures (US Dollars)",
    y = "Life Expectancy (Years)"
  ) +
  theme_calc()
```

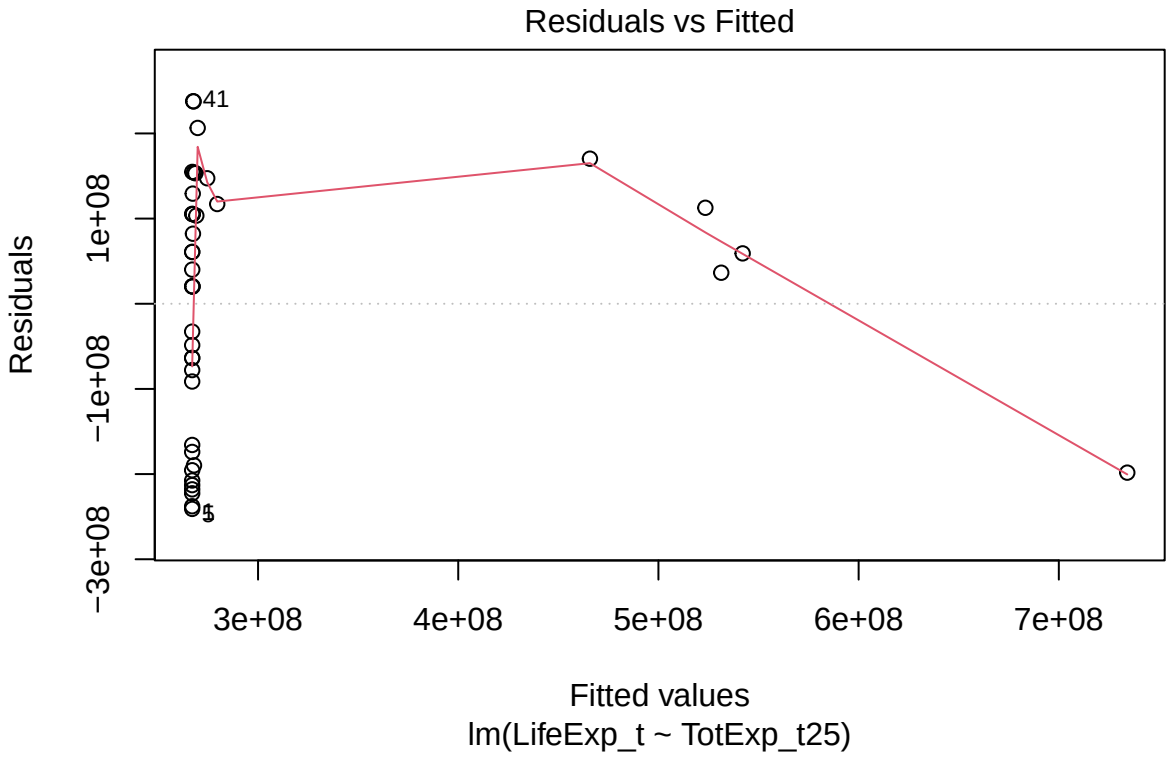



```
who_model_t25 <- lm(LifeExp_t ~ TotExp_t25, data = who_df)
summary(who_model_t25)
```

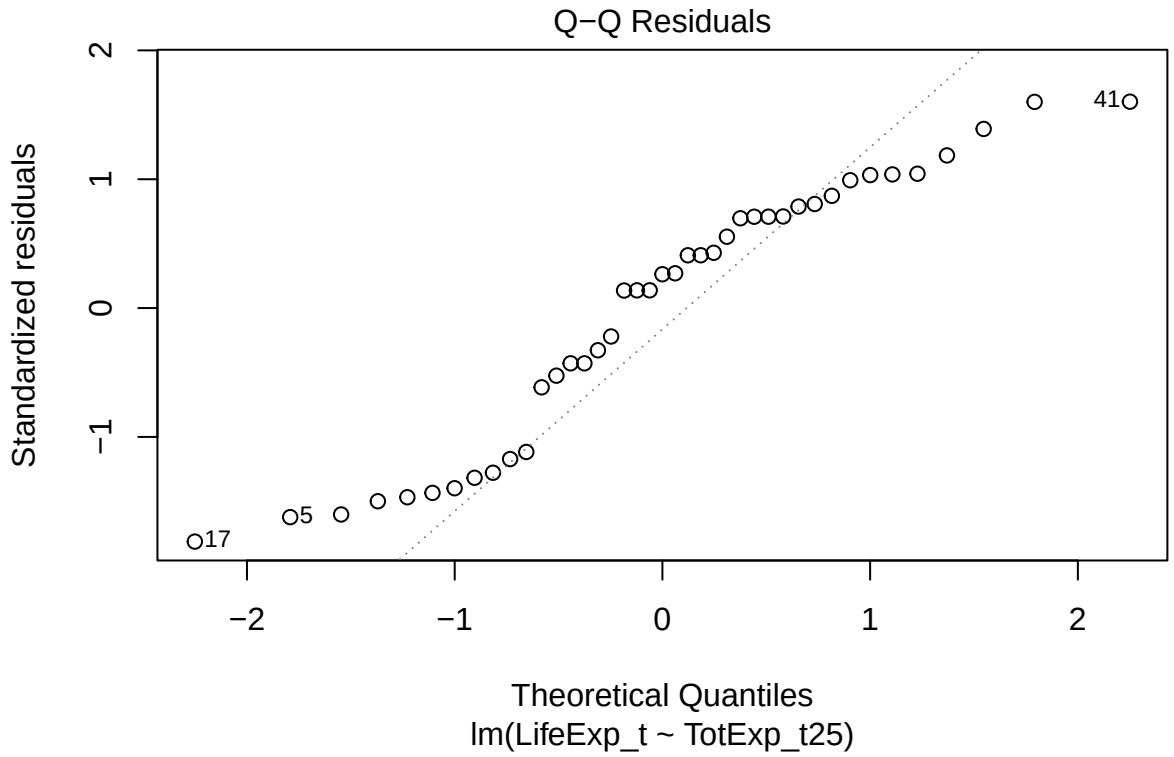
```
##
## Call:
## lm(formula = LifeExp_t ~ TotExp_t25, data = who_df)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -240868624 -165783689   36440609  112692262  237804522
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.671e+08  2.500e+07  10.683 3.81e-13 ***
## TotExp_t25  1.612e-05  3.768e-06   4.278 0.000118 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 150500000 on 39 degrees of freedom
##   (1 observation deleted due to missingness)
```

```
## Multiple R-squared:  0.3194, Adjusted R-squared:  0.302
## F-statistic:  18.3 on 1 and 39 DF,  p-value: 0.0001182
```

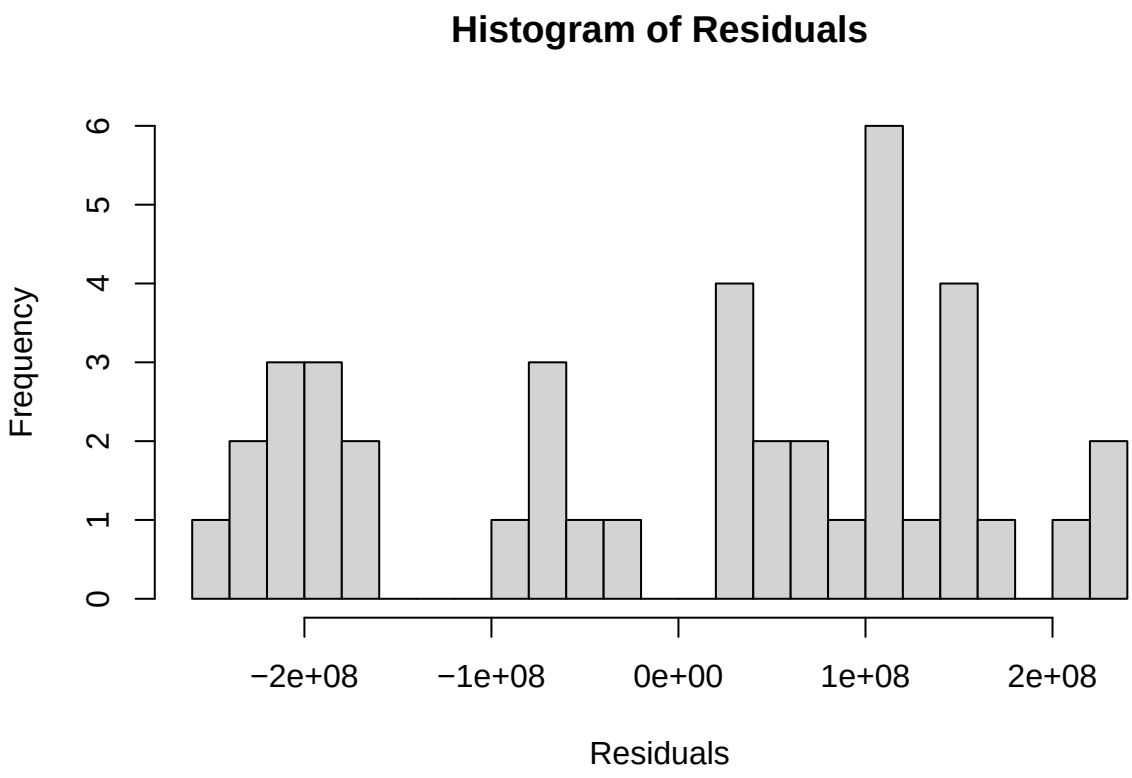
```r
#Residuals vs Fitted
plot(who_model_t25, which = 1)
```




```r
# Q-Q plot
plot(who_model_t25, which = 2)
```




```
# Histogram of residuals
hist(residuals(who_model_t25), breaks = 20, main = "Histogram of Residuals", xlab = "Residuals")
```



Discussion:

Discuss the implications of these forecasts for countries with different levels of healthcare spending. What do

these predictions suggest about the potential impact of increasing healthcare expenditures on life expectancy?

The results reveal that transformed healthcare expenditures positively correlate with life expectancy, emphasizing the role of healthcare spending in improving population health, but its not as simple as "spend more, live forever". In countries with lower levels of spending, increased investment in healthcare could yield substantial gains in life expectancy,but returns will diminish as expenditures grow larger. The residual standard error and adjusted R^2 values, indicate substantial unexplained variability and suggesting life expectancy is influenced by factors beyond spending. A multifaceted approach that targets spending and addresses other determinents would yield greater benefits to the analysis.

Question 4

Interaction Effects in Multiple Regression

Task

Build a multiple regression model to investigate the combined effect of the proportion of MDs and total healthcare expenditures on life expectancy. Specifically, use the model:

$LifeExp = b_0 + b_1 \cdot PropMD + b_2 \cdot TotExp + b_3 \cdot (PropMD \times TotExp)$

- interpret the F-statistic, R-squared value, standard error, and p-values.
- Evaluate the interaction term (PropMD * TotExp).

