

# **Electromagnetic Wave Propagation**

*Theory and Application to Bathymetric Lidar Simulation*

University of Colorado at Boulder  
ASEN 6849 Fall 2008

December 19, 2008

Steve Mitchell

## Table of Contents

<b>Introduction.....</b>	<b>3</b>
<b>Maxwell's Equations.....</b>	<b>3</b>
<b>Material Classification.....</b>	<b>4</b>
<b>Polarization.....</b>	<b>5</b>
<b>Boundary Between Media.....</b>	<b>6</b>
<b>Brewster Angle.....</b>	<b>12</b>
<b>Imaginary Component of Refractive Index.....</b>	<b>15</b>
<b>Penetration Depth.....</b>	<b>17</b>
<b>Graybody Approximation.....</b>	<b>18</b>
<b>Bathymetric Lidar Simulation.....</b>	<b>19</b>
<b>Conclusion.....</b>	<b>25</b>
<b>References.....</b>	<b>26</b>
<b>Appendix.....</b>	<b>27</b>

## Introduction

Calculation of distance measurements in laser altimetry requires knowledge of the propagation of electromagnetic radiation during ranging. Extinction caused by scattering and absorption during propagation, as well as reflection and transmission effects at target interfaces, must be considered when analyzing altimetry data. It is in this context of altimetry design and analysis that the propagation of electromagnetic radiation is discussed in the following pages. This discussion includes an application of electromagnetic wave propagation to the design and simulation of a bathymetric laser altimeter.

## Maxwell's Equations [1,2,3,4]

The propagation of electromagnetic (EM) waves is governed by Maxwell's equations, which encapsulate the connection between the electric field and electric charge, the magnetic field and electric current, and the bilateral coupling between the electric and magnetic field quantities. Maxwell's equations hold in any material, including free space, and at any spatial location in an arbitrary (x,y,z) coordinate system. The general form of Maxwell's equations are written as

$$\nabla \cdot D = \rho_v \quad (1)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (2)$$

$$\nabla \cdot B = 0 \quad (3)$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad (4)$$

where

$D$  = electric flux density

$E$  = electric field intensity

and  $D = \epsilon E$ , where  $\epsilon$  = electrical permittivity of the medium

$B$  = magnetic flux density

$H$  = magnetic field intensity

and  $B = \mu H$ , where  $\mu$  = magnetic permeability of the medium

$J$  = current density per unit area

$\rho_v$  = electric charge density per unit volume

From Maxwell's equations (1-4), the interactions between the electric and magnetic components of propagating waves, inseparably coupled and mutually sustaining, can be explained. These interactions, as will be shown in later sections, are of particular importance in laser ranging at target interfaces.

### **Material Classification [1]**

The electromagnetic parameters of a material effect the propagation of EM waves and include

$\epsilon$ , the material permittivity

$\mu$ , the material magnetic permeability

$\sigma$ , the material conductivity

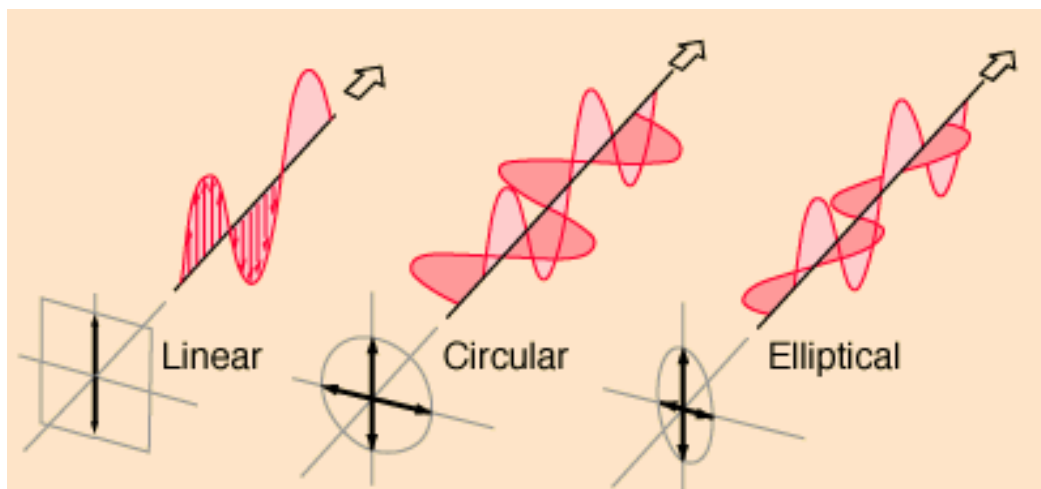
Materials are classified as either conductors (e.g. metals) or dielectrics (insulators) according to the magnitudes of their conductivities. A perfect dielectric is defined as a material with  $\sigma = 0$ , whereas a perfect conductor is defined as a material with  $\sigma = \infty$ . In a dielectric, the electrons are held tightly to their respective atoms. As a result, no current is capable of flowing through the material. An externally applied electric field cannot affect mass migration of charges since the electrons are held tightly to their respective atoms. However, the external field

can polarize the atoms in the material by distorting the center of the cloud and location of the nucleus. This induced electric field is referred to as the polarization field.

In a conductor, the material possesses a large number of loosely attached electrons in the outermost shells of the atoms. In the absence of an applied external electric field, these free electrons move in random directions with varying speeds, resulting in zero average current through the material. An externally applied electric field causes electrons to migrate from one atom to the next, resulting in a conduction current through the material.

### **Polarization [1,2,5,6]**

The polarization of a uniform plane wave describes the shape and locus of the tip of the electric field vector  $E$  at a given point in space as a function of time. A wave is linearly polarized if  $E_x(z,t)$  and  $E_y(z,t)$  are in phase ( $\delta=0$ ) or out of phase ( $\delta=\pi$ ). For circular polarization, the magnitudes of the x- and y-components of  $E(z)$  are equal and the phase difference is  $\delta = \pm\pi/2$ . From the point of view of the target looking at the transmitter, left-hand circular polarization has a phase difference of  $\delta=\pi/2$  and right-hand circular polarization has a phase difference of  $\delta=-\pi/2$ .



**Figure 1: Variations of EM wave polarization [6]**

As will be shown in the following sections, the polarization state of a propagating EM wave incident on a target in laser ranging must be fully understood to clearly analyze the reflected wave and resulting distance measurement.

### Boundary Between Media [1,2,7]

A key phenomena in laser ranging is the interaction of EM waves at target interfaces. For example, what happens to a propagating optical wave as it transitions from the atmosphere to a water target? To answer this question, look at the boundary between two lossless, homogeneous, dielectric media. Homogenous suggests a medium that is smooth and uniform on scales comparable to the wavelength of radiation. In reality, homogenous media are nonexistent in laser ranging; aerosols are present in the atmosphere, contaminants are present in water, etc. However, the use of homogenous media in the following descriptions provides the reader with an understanding of EM wave propagation at target interfaces.

Figure 2 illustrates the fundamental setup for an EM wave propagating across the interface between two media. The refractive indices of medium 1 and 2 are defined as  $n_1 = \sqrt{\mu_1 \epsilon_1}$  and  $n_2 = \sqrt{\mu_2 \epsilon_2}$ . Most often, laser altimetry deals with dielectrics for which  $\mu_1 \approx \mu_2 \approx \mu_0$ . Therefore, the primary material concern is the difference in electric permittivity  $\epsilon$  between the two media.

