2 Functions and equations

- A function is a to-one relationship.
- If the vertical line $x = a$ cuts the graph at one point only, then f is a function. If it cuts more than once, give an example.
- If a function passes the horizontal line test, it will have an inverse.
- The inverse is just a reflection of the graph in the line $y = x$.
- For inverse functions, $R_f = D_{f^{-1}}$ and $D_f = R_{f^{-1}}$.
- For gf to exist, $R_f \subseteq D_g$.
- $D_{gf} = D_f$.
- $R_{gf} = R_g | (D_g = R_f)$.
- $(g \circ f)^{-1}(x) = (f^{-1} \circ g^{-1})(x)$.
- $ff^{-1}(x)$ may not necessarily intersect with $f^{-1}f(x)$, it depends on the domain.
- For a periodic function, $f(x) = f(x + c)$.

2.1 Graphs

- To transform, TSST. (translate and stretch)x then (translate and stretch)y.
- For $y = |f(x)|$, retain $y \geq 0$, then reflect $y < 0$.
- For $y = f(|x|)$, retain $x \geq 0$, then reflect $x \geq 0$ to the left of the x-axis.
- For each transformation, you're allowed to replace x by something else.

2.2 Polynomials

- For a polynomial of degree n:
  - The sum of individual roots $= -\frac{a_{n-1}}{a_n}$
  - The sum of (choose 2) roots $= \frac{a_{n-2}}{a_n}$
  - The sum of (choose 3) roots $= -\frac{a_{n-3}}{a_n}$
  - The product of roots, i.e the sum of (choose n) roots $= (-1)^n \frac{a_o}{a_n}$
- For the special case of a quadratic: $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$
- A polynomial of degree n has a maximum of n roots, but some of these may be complex.

2.3 Circular functions and Trigonometry

- The ambiguous case of the sine rule occurs when the angle you are trying to find is opposite the longest side.
- $\sin(-\theta) = -\sin\theta \qquad \tan(-\theta) = -\tan\theta \qquad$ (odd functions).
- $\cos(-\theta) = \cos\theta \qquad$ (even function).
- For $\pi \pm \theta$ or $2\pi \pm \theta$: sin-sin, cos-cos, tan-tan.
- For $\frac{\pi}{2} \pm \theta$ or $\frac{3\pi}{2} \pm \theta$: sin-cos, cos-sin, tan-cot.
- $\tan x = \cot(\frac{\pi}{2} - x)$.
- $\sec x = \csc(\frac{\pi}{2} - x)$.
- The domain of $\arcsin x$ and $\arccos x$ are $[-1,1]$.
- $\cos(\arcsin x) = \sin(\arccos x) = \sqrt{1 - x^2}$.
- A circle with centre (h, k) and radius r is described by:

$$(x - h)^2 + (y - k)^2 = r^2$$

- To simplify an expression with trig, it may help to use the half angle formula.

$$\frac{\sin\theta}{1 + \cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1 + 2\cos^2\frac{\theta}{2} - 1} = \tan\frac{\theta}{2}$$