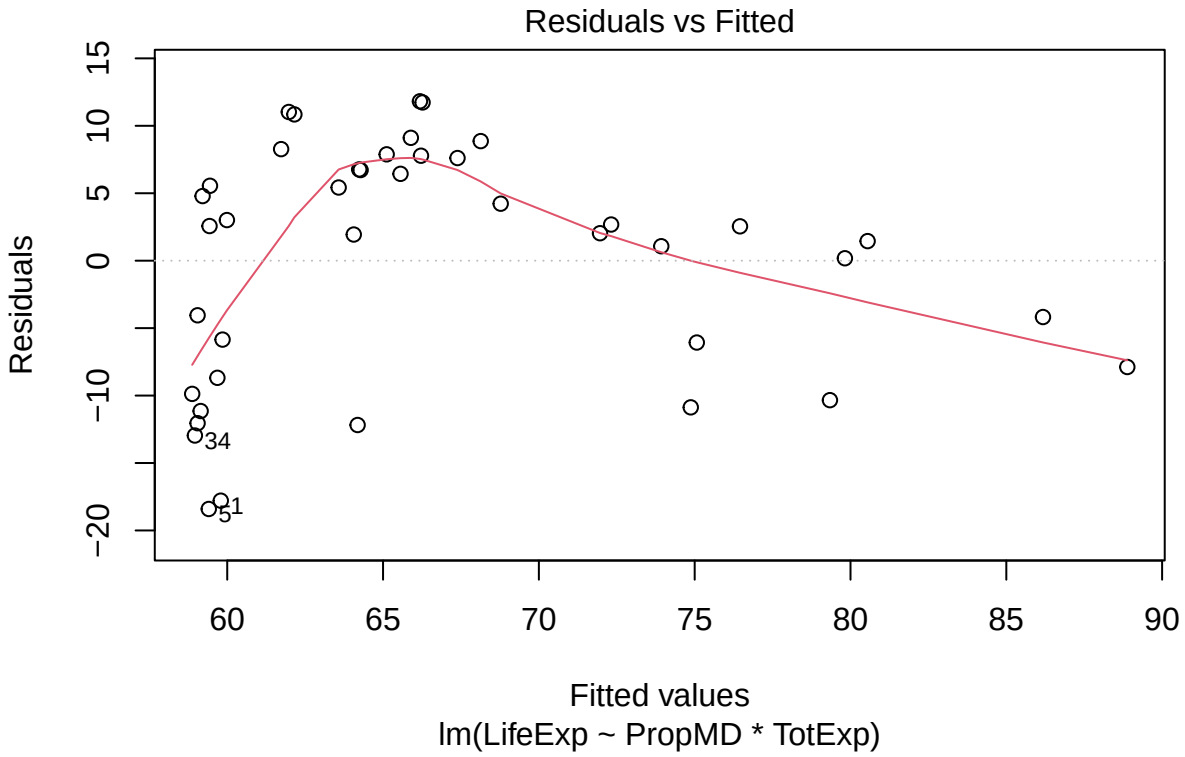
What does this interaction tell us about the relationship between the number of MDs, healthcare spending, and life expectancy?

First I'll create an interaction model and test for significance

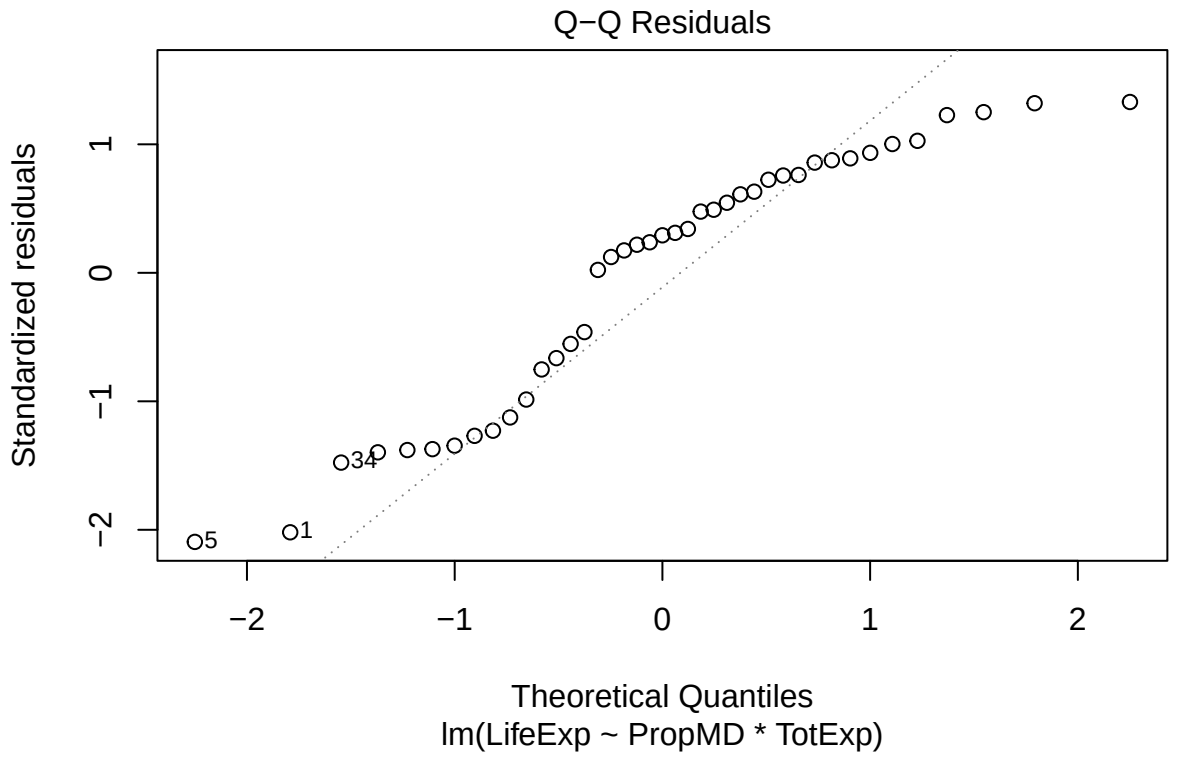
```
who_model_tMD_int <- lm(LifeExp ~ PropMD * TotExp, data = who_df)
summary(who_model_tMD_int)
```

```
##
## Call:
## lm(formula = LifeExp ~ PropMD * TotExp, data = who_df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -18.408  -7.883   2.546   6.765  11.816
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)    5.876e+01  2.097e+00  28.018  < 2e-16 ***
## PropMD         4.396e+03  1.360e+03   3.232  0.00258 **
## TotExp         2.050e-04  7.120e-05   2.880  0.00658 **
## PropMD:TotExp -4.935e-02  2.182e-02  -2.262  0.02969 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.016 on 