Assignment 2

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Problem 11.16.3.5(exemplar):-

A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution:-

Parameter	Value	Description
X	{1,2,3,4,5,6}	Number obtained on the die

Let m be any natural number such that $m \in \{1, 2, 3\}$.

$$\Pr(X = k) = \begin{cases} 2p, & if\ k = 2m - 1 \\ p, & if\ k = 2m \end{cases} \tag{1}$$

Since $1 \le X \le 6$,

$$\sum_{i=1}^{6} \Pr(X = i) = 1 \tag{2}$$

$$\implies \sum_{i=1}^{3} \Pr(X = 2i - 1) + \sum_{i=1}^{3} \Pr(X = 2i) = 1 \tag{3}$$

$$\implies \sum_{i=1}^{3} 2p + \sum_{i=1}^{3} p = 1 \tag{4}$$

$$\implies 6p + 3p = 1 \tag{5}$$

$$\implies 9p = 1 \tag{6}$$

$$\implies p = \frac{1}{9} \tag{7}$$

Let $F_X(k)$ be the cumulative distribution function such that,

$$F_X(k) = \Pr(X \le k) \tag{8}$$

If $k = 2m - 1$,

$$F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i) \tag{9}$$

$$= \sum_{i=1}^{m} \Pr(X = 2i - 1) + \sum_{i=1}^{m-1} \Pr(X = 2i) \tag{10}$$

$$= \sum_{i=1}^{m} 2\left(\frac{1}{9}\right) + \sum_{i=1}^{m-1} \left(\frac{1}{9}\right) \tag{11}$$

$$= \frac{2m}{9} + \frac{m-1}{9} \tag{12}$$

$$= \frac{3m-1}{9} \tag{13}$$

$$= \frac{3k+1}{18} \tag{14}$$

If $k = 2m$,

$$F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i) \tag{15}$$

$$= \sum_{i=1}^{m} \Pr(X = 2i - 1) + \sum_{i=1}^{m} \Pr(X = 2i) \tag{16}$$

$$= \sum_{i=1}^{m} 2\left(\frac{1}{9}\right) + \sum_{i=1}^{m} \left(\frac{1}{9}\right) \tag{17}$$

$$= \frac{2m}{9} + \frac{m}{9} \tag{18}$$

$$= \frac{m}{3} \tag{19}$$

$$= \frac{k}{6} \tag{20}$$

So,

$$F_X(k) = \begin{cases} \frac{3k+1}{18}, & if\ k = 2m - 1 \\ \frac{k}{6}, & if\ k = 2m \end{cases} \tag{21}$$

$$
\begin{aligned}
\Pr(G) &= \Pr(X > 3) && (22)\\
&= F_X(6) - F_X(3) && (23)\\
&= 1 - \frac{3(3)+1}{18} && (24)\\
&= 1 - \frac{5}{9} && (25)\\
&= \frac{4}{9} && (26)
\end{aligned}
$$