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An Ant Colony Optimization Algorithm for Knight's Tour Problem on the chessboard with holes

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***Abtract*—Knight's tour problem (KTP) on the chessboard with holes is an especial but interest tour problem. We improve the basic ant colony optimization,and introduce more better pheromone resource update and sharing mechanism,, and find its tour path solutions on the chessboard with holes with the help of colonies sharing the information resource by their intercommunion. We have used the algorithm to encumerate the knight's tours on different chessboard with holes, and made the particular analysis for produce. Experiment results show that the algorithm not only produce the path solutions quickly on the chessboard with holes, but also is easy to control the path starting square and path style.**

Keywords-KTP;chessboard with holes; ant colony optimization algorithm;muti-ant-colony optimization algorithm

I. INTRODUCTION

Knight's tour problem(KTP) is an age-old but interesting problem. Traditionally the "Knight's Tour" is a sequence of moves done by a knight on a chessboard. and, following the rules of chess(making **L**-shaped moves), must visit each square exactly once.Since the problem was proposed,a lot of famous mathematician such as Warnsdorff[1],Eulor etc had a farther study, and done prominent job. However, to existing documents, they were aimed at regular chessboard including $n\times n$ and $m\times n$ chessboard with the same and different length, respectively, few of which did some research on the chessboard with holes. Chessboads with holes are some exceptive boards formed by digging several squares from $m\times n$ chessboards. And a regular chessboard is looked as combination of several chessboard with holes. Thus, if KTP on chessboard with holes is resolved, KTP on the regular chessboard must be solved easily. And, knight's tour path on chessboard with holes has a bright application future for doing some special image encryption. So it is significant to do some research on KTP on the chessboard with holes.

So far, there has been many approaches for solving KTP, including intelligence searching-backtracking algorithm[2], a heurisitic approach with minimal outlet[3], depth-first searching algorithm(DFS)[1],'divide-conquer' method[4] and the recently optimisation algorithm applied widely such as neural networks algorithm(NN)[5], genetic algorithm(GA)[6],ant colony optimisation algorithm (ACO)[7,8] and so on. However, some approaches had poor produce rate, for instance, the produce rate of DFS , GA are only 0.000003 tours per attempt and 0.000642[8] tours per attempt, respectively. Only ACO's rate was acceptant, it's 0.076 tours per attempt. From approaches above, a conclusion can be drawn that the research object of approaches are regular chessboards, not chessboards with holes.

All things considered, the produce rate using ant colony optimisation algorithm is maximal. An ACO for knight's tour problem on the chessboard with holes is proposed. We introduce more better pheromone resource update and sharing mechanism, use furthersome heurisitic principle and multi-ant-colonies to get the knight's tour path efficiently. Experiments show that the improved ant colony optimisation algorithm (IACO) performs better, that is, it not only can get the knight's tour path on the regular board , but also is adapted to solve the KTP on board with holes.

II. ANT COLONY OPTIMISATION ALGORITHM AND ITS IMPROVEMENT

In this section, we describe the ant colony optimization algorithm that we designed to get the knight's tour path on chessboard with holes. We first review the basics of ant colony optimization[9](ACO), and present the improved ACO algorithm . It is similar to the well-known Ant Systems algorithm introduced by Dorigo et al.We then describe a new modification utilising multiple colonies.

A. Ant colony optimisation algorithm

Ant colony optimisation algorithms[9] are based on the natural phenomenon that ants, despite being almost blind and having very simple brains, are able to find their way to a food source and back to their nest, using the shortest route. Ant colony optimisation(ACO) algorithms were introduced by Marco Dorigo in his PhD thesis and later in the seminal paper in this area. In 1996, the algorithm is introduced by considering what happens when an ant comes across an obstacle and has to decide the best route to take around the obstacle. Initially, there is equal probability as to which way the ant will turn in order to negotiate the obstacle. Now consider a colony of ants making many trips around the obstacle and back to the nest. As they

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move, ants deposit a chemical (a *pheromone*) along their trail. If we assume that one route around the obstacle is shorter than the alternative route, then in a given period of time, a greater proportion of trips can be made over the shorter route. Thus, over time, there will be more pheromone deposited on the shorter route. Ants can increase their chance of finding the shorter route by preferentially choosing the one with more pheromone. There is positive feedback, in that the moresuccessful a behaviour proves to be, the more desirable it becomes. This form of behaviour is known as stigmergy or autocatalytic behaviour[8,9].