37 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.459,  Adjusted R-squared:  0.4152
## F-statistic: 10.47 on 3 and 37 DF,  p-value: 3.996e-05
```

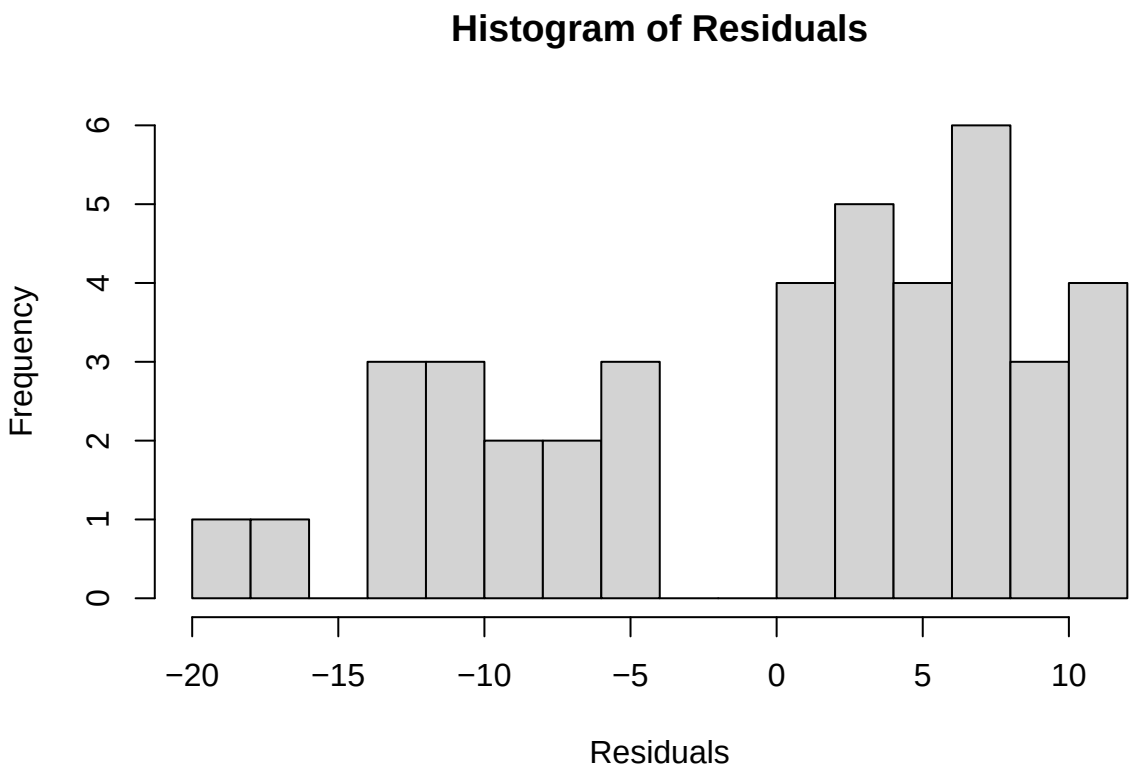
```
#Residuals vs Fitted
plot(who_model_tMD_int, which = 1)
```




```
# Q-Q plot
plot(who_model_tMD_int, which = 2)
```




```
# Histogram of residuals
hist(residuals(who_model_tMD_int), breaks = 20, main = "Histogram of Residuals", xlab = "Residuals")
```

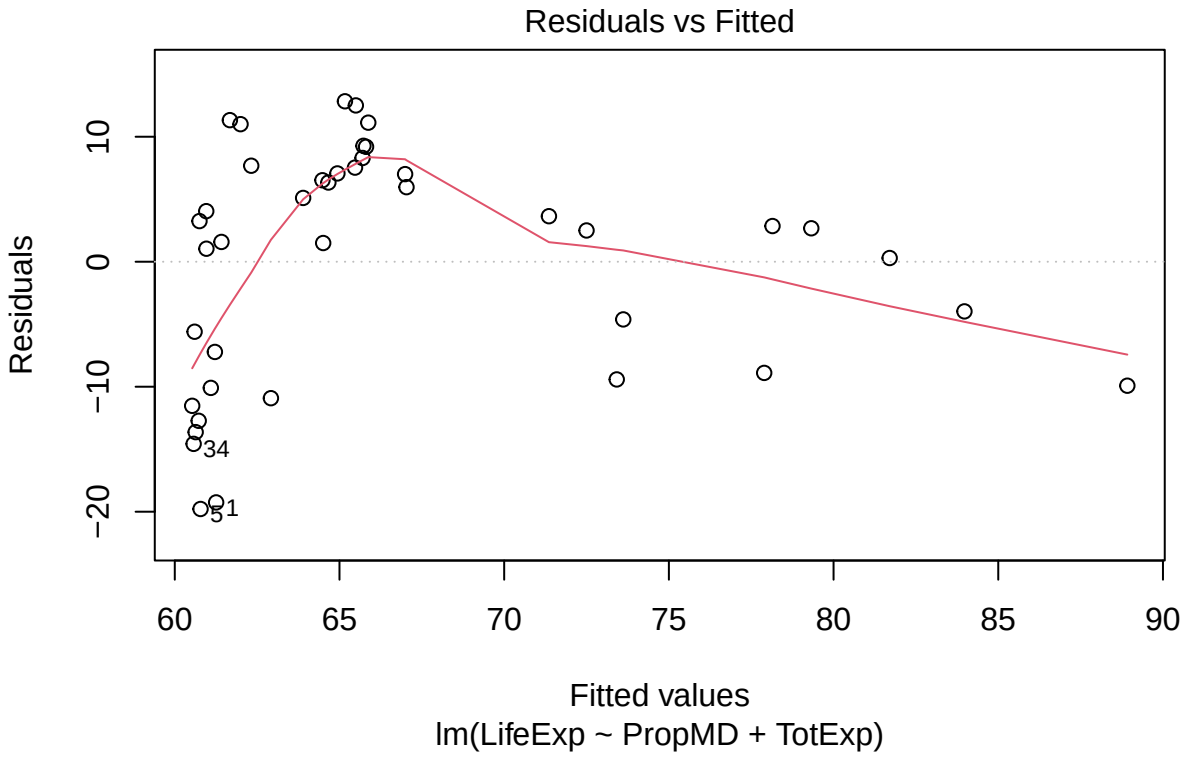


Considering the results I will create an additive model to interpret how they jointly affect LifeExp

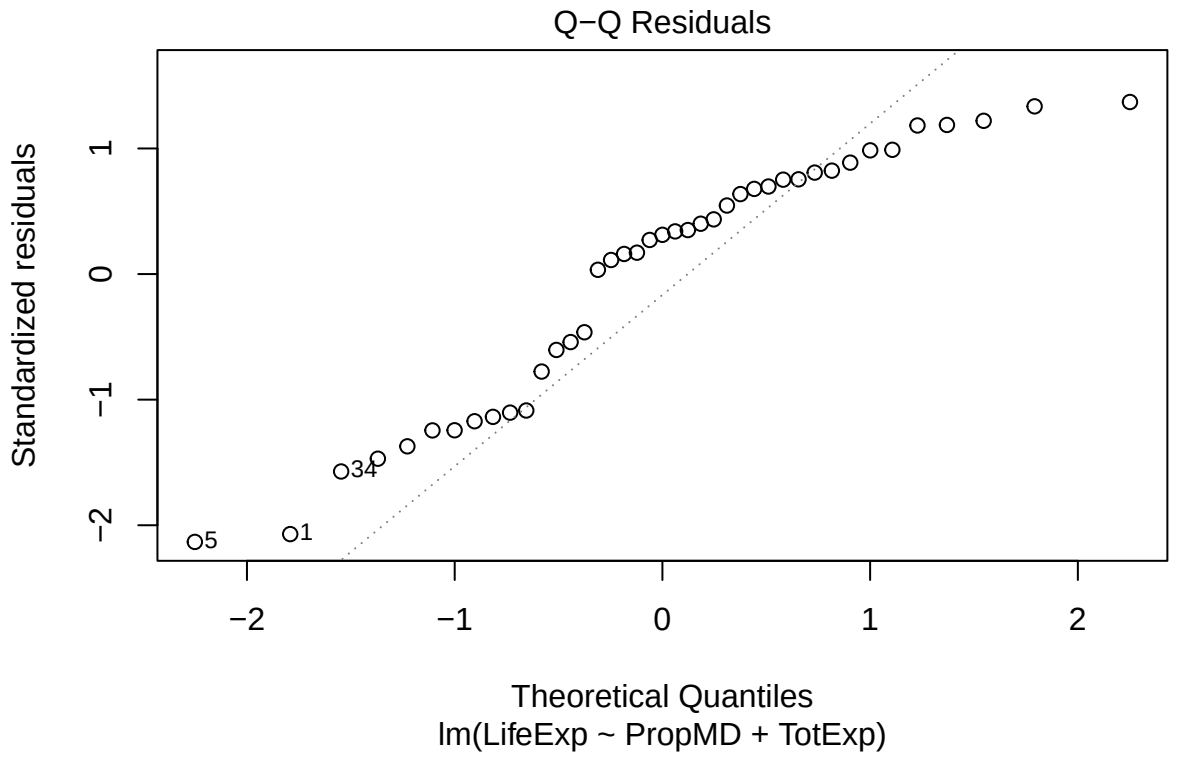
```
who_model_tMD_add <- lm(LifeExp ~ PropMD + TotExp, data = who_df)
summary(who_model_tMD_add)
```

```
##
## Call:
## lm(formula = LifeExp ~ PropMD + TotExp, data = who_df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -19.782  -8.894   2.676   7.063  12.834
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6.044e+01  2.066e+00  29.261   <2e-16 ***
