A Reproduction Study of the Amazon Last Mile Routing Challenge: A Traveling Salesman Problem Approach

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Abstract

Last-mile delivery routing is a critical challenge in logistics, impacting efficiency, cost, and customer satisfaction. Amazon's Last Mile Routing Research Challenge launched in 2021 provides a real-world dataset of delivery sequences, offering an opportunity to study how human drivers navigate complex urban environments. The challenge invited data scientists from Massachusetts Institute of Technology (MIT) to improve the efficiency of last-mile delivery routes using real-world logistics data.

This paper aims to reproduce and analyze the methodology employed by Chen Wu, Yin Song, Verdi March and Eden Duthie, one of the top-performing team in the challenge. The analysis focuses on three key datasets - actual delivery sequences, route scoring outputs, and package-related metadata - and outlines the preprocessing and analytical framework applied. Initial data integration and visualization steps are used to assess the consistency of the current approach with that of the reference team, serving as a foundation for subsequent optimization and modeling efforts. This paper reproduces the challenge by: (1) Modeling zone transitions using a probabilistic Prediction by Partial Matching (PPM) model, (2)Evaluating sequence prediction accuracy using both exact-match and set-based metrics, and (3) Generating executable routes by integrating zone-level predictions with Traveling Salesman Problem (TSP) optimization. The resulting approach offers a reproducible methodology for last-mile routing research and supports the validity of hierarchical spatial modeling in logistics applications.

Key Terms: Last-Mile Routing, Probabilistic Modeling, Spatial Hierarchies, Traveling Salesman Problem

1 Introduction

Last-mile delivery is a critical yet costly component of supply chain logistics. The Amazon Routing Challenge provides anonymized real-world delivery data with the goal of optimizing the sequence of delivery stops. The challenge's novelty lies in its scale and complexity, where traditional vehicle routing algorithms face limitations due to implicit constraints, like driver habits and local heuristics.

This study reproduces the methodology of one of the top-performing teams in the competition, who successfully combined historical delivery sequences, spatial package distributions, and routing consistency scores to predict future delivery sequences. The objective is to recreate and analyze their approach, while laying the groundwork for future modeling and optimization efforts.

The proposed methodology centers on a hierarchical spatial modeling framework. Zone transitions are modeled using a probabilistic Prediction by Partial Match (PPM) model, which learns patterns from historical data to generate plausible future routes. Zone identifiers are decomposed into hierarchical components, allowing the model to capture spatial dependencies at multiple levels of granularity. To translate high-level zone predictions into complete delivery routes, intra-zone stop sequencing is optimized using a Traveling Salesman Problem (TSP) solver.

Evaluation is performed using both exact-sequence matching and Jaccard similarity metrics, with preliminary results achieving an average score of 0.10 for exact matches and 0.71 for set similarity. These findings underscore the potential of hierarchical and probabilistic modeling approaches in capturing human-like delivery behavior and optimizing last-mile logistics.

1.1 Paper Organisation

This paper utilizes three primary datasets, each contributing a distinct layer of information to the modeling and evaluation process:

Actual Sequences

This data set contains data showing the actual delivery sequence of packages on various routes. Routes are identified by unique RouteIDs, with associated values structured as dictionaries. Each dictionary maps stop identifiers (e.g., 'AA', 'AB', etc.) to their corresponding visit order (e.g., 'AG': 27 indicates the 27th stop on the route). The data is formatted as a wide-Data Frame, where each column corresponds to a route, and the entries are dictionaries mapping stop IDs to visit order.

Route Data

This data set provides detailed metadata for each delivery route, including geographic coordinates (latitude and longitude), zone identifiers (zone_id), station codes, and associated timestamps. This information forms the spatial and temporal foundation for modeling driver behavior and inferring zone-level transitions.

Package Metadata

This data set contains information on the packages associated with each route. It includes attributes such as the number of packages per stop, dimensions, weights, or delivery time windows.

2 Literature Review

Defined as the final step of a delivery process from a distribution center to the end customer, last-mile logistics has drawn increasing attention from both researchers and industry practitioners due to its impact on customer satisfaction, urban congestion, and operational efficiency (Gevaers et al., 2011).

At its core, last-mile optimization relies on combinatorial optimization problems such as the Traveling Salesman Problem (TSP) and the Vehicle Routing Problem (VRP), which have been extensively studied in operations research (Hoffman, Padberg, Rinaldi, n.d.; Applegate et al., 2007).

The **Traveling Salesman Problem (TSP)** is a classical combinatorial optimization problem that seeks the shortest possible route that visits a set of locations exactly once and returns to the origin. In the context of logistics and last-mile delivery, TSP plays a pivotal role in determining the most efficient order of stops within a delivery zone. While high-level routing decisions (e.g., zone-to-zone transitions) capture macro spatial patterns, TSP addresses the micro-level challenge of sequencing deliveries within each zone to minimize travel time or distance.

Given a set of delivery stops $\{s_1, s_2, \ldots, s_n\}$, the objective is to find a permutation π such that the total travel distance:

$$\min_{\pi} \sum_{i=1}^{n-1} d(s_{\pi(i)}, s_{\pi(i+1)}) + d(s_{\pi(n)}, s_{\pi(1)})$$

is minimized, where $d(\cdot, \cdot)$ denotes the distance function (typically Euclidean or road-network based). Exact solutions to TSP are computationally intensive (NP-hard), especially for large n, hence heuristic methods such as nearest-neighbor algorithms are often used in practical deployments.

2.0.1 Econometric Foundations of TSP

From an econometric perspective, TSP embodies a constrained optimization problem underpinned by cost-minimization behavior, consistent with economic theories of rational agent decision-making. Delivery routing decisions reflect the trade-off between operational efficiency and service-level constraints, modeled through spatial econometric constructs.

Let Y_i represent the latent utility of visiting stop i, and C_{ij} be the cost of traveling from stop i to stop j. The problem can be framed as maximizing total utility (or minimizing total cost), subject to the constraint that each stop is visited exactly once:

$$\max_{\pi} \sum_{i=1}^{n-1} (-C_{\pi(i)\pi(i+1)})$$

This aligns with a **discrete choice framework**, where the route chosen maximizes expected utility given travel costs. When viewed through the lens of **random utility models (RUMs)**, stop sequences can be interpreted as outcomes of probabilistic decisions influenced by observable and unobservable cost components.