This idea has been transformed into various search algorithms, by augmenting probabilistic nature of ant movements following pheromone trails, with a problem specific heuristic. In the most famous example, ants can be used to search for solutions for the traveling salesman problem. Each ant in the colony traverses the city graph, depositing pheromone on edges between cities. High levels of pheromone on an edge indicate that it is part of relatively shorter tours found by previous ants. When deciding when to move from one vertex (city) to another, ants take into account the level of pheromone on the candidate edges along with a heuristic value (distance to the next city for the TSP). The combination of pheromone and heuristic probabilistically determines which city an ant moves to next. Many studies show that ACO has a good capability of finding better tour path,which not only uses the feedback principle to quicken evolution process of colonies,but also is a essential parallel algorithm.

B. Improved ant colony optimisation algorithm

Suppose that a knight (1,2) moves on the chessboard $m\times n$ digged H squares.If each square is regarded as a vertex of a graph G and edges between double squares can be shaped iff knight can move from one to the other by only one step,the brief-connected graph we get is named board-knight graph labelled as G=<V,E> (see in Fig.1)on chessboard with holes.Then, each vertex from (1,1)→(m,n) in graph G is scanned by antidromic Z-shape from the bottom up and labeled as 1,2,…,i,…j,…,mn-H by scanned order(see in Fig.2).In this way,it products a sequence of path vertexes.Therefore,vertex interrelation matrix **M** is constructed as follow:

$$M_{ij}=\begin{cases}1 & if\ (s-k)^2+(t-l)^2=1^2+2^2 and\ s\neq k,t\neq l\\ 0 & otherwise\end{cases}$$

Where, (s,t) denotes 2-dimensional coordinate of vertex i, (k,l) denotes 2-dimensional coordinate of vertex j.

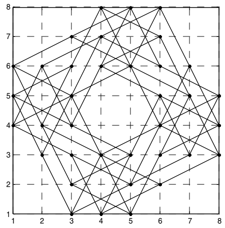


Fig.1

			34	35	36		
		31	32		33		
24	25	26	27	28	29	30	
19		20			21	22	23
14	15	16			17		18
	7	8	9	10	11	12	13
		4		5	6		
		1	2	3			

Fig.2

Fig.1 board-knight graph on the chessboard with holes
Fig.2 sketch map of vertexes scanning and label

Suppose that there are D_i squares connected with square i on the $m\times n$ chessboard with holes, the degree of vertex i is $D_j=\sum_j M_{ij}$. Suppose there are k ant colonies moving on the chessboard,we label them as $Colony_1$, $Colony_2$, … ,$Colony_k$, respectively.They divide the work orderly, work together, communicate with each other at regular intervals in the whole tour process, and share the pheromone imformation resource.

1) Initialising the chessboard

Initially, an amount of pheromone is laid on each edge. In our simulations we used $Ph_{l,p}$=0.5 for all edges corresponding to legal moves by ant colony $Colony_l$ (l=1,2,…,k).

2) Evaporating pheromones

Pheromones evaporate over time. This prevents the level of pheromone becoming unbounded, and makes the ant colony get newer information. In our algorithm, we implemented this by reducing the amount of pheromone on each edge once per cycle, using the update formula: $Ph_{l,p}=(1-\rho)\cdot Ph_{l,p}$, Where, $0<\rho<1$ is called the evaporation rate.

3) Starting ants from each colony

Each ant in ant colonies is placed on the squares of chessboard with holes,and owns a tabu list ***tabu*** which is the set of squares that the ant has visited so far and an allowable list ***allowed*** which is the set of squares that the ant is to visit.Initially, $tabu$ ={V_{start}}, V_{start} denotes the start vertex of G ,and each ant owns only one .In the tour process,every ant must remember its start square, its sequence of moves ***move*** and put squares visited in the tabu list ***tabu*** ever and again.Initially, ***moves*** =Ø , an empty list.

4) Improved heuristic principle for knight's tour

In the document [8], the heuristic principle was used for knight's tour as follow:

$$P_{ij}{}^{m}=\begin{cases}\dfrac{P_m}{\sum\limits_{S\notin tabu_m}P_j},P_m=Ph_{i,j}{}^{\alpha} & , \quad j,s\square allowed_m\ ,\\ 0 & otherwise\end{cases}$$

$allowed_m$={1,2,⋯,mn-H}-$tabu_m$.