## PropMD      3.533e+03  1.374e+03   2.571   0.0142 *
## TotExp      5.579e-05  2.816e-05   1.981   0.0548 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.491 on 38 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.3843, Adjusted R-squared:  0.3519
## F-statistic: 11.86 on 2 and 38 DF,  p-value: 9.965e-05
```

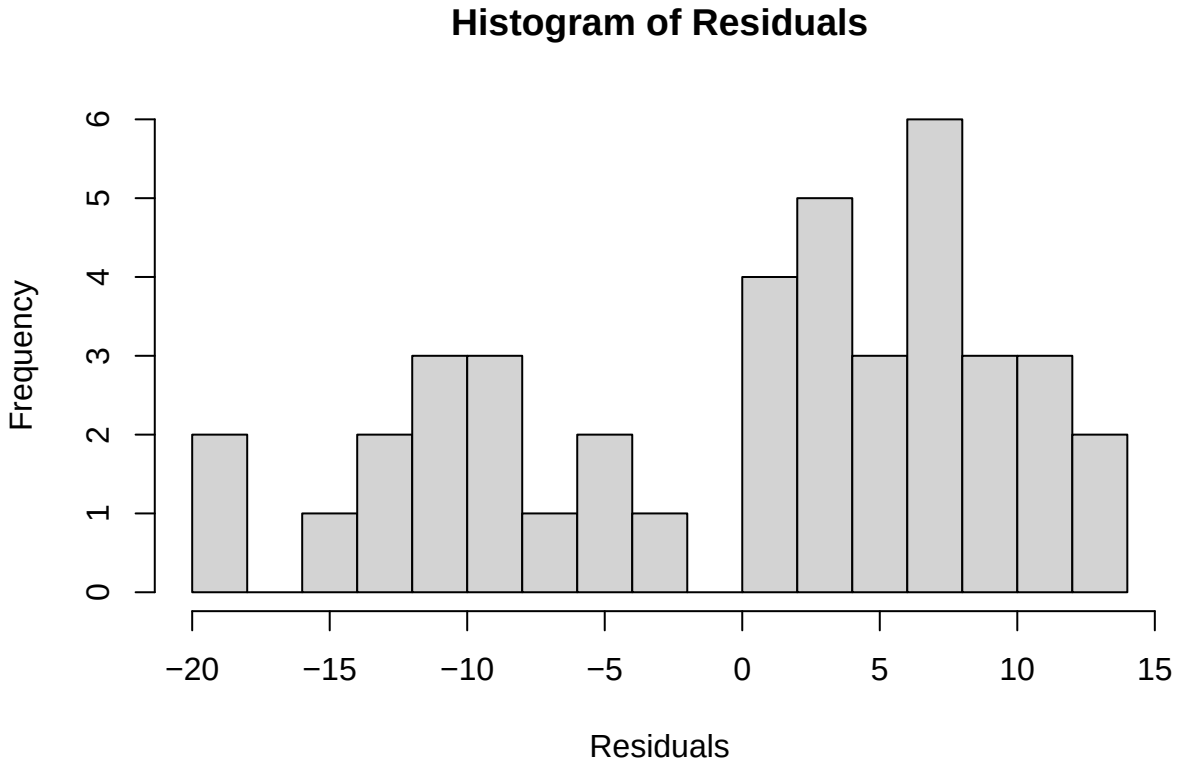
```
#Residuals vs Fitted
plot(who_model_tMD_add, which = 1)
```




```
# Q-Q plot
plot(who_model_tMD_add, which = 2)
```




```
# Histogram of residuals
hist(residuals(who_model_tMD_add), breaks = 20, main = "Histogram of Residuals", xlab = "Residuals")
```



Discussion

i

How does the presence of more MDs amplify or diminish the effect of healthcare expenditures on life expectancy?

