# On the thickness dependence of open circuit voltages of p-n junction solar cells

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A reduction of the thickness of solar cells with low surface recombination is known to result in enhanced open circuit voltages, provided that the short circuit current can be maintained sufficiently high by light trapping schemes. Lower volume recombination is generally assumed to cause this effect. We offer another interpretation: The voltage increases because thinning the cell at constant short circuit current enhances the minority carrier generation rate per unit volume and hence the steady state carrier concentration. Thermodynamically, an increased carrier concentration is equivalent to a reduction of the entropy production per photon, thus leading to larger voltage.

#### 1. Introduction

Extensive efforts are currently under way to increase the open circuit voltage  $V_{\rm oc}$  by thinning solar cells while maintaining high short circuit current density  $j_{\rm sc}$  [1–4]. This task is significant for attaining low-cost, high-efficiency thin-film solar cells, especially of silicon. The open circuit voltage is a much more critical indicator of cell efficiency than is the short-circuit current [5]; therefore lucid explanations of all influences on  $V_{\rm oc}$  are desirable. In the literature the increase in the open circuit voltage of an *illuminated* cell is explained with a reduction of the volume recombination current density  $j_0$  of the *non-illuminated* cell via the relation  $V_{\rm oc} \propto \ln(j_{\rm sc}/j_0)$  [6,7]. We feel that this argument conceals the fundamental physics of the effect. We suggest a thermodynamic interpretation, which does not refer to the cell in the dark to explain its voltage under illumination.

## 2. Conventional interpretation of open circuit voltage enhancement

The open circuit voltage

$$V_{\rm oc} = \frac{kT}{q} \ln \left( \frac{j_{\rm sc}}{j_0} + 1 \right) \tag{1}$$

of a p-n solar cell depends on the dark saturation current density  $j_0$  and short circuit current density  $j_{\rm sc}$  [6,8]. Here k is Boltzmann's constant, T is the cell temperature, and q is the elementary charge. In general  $j_{\rm sc}$  and  $j_0$  are both functions of the cell thickness. However, due to highly efficient light trapping schemes [9] the thickness dependence of  $j_{\rm sc}$  is much weaker than that of  $j_0$ . Therefore,  $j_{\rm sc}$  will be considered to be independent of thickness in this paper. For simplicity, we restrict the discussion to a cell with a p-type base and neglect current contributions from the n-type emitter. Within the usual one-dimensional model [8,10], the p-type base is characterized by a minority carrier diffusion length L, carrier lifetime  $\tau$  (that is diffusion constant  $D = L^2/\tau$ ), and the equilibrium minority carrier concentration  $n_0$ . The saturation current density

$$j_0 = qn_0 \frac{L}{\tau} \frac{(LS/D) \cosh(W/L) + \sinh(W/L)}{(LS/D) \sinh(W/L) + \cosh(W/L)}$$
(2)

of the solar cell in the dark depends on the recombination velocity S at the back surface and the thickness W of the base [6,8]. According to eqs. (1) and (2), the voltage  $V_{\rm oc}$  will only increase for reduced thickness W if the surface recombination velocity S is sufficiently low (S < D/L) and if the base thickness W is smaller than the diffusion length L [11]. Therefore, we discuss the limit of S = 0 and  $W \ll L$ . Then eq. (2) reads as

$$j_0 = q n_0 W / \tau, \tag{3}$$

which demonstrates, that a reduction of thickness W lowers  $j_0$  and hence increases  $V_{\rm oc}$ . Such a lowering of  $j_0$  represents a reduction of the thermal recombination current density in the volume of the *non-illuminated* cell. It has often been stated, that this reduction of volume recombination is responsible for an increase in open circuit voltage under illumination [4,6,7]. Such a statement is misleading. In fact the volume recombination current density of the *illuminated* cell is *not* reduced by thinning the cell: Under open circuit conditions and for zero surface recombination, the only remaining sink for electron hole pairs is volume recombination. Therefore the recombination current density equals the total generation current density which is  $j_{\rm sc}$  (neglecting the small thermal recombination). Hence the volume recombination current density does not depend on the film thickness provided  $j_{\rm sc}$  is maintained constant.

In the present paper we investigate the derivative  $dV_{oc}/dW$ . Since  $j_{sc} \gg j_0$ , and for  $j_{sc}$  independent of thickness, we find from eqs. (1) and (3)

$$\frac{\mathrm{d}V_{\mathrm{oc}}}{\mathrm{d}W} = -\frac{kT}{q}\frac{1}{W},\tag{4}$$

which demonstrates, that the change in voltage due to an infinitesimal increase in thickness is always negative and independent of the minority carrier life time. Eq. (4) describes the voltage enhancement under discussion and will be derived again from a purely thermodynamic consideration in the next section.