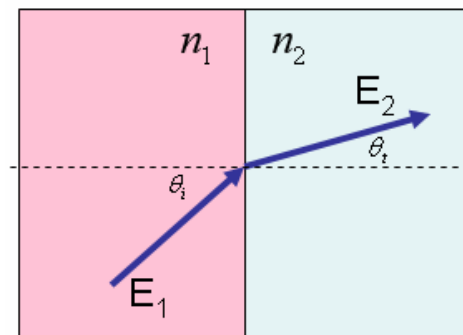
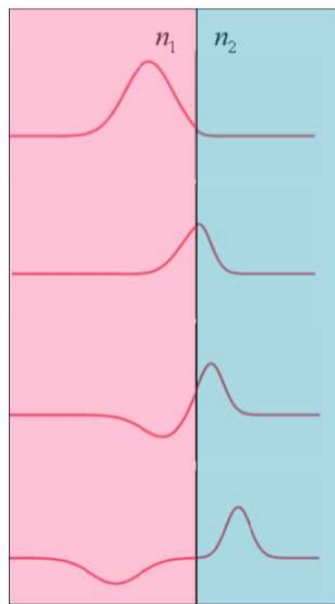


Figure 2: Boundary between two media with propagating EM wave [7]

As shown in Figure 3, when an electric field passes through the boundary between two different media, part of the wave is reflected, while the remainder of the wave is transmitted into the second medium. The figure illustrates the progression of an EM wave through two arbitrary, but different, media.



**Figure 3: Reflection and transmission of an EM wave at target interface [7]**

The reflection and transmission of an EM wave through two different media is described by the Fresnel coefficients. For normal incidence, the reflection coefficient  $r$  and transmission coefficient  $\tau$  of a boundary between two different media is independent of the polarization of the incident wave. This independence is due to the fact that the electric and magnetic fields of a normally incident plane wave are both always tangential to the boundary regardless of the wave polarization.

This is not the case for an EM wave incidence at a nonzero angle. As previously suggested, a wave with a specific polarization may be described as the superposition of two orthogonally polarized waves, one with its electric field parallel to the plane of incidence (“p-polarization”, “parallel polarization” or “transverse magnetic (TM)”) and another with its electric

field perpendicular to the plane of incidence (“s-polarization”, “perpendicular polarization” or “transverse electric (TE)”). The plane of incidence is subsequently defined as the plane containing the normal to the boundary and the direction of propagation of the incident wave. From this definition it is evident that p-polarized light is parallel to the boundary normal and s-polarized light is perpendicular to the normal.

For perpendicular polarization and the setup described in Figures 2 and 3, the reflection and transmission amplitude coefficients reduce to

$$r_{\perp} = \frac{E_{\perp 0}^r}{E_{\perp 0}^i} = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad (5)$$

$$\tau_{\perp} = \frac{E_{\perp 0}^{\tau}}{E_{\perp 0}^i} = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad (6)$$

where

$$r_{\perp} = 1 - \tau_{\perp} \quad (7)$$

and  $\theta_t$  is determined using Snell’s Law

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{n_1}{n_2} \quad (8)$$

Alternatively, for parallel polarization the reflection and transmission amplitude coefficients reduce to

$$r_{\parallel} = \frac{E_{\parallel 0}^r}{E_{\parallel 0}^i} = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad (9)$$

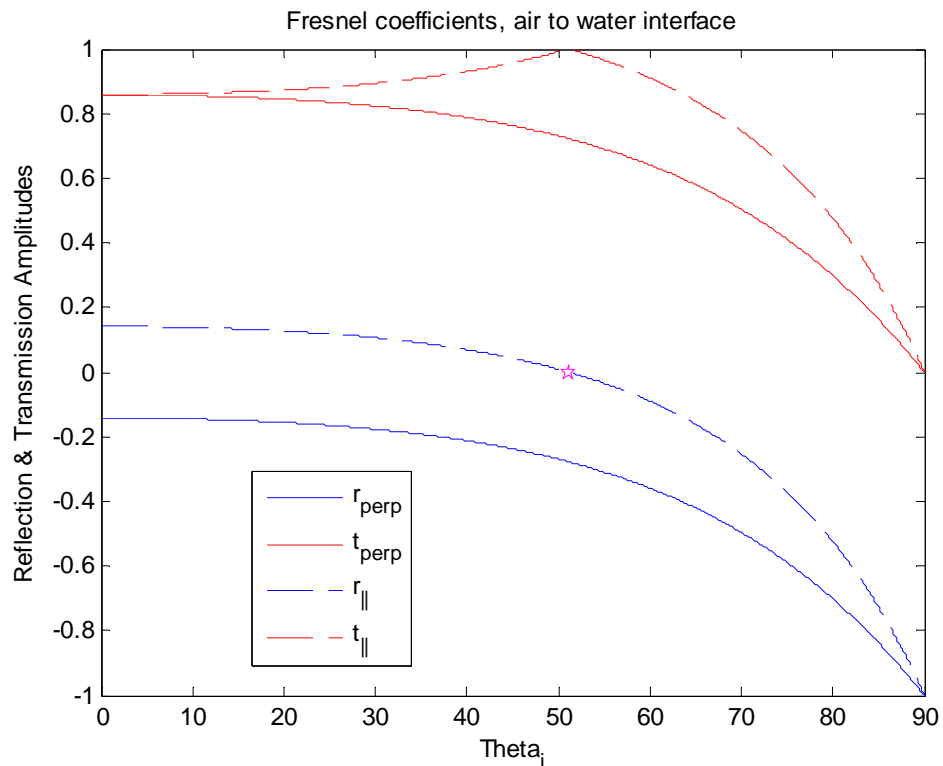
$$\tau_{\parallel} = \frac{E_{\parallel 0}^{\tau}}{E_{\parallel 0}^i} = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad (10)$$

where

$$r_{\parallel} = 1 - \tau_{\parallel} \quad (11)$$



Figure 4 illustrates the Fresnel amplitude coefficients for an air/water interface, common to bathymetric laser altimeters. The reflection and transmission amplitude coefficients are shown for both perpendicular and parallel polarizations of EM waves incident from normal to 90 degrees. The MATLAB code used to create the figure is also provided for the reader.



