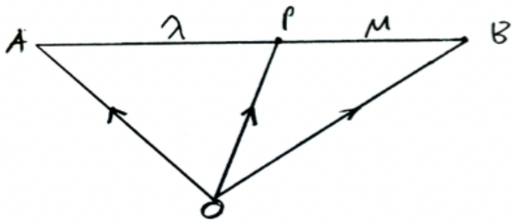
3 Vectors

- A vector $\overrightarrow{AB}$ can be represented by a straight line, with an arrow, joining A and B.
- A vector can also be denoted with a lower case letter, e.g **a**, which is written with a tilde below it.
- A position vector defines the position of a point relative to the origin. $\mathbf{a} = \overrightarrow{OA}$.
- The Cartesian form of a vector: $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, or $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$
- $|\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$
- A unit vector: $\hat{\mathbf{r}} = \frac{\mathbf{r}}{|\mathbf{r}|}$
- The Ratio Thoerem: $\overrightarrow{OP} = \frac{\mu\overrightarrow{OA} + \lambda\overrightarrow{OB}}{\mu + \lambda}$



3.1 Scalar products

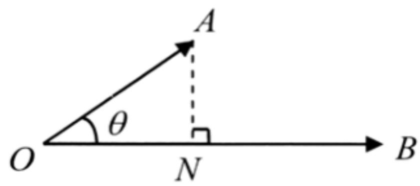
- The scalar product of two vectors is defined as $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$.
- The vectors must both converge or diverge from one point.
- $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$.
- Most algebra works, except for cancellation and division.

- $\mathbf{a} \perp \mathbf{b} \iff \mathbf{a} \cdot \mathbf{b} = 0$.
- $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$.

3.2 Vector products

- The vector product of two vectors is defined as $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta \hat{\mathbf{n}}$.
- $\hat{\mathbf{n}}$ is a unit vector perpendicular to $\mathbf{a}$ and $\mathbf{b}$.
- $(\mathbf{a} \times \mathbf{b}) = -(\mathbf{b} \times \mathbf{a})$.
- $(\lambda\mathbf{a}) \times (\mu\mathbf{b}) = (\lambda\mu)(\mathbf{a} \times \mathbf{b})$.
- $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$.
- $\mathbf{a} \parallel \mathbf{b} \iff \mathbf{a} \times \mathbf{b} = 0$, hence $\mathbf{a} \times \mathbf{a} = 0$.
- $\mathbf{a} \perp \mathbf{b} \iff \mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|$.
- $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$.
- $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - b_2a_3 \\ -(a_1b_3 - b_1a_3) \\ a_1b_2 - b_1a_2 \end{pmatrix}$. Cover top find det, cover mid find negative det, cover bot find det.
- Area $\Delta ABC = \frac{1}{2}|\overrightarrow{AB}||\overrightarrow{AC}| \sin\theta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$.

3.3 Projections and resolving vectors



- The length of the horizontal projection of $\mathbf{a}$ onto $\mathbf{b} = \overrightarrow{ON} = |\mathbf{a}||\hat{\mathbf{b}}| \cos\theta = \mathbf{a} \cdot \hat{\mathbf{b}}$
- The length of the vertical projection is given by $|AN| = |\mathbf{a} \times \hat{\mathbf{b}}|$
- The horizontal projection vector is then $\mathbf{u} = (\mathbf{a} \cdot \hat{\mathbf{b}})\hat{\mathbf{b}}$, which is the same as the resolved component of $\mathbf{a}$ parallel to $\mathbf{b}$.
- The perpendicular component of $\mathbf{a}$ is $\mathbf{v} = \mathbf{a} - \mathbf{u}$.

3.4 Straight lines

$$l: \quad \mathbf{r} = \mathbf{a} + \lambda\mathbf{d}, \qquad \lambda \in \mathbb{R}.$$

- The vector equation of a line uses a position vector $\mathbf{a}$ of a fixed point on l, and a direction vector $\mathbf{d}$ parallel to l, to find the position vector of any point on the line ($\mathbf{r}$).
- λ is a real parameter, which means that the vector equation of a line is not unique.

- If not parallel, it will intersect at a point, which can be found by substituting the line equation into the plane equation.
- The acute angle between l and Π: $\sin\theta = |\frac{\mathbf{d}\cdot\mathbf{n}}{|\mathbf{d}||\mathbf{n}|}|$
- When planes intersect, their Cartesian forms can be combined to form a system of simultaneous equations
  - If there is a unique solution, the planes intersect at a point.
  - If there are infinitely many solutions, the planes intersect in a line.
  - If there are no solutions, the three planes do not intersect.