- Positive coefficient 4.396×10^3 from the interaction model suggests that on average, an increase of proportion of MDS leads to higher life expectancy.
- Positive coefficient 2.050×10^{-4} implies increased healthcare expenditures is associated with higher life expectancy, assuming PropMD stays constant.
- Negative coefficient -4.935×10^{-2} suggests PropMD and LifeExp depends on TotExp, and vice-versa, with the positive effect of PropMD and LifeExp diminishing as TotExp increase, vice-versa.
- This generally implies diminishing return on increase of Expenditures and MDs.
- The p-value for $PropMD$ and $TotExp$ which is 0.02969 suggests a meaningful relationship.
- For the additive model the coefficients PropMD(3.533×10^3) and - TotExp(5.579×10^{-5}) are still positive but smaller than the interaction model.
- The $R^2 = 0.452$ from the interaction model reducing to $R^2 = 0.3519$ in the additive, suggests less visibility with the additive.

ii

What policy recommendations can be drawn from this analysis?

Since the results basically describe diminishing returns on excessive increase of MDs or Expenditures, a balance between the two is ideal. For low-spending countries, increasing MDs can offset decreased life expectancy, while in high spending countries, increasing MDs may not produce substantial improvement in life expectancy.

Question 5

Forecasting Life Expectancy with Interaction Terms

Task

Using the multiple regression model from Question 4, forecast the life expectancy for a country where:

The proportion of MDs is 0.03 (PropMD = 0.03). The total healthcare expenditure is 14 (TotExp = 14).

In order to forecast I will use the *predict*() function which takes in two parameters: a fitted model object and a dataframe. For the prediction, we are specifically interested in the interaction model. The use of the interaction model is because it captures the complexity of this nuance relationship, including the diminishing returns, while maintaining a relatively high level of statistical significance.

```
who_para <- data.frame(PropMD = 0.03, TotExp = 14)
pred_life_exp <- predict(who_model_tMD_int, newdata = who_para)
pred_life_exp
```

```
##        1
## 190.6272
```

Discussion

i

Does this forecast seem realistic?

No it does not.

ii

Why or why not?

Life span of the oldest person in history was only 122 in 1875, and it is very uncommon for individuals with the most optimal healthcare available, to live many years past 100.

iii

Consider both the potential strengths and limitations of using this model for forecasting in real-world policy settings.

The data is suited for a curve model, as noted before. The data not actually being linear, potential outliers and the impact of the interaction between the variables magnitude effectively created this unrealistic prediction.

Problem 3

Question 1

Inventory Cost

Scenario

A retail company is planning its inventory strategy for the upcoming year. They expect to sell 110 units of a high-demand product. The storage cost is $3.75 per unit per year, and there is a fixed ordering cost of $8.25 per order. The company wants to minimize its total inventory cost.

Task

Using calculus, determine the optimal lot size (the number of units to order each time) and the number of orders the company should place per year to minimize total inventory costs. Assume that the total cost function is given by:

$C(Q) = \frac{D}{Q} \times S + \frac{Q}{2} \times H$

Where:

- D is total demand (110 units).
- Q is the order quantity.
- S is the fixed ordering cost per order ($8.25).
- H is the holding cost per unit per year ($3.75).

NOTE

- $Ordering\ Cost = \frac{D}{Q} \times S$
- $Holding\ Cost = \frac{Q}{2} \times H$

As per the equation and description

We can substitute and calculate for:

$C(Q) = \frac{D}{Q} \times S + \frac{Q}{2} \times H$

to get

$\rightarrow C(Q) = \frac{110}{Q} \times 8.25 + \frac{Q}{2} \times 3.75$

and simplify to

$\rightarrow C(Q) = \frac{907.5}{Q} + \frac{3.75Q}{2}$

the derivative for both is going to essentially be the power rule $f'(x) = n \cdot x^{n-1}$

$\frac{d}{dQ}(\frac{907.5}{Q}) \rightarrow \frac{d}{dQ}(907.5 \times Q^{-1})$

which becomes

$-907.5 \times Q^{-2} = -\frac{907.5}{Q^2}$

and derivative for

$\frac{d}{dQ}(\frac{3.75Q}{2}) \rightarrow \frac{d}{dQ}\frac{3.75}{2} \times Q^1$

becomes just $\frac{3.75}{2}$

so our equation becomes

$\frac{dC(Q)}{dQ} = -\frac{907.5}{Q^2} + \frac{3.75}{2}$

To find our critical point we can set to 0

$-\frac{907.5}{Q^2} + \frac{3.75}{2} = 0 \rightarrow \frac{907.5}{Q^2} = \frac{3.75}{2} \rightarrow 907.5 = \frac{3.75}{2} \times Q^2 \rightarrow 1815 = 3.75 \times Q^2 \rightarrow Q^2 = 484$

finally when we find the square root we get

$Q = \sqrt{484} = 22$

So a critical point given is $Q = 22$

We can use the 2nd derivative to confirm if this is a minimum or maximum. In theory, but confirming if the 2nd derivative is positive, our graph would essentially concave up, ensuring positive values and therefore confirming $Q = 22$ is a minimum quantity to meet demand. A negative equation from the 2nd derivative would indicate a concave down suggesting the opposite and hence it is a maximum.