**Figure 4: Fresnel amplitude coefficients for an air/water interface**

```
% Refractive indices
n1 = 1.00; n2 = 1.33;

theta_i = 0:pi/10000:pi/2; theta_t = (n1/n2)*sin(theta_i);

% AMPLITUDE COEFFICIENTS
% Perpendicular polarization
for i = 1:length(theta_i)
    r1(i) = [n1*cos(theta_i(i))-n2*cos(theta_t(i))]/[n1*cos(theta_i(i))+n2*cos(theta_t(i))];
end
t1 = 1 - abs(r1);
theta_i = rad2deg(theta_i);

figure(1); hold on
plot(theta_i,r1,'b',theta_i,t1,'r')

theta_i = deg2rad(theta_i);
```

```

% Parallel polarization
for i = 1:length(theta_i)
r1(i) = [n2*cos(theta_i(i))-n1*cos(theta_t(i))]/[n1*cos(theta_t(i))+n2*cos(theta_i(i))];
end
t1 = 1 - abs(r1);
theta_i = rad2deg(theta_i);

plot(theta_i,r1,'b--',theta_i,t1,'r--',51.21,0,'mp')
title('Fresnel coefficients, air to water interface')
xlabel('Theta_i'); ylabel('Reflection & Transmission Amplitudes')
axis([0 90 -1 1]); legend('r_p_e_r_p','t_p_e_r_p','r_perp','t_perp')

```

With an understanding of the Fresnel equations (5-7) and (9-11), it is important to note that  $r$  and  $\tau$  refer to the amplitude of the reflected and transmitted EM waves. Since optical detectors measure power, or the square of the electric field, the reflectance and transmittance coefficients are defined as

$$R_{\perp} = r_{\perp}^2 \quad (12)$$

$$R_{\parallel} = r_{\parallel}^2 \quad (13)$$

and

$$T_{\perp} = \left( \frac{n_2 \cos \theta_t}{n_1 \cos \theta_i} \right) \tau_{\perp}^2 \quad (14)$$

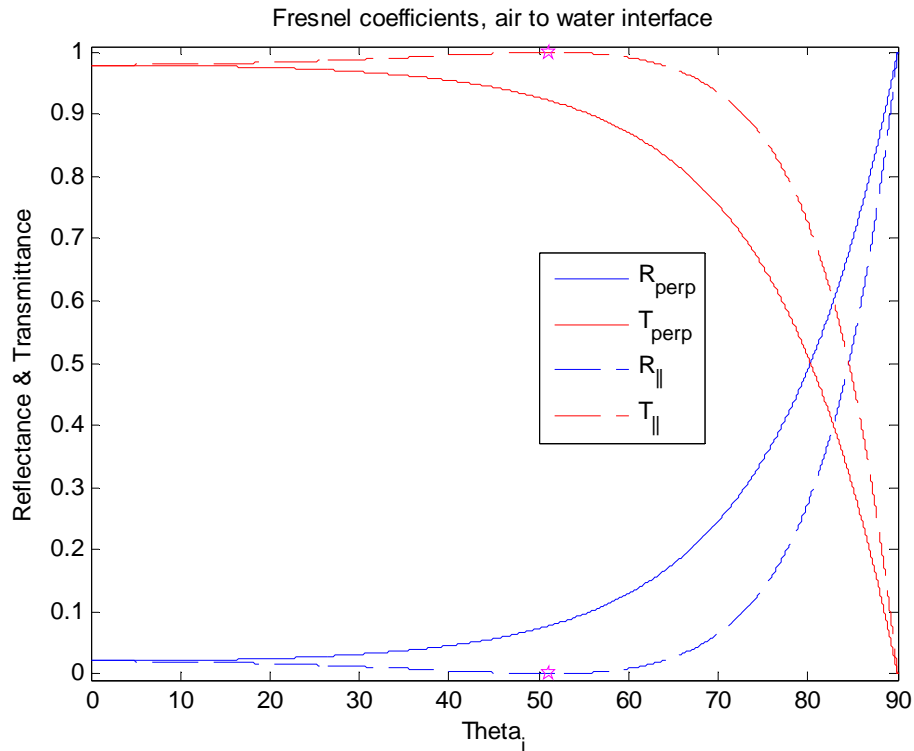
$$T_{\parallel} = \left( \frac{n_2 \cos \theta_t}{n_1 \cos \theta_i} \right) \tau_{\parallel}^2 \quad (15)$$

Furthermore, as in equations (7) and (11), it can be shown that

$$R_{\perp} = 1 - T_{\perp} \quad (16)$$

$$R_{\parallel} = 1 - T_{\parallel} \quad (17)$$

Figure 5 illustrates a plot of the reflectance and transmittance for an air/water interface identical to that of Figure 4. The MATLAB code used to generate the plot is also provided for the reader.



**Figure 5: Fresnel power coefficients for air/water interface**

```
% POWER COEFFICIENTS
% Refractive indices
n1 = 1.00; n2 = 1.33;

theta_i = 0:pi/10000:pi/2; theta_t = (n1/n2)*sin(theta_i);

% Perpendicular polarization
for i = 1:length(theta_i)
    r1(i) = [n1*cos(theta_i(i))-n2*cos(theta_t(i))]/[n1*cos(theta_i(i))+n2*cos(theta_t(i))];
end
r1 = abs(r1);
t1 = 1 - r1;
R1 = r1.^2;
T1 = 1 - R1;

theta_i = rad2deg(theta_i);

figure(2); hold on
plot(theta_i,R1,'b',theta_i,T1,'r')

theta_i = deg2rad(theta_i);

% Parallel polarization
for i = 1:length(theta_i)
    r1(i) = [n2*cos(theta_i(i))-n1*cos(theta_t(i))]/[n1*cos(theta_t(i))+n2*cos(theta_i(i))];
end
r1 = abs(r1);
```

```

t1 = 1 - r1;
R1 = r1.^2;
T1 = 1 - R1;

theta_i = rad2deg(theta_i);

plot(theta_i,R1,'b--',theta_i,T1,'r--')
plot(theta_i(find(R1==min(R1))),min(R1),'mp',theta_i(find(T1==max(T1))),max(T1),'mp')
title('Fresnel coefficients, air to water interface')
legend('R_p_e_r_p','T_p_e_r_p','R_s','T_s')
xlabel('Theta_i')
ylabel('Reflectance & Transmittance')
axis([0 90 -0.01 1.01])

```

### Brewster Angle [1,2,8]

From Figure 5, it is evident that a single angle exists for an incident parallel-polarized EM wave for which the wave is completely transmitted from medium 1 into medium 2. The angle, known as the Brewster angle, is defined as the incidence angle at which the Fresnel reflection coefficient  $r = 0$ . For EM waves of variable polarization incident at this angle, only the parallel-polarized component of the wave is completely transmitted into medium 2.

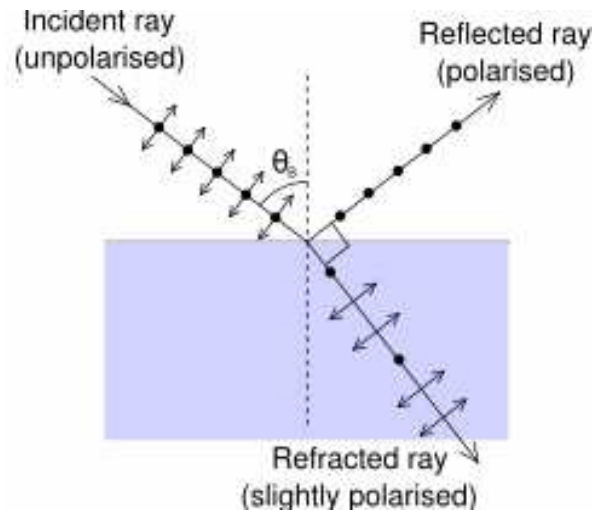


Figure 6: Parallel-polarized light is transmitted at the Brewster angle [8]

The physical mechanism for the Brewster angle can be qualitatively understood from the manner in which electric dipoles in the media respond to p-polarized light. Imagine a scenario in

which light incident on a surface is absorbed, and then reradiated by oscillating electric dipoles at the interface between the two media. The polarization of freely propagating light is always perpendicular to the direction in which the light is traveling. The dipoles that produce the transmitted (refracted) light oscillate in the polarization direction of that light. These same oscillating dipoles also generate the reflected light. However, dipoles do not radiate any energy in the direction along which they oscillate. Consequently, if the direction of the refracted light is perpendicular to the direction in which the light is predicted to be specularly reflected, the dipoles will not create any reflected light. Since, by definition, the s-polarization is parallel to the interface, the corresponding oscillating dipoles will always be able to radiate in the specular-reflection direction. This explains why there is no Brewster angle for s-polarized light.