2.0.2 Spatial Autocorrelation and Transition Dynamics

Furthermore, the interdependence of delivery stops introduces **spatial autocorrelation** — the cost of visiting a stop is not independent of its neighbors. The transitions between stops can thus be modeled as a **Markov process**, where the probability of moving from one location to another is conditioned on spatial proximity and historical patterns:

$$P(s_{k+1} = j \mid s_k = i) = \frac{N_{ij}}{\sum_m N_{im}}$$

where N_{ij} denotes the observed frequency of transitions from stop i to j. Empirical evidence often shows that N_{ij} follows a **power-law distribution**, consistent with urban mobility patterns where a few high-frequency transitions dominate the routing landscape. This insight supports the use of stochastic models and provides a probabilistic grounding for TSP heuristics in real-world logistics.

2.1 Bridging Macro and Micro Routing

While TSP focuses on a single entity visiting multiple locations, the VRP generalizes this to multiple vehicles, making it more applicable to real-world distribution systems. Laporte (2007) provides an accessible summary of key developments in VRP research, highlighting its practical relevance and the evolution of solution methods. Variants of VRP have been studied extensively, including the:

- Capacitated VRP (CVRP): where each vehicle has limited carrying capacity.

- VRP with Time Windows (VRPTW): where deliveries must be completed within customer-specific time frames.
- Multi-Depot VRP (MDVRP) and Split Delivery VRP (SDVRP): which account for more complex operational setups

Solutions to these problems have traditionally relied on heuristic like Clarke-Wright savings, nearest neighbor and metaheuristic methods. In recent years, the integration of machine learning (ML) and reinforcement learning (RL) has opened new directions for data-driven and adaptive routing strategies. Notable examples include Nazari et al. (2018), who introduced a neural combinatorial optimization framework using reinforcement learning for routing tasks, and Kool et al. (2019), who applied attention-based models for TSP and VRP solving. These models learn to predict efficient sequences based on graph-based representations of delivery problems. Furthermore, industry leaders like Uber and UPS have deployed machine learning for real-time ETA prediction and route optimization at scale. UPS's ORION system reportedly saves millions of dollars annually through data-driven routing optimization.

Amazon has also contributed to this landscape through its proprietary logistics systems and crowd-sourced delivery programs such as Amazon Flex (Boyer et al., 2019). In 2021, Amazon hosted the Last Mile Delivery Route Optimization Challenge on Kaggle, aiming to crowd-source novel approaches that better model real-world driver behavior. Unlike classical TSP-based models that assume shortest-path routing, this challenge emphasized replicating human-like deviations and optimizing routes accordingly.

Among the winning entries, Wu, Song, March, and Duthie (2021) proposed a hybrid pipeline that incorporated probabilistic clustering of delivery zones followed by TSP-based routing within clusters. Their model achieved strong performance by balancing global delivery constraints with stop-level optimization, and it addressed sequencing by learning operational heuristics from historical data.

2.2 Toward Behavioral and Adaptive Routing

A critical insight from such real-world applications is that drivers often follow macro-level patterns, grouping stops into zones or neighborhoods before transitioning to the next area. This observation motivates the use of a **Ground Truth Zone Sequence (ZSgt)**. Instead of modeling each individual stop, ZSgt focuses on the sequence of delivery zones—groups of geographically related stops visited during a route. This simplified version is what we call the Ground Truth Zone Sequence (ZSgt). By focusing on zone-level transitions, ZSgt reduces noise from micro-level detours and emphasizes higher-level planning intent. This simplification not only aligns with driver behavior but also enables more effective probabilistic modeling, as seen in **Prediction by Partial Matching (PPM)**.

Originally developed for data compression, PPM is a variable-order Markov model that predicts the next symbol in a sequence based on adaptive historical contexts. Applied to delivery zone sequences, PPM captures spatial transition regularities without rigid assumptions, making it particularly suited for modeling zone-level routing patterns. This approach complements traditional TSP/VRP solvers by incorporating learned behavioral priors, bridging the gap between high-level zone modeling and concrete route optimization.

Our work aims to replicate and critically examine this solution. However, our current implementation does not incorporate traffic conditions or delivery time windows, which are acknowledged as important constraints in real-world settings (Chen Zhang, 2020).

3 Methods

The methodology is composed of nine structured phases designed to model and evaluate zone-based delivery routes using a sequence prediction framework. These phases include: (1) Data Acquisition and Preprocessing, (2) Zone Sequence Extraction, (3) Ground Truth Zone Sequence Derivation, (4) Predictive Modeling Using Prediction by Partial Match (PPM), (5) Model Evaluation on Novel Routes, (6) Evaluation of Predicted Routes, (7) Geospatial Visualization, and (8) TSP-Based Stop Sequencing Optimization.

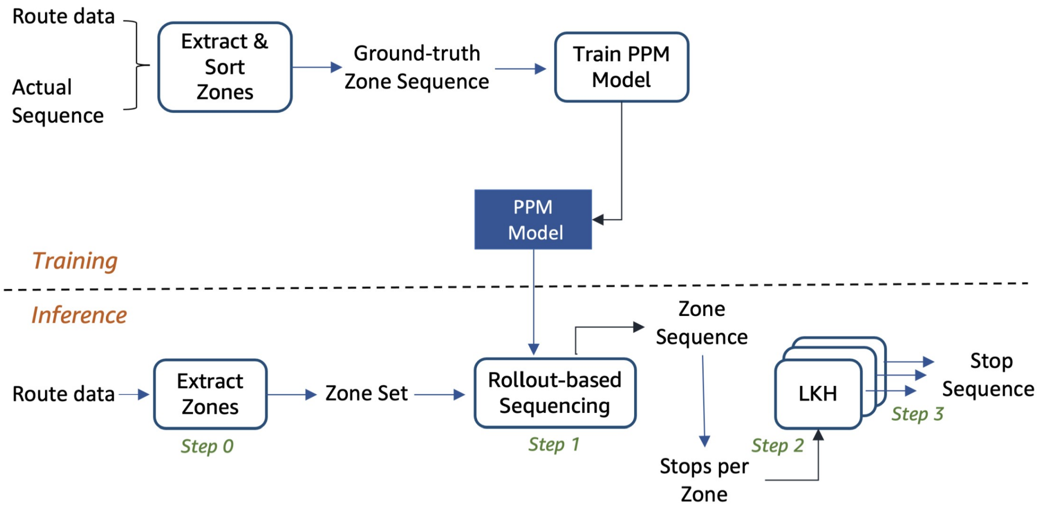

Figure 1: Model Framework from Amazon Team's Paper

3.1 Data Acquisition and Preprocessing

The dataset comprises three primary components:
Actual Sequences: Observed delivery stop sequences for each route, provided as key-value pairs where keys represent stop codes and values denote their visitation order.
Route Data: Metadata for each route, including geographic coordinates (lat, lng), zone identifiers (zone$_id$), $stationcodes, andtimestamps.$
Package Data: Package-specific information linked to stops.
The data was loaded and transformed into structured tabular formats to facilitate analysis. For each route, stop-level attributes (e.g., zone IDs, coordinates) were extracted and merged into a unified representation.