Considering that the earlier the square with smaller degree is visited ,the more quickly ants can find the legal tour path,ant m can choose its searching direction depending on the pheromone's quantity $Ph_{l,p}$ on each path of board-knight graph G .So the probability P_{ij}^{m} moving from vertex i to vertex j by ant m is defined as formula:

$$P_{ij}{}^{m}=\begin{cases}\dfrac{P_m}{\sum\limits_{S\notin tabu_m} P_j},P_m=Ph_{i,j}{}^{D_j} & , \quad j,s\square allowed_m , \\ 0 \quad otherwise\end{cases}$$

$allowed_m$={1,2,⋯,mn-H}-$tabu_m$,where, $allowed_m$ denotes the square ant m visits next, $tabu_m$ denotes ant m's tabu list.

As $0<Ph_{i,j}<1$,the greater D_j is,the less the probability choosing vertex j is.Thus,this overcomes shortcoming with uncertain value α,and intensifies selective self- adaptability and intelligence.

5) Update ants' moving list

If the ant has chosen next square ***next***,it will put the square in its own tabu list ***tabu*** in time,and delete the square from allowable list ***allowed*** in order to avoid squares on the chessboard being visited repeatedly.So we can get :

tabu→tabu U {***next***},***allowed→allowed***-{***next***},

and ***moves=moves***+{***next***}.

6) Improved updating style of pheromone information

When she has finished her attempted tour, the ant retraces its steps and adds pheromone to the edges that it traverses. In order to ensure that ants can obtain lastest pheromone information firstly,we must keep pheromones on each edge the newest to avoid ants making improper choice of tour path. Consequently,it is indispensable to update pheromone at a time.

It defined ,for each square i and j ,for each ant a in each colony, in document [8]:

$$\Delta Ph_{a,i,j}=\begin{cases}\dfrac{Q(|moves|-a)}{mn-a} & if\ ant\ a's\ r^{th}\ move\ from\ i\ to\ j , \\ 0 \quad otherwise\end{cases}$$

where, $m\times n$ are size of the chessboard.

Each element's value of vertex interrelation matrix **M** is only 1 or 0, which stands for accessibility or not. So a certain amount of pheromones between two accessible squares are laid. It defined ,for each square i and j ,for each ant a in each colony as follows:

$$\Delta Ph_{a,i,j}=\begin{cases}C & if\ ant\ a's\ r^{th}\ move\ from\ i\ to\ j \\ 0 & otherwise\end{cases}$$

Where, the parameter C is the update rate and constant because the length of every legal step is certain.If each ant in each colony finished one cycle,the pheromones left on the chessboard are updated as formula:

$$Ph_{i,j}=(1-\rho)\cdot Ph_{i,j}+\sum_{a}\Delta Ph_{a,i,j}$$

It is easy to plunge the local optimum solution because of not newest pheromone when ants in each colony are searching the legal tour path.So in this paper,information communication and share strategy are all introduced to enhance the efficiency of finding the legal path sequence and avoid ants plunging the local optimum solution.

7) Uptodate information communication and share strategy for each colony

It is difficult to assure that tours by ants in their colony are all legal because ants in each colony do their job alone. So it is supposed to make each colony work together and share pheromone information with communication when ants in each colony has finished one attempt. Suppose that ants in each colony communicate each other once at a certain time T (the time which can finish w cycles) and share information resource.We define each pheromone matrix for ants in each colony $Colony_l$ (l=1,2,…,k) is $\mathbf{Ph}_z$ (z=1,2,…,k), then the pheromone matrix **PH** after ants' communication and sharing information is listed as formula: $\mathbf{PH}=\|\mathbf{Ph}_1,\mathbf{Ph}_2,\ldots,\mathbf{Ph}_k\|_F$.

As is known, the amount of information obtained by ants in each colony is the same after their communication and information sharing, we can set: $\mathbf{PH}_1=\mathbf{PH}_2=\ldots=\mathbf{PH}_k=\mathbf{PH}$

F is called info communication intensity factor. where, $\|(\bullet)\|_F$ denotes doing F-norm operation for 1-dimensional vector composed of elements on corresponding position of matrix group in "()",that is:

$PH_{ij}=[\sum\limits_{t=1}^{k}\dfrac{(Ph_{t,ij})^F}{k}]^{\frac{1}{F}},(F=1,2,\cdots)$, if F=-∞ and F=∞, $PH_{ij}=\min\limits_{t=1}^{k}(Ph_{t,ij})$ and $PH_{ij}=\max\limits_{t=1}^{k}(Ph_{t,ij})$.