For perpendicular polarization, the Brewster angle is obtained by setting the numerator of the expression for  $r_{\perp}$  in equation (5) equal to zero, resulting in

$$\sin \theta_{B\perp} = \sqrt{\frac{1 - (\mu_1 \epsilon_2 / \mu_2 \epsilon_1)}{1 - (\mu_1 / \mu_2)^2}} \quad (18)$$

For parallel polarization, the Brewster angle is obtained by setting the numerator of the expression for  $r_{\parallel}$  in equation (9) equal to zero, resulting in

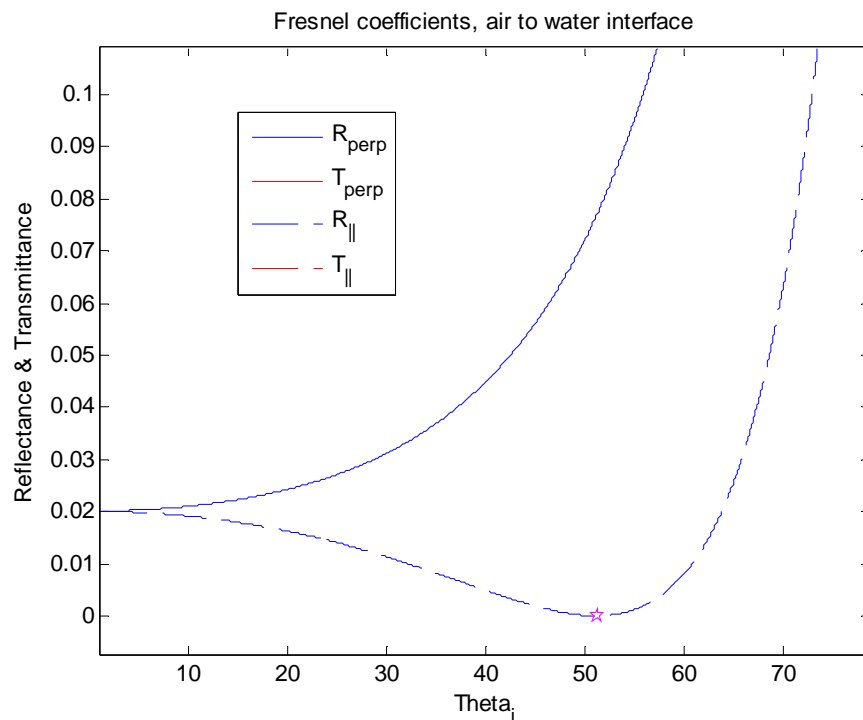
$$\sin \theta_{B\parallel} = \sqrt{\frac{1 - (\epsilon_1 \mu_2 / \epsilon_2 \mu_1)}{1 - (\epsilon_1 / \epsilon_2)^2}} \quad (19)$$

The Brewster angle can also be defined by the indices of refraction of the two media, according to

$$\tan \theta_B = \frac{n_2}{n_1} \quad (20)$$

The Brewster angle is also called the polarizing angle. If an EM wave composed of both perpendicular and parallel polarization components is incident upon a nonmagnetic surface at the Brewster angle  $\theta_B$ , the parallel-polarized component is completely transmitted into the second medium, and the perpendicular-polarized component is reflected by the surface. Sunglass manufacturers have taken advantage of this phenomena in the production of polarizing sunglasses.

For an air/water interface, the Brewster angle is  $52.64^\circ$ . A plot of the Brewster angle in power coefficient form is shown in Figure 7, with the Brewster angle illustrated by a magenta star. The MATLAB code used to determine the Brewster angle is also provided for the reader.



**Figure 7: Depiction of the Brewster angle for an air/water interface**

```
n1 = 1.00; % Air
n2 = 1.31; % Water
```

```
BrewsterAngle = atan2(n2,n1);
```

```
BrewsterAngle = rad2deg(BrewsterAngle)
```

## Imaginary Component of Refractive Index [9,10]

For a comprehensive understanding of EM wave propagation, phenomena associated with absorption and scattering must be considered. To this point, the index of refraction  $n$  has been assumed to consist solely of a real component that controls the effective phase speed of EM waves propagating through a medium. Therefore, the speed of light is reduced when propagating through water ( $n = 1.33$ ) compared to propagation through air ( $n = 1.00$ ). However, the index of refraction is in fact composed of a real and imaginary component,  $n_r$  and  $n_i$ , respectively.

The imaginary component describes the rate of absorption of the EM wave during propagation through the medium. The total index of refraction  $N$  is defined according to

$$N = n_r + n_i i \quad (21)$$

It is important to keep in mind that  $N$  is not constant for a medium, but depends strongly on the EM wavelength and, to a lesser degree, temperature, pressure and other variables. For example, it is the variation of  $n_r$  with wavelength in raindrops that gives rise to rainbows; it is a sharp variation of  $n_i$  with wavelength that makes wine red. Also, note that  $n_r$  and  $n_i$  are not free to vary independently of one another, but are tightly coupled.

In a nonabsorbing medium, the imaginary part of the index of refraction  $n_i = 0$ . However, when  $n_i$  is nonzero, absorption of the EM wave occurs as it propagates through the medium. For example, the  $n_i$  for water and ice is extremely small in the visible band but increases sharply as the EM wavelength moves into the IR band. This explains why Jim Maslanik's IR altimeter is unable to penetrate water to a useful depth for ranging on the ice sheet.

The rate of power attenuation per unit distance is described by the absorption coefficient  $\beta_a$ , which is related to  $n_i$  by