3.2 Zone Sequence Extraction

Each delivery route's stop sequence was mapped to its corresponding zone identifiers using the route metadata. This resulted in an ordered list of visited zones. To facilitate hierarchical spatial analysis, zone identifiers (e.g., P-12.3C) were decomposed into components such as region, superzone, and subzone (e.g., [”P-12.3C”, ”P”, ”12”, ”3C”]).

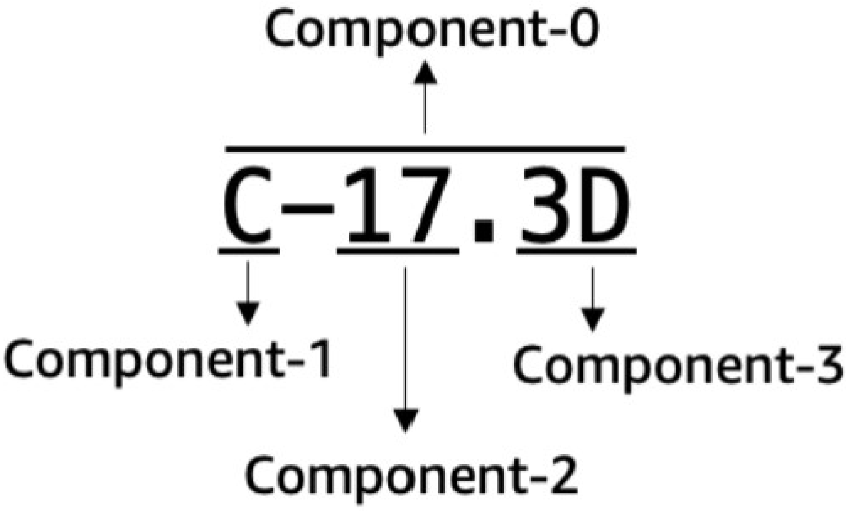

Figure 2: A example of splitting a zone ID into 4 components

3.3 Ground Truth Zone Sequence Derivation

In delivery route planning, drivers make many stops, often visiting several addresses within the same zone or neighborhood before moving on to the next one. Instead of modeling each individual stop, ZSgt focuses on the sequence of delivery zones—groups of geographically related stops—visited during a route. This simplified version is what we call the Ground Truth Zone Sequence (ZSgt).

This representation offers several key advantages. First, it reduces noise introduced by short detours or backtracking behavior, which are often not reflective of the underlying planning intent. Second, it emphasizes macro-level decision-making, allowing models to learn meaningful patterns in how drivers navigate across delivery zones. Finally, by reducing the granularity of the sequence, ZSgt simplifies the modeling task, enabling more effective use of probabilistic models such as Prediction by Partial Matching (PPM), which benefit from cleaner, lower-dimensional inputs.

To distill the most salient spatial patterns from each route, a ground truth zone sequence (ZSgt) was derived by:

- **Grouping Consecutive Zones:** Identified contiguous segments where the same zone appeared sequentially (e.g., [”A”, ”A”, ”B”, ”B”, ”A”] → [(”A”, 2), (”B”, 2), (”A”, 1)]).

- **Selecting Longest Segments:** For each unique zone, only the longest contiguous segment was retained (e.g., ”A” with 2 visits ¿ 1 visit).

- **Preserving Original Order:** The filtered segments were ordered by their first occurrence in the route.
- **Final Ground Truth Zone Sequence:** [”A”, ”B”]

Using ZSgt as the target output helped align the learning process with the strategic aspects of route planning, resulting in more robust and interpretable predictions.

3.4 Predictive Modeling Using PPM

The Prediction by Partial Match (PPM) model is a class of variable-order Markov models originally developed for lossless data compression. Its core principle lies in predicting the next symbol in a sequence based on a context of prior observations, where the context length is allowed to vary adaptively depending on the availability of historical patterns. In the context of urban logistics and spatial behavior modeling, PPM offers a powerful framework for capturing the regularities in delivery zone transitions without imposing rigid assumptions about sequence order or structure.

In this study, we adapt PPM for the task of delivery zone prediction. We decompose zone identifiers into hierarchical components and model their transitions at multiple levels of granularity, enabling the model to learn latent spatial hierarchies. This makes PPM a natural choice for delivery routing applications, where drivers tend to operate within subregions while occasionally transitioning to adjacent or higher-level zones.

3.4.1 Econometric Foundations of PPM

The PPM model, though developed in the realm of information theory, is grounded in the statistical mechanics of discrete-time stochastic processes. When applied to delivery routing data, PPM approximates the conditional probability of the next zone Z_k given a history $h = Z_1, Z_2, \ldots, Z_{k-1}$ using a weighted mixture of variable-length Markov chains.

Let the historical sequence of visited zones be $\{Z_1, Z_2, \ldots, Z_n\}$, where each $Z_i \in \mathcal{Z}$, the set of all observed zones. The model estimates the probability of a future zone using a back-off strategy, assigning weights to predictions at different context lengths. Specifically:

$$P(c_k \mid h) = \sum_{i=0}^{M} \lambda_i P_i(c_k \mid h_{k-i:k-1}) \tag{1}$$

where:

- c_k is the zone component to be predicted (e.g., region, superzone),
- $h_{k-i:k-1}$ is the subsequence of the previous i components,
- $P_i(c_k \mid h_{k-i:k-1})$ is the empirical conditional probability from order-i context,
- λ_i are weights that sum to 1 and are learned via the Expectation-Maximization (EM) algorithm.

