Assignment 2

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Problem 11.16.3.5(exemplar):-

A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution:-

Parameter	Value	Description
X	$\{1, 2, 3, 4, 5, 6\}$	Number obtained on the die

Let m be any natural number such that $m \in \{1, 2, 3\}$.

$$\Pr(X = k) = \begin{cases} 2p, & if\ k = 2m - 1 \\ p, & if\ k = 2m \end{cases} \tag{1}$$

Let $F_X(k)$ be the cumulative distribution function such that,

$$F_X(k) = \Pr(X \leq k) \tag{2}$$

If $k = 2m - 1$,

$$F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i) \tag{3}$$
$$= \sum_{i=1}^{m} \Pr(2i - 1) + \sum_{i=1}^{m-1} \Pr(2i) \tag{4}$$
$$= \sum_{i=1}^{m} 2p + \sum_{i=1}^{m-1} p \tag{5}$$
$$= (m)(2p) + (m - 1)(p) \tag{6}$$
$$= p(3m - 1) \tag{7}$$
$$= \frac{p(3k + 1)}{2} \tag{8}$$

If $k = 2m$,

$$F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i) \tag{9}$$
$$= \sum_{i=1}^{m} \Pr(2i - 1) + \sum_{i=1}^{m} \Pr(2i) \tag{10}$$
$$= \sum_{i=1}^{m} 2p + \sum_{i=1}^{m} p \tag{11}$$
$$= (m)(2p) + (m)(p) \tag{12}$$
$$= p(3m) \tag{13}$$
$$= \frac{3pk}{2} \tag{14}$$

So,

$$F_X(k) = \begin{cases} \frac{p(3k+1)}{2}, & if\ k = 2m - 1 \\ \frac{3pk}{2}, & if\ k = 2m \end{cases} \tag{15}$$

Since $1 \leq X \leq 6$,

$$F_X(6) = 1 \tag{16}$$
$$\implies \frac{3p(6)}{2} = 1 \tag{17}$$
$$\implies p = \frac{1}{9} \tag{18}$$

$$\Pr(G) = \Pr(X > 3) \tag{19}$$
$$= F_X(6) - F_X(3) \tag{20}$$
$$= \frac{3p(6)}{2} - \frac{p\{3(3) + 1\}}{2} \tag{21}$$
$$= 9p - 5p = 4p \tag{22}$$
$$= \frac{4}{9} \tag{23}$$