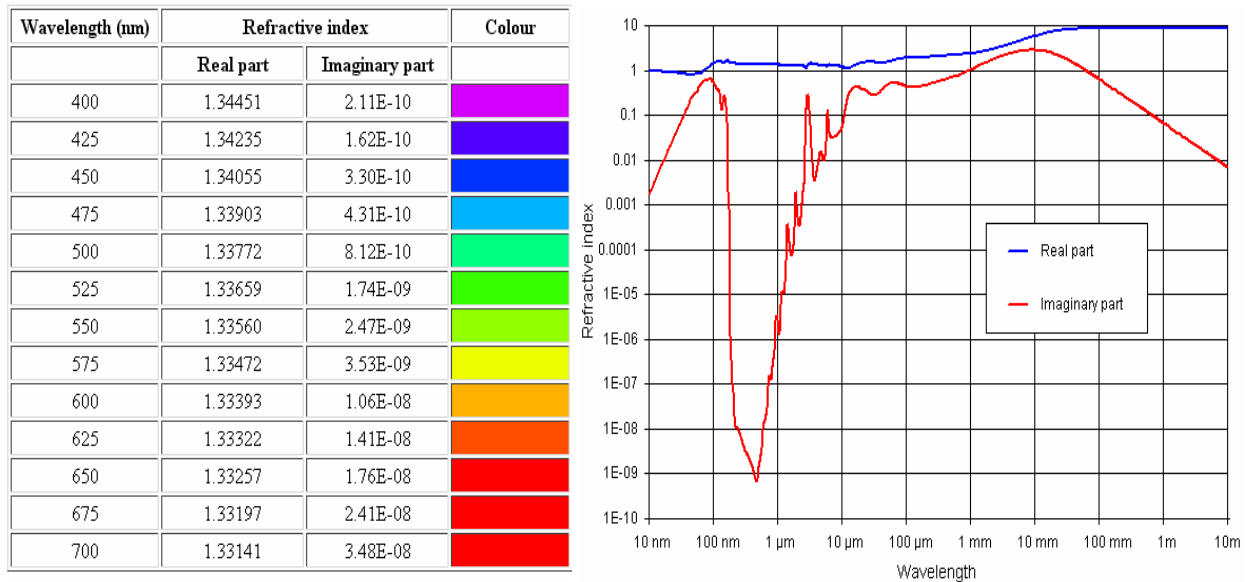
$$\beta_a = \frac{4\pi n_i}{\lambda} \quad (22)$$

and is related to the intensity propagation and overall transmittance  $t(x)$  by Beer's Law,

$$t(x) = \frac{I(x)}{I_0} = e^{-\beta_a x} \quad (23)$$

The transmittance  $t(x)$  is defined as the fraction of radiation that survives propagation over a distance  $x$ . The fraction of the initial radiation not transmitted is either absorbed and converted to another form of energy, such as heat, or scattered by molecules or containments along the propagation path.

It is important to note that even small values of  $n_i$  can result in strong absorption within a medium. For example, if a 1mm thick sheet of material transmits only 1% of radiation having a wavelength of 500 nm, this implies  $n_i = 1.8 \times 10^{-4}$ . Illustrated in Figure 8 is a plot of the complex index of refraction of water for various optical EM wavelengths.



**Figure 8: Index of refraction for optical EM radiation in water [11]**



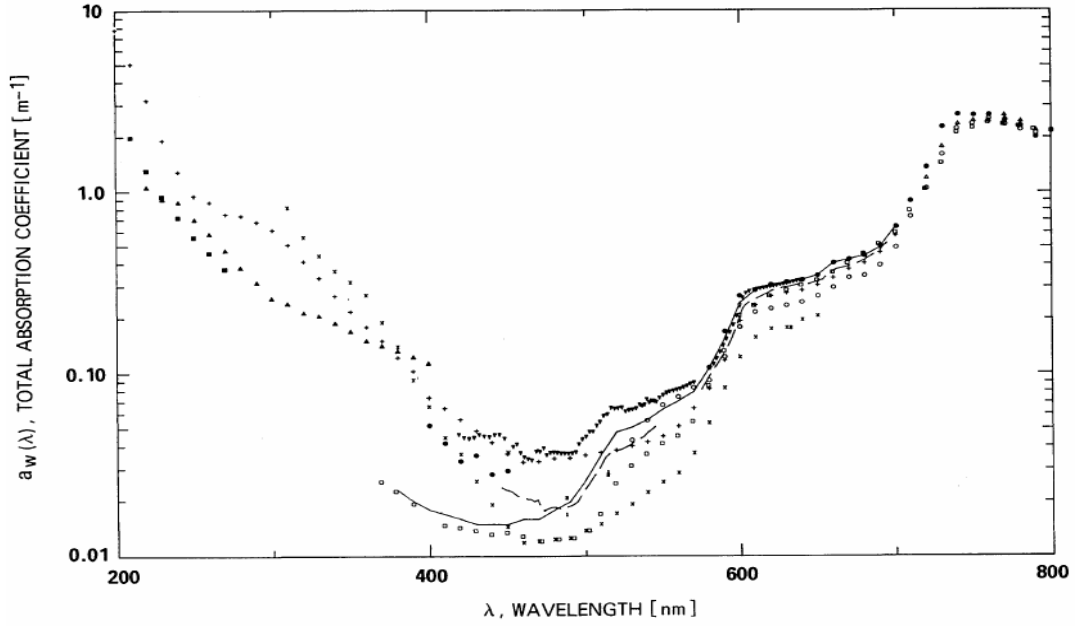


Figure 9: Absorption of EM radiation in water; compare to Figure 8 [12]

As previously suggested, each EM pulse transmitted by a laser altimeter experiences extinction along the transmit and receive paths. The total extinction  $\beta$ , composed of absorption from the imaginary component of the refractive index and scattering, reduces the amount of EM energy along the propagation path, and is defined as

$$\beta = \beta_{\text{absorption}} + \beta_{\text{scattering}} \quad (24)$$

Therefore, it is important to incorporate scattering into the total extinction  $\beta$  when analyzing EM wave losses during propagation, as defined by equation (23).

### Penetration Depth [9]

The propagation of light and losses due to absorption and scattering is defined by the transmittance  $t(x)$  in equation (23), which defines the amount of light transmitted through a distance  $x$  in a given medium. The transmittance of EM waves leads to a definition of the

penetration depth  $D$ , the value of  $x$  for which the transmittance  $t(x) = e^{-1} = 37\%$ . The penetration depth is the reciprocal of the total extinction coefficient in equation (24) and is defined by

$$D = \frac{1}{\beta} \quad (25)$$

The penetration depths of water and ice are given in [9] for optical EM radiation at 532 nm. Based on these values, the extinction coefficient  $\beta$  is evaluated according to

$$D_{\text{water}} = 50m = e^{-\beta(50m)} = 0.37$$

$$\ln[0.37 = e^{-\beta(50m)}]$$

$$-0.9943 = -\beta(50)$$

$$\beta_{\text{water}} = \frac{-0.9943}{-50} = 0.0199$$

for water. In the case of propagation through the atmosphere, the penetration depth is assumed to be 2 times greater than water, as a worst-case scenario for laser altimetry. Similar calculations to those above result in an extinction coefficient for air of  $\beta_{\text{air}} = 0.0099$ .

### Graybody Approximation [9]

As is often the case in laser altimetry, the materials through which EM waves propagate are inhomogeneous. Under such circumstances, the medium is assumed to appear “gray” over a broad range of wavelengths. Using the graybody approximation, a single average absorptivity is taken to be representative of the entire wavelength band. Illustrated in Table 1 are amplitude coefficients for the graybody reflectivity of surfaces commonly found in laser altimetry. As in equations (12-13), the values in Table 1 represent the reflection amplitudes; power coefficients require these values to be squared.

Table 1: Graybody percent reflectivity of common surfaces [9]	
Fresh, dry snow	70-90
Sand, desert	25-40
Dry vegetation	20-30
Grass	15-25
Bare soil	10-25

## Bathymetric Lidar Simulation

Utilizing the theory discussed in the previous sections, knowledge of EM wave propagation can be applied to the design and simulation of a bathymetric laser altimeter. In bathymetry, a pulse of optical EM radiation is transmitted from an instrument towards a target of interest, typically a column of water. The pulse is subjected to extinction from absorption and scattering in the atmosphere and water along the propagation route, as well as Fresnel reflections from the water column's surface and floor.