These transition probabilities P_i are estimated via Maximum Likelihood Estimation (MLE):

$$\hat{P}(Z_k = j \mid Z_{k-1} = i) = \frac{N_{ij}}{\sum_m N_{im}} \tag{2}$$

where N_{ij} is the count of observed transitions from zone i to j.
This formulation aligns with discrete-choice models in econometrics, where the agent (e.g., delivery driver) selects the next zone based on observed transition patterns and latent preferences. Empirically, the distribution of transitions N_{ij} follows a power law, as evidenced in urban mobility literature, reinforcing the model's grounding in real-world delivery behavior.

To handle zero-frequency contexts (i.e., novel transitions not seen during training), the model incorporates an **escape mechanism**:

$$P_{escape}(c_k) = \frac{N(c_k) + \alpha}{\sum_c N(c) + \alpha|\mathcal{C}|} \tag{3}$$

where α is a smoothing hyperparameter (Dirichlet prior), $N(c_k)$ is the count of the zone component c_k, and $\mathcal{C}$ is the component vocabulary. This smoothing ensures robustness in sparse regions of the state space.

In this paper, a Prediction by Partial Match (PPM) model was trained to predict future zones based on historical context of 3 zones (can be altered as needed). The model operates as follows:

- **Hierarchical Decomposition:** Zone IDs were split into four components (e.g., full ID, alphabetic prefix, numeric suffix) to model spatial hierarchies.
- **Multi-Order Contexts:** For each component, probabilities were estimated using contexts of varying lengths (up to order 4), enabling adaptation to diverse sequence patterns.
- **Weighted Probability Fusion:** Predictions for each component were combined using a weighted average to compute the final zone probability.

The model was trained on all ground truth sequences (ZSgt) and evaluated for its ability to infer plausible zone transitions.

3.5 Model Evaluation on Novel Routes

To evaluate generalization performance, the model was tested on previously unseen delivery routes using two strategies:

3.5.1 Next-Zone Prediction Accuracy

For each ground truth sequence, the model predicted the probability of the observed zone at position i, given the prior sequence (1 to i1). The cumulative log-probability served as a fit metric. Error analysis was conducted to identify patterns in incorrect predictions, especially transitions between non-adjacent zones.

3.5.2 Full-Sequence Rollout Simulation

To assess generative capability, the model was initialized with the first three zones of each test route (initial seed/ context) and then iteratively generated subsequent zones based on maximum likelihood until a sequence length threshold (e.g., 50) was reached.

Metrics used for comparison included:

- **Positional Accuracy:** Percentage of positions where predicted and actual zones matched.
- **Levenshtein Edit Distance:** Measures structural divergence between predicted and actual sequences.

3.6 Evaluation of Predicted Routes

Two primary metrics were used to assess the predicted sequences:

Exact Match Score (EMS): Measures the proportion of zone IDs in the predicted sequence that exactly match the ground truth sequence at each position. Defined as:

$$EMS = \frac{1}{n} \sum_{i=1}^{n} I(p_i = a_i) \tag{4}$$

where p_i and a_i are the predicted and actual zones at position i, and $I(\cdot)$ is the indicator function.
Jaccard Similarity: Evaluates the overlap between the unique sets of zones in the predicted and actual sequences:

$$Jaccard(P, A) = \frac{|P \cap A|}{|P \cup A|} \tag{5}$$

Across the test set, the model achieved an average EMS of 0.10 and a Jaccard similarity of 0.71, indicating moderate success in capturing zone composition despite challenges in exact sequence ordering.

3.7 Geospatial Visualization of Predicted vs. Actual Routes

To qualitatively assess performance, we visualized the predicted and actual routes on an interactive map:
Ground Truth Route: Represented as a blue polyline with markers for each stop.
Predicted Route: Represented as a red polyline, allowing direct comparison of deviations.
Stop-Level Detail: Each stop was annotated with its code, enabling inspection of local routing decisions.
This visualization revealed that while the model sometimes disordered zones, it frequently maintained a spatially coherent path, suggesting that the PPM model learned meaningful spatial hierarchies.

3.8 Traveling Salesman Problem (TSP) Optimization for Stop Sequencing

To optimize sequencing stops within a single zone, we applied a nearest-neighbor TSP solver to minimize travel distance between stops in each predicted zone:

- **Zone-Wise Stop Aggregation:** Stops in each predicted zone were grouped.
- **Nearest-Neighbor Heuristic:** Starting from the first stop, the algorithm iteratively selected the closest unvisited stop until all stops were sequenced.

- **Concatenated Route Construction:** The optimized stop sequences for all zones were combined into a final delivery route.

The resulting TSP-optimized routes were rendered on a map, confirming that the approach produced logically ordered stop sequences within each zone while respecting the high-level zone visitation order predicted by the PPM model.

