

Assignment 2

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Problem 11.16.3.5(exemplar):-

A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find $P(G)$, where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution:-

Parameter	Value	Description
X	$\{1,2,3,4,5,6\}$	Number obtained on the die

Let m be any natural number such that $m \in \{1, 2, 3\}$.

$$\Pr(X = k) = \begin{cases} 2p, & \text{if } k = 2m - 1 \\ p, & \text{if } k = 2m \end{cases} \quad (1)$$

Let $F_X(k)$ be the cumulative distribution function such that,

$$F_X(k) = \Pr(X \leq k) \quad (2)$$

If $k = 2m - 1$,

$$F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i) \quad (3)$$

$$= \sum_{i=1}^m \Pr(2i - 1) + \sum_{i=1}^{m-1} \Pr(2i) \quad (4)$$

$$= \sum_{i=1}^m 2p + \sum_{i=1}^{m-1} p \quad (5)$$

$$= (m)(2p) + (m - 1)(p) \quad (6)$$

$$= p(3m - 1) \quad (7)$$

$$= \frac{p(3k + 1)}{2} \quad (8)$$

If $k = 2m$,

$$F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i) \quad (9)$$

$$= \sum_{i=1}^m \Pr(2i - 1) + \sum_{i=1}^m \Pr(2i) \quad (10)$$

$$= \sum_{i=1}^m 2p + \sum_{i=1}^m p \quad (11)$$

$$= (m)(2p) + (m)(p) \quad (12)$$

$$= p(3m) \quad (13)$$

$$= \frac{3pk}{2} \quad (14)$$

So,

$$F_X(k) = \begin{cases} \frac{p(3k+1)}{2}, & \text{if } k = 2m - 1 \\ \frac{3pk}{2}, & \text{if } k = 2m \end{cases} \quad (15)$$

Since $1 \leq X \leq 6$,

$$F_X(6) = 1 \quad (16)$$

$$\implies \frac{3p(6)}{2} = 1 \quad (17)$$

$$\implies p = \frac{1}{9} \quad (18)$$

$$\Pr(G) = \Pr(X > 3) \quad (19)$$

$$= F_X(6) - F_X(3) \quad (20)$$

$$= \frac{3p(6)}{2} - \frac{p\{3(3) + 1\}}{2} \quad (21)$$

$$= 9p - 5p = 4p \quad (22)$$

$$= \frac{4}{9} \quad (23)$$