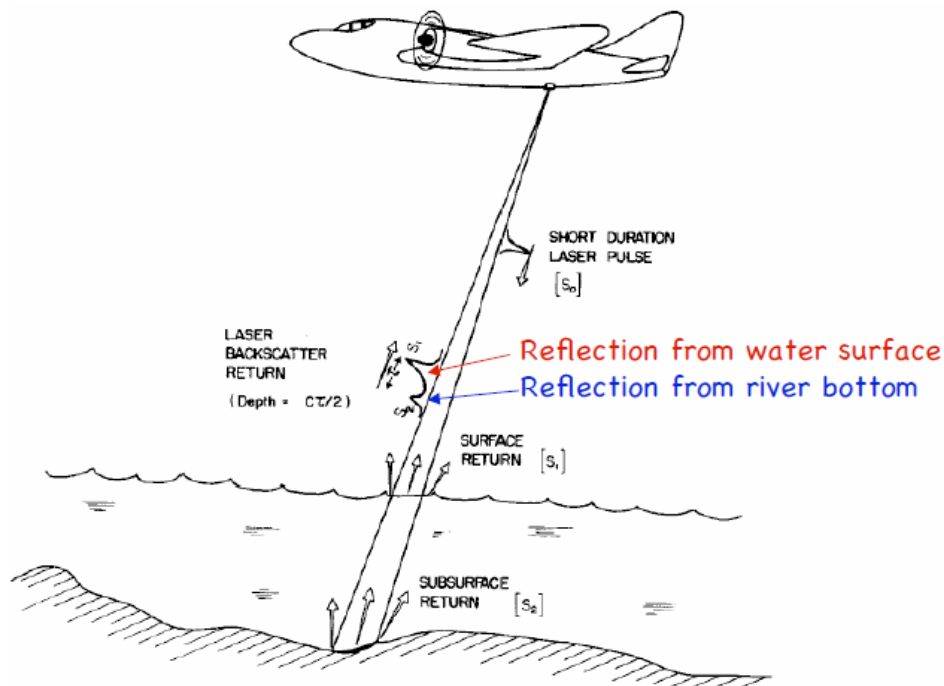


Figure 10: Setup for lidar bathymetry [13]

Design of a bathymeter begins with an evaluation of the fundamental lidar equation, illustrated below.

$$N_S(\lambda, R) = N_L(\lambda_L) \cdot [\beta(\lambda, \lambda_L, \theta, R) \Delta R] \cdot \frac{A}{R^2} \cdot [T(\lambda_L, R) T(\lambda, R)] \cdot [\eta(\lambda, \lambda_L) G(R)] + N_B$$

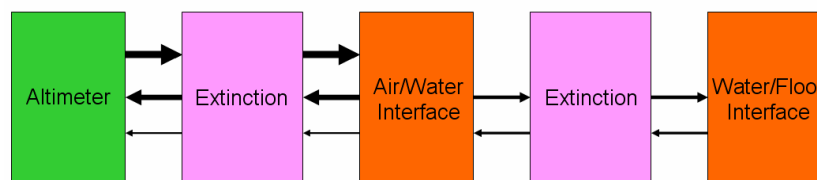
**Figure 11: Fundamental lidar equation [14]**

The key terms of the lidar equation include

- $N_s$  – *the expected photon counts at wavelength  $\lambda$  and range  $R$ .* This term represents the expected counts output from the lidar detector.
- $1^{st}$  term – *the transmitted photon number.* The number of transmitted photons, dependent on the laser pulse energy and wavelength.
- $2^{nd}$  term – *probability of a transmitted photon to be scattered into a unit solid angle.* The scattering processes experienced during laser altimetry are based on the reflection and transmission of photons, as explained by the Fresnel equations (5-17) for target interfaces and equations (21-24) for scattering elsewhere.
- $3^{rd}$  term – *probability of a scattered photon to be collected by the receiving telescope.* Due to the  $R^2$  dependence of signal loss, the probability of collection decreases as range increases. The probability of collecting a scattered photon is increased, however, by increasing the telescope primary diameter.
- $4^{th}$  term – *light transmission through a medium for the transmitted and return signal photons.* The transmitted laser pulse experiences extinction during flight through the atmosphere and water according to equations (21-24).
- $5^{th}$  term – *overall system efficiency.* The efficiency  $\eta$  of the telescope and supporting optics, as well as the quantum efficiency of the detector, and geometric overlap factor  $G$  contribute to the instrument detection efficiency of reflected photons.

- $N_B$  – background and detector noise counts. Detector dark counts and solar radiation decrease a laser altimeter’s signal-to-noise ratio. Noise is increasingly problematic when analyzing floor returns due to their weak signal strength compared to surface returns.

With an understanding of the lidar equation and its interactions between each term, design of a bathymeter continues with a depiction of the physical setting in which the instrument operates. As illustrated in Figure 10, the bathymeter is designed to fly onboard an aircraft, ranging to the water column surface and floor. Therefore, each transmitted EM wave will undergo extinction en route to the water surface. A portion of the EM energy will be reflected from the air/water interface and subjected to extinction during return to the altimeter, while the remainder of the beam will be transmitted into the water.



**Figure 12: Physical interactions of simulated bathymeter signals**

The portion of EM radiation that is transmitted into the water will undergo extinction during transit to the floor. At this interface, a portion of the beam will be reflected back toward the altimeter, while the remainder is transmitted into the floor, likely to be absorbed into the medium. The beam undergoing reflection off the floor interface is subject to extinction in the water and air, as well as a transmission loss at the water/air interface, en route to the altimeter.

While Figure 12 illustrates the physical interactions of the bathymeter signals, Figure 13 ties the governing lidar equation and physical phenomena together with the expected signal returns. An initial pulse of EM radiation is generated and emitted from the laser and is time stamped to begin time-of-flight calculations. The detector begins to count return pulses and

records the time between the initial pulse, the surface return, and the return from the floor. The return pulses are converted into range measurements based on the roundtrip propagation time and the speed of light. The surface and floor returns are of substantially less power than the initial pulse, due to extinction and interface losses.

