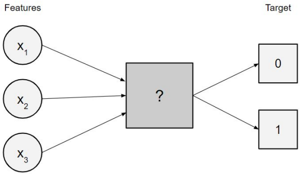
Introduction to Machine Learning

Introduction: Data

compstat-lmu.github.io/lecture_i2ml

DATA IN MACHINE LEARNING

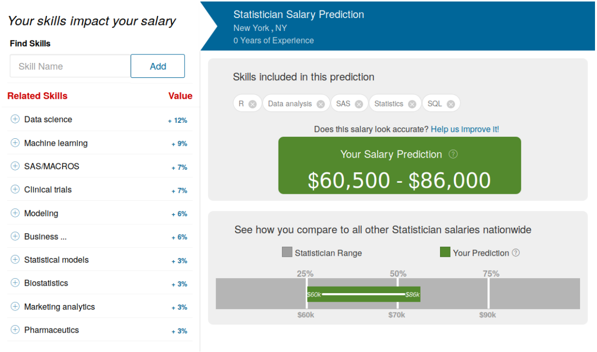
- The data we deal with in machine learning usually consists of observations on different aspects of objects:
  - **Target** variable(s): the attribute(s) of interest
  - **Features**: measurable properties that provide a concise description of the object
  - Both features and target variables may be of different data types (categorical, numeric, ...).
- We assume some kind of relationship between the features and the target, in a sense that the value of the target variable can be explained by a combination of the features.



Features x				Target y
Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
4.3	3.0	1.1	0.1	setosa
5.0	3.3	1.4	0.2	setosa
7.7	3.8	6.7	2.2	virginica
5.5	2.5	4.0	1.3	versicolor

DATA IN MACHINE LEARNING

- For instance, it is reasonable to assume a relationship between certain features of a job-seeker, such as their field of expertise, academic qualifications and previous job experiences, and their salary.



- In practical applications we frequently encounter high-dimensional data, i.e., data with many features and/or observations.

DATA LABELS

- We distinguish two basic forms our data may come in:
  - For **labeled** data we have already observed the target values *(labels)*.
  - For **unlabeled** data these remain unknown.
- It is easy to see how labeled data are vastly more informative.
- In practice, however, we will much more frequently encounter the unlabeled sort.
- Figure from slide 1 with some target values, some "?"

NOTATION FOR DATA

In formal notation, the data sets we are given are of the following form:

$$\mathcal{D} = \left\{ \left(\mathbf{x}^{(1)}, y^{(1)} \right), \ldots, \left(\mathbf{x}^{(n)}, y^{(n)} \right) \right\} \subset (\mathcal{X} \times \mathcal{Y})^n .$$

We call

- $\mathcal{X}$ the input space with $p = \dim(\mathcal{X})$ (for now: $\mathcal{X} \subset \mathbb{R}^p$),
- $\mathcal{Y}$ the output / target space,
- the tuple $\left(\mathbf{x}^{(i)}, y^{(i)}\right) \in \mathcal{X} \times \mathcal{Y}$ the *i*-th observation,
- $\mathbf{x}_j = \left(x_j^{(1)}, \ldots, x_j^{(n)} \right)^T$ the j-th feature vector.

DATA-GENERATING PROCESS

- We assume the observed data $\mathcal{D}$ to be generated by a process that can be characterized by some probability distribution

$$\mathbb{P}_{xy},$$

defined on $\mathcal{X} \times \mathcal{Y}$.

- Depending on the context, we denote the random variables following this distribution by $\mathbf{x}$ and y.
- It is important to understand that the true distribution is essentially **unknown** to us.

DATA-GENERATING PROCESS

- Usually we assume the data to be drawn *i.i.d.* from the joint probability density function (pdf) / probability mass function (pmf) $p(\mathbf{x}, y)$.
  - i.i.d. stands for **i**ndependent and **i**dentically **d**istributed.
  - We presuppose that all samples are drawn from the same distribution and are mutually independent – the i-th realization does not depend on the previous $i-1$ ones.
  - It is a strong yet crucial assumption that is precondition to many theoretical implications (e.g., the Central Limit Theorem).
- FIGURE

DATA-GENERATING PROCESS

Remarks:

- With a slight abuse of notation we write random variables, e.g., $\mathbf{x}$ and y, in lowercase, as normal variables or function arguments. The context will make clear what is meant.
- Often, distributions are characterized by a parameter vector $\boldsymbol{\theta} \in \Theta$. We then write $p(\mathbf{x}, y \mid \boldsymbol{\theta})$.
- This lecture mostly takes a frequentist perspective. Distribution parameters $\boldsymbol{\theta}$ appear behind the | for improved legibility, not to imply that we condition on them in a probabilistic Bayesian sense. So, strictly speaking, $p(\mathbf{x}|\boldsymbol{\theta})$ should usually be understood to mean $p_{\boldsymbol{\theta}}(\mathbf{x})$ or $p(\mathbf{x}, \boldsymbol{\theta})$ or $p(\mathbf{x}; \boldsymbol{\theta})$.