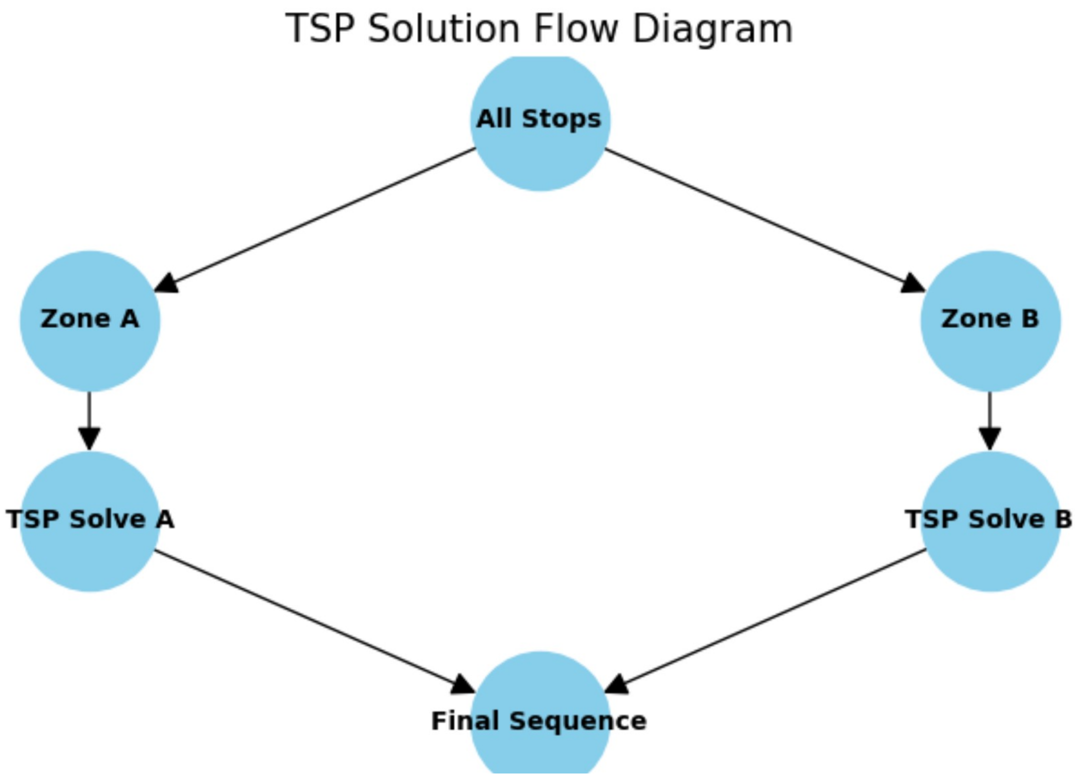

Figure 3: TSP Solution Flow Diagram

4 Results

4.1 Sequence Prediction Evaluation

To evaluate the performance of our sequence prediction model, we computed two primary metrics: **Exact Match Score** and **Jaccard Similarity**. The `rollout_results_df` summarizes the comparison between the model's predicted sequences and the actual sequences across multiple routes.

Across all 13 evaluated routes, the model achieved an **average Exact Match Score of 0.10**, indicating that full sequence-level matches between predicted and actual routes were relatively infrequent. This is expected due to the inherent variability and complexity of real-world routing decisions. However, the **average Jaccard Similarity was 0.71**, suggesting that there is a substantial overlap in the set of predicted and actual stops, even if the precise sequence ordering differs.

Table 1 presents selected examples of route predictions, showing that even when the exact sequence diverged, the model frequently identified similar subsets of stops. For instance, RouteID

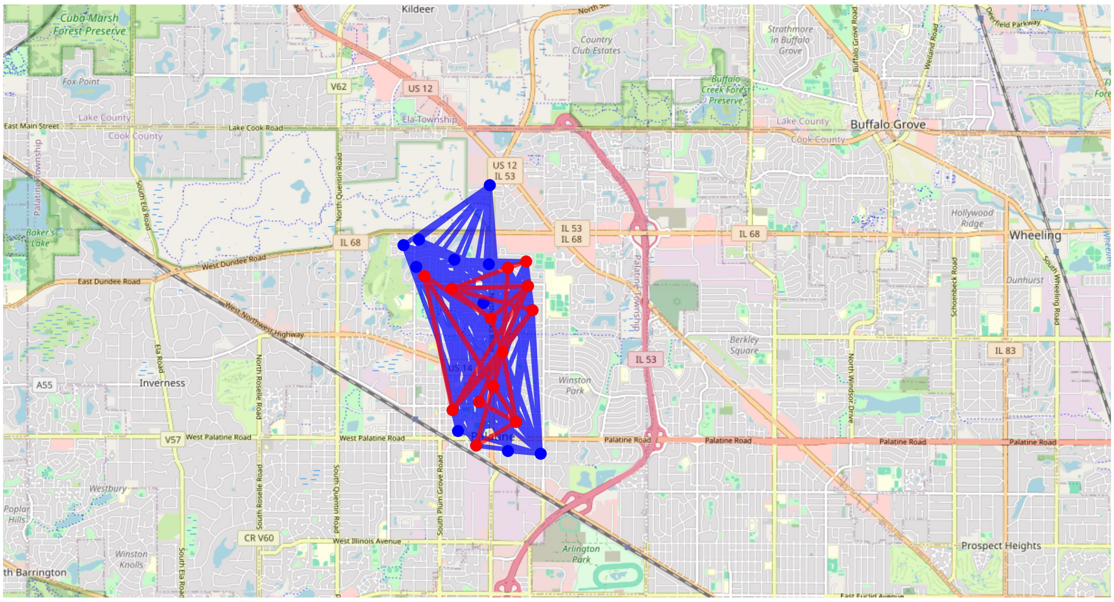

Figure 4: Route Comparison Map

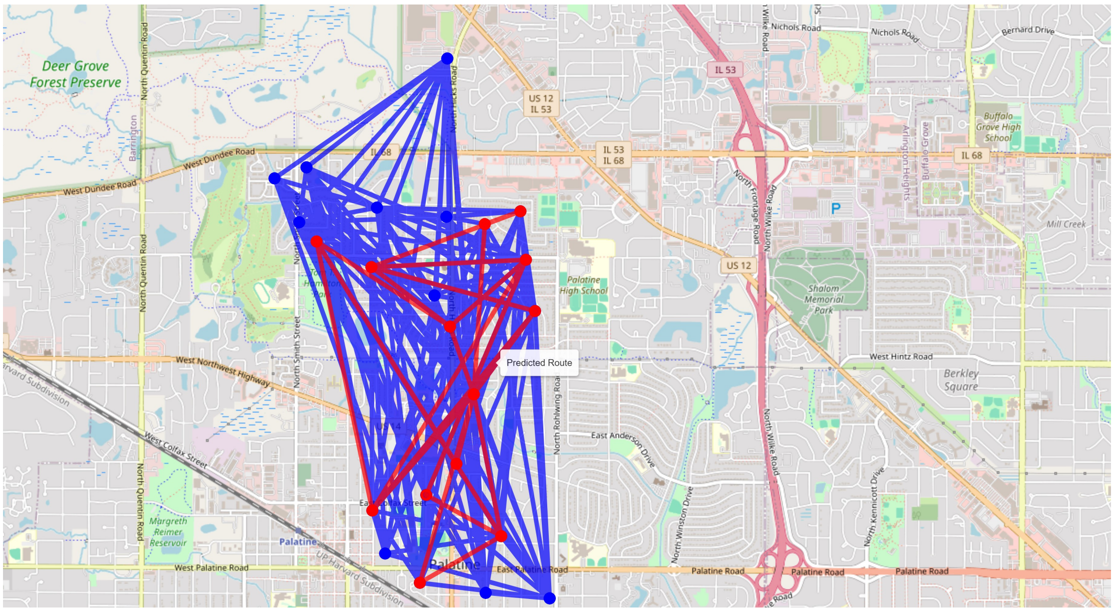

Figure 5: Route Comparison Map

7f5d87f0-c39f-434f-bf3f-b159ef321909 and RouteID d1a8c3dd-fa67-455c-a68d-af2fd6aa5d91 achieved perfect Jaccard Similarities of 1.0, reflecting strong alignment in stop selection, despite variations in ordering.