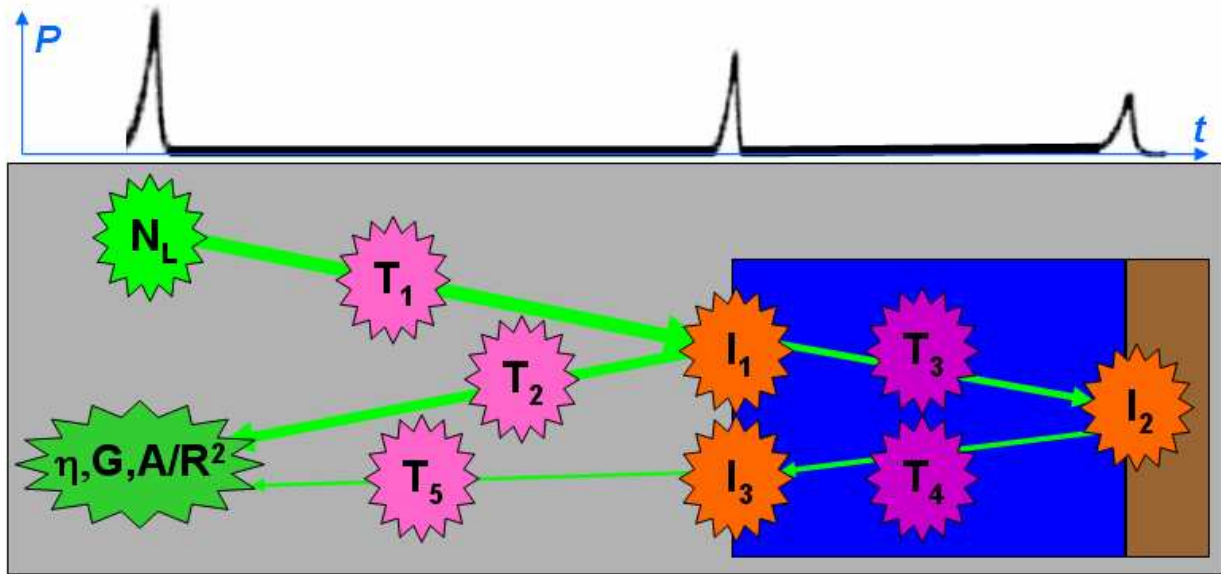
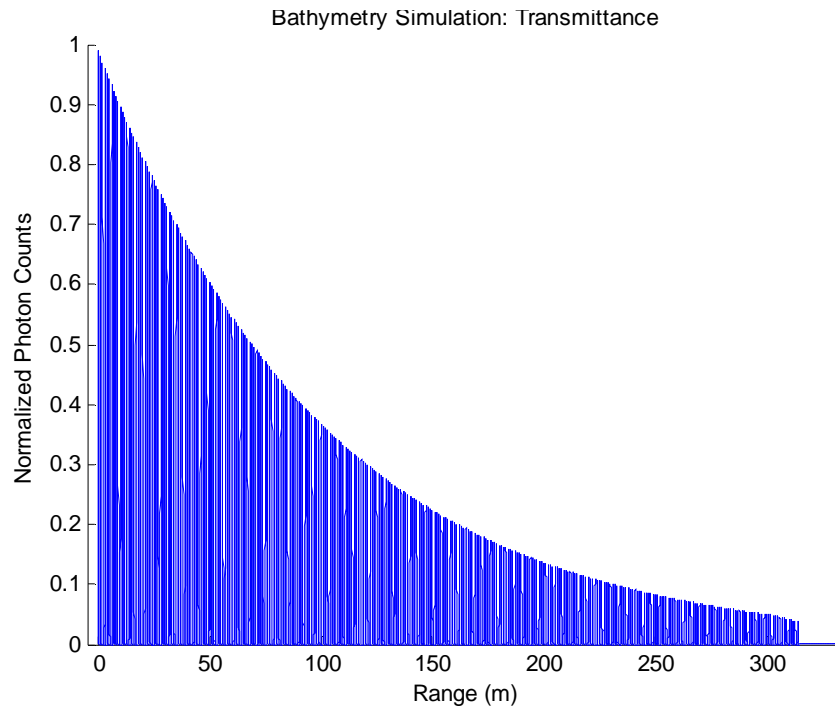


Figure 13: Physical bathymetry setup incorporating lidar equation; T = transmission; I = interface

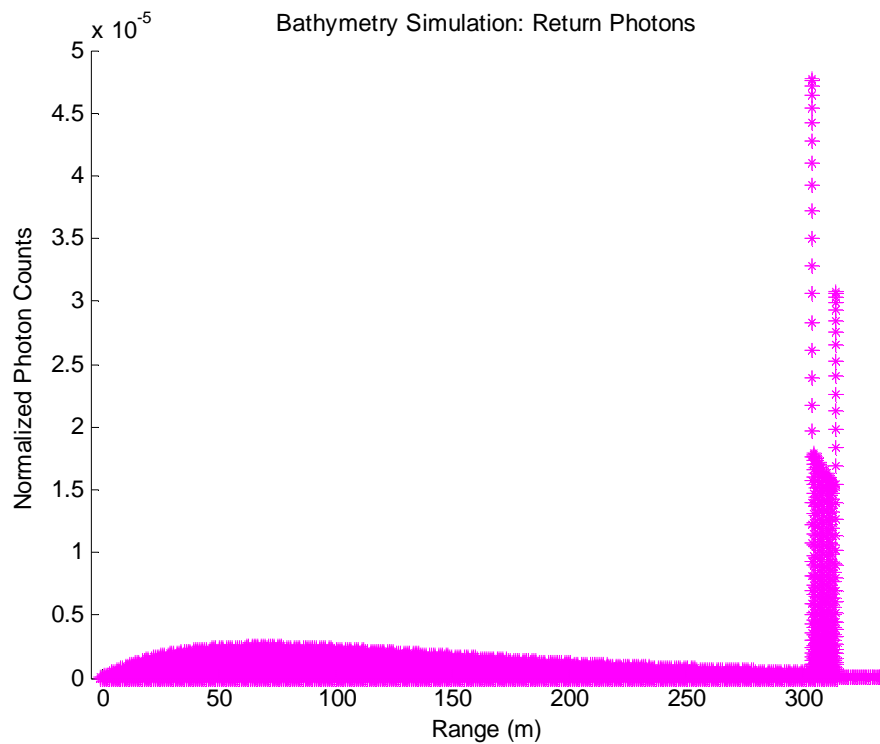
While the reader is directed to the Appendix for a comprehensive list of simulation details, the essential parameters are provided below in Table 2.

Table 2: Bathymeter simulation parameters	
Laser wavelength	532 nm (Nd:YAG laser & Fig 9)
Laser pulse energy	2.5 $\mu\text{J}$
$\eta_{\text{transmitter}}$	96%
$\eta_{\text{receiver}}$ (including detector QE)	10%
Atmospheric extinction	0.0099
Water extinction	0.0199
Refractive index of air	1.00
Refractive index of water	1.33
Angles of incidence on water surface	Normal
Target slope	$0^\circ$

