Assignment 2

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Problem 11.16.5.8(exemplar):-

A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution:-

Let X be a random variable denoting the number obtained on the die.

Let m be any natural number such that $m \in \{1, 2, 3\}$.

$$
\Pr(X = k) = \begin{cases} 2p, & if\ k = 2m - 1 \\ p, & if\ k = 2m \end{cases} \quad (1)
$$

Let $F_X(k)$ be the cumulative distribution function such that,

$$
F_X(k) = \Pr(X \le k) \quad (2)
$$

If $k = 2m - 1$,

$$
F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i) \quad (3)
$$

$$
= \sum_{i=1}^{m} \Pr(2i - 1) + \sum_{i=1}^{m-1} \Pr(2i) \quad (4)
$$

$$
= \sum_{i=1}^{m} 2p + \sum_{i=1}^{m-1} p \quad (5)
$$

$$
= (m)(2p) + (m - 1)(p) \quad (6)
$$

$$
= p(3m - 1) \quad (7)
$$

$$
= \frac{p(3k + 1)}{2} \quad (8)
$$

If $k = 2m$,

$$
F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i) \quad (9)
$$

$$
= \sum_{i=1}^{m} \Pr(2i - 1) + \sum_{i=1}^{m} \Pr(2i) \quad (10)
$$

$$
= \sum_{i=1}^{m} 2p + \sum_{i=1}^{m} p \quad (11)
$$

$$
= (m)(2p) + (m)(p) \quad (12)
$$

$$
= p(3m) \quad (13)
$$

$$
= \frac{3pk}{2} \quad (14)
$$

So,

$$
F_X(k) = \begin{cases} \frac{p(3k+1)}{2}, & if\ k = 2m - 1 \\ \frac{3pk}{2}, & if\ k = 2m \end{cases} \quad (15)
$$

Since $1 \le X \le 6$,

$$
F_X(6) = 1 \quad (16)
$$

$$
\implies \frac{3p(6)}{2} = 1 \quad (17)
$$

$$
\implies p = \frac{1}{9} \quad (18)
$$

$$
\Pr(G) = \Pr(X > 3) \quad (19)
$$

$$
= F_X(6) - F_X(3) \quad (20)
$$

$$
= \frac{3p(6)}{2} - \frac{p\{3(3) + 1\}}{2} \quad (21)
$$

$$
= 9p - 5p = 4p \quad (22)
$$

$$
= \frac{4}{9} \quad (23)
$$