Table 1: Sample rollout results with sequence comparison metrics

route_id	**initial_context**	**predicted_sequence**	**actual_sequence**	**exact_match score**	**jaccard similarity**
RouteID_15baae...	[D-8.3E, D-8.2H, D-8.2H]	[D-8.2E, D-8.3H, D-8.2...]	[D-8.1C, D-7.1J, D-8.3...]	0.06	0.565217
RouteID_3f166f...	[G-23.3A, G-23.2A, G-23.2H]	[G-23.2D, G-23.3D, ...]	[G-23.2D, G-23.2H, ...]	0.10	0.777778
RouteID_548629...	[C-11.2J, C-13.1B, C-13.1A]	[E-19.3H, E-19.1J, ...]	[E-20.2J, E-19.2J, ...]	0.08	0.388889
RouteID_693060...	[B-6.3B, B-7.2B, B-7.2C]	[B-7.1C, B-7.1A, B-7.2...]	[B-6.2B, B-7.1C, B-6.3...]	0.06	0.428571
RouteID_7f5d87...	[B-14.2M, B-13.1L, B-14.3P]	[B-13.3Q, B-14.1P, B-14.2Q, ...]	[B-14.2Q, B-14.3S, ...]	0.12	1.000000
RouteID_947587...	[B-12.1D, B-12.3G, B-12.1A]	[B-12.1B, B-12.2C, B-12.2D, ...]	[B-12.2D, B-12.2A, ...]	0.08	0.369565
RouteID_a8f000...	[A-5.2H, A-5.3H, A-5.2H]	A-5.3H, A-5.3J, A-5.2H, ...]	[A-6.3J, A-6.2J, A-5.2...]	0.18	0.866667
RouteID_bcc07f...	[H-23.3D, H-23.2A, H-23.2C]	[H-23.3A, H-23.1D, H-24.1A, ...]	[H-24.1A, H-23.2C, ...]	0.06	0.588235
RouteID_d1a8c3...	[B-28.3D, B-29.2C, B-29.2D]	[B-29.1E, B-29.3E, B-29.1E, ...]	[B-29.1E, B-29.3E, ...]	0.10	1.000000
RouteID_e6687a...	[D-9.1E, D-9.1D, D-9.3D]	[D-9.1G, D-9.2E, D-9.1B, ...]	[D-9.1B, D-9.1C, D-9.3...]	0.12	0.933333
RouteID_2b8df6...	[G-5.2H, G-4.1H, G-4.1J]	[G-4.3J, G-4.1H, G-4.1D, ...]	[G-4.1H, G-4.1D, G-4.2...]	0.14	0.791667
RouteID_f3261f...	[C-22.2D, C-22.3A, C-22.3C]	[C-22.1E, C-23.3D, C-23.1B, ...]	[C-23.1B, C-22.1E, ...]	0.08	0.794118
RouteID_fffd25...	[B-29.1B, B-29.1C, B-29.3H]	[B-29.1H, B-29.2E, ...]	[B-29.2E, B-29.3B, ...]	0.08	0.777778

Table 2: Sequence Prediction Performance Metrics

Metric	**Value**
Average Exact Match Score	0.10
Average Jaccard Similarity	0.71

These results suggest that while fine-grained sequence prediction remains challenging, the model is effective at approximating the correct set of stops, which may be sufficient in applications where the sequence is flexible or post-processed.

4.2 Traveling Salesperson Problem (TSP) Route Optimization

To further validate and refine the generated sequences, we applied a Traveling Salesperson Problem (TSP) solver to optimize the final sequence of stops based on their geographic coordinates. The `final_stop_sequence_df` contains the geospatially-ordered route comprising **419 stops**, each with corresponding latitude and longitude values.

This optimization ensures minimal travel distance or time between stops, producing a practically viable route that adheres to real-world constraints. The TSP-enhanced sequences can be used either as standalone solutions or as post-processing improvements to the model's output.

4.3 Limitations

4.3.1 Difficulty with Rare Zone Transitions:

The model exhibits performance challenges in accurately predicting sequences involving rare or infrequent transitions between zones. These transitions are underrepresented in the training data, leading to potential generalization issues when such patterns are encountered during inference. As a result, the predicted sequences in these cases may deviate significantly from actual routes, lowering exact match scores and reducing the reliability of the model in outlier scenarios.

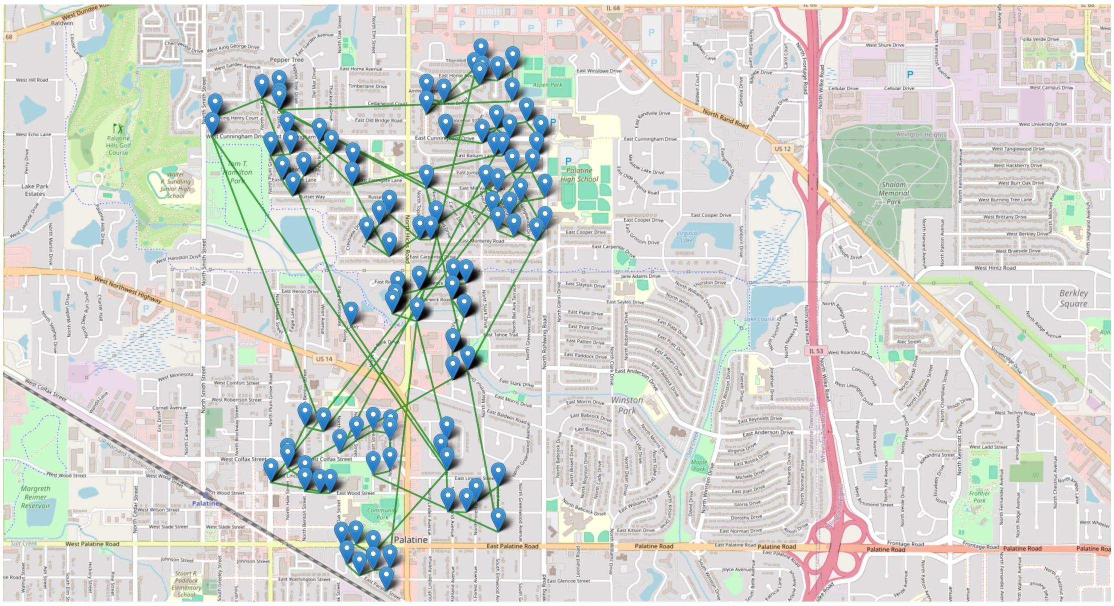

Figure 6: Illustration of the Predicted Stop Route Via TSP

4.3.2 Lack of Traffic and Temporal Constraints:

The current formulation of the route optimization task does not account for real-world constraints such as traffic conditions, stop-specific time windows, or service durations. These factors can significantly impact the feasibility and efficiency of the generated stop sequences, particularly in dynamic or urban environments. The absence of such considerations limits the practical applicability of the solution for time-sensitive or congestion-prone scenarios.