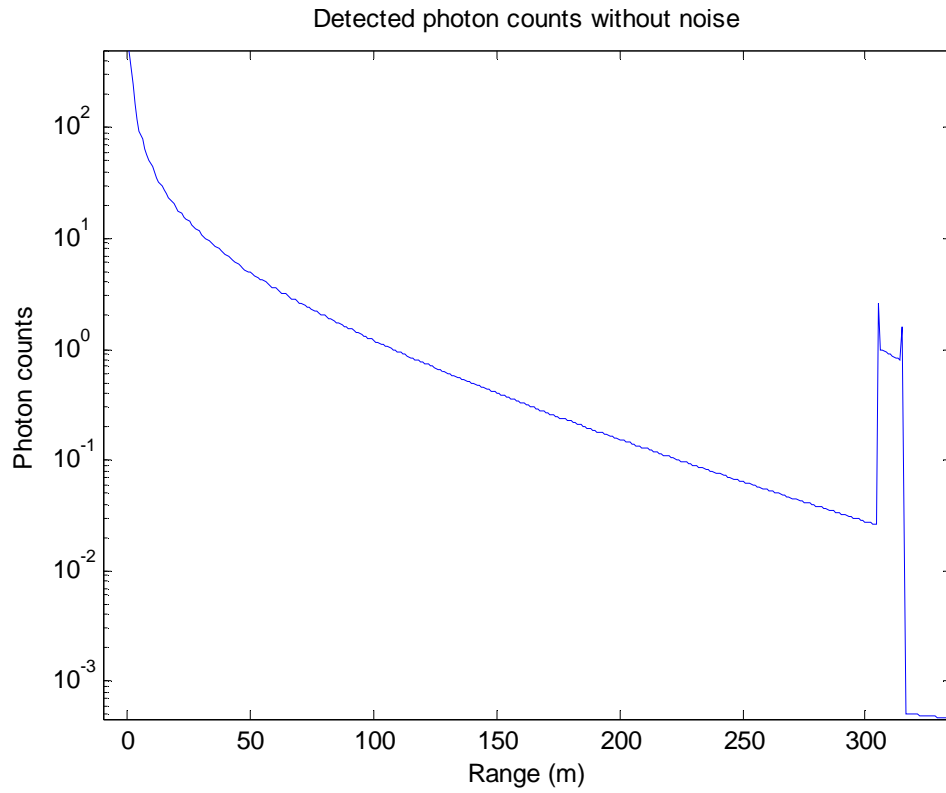
The results of the bathymetric lidar simulation are provided in the following figures.



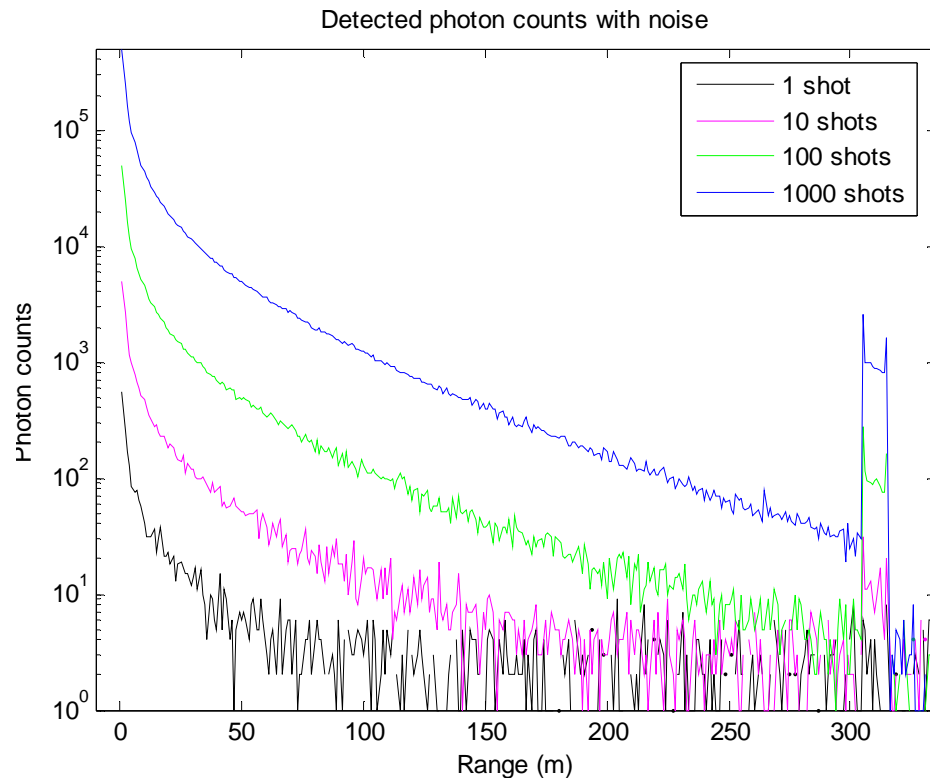
**Figure 14: Transmittance of simulated bathymeter**



**Figure 15: Photons received from scattering and Fresnel reflections at the water surface and floor**



**Figure 16: Ideal measurements from the bathymeter**



**Figure 17: Bathymetric measurements integrating over a range of shot counts**



## **Conclusion**

This document illustrates phenomena associated with the propagation of EM waves through various media. Of primary interest to the author are effects associated with absorption, scattering, and Fresnel reflections of EM waves during propagation. In an effort to illustrate these effects, the application of bathymetric lidar simulation has been provided for the reader. The use of laser ranging for distance measurements from an airborne platform to a water column's surface and floor requires an understanding of EM wave propagation. The simulation, code for which is provided for the reader in the Appendix, attempts to apply such an understanding to increase the precision of bathymetric lidar measurements.

## References

- [1] Ulaby, F. *Fundamentals of Applied Electromagnetics*, 5<sup>th</sup> ed. Pearson Prentice Hall, 2007.
- [2] Hecht, E. *Optics*, 4<sup>th</sup> ed. Addison Wesley Publishing, 2002.
- [3] “Maxwell’s Equations.” [http://en.wikipedia.org/wiki/Maxwell's\\_equations](http://en.wikipedia.org/wiki/Maxwell's_equations).
- [4] “Maxwell’s Equations and Electromagnetic Waves.”  
[http://galileo.phys.virginia.edu/classes/109N/more\\_stuff/Maxwell\\_Eq.html](http://galileo.phys.virginia.edu/classes/109N/more_stuff/Maxwell_Eq.html).
- [5] “Classification of Polarization.”  
<http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polclas.html#c4> .
- [6] “Polarization.” <http://en.wikipedia.org/wiki/Polarisation>
- [7] “Fresnel Equations.” [http://en.wikipedia.org/wiki/Fresnel\\_equations](http://en.wikipedia.org/wiki/Fresnel_equations).
- [8] “Brewster’s Angle.” [http://en.wikipedia.org/wiki/Brewster's\\_angle](http://en.wikipedia.org/wiki/Brewster's_angle).
- [9] Petty, G. *A First Course in Atmospheric Radiation*. 2<sup>nd</sup> ed. Sundog Publishing, 2006.
- [10] “Beer-Lambert Law.” [http://en.wikipedia.org/wiki/Beer-Lambert\\_law](http://en.wikipedia.org/wiki/Beer-Lambert_law).
- [11] “Refractive Index.” <http://www.philiplaven.com/p20.html>.
- [12] Smith, R., and K. Baker. “Optical properties of the clearest natural waters (200-800 nm).”  
*Applied Optics*, vol 20(2), 15 Jan 1981.
- [13] Chu, X. “Lecture 35.” *ASEN 6519: Lidar Remote Sensing*. University of Colorado at Boulder, Fall 2008. <http://cires.colorado.edu/science/groups/chu/classes/lidar2008/>.
- [14] Chu, X. “Lecture 4.” *ASEN 6519: Lidar Remote Sensing*. University of Colorado at Boulder, Fall 2008. <http://cires.colorado.edu/science/groups/chu/classes/lidar2008/>.

## **Appendix**

The following code calculates and plots the Fresnel coefficients in both amplitude and power for an air/water interface.

Please contact author for code – [Steven.E.Mitchell@gmail.com](mailto:Steven.E.Mitchell@gmail.com)