Introduction to Machine Learning

Introduction: Learners

compstat-lmu.github.io/lecture_i2ml

COMPONENTS OF A LEARNER

Summarizing what we have seen before, nearly all supervised learning algorithms can be described in terms of three components:

Learning = Hypothesis Space + Risk + Optimization

- **Hypothesis Space:** Defines (and restricts!) what kind of model f can be learned from the data.
- **Risk:** Quantifies how well a specific model performs on a given data set. This defines how to compare observed values to predictions and allows us to rank candidate models in order to choose the best one.
- **Optimization:** Defines how to search for the best model in the **hypothesis space**, typically guided by the metric used for the **risk**.

SUPERVISED LEARNING, FORMALIZED

A **learner** (or **inducer**) $\mathcal{I}$ is a *program* or *algorithm* which

- receives a **training set** $\mathcal{D} \in \mathcal{X} \times \mathcal{Y}$, and,
- for a given **hypothesis class** $\mathcal{H}$ of **models** $f : \mathcal{X} \to \mathbb{R}^g$,
- based on a **risk** function $\mathcal{R}_{\text{emp}}(f)$ that quantifies the performance of $f \in \mathcal{H}$ on $\mathcal{D}$,
- uses an **optimization** procedure to find

$$\hat{f} = \underset{f \in \mathcal{H}}{\arg\min}\, \mathcal{R}_{\text{emp}}(f).$$

As before, we can also adapt this concept to finding $\hat{\boldsymbol{\theta}}$ for parametric models.

(This does not cover all special cases, but it's a useful framework for most supervised ML problems.)

LEARNING AS EMPIRICAL RISK MINIMIZATION

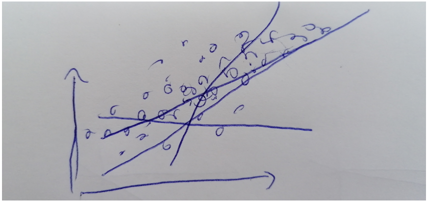
- By decomposing learners into these building blocks,
  - we have a framework to understand how they work,
  - we can more easily evaluate in which settings they may be more or less suitable, and
  - we can tailor learners to specific problems by clever choice of each of the three components.
- There will, for instance, be optimization procedures that work well for a certain combination of hypothesis space and risk function but perform poorly on others.
- In fact, it is a commonly acknowledged problem that no universally best learner exists.

EXAMPLE OF A LEARNER

So what could a learner look like? Let us consider a linear regression task with a single feature and a single target variable.

- The **hypothesis space** in univariate linear regression is the set of all linear functions, with $\boldsymbol{\theta} = (\theta_0, \theta)$:

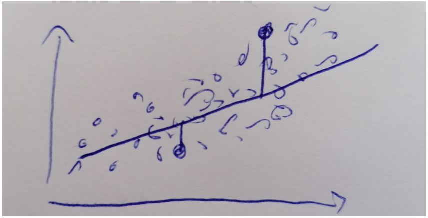
$$\mathcal{H} = \{f(\mathbf{x}) = \theta_0 + \theta\mathbf{x} : \theta_0, \theta \in \mathbb{R}\}$$



EXAMPLE OF A LEARNER

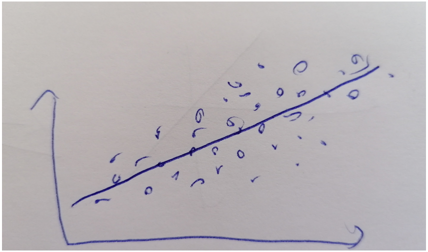
- We might use the mean squared error as loss function to our **risk**, punishing larger distances between observations and regression line more severely:

$$\mathcal{R}_{\text{emp}}(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n}(y^{(i)} - \theta_0 - \theta\mathbf{x}^{(i)})^2$$



EXAMPLE OF A LEARNER

- **Optimization** will usually mean deriving the ordinary-least-squares (OLS) estimator $\hat{\theta}$ analytically. We might, however, also use gradient descent or some other optimization procedure.



VARIETY OF LEARNING COMPONENTS

Hypothesis Space :
- Step functions
- Linear functions
- Sets of rules
- Neural networks
- Voronoi tesselations
- ...

Risk :
- Mean squared error
- Misclassification rate
- Negative log-likelihood
- Information gain
- ...

Optimization :
- Analytical solution
- Gradient descent
- Combinatorial optimization
- Genetic algorithms
- ...