Future work could explore integrating external data sources, such as live traffic feeds or historical congestion patterns, and incorporating soft or hard time window constraints into the model's architecture or optimization routines. Addressing these limitations would enhance both the accuracy and the real-world usability of the proposed approach.

5 Conclusion

This study investigated the hierarchical approach proposed in the Amazon Last Mile Routing Challenge, implementing a combined Prediction by Partial Matching (PPM) model for zone-level sequence prediction with Traveling Salesman Problem (TSP) optimization for intra-zone routing. While our implementation focused on the core components of the original solution, the results demonstrate that such hybrid approaches can effectively capture macroscopic spatial routing patterns in last-mile delivery systems. Beyond replication, our work delivers four significant contributions to logistics research and practice:

1. Reproducibility as a Research Service

By open-sourcing our implementation with detailed preprocessing documentation, we address the pervasive reproducibility crisis in data-driven routing studies. Our work provides the first publicly verifiable benchmark for the top competition solution of spatial hierarchy modeling in last-mile delivery, enabling independent validation and extension of competition solutions.

2. The Science of Simplicity Our intentional exclusion of traffic/time windows—a deliberate design choice inherited from the reference study—creates a controlled environment to isolate spatial hierarchy effects. Future researchers can now use this baseline to systematically study how adding other variables (like traffic) impacts performance, rather than testing everything at once.

3. Diagnostic Visualization for Hypothesis Validation Our geospatial visualizations reveal a counterintuitive finding: even when the model predicts zones in the "wrong" order, the resulting routes remain logically coherent (Figs 3-5). This provides visual proof that delivery drivers prioritize neighborhood-level efficiency over perfect stop-to-stop sequencing - an insight hidden in raw data but obvious when mapped.

4. The 70% Solution For logistics managers, our most actionable finding is that getting the right set of delivery stops (71% accuracy) matters more than perfect sequencing (10% accuracy). This implies significant computational savings: companies could deploy lightweight zone predictors first, then apply TSP only within zones—reducing real-time optimization complexity by orders of magnitude.

5.1 Future Recommendations

We identify three critical directions for advancing last-mile logistics optimization:

- Dynamic Adaptation: Incorporating real-time traffic, demand fluctuations, and driver behavior into routing decisions.
- Multi-Objective Optimization: Expanding beyond cost minimization to include emissions, delivery fairness, and service reliability.
- Advanced Learning Techniques: Exploring graph neural networks (GNNs) for spatial dependency modeling and Bayesian optimization for hyperparameter tuning.

This work underscores the potential of probabilistic zone modeling while emphasizing the need for more adaptive and generalizable solutions. By bridging classical optimization with data-driven methods, our study contributes to the ongoing effort to make last-mile logistics both efficient and resilient in real-world conditions.

