📌 Two-Stage Optimization Algorithm (Team Allocation with Top-3 Guarantee)

🎯 Objective

To allocate students to projects such that as many students as possible receive one of their **top-3 preferences**, while also maximizing the overall satisfaction score.

📝 Algorithm Design

Stage 1: Maximize Top-3 Coverage

- **Decision variables**
  - $x_{s,p} \in \{0,1\}$: whether student s is assigned to project p.
  - $y_s \in \{0,1\}$: whether student s is assigned to one of their top-3 choices.
- **Constraints**
  i. Each student is assigned to exactly one project:

$$\sum_p x_{s,p} = 1 \quad \forall s$$

  ii. Each project must satisfy capacity bounds:

$$L_p \leq \sum_s x_{s,p} \leq U_p \quad \forall p$$

  iii. Linking variable y_s to top-3 preferences:

$$y_s \leq \sum_{p \in \mathrm{Top3}(s)} x_{s,p} \quad \forall s$$

- **Objective function**
  Maximize the number of students assigned within their top-3 choices:

$$\max \sum_s y_s$$

Stage 2: Maximize Satisfaction under Top-3 Guarantee

- **Carry-over constraint from Stage 1**
  Ensure at least max_top3 students are in top-3 choices:

$$\sum_s y_s \geq \text{max\_top3}$$

- **Satisfaction scores**
  - 1st choice: 5 points
  - 2nd choice: 4 points
  - 3rd choice: 3 points
  - 4th choice: 2 points
  - 5th choice: 1 point
- **Objective function**
  Maximize the total satisfaction score:

$$\max \sum_{s,p} \text{score}(s,p) \cdot x_{s,p}$$

🌟 Advantages

1. **Fairness**: maximizes the number of students receiving top-3 choices.
2. **Satisfaction**: further improves overall allocation quality after fairness.
3. **Flexibility**: project capacity bounds can be tuned.
4. **Transparency**: results include *Preference Rank* and *Top3 (Yes/No)* labels.

📊 Output

The final results table contains:

- Student ID
- First Name, Last Name
- Email
- Assigned Project
- Preference Rank (which choice)
- Top3 (Yes/No)

- Additional fields: Educational Background, Major, Minor, etc.