8) **Gain path solutions**

Keep ants in each colony working until they finished the searching. Then we screen out legal solutions from path solutions found by ants in each colony.If the length of ***tabu*** is mn-H-1 or mn-H, each square on the chessboard is visited once and exactly once and a valid knight's tour has been succeeded in finding by ants. If not, a partial tour has been completed . Eventually, a knight's tour on 2-dimensional chessboard is gained.

III. DECISION OF TOUR START SQUARE AND PATH TYPE

A. Method of deciding tour start square

Each knight's tour path has a start square visited first.The start square of tour path can be estimated. And the principle of estimation is majority rule, that is, the square where amount of ants placed is the largest is supposed to be the start square. Indeed, the start square must be stochastic if ants are placed on the board at random. Especially, if all ants are placed on only one square , the square must be the start square of tour path .

B. Method of deciding tour type

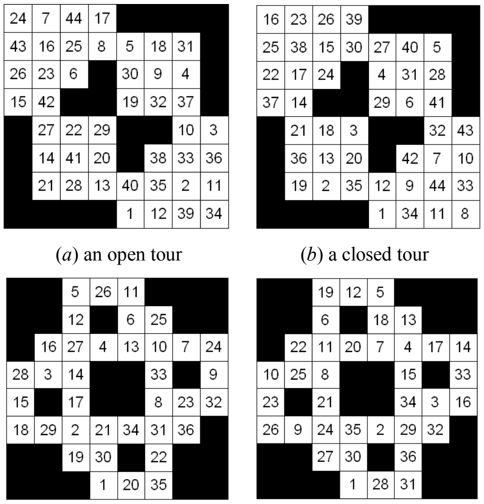
There are two types of knight's tour. The first one is merely an open tour while the second is called a closed tour, as the knight could complete a circuit with one more move.We can find that the closed tour is special open tour and the difference between each other is the last move of tour by comparison. So we can determine the type of tour is open or not arbitrarily. In the searching process by ants in colonies, if the pheromone deposited on edge from the last square and start square is not updated, that is , $\triangle Ph_{a,end,start}$=0, the length of the tour obtain from tabu list must be *mn*-*H*-1 and it must be open . Contrarily, the tour is closed .

IV. EXPERIMENTAL RESULTS AND ALGORITHM ANALYSIS

A. Experiments and choosing parameters

1) Experiment analysis

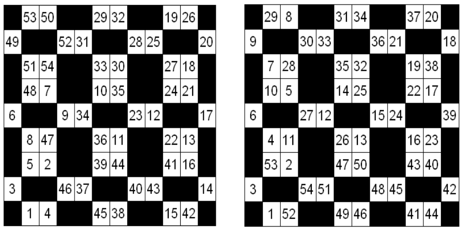
In our simulation,we set ρ , *C*, *w*, *F* and *k* to 0.45,0.1,10,-∞ and 3. In order to give expression to tour solutions by our algorithm conveniently, some experimental sketch map of knight's tour matrixesare displayed in the view of Eulor's algorithm[10] (The number "*i*" in figure below denotes position of the i^{th} step knight moved) .Several figures blelow are given and chessboards from Fig.3 to Fig.7 are acquired from http://www.archimedes-lab.org /knight_tour.html .