$\frac{d^2C(Q)}{dQ^2}(-\frac{907.5}{Q^2} + \frac{3.75}{2})$

can be solved like the first equation by finding the derivative for each, however the derivative of a constant (the 2nd term) is just 0, wo we will just solve for the first term.

$\frac{d^2C(Q)}{dQ^2}(-\frac{907.5}{Q^2}) \rightarrow \frac{d^2C(Q)}{dQ^2}(-907.5 \times Q^{-2}) \rightarrow -2 \times -907.5 \times Q^{-3}$

which simplified is

$\frac{1815}{Q^3}$ showing that for $Q > 0$ the second derivative is always positive, confirming $Q = 22$ is a minimum.

The minimize total inventory costs is just solving for

$\frac{D}{Q} \rightarrow \frac{110}{22} = 5$

Answer: $\therefore Optimal\ Order\ Quantity(Q) = 22\ units$

$No.\ Annual\ Orders = 5\ orders$

Question 2

Revenue Maximization

Scenario

A company is running an online advertising campaign. The effectiveness of the campaign, in terms of revenue generated per day, is modeled by the function: $R(t) = -3150t^{-4} - 220t + 6530$ Where:

- $R(t)$ represents the revenue in dollars after t days of the campaign.

Task

Determine the time t at which the revenue is maximized by finding the critical points of the revenue function and determining which point provides the maximum value. What is the maximum revenue the company can expect from this campaign?

Question 3

Demand Area Under Curve

Scenario

A company sells a product at a price that decreases over time according to the linear demand function: Where: $P(x) = 2x - 9.3$

$P(x)$ is the price in dollars, and x is the quantity sold.

Task

The company is interested in calculating the total revenue generated by this product between two quantity levels, $x_1 = 2$ and $x_2 = 5$, where the price still generates sales. Compute the area under the demand curve between these two points, representing the total revenue generated over this range.

Question 4

Profit Optimization

Scenario

A beauty supply store sells flat irons, and the profit function associated with selling x flat irons is given by:

$\Pi(x) = xln(9x) - \frac{x^6}{6}$

Where:

- $\Pi(x)$ is the profit in dollars.

Task

Use calculus to find the value of x that maximizes profit. Calculate the maximum profit that can be achieved and determine if this optimal sales level is feasible given market conditions.

Question 5

Spending Behavior

Scenario

A market research firm is analyzing the spending behavior of customers in a retail store. The spending behavior is modeled by the probability density function: $f(x) = \frac{1}{6x}$

Where x represents spending in dollars.

Task

Determine whether this function is a valid probability density function over the interval $[1, e^6]$. If it is, calculate the probability that a customer spends between \$1 and $\$e^6$.

Question 6

Market Share Estimation

Scenario

An electronics company is analyzing its market share over a certain period. The rate of market penetration is given by: $\frac{dN}{dt} = \frac{500}{t^4+10}$

Where $N(t)$ is the cumulative market share at time t.

Task

Integrate this function to find the cumulative market share $N(t)$ after t days, given that the initial market share $N(1) = 6530$. What will the market share be after 10 days?

Problem 4

Business Optimization

As a data scientist at a consultancy firm, you are tasked with optimizing various business functions to improve efficiency and profitability. Taylor Series expansions are a powerful tool to approximate complex functions, allowing for simpler calculations and more straightforward decision-making. This week, you will work on Taylor Series expansions of popular functions commonly encountered in business scenarios.

Question 1

Revenue and Cost

Scenario

A company's revenue from a product can be approximated by the function $R(x) = e^x$, where x is the number of units sold. The cost of production is given by $C(x) = ln(1 + x)$. The company wants to maximize its profit, defined as $\Pi(x) = R(x) - C(x)$.

Task

1. **Approximate the Revenue Function:** Use the Taylor Series expansion around $x = 0$ (Maclaurin series) to approximate the revenue function $R(x) = e^x$ up to the second degree. Explain why this approximation might be useful in a business context.
2. **Approximate the Cost Function:** Similarly, approximate the cost function $C(x) = ln(1 + x)$ using its Maclaurin series expansion up to the second degree. Discuss the implications of this approximation for decision-making in production.

3. **Linear vs. Nonlinear Optimization:** Using the Taylor Series expansions, approximate the profit function $\Pi(x)$. Compare the optimization results when using the linear approximations versus the original nonlinear functions. What are the differences, and when might it be more appropriate to use the approximation?

Submission

Provide your solutions using R-Markdown. Include the Taylor Series expansions, the approximated functions, and a discussion of the implications of using these approximations for business decision-making.

Question 2

Financial Modeling

Scenario

A financial analyst is modeling the risk associated with a new investment. The risk is proportional to the square root of the invested amount, modeled as $f(x) = \sqrt{x}$, where x is the amount invested. However, to simplify calculations, the analyst wants to use a Taylor Series expansion to approximate this function for small investments.