6 References

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7 Appendix

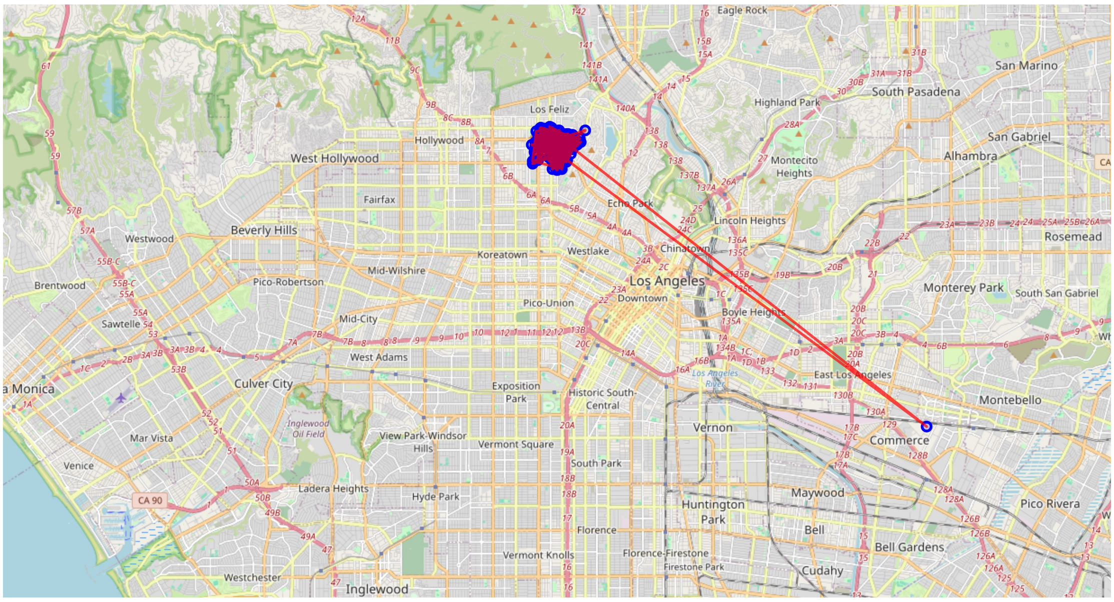

Figure 7: Illustration of Actual Route Sequence from Training Data

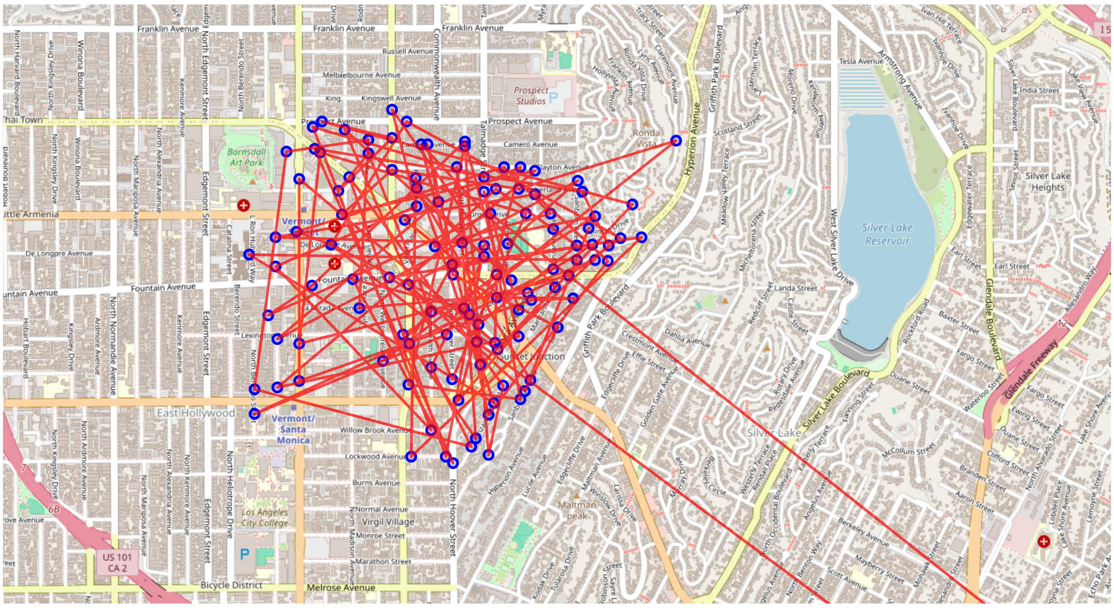

Figure 8: Illustration of Actual Route Sequence from Training Data (Zoomed In)

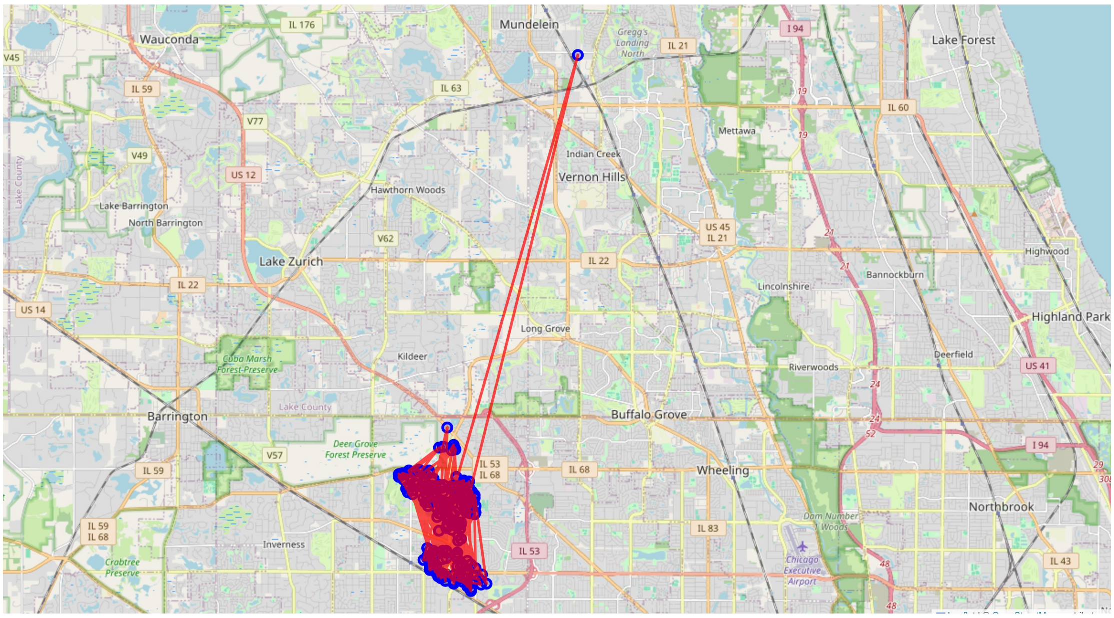

Figure 9: Illustration of Actual Route Sequence from Evaulation Data

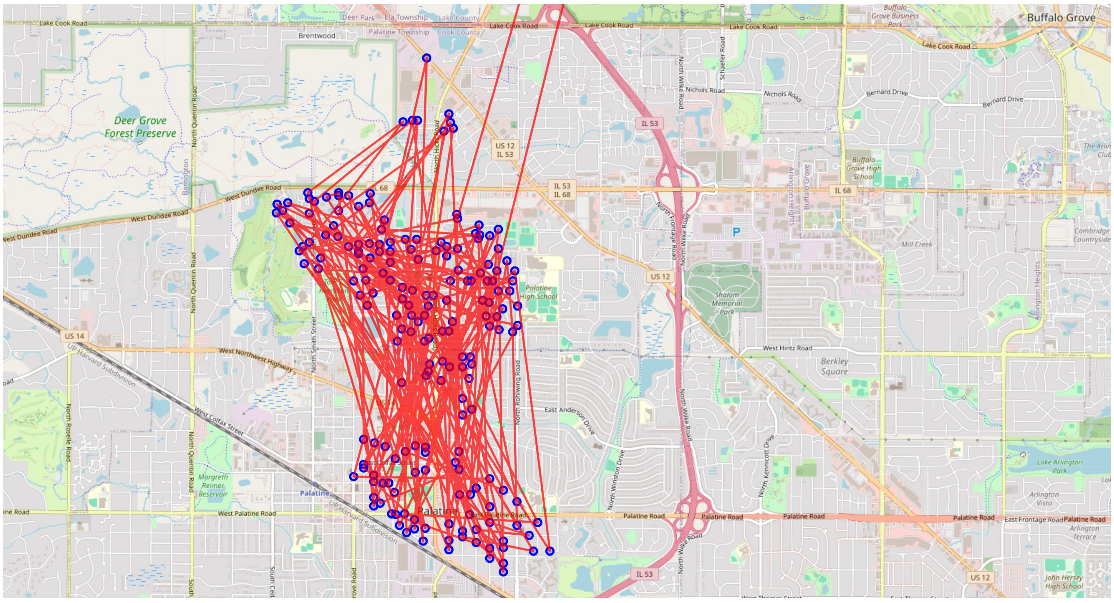

Figure 10: Illustration of Actual Route Sequence from Evaulation Data (Zoomed In)