# 1 Algebra

## 1.1 Sequences and Series

Arithmetic progressions

- $T_n = U_n = a + (n-1)d$.
- A sequence is an A.P if $T_n - T_{n-1} = d =$ constant.
- $S_n = \frac{n}{2}(a+l) = \frac{n}{2}(2a + (n-1)d)$.
- $T_n = S_n - S_{n-1}$.

Geometric progressions

- $T_n = ar^{n-1}$.
- A sequence is a G.P if $\frac{T_n}{T_{n-1}} = r =$ constant.
- $S_n = \frac{a(1-r^n)}{1-r}$.
- $|r| < 1 \implies S_\infty = \frac{a}{1-r}$.
- $|r| > 1 \implies$ divergent.

## 1.2 Summation

For $\sum\limits_{r=m}^{n} u_r$, the number of terms is $(n-m+1)$.

$$\sum_{r=1}^{n}(x_r \pm y_r) = \sum_{r=1}^{n} x_r \pm \sum_{r=1}^{n} y_r$$

$$\sum_{r=1}^{n} ku_r = k\sum_{r=1}^{n} u_r$$

$$\sum_{r=m}^{n} u_r = \sum_{r=1}^{n} u_r - \sum_{r=1}^{m-1} u_r$$

Useful sums:

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = (\sum_{r=1}^{n} r)^2 = \frac{1}{4}n^2(n+1)^2$$

## 1.3 Permutations and combinations

$$^nC_r = \frac{n!}{r!(n-r)!} \qquad ^nP_r =^n C_r \cdot r!$$

If $m$ objects are identical and the remaining are distinct (a total of $n$ objects), permutations $= \frac{n!}{m!}$

## 1.4 The Binomial Thoerem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

## 1.5 Mathematical induction

1. Let $P_n$ be the statement: *ello* for all $n \in \mathbb{Z}^+$.
2. For $n = 1$: LHS = *something*. RHS = *something* $\implies P_1$ is true.
3. Assume $P_k$ is true for some $k \in \mathbb{Z}^+$.
4. Showing that $P_{k+1}$ is true: *it is true!*
5. Since $P_1$ is true, and $P_k$ is true $\implies P_{k+1}$ is true, by Mathematical Induction, $P_n$ is true for all $n \in \mathbb{Z}^+$.

To do the inductive step:

- $\displaystyle\sum_{r=1}^{k+1} u_r = u_{k+1} + \sum_{r=1}^{k} u_r$
- $\displaystyle\frac{d^{k+1}y}{dx^{k+1}} = \frac{d}{dx}(\frac{d^k y}{dx^k})$
- For divisibility, let the expression = a multiple of $m$. You can always rearrange the inductive hypothesis.