Task

1. **Maclaurin Series Expansion:** Derive the Taylor Series expansion of $f(x) = \sqrt{x}$ around $x = 0$ up to the second degree.
2. **Practical Application:** Use the derived series to approximate the risk for small investment amounts (e.g., when x is small). Compare the approximated risk with the actual function values for small and moderate investments. Discuss when this approximation might be useful in financial modeling.
3. **Optimization Scenario:** Suppose the goal is to minimize risk while maintaining a certain level of investment return. Using the Taylor Series approximation, suggest an optimal investment amount x that balances risk and return.

Submission

Present your results in R-Markdown, including the Taylor Series expansions, comparisons between the original and approximated functions, and your recommendations based on the analysis.

Question 3

Inventory Management

Scenario

In a manufacturing process, the demand for a product decreases as the price increases, modeled by $D(p) = 1 - p$, where p is the price. The cost associated with producing and selling the product is modeled as $C(p) = e^p$. The company wants to maximize its profit, which is the difference between revenue and cost.

Task

1. **Taylor Series Expansion:** Expand the cost function $C(p) = e^p$ into a Taylor Series around $p = 0$ up to the second degree. Discuss why approximating the cost function might be useful in a pricing strategy.
2. **Approximating Profit:** Using the Taylor Series expansion, approximate the profit function $\Pi(p) = pD(p) - C(p)$. Compare the results when using the original nonlinear cost function versus the approximated cost function. What differences do you observe, and when might the approximation be sufficient?

3. **Pricing Strategy:** Based on the Taylor Series approximation, suggest a pricing strategy that could maximize profit. Explain how the Taylor Series approximation helps in making this decision.

Submission

Include your analysis in R-Markdown, with Taylor Series expansions, comparisons of the approximated and original functions, and a discussion of the implications for pricing strategy.

Question 4

Economic Forecasting

Scenario

An economist is forecasting economic growth, which can be modeled by the logarithmic function $G(x) = ln(1 + x)$, where x represents investment in infrastructure. The government wants to predict growth under different levels of investment.

Task

i.

Maclaurin Series Expansion: Derive the Maclaurin Series expansion of $G(x) = ln(x+1)$ up to the second degree. Explain the significance of using this approximation for small values of x_x in economic forecasting.

ii.

Approximation of Growth: Use the Taylor Series to approximate the growth for small investments. Compare this approximation with the actual growth function. Discuss the accuracy of the approximation for different ranges of x_x.

iii.

Policy Recommendation: Using the approximation, recommend a level of investment that could achieve a target growth rate. Discuss the limitations of using Taylor Series approximations for such policy recommendations.

Submission

Provide your answers in R-Markdown, with the Taylor Series expansions, comparisons between the approximated and original functions, and your investment recommendations.

Problem 5

Profit, Cost, & Pricing

Question 1

Profit Maximization

Scenario

A company produces two products, A and B. The profit function for the two products is given by $\Pi(x, y) = 30x - 2x^2 - 3xy + 24y - 4y^2$

Where:

- x is the quantity of Product A produced and sold.
- y is the quantity of Product B produced and sold.

- $\Pi(x, y)$ is the profit in dollars.

Task

Find all local maxima, local minima, and saddle points for the profit function $\Pi(x, y)$. Write your answer(s) in the form $(x, y, \Pi(x, y))$. Separate multiple points with a comma.

Discussion

Discuss the implications of the results for the company's production strategy. Which production levels maximize profit, and what risks are associated with the saddle points?

Question 2

Pricing Strategy

Scenario

A supermarket sells two competing brands of a product: Brand X and Brand Y. The store manager estimates that the demand for these brands depends on their prices, given by the functions:

- Demand for Brand X: $D_x(x, y) = 120 - 15x + 10y$
- Demand for Brand Y: $D_y(x, y) = 80 + 5x - 20y$

Where:

- x is the price of Brand X in dollars.
- y is the price of Brand Y in dollars.
- $D_x(x, y)$ and $D_x(x, y)$ are the quantities demanded for Brand X and Brand Y, respectively.

Task

1. **Revenue Function:** Find the revenue function R(x,y) for both brands combined.
2. **Optimal Pricing:** Determine the prices x and y that maximize the store's total revenue. Are there any saddle points to consider in the pricing strategy?

Discussion

Explain the significance of the optimal pricing strategy and how it can be applied in a competitive retail environment.

Question 3

Cost Minimization

Scenario

A manufacturing company operates two plants, one in New York and one in Chicago. The company needs to produce a total of 200 units of a product each week. The total weekly cost of production is given by $C(x, y) = \frac{1}{8}x^2 + \frac{1}{10}y^2 + 12x + 18y + 1500$

Where:

- x is the number of units produced in New York.
- y is the number of units produced in Chicago.
- $C(x, y)$ is the total cost in dollars.

Task

1. Determine how many units should be produced in each plant to minimize the total weekly cost.
2. What is the minimized total cost, and how does the distribution of production between the two plants affect overall efficiency?

Discussion

Discuss the benefits of this cost-minimization strategy and any practical considerations that might influence the allocation of production between the two plants.

Question 4

Marketing Mix

Scenario

A company is launching a marketing campaign that involves spending on online ads (x) and television ads (y). The effectiveness of the campaign, measured in customer reach, is modeled by the function $E(x, y) = 500x + 700y - 5x^2 - 10xy - 8y^2$

Where:

- x is the amount spent on online ads (in thousands of dollars).
- y is the amount spent on television ads (in thousands of dollars).
- $E(x, y)$ is the estimated customer reach.

Task

1. Find the spending levels for online and television ads that maximize customer reach.
2. Identify any saddle points and discuss how they could affect the marketing strategy.

Discussion

Explain how the results can be used to allocate the marketing budget effectively and what the company should consider if it encounters saddle points in the optimization.

As always, include a professional pdf only with all code, text, graphics, explanations, etc.