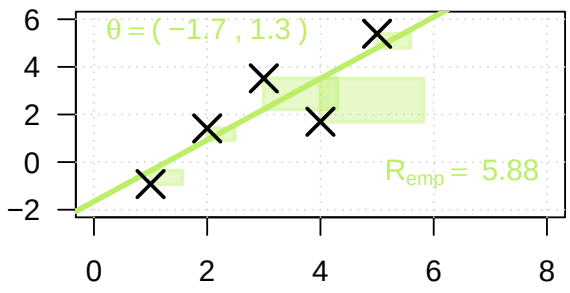
# Introduction to Machine Learning

# Introduction: Learners




## Learning goals

- Understand the components of a learner
- Understand the formalization of supervised learning
- Be able to apply the concept of a learner to a supervised learning task

# COMPONENTS OF A LEARNER

Summarizing what we have seen before, many supervised learning algorithms can be described in terms of three components:

**Learning = Hypothesis Space + Risk + Optimization**

- **Hypothesis Space:** Defines (and restricts!) what kind of model $f$ can be learned from the data.
- **Risk:** Quantifies how well a specific model performs on a given data set. This allows us to rank candidate models in order to choose the best one.
- **Optimization:** Defines how to search for the best model in the **hypothesis space**, i.e., the model with the smallest **risk**.

# COMPONENTS OF A LEARNER - ADVANCED

This concept can be extended by the concept of **penalization**, where the model complexity is accounted for in the risk:

**Learning = Hypothesis Space + Risk + Optim**
**Learning = Hypothesis Space + Loss (+ Penalization) + Optim**

- We will not treat penalization in this course, so for the moment you can just think of the risk as sum of the losses.
- While this a useful framework for most supervised ML problems, it does not cover all special cases, because some ML methods are not defined via risk minimization and for some models, it is not possible (or very hard) to explicitly define the hypothesis space.

# SUPERVISED LEARNING, FORMALIZED

A **learner** (or **inducer**) $\mathcal{I}$ is a *program* or *algorithm* which

- receives a **training set** $\mathcal{D} \in \mathcal{X} \times \mathcal{Y}$, and,
- for a given **hypothesis class** $\mathcal{H}$ of **models** $f : \mathcal{X} \to \mathbb{R}^g$,
- based on a **risk** function $\mathcal{R}_{\text{emp}}(f)$ that quantifies the performance of $f \in \mathcal{H}$ on $\mathcal{D}$,
- uses an **optimization** procedure to find

$$\hat{f} = \arg\min_{f \in \mathcal{H}} \mathcal{R}_{\text{emp}}(f).$$

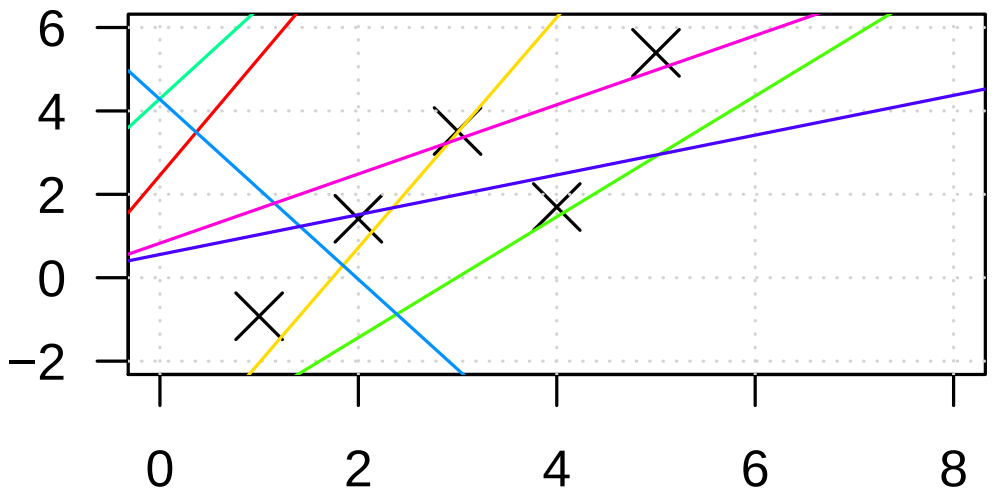
As before, we can also adapt this concept to finding $\hat{\boldsymbol{\theta}}$ for parametric models.

# EXAMPLE OF A LEARNER

Let us consider a linear regression task with a single feature and a single target variable.

- The **hypothesis space** in univariate linear regression is the set of all linear functions, with $\boldsymbol{\theta} = (\theta_0, \theta)^\top$:

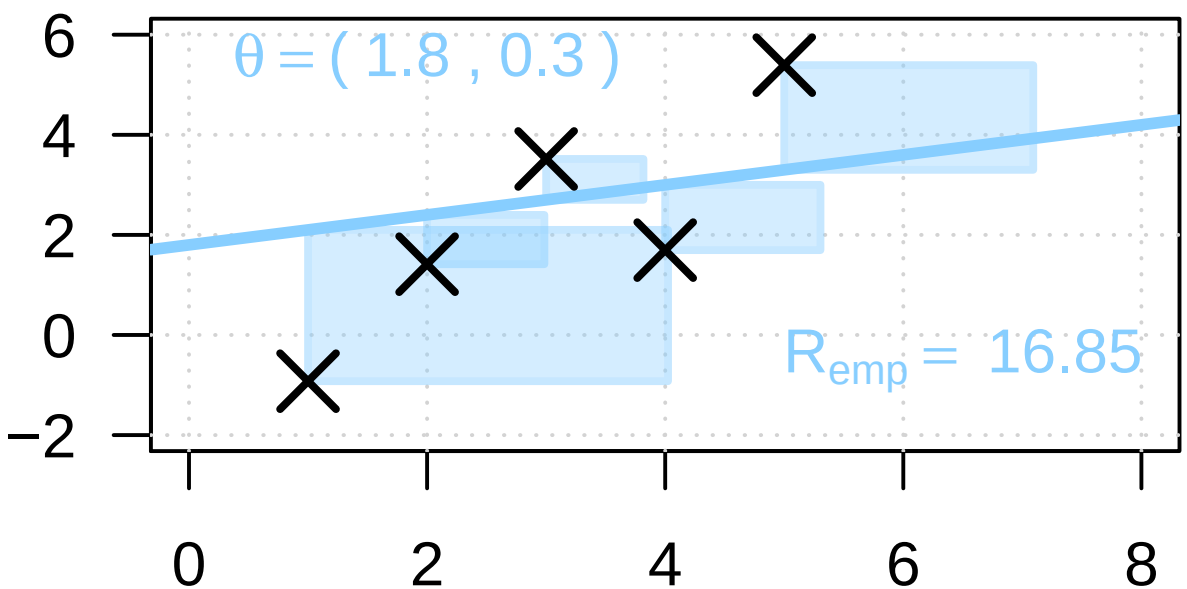
$$\mathcal{H} = \{f(\mathbf{x}) = \theta_0 + \theta\mathbf{x} : \theta_0, \theta \in \mathbb{R}\}$$



# EXAMPLE OF A LEARNER

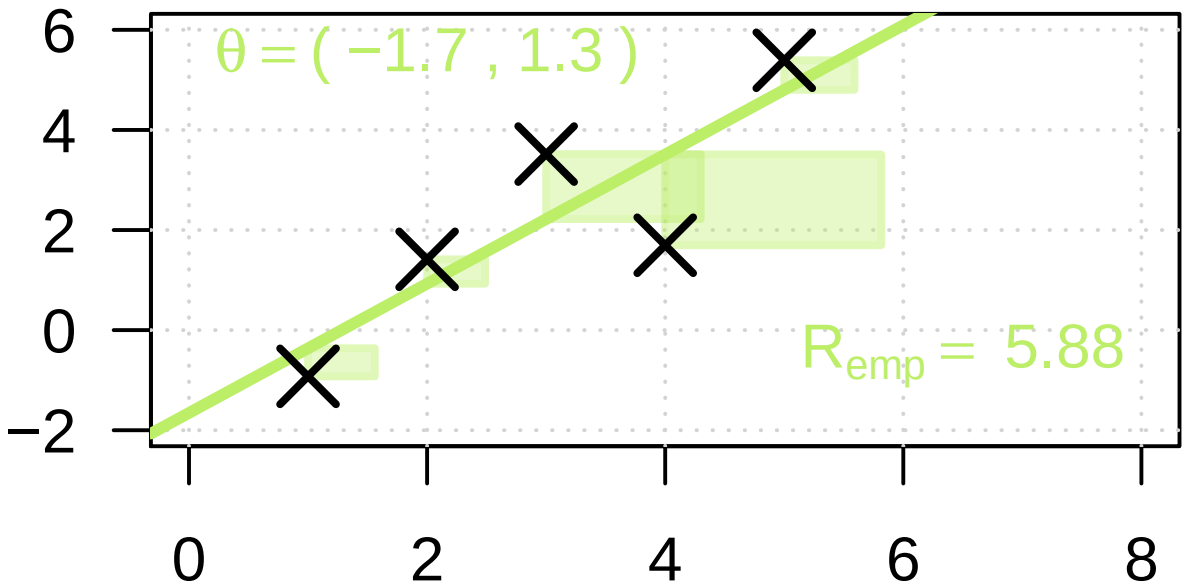
- We might use the mean squared error as loss function to our **risk**, punishing larger distances between observations and regression line more severely:

$$\mathcal{R}_{\text{emp}}(\boldsymbol{\theta}) = \sum_{i=1}^{n} (y^{(i)} - \theta_0 - \boldsymbol{\theta}\mathbf{x}^{(i)})^2$$



# EXAMPLE OF A LEARNER

- **Optimization** will usually mean deriving the ordinary-least-squares (OLS) estimator $\hat{\theta}$ analytically. We might, however, also use gradient descent or some other optimization procedure.



# VARIETY OF LEARNING COMPONENTS

**Hypothesis Space** :
- Step functions
- Linear functions
- Sets of rules
- Neural networks
- Voronoi tesselations
- ...

**Risk** / **Loss** :
- Mean squared error
- Misclassification rate
- Negative log-likelihood
- Information gain
- ...

**Optimization** :
- Analytical solution
- Gradient descent
- Combinatorial optimization
- Genetic algorithms
- ...

# LEARNING AS EMPIRICAL RISK MINIMIZATION

- By decomposing learners into these building blocks,
  - we have a framework to understand how they work,
  - we can more easily evaluate in which settings they may be more or less suitable, and
  - we can tailor learners to specific problems by clever choice of each of the three components.
- There will, for instance, be optimization procedures that work well for a certain combination of hypothesis space and risk function but perform poorly on others.
- In fact, it is a commonly acknowledged problem that no universally best learner exists.