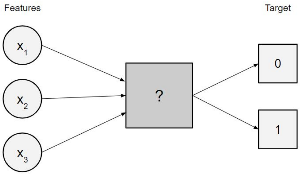
Introduction to Machine Learning

Introduction: Data

compstat-lmu.github.io/lecture_i2ml

DATA IN MACHINE LEARNING

- The data we deal with in machine learning usually consists of observations on different aspects of objects:
  - **Target** variable(s): the attribute(s) of interest
  - **Features**: measurable properties that provide a concise description of the object
  - Both features and target variables may be of different data types (categorical, numeric, ...).
- We assume some kind of relationship between the features and the target, in a sense that the value of the target variable can be explained by a combination of the features.



Features x				Target y
Sepal.Length	**Sepal.Width**	**Petal.Length**	**Petal.Width**	**Species**
4.3	3.0	1.1	0.1	setosa
5.0	3.3	1.4	0.2	setosa
7.7	3.8	6.7	2.2	virginica
5.5	2.5	4.0	1.3	versicolor

DATA LABELS

- We distinguish two basic forms our data may come in:
  - For **labeled** data we have already observed the target values *(labels)*.
  - For **unlabeled** data these remain unknown.
- It is easy to see how labeled data are vastly more informative.
- In practice, however, we will much more frequently encounter the unlabeled sort.

	Features x				Target y
	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
labeled data	4.3	3.0	1.1	0.1	setosa
	5.0	3.3	1.4	0.2	setosa
	7.7	3.8	6.7	2.2	virginica
	5.5	2.5	4.0	1.3	versicolor
unlabeled data	5.9	3.0	5.1	1.8	?
	4.4	3.2	1.3	0.2	?

NOTATION FOR DATA

In formal notation, the data sets we are given are of the following form:

$$\mathcal{D} = \left\{ \left(\mathbf{x}^{(1)}, y^{(1)}\right), \ldots, \left(\mathbf{x}^{(n)}, y^{(n)}\right) \right\} \subset (\mathcal{X} \times \mathcal{Y})^n .$$

We call

- $\mathcal{X}$ the input space with $p = \dim(\mathcal{X})$ (for now: $\mathcal{X} \subset \mathbb{R}^p$),
- $\mathcal{Y}$ the output / target space,
- the tuple $\left(\mathbf{x}^{(i)}, y^{(i)}\right) \in \mathcal{X} \times \mathcal{Y}$ the *i*-th observation,
- $\mathbf{x}_j = \left(x_j^{(1)}, \ldots, x_j^{(n)}\right)^T$ the j-th feature vector.

DATA-GENERATING PROCESS

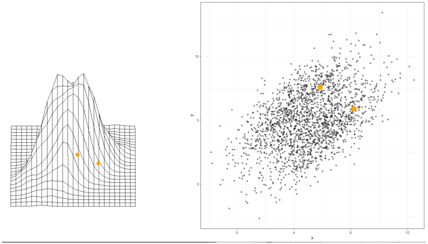
- We assume the observed data $\mathcal{D}$ to be generated by a process that can be characterized by some probability distribution

$$\mathbb{P}_{xy},$$

  defined on $\mathcal{X} \times \mathcal{Y}$.
- Depending on the context, we denote the random variables following this distribution by $\mathbf{x}$ and y.
- It is important to understand that the true distribution is essentially **unknown** to us.

DATA-GENERATING PROCESS

- Usually we assume the data to be drawn *i.i.d.* from the joint probability density function (pdf) / probability mass function (pmf) $p(\mathbf{x}, y)$.
  - i.i.d. stands for **i**ndependent and **i**dentically **d**istributed.
  - We presuppose that all samples are drawn from the same distribution and are mutually independent – the i-th realization does not depend on the previous $i-1$ ones.
  - It is a strong yet crucial assumption that is precondition to many theoretical implications (e.g., the Central Limit Theorem).



DATA-GENERATING PROCESS

Remarks:

- With a slight abuse of notation we write random variables, e.g., $\mathbf{x}$ and y, in lowercase, as normal variables or function arguments. The context will make clear what is meant.
- Often, distributions are characterized by a parameter vector $\boldsymbol{\theta} \in \Theta$. We then write $p(\mathbf{x}, y \mid \boldsymbol{\theta})$.
- This lecture mostly takes a frequentist perspective. Distribution parameters $\boldsymbol{\theta}$ appear behind the | for improved legibility, not to imply that we condition on them in a probabilistic Bayesian sense. So, strictly speaking, $p(\mathbf{x}|\boldsymbol{\theta})$ should usually be understood to mean $p_{\boldsymbol{\theta}}(\mathbf{x})$ or $p(\mathbf{x}, \boldsymbol{\theta})$ or $p(\mathbf{x}; \boldsymbol{\theta})$.