## 3. Thermodynamic interpretation of the voltage enhancement

A solar cell can be considered as a thermodynamic machine connected to two heat reservoirs: It receives radiation energy at the temperature of the sun and gives off energy close to room temperature by heat conduction and luminescent radiation. The second law of thermodynamics allows only a fraction of the high temperature input energy to be converted into electrical work; the less entropy is produced, the more work can be done. A reduction of thickness confines the light-generated minority carriers to a smaller volume. Thus less entropy is produced per absorbed photon. Since less entropy is produced per absorbed photon, more work can be done per photon created charge: the voltage  $V_{\rm oc}$  increases.

To calculate the entropy we describe the electrons in the conduction band of the base of thickness W and area A as a non-degenerated ideal gas of N particles at temperature T. The entropy per particle

$$s = k \left( \frac{5}{2} - \ln \left( \frac{n}{N_{c}(T)} \right) \right) \tag{5}$$

of an ideal gas depends logarithmically on the electron density n = N/(AW) [12]. Here  $N_c(T) = 2(2\pi mkT/h^2)^{3/2}$  denotes the effective density of states; m is an effective electron mass, and h is Planck's constant.

The entropy per electron s is the only thickness dependent contribution to the entropy production per incoming photon. This entropy s is reduced by (ds/dW) dW if the cell is thinned by dW. Therefore, the thermodynamic machine "solar cell" can perform an amount of -T(ds/dW) dW extra work. This extra amount of work per photon generated charge may also be expressed as a change in voltage:  $q(dV_{oc}/dW) dW$ . From eq. (5) we find that the contribution of the gas of electrons to the change in voltage

$$q\frac{\mathrm{d}V_{\mathrm{oc}}}{\mathrm{d}W} = -T\frac{\mathrm{d}s}{\mathrm{d}W} = kTn^{-1}\frac{\mathrm{d}n}{\mathrm{d}W} \tag{6}$$

is inversely proportional to the electron concentration n. Similarly, the contribution of the gas of holes is inversely proportional to the concentration of holes, and therefore it can be neglected in a p-type base at low injection.

Next we determine the steady state electron concentration n(W) as a function of thickness W under the assumptions made in section 2: no surface recombination,  $j_{sc} \gg j_0$ ,  $W \ll L$ ,  $j_{sc}$  independent of thickness, and open circuit condition. Because of  $W \ll L$ , the charge collection efficiency is essentially unity. Hence, the average light induced generation rate  $\bar{g}$ , measured as electron-hole pairs generated per unit volume, is equal to  $j_{sc}/(qW)$  and increases inversely proportional to the thickness W. For  $W \ll L$  the electron concentration n is constant throughout the volume of the base and is given as

$$n(W) = \bar{g}(W)\tau = j_{sc}\tau/(qW). \tag{7}$$

Inserting eq. (7) into eq. (6), we arrive again at eq. (4) which expresses the voltage enhancement under discussion. The basic physical effect that is responsible for the

voltage increase is therefore identified: The increase of the steady state minority carrier concentration n, which then leads to a decrease of the entropy production of the electron gas  $^{\#1}$ .

The assumption of constant life time independent of thickness breaks down for very thin cells due to Auger recombination [5]. However, our thermodynamic interpretation remains valid for high injection. The open circuit voltage

$$V_{\rm oc} = \frac{kT}{q} \ln \left( \frac{np}{n_i^2} \right) \tag{8}$$

is a function of electron concentration n, hole concentration p, and intrinsic carrier concentration  $n_i$  for low and high injection [5,10]. The derivative

$$\frac{\mathrm{d}V_{\mathrm{oc}}}{\mathrm{d}W} = \frac{kT}{a} \left( n^{-1} \frac{\mathrm{d}n}{\mathrm{d}W} + p^{-1} \frac{\mathrm{d}p}{\mathrm{d}W} \right) \tag{9}$$

of eq. (8) expresses the change of open circuit voltage with thickness, and can be understood to originate from the change in entropy production per photon by applying eq. (6) and a corresponding equation for holes. Hence our interpretation of the voltage enhancement is not limited to low injection. Our discussion is, however, limited to non-degeneracy.

#### 4. Conclusion

We have demonstrated that the thickness dependence of the open circuit voltage of solar cells can be interpreted as a thermodynamic effect. Reduction of the thickness at constant short circuit current increases the minority carrier generation rate per unit volume and hence the carrier concentration; a larger carrier concentration reduces the entropy production per photon, and consequently enhances the open circuit voltage. We have thus explained the increased voltage without referring to a reduction of volume recombination. In fact the volume recombination current density of the illuminated cell under open circuit condition is *not* reduced by thinning the cell for zero surface recombination and thickness independent short circuit current.

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<sup>#1</sup> The increase in voltage may also be considered as a consequence of a change in the chemical potential  $\mu$ . From  $E = \frac{3}{2}NkT$  and  $\mu = -T \frac{\partial S(N, E)}{\partial N}$  the chemical potential can be calculated:  $\mu = kT \ln(n/N_c(T))$ . Since the *electro*-chemical potential is fixed to its value in the emitter, an increase of the electron concentration n has to be compensated by a change of the potential energy, that is a change in voltage.

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