(*a*) an open tour (*b*) a closed tour

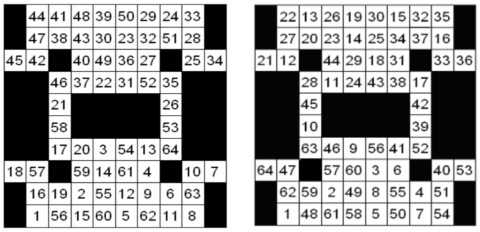
(*c*) an open tour (*d*) a closed tour

Fig.3 Knight's tour matrixes on 8 by 8 board



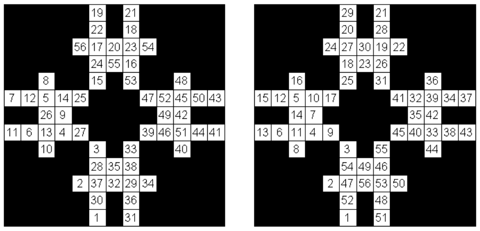
(*a*) an o pen tour (*b*) a closed tour

Fig.4 Knight's tour matrixes on 9 by 12 board



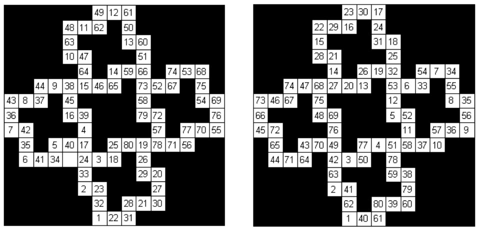
(*a*) an open tour (*b*) a closed tour

Fig.5 Knight's tour matrixes on 10 by 10 board



(*a*) an open tour (*b*) a closed tour

Fig.6 Knight's tour matrixes on 13 by 13 board



(*a*) an open tour (*b*) a closed tour

Fig.7 Knight's tour matrixes on 15 by 15 board

A conclusion is drawn from figures above that our algorithm not only can acquire open tours quickly,(see Fig.3(*a*,*c*) ,Fig.4 (*a*), Fig.5 (*a*), Fig.6(*a*) and Fig.7(*a*)),but also can obtain closed tours.(see Fig.3(*b*,*d*) ,Fig.4(*b*), Fig.5 (*b*), Fig.6 (*b*) and Fig.7(*b*)).

2) choosing parameters

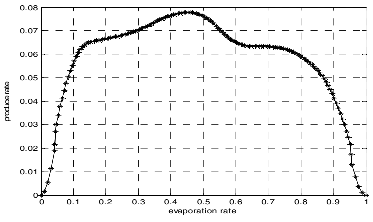


Fig.8 correlation of produce rate and evaporation rate

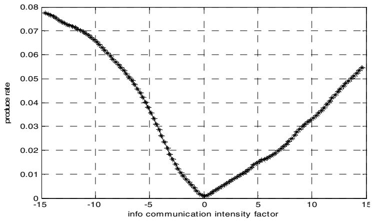


Fig.9 correlation of produce rate and info communication intensity factor

- *correlation of evaporation rate ρ and produce rate*

Figure 8 shows the produce rate versus evaporation rate *ρ* for our algorithms .The ant algorithm is initially very fast, but tails off later. Figure 10 and figure 11 illustrate the method for an 8 by 8 chessboard,where the best rate was approximately 0.0782 tours per attempted tour, which occurs with a *ρ* value of 0.45. Note that when evaporation rate *ρ* is between 0 and 0.05, the produce rate is pimping because pheromone information ants got is not new enough though amount of pheromone deposited on squares is large resulting from little evaporation rate *ρ*, and when evaporation rate *ρ* is between 0.95 and 1, the produce rate is low all the same because amount of pheromone deposited on squares is little resulting from large evaporation rate *ρ* though pheromone information ants got is new enough.So we set *ρ*=0.45 in our experiments.

- correlation of info communication intensity factor *F* and produce rate

Figure 9 illustrates that the produce rate takes on tendency of incessant augment as a whole whether *F* is positive number or not, but influence of the produce rate is distincter when *F* is negative comparing with plus *F* . Peculiarly, rate versus *F* at vicinity of zero is low because *F* is adjacent to zero and the amount of the information ants acquired is almost zero when ants in colonies share information resource. In other word, ants' search is restricted and it resulted in low rate ,this is the reason of setting F to -∞ in our simulation.

B. Algorithm analysis

Table 1 produce rate table by algorithm proposed

algorithm	DFS [1]	GA[6]	ACO[7]	our algorithm
produce rate	0.000003	0.000642	0.076	0.0782

It is shown that in table 1 that produce rate using improved ant colony optimization algorithm is out and away larger than that of DF[1] and GA[6], and is 0.0022 larger than ACO[7],which is thousands upon thousands times with the algorithm proposed above.

In our algorithm, a new heuristic principle for knight's tour and a new updating style of pheromone information are introducted properly, and information resources are shared rationally. All those can help ants to find tours on large-scale boards more exactly and quickly which is much better than DFS[1], searching-backtracking algorithm[2,11] and NN[5]. In addition, tours on the small-scale board by the algorithm can be initial solutions for 'divide-conquer' method[4], and our algorithm is used with GA[6] together and achieved by program easily.

V. CONCLUSION

In this paper, we use improved multi-ant-colony optimization algorithm to get solutions for knight's tour problem on chessboard with holes. Experiment results show the algorithm not only produce the path solutions quickly on the chessboard with holes, but also is easy to control the path starting square and path style(open or closed). In future work, we intend to explore some of these options in order to tackle several related problems:one is how to solve and validate the existence of knight's tour on board with holes which is a part and parcel,the other is how to estimate and get the elaborate amount of tours,which is what we do next.

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