Assignment 2

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Problem 11.16.5.8(exemplar):-

A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution:-

Let X be a random variable denoting the number obtained on the die.

Let m be any natural number such that $m \in \{1, 2, 3\}$.

$$\Pr(X = x) = \begin{cases} 2p, & if \ x = 2m - 1 \\ p, & if \ x = 2m \end{cases} \tag{1}$$

Let $F_X(x)$ be the cumulative distribution function such that,

$$F_X(x) = \Pr(X \le x) \tag{2}$$

If $x = 2m - 1$,

$$F_X(x) = \underbrace{\Pr(X = 1) + \Pr(X = 3) + \dots \Pr(X = 2m - 1)}_{m \text{ terms}} + \underbrace{\Pr(X = 2) + \Pr(X = 4) + \dots \Pr(X = 2m - 2)}_{m - 1 \text{ terms}} \tag{3}$$

$$= (m)(2p) + (m - 1)(p) \tag{4}$$

$$= p(3m - 1) \tag{5}$$

$$= \frac{p(3x + 1)}{2} \tag{6}$$

If $x = 2m$,

$$F_X(x) = \underbrace{\Pr(X = 1) + \Pr(X = 3) + \dots \Pr(X = 2m - 1)}_{m \text{ terms}} + \underbrace{\Pr(X = 2) + \Pr(X = 4) + \dots \Pr(X = 2m)}_{m \text{ terms}} \tag{7}$$

$$= (m)(2p) + (m)(p) \tag{8}$$

$$= p(3m) \tag{9}$$

$$= \frac{3px}{2} \tag{10}$$

So,

$$F_X(x) = \begin{cases} \frac{p(3x+1)}{2}, & if \ x = 2m - 1 \\ \frac{3px}{2}, & if \ x = 2m \end{cases} \tag{11}$$

Since $1 \le X \le 6$,

$$F_X(6) = 1 \tag{12}$$

$$\implies \frac{3p(6)}{2} = 1 \tag{13}$$

$$\implies p = \frac{1}{9} \tag{14}$$

$$\Pr(G) = \Pr(X > 3) \tag{15}$$

$$= F_X(6) - F_X(3) \tag{16}$$

$$= \frac{3p(6)}{2} - \frac{p\{3(3) + 1\}}{2} \tag{17}$$

$$= 9p - 5p = 4p \tag{18}$$

$$= \frac{4}{9} \tag{19}$$